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ARTICLE

Epistemology in Practice: Ernst Mach's Experiments on Shock Waves and The Place of Philosophy

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Abstract
The present paper studies Ernst Mach's experimental work with "spark waves" and other types of shock waves, which brought him to the 1887–88 famous schlieren photographs of supersonic phenomena triggered by bullets shot at high speed. Against what it is traditionally argued, I show (1) that Ernst Mach's visualization attempts do not depend on his commitment to any particular philosophical doctrine about the role of sensations as the foundation of empirical science, and (2) that his inclination toward experimental research may have crucially contributed to the development of his epistemological and ontological views.

Keywords Ernst Mach · Shock waves · Schlieren · Phenomenalism · Neutral monism

1 Introduction

There is a striking paradox surrounding Ernst Mach. As a matter of fact, he developed very early an interest for psychophysics and psychology as well as for the history of science. In addition, in 1895 he was appointed as *Ordinarius* of philosophy, "especially for the history and theory of inductive sciences" (Blackmore 1972, 145–154). Undoubtedly, these strands were crucial to the development of his philosophical views. On the other hand, if one looks at his professional career, for most of his academic life Mach was an experimental physicist. Then, one may well expect that his activity in such context also influenced his epistemology to a considerable extent. However —and here is the paradox—, virtually no attention has been payed to this issue, whereas the connections between Mach's theory of knowledge and his interest for psychophysics and psychology on the one hand and the history of science on the other hand have been studied in detail.[1] Contrarily, scholarship

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[1] The literature is huge, traces back to the early twentieth century and includes noticeable contributions in German, English, French, and Italian at least. Here I just provide a selection, from the English-speaking area, of significant papers and books in the last decades. On the relationship between psychophysics, psychology and philosophy in Mach's thought see, in particular, Cohen (1968); Blackmore (1972); Banks (2003); Heidelberger (2010); Banks (2014); Preston (2021). On Mach about history and philosophy of science see Giere (1973); Hiebert (1970); Feyerabend (1984); Blackmore (1992); Guzzardi (2021).

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has often considered Mach's experimental activity in the context, or even as an effect, of his phenomenalist philosophy,[2] i.e. the view according to which our statements about the objects are statements about our perceptions of the objects (I will come back to the kind of phenomenalism that is frequently, but not always, ascribed to Mach in Sect. 2).

In the present paper I take the opposite approach. I do not take for granted that Mach should always have possessed philosophical views that must have shaped his science but probe the possibility that, particularly at the beginning of his career, his experimental work was largely independent from any kind of ontological commitment he could have retained at that time. Conversely, I will suggest that this reveals an approach which might have shaped both his epistemological and ontological views, particularly the "theory of the elements" as it is exposed in *The Analysis of Sensations*.

2 Methodology vs Metaphysics

Traditional interpretations of Mach's epistemology have often emphasized his insistence on sensations as the ultimate source of knowledge. Noticeable critics such as Max Planck, Albert Einstein, and Karl Popper treated Mach as a sort of Berkeleyan anti-realist, who maintained that sensations are "the actual and sole elements of the world" (Planck 1909, 25), or that they are "the building blocks of the real world" (Einstein in a letter to Michele Besso, 6 January 1948: Einstein and Besso 1979, Letter 153), or that "there is no physical world ... behind the world of physical appearances" (Popper 1953, 173). Popper was one of the firsts to stick on Mach the label of phenomenalist that remained since then attached to his name —namely, a supporter of "the view that physical things are bundles, or complexes, or constructs of phenomenal *qualities*, of particular experienced colours, noises, etc." (1953, 173, *emphasis in original*)

Apparently, the above-quoted claims blame Mach for an authentically ontological version of phenomenalism: sensations not only are the vehicles of our knowledge of the world; they are, indeed, all the world we have, and we cannot have another world than our sensations. As strong as this version might be, it appears to agree, at first, with Mach's rejection of the "antithesis between appearance and reality", which includes, e.g., an argument as reasonable as that of the pencil dipped in water. According to Mach, this *is*, and not only *appears*, crooked: more precisely, the pencil "is optically crooked but ... tactually and metrically straight" (Mach 1886/1959, 8/10).[3] But Mach's ontic phenomenalism sometimes shows an uglier, iconoclastic face:

> We see an object having a point S. If we touch S, that is, bring it into connection with our body, we receive a prick. We can see S, without feeling the prick. But as soon as we feel the prick we find S on the skin. The visible point, therefore, is a *permanent nucleus*, to which the prick is annexed, according to circumstances, as something accidental. From the frequency of analogous occurrences we ultimately accustom ourselves to regard *all* properties of bodies as "effects" proceeding from permanent

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[2] To limit ourselves to studies involving the experiments on shock waves, see, for example, Merzkirch (1970); Seeger (1970); Blackmore (1972); Cekic (1992); Hentschel (2014); de Waal & ten Hagen (2020).

[3] When possible, quotes from Mach's works are taken from the "Ernst Mach Studienausgabe" (Xenomoi: Berlin, 2008 ff.), which also includes reference to the original pagination. Translations usually follow the English edition in use. Where this is not completely satisfying in light of recent research, I allow me to introduce some changes. Emphasized words and passages follow the original German texts, since they are the same as in the English translations.

> nuclei and conveyed to the ego through the medium of the body; which effects we call *sensations*. By this operation, however, these nuclei are deprived of their entire sensory content, and converted into mere mental symbols. The assertion, then, is correct that the world consists *only* of our sensations. And thus, we do have knowledge only of sensations, and the assumption of the nuclei referred to, or of a reciprocal action between them, from which sensations proceed, turns out to be quite idle and superfluous. (1886/1959, 9-10/12, emphasis in original).

The phenomenalist readings of Mach can be criticized in many ways—and the label itself is an ambiguous one because it was used in Mach's times with a different meaning, as instantiated by Kleinpeter (1913). The most important alternative to phenomenalism is another tradition that has always been related to Mach, namely neutral monism —a view that associates him with philosophers like William James and Bertrand Russell or, to come to more recent philosophers, Herbert Feigl and Wilfrid Sellars. It holds, in first approximation, that "objects and the human mind are both concatenations of neutral elements ... neither exclusively mental nor physical in nature" (Banks 2018, 78). If the elements are considered in their own mutual connection, without reference to the mind, then Mach would say that those connections are physical; if they appear in connection with the mind, they are said to be psychical. A striking example of what the neutral monist would consider the focal point of Mach's conception is to be found in the final retrospective of his own theory of knowledge, which he wrote in 1910 as an answer to Planck's assault:

> Let us call *ABCDE*... the sensible elementary components of the environment, *U* the boundary of our body with respect to the environment, and *KLMN*... the sensible elementary components we find within the closed surface *U*. Thus, each element of the former group, e.g. *A* (the green of a leave) *depends* on other elements of the same group, e.g. *B* (the sunlight containing green light). But it also depends on elements of the latter group, such as *K* (the eyes being open) or *N* (the sensibility of the retina) ... Everyone will recognize the mutual dependence amongst the elements of the former group as a *physical* one, that amongst the elements of the latter group, which is completely different and transcends the boundary *U*, as a *physiological*. (Mach 1910, 236.)

Notwithstanding their differences, phenomenalist and neutral monist readers of Mach share an important point. Both suggest that his alleged ontology, namely the theory of the elements, should be viewed as the foundation of his epistemology—they disagree on the meaning of the theory but agree on its function.

Phenomenalist readers tend to consider Mach's experimental methods a more or less direct outcome of a sensualist philosophy. An often-quoted example in this connection is the series of experiments with supersonic bullets through which Mach and collaborators, in the 1880s, *made visible* shock waves in fluids (particularly in air). According to Hentschel (2014, 384–385), "as a rigorous phenomenalist, he restricted his ontology to directly observable things or processes... Everything else was 'metaphysical' to him —and to him that meant that it had to be ignored. The ability to photograph the V-shaped cones of compressed air in front of a projectile as well as the turbulence behind it had special significance. Both of these features ... finally became 'real' processes of the world to him." For Hentschel and many others,[4] it was Mach's ontology (something similar to that of "Bishop Berkeley before him", Hentschel adds, 2014, 384–385, clearly reminiscent of Popper 1953) that guided him in these experiments. On the other hand, neutral monists conceive the theory of the elements as a "good metaphysics", or a "scientific metaphysics, or natural philosophy". More precisely, it is a metaphysics "in its traditional Aristotelian sense", that Mach failed to recognize because he was not trained in philosophy, but through which, as per a meta-scientific theory, he intended to unify different sciences, most of all physics, sense physiology, and psychology (Banks 2013, 58, 63; but also see Banks 2004; 2014; 2018).

Preston (2021) has proposed that the phenomenalist as well as the neutral monist approach may reveal some aspect of Mach's epistemology, but Mach himself was neither a phenomenalist nor a neutral monist, because he did not endorse *any* metaphysics at all. In the end, the problem with both interpretations is that they put the cart before the horse. They tend to make of him a philosopher in the first place, who —in the second place— extrapolates his methods for testing theories from a philosophy that has its roots in itself. In doing this, phenomenalists and neutral monists also make an implicit historiographic assumption—they assume that Mach's philosophy (no matter what it is) is an immutable, hyperuranical substance, something that has not historically developed but always existed (or, at least, that was created consubstantially with Mach himself). So, Mach was a phenomenalist (or a neutral monist) even before expressing phenomenalism (or neutral monism) in his epistemological works.

The landscape that one senses when reading Mach is clearly more colorful and nuanced than this philosophical dichotomy allows to see. To mention few examples, Eric Banks (2003) himself has convincingly shown to what extent Mach's ontology of the elements is deep-rooted in his acquaintance with psychophysics in the first place. More recent scholarship has highlighted the significance of Mach's studies in that domain for the development of his meta-scientific conceptions as well as for shaping his understanding of specific issues in other scientific fields, beginning with mechanics (Staley 2018/2021).[5] Alexandra Hui (2013, 89–121; 2021, esp. 15–17) has argued that Mach's monism as asserted in the *Analysis of Sensations* was strongly connected with his interest for the theory of hearing and the experimental studies on accommodation in hearing during the 1860s and early 1870s.[6] It is also worthwhile noticing that Mach's occupation with psychophysics since

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[4] For an overview and a list of significant phenomenalist readers of Mach, both the sympathetic and the critics, see Preston (2021).

[5] In a certain sense, this has always been highly plausible and, in a certain sense, obvious but remained strangely overlooked or under-investigated for much time. After all, Chapter V of Mach's *Science of Mechanics*, first published in 1883, is entitled to "the relation of mechanics to the other department of knowledge" (meaning physics and physiology, but also including psychology and psychophysics), and the Endnotes to Mach (1872/1911, 91–92) close with a "General remark" on the connection between psychology, physics, and psychophysics.

[6] Footing on Mach's historicist conception of sound sensations, as it emerges both from his works and his correspondence with the writer and musical critic Eduard Kulke, Hui also suggests that "Mach's early studies of accommodation in hearing were foundational for his later historical epistemology" (Hui 2021, 25; on this, see also Hui 2013, 120–121). As reported by Hui (2021, 21), in 1872, while experimenting on accommodation in hearing, "Mach asked Kulke if he believed it possible for listeners in the present to hear what the ancient Greeks had heard." That same year, Mach articulated his scientific historicism in *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit*. I find this hypothesis very evocative and appealing, but not entirely convincing. I do think that there is a connection between historicizing scientific ideas and practices and the view that hearing is historical, but it seems to me that chronology does not suffice to prove that Mach modelled or even just envisaged his historicism on science because of his historicism on hearing. (Also, note that *Die Geschichte und die Wurzel…* is the published version of a public lecture Mach held on 15. November 1871, see Mach 1872/1911).

the early 1860s, while providing a basis from which he developed his ontology of the elements in the medium or long run, might have played a pivotal role in shaping his epistemology as one in which experiments are primarily aimed to provide precise measurements of the phenomena involved (I will come back to this issue in the last section).

In what follows I shall avoid making assumptions on what kind of philosophical or meta-scientific theories would be implied in Mach's discourse in view of conceptions that he only expressed later. I would suggest that he could arguably have developed these in strong connection with the practices in which he was already immersed.

In particular, I will take into consideration the experimental techniques he used during the 1870s and until mid-1880s for testing hypotheses on phenomena associated with what we now call shock waves. The approach Mach ultimately employed, which involved visualization and snapshot photography of high-speed phenomena, is often seen as derived from his belief in sensations as a standard of genuine, anti-metaphysical knowledge. I will try to revert this view upside down. In doing this, I will not argue that while experimenting on shock waves he did not actually *have* any ontological or meta-scientific commitment; as the above-quoted literature convincingly shows, by that time his ideas both on sensations and on the role and functioning of scientific knowledge had already taken a somewhat definite direction. I will argue, however, that such commitments did not play a relevant role in Mach's choice for the experimental method, which is guided by other factors. In turn, as I will suggest in the very last paragraphs of this paper, some epistemological tools he developed or used in his experimentation may have indirectly contributed to shape his anti-metaphysical appeal to sensations, which was so crucial for the "theory of elements" he developed later on.

3 Visualization Techniques in Mach's Experimental Practice

There is general consensus (e.g. Merzkirch 1970; Blackmore 1972; Krehl 2009) that Mach played a remarkable role in posing the foundations of gas dynamics, particularly supersonic aerodynamics. The culmination point of his investigations was reached in 1887, when he and Peter Salcher succeeded in catching on a photographic plate a bullet at supersonic speed, together with a thin layer of compressed air in front of it and other effects associated with the supersonic regime (Mach and Salcher 1887).[7] Indeed, the story traces back to Mach's scientific activity in experimental physics at the university of Prague during the 1870s. One of his chief interests since the beginnings of the decade was the study of sound, both in physiological and in physical terms. At a certain point, his curiosity was aroused by the blast caused by electric sparks (a so called "*Funkenwelle*", or "spark wave"), which he quite naturally interpreted at first as an acoustic soundwave that exerted its mechanic effects on the surrounding air (Merzkirch 1970, 43–44).

Holding the chair of experimental physics, Mach's duties included the direction of the physical laboratory. Experiments were usually conducted with the help of a laboratory technician and instrument maker (a so-called *Mechaniker*) and in collaboration with his

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[7] This episode has been recounted many times, beginning with Merzkirch (1970) and Blackmore (1972). The most complete reconstruction is probably Hoffmann and Berz (2001). The experiments were then repeated in the following years with varied setups. For technical details and the physical interpretation of the experiments, see Krehl (2009, esp. 918–921) and Settles (2001, 11–13).

students. They mostly resulted in papers that Mach co-authored (not an obvious practice at the time) with several different young scholars (see, on this, Blackmore 1972, 41–46).

For studying spark waves and similar phenomena, Mach and collaborators essentially employed three methods, that could occasionally be combined. The first one consisted in visualizing the wave effects on air through layers of dust or soot. This was, at that time, an established technique. Since years, experiments for studying the effects of soundwaves in air had mostly consisted in visualization techniques that made use of fine dust in a variety of experimental arrangements. In 1866 August Kundt advanced a method consisting in filling a transparent tube with lycopodium powder (obtained from the spores of lycopodium, a kind of ground-pine) and studying the figures produced by sounds entering the tube. Other methods employed layers of powder on flat surfaces, and these had been used long before in order to generate figures through electric discharges. In 1870, a series of experiments that took place at the Prague institute of experimental physics directed by Mach, showed that the figures produced through electric sparks and those produced through sound were, in fact, very similar and probably due to the same phenomenon.[8]

In 1873 the Hungarian schoolteacher Károly Antolik used thin layers of soot on a glass plate in order to visualize the waves generated by electric discharges. But visualization was only one aspect. In the hands of Mach and his collaborators, improved versions of Antolik's method were utilized to provide accurate descriptions of important effects (Krehl 2009, 130; Krehl and Geest 2019, 447–448). The most remarkable experimental result was probably the quantitative account of the irregular reflection of shock waves (Merzkirch 1970, 52). Moreover, in a paper resulting from the soot-experiments series, Mach and Wosyka (1875) made clear that the mechanical effects of electric sparks obtained through Antolik's technique, i.e. the soot figures, are not characteristic of electric discharges—in fact, "in no way they are of electric nature, but can be produced through any kind of explosion causing the corresponding air motion" (Mach and Wosyka 1875, 412). In one experiment of the series, they measured the velocity at which the figures were formed—an indirect way to determine the velocity of propagation of the waves. These turned out to be "at least of the order of the velocity of sound" (1875, 415). Later on, Mach et al. (1878) obtained precise velocity diagrams for the propagation of shock waves from spark discharges (Merzkirch 1970, 48). Noticeably, the measurements of supersonic velocities for wave propagation contributed to let Mach soon change his mind about the nature of the mechanical waves they were observing. He and his collaborators initially identified them as acoustic waves, to the extent that, after repeating Antolik's experiments, they became convinced to "have shown that, in all probability, the soot figures can be explained through air motions, in particular through sound motions" (Mach and Wosyka 1875, 416). However, the accurate velocity measurements that took place beginning with 1876 (they were first reported in two short notes published in 1876 by the Imperial Academy of Sciences in Wien) let them conclude that "all blast waves from explosions, and not only those generated by electric discharges, display peculiarities in their mode of action [*Wirkung*] that let infer deviations from the known properties of the usual soundwaves" (Mach and Sommer 1877, 102).

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[8] In particular, the experiments showed that "the figures generated by Kundt through sound are strongly akin to, if not identical with, those produced long before by Abria through electric discharges" (Neumann 1870, 221).The paper of the Bordeaux professor of physics Jean-Joseph-Benoît Abria was originally published with the title "Mémoire sur quelques phénomènes mécaniques qui accompagnent les décharges électriques" in the *Annales de chimie et de physique*, 74, 1840, then reprised in the *Annalen der Physik und der Chemie*, 53, 1841, pp. 589–602. Electric dust figures, however, were already known at least since Lichtenberg's times.

The second method Mach and collaborators employed, was based on a modified Jamin interferometer. The original design essentially consists of two thick mirrors parallel to one another. Let us imagine concentrating a beam of light and to throw it onto the first mirror at a certain angle (say, 45°). This will split into two rays, parallel to each other and displaced by a certain amount depending on the thickness of the mirror. Being reflected, the two rays will continue their path toward the second mirror, where they will be recombined to be finally imaged onto a screen. Here they display an interference pattern. Occasionally, a vessel filled, e.g., with gas may be placed between the mirrors, and its refraction index may be determined through the displacement of the final fringes. Analogously, a prism may also be added after the second mirror, so that spectral interference fringes appear on the screen. Working on this basic setup, Mach and von Weltrubský (1878) introduced, between the two mirrors, a vessel closed by thick optical flats and divided into two parts by means of a vertical wall, so that each ray passes through one part only. They put electrodes on the ends of the vessel in order that two spark waves can be transmitted. Since spark waves cause periodic condensation and rarefaction of air particles, it turns out that the refraction index of air is altered accordingly, and the changes are made visible in the final (spectral) interference pattern.

Notwithstanding many technical difficulties, Mach and von Weltrubský (1878) were able to attain at least one quantitative result by means of the refraction-interferometric method. They observed the "condensation curve" that the spark wave caused in the air particles contained in the vessel and assumed it would follow, although not precisely, Riemann's equations for the propagation of plane (non-linear) air waves of finite amplitude. Finally, they determined the maximum condensation of air when a certain spark wave passes through and noted "how tremendously the condensation of spark waves exceeds that of the usual soundwaves" (Mach and von Weltrubský, 1878, 559).

A third approach was occasionally used by Mach and collaborators, particularly for repeating the soot experiments and checking the results obtained (see, e.g., Mach and Wosyka 1875, 415; Mach and Sommer 1877, 125–127). This was the so-called "*Schlieren*" (i.e., "striation") method, developed by the German physicist August Toepler between 1859 and 1864, whose earliest versions, however, trace back to the late 17th century (Krehl and Engemann 2001; Settles 2001, 1–15). As described by Hoffmann (2009, 18), Toepler's apparatus "consisted of a lens system and a small moveable opaque screen (as an optical diaphragm), which together made the density differences in a gas-like medium visible." A strong light source, called the "*Illuminator*", beamed across the system of lenses (the "*Schlierenkopf*"), causing the density gradients of air (or any other gas) to appear as striations. These were then examined by the experimenter through a telescope (the "*Analysator*"). Since mechanical waves produce inhomogeneities in the medium they pass through, Toepler's apparatus allowed to study waves generated in air by electric sparks (in fact, this was one of its earliest applications: see Toepler 1867), but could easily be applied to every kind of blast waves as well.

Mach and collaborators tended to consider the schlieren-technique as an ancillary method that could be used for confirming results. They also were not sure, but considered as highly probable, that the interference pattern they observed on the soot-covered plates and the schlieren-figures in their Toepler's apparatus were due to the same phenomenon (Mach 1876, 194; Mach and Sommer 1877, 127). Most of all, they doubted that the schlieren-technique, although it was the method by which "Toepler first observed *optically* the spark waves", could really allow reliable measurements; on the contrary, they complained that Toepler's apparatus "can provide, by its very nature, none but incomplete information about quantitative relations" (Mach and von Weltrubský 1878, 551).

In fact, there is a key difference in the information provided by the techniques employed in Mach's Prague laboratory for the study of shock waves. The soot figures are not designed to capture the actual process of wave propagation, but they are susceptible in an obvious way to quantification. As described by Krehl (2009, 130), when using soot or dust figures for studying waves, one deals with time-integrating recording methods, meaning that the effects of the waves appear as "summed up" in the final configurations the dust or the soot layers take after an electric discharge or an explosion has happened. The soot *records* the figure that can be studied with a certain precision, bearing to quantitative relations whose accuracy depends on the experimental setup, which in turn can be improved. Similar conditions apply to the refraction-interferometric method as well; quantification in this case was based on the interference pattern that was finally imaged on a screen. Conversely, the schlieren-technique was primarily conceived by Mach and collaborators as a mere visualization method. It made perceptible how waves propagated and interacted, but quantitative aspects, albeit certainly present (see, e.g., Mach and Sommer 1877, 125–126), were poor if compared to their other techniques. This can be puzzling in light of the developments of schlieren photography throughout the twentieth century. However, Toepler —and Mach, initially— used the schlieren-technique as a "live performance": in studying high-speed phenomena as sound or spark waves, a skilled experimenter should observe a phenomenon whose duration is in the range of few milliseconds and try, after the experiment took place, to fix in drawings the momentary impressions on her or his retina. This as well as the complicated fine-tuning of the instrument contributed to make difficult the task of extrapolating exact measurements from processes observed with Toepler's apparatus (Hoffmann 2009, 18–19.)

All in all, Mach's "three methods" were different, gave different information and could be designed for different purposes. To the aims that he and collaborators set during the late 1870s, which strongly involved quantification, the soot-technique and the refraction-interferometric method reasonably seemed more appropriate; the schlieren-technique, instead, appeared to have a more limited application. But, in the end, which method is employed for a certain goal largely depends on the goal itself. If this is changed, one should expect that the method chosen would also be modified accordingly.

4 Catching the Bullet—or, Why and How Mach Made Things Visible

According to a widespread narrative, Mach more or less deliberately chose the schlieren method in order to make visible the effects of a high-speed projectile on the surrounding air. In this manner, in 1887 he succeeded to visualize many characteristic phenomena of the supersonic regimes, which are now frequently associated with his name in their modern labels —e.g., "Mach cone", "Mach angle", "Mach wave", etc.

Up to a certain extent, this narrative finds support in a later account given by Mach himself in a popular lecture of 1897:

> Modern science strives to construct its picture of the world not from speculations but so far as possible from observed facts. It re-examines its constructs by recourse to observation. Every newly observed fact completes this world-picture, and every divergence of a construct from observation points to some imperfection, to some lacuna in it. What is seen is examined and completed through what is thought, which again is only the outcome of things previously seen. It is always peculiarly fascinating, therefore, to make directly accessible to the examination through observation,

> i.e. perceptible, something which was merely theoretically deduced or theoretically conjectured. (Mach 1923/1898, 356-357/310, with some changes in the translation).[9]

However, this narrative is unsatisfying in more than one respect. Of course, it is true that Mach, together with Peter Salcher —a former pupil of Toepler, then professor of physics at the Naval Academy of Fiume (now Rijeka in Croatia)— succeeded in visualizing shock waves from projectiles.[10] And of course, the narrative is based on Mach's own memories. However, when he held his lecture in 1897, ten years had passed from his experiments with bullets and more than fifteen years from the events that drew his attention towards the phenomena in air associated with high-speed bullets.

I am neither claiming that his memories are necessarily spurious nor that he and Salcher somehow failed in the correct understanding of the processes. However, it is plausible that what Mach recounts is not the whole story. In that lapse of time, he gave his most important contributions to ballistics (Kutterer 1966). With all his and Salcher's results in the hands, many difficulties of the experiments faded out; the same applies to the hesitations possibly connected with the interpretation of the observed phenomena, that might have been susceptible, at that time, to a plurality of interpretations.

Reading Mach's 1897 account, one has the impression that its most remarkable concepts simply and univocally followed from the photographs. Here I try to give a more accurate reconstruction of why and how Mach became involved in experiments with bullets, realizing that he could investigate their behavior and the effects on the surrounding air with a Toepler-apparatus.

4.1 An Elusive Entity

The story began back in 1879 in a somewhat serendipitous manner, when Mach was elected rector of the University of Prague, an office that for some time must have brought him far from research and experiments. However, his recognition within the community of the "electricians", together with the circumstance of his rectorship, gained him an invitation to the first International Electrical Exhibition in Paris (scheduled from August to November 1881) as one of the few members of the Austrian delegation. During his stay, he and other foreign scientists were requested to present their current research during a special session of the French Society of Physics (Mach's resulting paper was then published as "Sur les ondes produites par les étincelles électriques", see Mach, 1881). By this

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[9] I have modified very much the translation of this passage, since I do not find entirely convincing McCormack's English version in Mach's *Popular Scientific Lectures* (1898), which —after more than a century— is well in use in the one-sided academic English-speaking (and English-quoting) world. In particular, I find unsatisfying his rendering of the German terms *prüfen*, *Prüfung*, with *verify*, *verification*, which are loaded with assumptions indebted to early logical positivism. Here is the original text: "*Die heutige Naturwissenschaft ist bestrebt, ihr Weltbild nicht auf Spekulationen, sondern nach Möglichkeit auf beobachtete Tatsachen aufzubauen: sie prüft ihre Konstruktionen wieder durch die Beobachtung. Jede neu beobachtete Tatsache ergänzt dieses Weltbild, und jede Abweichung einer Konstruktion von der Beobachtung macht auf eine Unvollkommenheit, auf eine Lücke desselben aufmerksam. Das Gesehene wird durch das Gedachte, welches selbst nur das Ergebnis des vorher Gesehenen ist, geprüft und ergänzt. Es hat deshalb einen besonderen Reiz, das, was man nur theoretisch erschlossen hat, oder theoretisch vermutet, der Prüfung durch die Beobachtung unmittelbar zugänglich, d. h. wahrnehmbar zu machen.*".

[10] On Salcher and the Mach-Salcher collaboration, see esp. Merzkirch (1970); Hoffmann and Berz (2001); Alebic-Juretic (2019). See also Blackmore (1972, 110–115). The Mach-Salcher correspondence is partly published in Pohl and Salcher (2001).

occasion, he could attend a talk by the Belgian physicist Louis-Henri-Frédéric Melsens, who—speaking immediately after Mach in the session—reported on the explosive effects of bullets from firearms, giving a plausible explanation of the increasing seriousness of gunshot wounds in recent wars.

Melsens's presentation dealt with an issue that had been in the center of the public debate for years—and this might have contributed to arouse Mach's curiosity. Particularly during the Franco-Prussian War of 1870–71 it was observed that gunshot wounds caused by Chassepot rifles, in use in the French army, were much more severe than war injuries experienced so far. According to a report published on the medical Journal *The Lancet*, "the wound produced by the entrance of the chassepôt bullet is very small ...; the exit orifice of the ball, however, is large and ragged ... In 1866, Büttner and Gleisberg found in 100 wounds caused by needle-gun bullets, 58 fractures of bone, and 42 flesh rents. The proportion of the latter in wounds produced by the chassepôt bullet is far larger" (Anonymous 1871).

The cause of the increased damage was not completely clear, but descriptions like these sufficed to make the victorious German suspect that the enemy had employed explosive bullets in violation of the international treaties. Others conjectured that an explosive or an expanding effect could be due to the high velocity that a Chassepot bullet reached when hitting or penetrating a target, which caused fragmentation and fusion of the alloy that the bullet was made of and thus produced severe body wounds. Based on strict physical calculations, Melsens (1872) rejected the theory of the "fragmenting-fusing" bullet but also gave an alternative, sophisticated explanation of the apparent explosive effects. According to him, these were due to the fact that projectiles shot by firearms push ahead a mass of compressed air. As Melsens (1872, 57) put it, bullets shot from guns are not at all "simple" but "composed by a solid projectile and an *air projectile*" (emphasis in original). Therefore,

> in gunshot wounds the effects are produced by two bullets that simultaneously hit the target: the solid bullet that is deformed without experiencing substantial changes in its volume, and the gaseous bullet [*projectile gazeux*] which, being compressed ahead of the solid projectile, tends to get back to its previous volume corresponding to the atmospheric pressure. Meanwhile, it loses its living force and produces peculiar lacerations that, in the case at hand, can result into a similar effect an explosive bullet would produce. (Melsens 1872, 59)

In the extended paper resulting from the talk at the 1881 International Electrical Exhibition, Melsens described an experimental setup capable of collecting the air ahead of the projectile (Melsens 1882). The experiment consisted in shooting a spherical bullet (of different materials and with different initial velocities) into a conic hole dug in a cast iron block. The conic hole contained a mobile, open metal cone, whose size is much smaller than the bullet diameter, and terminated into a narrow steel cylinder. This was, in turn, connected with a rifle barrel linked with a water tank and, immersed in it, a cloche of glass or other materials (such as sealed glass or metal) for collecting the compressed air. The barrel, the tank, and the cloches were filled with water. When bullets were shot into the hole, a small portion of it passed through, plugging the cone and pushing air into water. What Melsens observed was that the barrel, the cloches, and the connections between them were cracked after the shots. In the case of the glass cloches, these went completely broken.

Melsens also noticed that the air tends to decompress while the velocity of the projectile diminishes. Based on this, he conjectured that "the front structure [*forme antérieure*] of the total mass of air moving along with the bullet gathers together, so that it is changed into an ogival-shaped mass ... As soon as the velocity decreases, the front air is dispersed"

(Melsens 1882, 398). This ogival-shaped mass of gas immediately ahead of the solid projectile was what he called the *projectile-air*. Still, up to that point this was not but an intriguing evidence-based conjecture. Its supposed effects could easily be observed, but the gaseous bullet seemed to back out of direct experimentation. In this sense, it remained elusive.

4.2 Mach's Pistols

By the mid of the 1870s Mach and collaborators also became interested in explosions and devoted some attention to those explosions occurring in gunshots. Of course, their interest was not guided by ballistic considerations, as in Melsens's case, but developed from their study of *Funkenwellen* (spark waves) and remained focused on undulatory phenomena. In a series of experiments that took place in 1875–76, they established a strong connection between "spark soundwaves" and "explosion soundwaves" (both of which we would now catalogue under the common label of shock waves). To produce explosions in their varied experimental setups they mainly used a high explosive (*Knallsilber*, or silver fulminate) but also utilized pistols to compare the velocity of a bullet with that of the shock wave produced by the gunshot itself. The most comprehensive results are presented by Mach and Sommer (1877). They begin reminding the above-mentioned conclusion of Mach and Wosyka (1875, 412) that the soot figures could be obtained by any kind of induced mechanical motion in the air. As Mach and Sommer (1877) put it, "whenever an air interval [*Luftstrecke*] is breached, whenever it is shaken by an explosion, the phenomena that occur are essentially the same" (Mach and Sommer 1877, 101). The most important outcomes are summarized at the end of the paper (1877, 127): first, they improved a technique in order to have an accurate measure of the velocity of the projectile through the soot pattern of interference; second, they compared the velocity of the projectile with that of the propagation of the wave, finding that at the places where the explosion occurs "the propagation velocity of the motion producing the [soot] stripes … is of the same order of the velocity of sound", whereas it diminishes more and more as the distance increases; and third, the velocity of propagation depends on the type and the intensity of the explosion. Remarkably, they also observed the wave propagation with the schlieren-technique and concluded that the phenomena observed in the schlieren-apparatus—i.e., differences in air condensation—and those observed on the soot-covered plate (namely, the formation of the soot figures) were produced by the same mechanical process: the "shaken air" caused by explosions of *any kind*, including e.g. pistol shots, as well as by electric discharges.

All this considered, Mach (1923/1898, 357/310) had a point when, in his 1897 retrospective lecture, he reminded that on hearing Melsens's conjecture, his desire to experimentally testing it "was the stronger as … all the means for realizing it existed, and [he] had in part already used and tested them for other purposes." As a matter of fact, he and collaborators used two of their methods —i.e., the soot and the schlieren techniques— for observing processes in air shaken by pistol shots: they chose or varied the visualization technique depending on the experimental situation. Now, the patterns of interference registered through the refraction-interferometric method or on soot-covered plates provide traces of an energy transfer in air, no matter if this is caused by sparks or explosions, and as such have an immediate wave-like meaning. However, to catch an air bullet like that of Melsens, the only viable option was the Toepler-apparatus. For, the purpose was not to visualize the effects of an explosion on the surrounding air immediately after a bullet was shot, but to visualize something that is supposed to exist in the air at the front of the material projectile, something that this brings along in its front part. This would make both the soot technique and the refraction-interferometric method useless, since what is required is not only to observe the effects of a shot on the surroundings but, at the same time, to image the solid bullet together with its hypothesized compressed air-forepart.

Thus, the schlieren-technique was a flexible one, but their results were potentially ambiguous. It could directly image a material object moving through space like a bullet as well as inhomogeneities in air condensation. The status of the latter, however, was questionable. Without appropriate background knowledge, they could be interpreted as due to compressed air transported through space, hence following Melsens's hypothesis, or to local disturbances of relatively stationary air, like in wave formation. As we have seen, starting from ballistic considerations Melsens had provided a clear "matter-like understanding" of what is going on when a bullet strikes a body. The damage caused by a Chassepot rifle, he argued, happened to be increased because high-speed projectiles are more effective in compressing air. Since, under these conditions, compression depends on the speed of the bullet and its cross-section, the higher the velocity, the higher is the air compression, and the more severe will be the wounds. In the injuries caused by powerful gunshots, Melsens saw the effects of matter (compressed air) carried about by other matter (solid bullet). The schlieren-technique that Mach would apply to the flying bullet-case, on the other hand, was based on his experience in the field of shock waves. It would not force him to a "wave-like understanding" of the phenomena he would observe in the schlieren-apparatus—but it was open to it. This does not mean that Mach's later interpretation of his and Salcher's experiments with bullets in terms of shock waves was already there at this stage; it signals, however, that essential epistemic resources in order that such an interpretation could be developed were already in place.

5 Hybridizations

Mach must have realized the potential of the schlieren-approach as early as in his 1884-1885 experiments with Wentzel on "the mechanics of explosions". The resulting paper (Mach and Wentzel 1885) was a very broad study that brought in connection some counterintuitive mechanical effects of explosions with the high velocity of propagation of the released shock waves, which is typically (much) higher than that of sound. For example, they observed that, when a relatively high-speed bullet hits a glass sheet, it produces an approximately round, "funnel-shaped" hole with sharp edges but does not necessarily smash the glass sheet completely. They noticed that holes produced in glass sheets by electric sparks or by small explosive charges display the same characteristics, and suggested that the particular funnel shape of the holes "may be explained if one considers that a longitudinal soundwave [shock wave] propagates from the point of impact with a velocity in any case very high" (1885, 636; see also pp. 634-635 for a quantitative explanation of the phenomena involved).

In this context they included a test of Melsens's prediction of the air bullet. In their own words, they "nurtured the hope that [they] would succeed in making visible the masses of air [brought along by a high-speed projectile] through the schlieren-technique and record them by photography" (1885, 638).[11] They arguably did not have a clear expectation about

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[11] In the following pages I will not consider the role of photography in Mach and Wentzel's and in Mach and Salcher's experiments. On the one hand, there are excellent studies on this and the importance that photography acquired since then in the development of experimental practices (see, e.g., Hoffmann 1996; Stie-

the possible outcomes of the experiment but reasonably assumed that a sufficiently powerful explosion-by-gunshot, occurring with a velocity comparable with that of sound, could arouse novel, maybe unexpected processes in the air traversed. Probably for this reason they were not particularly surprised as their schlieren-apparatus failed in revealing any striations but promptly suggested that the lack of traces was due to the poor velocity and the small size of pistol projectiles they had to employ in the limited experimental facilities they had available. While affirming the purpose to "make visible such masses of air" in the future, they decided to subdivide the original task (to photograph the projectile together with the air masses it brings along) into two separated parts as a preliminary exercise: "to photograph flying pistol bullets" and to photograph "soundwaves" [i.e., shock waves] from sparks (1885, 638–639). This does not necessarily imply that Mach was already convinced of the role of shock waves in this context nor that he expected that a high-speed flying bullet comes associated with shock waves in the manner he and Salcher later explained. Rather, at this stage Mach and Wentzel wanted to make sure that they were capable of taking reliable schlieren-snapshots both of the bullet and of a "mass of air" (i.e., a portion of relatively condensed air) of any sort, no matter how it is produced.

This provides a background for the letter with which Mach contacted Salcher on 25 January 1886. Here he proposed "to prove optically [*optisch nachzuweisen*], or else to photograph, the mass of air which is brought along by a small projectile flying at 500-600 m/s. Since the impression of soundwaves [i.e., shock waves] can be obtained with minor difficulties only, this should be feasible as well, given that the condensations are so considerable that they will produce substantial mechanical effects" (Mach to Salcher, in Pohl and Salcher 2001, 131). In a subsequent letter (16 February 1886) Mach made a sketch of the expected result: "I expect that the projectile would bring along a shell of condensed air in the form [of Fig. 1a]. The aperture of the smoothed cone will certainly depend on the ratio of the velocity of sound to the velocity of the projectile" (Mach to Salcher, in Pohl and Salcher 2001, 133).

Mach's prediction was confirmed by the experiments performed by Salcher during 1886, and the expected "smoothed cone" finally appeared (see Fig. 1b for one example) in the schlieren-images annexed to Mach and Salcher (1887, Tafel). Apparently, the brief description of the phenomenon reported above might be applicable both to a matter-like and a wave-like understanding; after all, Melsens's air bullet was "a shell of condensed air" that the material bullet brings along. Of course, Mach could hardly doubt that shock waves (he would have called them *Explosionsschallwellen*, or "explosion soundwaves") were released in the instant of the gunshot. But that waves would accompany the supersonic motion of the projectile was a different issue. This required a background knowledge in the general theory of waves and possibly compared cases from other domains involving wave motion (in particular, water waves: see Hoffmann 2001, esp. 259–269). For Mach and Salcher, this meant some months of reflection and study during 1886 that eventually resulted in the wave-like explanation provided in Section 5 of their 1887 fundamental paper:

Footnote 11 (continued)

gler 1998; Karallus 2001; Daston and Galison 2007; Hoffmann 2009; 2016). On the other hand, it is important to realize that the schlieren apparatus and the photographic camera, joined together in the experimental arrangement, have a different epistemic significance. As Hoffmann (2009, 18) summarizes it: "Whether the image from the schlieren apparatus fell into the eyepiece of a telescope or onto a photographic plate only made a difference in the persistence of the impression" (Hoffmann 2009, 18). This could highly simplify or even modify the analysis of the results and the exchange between two collaborators working at a distance like Salcher and Mach but leaves untouched the essence of the schlieren-approach that is at stake here.

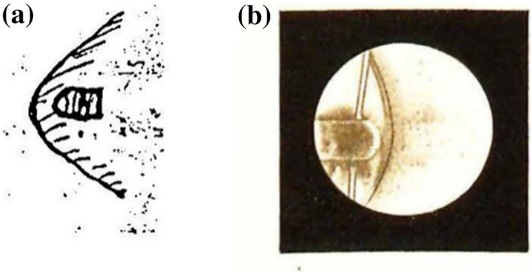

Fig. 1 **a** (left): The "smoothed cone" [*stumpfen Kegel*] of compressed air in front of a high-speed flying bullet (that Mach and Salcher later interpreted as the "head wave") as expected by Mach in a sketch included in a letter to Salcher on 16 February 1886 (from Pohl and Salcher 2001, 133). **b** (right): The cone as photographed by Salcher through the schlieren-apparatus. The negatives were then published as lithographs in the annexed plate of Mach and Salcher's paper (this image is Fig. 3 of Mach and Salcher 1887, Tafel), a circumstance that was considered as problematic, in the end, by Mach himself: see Hoffmann (2016; 2019)

> If a body of finite cross-section moves in air, it produces finite condensations [*Verdichtungen*], which become substantial at greater velocity. As results from theoretical investigations (Lagrange, Poisson, Stokes, Earnshaw, Riemann, Tumlirz) and experiments have taught (Regnault, Mach), they propagate with a velocity which exceeds the normal velocity of sound … As the wave expands, the condensation, and with it the velocity of propagation, diminish. If we apply this remark to our case, we see that the condensation before the projectile must grow with the latter's velocity, which in turn exceed the normal velocity of sound, until the velocity of propagation equals that of the projectile. At this point, the reason for further modification disappears. The condensation before the projectile preserves its own form and size. If we think of a projectile moving with constant velocity since infinitely long time, this brings along a kind of stationary soundwave, which preserves unaltered its form and density. Now, since the maximum condensation will be found at the head of the projectile, and its velocity of propagation is the same as the projectile; moreover, since the density and velocity diminish with the expansion of the wave, the meridian of the interface surface cannot be a broken straight line. Let us follow the curve backwards starting from the vertex, the angle … must decrease gradually … as is actually shown in the pictures. Thus, the meridian resembles the arc of a hyperbola … From the mere sight of the images one learns that the condensations of air before the projectile must be very considerable. In any case, they are of the same order like those of the spark waves, for which Mach [and Sommer 1877] in previous experiments detected a velocity of propagation until 700 m/s and for the weakest of which Mach and von Weltrubský [1887, 559] observed a value of condensation of 0,15 of an atmosphere. (Mach and Salcher 1887, 770-773)[12]

What did Mach and Salcher really observe? Following a previously quoted suggestion by Hentschel (2014), they photographed "V-shaped cones of compressed air in front of a projectile" (and other processes behind it). However, in contrast with Hentschel's

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[12] As Mach and Salcher (1887, 773) note, these and the subsequent remarks also serve as a comment to the photographs. In particular, whereas Sect. 5 deals with the processes at the forepart of the projectile, Sect. 6 is devoted to the puzzling phenomena at the rear, i.e. "the peculiar small clouds forming in the shooting channel [*Schusscanal*] behind a high-speed projectile." It is above the aim of the present paper to go deeper into this matter.

phenomenalist reading, Mach certainly did not "restrict his ontology to directly observable things or processes" so that "everything else was 'metaphysical' to him". To be sure, Mach finally gave a wave-like interpretation of the V-shape cone in analogy with his previous studies of shock waves from electric discharges and explosions. For him, waves were clearly not metaphysical but part of the physical ontology —but, literally, he did not see waves directly. He did not, because this is simply impossible. Let us take by analogy a water wave. This is conceivable as an energy transfer, a disturbance of relatively stationary water particles whose kinetic energy is transferred from point to point in space. Analogously, what the Toepler-apparatus can image —and what any observer sees— are consecutive states of condensation and rarefaction of air appearing as "striations" on a photographic plate.

In this sense, Melsens and Mach observed the same thing: condensations of air. But for Melsens this was just a mechanical effect of the pressure exerted by the material bullet on the air before it; for Mach, it was the effect of high-speed disturbances that are transmitted by air particles vibrating in the same direction of motion as the projectile —in other words, longitudinal waves. Still, it is Mach's wave-like interpretation of the process, not "the mere sight of the images", that let Melsens's matter-like understanding evaporate. It is this theoretical interpretation, not the photographs alone, that turned the *directly unobservable* shock waves, which are made experimentally measurable through the effects on the surrounding air, into entities included in the physical ontology.

Mach and Salcher (1887, 780) dubbed "head wave" (*Kopfwelle*) the condensation of air at the forepart of the projectile. As Mach would remember in his 1897 retrospective: "No physicist who has ever studied waves of sound or photographed them will have the least doubt regarding the sound-wave character of the atmospheric condensation encompassing the head of a flying projectile. We have therefore, without ado, called this condensation the head-wave" (Mach 1923/1898, 373/327). Curiously, this term and its cognate concept "tail wave" (*Achterwelle*) had only one occurrence in the paper. "Head wave", however, appears many times in Mach's correspondence with Salcher as well as in his diary notes from 1887 (Hoffmann and Berz 2001, 47–141) and later became of ample usage (Krehl and Engemann 2001; Krehl 2009).

Since these phrases —particularly the tail wave— were introduced by analogy with waves created by ships in their motion in water, one has the impression that they are mere descriptions of observed processes. In fact, they are hybrid concepts.[13] On the one hand, they are certainly descriptions of observed facts, but the fact that is described is a certain alternating, forward-moving state of condensation-rarefaction of air. On the other hand, thinking of such a state in terms of a wave means to include properties that derive from theoretical conceptions, which are intertwined with experimental arrangements developed, in turn, to study wave-like phenomena, namely soundwaves and spark waves. Thus, the hybrid nature of Mach's head wave ultimately relies on the hybrid nature of the experimental systems as advanced by Hagner and Rheinberger (1998, 359): "they mix up elements" such as "research objects, theories, experimental arrangements, instruments as well as disciplinary, institutional, social, and cultural *dispositifs*".

As Hagner and Rheinberger continue, the resulting hybrids are "amalgams of every conceivable gradation"; thus, philosophical elements may also concur to the uninterrupted,

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[13] For a detailed reconstruction of this process of hybridization in Mach and Salcher's experiments and the crucial role of the analogy with water waves produced by ships, see Hoffmann (2001).

growing process of hybridization. However, there is no standard recipe for hybridizations and every one depends on specific conditions that historians and philosophers of science should investigate. To argue that Mach's experimentalism and his achievements in the realm of shock waves can be explained as effects of his ontology of the elements is probably not a good hybridization. By the time of his experiments on shock waves, Mach had certainly developed some ideas that he later included in his theory of the elements. They are to be found, for example, in *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit*, which he considered "the first attempt to give an adequate exposition of [his] epistemological standpoint ... based on a study of the physiology of the senses" (Mach 1872/1911, III/9). However, the experiments in gas dynamics of the late 1870s as well as those on projectiles are quite independent from any conception about the role of sensations he could have retained in that period. To put it more clearly: Mach's philosophical commitments—no matter if interpreted as phenomenalist or neutral monist—did not play any relevant role either in the preparation of the experiments or in the interpretation of the results.

By contrast, the approach he ultimately selected for his experiments with bullets—visualization of certain phenomena through the schlieren-technique—was chosen because it was the best option for visualizing both a projectile and the surrounding disturbances; by doing this, he implicitly put in the foreground a wave-like image and brought into the picture certain theoretical components from the theory of waves with which he had become familiar; the results he reached (the visualization of a head wave and a tail wave) were hybrids. They were the outcome of an experiment-based theoretical interpretation that stands even at odds with a phenomenalist conception. But after all, to develop an interpretation of his schlieren-images, Mach did not need to be committed to a particular ontology. No matter if he actually had ontological or —more in general— meta-scientific commitments at that time, what he surely needed, and had, was competence in the theory of waves associated with a strong experimental background.

It can easily be seen that exactly this experimental background transpires from many passages of *Knowledge and Error*. This is the case, for example, of Mach's remarks on the possibility to replace one sense with another—e.g., "optical devices can make soundwaves visible and light waves audible" —, which is directly connected with his experiments with shock waves (Mach 1905/1976, 147/106). Mach's experimentalism is also instrumental to his treatment of thought experiment (Chapter 11) and re-emerges throughout Chapter 12, "Physical Experiment and its Leading Features". It is involved in the many examples reported in Chapter 17, "Pathways of Enquiry", and certainly in many other places that cannot be explored in this limited study.

In the same vein, I would like to venture, as a working hypothesis which should be subject to further analytical examination, that Mach's experimental works in diverse fields not only helped him shape his late epistemology but also might have contributed to his ontology in a somewhat indirect manner. As noted by Hui (2021, 11), he began his *Vorträge über Psychophysik* (Mach, 1863, 146–147) asking what makes a science an *exact* science. His answer was that exactness is not an inherent quality to any particular discipline but only denotes a certain developmental step. In the second lecture is made clear that the development which sciences —in particular, psychology and psychophysics— are expected to follow in order that they can turn into fully exact doctrines is the application of a particular version of the experimental method (that Mach here identifies with the Baconian method):

> This consists in the variation of conditions that are conjectured to be connected. Let x be the load and y the produced elongation of a wire, different values in sequence will be ascribed to x and the corresponding magnitudes of y are observed. Then, one can either express the connection of x and y in words, or make it evident in a table, or represent it graphically with a curve, or finally express it through a mathematical formula (Mach 1863, 167).

This is clearly reminiscent of Mach's insistence on the method of variation, which is practically omnipresent in *Knowledge and error*, beginning with the first chapter: "If we have to investigate a set of multiply interdependent elements there is only one method at our disposal: the method of variation. We simply have to observe the change of every element for changes in any other" (Mach 1905/1976, 17/10). The reference to analytical representations into curves or equations makes it also reminiscent of Mach's mathematically-based concept of function, i.e. "*the dependence of phenomena on one another*, or, more accurately, *the dependence of the characteristics of phenomena on one another*", as is described in *The Analysis of Sensations* (Mach 1886/1959, 74/89, emphasis in original). Here this acquires a particular flavor. Mach emphasizes that all we know are not the elements in themselves but their functional relations with one another: the variation of a certain group of elements, provided that another group is varied. In other words, elements—as "*ultimate component parts* [letzte Bestandteile]" of complex experiences (Mach 1886/1959, 4/5, emphasis in original), but only "in the sense that no further resolution has as yet been made of them" (1886/1959, 34/42)—are found through, and can be subject to, possible variations of conditions, no matter if this is performed in a laboratory or as a psychological, more or less informal exercise. Note that both kinds of examples are recurrent in *The Analysis of Sensations* (e.g., Mach 1886/1959, 2–4/2–6, 33–34/41–42) and in *Knowledge and error* (e.g., Mach 1905/1976, 7–15/5–9).

Now, these few remarks should suffice to highlight that concepts such as the method of variation and that of a functional relation among varying quantities provided a basis, in the medium and long run, onto which Mach built his ontology: the theory of the elements. However, as I instantiated above, they were originally developed with reference to an ideal of experimentation that he thought to be applicable to psychophysics, making of it an exact science in the same manner as physics, for example. In this sense, Mach drew his conclusions about the method of variation and its importance reflecting on his activity in the domains in which he was experimentally involved—so to speak, from his experimental life as a whole.

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Conflict of interest I certify that there is no actual or potential conflict of interest, nor there are competing interests, in relation to this article.

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