Educational Studies in Mathematics (2023) 112:141–174
https://doi.org/10.1007/s10649-022-10173-1

"How to meme it": reverse engineering the creative process of mathematical Internet memes

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Accepted: 11 July 2022 / Published online: 29 September 2022

Abstract

Mathematical Internet memes are examples of how the creative thrust characterising the Web 2.0 environment reaches the field of mathematics, translating mathematical statements into a new digital form endowed with an epistemic potential that is capable of initiating a process of mathematical argumentation. The research presented in this paper aims to shed light on the creative process of mathematical memes, contributing to building a body of knowledge on mathematical memes that, prospectively, could enable educators to profit from these objects in their teaching. Theoretically, this is based on a widened concept of creativity that focuses on the connection linking digital culture with mathematics, and on distinguishing and merging three perspectives to disclose the meanings of mathematical memes. Methodologically, the process of mathematical memes' creation is investigated through a reverse engineering approach on a dataset of about 2100 items collected in a 3-year-long ethnographic observation within online communities. The result is a heuristic action model of the creation process, that is validated by creating two new mathematical Internet memes that are shared online within the observed communities to explore if they retain the mathematical and epistemic characteristics of Web-found ones.

Keywords Internet meme · Mathematics · Creativity · Reverse engineering · Mathematical statement · Web 2.0 culture

1 Introduction

In September 2019, the image in Fig. 1 was posted on the social media Web site Reddit in a community dedicated to exam tips for the UK General Certificate of Secondary Education.

The post was tagged "revision resources" and sparked a lively thread of comments, an excerpt of which is reported here (comments are unredacted, and for ethical reasons anonymised, and no URL is provided):

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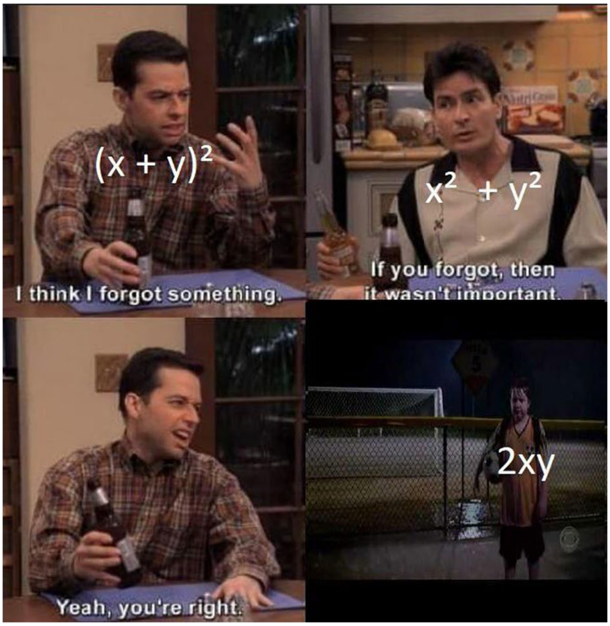


Fig. 1 *I think I forgot something* mathematical meme (source Reddit)

1-Commenter 1: Is it bad that I'm doing A level maths and dont get it?
2-Commenter 2: Not gonna say yes or no but it's basically when expanding $(x+y)^2$ people always forget to multiply x by y and y by x so miss out the 2xy
3-Commenter 1: Ohhh yeah I get what it means now thanks
4-Author: I'm doing A-Level and it's been two weeks. Yet I barely understand anything. Basically $(x+2)^2$ means $(X+2)(x+2)$ which then expands to x^2+4x+4. But because it's $(x+2)^2$ instead of $(X+2)(x+2)$ people forget

This is an example of a *mathematical Internet meme:* it is an *Internet meme* (or simply a *meme*), a designed mutation of an existing image (Davison, 2012; Dawkins, 2013; Shifman, 2014) created by Internet users adding an original text envisaged to convey the user's feelings or opinion on a certain subject, and it is *mathematical* because the added texts [the symbols $(x+y)^2$, x^2+y^2, $2xy$] address a mathematical topic, identified by a coalescing of mathematical signs (Love & Pimm, 1996; O'Halloran, 2005). In this example, the existing image is a four-panel composition of screenshots from a TV sitcom, already incorporating the subtitles "I Think I Forgot Something – If you forgot, then it wasn't important – Yes, you're right". The image is used to represent metaphorically the idea of forgetting something vitally important, which is identified by the added texts, the mathematical signs addressing the "freshman's dream" mistake of expanding the quadratic binomial forgetting the middle term $2xy$, personified by the child left at soccer practice in the lower-right panel.

Memes are so popular that there are Web sites to generate them (Imgflip,[1] Kapwing[2]), and specialised online encyclopaedias such as KnowYourMeme[3] (KYM) or Meming Wiki[4] (MW). In particular, mathematical memes are so widespread that online

1 https://imgflip.com/memegenerator
2 https://www.kapwing.com/meme-maker
3 https://www.knowyourmeme.com
4 https://en.meming.world/wiki/Main_Page

communities are dedicated to them, constituting the mathematical subset of the *memesphere* (Stryker, 2011), the set of Web communities where memes are created, mutated, shared and commented.

A recent ethnographic study (Bini et al., 2022) looked into mathematical memes analysing the threads of comments initiated by these digital objects within dedicated online communities. This analysis is described as *cultural* because it focuses on the investigation of the environmental culture emerging from these comments. This cultural analysis revealed that, pairing with mathematical content, memes move from their original purpose of disseminating feelings and opinions to that of representing *mathematical statements*, and that they are endowed with the potential to activate an epistemic culture in the community. This epistemic culture is made up of different aspects: less knowledgeable users comment showing *epistemic needs* (Kidron et al., 2011), which are met by more knowledgeable users who spontaneously offer *informal mentorship* until the process of meaning making is completed, and the epistemic needs are fulfilled. Thus, mathematical meanings are negotiated in the culture, "resulting in proofs and clarifications around the mathematical idea in the meme" (Bini et al., 2022, p. 1288). Observing the example in Fig. 1 through this lens, we can interpret the represented mathematical statement as the formula of the square of the binomial $(x + y)^2 = x^2 + y^2 + 2xy$. In the excerpt, we note that Commenter 1 shows epistemic needs (#1 "dont get it"), and that informal mentorship is spontaneously offered by Commenter 2 (#2 "when expanding $(x+y)^2$…") until meaning making is completed (#3 "…I get what it means now thanks"); the proof of the mathematical statement is then reinforced by the meme's author (#4 "$(x+2)^2$ means (X+2)(x+2)…"). Since the mathematical content of this example focuses on a typical mistake, some behavioural patterns (#2 "people always forget" #4 "people forget") are discussed in addition to the proof.

In a nutshell, a mathematical meme is a cultural product where mathematics is knowingly incorporated and not simply *hidden* in the sense of Gerdes (1988), like in traditional techniques such as tapestry weaving or basket making. It is the awareness of the mathematical content that endows mathematical memes with an epistemic potential capable of initiating a process of mathematical argumentation.

Examples in Bini et al. (2022), and our opening case show that mathematical rules are preserved in the transition from the mathematical environment to the memesphere. We can expect this also to happen in the classroom: indeed, recent research envisaged that mathematical memes could be fruitfully exploited to design educational experiences where digital culture meets school culture, "emphasiz[ing] the movements and connections between mathematics education and other practices" (Bakker et al., 2021, p. 8).

Educational potentialities of mathematical memes are being recognised by research in mathematics education, with studies reporting the positive effects of their use on students' engagement, observed during in-person school experiments (Beltrán-Pellicer, 2016; Bini & Robutti, 2019a, 2019b; Brito et al., 2020; Felcher & Folmer, 2018; Friske, 2018, 2020; Friske & Rosa, 2021) and distance-learning activities (Abrams, 2021), and by studies in cognitive science, showing that mathematical memes can engage students in educational contexts (Van, 2021).

All cited studies in mathematics education sample mathematical memes' didactic potentialities in activities where students create their own memes, possibly accompanied by the discussion of other memes sourced from the Web or created by researchers. This choice of focusing on students' creations grounds on the awareness of the importance of "creating and sharing" in the participatory culture (Jenkins, 2009, p. xi) that permeates the Web 2.0 culture epitomised by memes (Shifman, 2014). Nevertheless, this setting requires educators to be able to understand students' creations, interpreting mathematical statements represented by memes "in a new hybrid language which merges mathematical elements with memetic elements" (Bini et al., 2022, p. 1290). As shown in the opening example, the interpretation of the represented statement is not straightforward and requires teachers to be proficient in memes' hybrid language to grasp and unfold the connection between elements pertaining to different cultures. This aspect is left aside by existing research: in fact, in all cited studies, students' memes are analysed by dedicated researchers, who are clearly skilled in memes' language rules and signs, but we cannot expect the same proficiency from teachers.

To enable teachers to take advantage of memes' educational potential, we must provide them with guidance about how mathematical memes can be understood. Embracing Richard Feynman's legacy stating "what I cannot create, I do not understand" (1988)[5], we start from the assumption that this understanding is achieved by showing how mathematical memes can be created and thus providing heuristics.

This paper aims to close this research gap. For that, we look into the creative process of mathematical memes to contribute to building a body of knowledge on mathematical memes that, prospectively, could enable researchers, teachers, and educators to profit from these objects in their teaching.

2 Rationale: rethinking creativity in mathematics education

The role of memes as epitomising objects of the Web 2.0 creative thrust (Shifman, 2014) prompts a reflection on creativity in mathematics education.

Presently, creativity in mathematics is defined as the act of discovering something new (or subjectively new) in mathematics: new concepts, new solutions to open questions, or new angles of observation of problematic situations (Nadjafikhah et al., 2012; Sriraman, 2009; Sriraman et al., 2011; van Hiele, 1986). This definition implies that a creative act in mathematics involves skills that typically are the prerogative of gifted students. Indeed, studies about mathematical creativity are usually paired with studies on giftedness, as in an article by Sriraman (2005) and in the recent ICME-13 Monograph (Singer, 2018).

Whereas, in the field of general education, creativity is more broadly defined as the ability of "recombining ideas or seeing new relationships among ideas" (Torrance, 1969, p. 4). This broader definition is shared by other research fields: psychological studies define creativity as the aptitude of "making unfamiliar combinations of familiar ideas" (Boden, 2004, p. 3), neurological studies affirm that "creative thinking involves searching memory to connect concepts that are 'farther away' from each other in memory" (Beaty & Kenett, 2020, p. 219) and machine intelligence studies reckon that "moments of insight are merely the result of the brain making connections between weakly and strongly activated bits of information, and then bringing them to consciousness" (Carpenter, 2019, p. 1).

The latter and broader definition is more aligned with the recommendations of OECD's "Future of Education and Skills 2030"[6] project and of the PISA 2021[7] mathematics framework, both enlisting creativity among the 21st Century core capacities to be powered *in all students*, following Vygotsky's idea that "creativity creates a lifelong zone of proximal development" (Moran & John-Steiner, 2003, p. 3) and Piaget's convincement that "the principal goal of education is to create [people] who are capable of doing new things, not simply of repeating what other generations have done" (as cited in Duckworth, 1964, p. 499).

5 From Feynman's last blackboard https://digital.archives.caltech.edu/islandora/object/image%3A2545

6 https://www.oecd.org/education/2030/E2030%20Position%20Paper%20(05.04.2018).pdf

7 https://pisa2021-maths.oecd.org/

Therefore, efforts should be aimed at rethinking creativity in mathematics education, and at designing educational activities that widen the range of students who can experience creativity in mathematics, as already advocated by Baker (2016), Czarnocha et al. (2016) and Prabhu and Czarnocha (2014).

This can be done by merging the definitions recalled above, and considering as creative not only acts that produce new mathematical objects, but also acts that *create new representations of known objects through new connections*. In this way, we could count mathematical memes as manifestations of creativity since they are representations of known mathematical objects (statements) created through new connections between digital culture and mathematics. They are mathematical mutations of existing images where "the originality is in the combination, not [in] the elements" (Robinson, 2011, para. 4).

Our purpose is therefore to produce an understanding of the creative process of mathematical memes that could support a future transformation of mathematical memes from cultural Internet objects into educational objects (as addressed by Friesen, 2001) to be shared and used by educators.

Our research is guided by the following research question:

RQ How can mathematical memes be created?

With the generality of this research question, we acknowledge that there may be various ways to create memes and simultaneously this research question guides us in producing a heuristic action model as an answer. To answer this question, we are challenged by crucial practical limitations: we only have access to the finished objects (mathematical memes on the Web), and we cannot directly investigate the creative process, as creators do not disclose it, nor can we approach memes' creators who usually hide behind nicknames on social media. Thus, we choose to adopt a *reverse engineering approach* to work our way back from the finished product to the possible creation stages.

The term *reverse engineering* refers to the process of extracting design knowledge from anything human-made, i.e., anything that has been *engineered* by a human being (Eilam, 2011; Friesinger & Herwig, 2014; Müller et al., 2000; Rekoff, 1985; Samuelson & Scotchmer, 2002). Reverse engineering starts from a finished object to produce design knowledge. It is the inverse process of forward engineering (Baxter & Mehlich, 2000) that starts from design knowledge to produce a finished object. The reverse engineering strategy is usually adopted "to obtain missing knowledge, ideas, and design philosophy when such information is unavailable" (Eilam, 2011, p. 3). Thus, this approach fits with our purpose to gain insight into the process of creation of mathematical memes, since these objects are human-made and information about their creation is not directly available.

Reverse engineering aims at determining how the original product was designed to gain knowledge suitable to build a new—and possibly enhanced—version. It is a shared idea in literature that through reverse engineering "we do not only learn how to replicate technology in detail (…) but we also learn how to modify, create and design" (Friesinger & Herwig, 2014, p. 10). Therefore, this approach fits also with our idea that this understanding enables the creation of new mathematical memes.

To pursue this line of research we need theoretical and methodical tools. Theoretically, we conceptualize creativity using Koestler's *bisociation* theory of the creative act (1964), which coherently frames our broader definition of creativity, and we use Bini and Robutti's *triple-s construct* (2019a) to distinguish the different levels of meanings carried by mathematical memes. Methodically, we face the lack of an existing *reverse engineering methodology* suitable for our purpose, which we overcome by extracting from and adapting to our aim the current literature of mechanical and software reverse engineering. This effort produces an original reverse engineering method that is applied to our dataset of about 2100 mathematical memes collected in a 3-year-long ethnographic study. It allows us to reach our goal of understanding which components mathematical memes are built on and how these components are connected. This understanding is then summarised in a validated heuristic action model.

3 Theoretical background

We summarise here Koestler's *bisociation* theory of the creative act (1964), which we use to frame theoretically our widened definition of creativity as the result of new connections, and the *triple-s* construct (Bini & Robutti, 2019a), that we use to understand in what parts and cultural realms mathematical memes can be analytically separated into and how these parts relate to each other.

3.1 Koestler's bisociation theory

According to Koestler, the creative act "does not create something out of nothing; it uncovers, selects, re-shuffles, combines, [and] synthesizes already existing facts, ideas, faculties, skills" (1964, p. 120). The creative act, therefore, lies in producing an original connection between two apparently estranged frames of reference through a *linking idea* that is *bisociated*, i.e., perceived in the two frames of reference at once. In this process, each frame of reference (indicated by FR1 and FR2 in Fig. 2) retains its identifying rules and logic. When two frames are linked in a creative act, they "can be adapted to environmental conditions; but the rules (…) must be observed and set a limit to flexibility" (p. 38). In other words, the new object that is the product of a bisociated creative act is not accompanied by new epistemology and

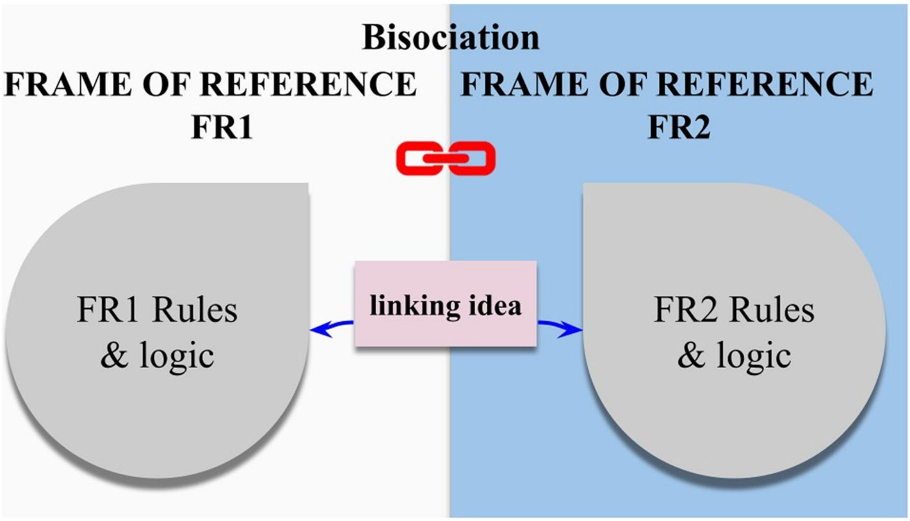


Fig. 2 Koestler's bisociation theory (1964)

rules—as happens in conceptual blending (Fauconnier & Turner, 2008)—on the contrary, it preserves and incorporates the rules of the two originating frames of reference.

Koestler's stance is therefore fitting to investigate mathematical memes, because these objects are *designed mutations* connecting already existing and apparently estranged things (popular images and mathematical ideas), and because these mutations are created preserving mathematical and memetic rules independently, as shown by Bini et al. (2022).

Koestler names his theory *bisociation*, a neologism that reflects his view that the linking idea is "not merely linked to one associative context, but bisociated with two" (Koestler, 1964, p. 35). Thus, *bisociation* is a counterpoint to the known concept of *association*, which Koestler interprets as the standard connection among elements from a single frame of reference.

He also reckons that the act of understanding a creative product corresponds to re-creating it in our mind and is therefore a creative act in itself (p. 263). This is not an ordinary use of the concept of creativity, it makes this theory even more aligned with our declared research aim to provide teachers with the knowledge to understand mathematical memes by showing how they can be created.

To exemplify his theory in science, Koestler mentions the frames of reference of magnetism and electricity, physics and chemistry, and corpuscles and waves, all initially "developed separately and independently, both in the individual and the collective mind, until the frontiers broke down" (p. 658), shattered by a creative act that generates a new scientific idea. The two separated frames of reference pre-existed, and the new linking idea came into being only when "the time was ripe for that particular synthesis" (p. 658). In our case, bisociation explains memes' hybridity, fostered by the Web 2.0 socio-cultural environment, and the superposed perceptions of mathematical memes as Internet memes as well as mathematical statements, revealed by investigations inside dedicated communities (Bini et al., 2022). As memes, they make the reader laugh and provide bonding experiences, and as mathematical statements, they engage the reader in a process of argumentation to determine or justify their truth value.

3.2 The triple-s construct of partial meanings of a meme

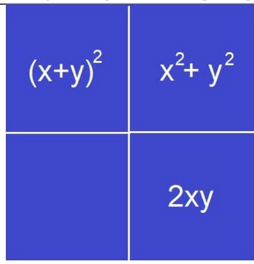
Memes' narrative strategies are evident to readers accustomed to their hybrid language but can be less evident to those not familiar with it. Yet, to apply Koestler's (1964) theoretical framework to mathematical memes, we need to unveil these narrative strategies to understand what is bisociated and how. To do this, we adopt the *triple-s construct of partial meanings* (Bini & Robutti, 2019a): according to this construct, the interpretation of a meme is achieved through the understanding of *three partial meanings* and their subsequent interconnection to build the *full meaning* of the meme which, in our case, is the mathematical statement.

The *three partial meanings* are described in the first two columns of Table 1. The third column exemplifies the effect of the triple-s construct when applied to the opening example (Fig. 1), distinguishing the three levels of partial meanings.

In the memesphere, the pictorial component carrying the *social meaning* is known as *template*, a terminology we shall adopt from now on. A template is *something that is used as a pattern for producing other similar things*[8]: this term reveals that the image in the meme is not simply an image, it is a picture already containing the seed of its reproducibility through mutations. Such templates are images taken from the Web, but not all images become templates: they achieve the status of templates when they become signs, loaded with a recognisable meaning, assigned

[8] https://dictionary.cambridge.org/dictionary/english/template

Table 1 The triple-s construct of the partial meaning of an image-based meme (Bini & Robutti, 2019a)

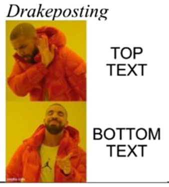
Partial meaning	Description	Application of the triple-s construct to the mathematical mutation in the example in Fig. 1
Social	Information carried by the value conventionally attributed in the memesphere to the pictorial component in the meme	 Social meaning of the pictorial component: forgetting something vitally important (KYM).
Structural	Information carried by the value conventionally attributed in the memesphere to the graphical composition of the meme: font style, position, phrasal pattern of the captions, overall arrangement of the composition	 Structural meaning of the graphical composition: characters embody the object in the superimposed text
Specialised	Information carried by texts, pictorial elements, addition or alterations of the original image referring to a specific topic	 Specialised meaning of the disciplinary signs: algebra, the square of the binomial

through a process of collective semiosis (Osterroth, 2015). Rules that codify the social meaning develop spontaneously in the memesphere; once established, templates acquire names, categorised in meme encyclopaedias and meme generator websites: for example, the template of the meme in Fig. 1 can be found in all websites as *I Think I Forgot Something*. Templates are used to create mutations shared inside Web-based communities, which harshly condemns their misuse with a vocabulary that "marks the 'breaches' as outsiders" (Nissenbaum & Shifman, 2017, p. 491).

Conventions established within the memesphere, and legitimated by meme encyclopaedias, dictate also the rules for the graphical composition of each template, corresponding to the *structural* meaning. These rules are so ingrained in the memesphere that meme-generator Web sites provide automatically the established structural set-up according to the chosen template. In Table 2, we summarise and exemplify the currently (September 2021) known instances of the structural meaning, assuming the terminology from KYM.

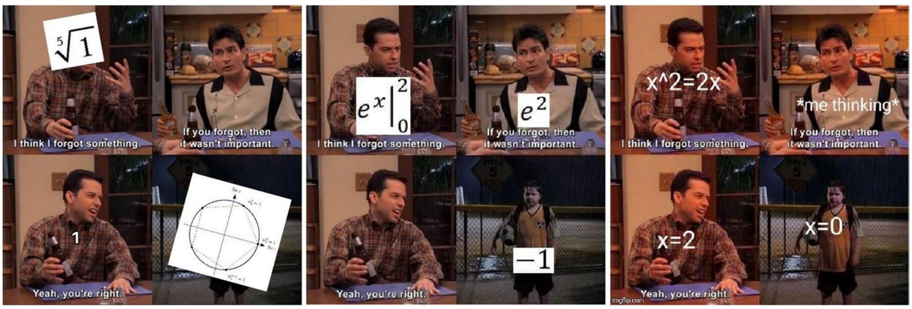
As evident from Table 2, the structural meaning follows some overarching rules: added textual elements must be clearly recognisable (hence the font), readable (hence the font size) and captivating (hence the brevity of the text). Moreover, the narrative of the

Table 2 Structural meaning values (source KYM)

<table>
<tr><td>Reaction images</td><td>Object labelling</td><td>Exploitable</td><td>Multi-pane</td><td>Interior Monologue Captioning</td></tr>
<tr><td>Top or top/bottom text arrangement, usually in Impact or Arial font. Brief texts possibly following a phrasal template (Zappavigna, 2012) with fixed parts to be completed with suited variable elements</td><td>Brief texts or symbols are superimposed onto the image, with elements in the picture embodying the object in the superimposed label, according to the social meaning of the template</td><td>Brief texts or symbols are inserted in specific empty spots of the frame, according to the social meaning of the template</td><td>Different lines of brief texts or symbols are inserted in each panel, usually in Impact or Arial font uppercase and lowercase, in a sequence modulated according to the social meaning of the template</td><td>Randomly dispersed brief texts or symbols representing what the subject is thinking or feeling, usually in Comic Sans font uppercase and lowercase</td></tr>
<tr><td>Example 1: Annoyed Picard
</td><td>Example 2: I think I forgot something
</td><td>Example 3: Two buttons
</td><td>Example 4: Drakeposting
</td><td>Example 5:Obama Cell Phone Photo
</td></tr>
</table>

meme always develops in a top/bottom direction, leaving to the bottom part the role of the punchline. We interpret these aesthetic features as imposed by practical needs of readability via the quick top/down scrolling gesture on mobile devices, which are the natural habitat of memes. In other words, it is the technology providing the support for the meme that imposes these structural rules, to which the meme adapts. This causation between technology and patterns of human interactions has been theorised by McLuhan in his famous saying "the medium is the message" (1964, p. 9): here, the medium is the touch-screen technology and the message is the structural meaning of the meme.

The last partial meaning is *specialised*, framing the meme in a disciplinary domain, in our case mathematics. This meaning can leverage on elements of the template, alterations of the template that do not prevent its recognition, or additions as symbols, diagrams, other images or textual elements (as in the example in Fig. 1). In Fig. 3, we see three mutations of the *I Think I Forgot Something* template used in Fig. 1, where the social meaning of forgetting something vitally important matches with different specialised meanings in the topics of complex numbers (Fig. 3a), definite integrals (Fig. 3b) and quadratic equations (Fig. 3c).



a **b** **c**

Fig. 3 **a**–**c** Mutations of *I Think I Forgot Something* (sources Reddit and Facebook)

The understanding of the three partial meanings is a necessary but not sufficient step to grasp the *full meaning* of a meme, corresponding to the translation of the represented mathematical statement from the hybrid language of memes to the formal language of mathematics. To *see the statement*, the reader has to take the different meanings and combine them into a single unit. It is a process of synthesis in the sense of Kant, where a collection of elements is "gone through, taken up and combined in a certain way in order for a cognition to be made out of it" (Kant, 1998, A77/B103). It is "the sudden emergence of a new insight, or [the] short-circuits of reasoning" (Koestler, 1964, p. 211) that characterises the act of creating a meme as well as the act of understanding it.

4 Methodology and methods

This research is imprinted to an approach that merges ethnography (Eisenhart, 1988; Harwati, 2019) with grounded theory (Glaser & Strauss, 1967; Strauss & Corbin, 1998; Teppo, 2015). This methodological approach is well established in studies on memes (Czarnocha et al., 2016; Denisova, 2019; Katz & Shifman, 2017; Nissenbaum & Shifman, 2017; Stöckl et al., 2019), taking into account the fact that data are openly available in the field and do not come from structured experiments. Following a basic principle of anthropology (Powdermaker, 1967), we have made an effort to balance our *inside* and *outside* stance on the observed culture. The first author is an observer *inside* online communities dedicated to mathematical memes, while the other authors remain *outside* acting as controllers questioning the first author's reporting. The fieldwork started in February 2018 and is still ongoing; throughout it, chronological field notes and memos are taken in real time by the first author, following the guidelines of ethnographic research (Sunstein & Chiseri-Strater, 2012), noting key cases, norms, impressions and behaviours, insider phrases, and rituals of the observed communities. These observations are shared between authors, following a reflexive approach to grounded theory (Mruck & Mey, 2007), enabling us to harness new ideas as the research proceeds.

We address the research question by observing data collected through the ethnographic investigation which had been previously coded and categorised following the grounded theory approach, with a reverse engineering based on Koestler's bisociation theory and the triple-s construct. This novel approach in the field of meme studies suits our purpose to unveil the process of design behind memes, which can then be remixed to create new original memes, and not to plagiarise existing memes. To validate the knowledge obtained, we use it to create new memes and then share them on the Web to explore if they activate and guide interactions among members of online communities as their wild counterparts investigated by Bini et al. (2022).

4.1 The ethnographic investigation: data collection, coding and categorising

The fieldwork was preceded by a plan that consisted of the following: (1) mapping the territory for websites and community identification and selection and (2) collecting the dataset. The first step was guided by criteria of popularity, expected to imply richer habitats to explore: we selected 25 of the most popular communities (in terms of members) sharing mathematical memes within three of the most popular social media websites[9] (Facebook, Instagram and Reddit). The second step was guided by the long-term purpose of using the results of this research to inform the teaching of mathematics. Consequently, we collected memes representing topics from the 8th to 13th grade mathematics syllabus (according to

[9] https://www.statista.com/statistics/272014/global-social-networks-ranked-by-number-of-users/

the Italian national curriculum[10]). We also collected meta-memes, i.e., memes about memes, which reveal how the practice of meme creation is perceived inside the memesphere. Data collection entailed downloading memes to gather a *meme pool* and saving the associated active links to gather a *comment pool* that gives access to the threads of comments, where discussions about the mathematics in the meme take place. This gave us insight into how the specialised meaning of the meme is approached inside the community, as in the opening example. In the period from February 2018 to September 2021, we collected a meme pool of about 2100 memes (filtering reposts) and a corresponding comment pool. All memes and threads of comments in the dataset were collected from publicly available groups and pages, and no personal information was the object of research, thus abiding by the ethical use of social media data in research (Sloan & Quan-Haase, 2016; Townsend & Wallace, 2016).

Following the grounded theory approach, memes were coded as they were collected. Each meme was saved and renamed with a two-part code: the first part identifies the template via its social name reported in the encyclopaedias KYM or MW (or by another clear identifier), and the second part identifies the mathematical topic: thus, the first part is connected to the social and structural meanings and the second part to the specialised meaning. For example, the code for the meme in Fig. 1 is *I think I forgot something binomial square.* The choice for the first part of the code ensures that memes with the same template are archived close to each other so that the designed mutations can be observed. Constant comparison (Glaser & Strauss, 1967, p. 102) of new and old data allowed this coding activity to proceed coherently.

Codes of the mathematical memes collected from February 2018 to September 2021 were then combined and related to one another, focusing on the second part of the code until the process saturated theoretically (Strauss & Corbin, 1998, p. 143) leading to the emergence of 14 *categories* (e.g. Properties, Objects or Typical Mistakes, see Fig. 7). Categories were subsequently clustered into four *macro-themes,* as Mathematical Concepts or Mathematical Knowledge (see Fig. 7), allowing us to gather a holistic view of the mathematics addressed in the meme pool.

Finally, we chose exemplary cases representing each category, following the *diverse case method* (Seawright & Gerring, 2008), with each case being illustrative of the corresponding category. This strategy suits the exploratory nature of this research, fits the need for ordering in the ethnographic approach, and enhances the representativeness of our sample of cases showing the "maximum variance along relevant dimensions" (p. 300), which in our case is the mathematical content of the meme.

4.2 The reverse engineering investigation

In this section, we introduce the methods we developed and the tools we used to perform the reverse engineering investigation.

4.2.1 Developing a suitable reverse engineering strategy

Reverse engineering is a novel approach in the investigation of memes, although already addressed by Gosztonyi (2019) in mathematics education research. It is mostly used in the fields of mechanical and software engineering, where it is developed along paths that do not fit our case. To adapt this research strategy to our purpose we approached the literature in the technical and educational field to conceptualize the underlying common structure.

Educational mechanical reverse engineering studies usually adopt the DAA (disassemble, analyse, assemble) approach (Calderon, 2010; Ogot et al., 2008) that revolves around

[10] http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_Liceo%20scientifico.pdf

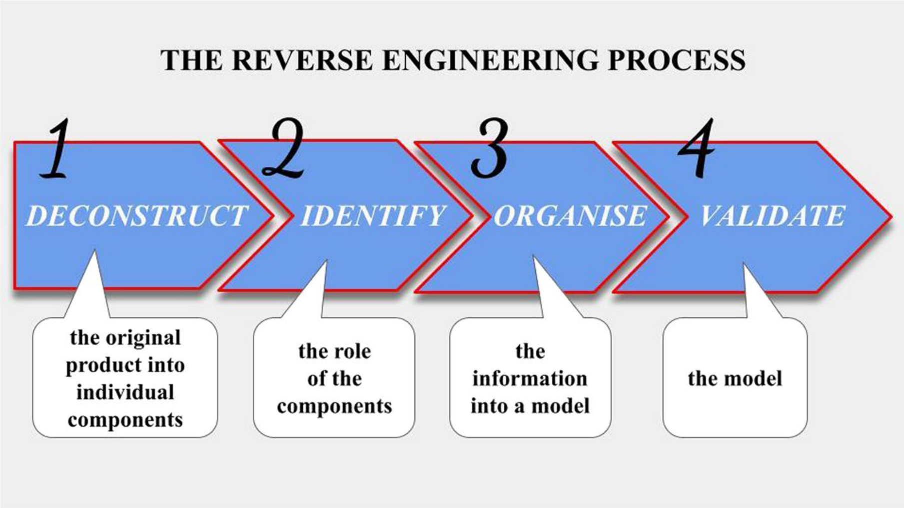


Fig. 4 Our reverse engineering process

the dissection, analysis and subsequent reassembling of a product. Technical studies in mechanical reverse engineering (Rekoff, 1985) use the information gathered in the disassembling and analysing steps to produce a set of specifications that enables the construction of a new object, instead of simply reassembling the existing one. Software reverse engineering studies (Eilam, 2011) start with similar initial steps, yielding the production of a model of the reverse-engineered program, whose reliability is then validated using it to create a new program that should emulate the output of the initial one.

Merging and adapting these approaches to our case, we extracted four steps for our reverse engineering process, systematised in Fig. 4.

The four steps involve:

1. *Deconstruct* a finished product into its individual components, i.e. until "further decomposition is not possible without destroying the integrity of a part" (Rekoff, 1985, p. 245);
2. *Identify* the role of the individual components and how they work together;
3. *Organise* the information gathered into a model;
4. *Validate* the model, using it to create a new object that is then tested in a scenario that is similar to that of the initial product.

4.2.2 Reverse engineering tools and their use

As Eilam notes "reversing is all about the tools [and] many of these tools were not specifically created as reversing tools, but can be quite useful nonetheless" (2011, p. 14). Due to the novelty of this approach in the field of meme studies, there are no specifically designed reversing tools. Thus, we employed existing tools, summarised in Fig. 5 that proved useful to perform the steps of our reverse engineering.

Koestler's bisociation (1964) and the triple-s construct (Bini & Robutti, 2019a) were used to perform the deconstruct and identify steps. Assuming Koestler's theory as our guide, we used the triple-s to (1) deconstruct mathematical memes into their individual components, connecting these components to the frames of reference linked in bisociation.

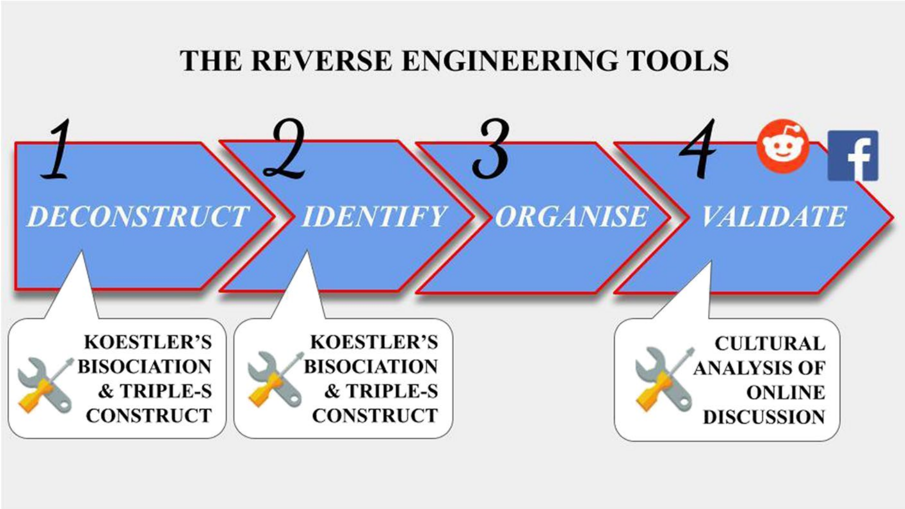


Fig. 5 The reverse engineering tools

This was done, following Koestler's methodical instruction, by "discovering the type of logic, the rules of the game, which govern each [frame of reference]" (1964, p. 63).

Then, we proceeded (2) to identify the role of the individual components and how they work together. This was done using the triple-s construct to identify the meme's partial meanings; figuring out how the individual components work together by interpreting the full meaning (i.e., the represented statement) and "find[ing] the 'link'—the focal concept, word, or situation which is bisociated with both [frames of reference]" (p. 63). We performed this analysis on all 2100 memes of our dataset: for reasons of space constraints, we detail here this step only for the memes chosen as exemplary cases for the 14 categories that emerged from the ethnographic investigation.

Subsequently, the information gathered was organised (3) to produce a heuristic model that took into account the individual components, how they are related and how they inform each other.

Finally, the model was (4) validated, using it to forward engineer two new mathematical memes that were shared on the Web within two dedicated communities, to explore if members interacted with our memes as with their Web-born counterparts studied by Bini et al. (2022). Following the methodology of Bini et al., we chose our communities based on their popularity, expressed by the number of members. We shared our memes in Mathematical Mathematics Memes, the most popular community on Facebook (329.1 k members in September 2021), and in r/mathmemes, the most popular community on Reddit (310 k members in September 2021). Monitoring the interaction around our memes, we analysed the threads of comments to verify if the community perceived them as mathematical statements endowed with an epistemic potential. We shaped our analysis following Bini et al.'s (2022) *cultural analysis* introduced at the beginning of this work, distinguishing different levels of information: *explicit* (openly expressed by the text), *implicit* (about the commenters and their relationship with the meme); *environmental culture* (about the emergence of epistemic needs and informal mentorship) and finally synthesising the *mathematical themes* emerging from the online discussion. To comply with privacy regulations and ethical principles, we pseudonymised all excerpts with the codes A = author of the mathematical meme, C1 C2 C3… = commenters. All comments are unredacted and originally in English.

5 Findings

5.1 Deconstruct the original product into its individual components

The triple-s construct allows us to perform the first deconstructing step of our reverse engineering, splitting memes into a social, structural and specialised partial meaning. Since these partial meanings cannot undergo further decomposition "without destroying the integrity of a part" (Rekoff, 1985, p. 245), we interpret them as the memes' individual components.

Then, we use this knowledge to deconstruct the two frames of reference (FR1 and FR2), which are connected through the bisociative act in a mathematical meme. This is pursued, following Koestler's suggestion, "by discovering the type of logic, the rules of the game" (1964, p. 63) governing each frame of reference. Since the social and structural meanings of a mathematical meme follow the logic of the digital culture, while the specialised meaning respects the rules of mathematics, we can conclude that FR1 is Digital Culture and FR2 is Mathematics as illustrated in Fig. 6. The specific idea linking FR1 and FR2 depends on the partial meanings of the mathematical meme taken into consideration: we will detail this level of analysis in the *identifying* step of the reverse engineering.

5.2 Identify the role of the components and how they work together

As a second step, we analyse memes in our dataset based on the deconstructing step described above. We focus on the categorisation that emerged through the process of comparison of the second part of the code (the specialised meaning), which we can now interpret as the individual components referring to FR2 (Mathematics). Following the inductive nature of grounded theory methodology, category names have been chosen from the clustering codes, and subsequently organised into four macro-themes (Mathematical Relationships, Mathematical Content, Mathematical Signs and Mathematical Knowledge). Figure 7 shows the result of the categorization process, with each category accompanied by its exemplary meme (enlarged images are visible in Table 4).

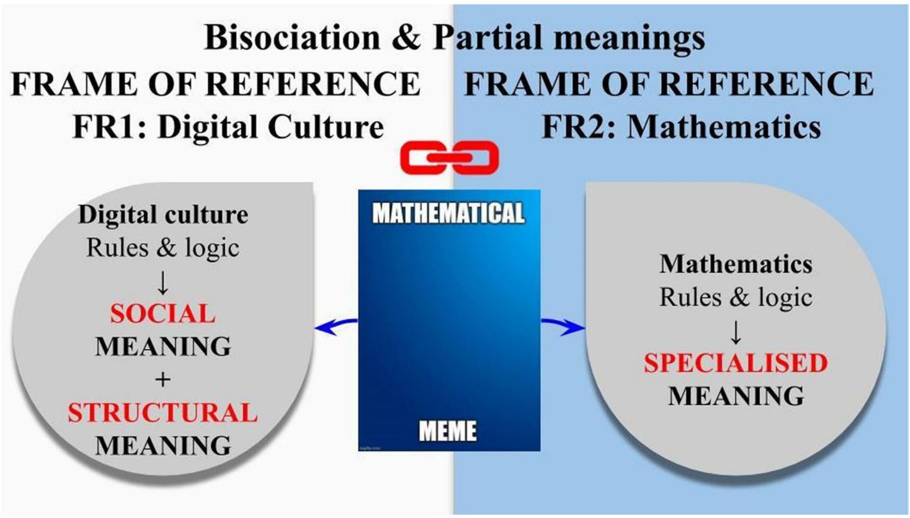


Fig. 6 Deconstructing mathematical memes' individual components

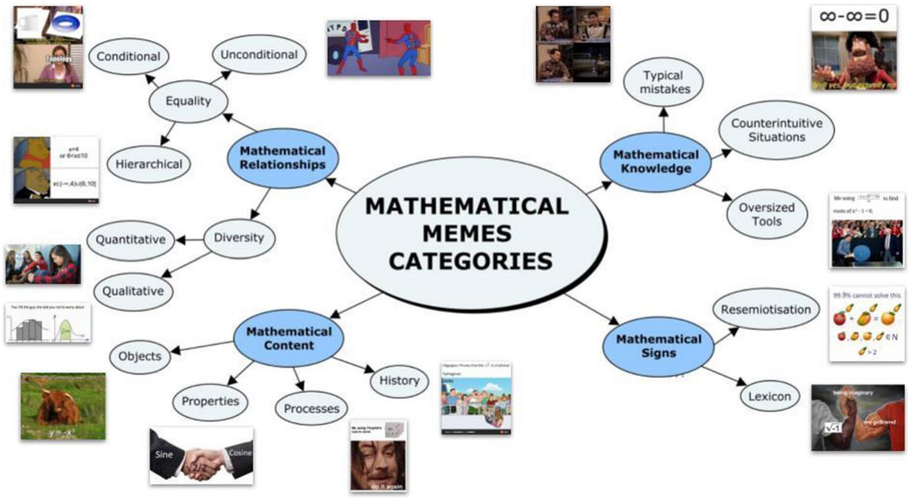


Fig. 7 Categories of mathematical memes in the meme pool with exemplary cases

Firstly, we use the triple-s construct to identify the *roles of the individual components*, i.e., the specific instances of the partial meanings. This is done drawing from KYM and MW for the social and structural meanings, and from the authors' mathematical knowledge for the specialised meaning. Then, we use this knowledge to identify *how the components work together*, finding the represented statement and, following Koestler, identifying the *linking idea*, "the *focal concept* (…) which is bisociated [and] made to vibrate simultaneously on two different wavelengths" (Koestler, 1964, p. 63, emphasis added).

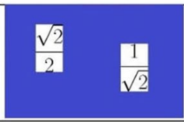
We will illustrate the reasoning leading to the identifying step for the meme exemplifying *Unconditional Equality* (Fig. 8) and we will then list the results identified similarly for the remaining exemplary memes.

We start by identifying the role of the components deconstructed in the previous step, represented by the social and structural meanings in FR1 (Digital Culture) and by the specialised meaning in FR2 (Mathematics): this leads to the identification of what Bini and Robutti (2019a, 2019b) call the *full meaning* of the meme, represented by the mathematical statement as illustrated in Table 3.

Then, we look deeper into how these individual components work together to reverse engineer the linking idea. This is done by finding the focal concept addressed by the

Fig. 8 Exemplary case for *Unconditional Equality* category

Table 3 Identifying the role of the individual components and the full meaning

FR1 (Digital Culture)		FR2 (Mathematics)	Mathematical Meme
Social Meaning: represents situations where similar things meet	Structural Meaning: object labelling (see Table 2)	Specialised Meaning: arithmetic, equivalence of rationalised fractions	Full meaning: Mathematical statement $\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$
			
Template: *Spiderman pointing*	Text positioning	Mathematical additions	Finished mathematical meme (see enlargement in Fig. 8)

meme in FR1 (Digital Culture) that is linked in bisociation to the focal concept in FR2 (Mathematics). The process is illustrated in Fig. 9: on the left is the *focal concept* in FR1, corresponding to the idea of similar things. On the right is the *focal concept* in FR2, corresponding to the idea of mathematical objects that share a mathematical meaning, in this case the fact of representing the same irrational number. The connection between these focal concepts corresponds to the linking idea, which in this case we identified as *sameness*.

The reverse engineering of this example suggests that the idea linking the two frames of reference does not depend on the specific couple of mathematical objects added by the author who created the meme. This fact indicates that the linking idea is not connected to a specific mutation of the template, but is more general and corresponds to the focal concept identified by the template that is linked to the focal concept addressed by the specialised mathematical meaning. Therefore, we hypothesise that each template carries a focal concept identifying a specific linking idea that remains unaltered in all mutations. To validate this hypothesis, we reverse

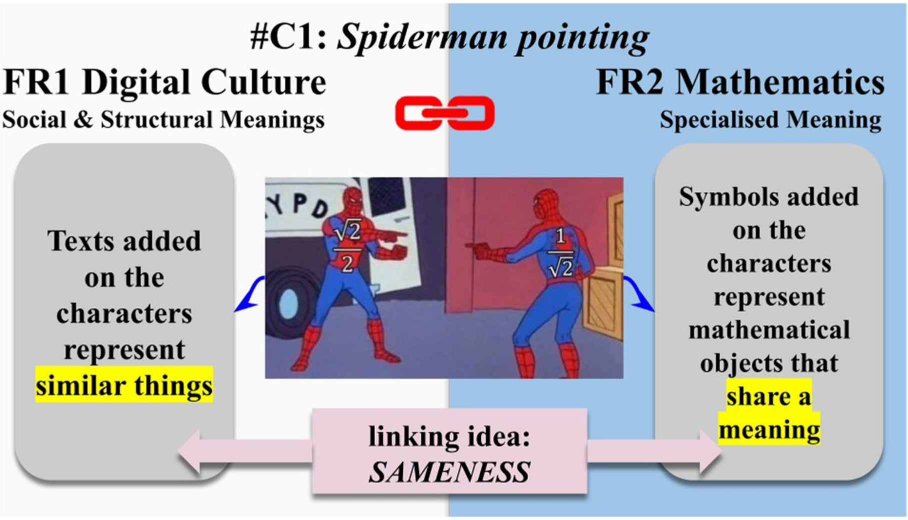
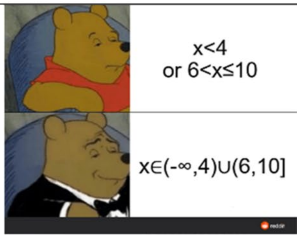


Fig. 9 Exemplary case: identifying the linking idea

Table 4 Reverse engineering analyses of mathematical memes: the *identifying* step

<table>
<tr><th colspan="2" rowspan="2">Macro-themes and Categories</th><th rowspan="2">Description of the represented statement</th><th rowspan="2">Entries</th><th colspan="3">Exemplary case</th></tr>
<tr><th>FR1: Digital culture</th><th>Mathematical Meme and coding</th><th>FR2: Mathematics</th></tr>
<tr><td rowspan="12">Mathematical Relationships</td><td rowspan="4">Unconditional Equality</td><td rowspan="4">The elements in the meme are equal</td><td rowspan="4">125</td><td>Social: represent situations where similar things meet</td><td rowspan="2">

Spiderman pointing / rationalisation (source Facebook)</td><td rowspan="2">Specialised: arithmetic, equivalence of rationalised fractions</td></tr>
<tr><td>Structural: object labelling</td></tr>
<tr><td colspan="3">Mathematical statement: $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$</td></tr>
<tr><td colspan="3">Linking idea: Sameness</td></tr>
<tr><td rowspan="4">Conditional Equality</td><td rowspan="4">Under specific circumstances, the elements in the meme are equal</td><td rowspan="4">69</td><td>Social: represent situations where an opinion is expressed</td><td rowspan="2">

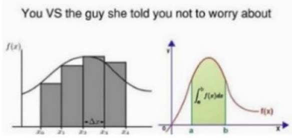
They're the same picture / topology (source Reddit)</td><td rowspan="2">Specialised: geometry, invariants in topology</td></tr>
<tr><td>Structural: exploitable</td></tr>
<tr><td colspan="3">Mathematical statement:
In topology, a mug and a torus are equivalent</td></tr>
<tr><td colspan="3">Linking idea: Reasoning</td></tr>
<tr><td rowspan="4">Hierarchical Equality</td><td rowspan="4">The elements in the meme are equal, with nuances of mathematical finesse or correctness</td><td rowspan="4">347</td><td>Social: represent equal ideas expressed in progressing finesse</td><td rowspan="2">

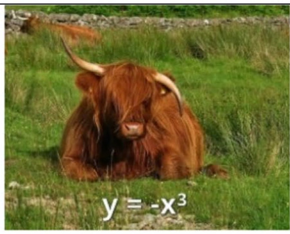
Tuxedo Pooh / interval notation (source Reddit)</td><td rowspan="2">Specialised: real numbers, equivalent notations for subsets</td></tr>
<tr><td>Structural: multi-pane</td></tr>
<tr><td colspan="3">Mathematical statement: The two expressions represent equal subsets in different notations (inequality and interval)</td></tr>
<tr><td colspan="3">Linking idea: Elegance</td></tr>
</table>

engineered memes sharing the same template in our meme pool and we verified that they are built on the same linking idea. Subsequently, we adopted this hypothesis to identify the linking ideas of other memes in our dataset. In Table 4, we present the

Table 4 (continued)

<table>
<tr><td rowspan="8"></td><td rowspan="4">Quantitative Diversity</td><td rowspan="4">The output of one of the operations in the meme is different from that of the others</td><td rowspan="4">36</td><td>Social: single out one element based on a specific diversity</td><td rowspan="2">

Bullied girl / 1+1 (source Instagram)</td><td rowspan="2">Specialised: arithmetic, different outputs of operations</td></tr>
<tr><td>Structural: object labelling</td></tr>
<tr><td colspan="3">Mathematical statement: $1^1 = 1 \times 1 = \frac{1}{1} \neq 1 + 1$</td></tr>
<tr><td colspan="3">Linking idea: Separation</td></tr>
<tr><td rowspan="4">Qualitative Diversity</td><td rowspan="4">The nature of one of the operations in the meme is different from that of the others</td><td rowspan="4">75</td><td>Social: compare two different but related elements, with the one on the right being the better one</td><td rowspan="2">

VS / Riemann sums (source Facebook)</td><td rowspan="2">Specialised: calculus, connection between Riemann sums and Riemann integral</td></tr>
<tr><td>Structural: exploitable</td></tr>
<tr><td colspan="3">Mathematical statement: The Riemann integral of a function gives the actual area underneath the graph of f, while the Riemann sums give an approximation</td></tr>
<tr><td colspan="3">Linking idea: Comparison</td></tr>
<tr><td rowspan="8">Mathematical Content</td><td rowspan="4">Objects</td><td rowspan="4">A mathematical object is directly represented or hinted by some element of the meme</td><td rowspan="4">226</td><td>Social: part of a series where the shapes of the cow's horns refer to functions' graphs</td><td rowspan="2">

Cow / Cubic function (source Instagram)</td><td rowspan="2">Specialised: pre-calculus, graphs of polynomials</td></tr>
<tr><td>Structural: object labelling</td></tr>
<tr><td colspan="3">Mathematical statement: The graph of the function $y = -x^3$ is</td></tr>
<tr><td colspan="3">Linking idea: Shape</td></tr>
<tr><td rowspan="4">Properties</td><td rowspan="4">A property of one or more mathematical objects is represented in the meme</td><td rowspan="4">290</td><td>Social: represent situations where two subjects share something</td><td rowspan="2">

Business handshake / trig functions (source Facebook)</td><td rowspan="2">Specialised: pre-calculus, common values of goniometric functions'</td></tr>
<tr><td>Structural: object labelling</td></tr>
<tr><td colspan="3">Mathematical statement: The functions $y = sinx$ and $y = cosx$ intersect for $y=\frac{\sqrt{2}}{2}$</td></tr>
<tr><td colspan="3">Linking idea: Commonality</td></tr>
</table>

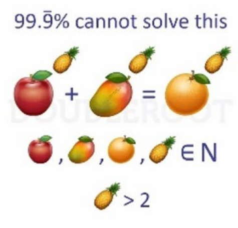
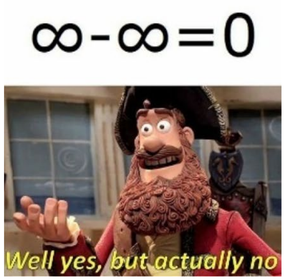
results of the *identifying* step for the 14 exemplary cases, giving some overall information about each category: a general description of the represented statement and the total number of entries in the dataset.

Table 4 (continued)

	Processes	A mathematical procedure or operation (possibly accompanied by its output) is represented	340	Social: represent situations where an action has to be repeated	 Do it again / Hopital (source Facebook)	Specialised: calculus, solving limits applying de L'Hôpital's theorem
				Structural: reaction image		
				Mathematical statement: Solving the limit $\lim_{x \to \infty} \frac{e^x}{x^3}$ requires applying de L'Hôpital's theorem three times		
				Linking idea: Repetition		
	History	The meme represents a famous anecdote in the history of mathematics	36	Social: represent situations where someone gets rid of something annoying	 Griffin / Pythagoras & Hippasus (source Reddit)	Specialised: algebra, irrational numbers
				Structural: object labelling		
				Mathematical statement: According to the legend, Pythagoras throws Hippasus off a ship to keep irrationality secret.		
				Linking idea: Rejection		
Mathematical Signs	Lexicon	The statement revolves around the polysemy of a mathematical term	141	Social: represent situations where two subjects share something	 Epic handshake / imaginary (source Instagram)	Specialised: pre-calculus, imaginary unit in complex numbers
				Structural: object labelling		
				Mathematical statement: $\sqrt{-1} = i$, the imaginary unit in the complex field		
				Linking idea: Commonality		

The results of the identifying step in Table 4 show that our meme pool covers a broad variety of mathematical ideas, of which a vast majority (98.5%) are correct and correctly expressed. Some preferred categories stand out, as *Hierarchical Equality*

Table 4 (continued)

	Resemiotisation	The statement of a famous mathematical theorem is represented, with mathematical elements replaced by other objects	40	Social: part of a series that parodies the "fruit salad" approach to algebra teaching (MacGregor, 1986) replacing elements of higher mathematical statements with fruits and flowers Structural: exploitable	 Fruits and flowers / Fermat (source Reddit)	Specialised: number theory, Fermat's last theorem
				Mathematical statement: Fermat's last theorem		
				Linking idea: Replacement		
Mathematical Knowledge	Oversized Tools	A situation /problem can be dealt with a simpler tool/ procedure than the one represented in the meme	58	Social: represent situations where a more powerful tool than needed is used Structural: reaction image	 Giant ping pong paddle / quadratic formula (source Reddit)	Specialised: algebra, quadratic equations solving techniques
				Mathematical statement: The equation $x^2 - 1 = 0$ can be solved by factorization		
				Linking idea: Exaggeration		
	Counterintuitive Situations	The result of the procedure/ operation in the meme is different than expected	105	Social: represent unexpected situations Structural: reaction image	 Actually no / infinity - infinity (source Facebook)	Specialised: calculus, indeterminate forms
				Mathematical statement: When computing limits, $[\infty - \infty]$ is an indeterminate form		
				Linking idea: Puzzlement		

(347 entries), *Processes* (340), *Properties* (290), and some *niche* categories such as *Resemiotisation* (40) and *History* (36). A higher number of entries can depend either on the popularity of a template or of a mathematical topic; for instance, *Hierarchical Equality* is a very popular category thanks to the diffusion of the *Tuxedo Pooh*

Table 4 (continued)

	Typical Mistakes	The result of the procedure/ operation in the meme is wrong (possibly accompanied by its rectification)	226	Social: represent situations where something vitally important is forgotten Structural: object labelling	 I think I forgot something / binomial square (source Facebook)	Specialised: algebra, the square of the binomial
				Mathematical statement: $(x+y)^2 = x^2 + y^2 + 2xy$		
				Linking idea: Carelessness		

template that counts 65 mutations in our meme pool, while *Processes* owes its popularity to that of de L'Hôpital's theorem, which appears 53 times as specialised meaning in our meme pool. These observations show that mathematical memes in our meme pool balance the two frames of reference they are bisociated into, and suggest that the creative process from which a mathematical meme emerges can initiate equally from the frame of digital culture (a popular template) or from the frame of mathematics (a popular mathematics subject), eventually reaching for the other frame and linking the two in the final object.

We notice also that each template has specific parts that combine with explicit mathematical elements added by the creator to represent the mathematical statement. For example, in *Spiderman pointing* (*Unconditional Equality*), the template represents the sign = connecting the two expressions $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$, as corroborated by the comments in Fig. 10, from another mathematical meme using the same template. In *Bullied girl* (*Quantitative Diversity*), the girls in the template background represent the sign = connecting the expressions 1^1, 1×1, $\frac{1}{1}$ while the girl in the foreground represents the sign ≠ linked to the $1+1$.

These shared interpretations of templates show that the culture of the mathematical memesphere forges the hybrid language of mathematical memes, implementing the connections between memetic and mathematical signs. In addition to that, the memetic component emphasises parts of the statement with an emotional charge carried by the social meaning of the template, in the entertaining way that characterises this means of communication (Shifman, 2014).

Fig. 10 Comments evidencing the contribution of the template (source Reddit)

5.3 Organise the information into a heuristic model

We now organise the information gathered in the deconstructing and identifying steps of reverse engineering to produce an action model for the creation of a mathematical meme. This model summarises the design strategies gathered in the previous steps and can be used as a heuristic.

Our previous analyses suggest that the creation process revolves around a linking idea that connects a template with a mathematical statement. The linking idea connects the focal concept of the template's social meaning to the focal concept of the mathematical statement, which assumes the role of the specialised meaning of the meme. The connection is implemented through textual or pictorial additions complying with the structural meaning of the template.

We can assume that the creative process can be initiated by a specific template in FR1 (Digital Culture), or by a particular statement in FR2 (Mathematics): this suggests that we can model two parallel creative strategies starting from either of the two frames of reference, as schematised by the two columns in Fig. 11. The process of creating a mathematical meme implies accessing the model through one column (Digital Culture *or* Mathematics) and then proceeding from top to bottom in the selected column. This process may require applying fundamental heuristics as forward/backward reasoning (Polya, 1945), which in this case means moving back and forth in the column until the linking idea between the focal concepts emerges, as a creative act connecting the two cultures while respecting both sets of rules.

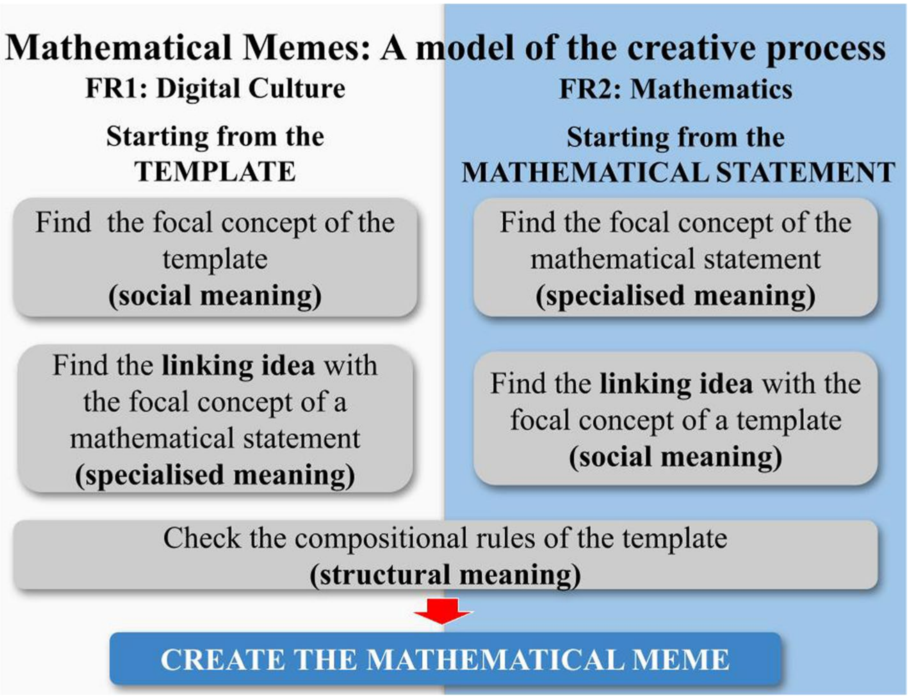


Fig. 11 A model of the creative process of a mathematical meme

5.4 Validate the model

Borrowing Polya's words, our model "is only heuristically assumed, not logically proven […]. It is a guess, not a proof" (1945, p. 158). As a final step of our reverse engineering process, we want to validate it. To do this, we use it to create new mathematical memes that are subsequently shared online in a scenario that is similar to that of the initial product (i.e., a mathematical memes' online community) to investigate how members of the community interact with them. We will present two memes, both generated with the Imgflip website, corresponding to the two possible starting points in the model in Fig. 11, and the related cultural analyses (Bini et al., 2022) of the threads of comments.

5.4.1 Experiment 1: starting from the template

The creative process for the meme in Experiment 1 originates in FR1 (Digital culture). The starting point is the *Stereotype me* template in Fig. 12, featuring young women holding signs with various stereotypes: "I'm Punk So I Must Rebel", "I'm Asian So I Must

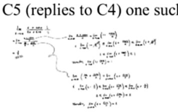
Fig. 12 Experiment 1—the original template

Table 5 Experiment 1—starting from the template

Find the focal concept of the template **(social meaning)**	The focal concept of the template is symbolised by the signs held by the three young women, describing situations where an initial condition leads automatically to a pedestrian conclusion, without further inspection of the specific case.
Find the **linking idea** with the focal concept of a mathematical statement **(specialised meaning)**	The focal concept of the template resonates with the diffused misuse of de L'Hôpital's theorem for any limit presenting an $\frac{\infty}{\infty}$ indeterminate form, without prior verification of the validity of other hypotheses. This corresponds to the mathematical statement: "In evaluating a limit, the presence of an indeterminate form of the $\frac{\infty}{\infty}$ kind is a necessary but not sufficient condition to apply de L'Hôpital's theorem". The connection between these focal concepts is the linking idea, identified in *stereotype*.
Check the compositional rules of the template **(structural meaning)**	The template is a multi-pane exploitable (see Table 2), with text to be added in the lower right panel. The spelling of *de L'Hôpital* has been simplified to *Hopital*, as commonly referred to in the memesphere.

CREATE THE MATHEMATICAL MEME

Table 6 Experiment 1: cultural analysis

Comments	Explicit information	Implicit information	Environmental culture	Emerging mathematical themes
1-C1 so what are the requirements for l hospitals	C1 asks for elucidations about the complete hypotheses needed to apply de L’Hôpital’s theorem	C1 shows knowledge, recognizing that the problem lays in some missing “requirements” for de L’Hôpital’s theorem	C1 shows an epistemic need and addresses the community as a knowledge holder	Necessary conditions for de L’Hôpital’s theorem
2-C2 (replies to C1) de l'hospital's just workes the other way around. IF the limit of f'(x)/g'(x) exists, THEN it is equal to the limit of f(x)/g(x). But that limit might not exist while the f(x)/g(x) limit still does. So in practice you just need to try using it and then see if the limit behaves itself. Here it doesn't, even though the f(x)/g(x) limit exists and it's clearly 1/2, but the f'(x)/g'(x) limit does not exist, neither does any subsequent one.	C2 provides the requested elucidations, explaining how de L’Hôpital’s theorem works	C2 gives a first reply, stressing the logic behind the rule (“IF” and “THEN” capitalised) C2 aims at fulfilling C1’s epistemic need by explaining what should exist but does not exist in this case	C2 is self-positioning as an expert in the community, offering informal mentorship	
3-C1 (replies to C2) i see that the limit is ½ but idk how to write the proof🤔	C1 asks for further support to prove the result	C1 moves the attention to the result of the limit. C1 grasps the result, but needs help to produce the argumentation that supports it	C1 accepts C2 expertise and addresses C2 as a knowledge holder	Proof of the limit using the squeeze theorem
4-C2 (replies to C1) if you are unconvinced by any particular method, you can always try doing it directly using the definition. With epsilon and the vicinity etc. But an easier way is to notice that (x-1)/(2x+1) < f(x) < (x+1)/(2x-1) For all big x, and the limits on both sides when x goes to infinity are equal to 1/2 so the limit of the thing in the middle also is.	C2 details the initial steps of the procedure that leads the result anticipated by C1	C2 shows mathematical expertise and desire to fulfil C1’s epistemic needs by giving two different explanations	C2 accepts the expert role and practices informal mentorship	
5-C3 (replies to C2) there are actually a couple of other assumptions, like that g'(x) must not be 0 within a neighbourhood of the value x is approaching; see Wikipedia for counterexamples	C3 follows up C2’s reply to C1’s original question about the requirements	C3 goes back to C1’s initial epistemic need, remarking that C2’s reply was not complete and focuses the answer on the missing necessary conditions	C3 is comparing expertise with C2, giving explanations and practising informal mentorship towards C1	Necessary conditions for de L’Hôpital’s theorem
[…]				
6-C4 How would you prove that the result is 1/2 then?	C4 asks for more detailed support to prove the result	Since C2 has only hinted at the first step of the proof, C4 needs more explanations to grasp the whole development	C4 shows an epistemic need and addresses the community as a knowledge holder	Proof of the limit using the squeeze theorem
7- C5 (replies to C4) one such  way (Enlargement in Fig. 13)	C5 posts an image of the complete written proof of the limit	C5 shows expertise and responds to C4’ epistemic need, using an image, that allows more detailed explanations than simply typing the answer in the comment box	The epistemic need is fulfilled, meaning making is complete	

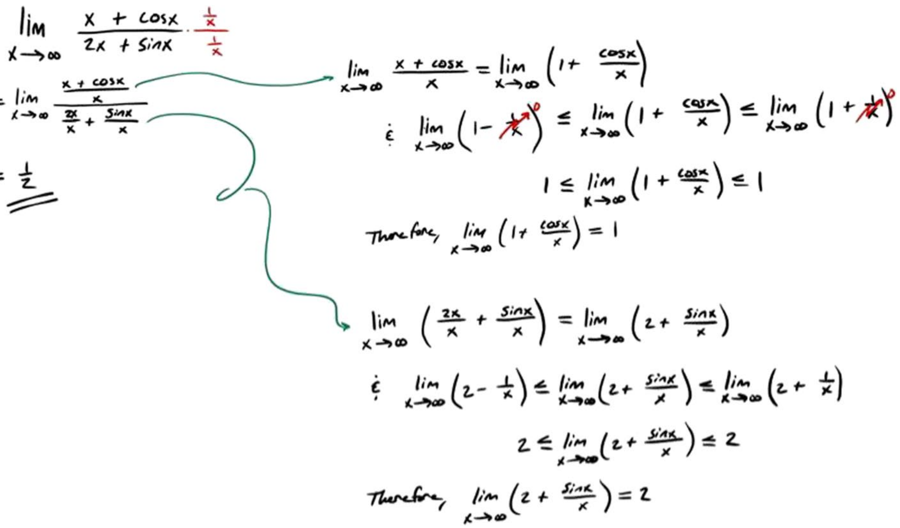


Fig. 13 Experiment 1—C5’s comment

Like Maths”, “I Dye My Hair Crazy Colours So I Must Be Looking For Attention”. The template is used in the memesphere to mock conventional opinions (source KYM).

This template inspired the creation of a new mathematical meme, through the process in Table 5.

Once created, the meme has been shared online on Facebook in the community Mathematical Mathematics Meme, to observe how users interact with it. The meme has been well received and gained 713 likes, 62 shares and 91 comments. We present here in Table 6 the cultural analysis (Bini et al., 2022) of an excerpt of the thread of comments, aimed at validating our model through the observation of the nature of the interaction.

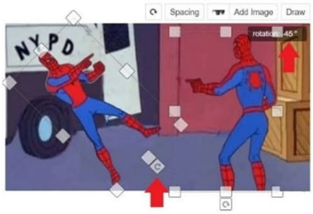
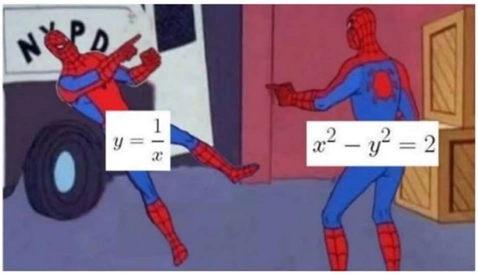
The cultural analysis of this excerpt validates our model, showing that the meme created following the model is capable of activating the epistemic culture in the community as observed in Bini et al. (2022). In particular, we note that comments focus on “the mathematical idea encoded in the meme [which] is reconstructed producing and using information and knowledge by commenters” (p. 27). We also note that commenters enact the roles detected by Bini et al., showing epistemic needs (#1, #3, #6) and making efforts to offer informal mentorship (#2, #4, #5¸ #7) as clearly as possible, as we can see in the image posted by C5 in Fig. 13. There the proof of the limit is detailed with clarifying signs (arrows) and colours. We can conclude that the reaction of the community shows that the mathematical meme created using our model retains the epistemic potentialities to initiate a process of mathematical argumentation.

5.4.2 Experiment 2: starting from the mathematical statement

The meme in the second experiment originates in FR2 (Mathematics). The starting point is the statement “In a Cartesian plane, a 45° counterclockwise rotation around the origin transforms the hyperbola $x^2 - y^2 = 2$ into the hyperbola $y = \frac{1}{x}$”, which inspired the creation of a mathematical meme following the process in Table 7.

The meme has been shared online on Reddit in the community r/mathmemes. It received 187 upvotes and 13 comments. In this experiment, the first author intervened as

Table 7 Experiment 2—starting from the statement

Find the focal concept of the mathematical statement **(specialised meaning)**	The focal concept of the statement is congruence, in this case with respect to a rigid motion (specifically, a rotation).
Find the **linking idea** with the focal concept of a template **(social meaning)**	The focal concept of the template resonates with the focal concept of the *Spiderman pointing* template.  The linking idea is *sameness*.
Check the compositional rules of the template **(structural meaning)**	The template is an object labelling meme (see Table 2): the two equations $y=\frac{1}{x}$ and $x^2-y^2=2$ must be superimposed onto the two Spidermen. To represent the rotation, the $y=\frac{1}{x}$ Spiderman has to be tilted 45° counterclockwise. This designed mutation of the original template can be obtained using the tools provided by the meme generator website. 
CREATE THE MATHEMATICAL MEME  	

a commenter (A), to probe the community reaction to the linking idea "sameness". As for the first meme, we present here in Table 8 the cultural analysis (Bini et al., 2022) of an excerpt of the thread of comments.

Table 8 Experiment 2: cultural analysis

Comments	Explicit information	Implicit information	Environmental culture	Emerging mathematical themes
1-C1 Both are called hyperbolas!	C1 explicates the name of the conics represented by the two equations	C1 wants to show some mathematical knowledge	C1 is self-positioning as an expert	Congruence with respect to rigid motion
2-A and they are the same hyperbola!	A intervenes highlighting that the two hyperbolas are related	A uses the term "same" to provoke a reaction	A is self-positioning at the same level of expertise as C1	
3-C1 I disagree. This may be parochial but to me rotating something makes it no longer 'the same'. Guess we have to define what 'the same' means.	C1 disagrees and clarifies that rotation changes the object	C1 react to A's use of the term "same", showing an epistemic need about the definition of "sameness"	C1 discuss with the community in a peer-to-peer way	
4-C2 You could say they are congruent, in the sense that one could fit the first hyperbola perfectly on top of the second by using only translations and rotations. But yeah, they aren't truly equal, calling things "the same" is confusing sometimes.	C2 clarifies in which sense the two hyperbolas are related	C2 moves the language to a more mathematical level, using terms as "congruent" and translations and rotations"	The peer-to-peer exchange makes C2 feel entitled to step in with clarifications, practising informal mentorship and mediating between C1 and A	
5-A Correct, isometric is much better than "same"	Acknowledgement	A accepts C2's mediation	Meaning making is complete	
[…]				
6-C3 Is there any proof to this or reason why it happens?	C3 asks explicitly for proofs of the congruence	C3 needs to know why "it happens", showing an epistemic need	C3 seeks help from the community for the meaning-making, addressing the community as a knowledge holder	Analytic equations of the rotation in the Cartesian plane
7-C4 The second is the first rotated by 45 degrees. Rotation is done by sending points (x,1/x) to (cos(45)x-sin(45)/x, sin(45)x+cos(45)/x). Since cos(45)=sin(45)=sqrt(1/2), plugging these coordinates into the second equation simplifies to 2. This shows that the first is congruent to a subset of the second. The converse can be shown in a similar way.	C4 gives a detailed description of the steps of the requested analytic proof	C4 shows experience, both in expressing the proof and in managing to represent mathematical functions using only keyboard signs (sqrt). The direction of the rotation is imprecise (45° instead of -45°), but the calculations are correct.	C4 explains, hence, practices informal mentorship	
8-C3 Thanks!	Acknowledgement	C3 does not ask for further questions, indicating that the epistemic need is fulfilled	Meaning making is complete	

The second experiment also validates our model, proving that the meme created following the model is perceived by the community in the same way as spontaneous creations. In particular, it retains memes' epistemic potentialities to activate the community with requests for clarification (#3, #6) and offers of informal mentorship (#4, #7). The discussion about the use of the term "same" (#2, #3, #4, #5) shows also that the linking idea is properly *bisociated* between the two frames of reference, and is perceived as congruence in the frame of reference of Mathematics.

6 Summary of results, discussion and conclusion

This investigation started with the purpose of shedding light on the creation of mathematical memes to expand the body of knowledge on this subject initiated by Bini et al. (2022). Our research question focused on *how* mathematical memes can be created. In Fig. 14, we summarise our reverse engineering research process: starting with Web-found mathematical memes collected through extensive ethnographic research, we performed the deconstructing and identifying steps, gathering information which we organised in the heuristic action model presented in Fig. 11 and on the right in Fig. 14. Then, we validated our model, showing how it can be used to create original memes that retain the mathematical and epistemic characteristics of Web-found memes.

The heuristic model is an original outcome of this research that provides possibilities to create mathematical memes with some degree of freedom. As mathematical memes are creative acts, they cannot be created by an algorithm and therefore such a heuristic model is the closest we can get for creating them. Nevertheless, the model can inform researchers and educators about how to create a mathematical meme, and anticipate what can be expected when students create one. This constitutes a first relevant step towards enabling teachers to create and implement these objects in the mathematics classroom, nurturing an environment that *fosters creativity in all students* as advocated by OECD 2030 and PISA 2021 (see Sec. 2). The model shows also that, notwithstanding memes' *hybrid language*, the creative process can be initiated within whatever frame of reference the creator is more familiar with, giving educators a key to enter the mathematical memesphere through the mathematical statement they might feel more comfortable with.

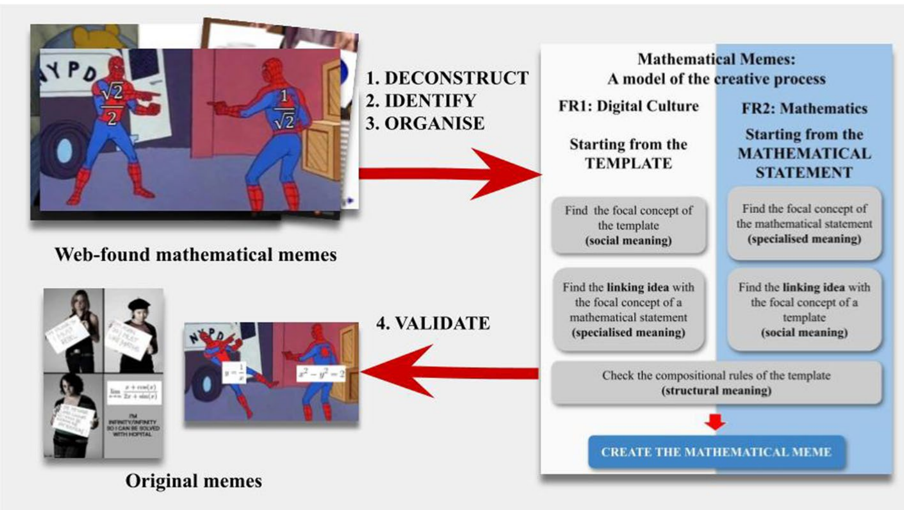


Fig. 14 A summary of the research process

This result testifies the appropriateness of our two choices, the theoretical choice to use Koestler's bisociation and the methodological choice to reverse engineer Web-found memes collected through an ethnographic investigation.

Koestler's bisociation theory allowed us to interpret mathematical memes as results of creative acts, implementing innovative connections that go beyond the traditional concept of creativity in mathematics by linking mathematics and digital culture. As products of bisociation, mathematical memes are created "rigorously abiding by the rules of both fields" (Koestler, 1964, p. 33), which are the rules shared in the memesphere that regulate the social and structural meaning (Bini et al., 2022) and the rules ensuring that a meme's specialised meaning is mathematically correct. This fact implies that these rules must be understood in their essence and not only reproduced automatically, as summarised by Koestler's description of the creative act as "the defeat of habit by originality" (p. 96). This means that interacting with (i.e., creating *or* understanding) a mathematical meme fosters in-depth conceptual knowledge of the topic, a fact that is widely recognized as important for success in mathematics (Canobi, 2009; Rittle-Johnson & Schneider, 2015).

The reverse engineering has proved itself effective to work our way back from the object to the underlying design knowledge. Its grounding on ethnographic data allowed us to produce knowledge about the culture of mathematical memes that informed the reverse engineering itself. This innovative methodology can now be more broadly used to reconstruct educational material where the creation process is not accessible. Pairing reverse engineering with our original use of ethnography, we are now able to reconstruct a new phenomenon in the digital culture where mathematics plays a prominent role.

Going back to the research question of how to create a meme, we see that this "how" has a two-fold meaning: it led on to the result, the heuristic model that tells us *how to create* memes, and it led on to the method of reverse engineering that tells us *how to gain* this result.

The generality of the research question acknowledges that there may be various ways of creating mathematical memes. We have answered it by a case of reverse engineering, in which we theorized the creative act of meme construction based on a conceptual decomposition of memes using the triple-s-construct. In this way, we have developed and validated one possible heuristic that may guide teachers or students in the process of creating mathematical memes. Future research may disclose other ways of meme creation. The generality of the research question had a certain methodical advantage. It allowed us to search in and learn from a broader design area to identify how our problem is solved outside the field of mathematics education. This way, we were able to translate and elaborate the method of reverse engineering.

Although the study has reached its purpose of gaining an understanding of mathematical memes' creative process, we must acknowledge its limitations. First, we examined in an in-depth manner only a small number of exemplary cases within a more comprehensive ethnography. However, these cases are carefully and systematically chosen as representatives of the inspected categories to gain theoretical knowledge in a completely new research area in mathematics education. Second, we banked on data gathered during a time-consuming ethnographic research, whose interpretation and analysis rely mainly on the knowledge gained by the researcher during the fieldwork. Lastly, we validated the reverse-engineered heuristic model ourselves, and we validated it inside the natural habitat of memes, showing that in this environment our memes activate the expected epistemic culture. The question of whether teachers will be able to create mathematical memes using this model, or whether the same epistemic culture will emerge in the class remains open for further research, as well as the investigation of the effectiveness of mathematical memes in fostering creativity in all the students.

Mathematical memes are mathematical statements, yet at the same time they are something more. They are the outcome of a contamination, intended in a positive sense as "the crossing, the exchange, the *métissage*" between cultures as described by Cambi (2011, p. 157). This positive contamination between mathematics and memes empowers both cultural realms: it upgrades the use of memes beyond their original subculture, enriching them with an epistemic power that nurtures mathematical discussions, and it expands the range of mathematical signs traditionally pertaining to the domain of school.

Author contribution Not applicable.

Funding Open access funding provided by Università degli Studi di Torino within the CRUI-CARE Agreement.

Declarations

Conflict of interest The authors declare no competing interests.

Availability of data and material Not applicable.

Code availability Not applicable.

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