Two-loop QCD Correction to Massive Spin-2 Resonance $\rightarrow q\ \bar{q}\ g$

Taushif Ahmed,[a] **Goutam Das,**[b] **Prakash Mathews,**[b] **Narayan Rana**[a] **and V. Ravindran**[a]

[a] *The Institute of Mathematical Sciences,*
IV Cross Road, CIT Campus, Chennai 600 113, India
[b] *Theory Division, Saha Institute of Nuclear Physics,*
1/AF, Bidhannagar, Kolkata 700 064, India

E-mail: taushif@imsc.res.in, goutam.das@saha.ac.in,
prakash.mathews@saha.ac.in, rana@imsc.res.in, ravindra@imsc.res.in

Abstract: Two-loop QCD correction to massive spin-2 Graviton decaying to $q + \bar{q} + g$ is presented considering a generic universal spin-2 coupling to the SM through the conserved energy-momentum tensor. Such a massive spin-2 particle can arise in extra-dimensional models. The ultraviolet and infrared structure of the QCD amplitudes are studied. In dimensional regularisation, the infrared pole structure is in agreement with Catani's proposal, confirming the universal factorization property of QCD amplitudes, even with the spin-2 tensorial coupling. This computation now completes the full two-loop QCD corrections for the production of a spin-2 in association with a jet.

Keywords: Perturbative QCD, Spin-2

Contents

1 Introduction

After the discovery of the Higgs boson one of the main motivation of Large Hadron Collider (LHC) is to search for new physics beyond the Standard Model (BSM). It is well understood that the Standard Model (SM) is not complete on many accounts. One of the main motivation to look for BSM physics is the large scale hierarchy between the Electroweak scale and the Planck scale, which is commonly known as the hierarchy problem. There are a plethora of models to address this issue. The models with spin-2 particles in brane world scenarios are particularly interesting. Recently there has been a renewed interest in search of spin-2 particle in the context of the Higgs boson discovery. Of these models, theories with large extra dimension like Arkani-Hamed-Dimopoulos-Dvali (ADD) [1–3] and warped extra dimension like Randall-Sundrum (RS) [4] have gained a lot of attention as their signature can be tested at LHC energies. In fact, dedicated groups at the LHC are studying the search potentials of such exotic particles. In the simplest of these models gravity is allowed to explore the full space-time dimensions. The effect of gravity on the 3-brane where the SM resides is realized by a tower of spin-2 Kaluza-Klein (KK) excitations. When coupled to the SM, these KK modes may provide important signatures about these models to be ultimately tested by the LHC. The channels like $\ell^+\ell^-, \gamma\gamma, ZZ, W^+W^-$ and di-jet are the main channels to look for any deviations from the SM as a result of the virtual spin-2

exchange. At hadron colliders, QCD corrections to these processes are necessary to make precise quantitative predictions and to have control over the theoretical uncertainties. To next-to-leading-order (NLO) accuracy, these processes have been studied in the context of ADD [5–10] and RS [11–15]. The effect of parton shower (PS) has also been considered [16–18]. Moreover the signature can also be found in tri-gauge boson productions [19–23]. Recently all the important processes are fully automatized [24] in MadGraph5aMC framework [25] with NLO+PS accuracy for a generic spin-2 particle. Nevertheless the NLO corrections are often very large, resulting in large K-factors and large scale uncertainties. To have precise predictions the next stage is to compute full next-to-next-to-leading-order (NNLO) correction. Higher order correction involves pure virtual contributions, pure real corrections and appropriate interference terms at each order. For the spin-2 case, the quark and gluon form-factors have been calculated at two-loop level [26] and at three-loop level [27] in peturbative QCD (pQCD). An attempt to consider the soft and collinear contribution at NNLO accuracy was made [28] in the context of spin-2. This reduces the unphysical scale uncertainties, thereby improving the predictions. Recently the full NNLO correction has been studied [29] for Drell-Yan process with spin-2 mediators in the context of ADD.

The processes with missing energy associated with a SM particle can also give significant information about new physics. Particularly, jet + missing energy is one important process studied at the LHC to look for BSM signature like dark matter in simplified models (see for example [30] and references therein). A massive spin-2 particle which goes undetected could be an important dark matter candidate. The NLO correction for this process in large extra dimension has been considered in [31] and it is shown that the QCD corrections for this process could be as large as 50% and suffer from large scale uncertainties. Therefore a full NNLO correction is important for such process to have accurate prediction of cross-section and distributions and also to have the scale uncertainties under control. An attempt has been made towards this direction in [32], where the virtual NNLO QCD correction has been studied for massive spin-2 $\to g+g+g$. Although the gg initiated subprocess is the dominant contribution at the LHC, as the perturbative order increases the other sub-processes like $q\bar{q}, q(\bar{q})g$ begin to contribute significantly. In fact for full NNLO correction one needs to have the virtual contributions from all the sub-processes, the real-virtual piece and the pure real corrections. In this article we have considered the two loop virtual QCD correction to the process massive spin-2 $\to \quad q+\bar{q}+g$. After appropriate analytical continuation of the kinematical variables to the respective regions [33], the result in this paper can be used for other scattering sub-processes *viz.* $q+\bar{q} \to G+g$ and $q(\bar{q})+g \to G+q(\bar{q})$, where G denotes the spin-2 field. This computation along with the $G \to g+g+g$ [32] to two-loop now completes the full two-loop QCD corrections to the production of spin-2+jet at a hadron collider.

Our main motivation for this work is two-fold; first to probe the structure of Quantum Field Theory in the presence of a spin-2 field, to check the universality of infrared (IR) pole structure in QCD [34]. The correct IR pole structure has been already realized in the case of spin-2$\to g+g+g$ [32]. Here we demonstrate the same for spin-2$\to q+\bar{q}+g$. Secondly, we present one of the important ingredients for full two-loop QCD correction for real graviton production associated with a jet. We consider a minimal universal coupling of

spin-2 to the SM through energy-momentum tensor. Due to the conserved spin-2 current, no additional ultraviolet (UV) renormalization is needed other than the QCD one. We encounter more than 800 Feynman diagrams with higher tensorial integrals. The rank-2 nature of spin-2 increases the complexity of our calculation. Using Integration-By-Parts (IBP) identities [35, 36] and Lorentz Invariance (LI) identities [37], we are able to reduce all the scalar integrals to a fewer set of Master Integrals (MI). These MIs are available in [37–42]. Finally we observe the universality of infrared factorization of QCD amplitudes as predicted by Catani [34] (see also [43]).

The paper is organized as follows: in section 2, we discuss the theoretical framework for our work, particularly the spin-2 action of interaction with the SM, the notation used, the procedure of UV renormalization and infrared factorization. In section 3, we discuss our approach to the computation. Section 4 and appendix are devoted to present the results. We conclude in section 5.

2 Theoretical Framework

We consider a generic spin-2 particle minimally coupled to the SM fields through the conserved SM energy-momentum tensor. The effective action which describes a spin-2 particle ($G^{\mu\nu}(x)$) interacting with colored particles is given by [1–4, 44, 45]

$$\mathcal{S}_{int} = -\frac{\kappa}{2} \int d^4x \; T^{QCD}_{\mu\nu}(x) \; G^{\mu\nu}(x), \tag{2.1}$$

where κ is a dimensionful universal coupling which determines the strength of graviton coupling to the SM. $T^{QCD}_{\mu\nu}$ is given by

$$\begin{aligned} T^{QCD}_{\mu\nu} =& - g_{\mu\nu}\mathcal{L}_{QCD} - F^a_{\mu\rho}F^{a\rho}_{\nu} - \frac{1}{\xi}g_{\mu\nu}\partial^\rho(A^a_\rho\partial^\sigma A^a_\sigma) + \frac{1}{\xi}(A^a_\nu\partial_\mu(\partial^\sigma A^a_\sigma) + A^a_\mu\partial_\nu(\partial^\sigma A^a_\sigma)) \\ &+ \frac{i}{4}\Big[\bar\psi\gamma_\mu(\partial_\nu - ig_sT^aA^a_\nu)\psi - \bar\psi(\partial_\nu + ig_sT^aA^a_\nu)\gamma_\mu\psi + (\mu \leftrightarrow \nu)\Big] \\ &+ \Big[\partial_\mu\bar\omega^a(\partial_\nu\omega^a - g_sf^{abc}A^c_\nu\omega^b) + (\mu \leftrightarrow \nu)\Big], \end{aligned} \tag{2.2}$$

where g_s is the strong coupling constant and ξ is gauge fixing parameter. Here ω^a is the ghost field introduced in order to cancel the unphysical degrees of freedom associated with the gluon fileds (A^a_μ). T^a and f^{abc} represent the generator and structure constants of $SU(N)$ gauge group, respectively. Throughout the computation, we consider $SU(N)$ as our gauge group and the QCD corresponds to $N = 3$.

The decay process considered is

$$G(Q) \to q(p_1) + \; \bar q(p_2) + \; g(p_3). \tag{2.3}$$

The corresponding Mandelstam variables for this process are defined as

$$s \equiv (p_1 + p_2)^2, \qquad t \equiv (p_2 + p_3)^2, \qquad u \equiv (p_3 + p_1)^2. \tag{2.4}$$

They satisfy the following relation

$$s + t + u = M_G^2 \equiv Q^2. \tag{2.5}$$

Here M_G is the mass of the spin-2 field. The following dimensionless invariants which appear in the argument of harmonic polylogarithms (HPL) [46] and two-dimensional HPLs [41] are also defined:

$$x \equiv s/Q^2, \qquad y \equiv u/Q^2, \qquad z \equiv t/Q^2. \tag{2.6}$$

Accordingly, Eq. (2.5) becomes

$$x + y + z = 1. \tag{2.7}$$

2.1 Ultraviolet Renormalization

Beyond leading order in perturbation theory, the on-shell QCD amplitudes develop both ultraviolet and infrared divergences. The spin-2 coupling to the SM particles κ is free from such ultraviolet renormalization, which is due to the fact that spin-2 couples universally to the SM through conserved current. So the only UV renormalization required is for the strong coupling constant $\hat{g}_s$. Before performing the renormalization, we need to regularize the theory in order to identify the true nature of the divergences. We regularize the theory under dimensional regularization where the space-time dimension is chosen to be $d = 4+\epsilon$. Expanding the scattering amplitude in powers of $\hat{a}_s = \hat{g}_s^2/16\pi^2$, the matrix element (ME) is given by:

$$|\mathcal{M}\rangle = \left(\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon\right)^{\frac{1}{2}} \left(|\hat{\mathcal{M}}^{(0)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon\right) |\hat{\mathcal{M}}^{(1)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon\right)^2 |\hat{\mathcal{M}}^{(2)}\rangle + \mathcal{O}(\hat{a}_s^3) \right) \tag{2.8}$$

where $\hat{g}_s$ is the unrenormalized strong coupling constant, $S_\epsilon = \exp[\ \frac{\epsilon}{2}(\gamma_E - \ln\ 4\pi)]$ and $\gamma_E = 0.5772\ \ldots$ is the Euler constant. $|\hat{\mathcal{M}}^{(i)}\rangle$ is the unrenormalized color-space vector representing the i^{th} loop-amplitude. μ_0 is a mass scale introduced to make the strong coupling constant ($\hat{g}_s$) dimensionless in d-dimension. We work within the $\overline{\text{MS}}$ scheme for performing the UV renormalization, in which the renormalized coupling constant $a_s \equiv a_s(\mu_R^2)$ is defined at the renormalization scale μ_R and is related to the unrenormalized $\hat{a}_s$ by

$$\begin{aligned} \frac{\hat{a}_s}{\mu_0^\epsilon}\ S_\epsilon &= \frac{a_s}{\mu_R^\epsilon}\ Z(\mu_R^2), \\ &= \frac{a_s}{\mu_R^\epsilon}\left[1 + a_s \frac{2\beta_0}{\epsilon} + a_s^2 \left(\frac{4\beta_0^2}{\epsilon} + \frac{\beta_1}{\epsilon}\right) + \mathcal{O}(a_s^3)\right], \end{aligned} \tag{2.9}$$

where

$$\beta_0 = \left(\frac{11}{3} C_A - \frac{4}{3} T_F\ n_f\right), \qquad \beta_1 = \left(\frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F\ n_f - 4 C_F T_F\ n_f\right). \tag{2.10}$$

Here $C_A = N$ and $C_F = (N^2-1)/2N$ are the quadratic Casimir of the SU(N) group. $T_F = 1/2$ and n_f is the number of light active quark flavors.

The matrix element can also be expressed as a power series of renormalized strong coupling constant with UV finite matrix elements $|\mathcal{M}^{(i)}\rangle$,

$$|\mathcal{M}\rangle = (a_s)^{\frac{1}{2}} \left(|\mathcal{M}^{(0)}\rangle + a_s |\mathcal{M}^{(1)}\rangle + a_s^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(a_s^3) \right) \tag{2.11}$$

where

$$\begin{aligned}
|\mathcal{M}^{(0)}\rangle &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{1}{2}} \; |\hat{\mathcal{M}}^{(0)}\rangle \,, \\
|\mathcal{M}^{(1)}\rangle &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{3}{2}} \left[|\hat{\mathcal{M}}^{(1)}\rangle + \mu_R^\epsilon \frac{r_1}{2} \; |\hat{\mathcal{M}}^{(0)}\rangle \right], \\
|\mathcal{M}^{(2)}\rangle &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{5}{2}} \left[|\hat{\mathcal{M}}^{(2)}\rangle + \mu_R^\epsilon \frac{3r_1}{2} \; |\hat{\mathcal{M}}^{(1)}\rangle + \mu_R^{2\epsilon} \left(\frac{r_2}{2} - \frac{r_1^2}{8}\right) \; |\hat{\mathcal{M}}^{(0)}\rangle \right]
\end{aligned} \tag{2.12}$$

with

$$r_1 = \frac{2\beta_0}{\epsilon}, \qquad r_2 = \left(\frac{4\beta_0^2}{\epsilon^2} + \frac{\beta_1}{\epsilon}\right) \tag{2.13}$$

2.2 Infrared Factorization

In higher order calculation, the UV renormalized matrix elements contain singularities of infrared origin. Generally two kinds of singularities arise – the soft and collinear while working with massless QCD. According to the KLN theorem, these singularities get canceled against the similar contribution from real emission Feynman diagrams, resulting in infrared safe observables. The IR divergences have a universal structure in dimensional regularization, which was predicted in [34] to two loop, except the two loop single pole in ϵ. In [43, 47, 48] the infrared structure of scattering amplitudes are studied and connection of the single pole to soft anomalous dimension matrix is predicted. The factorization of the single pole in quark and gluon form factors in terms of soft and collinear anomalous dimensions was demonstrated to two-loop level [49] whose validity at three-loop was later established in [50]. The proposal by Catani was generalized beyond two loops in [51, 52].

According to Catani's prediction [34], the renormalized amplitude factorizes in dimensional regularization. The ME at a given order $|\mathcal{M}^{(i)}\rangle$ can be expressed as the sum of the lower order MEs times appropriate insertion operators ($\mathbf{I}_q^{(i)}(\epsilon)$) and a finite piece $|\mathcal{M}^{(i)fin}\rangle$. These insertion operators contain the infrared pole structure which are universal. For the present case, we have two external massless quarks and a gluon, for which the one-loop and the two-loop ME can be written in the following form,

$$\begin{aligned}
|\mathcal{M}^{(1)}\rangle &= 2\, \mathbf{I}_q^{(1)}(\epsilon) \; |\mathcal{M}^{(0)}\rangle \; + \; |\mathcal{M}^{(1)fin}\rangle \,, \\
|\mathcal{M}^{(2)}\rangle &= 2\, \mathbf{I}_q^{(1)}(\epsilon) \; |\mathcal{M}^{(1)}\rangle \; + \; 4\, \mathbf{I}_q^{(2)}(\epsilon) \; |\mathcal{M}^{(0)}\rangle \; + \; |\mathcal{M}^{(2)fin}\rangle
\end{aligned} \tag{2.14}$$

where the one-loop and two-loop insertion operators are given by

$$\begin{aligned}
\mathbf{I}_q^{(1)}(\epsilon) &= \frac{1}{2}\,\frac{e^{-\frac{\epsilon}{2}\gamma_E}}{\Gamma(1+\frac{\epsilon}{2})}\Bigg\{\Big(\frac{4}{\epsilon^2}-\frac{3}{\epsilon}\Big)(C_A-2C_F)\Big[\Big(-\frac{s}{\mu_R^2}\Big)^{\frac{\epsilon}{2}}\Big] \\
&\quad+\Big(-\frac{4C_A}{\epsilon^2}+\frac{3C_A}{2\epsilon}+\frac{\beta_0}{2\epsilon}\Big)\Big[\Big(-\frac{t}{\mu_R^2}\Big)^{\frac{\epsilon}{2}}+\Big(-\frac{u}{\mu_R^2}\Big)^{\frac{\epsilon}{2}}\Big]\Bigg\}, \\
\mathbf{I}_q^{(2)}(\epsilon) &= \frac{1}{2}\,\mathbf{I}_q^{(1)}(\epsilon)\,\Big[\mathbf{I}_q^{(1)}(\epsilon)-\frac{2\beta_0}{\epsilon}\Big]+\frac{e^{\frac{\epsilon}{2}\gamma_E}\,\Gamma(1+\epsilon)}{\Gamma(1+\frac{\epsilon}{2})}\Big[-\frac{\beta_0}{\epsilon}+K\Big]\,\mathbf{I}_q^{(1)}(2\epsilon) \\
&\quad+\Big(2\mathbf{H}_q^{(2)}(\epsilon)+\mathbf{H}_g^{(2)}(\epsilon)\Big)
\end{aligned} \tag{2.15}$$

where

$$K=\Big(\frac{67}{18}\;-\;\frac{\pi^2}{6}\Big)C_A\;-\;\frac{10}{9}T_F\;n_f. \tag{2.16}$$

The functions $\mathbf{H}_q^{(2)}(\epsilon),\mathbf{H}_g^{(2)}(\epsilon)$ are dependent on the renormalization scheme. In the $\overline{\text{MS}}$ scheme these are given by

$$\begin{aligned}
\mathbf{H}_q^{(2)}(\epsilon) &= \frac{1}{\epsilon}\Bigg\{C_AC_F\Big(-\frac{245}{432}+\frac{23}{16}\zeta_2-\frac{13}{4}\zeta_3\Big)+C_F^2\Big(\frac{3}{16}-\frac{3}{2}\zeta_2+3\zeta_3\Big) \\
&\quad+C_F\;n_f\Big(\frac{25}{216}-\frac{1}{8}\zeta_2\Big)\Bigg\}, \\
\mathbf{H}_g^{(2)}(\epsilon) &= \frac{1}{\epsilon}\Bigg\{C_A^2\Big(-\frac{5}{24}-\frac{11}{48}\zeta_2-\frac{1}{4}\zeta_3\Big)+C_A\;n_f\Big(\frac{29}{54}+\frac{1}{24}\zeta_2\Big)+\frac{1}{4}C_F\;n_f-\frac{5}{54}n_f^2\Bigg\}.
\end{aligned} \tag{2.17}$$

Here ζ_i is the Riemann Zeta function.

3 Calculation of Amplitudes

In this section we discuss the calculational details of the amplitudes $|\hat{\mathcal{M}}^{(i)}\rangle$ for the process $G\to q\;+\;\bar{q}\;+\;g$ to two-loop level in pQCD. Particularly we calculate the squared matrix elements $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$ and $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle$. Due to the tensorial coupling of spin-2 with the SM, the computational procedure becomes tedious. Starting from the generation of Feynman amplitudes, we systematically automatize the calculational procedure using in-house codes based on FORM [53], Mathematica, Reduze 2 [54] and LiteRed [55, 56].

3.1 Generation of Feynman Diagrams and Simplification

QGRAF [57] is used to generate all the Feynman amplitudes in terms of symbolic expressions. For the process under consideration, we have 4 Feynman diagrams in the Born, 43 in the one-loop and 847 in the two-loop level; where we have excluded all the tadpoles and self-energy corrections to the external legs. The raw QGRAF output is then manipulated using in-house FORM routines to incorporate the Feynman rules [44, 45] and to take care of the color and Dirac matrix ordering. For the internal gluons Feynman gauge is used and

the ghost-graviton interactions [58] are introduced in the Lagrangian (see Eq. (2.2)) as is necessary for higher order computations. For the external gluon the physical polarisations are summed using

$$\sum_{\lambda=\pm 1} \epsilon^{\mu}(p_3,\lambda)\epsilon^{\nu *}(p_3,\lambda) = -g^{\mu\nu} \ + \ \frac{p_3^{\mu} n^{\nu} + n^{\mu} p_3^{\nu}}{p_3.n}. \tag{3.1}$$

Here λ is the helicity and p_3 is the momentum of the external gluon. n is an arbitrary light-like 4-vector. We choose $n = p_1$, one of the external fermion momenta without loss of generality. The spin-2 polarisation sum in d-dimension is given by [44, 58]:

$$\begin{aligned} B^{\mu\nu;\rho\sigma}(q) \ = \ & \left(g^{\mu\rho} - \frac{q^{\mu}q^{\sigma}}{q.q}\right)\left(g^{\nu\sigma} - \frac{q^{\nu}q^{\sigma}}{q.q}\right) + \left(g^{\mu\sigma} - \frac{q^{\mu}q^{\sigma}}{q.q}\right)\left(g^{\nu\rho} - \frac{q^{\nu}q^{\rho}}{q.q}\right) \\ & - \frac{2}{d-1}\left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q.q}\right)\left(g^{\rho\sigma} - \frac{q^{\rho}q^{\sigma}}{q.q}\right). \end{aligned} \tag{3.2}$$

where the metric $g_{\mu\nu} = Diag(1,-1,-1,-1)$. The squared matrix elements are further processed using in-house codes based on LiteRed and Mathematica.

3.2 Reduction of Tensor Integrals

Two loop calculation involves a large number of higher rank Feynman integrals, particularly in the present case it contains tensorial integrals. The conventional approach is to convert the different Feynman integrals into scalar integrals. This generates thousands of scalar integrals which need to be properly classified. The idea is to connect all the scalar integrals to belong to a particular basis. The basis is chosen keeping in mind that any scalar products of loop momenta and external momenta can be expressed only in terms of linear combinations of the propagators. In the case of one-loop there are four different scalar products which are written in terms of four propagators. The choice of basis for one-loop and two-loop case is given in [32], for completeness we present those here. It is straight forward to choose the following set as the basis for the one-loop case,

$$\begin{aligned} B_{11} &= \{\mathcal{D}_1, \mathcal{D}_{1;1}, \mathcal{D}_{1;12}, \mathcal{D}_{1;123}\}, \\ B_{12} &= \{\mathcal{D}_1, \mathcal{D}_{1;2}, \mathcal{D}_{1;23}, \mathcal{D}_{1;123}\}, \\ B_{13} &= \{\mathcal{D}_1, \mathcal{D}_{1;3}, \mathcal{D}_{1;31}, \mathcal{D}_{1;123}\} \end{aligned} \tag{3.3}$$

where

$$\mathcal{D}_1 = k_1^2, \ \mathcal{D}_{1;i} = (k_1 - p_i)^2, \ \mathcal{D}_{1;ij} = (k_1 - p_i - p_j)^2, \ \mathcal{D}_{1;ijk} = (k_1 - p_i - p_j - p_k)^2 \tag{3.4}$$

with $i,j,k = 1,2,3$. At two-loop one has 9 independent scalar products of loop momenta and external momenta, *viz.* $\{(k_\alpha \cdot k_\beta), (k_\alpha \cdot p_i)\}$; $\alpha,\beta = 1,2$ and $i = 1,2,3$. The physical diagrams contain at most 7 different propagators. Hence we need to increase the number of propagators to 9. With the help of Reduze 2, all the two-loop diagrams are classified

into six different auxiliary topologies presented below:

$$\begin{aligned}
B_{21} &= \{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;1},\ \mathcal{D}_{2;1},\ \mathcal{D}_{1;12},\ \mathcal{D}_{2;12},\ \mathcal{D}_{1;123},\ \mathcal{D}_{2;123}\},\\
B_{22} &= \{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;2},\ \mathcal{D}_{2;2},\ \mathcal{D}_{1;23},\ \mathcal{D}_{2;23},\ \mathcal{D}_{1;123},\ \mathcal{D}_{2;123}\},\\
B_{23} &= \{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;3},\ \mathcal{D}_{2;3},\ \mathcal{D}_{1;31},\ \mathcal{D}_{2;31},\ \mathcal{D}_{1;123},\ \mathcal{D}_{2;123}\},\\
B_{24} &= \{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;1},\ \mathcal{D}_{2;1},\ \mathcal{D}_{0;3},\ \mathcal{D}_{1;12},\ \mathcal{D}_{2;12},\ \mathcal{D}_{1;123}\},\\
B_{25} &= \{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;2},\ \mathcal{D}_{2;2},\ \mathcal{D}_{0;1},\ \mathcal{D}_{1;23},\ \mathcal{D}_{2;23},\ \mathcal{D}_{1;123}\},\\
B_{26} &= \{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;3},\ \mathcal{D}_{2;3},\ \mathcal{D}_{0;2},\ \mathcal{D}_{1;31},\ \mathcal{D}_{2;31},\ \mathcal{D}_{1;123}\}
\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}
&\mathcal{D}_0 = (k_1-k_2)^2,\ \mathcal{D}_\alpha = k_\alpha^2,\ \mathcal{D}_{\alpha;i} = (k_\alpha - p_i)^2,\ \mathcal{D}_{\alpha;ij} = (k_\alpha - p_i - p_j)^2,\\
&\mathcal{D}_{0;i} = (k_1-k_2-p_i)^2,\ \mathcal{D}_{\alpha;ijk} = (k_\alpha - p_i - p_j - p_k)^2.
\end{aligned} \tag{3.6}$$

Although properly classified, these large number of scalar integrals are not all independent. In fact they are related by the IBP identities [35, 36] and LI identities [37] which follow from the Poincare invariance. At a fixed order, they result in a large linear system of equations for the integrals. The inclusion of LI identities accelerates the solution of the system of equations, although they are not independent from the IBP identities [59]. We generate the IBP relations and LI identities using Laporta algorithm [60] as implemented in LiteRed. Using LiteRed along with Mint [61, 62] we reduce all different scalar integrals to a fewer set of irreducible scalar integrals *i.e.* the MIs.

At one-loop, two kinds of MIs appear, *viz.* the *Bubble*- two-propagator MI and *Box*- four-propagator MI. In case of two-loop we find a total of 24 topologies, out of which 8 are non-planar topologies and 16 planar topologies. All the two-loop MIs in our calculation can be related to the MI computed in [41, 42]. At this point we would like to note that some of the MIs in our case do not appear as given in [41, 42]. The reason behind this is the different convention in the basis in LiteRed and in [41, 42]. Thus we found topologies containing higher power of propagators instead of the irreducible numerator in [41, 42]. Nevertheless those can be related by properly using the IBP and LI identities. In this way we reduce all the scalar integrals to the known set of MIs. We also found two extra topologies for the MI [32], *viz. Kite* and *GlassS* which are basically product of two one-loop MIs. *GlassS* is found to be the product of two *Bubbles*. *Kite* is the product of one *Bubble* and one *Box* one-loop MIs. Using all the MIs we finally find the unrenormalized one-loop $\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(1)}\rangle$ and two-loop $\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(2)}\rangle$ matrix elements which are presented in the next section.

4 Results

We have checked that the amplitudes are gauge invariant, which serves a crucial check on our computation. Following the renormalization prescription in section (2.1), we compute the UV renormalized matrix elements $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$ and $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle$ in terms of the

unrenormalized ones,

$$\begin{aligned}
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle &= \left(\frac{1}{\mu_R^{\epsilon}}\right)^2 \left[\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(1)}\rangle \;+\; \mu_R^{\epsilon}\frac{r_1}{2}\,\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(0)}\rangle\right] \\
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle &= \left(\frac{1}{\mu_R^{\epsilon}}\right)^3 \left[\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(2)}\rangle \;+\; \mu_R^{\epsilon}\frac{3r_1}{2}\,\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(1)}\rangle\right. \\
&\qquad\left. +\; \mu_R^{2\epsilon}\left(\frac{r_2}{2}-\frac{r_1^2}{8}\right)\langle\hat{\mathcal{M}}^{(0)}|\hat{\mathcal{M}}^{(0)}\rangle\right].
\end{aligned} \tag{4.1}$$

Similarly the renomalized matrix elements according to Catani's prescription can be written as

$$\begin{aligned}
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle &= 2\;\mathbf{I}_q^{(1)}(\epsilon)\;\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle \;+\; \langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)fin}\rangle \\
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle &= 2\;\mathbf{I}_q^{(1)}(\epsilon)\;\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle + 4\;\mathbf{I}_q^{(2)}(\epsilon)\;\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle \;+\; \langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)fin}\rangle\,.
\end{aligned} \tag{4.2}$$

From Eq. (4.1) we extract the coefficients of different poles *viz.* the $1/\epsilon^4, 1/\epsilon^3, 1/\epsilon^2, 1/\epsilon$ and we find that they exactly agree with the respective poles coming from Eq. (4.2). By comparing the $\mathcal{O}(\epsilon^0)$ terms from these two sets of equations (Eq. (4.1)) and (Eq. (4.2)), we obtain the unknown pieces $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)fin}\rangle$ and $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)fin}\rangle$.
The final result is written in the following form:

$$\begin{aligned}
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle &= \mathcal{F}_b\;\mathcal{A}^{(0)}, \\
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)fin}\rangle &= \mathcal{F}_b\Bigg\{\;\mathcal{A}_0^{(1)}\;\ln\!\left(-\frac{Q^2}{\mu_R^2}\right) + \left(\mathcal{A}_1^{(1)}\zeta_2 + \mathcal{A}_2^{(1)}\right)\Bigg\}, \\
\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)fin}\rangle &= \mathcal{F}_b\Bigg\{\;\mathcal{A}_0^{(2)}\;\ln^2\!\left(-\frac{Q^2}{\mu_R^2}\right) \\
&\qquad + \left(\;\mathcal{A}_1^{(2)}\zeta_3 + \;\mathcal{A}_2^{(2)}\zeta_2 + \mathcal{A}_3^{(2)}\right)\ln\!\left(-\frac{Q^2}{\mu_R^2}\right) \\
&\qquad + \left(\mathcal{A}_4^{(2)}\zeta_2^2 + \mathcal{A}_5^{(2)}\zeta_3 + \mathcal{A}_6^{(2)}\zeta_2 + \mathcal{A}_7^{(2)}\right)\Bigg\}
\end{aligned} \tag{4.3}$$

where

$$\mathcal{F}_b = 16\pi^2\kappa^2\big(N^2-1\big)\;Q^2, \tag{4.4}$$

$$\mathcal{A}^{(0)} = \frac{\Big(2+y^2\big(3-9z\big)-4z+3z^2-z^3+y^3\big(-1+4z\big)+y\big(-4+12z-9z^2+4z^3\big)\Big)}{4yz(1-y-z)}, \tag{4.5}$$

$$\begin{aligned}
\mathcal{A}_i^{(1)} &= \mathcal{A}_{i;C_A}^{(1)}\;C_A + \mathcal{A}_{i;C_F}^{(1)}\;C_F + \mathcal{A}_{i;n_f}^{(1)}\;n_f, \\
\mathcal{A}_i^{(2)} &= \mathcal{A}_{i;C_A^2}^{(2)}C_A^2 + \mathcal{A}_{i;C_F^2}^{(2)}C_F^2 + \mathcal{A}_{i;n_f^2}^{(2)}n_f^2 + \mathcal{A}_{i;C_AC_F}^{(2)}C_AC_F + \mathcal{A}_{i;C_An_f}^{(2)}C_An_f + \mathcal{A}_{i;C_Fn_f}^{(2)}C_Fn_f.
\end{aligned} \tag{4.6}$$

We notice that unlike the Higgs decay i.e. $H \to b + \bar{b} + g$ [63], there is no C_F term in the $\mathcal{A}_0^{(1)}$, which is due to the absence of Yukawa-like term in spin-2 case. All the one-loop and two-loop coefficients are presented in the Appendix B and C respectively except the $A_7^{(2)}$ term which is provided as the ancillary file in the arXiv.

5 Conclusions

In this article, we present the two-loop virtual QCD correction to massive $G \to q + \bar{q} + g$ considering the minimal and universal coupling between spin-2 and the SM particles. We confine ourselves within the framework of massless QCD where only the light quark degrees of freedom are taken into account. We employ the Feynman diagrammatic approach to achieve our goal. As expected, the computation becomes very tedious not only due to the presence of a large number of Feynman diagrams but also due to the involvement of a tensorial coupling. In-house codes and state-of-the-art techniques, in particular, IBP and LI identities, are employed extensively to execute the computation successfully. The bare matrix elements contain UV as well as IR divergences. The strong coupling constant renormalization is sufficient to make it UV finite. No extra UV renormalization is required for the spin-2 coupling as a consequence of the conserved SM energy-momentum tensor through which it couples universally to the SM fields. The UV finite matrix elements exhibit poles of infrared origin in dimensional regularization. The resulting infrared pole structures are in exact agreement with the Catani's prescription which ensures the universal factorization property of QCD amplitudes even in the presence of spin-2 field. This serves a crucial check on the correctness of our computation. The result presented here is an important piece, which now completes the full two-loop calculation of real graviton production associated with a jet. The full NNLO computation to this process needs additional inputs that we reserve for future study.

Acknowledgments

We sincerely thank Thomas Gehrmann for providing the necessary MIs which are used in this calculation. GD would like to thank R. N. Lee for help with LiteRed. GD also thanks Kuntal Nayek for help with Mathematica. GD is supported by funding from Department of Atomic Energy, India.

A Harmonic Polylogarithms

All our results are presented in terms of HPLs which are generalisation of Neilson's polylogarithms. Here we briefly discuss some important properties of HPLs. For more details one can see [41, 46]. The HPLs are represented by $H(\vec{m}_\omega; y)$; $\vec{m}_\omega$ being a ω-dimensional vector which belongs to the set {-1,0,1} through which we define the following rational functions

$$f(1;y) \equiv \frac{1}{1-y}, \qquad f(0;y) \equiv \frac{1}{y}, \qquad f(-1;y) \equiv \frac{1}{1+y}. \tag{A.1}$$

The weight-1 ($\omega = 1$) HPLs are then defined as

$$H(1,y) \;\equiv\; -\ln(1-y), \qquad H(0,y) \;\equiv\; \ln(y), \qquad H(-1,y) \;\equiv\; \ln(1+y). \tag{A.2}$$

For $\omega >\ 1$ HPLs are defined as

$$H(m,\vec{m}_\omega;y) \;\equiv\; \int_0^y dx\; f(m,x)\; H(\vec{m}_\omega;x), \qquad\qquad m \in 0,\pm 1. \tag{A.3}$$

Moreover the higher dimensional HPLs are defined following the Eq. (A.3) for the new elements 2,3 in $\vec{m}_\omega$, representing a new class of rational functions

$$f(2;y) \equiv f(1-z;y) \equiv \frac{1}{1-y-z}, \qquad f(3;y) \equiv f(z;y) \equiv \frac{1}{y+z}; \tag{A.4}$$

correspondingly the weight-1 ($\omega = 1$) two-dimensional HPLs are given by

$$H(2,y) \equiv -\ln\Big(1-\frac{y}{1-z}\Big), \qquad H(3,y) \equiv \ln\Big(\frac{y+z}{z}\Big). \tag{A.5}$$

Properties

HPLs follow some important properties

Shuffle algebra: Product of two HPLs with weights ω_1 and ω_2 of same argument y is a HPL of weight $(\omega_1+\omega_1)$ and argument y. This way all possible permutations of the elements of $\vec{m}_{\omega_1}$ and $\vec{m}_{\omega_2}$ are considered preserving the relative orders of the element of $\vec{m}_{\omega_1}$ and $\vec{m}_{\omega_2}$,

$$H(\vec{m}_{\omega_1};y)\ H(\vec{m}_{\omega_2};y) = \sum_{\vec{m}_\omega = \vec{m}_{\omega_1} \uplus \vec{m}_{\omega_2}} H(\vec{m}_\omega;y) \tag{A.6}$$

Integration-by-parts identities: Using the integration-by-parts identities the ordering of the elements of $\vec{m}_\omega$ inside the HPL can be reversed and in this process some products of two HPLs can be generated.

$$\begin{aligned} H(\vec{m}_\omega;y) \equiv H(m_1,m_2,...m_\omega;y) &= H(m_1;y)\ H(m_2,...m_\omega;y) \\ &- H(m_2,m_1;y)\ H(m_3,...m_\omega;y) \\ &+ ... + (-1)^{\omega+1}\ H(m_\omega,...,m_2,m_1;y). \end{aligned} \tag{A.7}$$

Equations A.6 and A.7 are very useful in writing the higher-weight HPLs in terms of lower-weight HPLs or products of HPLs, which have been used in the checks of the different poles in section 5. Below we provide the necessary relations of HPLs used in our computation.

$$\begin{aligned} H(0,2,0,y) &= H(0,y)H(0,2,y) - 2H(0,0,2,y), \\ H(1,0,2,y) &= H(1,y)H(0,2,y) - H(0,1,2,y) - H(0,2,1,y), \\ H(1,2,0,y) &= H(1,y)H(2,0,y) - H(2,1,y)H(0,y) + H(0,2,1,y), \\ H(2,0,0,y) &= H(2,y)H(0,0,y) - H(0,2,y)H(0,y) + H(0,0,2,y), \\ H(2,0,2,y) &= H(2,y)H(0,2,y) - 2H(0,2,2,y), \\ H(2,2,2,y) &= \frac{1}{3}H(2,y)H(2,2,y), \\ H(2,2,0,y) &= H(2,y)H(2,0,y) - H(2,2,y)H(0,y) + H(0,2,2,y), \\ H(3,0,2,y) &= H(3,y)H(0,2,y) - H(0,3,2,y) - H(0,2,3,y), \\ H(3,2,0,y) &= H(3,y)H(2,0,y) - H(2,3,y)H(0,y) + H(0,2,3,y), \\ H(3,3,2,y) &= H(3,y)H(3,2,y) - H(3,3,y)H(2,y) + H(2,3,3,y), \\ H(2,1,0,y) &= H(2,y)H(1,0,y) - H(1,2,y)H(0,y) + H(0,1,2,y), \\ H(2,3,2,y) &= H(2,y)H(3,2,y) - 2H(3,2,2,y) \end{aligned} \tag{A.8}$$

B One-loop Coefficients

$$\mathcal{A}_0^{(1)} = -\frac{\beta_0}{2}\,\mathcal{A}_0;$$

$$\mathcal{A}_{1;C_A}^{(1)} = -C_A\,\mathcal{A}_0; \qquad \mathcal{A}_{1;C_F}^{(1)} = 0; \qquad \mathcal{A}_{1;n_f}^{(1)} = 0;$$

$$\begin{aligned}
\mathcal{A}_{2;C_A}^{(1)} = \Big\{ & -6(-1+y)y^4(-1+z)(y+z) + 6(-1+y)y^5(-1+z)(y+z) + 84(-1+y)y^3(-1+z)z(y+z) - 54(-1+y)y^4(-1 \\
& + z)z(y+z) + 44(-1+y)y^5(-1+z)z(y+z) + 132(-1+y)y^2(-1+z)z^2(y+z) - 152(-1+y)y^3(-1+z)z^2(y \\
& + z) + 48(-1+y)y^5(-1+z)z^2(y+z) + 84(-1+y)y(-1+z)z^3(y+z) - 152(-1+y)y^2(-1+z)z^3(y+z) - 88(-1 \\
& + y)y^3(-1+z)z^3(y+z) + 144(-1+y)y^4(-1+z)z^3(y+z) - 6(-1+y)(-1+z)z^4(y+z) - 54(-1+y)y(-1 \\
& + z)z^4(y+z) + 144(-1+y)y^3(-1+z)z^4(y+z) + 6(-1+y)(-1+z)z^5(y+z) + 44(-1+y)y(-1+z)z^5(y+z) \\
& + 48(-1+y)y^2(-1+z)z^5(y+z) + 3(-1+y)y(-1+z)(y+z)^4(16y^3 + y^2(-41+12z) - 2(7-6z+3z^2) \\
& + 3y(13-9z+4z^2))H(0,y)^2 + 3(-1+y)(-1+z)z(y+z)^4(-14+39z-41z^2+16z^3+6y^2(-1+2z)+3y(4 \\
& - 9z+4z^2))H(0,z)^2 + 3(-1+y)(-1+z)(y+z)^4(4+16y^4-22z+45z^2-43z^3+16z^4+y^3(-43+20z) \\
& + 3y^2(15-17z+8z^2)+y(-22+48z-51z^2+20z^3))H(1,z)^2 + 2(-1+z)(y+z)^4H(0,y)(-10+30y-35y^2 \\
& + 20y^3-5y^4+20z-80yz+132y^2z-116y^3z+44y^4z-15z^2+60yz^2-72y^2z^2+24y^3z^2+5z^3 \\
& - 25yz^3+20y^2z^3+3(-1+y)(2+y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z-9z^2 \\
& + 4z^3))H(0,z) + 3(-1+y)(2+16y^4-4z+3z^2-z^3+2y^3(-21+8z)+6y^2(7-6z+2z^2)+y(-18+24z \\
& - 15z^2+4z^3))H(1,z)) + (-1+y)(-1+z)(y^7(-9+84z)+3y^6(9-83z+144z^2)-9z^4(-2+4z-3z^2+z^3) \\
& + 3y^5(-12+92z-373z^2+324z^3)+yz^3(-36-136z+276z^2-249z^3+84z^4)+y^2z^2(-60-292z+1053z^2 \\
& - 1119z^3+432z^4)+y^3z(-36-292z+1608z^2-2175z^3+972z^4)+y^4(18-136z+1053z^2-2175z^3 \\
& + 1248z^4))H(2,y) + 3(-1+y)(-1+z)(y+z)^4(4+16y^4-22z+45z^2-43z^3+16z^4+y^3(-43+20z)+3y^2(15 \\
& - 17z+8z^2)+y(-22+48z-51z^2+20z^3))H(2,y)^2 + 2(-1+y)(y+z)^4H(0,z)(-5(-1+z)^2(2-2z+z^2) \\
& + 5y^3(1-5z+4z^2)+3y^2(-5+20z-24z^2+8z^3)+4y(5-20z+33z^2-29z^3+11z^4)+3(-1+z)(y^3(-1 \\
& + 4z)+4y(-1+z)^2(-1+4z)+3y^2(1-5z+4z^2)+2(1-9z+21z^2-21z^3+8z^4))H(2,y)) + (-1+y)(-1 \\
& + z)H(1,z)(3y^7(-3+28z)+3y^6(9-83z+144z^2)-9z^4(-2+4z-3z^2+z^3)+3y^5(-12+92z-373z^2 \\
& + 324z^3)+yz^3(-36-136z+276z^2-249z^3+84z^4)+y^2z^2(-60-292z+1053z^2-1119z^3+432z^4) \\
& + y^3z(-36-292z+1608z^2-2175z^3+972z^4)+y^4(18-136z+1053z^2-2175z^3+1248z^4)-6(y+z)^4(4 \\
& + 16y^4-22z+45z^2-43z^3+16z^4+y^3(-43+20z)+3y^2(15-17z+8z^2)+y(-22+48z-51z^2 \\
& + 20z^3))H(3,y)) - 6(-1+y)y(-1+z)(y+z)^4(16y^3+y^2(-41+12z)-2(7-6z+3z^2)+3y(13-9z \\
& + 4z^2))H(0,0,y) - 6(-1+y)(-1+z)z(y+z)^4(-14+39z-41z^2+16z^3+6y^2(-1+2z)+3y(4-9z \\
& + 4z^2))H(0,0,z) - 6(-1+y)(-1+z)(y+z)^4(2+16y^4-4z+3z^2-z^3+2y^3(-21+8z)+6y^2(7-6z \\
& + 2z^2)+y(-18+24z-15z^2+4z^3))H(0,1,z) + 6(-1+y)(-1+z)(y+z)^4(2+16y^4-4z+3z^2-z^3 \\
& + 2y^3(-21+8z)+6y^2(7-6z+2z^2)+y(-18+24z-15z^2+4z^3))H(0,2,y) - 6(-1+y)y(-1+z)(y \\
& + z)^4(16y^3+y^2(-41+12z)-2(7-6z+3z^2)+3y(13-9z+4z^2))H(1,0,y) + 6(-1+y)(-1+z)(y+z)^4(2 \\
& + y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z-9z^2+4z^3))H(1,0,z) - 6(-1+y)(-1+z)(y \\
& + z)^4(4+16y^4-22z+45z^2-43z^3+16z^4+y^3(-43+20z)+3y^2(15-17z+8z^2)+y(-22+48z-51z^2 \\
& + 20z^3))H(1,1,z) + 6(-1+y)(-1+z)(y+z)^4(2+16y^4-4z+3z^2-z^3+2y^3(-21+8z)+6y^2(7-6z \\
& + 2z^2)+y(-18+24z-15z^2+4z^3))H(2,0,y) - 6(-1+y)(-1+z)(y+z)^4(4+16y^4-22z+45z^2-43z^3 \\
& + 16z^4+y^3(-43+20z)+3y^2(15-17z+8z^2)+y(-22+48z-51z^2+20z^3))H(2,2,y) - 6(-1+y)(-1+z)(y \\
& + z)^4(4+16y^4-22z+45z^2-43z^3+16z^4+y^3(-43+20z)+3y^2(15-17z+8z^2)+y(-22+48z-51z^2 \\
& + 20z^3))H(3,2,y)\Big\} \Big/ \Big(24(-1+y)y(-1+z)z(-1+y+z)(y+z)^4\Big);
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{2;C_F}^{(1)} = \Big\{ & 20(-1+y)y(-1+z)(y+z) - 59(-1+y)y^2(-1+z)(y+z) + 68(-1+y)y^3(-1+z)(y+z) - 39(-1+y)y^4(-1 \\
& + z)(y+z) + 10(-1+y)y^5(-1+z)(y+z) + 20(-1+y)(-1+z)z(y+z) - 138(-1+y)y(-1+z)z(y+z) + 316(-1 \\
& + y)y^2(-1+z)z(y+z) - 318(-1+y)y^3(-1+z)z(y+z) + 161(-1+y)y^4(-1+z)z(y+z) - 41(-1+y)y^5(-1 \\
& + z)z(y+z) - 59(-1+y)(-1+z)z^2(y+z) + 316(-1+y)y(-1+z)z^2(y+z) - 578(-1+y)y^2(-1+z)z^2(y+z) \\
& + 435(-1+y)y^3(-1+z)z^2(y+z) - 142(-1+y)y^4(-1+z)z^2(y+z) + 28(-1+y)y^5(-1+z)z^2(y+z) + 68(-1 \\
& + y)(-1+z)z^3(y+z) - 318(-1+y)y(-1+z)z^3(y+z) + 435(-1+y)y^2(-1+z)z^3(y+z) - 202(-1+y)y^3(-1
\end{aligned}$$

$$
\begin{aligned}
&+ z)z^3(y+z) + 20(-1+y)y^4(-1+z)z^3(y+z) - 39(-1+y)(-1+z)z^4(y+z) + 161(-1+y)y(-1+z)z^4(y+z)\\
&- 142(-1+y)y^2(-1+z)z^4(y+z) + 20(-1+y)y^3(-1+z)z^4(y+z) + 10(-1+y)(-1+z)z^5(y+z) - 41(-1\\
&+ y)y(-1+z)z^5(y+z) + 28(-1+y)y^2(-1+z)z^5(y+z) - (-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2+2y^3+4z\\
&- 3z^2+z^3+y^2(-6+4z)+y(6-8z+3z^2))H(0,y)^2 + (1-y)(-1+y)(y^3+3y^2(-1+z)+4y(-1+z)^2+2(-1\\
&+ z)^3)(-1+z)^2(y+z)^2(-1+4z)H(0,z)^2 + (1-y)(-1+y)(-1+z)^2(y+z)^2(4+8y^4-18z+33z^2-27z^3\\
&+ 8z^4+y^3(-27+20z)+3y^2(11-17z+8z^2)+y(-18+48z-51z^2+20z^3))H(1,z)^2 + (-1+z)^2(-1+y+z)(y\\
&+ z)^2H(0,y)(-(yz(8y^3+y(30-27z)+12(-1+z)+2y^2(-13+6z))) - 2(-1+y)^2(-1+4y)(2+2y^2+2y(-2\\
&+ z) - 2z+z^2)H(1,z)) + (1-y)(-1+y)(-1+z)^2(y^5(-3+20z)+y^4(9-73z+80z^2) - 3z^2(-2+4z-3z^2\\
&+ z^3) + 6y^3(-2+15z-34z^2+20z^3) + yz(-8-32z+90z^2-73z^3+20z^4) + 2y^2(3-16z+81z^2-102z^3\\
&+ 40z^4))H(2,y) + (1-y)(-1+y)(-1+z)^2(y+z)^2(4+8y^4-18z+33z^2-27z^3+8z^4+y^3(-27+20z)\\
&+ 3y^2(11-17z+8z^2)+y(-18+48z-51z^2+20z^3))H(2,y)^2 + (1-y)(-1+y)(-1+y+z)(y\\
&+ z)^2H(0,z)(yz(-12+30z-26z^2+8z^3+3y(4-9z+4z^2)) + 2(y^2+2y(-1+z)+2(-1+z)^2)(-1+z)^2(-1\\
&+ 4z)H(2,y)) + (-1+y+z)(-1+y+z-yz)^2H(1,z)(y^4(3-20z)+y^3(-6+50z-60z^2)+3z^2(2-2z+z^2)\\
&+ y^2(6-34z+94z^2-60z^3) - 2yz(4+17z-25z^2+10z^3) + 2(y+z)^2(-4+8y^3+14z-19z^2+8z^3\\
&+ y^2(-19+12z)+2y(7-10z+6z^2))H(3,y)) + 2(1-y)(1-5y+4y^2)(-y-z)(-1+z)^2(2y^4+6y^3(-1+z)\\
&+ y^2(6-14z+7z^2)+z(-2+4z-3z^2+z^3)+y(-2+10z-11z^2+4z^3))H(0,0,y) + 2(1-y)(-1+y)(-y\\
&- z)(-1+z)(1-5z+4z^2)(y^4+2(-1+z)^3z+2y(-1+z)^2(-1+3z)+y^3(-3+4z)+y^2(4-11z\\
&+ 7z^2))H(0,0,z) + 2(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2+2y^3+4z-3z^2+z^3+y^2(-6+4z)+y(6-8z\\
&+ 3z^2))H(0,1,z) - 2(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2+2y^3+4z-3z^2+z^3+y^2(-6+4z)+y(6-8z\\
&+ 3z^2))H(0,2,y) + 2(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2+2y^3+4z-3z^2+z^3+y^2(-6+4z)+y(6-8z\\
&+ 3z^2))H(1,0,y) + 2(-1+y)^2(-1+z)^2(y+z)^2(4+8y^4-18z+33z^2-27z^3+8z^4+y^3(-27+20z)\\
&+ 3y^2(11-17z+8z^2)+y(-18+48z-51z^2+20z^3))H(1,1,z) - 2(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2\\
&+ 2y^3+4z-3z^2+z^3+y^2(-6+4z)+y(6-8z+3z^2))H(2,0,y) + 2(-1+y)^2(-1+z)^2(y+z)^2(4+8y^4\\
&- 18z+33z^2-27z^3+8z^4+y^3(-27+20z)+3y^2(11-17z+8z^2)+y(-18+48z-51z^2+20z^3))H(2,2,y)\\
&+ 2(-1+y)^2(-1+z)^2(y+z)^2(4+8y^4-18z+33z^2-27z^3+8z^4+y^3(-27+20z)+3y^2(11-17z+8z^2)\\
&+ y(-18+48z-51z^2+20z^3))H(3,2,y)\Big\}\Big/\Big(4(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2\Big);
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(1)}_{2;n_f} = \Big\{ &- 24y^4z + 12y^5z - 8y^6z - 24y^3z^2 + 8y^4z^2 - 8y^5z^2 - 24y^2z^3 - 8y^3z^3 + 16y^4z^3 - 24yz^4\\
&+ 8y^2z^4 + 16y^3z^4 + 12yz^5 - 8y^2z^5 - 8yz^6 - (y+z)^4(2+y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)\\
&+ y(-4+12z-9z^2+4z^3))H(0,y) - (y+z)^4(2+y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z\\
&- 9z^2+4z^3))H(0,z) + 4yz(-5y^3+3y^4+yz^2+y^2(6+z-6z^2)+z^2(6-5z+3z^2))H(1,z) + 4yz(-5y^3\\
&+ 3y^4+yz^2+y^2(6+z-6z^2)+z^2(6-5z+3z^2))H(2,y)\Big\}\Big/\Big(24yz(-1+y+z)(y+z)^4\Big).
\end{aligned}
$$

C Two-loop Coefficients

$$\mathcal{A}_0^{(2)} = -\frac{3\beta_0^2}{8}\,\mathcal{A}_0;$$

$$\mathcal{A}^{(2)}_{1;C_A^2} = -\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{1;C_F^2} = 24\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{1;n_f^2} = 0; \qquad \mathcal{A}^{(2)}_{1;C_A C_F} = -26\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{1;C_A n_f} = 0; \qquad \mathcal{A}^{(2)}_{1;C_F n_f} = 0;$$

$$\mathcal{A}^{(2)}_{2;C_A^2} = \frac{55}{12}\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{2;C_F^2} = -12\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{2;n_f^2} = 0; \qquad \mathcal{A}^{(2)}_{2;C_A C_F} = \frac{23}{2}\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{2;C_A n_f} = -\frac{5}{6}\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{2;C_F n_f} = -\,\mathcal{A}_0;$$

$$\begin{aligned}
\mathcal{A}^{(2)}_{3;C_A^2} = \Big\{ & 8(-1+y)^2 y(-1+z)^2 z(y+z)(39y^6(-1+4z) + y^5(84-710z+204z^2) + 3z^3(26-41z+28z^2-13z^3) \\
& - 3y^4(41-372z+507z^2+56z^3) + 2yz^2(117-543z+558z^2-355z^3+78z^4) - 2y^3(-39+543z-1354z^2 \\
& + 850z^3+84z^4) + y^2 z(234-1662z+2708z^2-1521z^3+204z^4)) - 8(-1+y)^2 y(-1+z)^2 z(y+z)^4(56+500y^4 \\
& + 34z-396z^2+550z^3-244z^4 + y^3(-1415+1348z) + 3y^2(433-715z+180z^2) - 4y(110-219z+42z^2 \\
& + 38z^3))H(0,y)^3 + 8(-1+y)^2 y(-1+z)^2 z(y+z)^4(-56+244y^4+440z-1299z^2+1415z^3-500z^4+2y^3(-275 \\
& + 76z) + y^2(396+168z-540z^2) - y(34+876z-2145z^2+1348z^3))H(0,z)^3 - 32(-1+y)^2 y(-1+z)^2 z(y+z)^4(9 \\
& + 364y^4-203z+702z^2-872z^3+364z^4 + y^3(-872+564z) + 6y^2(117-167z+92z^2) + y(-203+606z-1002z^2 \\
& + 564z^3))H(1,z)^3 + (-1728y^{12}(-1+z)^2 + 216(-1+z)^5 z^6(-1+4z) + 24y^{11}(-1+z)^2(375+718z) + y^{10}(-19080 \\
& - 28289z+250122z^2-338949z^3+136304z^4) - y(-1+z)^2 z^5(-8082+36428z-80967z^2+91745z^3-40992z^4+1728z^5) \\
& + y^9(20808+63133z-748547z^2+1624467z^3-1359901z^4+400256z^5) + y^2(-1+z)^2 z^4(25944-171016z+438304z^2 \\
& - 585767z^3+364650z^4-73560z^5+864z^6) + y^8(-12024-69787z+1130000z^2-3636494z^3+4922656z^4-3035579z^5 \\
& + 701120z^6) + y^7(3312+52501z-1005911z^2+4577790z^3-9135618z^4+9138769z^5-4491307z^6+860032z^7) \\
& + y^3 z^3(32844-401168z+1810731z^2-4338377z^3+6108582z^4-5103134z^5+2392815z^6-537669z^7+35376z^8) \\
& + y^4 z^2(23424-374888z+2225864z^2-6614805z^3+11116384z^4-10963698z^5+6184528z^6-1787581z^7+190880z^8) \\
& + y^5 z(6066-175600z+1559775z^2-6176973z^3+12917254z^4-15269476z^5+10196989z^6-3546779z^7+488960z^8) \\
& + y^6(-288-26312z+553704z^2-3461777z^3+9763000z^4-14401588z^5+11527644z^6-4728907z^7+774416z^8))H(2,y)^2 \\
& - 32(-1+y)^2 y(-1+z)^2 z(y+z)^4(9+364y^4-203z+702z^2-872z^3+364z^4+y^3(-872+564z)+6y^2(117-167z \\
& + 92z^2) + y(-203+606z-1002z^2+564z^3))H(2,y)^3 + H(0,y)^2((y+z)^2(1728y^{10}(-1+z)^2 + 216(-1+z)^5 z^3(-1 \\
& + 4z) - 72y^9(-1+z)^2(149+87z) + y^8(28080-43945z-17990z^2+55603z^3-21640z^4) + y(-1+z)^2 z^2(144-3573z \\
& + 10219z^2-12168z^3+4234z^4+864z^5) - y^7(39888-94511z+2851z^2+137595z^3-104887z^4+19064z^5) + y^6(32832 \\
& - 125478z+90408z^2+113163z^3-158918z^4+46681z^5+1096z^6) - y^2(-1+z)^2 z(360-2430z-10125z^2+18134z^3 \\
& - 1460z^4-13140z^5+4320z^6) + y^5(-15336+94486z-141904z^2+19649z^3+66045z^4+13557z^5-53601z^6+17104z^7) \\
& + y^4(3600-38177z+94480z^2-70947z^3+25180z^4-94718z^5+142546z^6-75040z^7+13184z^8) + y^3(-288+7227z \\
& - 28965z^2+25131z^3+4151z^4+35390z^5-108748z^6+97128z^7-33618z^8+2592z^9)) + 48(-1+y)^2 y(-1+z)^2 z(y \\
& + z)^4(16y^4+4y^3(-11+6z)+6y^2(8-9z+2z^2)-3(-2+4z-3z^2+z^3)+y(-26+48z-33z^2+12z^3))H(0,z) \\
& + 144(-1+y)^2 y(-1+z)^2 z(y+z)^4(2+16y^4-4z+3z^2-z^3+2y^3(-21+8z)+6y^2(7-6z+2z^2)+y(-18 \\
& + 24z-15z^2+4z^3))H(1,z) + 48(-1+y)^2 y(-1+z)^2 z(y+z)^4(2+16y^4-4z+3z^2-z^3+2y^3(-21+8z) \\
& + 6y^2(7-6z+2z^2)+y(-18+24z-15z^2+4z^3))H(2,y)) + H(0,z)^2((y+z)^2(1728y^9(-1+z)^2(-1+3z) \\
& - 216(-1+z)^6 z^3(-1+4z) + y^8(9000-48470z+93492z^2-78258z^3+23552z^4) + 4y^7(-4770+28897z-69529z^2 \\
& + 79974z^3-42772z^4+8200z^5) + y(-1+z)^2 z^2(144-3501z+19081z^2-43239z^3+50375z^4-23976z^5+1728z^6) \\
& + y^6(20808-148735z+427794z^2-614728z^3+447358z^4-140145z^5+9016z^6) + y^5(-12024+107821z-367983z^2 \\
& + 619778z^3-507386z^4+132177z^5+54601z^6-26984z^7) - y^2(-1+z)^2 z(360-2934z-12651z^2+74708z^3-142537z^4 \\
& + 119942z^5-36936z^6+864z^7) + y^4(3312-41639z+174106z^2-315241z^3+188404z^4+184665z^5-356162z^6 \\
& + 199207z^7-37336z^8) + y^3(-288+7155z-40995z^2+60519z^3+92277z^4-437623z^5+633399z^6-444135z^7+146323z^8 \\
& - 16632z^9)) + 48(-1+y)^2 y(-1+z)^2 z(y+z)^4(y^3(-1+4z)+4y(-1+z)^2(-1+4z)+3y^2(1-5z+4z^2)+2(1 \\
& - 9z+21z^2-21z^3+8z^4))H(1,z) + 144(-1+y)^2 y(-1+z)^2 z(y+z)^4(y^3(-1+4z)+4y(-1+z)^2(-1+4z) \\
& + 3y^2(1-5z+4z^2)+2(1-9z+21z^2-21z^3+8z^4))H(2,y)) + H(1,z)^2(-1728y^{12}(-1+z)^2 + 216(-1 \\
& + z)^5 z^6(-1+4z) + 24y^{11}(-1+z)^2(375+718z) + y^{10}(-19080-28289z+250122z^2-338949z^3+136304z^4) - y(-1 \\
& + z)^2 z^5(-8082+36428z-80967z^2+91745z^3-40992z^4+1728z^5) + y^9(20808+63133z-748547z^2+1624467z^3 \\
& - 1359901z^4+400256z^5) + y^2(-1+z)^2 z^4(25944-171016z+438304z^2-585767z^3+364650z^4-73560z^5+864z^6) \\
& + y^8(-12024-69787z+1130000z^2-3636494z^3+4922656z^4-3035579z^5+701120z^6) + y^7(3312+52501z-1005911z^2 \\
& + 4577790z^3-9135618z^4+9138769z^5-4491307z^6+860032z^7) + y^3 z^3(32844-401168z+1810731z^2-4338377z^3
\end{aligned}$$

$$+ 6108582z^4 - 5103134z^5 + 2392815z^6 - 537669z^7 + 35376z^8) + y^4z^2(23424 - 374888z + 2225864z^2 - 6614805z^3$$
$$+ 11116384z^4 - 10963698z^5 + 6184528z^6 - 1787581z^7 + 190880z^8) + y^5z(6066 - 175600z + 1559775z^2 - 6176973z^3$$
$$+ 12917254z^4 - 15269476z^5 + 10196989z^6 - 3546779z^7 + 488960z^8) + y^6(-288 - 26312z + 553704z^2 - 3461777z^3$$
$$+ 9763000z^4 - 14401588z^5 + 11527644z^6 - 4728907z^7 + 774416z^8) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(13 + 380y^4 - 225z$$
$$+ 747z^2 - 915z^3 + 380z^4 + y^3(-915 + 584z) + 9y^2(83 - 117z + 64z^2) + y(-225 + 654z - 1053z^2 + 584z^3))H(2,y)$$
$$- 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2)$$
$$+ y(-22 + 48z - 51z^2 + 20z^3))H(3,y)) - 2(y+z)^2(1728y^{10}(-1+z)^2 + 216(-1+z)^5z^3(-1+4z) - 72y^9(-1$$
$$+ z)^2(149 + 87z) + y^8(28080 - 43945z - 17990z^2 + 55603z^3 - 21640z^4) + y(-1+z)^2z^2(144 - 3573z + 10219z^2$$
$$- 12168z^3 + 4234z^4 + 864z^5) - y^7(39888 - 94511z + 2851z^2 + 137595z^3 - 104887z^4 + 19064z^5) + y^6(32832 - 125478z$$
$$+ 90408z^2 + 113163z^3 - 158918z^4 + 46681z^5 + 1096z^6) - y^2(-1+z)^2z(360 - 2430z - 10125z^2 + 18134z^3 - 1460z^4$$
$$- 13140z^5 + 4320z^6) + y^5(-15336 + 94486z - 141904z^2 + 19649z^3 + 66045z^4 + 13557z^5 - 53601z^6 + 17104z^7) + y^4(3600$$
$$- 38177z + 94480z^2 - 70947z^3 + 25180z^4 - 94718z^5 + 142546z^6 - 75040z^7 + 13184z^8) + y^3(-288 + 7227z - 28965z^2$$
$$+ 25131z^3 + 4151z^4 + 35390z^5 - 108748z^6 + 97128z^7 - 33618z^8 + 2592z^9))H(0,0,y) - 2(y+z)^2(1728y^9(-1+z)^2(-1$$
$$+ 3z) - 216(-1+z)^6z^3(-1+4z) + y^8(9000 - 48470z + 93492z^2 - 78258z^3 + 23552z^4) + 4y^7(-4770 + 28897z - 69529z^2$$
$$+ 79974z^3 - 42772z^4 + 8200z^5) + y(-1+z)^2z^2(144 - 3501z + 19081z^2 - 43239z^3 + 50375z^4 - 23976z^5 + 1728z^6)$$
$$+ y^6(20808 - 148735z + 427794z^2 - 614728z^3 + 447358z^4 - 140145z^5 + 9016z^6) + y^5(-12024 + 107821z - 367983z^2$$
$$+ 619778z^3 - 507386z^4 + 132177z^5 + 54601z^6 - 26984z^7) - y^2(-1+z)^2z(360 - 2934z - 12651z^2 + 74708z^3 - 142537z^4$$
$$+ 119942z^5 - 36936z^6 + 864z^7) + y^4(3312 - 41639z + 174106z^2 - 315241z^3 + 188404z^4 + 184665z^5 - 356162z^6$$
$$+ 199207z^7 - 37336z^8) + y^3(-288 + 7155z - 40995z^2 + 60519z^3 + 92277z^4 - 437623z^5 + 633399z^6 - 444135z^7 + 146323z^8$$
$$- 16632z^9))H(0,0,z) + 8(-1+y)^2y(-1+z)z(528y^8(-1+z) - 3(-1+z)^2z^4(14 - 14z + 3z^2) + 6y^7(227 - 631z$$
$$+ 404z^2) + 6y^6(-223 + 1245z - 1758z^2 + 738z^3) + 3y^5(182 - 2102z + 5517z^2 - 4865z^3 + 1284z^4) + 2y^2z^2(-294$$
$$+ 1706z - 2858z^2 + 1581z^3 + 225z^4 - 354z^5) + yz^3(-384 + 1354z - 1582z^2 + 531z^3 + 261z^4 - 180z^5) - 4y^3z(96$$
$$- 1033z + 2746z^2 - 2667z^3 + 666z^4 + 180z^5) + y^4(-42 + 2398z - 11683z^2 + 18420z^3 - 10149z^4 + 1128z^5))H(0,1,z)$$
$$- 8(-1+y)y(-1+z)^2z(528y^9 + 6y^8(-323 + 492z) + 12y^7(237 - 866z + 609z^2) + 3z^4(-30 + 60z - 49z^2 + 19z^3)$$
$$+ 3y^6(-692 + 4784z - 7993z^2 + 3540z^3) + yz^3(-144 + 1364z - 1944z^2 + 1374z^3 - 405z^4) + y^5(732 - 9572z + 30498z^2$$
$$- 31161z^3 + 9960z^4) + y^2z^2(-204 + 3656z - 8678z^2 + 7902z^3 - 3411z^4 + 348z^5) + 2y^3z(-72 + 2320z - 8830z^2$$
$$+ 11118z^3 - 6117z^4 + 1086z^5) + y^4(-90 + 2804z - 18275z^2 + 34641z^3 - 24864z^4 + 6048z^5))H(0,2,y) + 264(-1$$
$$+ y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(1,0,y) - 8(-1$$
$$+ y)^2y(-1+z)z(-3(-1+z)^2z^4(30 - 30z + 19z^2) + y^7(57 - 405z + 348z^2) + 3y^6(-49 + 392z - 939z^2 + 592z^3)$$
$$+ 3y^5(60 - 516z + 1941z^2 - 2857z^3 + 1356z^4) + yz^3(-144 + 956z - 2180z^2 + 2505z^3 - 1581z^4 + 444z^5)$$
$$+ y^2z^2(-204 + 1868z - 6197z^2 + 9312z^3 - 6951z^4 + 2160z^5) + y^3z(-144 + 1808z - 8288z^2 + 15567z^3 - 13539z^4$$
$$+ 4548z^5) + y^4(-90 + 902z - 5345z^2 + 13260z^3 - 14271z^4 + 5472z^5))H(1,0,z) + 2(1728y^{12}(-1+z)^2 - 216(-1$$
$$+ z)^5z^6(-1+4z) - 24y^{11}(-1+z)^2(375 + 718z) + y^{10}(19080 + 28289z - 250122z^2 + 338949z^3 - 136304z^4) + y(-1$$
$$+ z)^2z^5(-8082 + 36428z - 80967z^2 + 91745z^3 - 40992z^4 + 1728z^5) - y^9(20808 + 63133z - 748547z^2 + 1624467z^3$$
$$- 1359901z^4 + 400256z^5) + y^8(12024 + 69787z - 1130000z^2 + 3636494z^3 - 4922656z^4 + 3035579z^5 - 701120z^6) - y^2(-1$$
$$+ z)^2z^4(25944 - 171016z + 438304z^2 - 585767z^3 + 364650z^4 - 73560z^5 + 864z^6) - y^7(3312 + 52501z - 1005911z^2$$
$$+ 4577790z^3 - 9135618z^4 + 9138769z^5 - 4491307z^6 + 860032z^7) + y^6(288 + 26312z - 553704z^2 + 3461777z^3 - 9763000z^4$$
$$+ 14401588z^5 - 11527644z^6 + 4728907z^7 - 774416z^8) + y^5z(-6066 + 175600z - 1559775z^2 + 6176973z^3 - 12917254z^4$$
$$+ 15269476z^5 - 10196989z^6 + 3546779z^7 - 488960z^8) + y^4z^2(-23424 + 374888z - 2225864z^2 + 6614805z^3 - 11116384z^4$$
$$+ 10963698z^5 - 6184528z^6 + 1787581z^7 - 190880z^8) + y^3z^3(-32844 + 401168z - 1810731z^2 + 4338377z^3 - 6108582z^4$$
$$+ 5103134z^5 - 2392815z^6 + 537669z^7 - 35376z^8))H(1,1,z) + H(3,y)(96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z$$
$$+ 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(0,1,z) + 96(-1$$
$$+ y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22$$
$$+ 48z - 51z^2 + 20z^3))H(1,0,z) + 192(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4$$
$$+ y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(1,1,z)) - 8(-1+y)y(-1+z)^2z(528y^9$$
$$+ 6y^8(-323 + 492z) + 12y^7(237 - 866z + 609z^2) + 3z^4(-30 + 60z - 49z^2 + 19z^3) + 3y^6(-692 + 4784z - 7993z^2$$
$$+ 3540z^3) + yz^3(-144 + 1364z - 1944z^2 + 1374z^3 - 405z^4) + y^5(732 - 9572z + 30498z^2 - 31161z^3 + 9960z^4)$$
$$+ y^2z^2(-204 + 3656z - 8678z^2 + 7902z^3 - 3411z^4 + 348z^5) + 2y^3z(-72 + 2320z - 8830z^2 + 11118z^3 - 6117z^4$$
$$+ 1086z^5) + y^4(-90 + 2804z - 18275z^2 + 34641z^3 - 24864z^4 + 6048z^5))H(2,0,y) + 2(1728y^{12}(-1+z)^2 - 216(-1$$
$$+ z)^5z^6(-1+4z) - 24y^{11}(-1+z)^2(375 + 718z) + y^{10}(19080 + 28289z - 250122z^2 + 338949z^3 - 136304z^4) + y(-1$$

$$+ z)^2 z^5(-8082 + 36428z - 80967z^2 + 91745z^3 - 40992z^4 + 1728z^5) - y^9(20808 + 63133z - 748547z^2 + 1624467z^3$$
$$- 1359901z^4 + 400256z^5) + y^8(12024 + 69787z - 1130000z^2 + 3636494z^3 - 4922656z^4 + 3035579z^5 - 701120z^6) - y^2(-1$$
$$+ z)^2 z^4(25944 - 171016z + 438304z^2 - 585767z^3 + 364650z^4 - 73560z^5 + 864z^6) - y^7(3312 + 52501z - 1005911z^2$$
$$+ 4577790z^3 - 9135618z^4 + 9138769z^5 - 4491307z^6 + 860032z^7) + y^6(288 + 26312z - 553704z^2 + 3461777z^3 - 9763000z^4$$
$$+ 14401588z^5 - 11527644z^6 + 4728907z^7 - 774416z^8) + y^5 z(-6066 + 175600z - 1559775z^2 + 6176973z^3 - 12917254z^4$$
$$+ 15269476z^5 - 10196989z^6 + 3546779z^7 - 488960z^8) + y^4 z^2(-23424 + 374888z - 2225864z^2 + 6614805z^3 - 11116384z^4$$
$$+ 10963698z^5 - 6184528z^6 + 1787581z^7 - 190880z^8) + y^3 z^3(-32844 + 401168z - 1810731z^2 + 4338377z^3 - 6108582z^4$$
$$+ 5103134z^5 - 2392815z^6 + 537669z^7 - 35376z^8))H(2,2,y) + H(0,z)(-88(-1+y)^2 y(-1+z)z(y+z)^4(-5(-1+z)^2(2$$
$$- 2z + z^2) + 5y^3(1 - 5z + 4z^2) + 3y^2(-5 + 20z - 24z^2 + 8z^3) + 4y(5 - 20z + 33z^2 - 29z^3 + 11z^4)) + 96(-1$$
$$+ y)^2 y(-1+z)^2 z(y+z)^4(1 + 8y^4 - 9z + 21z^2 - 21z^3 + 8z^4 + y^3(-21 + 8z) + 3y^2(7 - 7z + 4z^2) + y(-9 + 18z$$
$$- 21z^2 + 8z^3))H(1,z)^2 - 264(-1+y)^2 y(-1+z)^2 z(y+z)^4(y^3(-1+4z) + 4y(-1+z)^2(-1+4z) + 3y^2(1 - 5z$$
$$+ 4z^2) + 2(1 - 9z + 21z^2 - 21z^3 + 8z^4))H(2,y) + 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(8y^4 + 2y^3(-11 + 6z)$$
$$+ 12y^2(2 - 3z + 2z^2) + y(-13 + 42z - 57z^2 + 24z^3) + 3(1 - 9z + 21z^2 - 21z^3 + 8z^4))H(2,y)^2 + H(1,z)(32(-1$$
$$+ y)^2 y(-1+z)z(-6(-1+z)^2 z^4(1 - z + z^2) + 6y^7(1 - 10z + 9z^2) + 3y^6(-4 + 54z - 155z^2 + 104z^3) + 3y^5(4$$
$$- 52z + 279z^2 - 497z^3 + 262z^4) + yz^3(30 + 8z - 182z^2 + 321z^3 - 255z^4 + 78z^5) + y^2 z^2(48 + 38z - 650z^2$$
$$+ 1338z^3 - 1185z^4 + 408z^5) + 2y^3 z(15 + 28z - 475z^2 + 1158z^3 - 1185z^4 + 453z^5) + 2y^4(-3 + 22z - 301z^2$$
$$+ 981z^3 - 1260z^4 + 552z^5)) + 96(-1+y)^2 y(-1+z)^2 z^2(y+z)^4(-14 + 39z - 41z^2 + 16z^3 + 6y^2(-1 + 2z) + 3y(4$$
$$- 9z + 4z^2))H(2,y) - 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z)$$
$$+ 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,y)) - 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(16y^4 + 4y^3(-11$$
$$+ 6z) + 6y^2(8 - 9z + 2z^2) - 3(-2 + 4z - 3z^2 + z^3) + y(-26 + 48z - 33z^2 + 12z^3))H(0,0,y) - 96(-1+y)^2 y(-1$$
$$+ z)^2 z^2(y+z)^4(-14 + 39z - 41z^2 + 16z^3 + 6y^2(-1 + 2z) + 3y(4 - 9z + 4z^2))H(0,0,z) - 96(-1+y)^2 y(-1$$
$$+ z)^2 z(y+z)^4(2 + 16y^4 - 4z + 3z^2 - z^3 + 2y^3(-21 + 8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2$$
$$+ 4z^3))H(0,1,z) - 96(-1+y)^2 y(-1+z)^2 z(-1+y+z)(y+z)^4(-4 + 18z - 27z^2 + 16z^3 + y^2(-2 + 8z) + 2y(2$$
$$- 7z + 2z^2))H(0,2,y) + 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(2 + y^2(3 - 9z) - 4z + 3z^2 - z^3 + y^3(-1 + 4z) + y(-4$$
$$+ 12z - 9z^2 + 4z^3))H(1,0,z) - 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43$$
$$+ 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(1,1,z) - 96(-1+y)^2 y(-1+z)^2 z(-1+y+z)(y$$
$$+ z)^4(-4 + 18z - 27z^2 + 16z^3 + y^2(-2 + 8z) + 2y(2 - 7z + 2z^2))H(2,0,y) - 192(-1+y)^2 y(-1+z)^2 z(y$$
$$+ z)^4(8y^4 + 2y^3(-11 + 6z) + 12y^2(2 - 3z + 2z^2) + y(-13 + 42z - 57z^2 + 24z^3) + 3(1 - 9z + 21z^2 - 21z^3$$
$$+ 8z^4))H(2,2,y)) + 264(-1+y)^2 y(-1+z)^2 z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z)$$
$$+ 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,2,y) + H(0,y)(-88(-1+y)y(-1+z)^2 z(y+z)^4(y^4(-5$$
$$+ 44z) + 4y^3(5 - 29z + 6z^2) + 5(-2 + 4z - 3z^2 + z^3) - 5y(-6 + 16z - 12z^2 + 5z^3) + y^2(-35 + 132z - 72z^2$$
$$+ 20z^3)) + 48(-1+y)^2 y(-1+z)^2 z(y+z)^4(6 - 26z + 48z^2 - 44z^3 + 16z^4 + 3y^3(-1 + 4z) + 3y^2(3 - 11z + 4z^2)$$
$$+ 6y(-2 + 8z - 9z^2 + 4z^3))H(0,z)^2 + 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(3 + 24y^4 - 13z + 24z^2 - 22z^3 + 8z^4$$
$$+ 3y^3(-21 + 8z) + 3y^2(21 - 19z + 8z^2) + 3y(-9 + 14z - 12z^2 + 4z^3))H(1,z)^2 + 32(-1+y)y(-1+z)^2 z(y^8(-6$$
$$+ 78z) + 3y^7(6 - 85z + 136z^2) + 6z^4(-1 + 2z - 2z^2 + z^3) + 3y^6(-8 + 107z - 395z^2 + 302z^3) + 2yz^3(15 + 22z$$
$$- 78z^2 + 81z^3 - 30z^4) + 2y^5(9 - 91z + 669z^2 - 1185z^3 + 552z^4) + y^2 z^2(48 + 56z - 602z^2 + 837z^3 - 465z^4$$
$$+ 54z^5) + y^3 z(30 + 38z - 950z^2 + 1962z^3 - 1491z^4 + 312z^5) + y^4(-6 + 8z - 650z^2 + 2316z^3 - 2520z^4$$
$$+ 786z^5))H(2,y) + 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(1 + 8y^4 - 9z + 21z^2 - 21z^3 + 8z^4 + y^3(-21 + 8z) + 3y^2(7$$
$$- 7z + 4z^2) + y(-9 + 18z - 21z^2 + 8z^3))H(2,y)^2 + H(0,z)(-264(-1+y)^2 y(-1+z)^2 z(y+z)^4(2 + y^2(3 - 9z)$$
$$- 4z + 3z^2 - z^3 + y^3(-1 + 4z) + y(-4 + 12z - 9z^2 + 4z^3)) + 96(-1+y)^2 y(-1+z)^2 z(-1+y+z)(y+z)^4(16y^3$$
$$+ y^2(-27 + 4z) - 2(2 - 2z + z^2) + 2y(9 - 7z + 4z^2))H(1,z) + 96(-1+y)^2 y(-1+z)^2 z(-1+y+z)(y+z)^4(-4$$
$$+ 18z - 27z^2 + 16z^3 + y^2(-2 + 8z) + 2y(2 - 7z + 2z^2))H(2,y)) + H(1,z)(-264(-1+y)^2 y(-1+z)^2 z(y+z)^4(2$$
$$+ 16y^4 - 4z + 3z^2 - z^3 + 2y^3(-21 + 8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2 + 4z^3)) + 96(-1+y)^2 y^2(-1$$
$$+ z)^2 z(y+z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(2,y) - 96(-1+y)^2 y(-1$$
$$+ z)^2 z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z$$
$$- 51z^2 + 20z^3))H(3,y)) - 96(-1+y)^2 y^2(-1+z)^2 z(y+z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13$$
$$- 9z + 4z^2))H(0,0,y) - 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(6 - 26z + 48z^2 - 44z^3 + 16z^4 + 3y^3(-1 + 4z) + 3y^2(3$$
$$- 11z + 4z^2) + 6y(-2 + 8z - 9z^2 + 4z^3))H(0,0,z) - 96(-1+y)^2 y(-1+z)^2 z(-1+y+z)(y+z)^4(16y^3 + y^2(-27$$
$$+ 4z) - 2(2 - 2z + z^2) + 2y(9 - 7z + 4z^2))H(0,1,z) + 96(-1+y)^2 y(-1+z)^2 z(y+z)^4(2 + 16y^4 - 4z + 3z^2$$
$$- z^3 + 2y^3(-21 + 8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2 + 4z^3))H(0,2,y) - 96(-1+y)^2 y^2(-1+z)^2 z(y$$

$$+ z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(1,0,y) - 96(-1+y)^2y(-1+z)^2z(-1+y$$
$$+ z)(y+z)^4(16y^3 + y^2(-27 + 4z) - 2(2 - 2z + z^2) + 2y(9 - 7z + 4z^2))H(1,0,z) - 192(-1+y)^2y(-1+z)^2z(y$$
$$+ z)^4(3 + 24y^4 - 13z + 24z^2 - 22z^3 + 8z^4 + 3y^3(-21 + 8z) + 3y^2(21 - 19z + 8z^2) + 3y(-9 + 14z - 12z^2$$
$$+ 4z^3))H(1,1,z) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 16y^4 - 4z + 3z^2 - z^3 + 2y^3(-21 + 8z) + 6y^2(7 - 6z$$
$$+ 2z^2) + y(-18 + 24z - 15z^2 + 4z^3))H(2,0,y) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3$$
$$+ 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(2,2,y) - 96(-1+y)^2y(-1$$
$$+ z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z$$
$$- 51z^2 + 20z^3))H(3,2,y)) + H(2,y)(-44(-1+y)^2y(-1+z)^2z(y^7(-9 + 84z) + 3y^6(9 - 83z + 144z^2) - 9z^4(-2$$
$$+ 4z - 3z^2 + z^3) + 3y^5(-12 + 92z - 373z^2 + 324z^3) + yz^3(-36 - 136z + 276z^2 - 249z^3 + 84z^4) + y^2z^2(-60$$
$$- 292z + 1053z^2 - 1119z^3 + 432z^4) + y^3z(-36 - 292z + 1608z^2 - 2175z^3 + 972z^4) + y^4(18 - 136z + 1053z^2 - 2175z^3$$
$$+ 1248z^4)) - 96(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z$$
$$+ 4z^2))H(0,0,y) - 288(-1+y)^2y(-1+z)^2z(y+z)^4(y^3(-1 + 4z) + 4y(-1+z)^2(-1 + 4z) + 3y^2(1 - 5z + 4z^2)$$
$$+ 2(1 - 9z + 21z^2 - 21z^3 + 8z^4))H(0,0,z) - 96(-1+y)^2y(-1+z)^2z^2(y+z)^4(-14 + 39z - 41z^2 + 16z^3$$
$$+ 6y^2(-1 + 2z) + 3y(4 - 9z + 4z^2))H(0,1,z) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 16y^4 - 4z + 3z^2 - z^3$$
$$+ 2y^3(-21 + 8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2 + 4z^3))H(0,2,y) - 96(-1+y)^2y^2(-1+z)^2z(y$$
$$+ z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(1,0,y) - 96(-1+y)^2y(-1+z)^2z^2(y$$
$$+ z)^4(-14 + 39z - 41z^2 + 16z^3 + 6y^2(-1 + 2z) + 3y(4 - 9z + 4z^2))H(1,0,z) + 192(-1+y)^2y(-1+z)^2z(y$$
$$+ z)^4(13 + 380y^4 - 225z + 747z^2 - 915z^3 + 380z^4 + y^3(-915 + 584z) + 9y^2(83 - 117z + 64z^2) + y(-225 + 654z$$
$$- 1053z^2 + 584z^3))H(1,1,z) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 16y^4 - 4z + 3z^2 - z^3 + 2y^3(-21 + 8z)$$
$$+ 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2 + 4z^3))H(2,0,y) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z$$
$$+ 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(2,2,y) - 96(-1$$
$$+ y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22$$
$$+ 48z - 51z^2 + 20z^3))H(3,2,y)) + H(1,z)(-44(-1+y)^2y(-1+z)^2z(y^7(-9 + 84z) + 3y^6(9 - 83z + 144z^2)$$
$$- 9z^4(-2 + 4z - 3z^2 + z^3) + 3y^5(-12 + 92z - 373z^2 + 324z^3) + yz^3(-36 - 136z + 276z^2 - 249z^3 + 84z^4)$$
$$+ y^2z^2(-60 - 292z + 1053z^2 - 1119z^3 + 432z^4) + y^3z(-36 - 292z + 1608z^2 - 2175z^3 + 972z^4) + y^4(18 - 136z$$
$$+ 1053z^2 - 2175z^3 + 1248z^4)) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(13 + 380y^4 - 225z + 747z^2 - 915z^3 + 380z^4$$
$$+ y^3(-915 + 584z) + 9y^2(83 - 117z + 64z^2) + y(-225 + 654z - 1053z^2 + 584z^3))H(2,y)^2 + 264(-1+y)^2y(-1$$
$$+ z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z$$
$$- 51z^2 + 20z^3))H(3,y) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z)$$
$$+ 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(2,y)H(3,y) - 288(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 16y^4$$
$$- 4z + 3z^2 - z^3 + 2y^3(-21 + 8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2 + 4z^3))H(0,0,y) - 96(-1+y)^2y(-1$$
$$+ z)^2z^2(y+z)^4(-14 + 39z - 41z^2 + 16z^3 + 6y^2(-1 + 2z) + 3y(4 - 9z + 4z^2))H(0,0,z) - 96(-1+y)^2y(-1$$
$$+ z)^2z(y+z)^4(2 + 16y^4 - 4z + 3z^2 - z^3 + 2y^3(-21 + 8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2$$
$$+ 4z^3))H(0,1,z) - 96(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z$$
$$+ 4z^2))H(0,2,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z)$$
$$+ 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(0,3,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + y^2(3 - 9z)$$
$$- 4z + 3z^2 - z^3 + y^3(-1 + 4z) + y(-4 + 12z - 9z^2 + 4z^3))H(1,0,z) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4$$
$$- 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(1,1,z)$$
$$- 96(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41 + 12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(2,0,y)$$
$$+ 192(-1+y)^2y(-1+z)^2z(y+z)^4(13 + 380y^4 - 225z + 747z^2 - 915z^3 + 380z^4 + y^3(-915 + 584z) + 9y^2(83 - 117z$$
$$+ 64z^2) + y(-225 + 654z - 1053z^2 + 584z^3))H(2,2,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2$$
$$- 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(2,3,y) + 96(-1+y)^2y(-1$$
$$+ z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z$$
$$- 51z^2 + 20z^3))H(3,0,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43$$
$$+ 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,2,y)) + 48(-1+y)^2y(-1+z)^2z(y+z)^4(56$$
$$+ 596y^4 + 34z - 396z^2 + 550z^3 - 244z^4 + y^3(-1661 + 1420z) + 3y^2(511 - 769z + 204z^2) - 4y(131 - 237z + 51z^2$$
$$+ 38z^3))H(0,0,0,y) - 48(-1+y)^2y(-1+z)^2z(y+z)^4(-56 + 244y^4 + 524z - 1533z^2 + 1661z^3 - 596z^4$$
$$+ 2y^3(-275 + 76z) + y^2(396 + 204z - 612z^2) - y(34 + 948z - 2307z^2 + 1420z^3))H(0,0,0,z) + 96(-1+y)^2y(-1$$
$$+ z)^2z(y+z)^4(2 + 32y^4 - 4z + 3z^2 - z^3 + y^3(-83 + 28z) + 3y^2(27 - 21z + 8z^2) + y(-32 + 36z - 21z^2$$
$$+ 4z^3))H(0,0,1,z) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(32y^4 + y^3(-85 + 36z) + 3y^2(29 - 27z + 8z^2) - 3(-2 + 4z$$

$$
\begin{aligned}
&- 3z^2 + z^3) + y(-40 + 60z - 39z^2 + 12z^3))H(0,0,2,y) + 96(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41+12z)\\
&- 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(0,1,0,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(-2 + 16y^4 + 8y^3(-5+z)\\
&+ 4z - 3z^2 + z^3 + 6y^2(6 - 3z + 2z^2) + y(-10 + 3z^2 - 4z^3))H(0,1,0,z) + 96(-1+y)^2y(-1+z)^2z(y\\
&+ z)^4(32y^4 + y^3(-85+36z) + 3y^2(29 - 27z + 8z^2) - 3(-2 + 4z - 3z^2 + z^3) + y(-40 + 60z - 39z^2\\
&+ 12z^3))H(0,1,1,z) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(32y^4 + y^3(-85+36z) + 3y^2(29 - 27z + 8z^2) - 3(-2\\
&+ 4z - 3z^2 + z^3) + y(-40 + 60z - 39z^2 + 12z^3))H(0,2,0,y) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 32y^4 - 4z\\
&+ 3z^2 - z^3 + y^3(-83+28z) + 3y^2(27 - 21z + 8z^2) + y(-32 + 36z - 21z^2 + 4z^3))H(0,2,2,y) + 96(-1+y)^2y(-1\\
&+ z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43+20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z\\
&- 51z^2 + 20z^3))H(0,3,2,y) + 192(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41+12z) - 2(7 - 6z + 3z^2)\\
&+ 3y(13 - 9z + 4z^2))H(1,0,0,y) - 288(-1+y)^2y(-1+z)^2z(y+z)^4(2 + y^2(3-9z) - 4z + 3z^2 - z^3 + y^3(-1\\
&+ 4z) + y(-4 + 12z - 9z^2 + 4z^3))H(1,0,0,z) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 16y^4 - 4z + 3z^2 - z^3\\
&+ 2y^3(-21+8z) + 6y^2(7 - 6z + 2z^2) + y(-18 + 24z - 15z^2 + 4z^3))H(1,0,1,z) + 96(-1+y)^2y^2(-1+z)^2z(y\\
&+ z)^4(16y^3 + y^2(-41+12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(1,0,2,y) - 96(-1+y)^2y(-1+z)^2z(y\\
&+ z)^4(2 + y^2(3-9z) - 4z + 3z^2 - z^3 + y^3(-1+4z) + y(-4 + 12z - 9z^2 + 4z^3))H(1,1,0,z) + 96(-1+y)^2y(-1\\
&+ z)^2z(y+z)^4(30 + 776y^4 - 472z + 1539z^2 - 1873z^3 + 776z^4 + y^3(-1873 + 1188z) + 3y^2(513 - 719z + 392z^2)\\
&+ y(-472 + 1356z - 2157z^2 + 1188z^3))H(1,1,1,z) + 96(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41+12z) - 2(7\\
&- 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(1,2,0,y) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(32y^4 + y^3(-85+36z) + 3y^2(29\\
&- 27z + 8z^2) - 3(-2 + 4z - 3z^2 + z^3) + y(-40 + 60z - 39z^2 + 12z^3))H(2,0,0,y) - 96(-1+y)^2y(-1+z)^2z(y\\
&+ z)^4(2 + 32y^4 - 4z + 3z^2 - z^3 + y^3(-83+28z) + 3y^2(27 - 21z + 8z^2) + y(-32 + 36z - 21z^2\\
&+ 4z^3))H(2,0,2,y) + 96(-1+y)^2y^2(-1+z)^2z(y+z)^4(16y^3 + y^2(-41+12z) - 2(7 - 6z + 3z^2) + 3y(13 - 9z\\
&+ 4z^2))H(2,1,0,y) - 96(-1+y)^2y(-1+z)^2z(y+z)^4(2 + 32y^4 - 4z + 3z^2 - z^3 + y^3(-83+28z) + 3y^2(27\\
&- 21z + 8z^2) + y(-32 + 36z - 21z^2 + 4z^3))H(2,2,0,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(30 + 776y^4 - 472z\\
&+ 1539z^2 - 1873z^3 + 776z^4 + y^3(-1873 + 1188z) + 3y^2(513 - 719z + 392z^2) + y(-472 + 1356z - 2157z^2\\
&+ 1188z^3))H(2,2,2,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43+20z)\\
&+ 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(2,3,2,y) + 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4\\
&- 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43+20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,0,2,y)\\
&+ 96(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43+20z) + 3y^2(15 - 17z + 8z^2)\\
&+ y(-22 + 48z - 51z^2 + 20z^3))H(3,2,0,y) + 192(-1+y)^2y(-1+z)^2z(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3\\
&+ 16z^4 + y^3(-43+20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,2,2,y)\Big\}\Big/\Big(192(-1+y)^2y^2(-1\\
&+ z)^2z^2(-1+y+z)(y+z)^4\Big);
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{3;C_F^2} = \Big\{ &- 18(-1+y)^2y(-1+z)^2z(y+z)^2(2 + y^2(3-9z) - 4z + 3z^2 - z^3 + y^3(-1+4z) + y(-4 + 12z - 9z^2 + 4z^3))\\
&- 12(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-44 + 528y^3 + 66z - 99z^2 + 64z^3 + y^2(-829 + 412z) + 2y(187 - 176z\\
&+ 58z^2))H(0,y)^3 - 12(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-44 + 64y^3 + 374z - 829z^2 + 528z^3 + y^2(-99\\
&+ 116z) + y(66 - 352z + 412z^2))H(0,z)^3 - 12(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(4 + 72y^3 + 46z - 131z^2\\
&+ 72z^3 + y^2(-131 + 108z) + 2y(23 - 50z + 54z^2))H(1,z)^3 - 2(432y^{10}(-1+z)^2 - 54(-1+z)^5z^4(-1+4z)\\
&+ 2y^9(-1+z)^2(-1125 + 1321z) + 3y^8(1590 - 7130z + 10312z^2 - 5603z^3 + 822z^4) + y^7(-5202 + 29970z - 47113z^2\\
&+ 16349z^3 + 14834z^4 - 8838z^5) + y(-1+z)^2z^3(-1653 + 4845z - 5811z^2 + 4198z^3 - 2002z^4 + 432z^5) + y^6(3006\\
&- 18611z + 14103z^2 + 65692z^3 - 134393z^4 + 90425z^5 - 20168z^6) - y^2(-1+z)^2z^2(792 - 14109z + 32355z^2 - 27545z^3\\
&+ 10016z^4 - 1898z^5 + 216z^6) + y^5(-828 + 945z + 35948z^2 - 178242z^3 + 329525z^4 - 282687z^5 + 110081z^6 - 14742z^7)\\
&+ y^4(72 + 4209z - 39045z^2 + 174583z^3 - 372816z^4 + 403037z^5 - 220595z^6 + 54578z^7 - 4050z^8) + y^3z(-1401 + 13065z\\
&- 78255z^2 + 213625z^3 - 291633z^4 + 209152z^5 - 75802z^6 + 11847z^7 - 598z^8))H(2,y)^2 - 12(-1+y)^2y(-1+z)^2z(-1\\
&+ y+z)(y+z)^2(4 + 72y^3 + 46z - 131z^2 + 72z^3 + y^2(-131 + 108z) + 2y(23 - 50z + 54z^2))H(2,y)^3\\
&+ H(0,z)^2(-108(-1+z)^6z^3(-1+4z) - 4y^9(-1+z)^2(216 + 215z) + y^8(4500 - 8947z - 426z^2 + 9495z^3 - 4692z^4)\\
&+ y(-1+z)^3z^2(-72 + 2469z - 8130z^2 + 15633z^3 - 12928z^4 + 864z^5) - y^7(9540 - 30281z + 21427z^2 + 17363z^3\\
&- 25789z^4 + 7740z^5) - 2y^6(-5202 + 24062z - 33217z^2 + 1427z^3 + 33635z^4 - 28049z^5 + 7274z^6) - y^2(-1+z)^2z(180\\
&+ 114z - 13297z^2 + 48726z^3 - 88199z^4 + 76698z^5 - 22076z^6 + 432z^7) - 4y^5(1503 - 9820z + 19651z^2 - 5048z^3\\
&- 33560z^4 + 51210z^5 - 31336z^6 + 7400z^7) + y^4(1656 - 15965z + 40998z^2 + 3632z^3 - 193426z^4 + 388782z^5 - 361522z^6\\
&+ 164423z^7 - 28648z^8) - y^3(144 - 2787z + 8149z^2 + 23010z^3 - 166174z^4 + 387958z^5 - 474394z^6 + 314077z^7 - 100103z^8
\end{aligned}
$$

$$+ 10120z^9) + 48(-1+y)^2y(y^2+2y(-1+z)+2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(1,z) + 48(-1$$
$$+ y)^2y(y^2+2y(-1+z)+2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(2,y)) + H(0,y)^2(864y^{10}(-1+z)^2$$
$$+ 108(-1+z)^5z^3(-1+4z) - 4y^9(-1+z)^2(1341+1234z) + y^8(14040-14991z-32938z^2+54743z^3-20800z^4)$$
$$+ y^7(-19944+36561z+52567z^2-160807z^3+117263z^4-25640z^5) - y(-1+z)^2z^2(-72+2577z-8383z^2+13377z^3$$
$$- 10501z^4+3020z^5) - 4y^6(-4104+13505z+4189z^2-53569z^3+65725z^4-30346z^5+4627z^6) + y^2(-1+z)^2z(-180$$
$$- 366z+14249z^2-35168z^3+44729z^4-26794z^5+4744z^6) - 2y^5(3834-21096z+12153z^2+79661z^3-164736z^4$$
$$+ 132075z^5-49685z^6+7794z^7) + y^4(1800-16449z+20344z^2+84562z^3-275038z^4+340574z^5-219676z^6+73813z^7$$
$$- 9876z^8) - y^3(144-2823z+4349z^2+40704z^3-163328z^4+272002z^5-250136z^6+128747z^7-31815z^8+2156z^9)$$
$$+ 48(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(1,z) + 48(-1+y)^2y(-1$$
$$+ 4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(2,y)) + H(1,z)^2(-2(432y^{10}(-1+z)^2$$
$$- 54(-1+z)^5z^4(-1+4z) + 2y^9(-1+z)^2(-1125+1321z) + 3y^8(1590-7130z+10312z^2-5603z^3+822z^4)$$
$$+ y^7(-5202+29970z-47113z^2+16349z^3+14834z^4-8838z^5) + y(-1+z)^2z^3(-1653+4845z-5811z^2+4198z^3$$
$$- 2002z^4+432z^5) + y^6(3006-18611z+14103z^2+65692z^3-134393z^4+90425z^5-20168z^6) - y^2(-1+z)^2z^2(792$$
$$- 14109z+32355z^2-27545z^3+10016z^4-1898z^5+216z^6) + y^5(-828+945z+35948z^2-178242z^3+329525z^4$$
$$- 282687z^5+110081z^6-14742z^7) + y^4(72+4209z-39045z^2+174583z^3-372816z^4+403037z^5-220595z^6+54578z^7$$
$$- 4050z^8) + y^3z(-1401+13065z-78255z^2+213625z^3-291633z^4+209152z^5-75802z^6+11847z^7-598z^8)) - 60(-1$$
$$+ y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+56y^3+50z-109z^2+56z^3+y^2(-109+84z)+y(50-92z$$
$$+ 84z^2))H(2,y) - 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z)$$
$$+ 2y(7-10z+6z^2))H(3,y)) - 2(864y^{10}(-1+z)^2+108(-1+z)^5z^3(-1+4z)-4y^9(-1+z)^2(1341+1234z)$$
$$+ y^8(14040-14991z-32938z^2+54743z^3-20800z^4) + y^7(-19944+36561z+52567z^2-160807z^3+117263z^4-25640z^5)$$
$$- y(-1+z)^2z^2(-72+2577z-8383z^2+13377z^3-10501z^4+3020z^5) - 4y^6(-4104+13505z+4189z^2-53569z^3$$
$$+ 65725z^4-30346z^5+4627z^6) + y^2(-1+z)^2z(-180-366z+14249z^2-35168z^3+44729z^4-26794z^5+4744z^6)$$
$$- 2y^5(3834-21096z+12153z^2+79661z^3-164736z^4+132075z^5-49685z^6+7794z^7) + y^4(1800-16449z+20344z^2$$
$$+ 84562z^3-275038z^4+340574z^5-219676z^6+73813z^7-9876z^8) - y^3(144-2823z+4349z^2+40704z^3-163328z^4$$
$$+ 272002z^5-250136z^6+128747z^7-31815z^8+2156z^9))H(0,0,y) + 2(108(-1+z)^6z^3(-1+4z) + 4y^9(-1+z)^2(216$$
$$+ 215z) + y^8(-4500+8947z+426z^2-9495z^3+4692z^4) - y(-1+z)^3z^2(-72+2469z-8130z^2+15633z^3-12928z^4$$
$$+ 864z^5) + y^7(9540-30281z+21427z^2+17363z^3-25789z^4+7740z^5) + 2y^6(-5202+24062z-33217z^2+1427z^3$$
$$+ 33635z^4-28049z^5+7274z^6) + y^2(-1+z)^2z(180+114z-13297z^2+48726z^3-88199z^4+76698z^5-22076z^6$$
$$+ 432z^7) + 4y^5(1503-9820z+19651z^2-5048z^3-33560z^4+51210z^5-31336z^6+7400z^7) + y^4(-1656+15965z$$
$$- 40998z^2-3632z^3+193426z^4-388782z^5+361522z^6-164423z^7+28648z^8) + y^3(144-2787z+8149z^2+23010z^3$$
$$- 166174z^4+387958z^5-474394z^6+314077z^7-100103z^8+10120z^9))H(0,0,z) - 48(-1+y)^2y^2z^2(-1+y+z)(y$$
$$+ z)^2(-12+30z-26z^2+8z^3+3y(4-9z+4z^2))H(0,1,z) - 48y^2(-1+z)^2z^2(-1+y+z)(y+z)^2(8y^3+y(30$$
$$- 27z)+12(-1+z)+2y^2(-13+6z))H(0,2,y) - 48(-1+y)^2y^2z^2(-1+y+z)(y+z)^2(-12+30z-26z^2+8z^3$$
$$+ 3y(4-9z+4z^2))H(1,0,z) + 4(432y^{10}(-1+z)^2-54(-1+z)^5z^4(-1+4z)+2y^9(-1+z)^2(-1125+1321z)$$
$$+ 3y^8(1590-7130z+10312z^2-5603z^3+822z^4) + y^7(-5202+29970z-47113z^2+16349z^3+14834z^4-8838z^5)$$
$$+ y(-1+z)^2z^3(-1653+4845z-5811z^2+4198z^3-2002z^4+432z^5) + y^6(3006-18611z+14103z^2+65692z^3$$
$$- 134393z^4+90425z^5-20168z^6) - y^2(-1+z)^2z^2(792-14109z+32355z^2-27545z^3+10016z^4-1898z^5+216z^6)$$
$$+ y^5(-828+945z+35948z^2-178242z^3+329525z^4-282687z^5+110081z^6-14742z^7) + y^4(72+4209z-39045z^2$$
$$+ 174583z^3-372816z^4+403037z^5-220595z^6+54578z^7-4050z^8) + y^3z(-1401+13065z-78255z^2+213625z^3$$
$$- 291633z^4+209152z^5-75802z^6+11847z^7-598z^8))H(1,1,z) + 192(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4$$
$$+ 8y^3+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z+6z^2))H(3,y)H(1,1,z) + H(0,y)(96(-1+y)^2y(-1$$
$$+ 4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(1,z)^2 + 48y^2(-1+z)^2z^2(-1+y+z)(y$$
$$+ z)^2(8y^3+y(30-27z)+12(-1+z)+2y^2(-13+6z))H(2,y) + 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y$$
$$+ z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(1,z)H(2,y) - 192(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2$$
$$+ 2y^2+2y(-2+z)-2z+z^2)H(1,1,z)) - 48y^2(-1+z)^2z^2(-1+y+z)(y+z)^2(8y^3+y(30-27z)+12(-1+z)$$
$$+ 2y^2(-13+6z))H(2,0,y) + 4(432y^{10}(-1+z)^2-54(-1+z)^5z^4(-1+4z)+2y^9(-1+z)^2(-1125+1321z)$$
$$+ 3y^8(1590-7130z+10312z^2-5603z^3+822z^4) + y^7(-5202+29970z-47113z^2+16349z^3+14834z^4-8838z^5)$$
$$+ y(-1+z)^2z^3(-1653+4845z-5811z^2+4198z^3-2002z^4+432z^5) + y^6(3006-18611z+14103z^2+65692z^3$$
$$- 134393z^4+90425z^5-20168z^6) - y^2(-1+z)^2z^2(792-14109z+32355z^2-27545z^3+10016z^4-1898z^5+216z^6)$$
$$+ y^5(-828+945z+35948z^2-178242z^3+329525z^4-282687z^5+110081z^6-14742z^7) + y^4(72+4209z-39045z^2$$

$$
\begin{aligned}
&+ 174583z^3 - 372816z^4 + 403037z^5 - 220595z^6 + 54578z^7 - 4050z^8) + y^3z(-1401 + 13065z - 78255z^2 + 213625z^3 \\
&- 291633z^4 + 209152z^5 - 75802z^6 + 11847z^7 - 598z^8))H(2,2,y) + H(0,z)(96(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1 \\
&+ z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(2,y)^2 + H(1,z)(48(-1+y)^2y^2z^2(-1+y+z)(y+z)^2(-12+30z \\
&- 26z^2 + 8z^3 + 3y(4 - 9z + 4z^2)) + 96(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y \\
&+ z)^2(-1+4z)H(2,y)) - 192(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1 \\
&+ 4z)H(2,2,y)) + H(2,y)(-96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z \\
&+ z^2)H(0,0,y) - 96(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(0,0,z) \\
&- 96(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(0,1,z) + 96(-1+y)^2y(-1 \\
&+ 4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(0,2,y) - 96(-1+y)^2y(-1+4y)(-1 \\
&+ z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(1,0,y) - 96(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1 \\
&+ z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(1,0,z) + 120(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+56y^3 \\
&+ 50z - 109z^2 + 56z^3 + y^2(-109+84z) + y(50 - 92z + 84z^2))H(1,1,z) + 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y \\
&+ z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(2,0,y) - 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3 \\
&+ 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z + 6z^2))H(2,2,y) - 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y \\
&+ z)^2(-4+8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z + 6z^2))H(3,2,y)) + H(1,z)(-60(-1+y)^2y(-1 \\
&+ z)^2z(-1+y+z)(y+z)^2(-4+56y^3 + 50z - 109z^2 + 56z^3 + y^2(-109+84z) + y(50 - 92z + 84z^2))H(2,y)^2 \\
&- 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z \\
&+ 6z^2))H(2,y)H(3,y) - 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z \\
&+ z^2)H(0,0,y) - 96(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(0,0,z) \\
&- 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(0,1,z) - 96(-1 \\
&+ y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(0,2,y) - 96(-1+y)^2y(-1 \\
&+ z)^2z(-1+y+z)(y+z)^2(-4+8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z + 6z^2))H(1,1,z) - 96(-1 \\
&+ y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(2,0,y) + 120(-1+y)^2y(-1 \\
&+ z)^2z(-1+y+z)(y+z)^2(-4+56y^3 + 50z - 109z^2 + 56z^3 + y^2(-109+84z) + y(50 - 92z + 84z^2))H(2,2,y) \\
&+ 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z \\
&+ 6z^2))H(2,3,y) + 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) \\
&+ 2y(7 - 10z + 6z^2))H(3,2,y)) + 72(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-44+528y^3 + 66z - 99z^2 + 64z^3 \\
&+ y^2(-829+412z) + 2y(187 - 176z + 58z^2))H(0,0,0,y) + 72(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-44+64y^3 \\
&+ 374z - 829z^2 + 528z^3 + y^2(-99+116z) + y(66 - 352z + 412z^2))H(0,0,0,z) + 192(-1+y)^2y(-1+4y)(-1 \\
&+ z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(0,1,1,z) - 192(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y \\
&+ z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(0,2,2,y) + 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y \\
&+ z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(1,0,1,z) + 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2 \\
&+ 2y^2+2y(-2+z) - 2z + z^2)H(1,0,2,y) + 72(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-12+104y^3 + 102z \\
&- 207z^2 + 104z^3 + 3y^2(-69+52z) + 6y(17 - 30z + 26z^2))H(1,1,1,z) + 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y \\
&+ z)(y+z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(1,2,0,y) - 192(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y \\
&+ z)^2(2+2y^2+2y(-2+z) - 2z + z^2)H(2,0,2,y) + 96(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2 \\
&+ 2y^2+2y(-2+z) - 2z + z^2)H(2,1,0,y) - 192(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2 \\
&+ 2y(-2+z) - 2z + z^2)H(2,2,0,y) + 72(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-12+104y^3 + 102z - 207z^2 \\
&+ 104z^3 + 3y^2(-69+52z) + 6y(17 - 30z + 26z^2))H(2,2,2,y) + 96(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4 \\
&+ 8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z + 6z^2))H(2,3,2,y) + 192(-1+y)^2y(-1+z)^2z(-1+y \\
&+ z)(y+z)^2(-4+8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7 - 10z + 6z^2))H(3,2,2,y)\Big\}\Big/\Big(48(-1+y)^2y^2(-1 \\
&+ z)^2z^2(-1+y+z)(y+z)^2\Big);
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{3;n_f^2} = &\Big\{4(5y^7(-1+4z) + y^6(15 - 83z + 80z^2) - 5z^4(-2 + 4z - 3z^2 + z^3) + y^5(-20 + 147z - 273z^2 + 140z^3) + yz^3(40 \\
&- 154z + 147z^2 - 83z^3 + 20z^4) + y^2z^2(60 - 254z + 363z^2 - 273z^3 + 80z^4) + y^3z(40 - 254z + 462z^2 - 439z^3 \\
&+ 140z^4) + y^4(10 - 154z + 363z^2 - 439z^3 + 160z^4)) - 9(y+z)^4(2 + y^2(3 - 9z) - 4z + 3z^2 - z^3 + y^3(-1+4z) \\
&+ y(-4 + 12z - 9z^2 + 4z^3))H(0,y) - 9(y+z)^4(2 + y^2(3 - 9z) - 4z + 3z^2 - z^3 + y^3(-1+4z) + y(-4 + 12z - 9z^2 \\
&+ 4z^3))H(0,z) + 36yz(-5y^3 + 3y^4 + yz^2 + y^2(6 + z - 6z^2) + z^2(6 - 5z + 3z^2))H(1,z) + 36yz(-5y^3 + 3y^4 \\
&+ yz^2 + y^2(6 + z - 6z^2) + z^2(6 - 5z + 3z^2))H(2,y)\Big\}\Big/\Big(216yz(-1+y+z)(y+z)^4\Big);
\end{aligned}
$$

$$\mathcal{A}^{(2)}_{3;C_AC_F} = \Big\{ -4(-1+y)y(-1+z)z(y+z)((-1+z)^2z(5450-5153z+2725z^2)+y^5(2725-10952z+7336z^2)+y^4(-10603+43407z-37764z^2+4960z^3)+y^3(18481-86606z+117680z^2-53624z^3+4960z^4)+y(5450-38046z+86747z^2-86606z^3+43407z^4-10952z^5)+y^2(-16053+86747z-157946z^2+117680z^3-37764z^4+7336z^5))+36(-1+y)^2y(-1+z)^2z(y+z)^2(272+3128y^4-470z+417z^2-419z^3+200z^4+y^3(-8003+5572z)+3y^2(2355-3089z+936z^2)+y(-2462+4440z-2739z^2+772z^3))H(0,y)^3+36(-1+y)^2y(-1+z)^2z(y+z)^2(272+200y^4-2462z+7065z^2-8003z^3+3128z^4+y^3(-419+772z)+3y^2(139-913z+936z^2)+y(-470+4440z-9267z^2+5572z^3))H(0,z)^3+36(-1+y)^2y(-1+z)^2z(y+z)^2(60+1048y^4-1190z+3387z^2-3305z^3+1048z^4+y^3(-3305+2652z)+y^2(3387-5793z+3624z^2)+y(-1190+3984z-5793z^2+2652z^3))H(1,z)^3+9(2592y^{10}(-1+z)^2-324(-1+z)^5z^4(-1+4z)+4y^9(-1+z)^2(-3375+1232z)+y^8(28620-84611z+64366z^2+10459z^3-18996z^4)+y(-1+z)^2z^3(-10284+31286z-47769z^2+47069z^3-22936z^4+2592z^5)-y^7(31212-112231z+31885z^2+231313z^3-265795z^4+83616z^5)+y^6(18036-61763z-163792z^2+837127z^3-1162130z^4+669294z^5-136448z^6)-y^2(-1+z)^2z^2(8364-92610z+217312z^2-239107z^3+137650z^4-33236z^5+1296z^6)-y^5(4968+12031z-343845z^2+1384321z^3-2358789z^4+1969504z^5-787230z^6+119040z^7)+y^4(432+28202z-276976z^2+1189366z^3-2507872z^4+2799861z^5-1679342z^6+504259z^7-58092z^8)+y^3z(-8772+93570z-525300z^2+1423618z^3-2064667z^4+1697887z^5-784219z^6+182395z^7-14512z^8))H(2,y)^2+36(-1+y)^2y(-1+z)^2z(y+z)^2(60+1048y^4-1190z+3387z^2-3305z^3+1048z^4+y^3(-3305+2652z)+y^2(3387-5793z+3624z^2)+y(-1190+3984z-5793z^2+2652z^3))H(2,y)^3+H(0,z)^2(9(324(-1+z)^6z^3(-1+4z)+16y^9(-1+z)^2(162+73z)+y^8(-13500+34093z-26250z^2+5391z^3+1180z^4)+y^7(28620-106481z+143631z^2-77501z^3+5003z^4+6728z^5)-2y(-1+z)^2z^2(108-3068z+15065z^2-37089z^3+46088z^4-22256z^5+1296z^6)+y^6(-31212+163509z-312112z^2+240300z^3-456z^4-105981z^5+44124z^6)+2y^2(-1+z)^2z(270-1316z-14807z^2+70951z^3-142240z^4+124888z^5-36250z^6+648z^7)+y^5(18036-132635z+331953z^2-300344z^3-147828z^4+523715z^5-390745z^6+97848z^7)+y^4(-4968+54838z-176524z^2+144129z^3+393770z^4-1110625z^5+1146516z^6-538862z^7+92640z^8)+y^3(432-9848z+42974z^2+4423z^3-410951z^4+1156123z^5-1515467z^6+1025284z^7-326466z^8+33496z^9))-864(-1+y)^2y(-1+z)^2z(y+z)^2(y^3(-1+4z)+4y(-1+z)^2(-1+4z)+3y^2(1-5z+4z^2)+2(1-8z+18z^2-17z^3+6z^4))H(1,z)-1728(-1+y)^2y(-1+z)^2z(y+z)^2(2-15z+33z^2-30z^3+10z^4+y^3(-1+4z)+4y(-1+z)^2(-1+4z)+3y^2(1-5z+4z^2))H(2,y))+H(0,y)^2(-9(2592y^{10}(-1+z)^2+324(-1+z)^5z^3(-1+4z)-4y^9(-1+z)^2(4023+4486z)-6y^8(-7020+5368z+21840z^2-31731z^3+11516z^4)+y^7(-59832+90672z+223220z^2-565474z^3+397382z^4-85968z^5)-y(-1+z)^2z^2(-216+6244z-21180z^2+34761z^3-27075z^4+7648z^5)+y^6(49248-151456z-105440z^2+735113z^3-850650z^4+378865z^5-56004z^6)+y^2(-1+z)^2z(-540+1876z+30168z^2-80106z^3+101421z^4-58502z^5+11408z^6)-y^5(23004-128508z+71864z^2+470215z^3-932695z^4+701645z^5-235797z^6+30272z^7)+y^4(5400-53988z+88836z^2+166115z^3-638606z^4+766830z^5-456762z^6+139069z^7-16732z^8)-y^3(432-9956z+29272z^2+57505z^3-334959z^4+576238z^5-518670z^6+256651z^7-61569z^8+5056z^9))-432(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(0,z)-1728(-1+y)^2y(-1+z)^2z(y+z)^2(2+10y^4-4z+3z^2-z^3+2y^3(-15+8z)+3y^2(11-12z+4z^2)+y(-15+24z-15z^2+4z^3))H(1,z)-864(-1+y)^2y(-1+z)^2z(y+z)^2(2+12y^4-4z+3z^2-z^3+2y^3(-17+8z)+12y^2(3-3z+z^2)+y(-16+24z-15z^2+4z^3))H(2,y))+H(1,z)^2(9(2592y^{10}(-1+z)^2-324(-1+z)^5z^4(-1+4z)+4y^9(-1+z)^2(-3375+1232z)+y^8(28620-84611z+64366z^2+10459z^3-18996z^4)+y(-1+z)^2z^3(-10284+31286z-47769z^2+47069z^3-22936z^4+2592z^5)-y^7(31212-112231z+31885z^2+231313z^3-265795z^4+83616z^5)+y^6(18036-61763z-163792z^2+837127z^3-1162130z^4+669294z^5-136448z^6)-y^2(-1+z)^2z^2(8364-92610z+217312z^2-239107z^3+137650z^4-33236z^5+1296z^6)-y^5(4968+12031z-343845z^2+1384321z^3-2358789z^4+1969504z^5-787230z^6+119040z^7)+y^4(432+28202z-276976z^2+1189366z^3-2507872z^4+2799861z^5-1679342z^6+504259z^7-58092z^8)+y^3z(-8772+93570z-525300z^2+1423618z^3-2064667z^4+1697887z^5-784219z^6+182395z^7-14512z^8))+108(-1+y)^2y(-1+z)^2z(y+z)^2(124+1240y^4-1510z+4011z^2-3865z^3+1240z^4+y^3(-3865+2972z)+y^2(4011-6609z+4008z^2)+y(-1510+4752z-6609z^2+2972z^3))H(2,y)+1728(-1+y)^2y(-1+z)^2z(y+z)^2(4+12y^4-20z+39z^2-35z^3+12z^4+5y^3(-7+4z)+3y^2(13-17z+8z^2)+y(-20+48z-51z^2+20z^3))H(3,y))+18(2592y^{10}(-1+z)^2+324(-1+z)^5z^3(-1+4z)-4y^9(-1+z)^2(4023+4486z)-6y^8(-7020+5368z+21840z^2-31731z^3+11516z^4)+y^7(-59832+90672z+223220z^2-565474z^3+397382z^4-85968z^5)-y(-1+z)^2z^2(-216+6244z-21180z^2+34761z^3-27075z^4+7648z^5)+y^6(49248-151456z-105440z^2+735113z^3-850650z^4+378865z^5-56004z^6)+y^2(-1+z)^2z(-540+1876z+30168z^2-80106z^3+101421z^4-58502z^5+11408z^6)-y^5(23004-128508z+71864z^2+470215z^3$$

$$
\begin{aligned}
&- 932695z^4 + 701645z^5 - 235797z^6 + 30272z^7) + y^4(5400 - 53988z + 88836z^2 + 166115z^3 - 638606z^4 + 766830z^5 \\
&- 456762z^6 + 139069z^7 - 16732z^8) - y^3(432 - 9956z + 29272z^2 + 57505z^3 - 334959z^4 + 576238z^5 - 518670z^6 \\
&+ 256651z^7 - 61569z^8 + 5056z^9))H(0,0,y) - 18(324(-1+z)^6z^3(-1+4z) + 16y^9(-1+z)^2(162+73z) + y^8(-13500 \\
&+ 34093z - 26250z^2 + 5391z^3 + 1180z^4) + y^7(28620 - 106481z + 143631z^2 - 77501z^3 + 5003z^4 + 6728z^5) - 2y(-1 \\
&+ z)^2z^2(108 - 3068z + 15065z^2 - 37089z^3 + 46088z^4 - 22256z^5 + 1296z^6) + y^6(-31212 + 163509z - 312112z^2 \\
&+ 240300z^3 - 456z^4 - 105981z^5 + 44124z^6) + 2y^2(-1+z)^2z(270 - 1316z - 14807z^2 + 70951z^3 - 142240z^4 \\
&+ 124888z^5 - 36250z^6 + 648z^7) + y^5(18036 - 132635z + 331953z^2 - 300344z^3 - 147828z^4 + 523715z^5 - 390745z^6 \\
&+ 97848z^7) + y^4(-4968 + 54838z - 176524z^2 + 144129z^3 + 393770z^4 - 1110625z^5 + 1146516z^6 - 538862z^7 + 92640z^8) \\
&+ y^3(432 - 9848z + 42974z^2 + 4423z^3 - 410951z^4 + 1156123z^5 - 1515467z^6 + 1025284z^7 - 326466z^8 \\
&+ 33496z^9))H(0,0,z) - 216(-1+y)^2yz(88y^6(-1+z)^2 - (-1+z)^3z^2(14 - 14z + 3z^2) + y^5(-278 + 828z - 816z^2 \\
&+ 272z^3) - y(-1+z)^2z(-68 + 202z - 114z^2 - 61z^3 + 60z^4) + 2y^4(157 - 686z + 1012z^2 - 591z^3 + 114z^4) \\
&- y^3(138 - 984z + 2141z^2 - 1802z^3 + 433z^4 + 68z^5) + y^2(14 - 340z + 1129z^2 - 1351z^3 + 427z^4 + 293z^5 \\
&- 172z^6))H(0,1,z) + 216y(-1+z)^2z(88y^8 + y^7(-470 + 456z) + 2y^6(511 - 1064z + 482z^2) + z^2(30 - 60z + 49z^2 \\
&- 19z^3) + y^5(-1156 + 4028z - 3857z^2 + 1036z^3) + yz(20 - 342z + 666z^2 - 509z^3 + 162z^4) + y^4(716 - 3888z \\
&+ 6107z^2 - 3483z^3 + 564z^4) + y^2(30 - 432z + 1885z^2 - 2545z^3 + 1441z^4 - 273z^5) + y^3(-230 + 1944z - 4787z^2 \\
&+ 4380z^3 - 1557z^4 + 124z^5))H(0,2,y) - 2376(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2 \\
&+ 2y(-2+z) - 2z + z^2)H(1,0,y) + 432(-1+y)^2yz(-4(-1+z)^3z^2(1-z+z^2) + y^5(-4 + 48z - 87z^2 + 40z^3) \\
&+ 4y(-1+z)^2z(-3 - 5z + 27z^2 - 31z^3 + 13z^4) + y^4(8 - 128z + 407z^2 - 465z^3 + 172z^4) + y^3(-8 + 124z \\
&- 552z^2 + 1046z^3 - 889z^4 + 276z^5) + y^2(4 - 28z + 244z^2 - 760z^3 + 1079z^4 - 735z^5 + 196z^6))H(1,0,z) \\
&- 18(2592y^{10}(-1+z)^2 - 324(-1+z)^5z^4(-1+4z) + 4y^9(-1+z)^2(-3375 + 1232z) + y^8(28620 - 84611z + 64366z^2 \\
&+ 10459z^3 - 18996z^4) + y(-1+z)^2z^3(-10284 + 31286z - 47769z^2 + 47069z^3 - 22936z^4 + 2592z^5) - y^7(31212 \\
&- 112231z + 31885z^2 + 231313z^3 - 265795z^4 + 83616z^5) + y^6(18036 - 61763z - 163792z^2 + 837127z^3 - 1162130z^4 \\
&+ 669294z^5 - 136448z^6) - y^2(-1+z)^2z^2(8364 - 92610z + 217312z^2 - 239107z^3 + 137650z^4 - 33236z^5 + 1296z^6) \\
&- y^5(4968 + 12031z - 343845z^2 + 1384321z^3 - 2358789z^4 + 1969504z^5 - 787230z^6 + 119040z^7) + y^4(432 + 28202z \\
&- 276976z^2 + 1189366z^3 - 2507872z^4 + 2799861z^5 - 1679342z^6 + 504259z^7 - 58092z^8) + y^3z(-8772 + 93570z \\
&- 525300z^2 + 1423618z^3 - 2064667z^4 + 1697887z^5 - 784219z^6 + 182395z^7 - 14512z^8))H(1,1,z) + H(3,y)(-864(-1 \\
&+ y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z \\
&+ 6z^2))H(0,1,z) - 864(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z) \\
&+ 2y(7-10z+6z^2))H(1,0,z) - 3456(-1+y)^2y(-1+z)^2z(y+z)^2(4+12y^4-20z+39z^2-35z^3+12z^4 \\
&+ 5y^3(-7+4z)+3y^2(13-17z+8z^2)+y(-20+48z-51z^2+20z^3))H(1,1,z)) + 216y(-1+z)^2z(88y^8 \\
&+ y^7(-470+456z)+2y^6(511-1064z+482z^2)+z^2(30-60z+49z^2-19z^3)+y^5(-1156+4028z-3857z^2 \\
&+ 1036z^3)+yz(20-342z+666z^2-509z^3+162z^4)+y^4(716-3888z+6107z^2-3483z^3+564z^4)+y^2(30 \\
&- 432z+1885z^2-2545z^3+1441z^4-273z^5)+y^3(-230+1944z-4787z^2+4380z^3-1557z^4+124z^5))H(2,0,y) \\
&- 18(2592y^{10}(-1+z)^2 - 324(-1+z)^5z^4(-1+4z) + 4y^9(-1+z)^2(-3375+1232z) + y^8(28620 - 84611z + 64366z^2 \\
&+ 10459z^3 - 18996z^4) + y(-1+z)^2z^3(-10284 + 31286z - 47769z^2 + 47069z^3 - 22936z^4 + 2592z^5) - y^7(31212 \\
&- 112231z + 31885z^2 + 231313z^3 - 265795z^4 + 83616z^5) + y^6(18036 - 61763z - 163792z^2 + 837127z^3 - 1162130z^4 \\
&+ 669294z^5 - 136448z^6) - y^2(-1+z)^2z^2(8364 - 92610z + 217312z^2 - 239107z^3 + 137650z^4 - 33236z^5 + 1296z^6) \\
&- y^5(4968 + 12031z - 343845z^2 + 1384321z^3 - 2358789z^4 + 1969504z^5 - 787230z^6 + 119040z^7) + y^4(432 + 28202z \\
&- 276976z^2 + 1189366z^3 - 2507872z^4 + 2799861z^5 - 1679342z^6 + 504259z^7 - 58092z^8) + y^3z(-8772 + 93570z \\
&- 525300z^2 + 1423618z^3 - 2064667z^4 + 1697887z^5 - 784219z^6 + 182395z^7 - 14512z^8))H(2,2,y) + H(0,z)(1188(-1 \\
&+ y)^2y^2z^2(-1+y+z)(y+z)^2(-12+30z-26z^2+8z^3+3y(4-9z+4z^2)) - 864(-1+y)^2y(-1+z)^2z(y \\
&+ z)^2(4y^4+(-1+z)^3(-1+4z)+y^3(-13+8z)+3y^2(5-7z+4z^2)+y(-7+18z-21z^2+8z^3))H(1,z)^2 \\
&+ 2376(-1+y)^2y(y^2+2y(-1+z)+2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(2,y) - 864(-1 \\
&+ y)^2y(-1+z)^2z(y+z)^2(5+4y^4-39z+87z^2-81z^3+28z^4+y^3(-15+16z)+3y^2(7-17z+12z^2) \\
&+ y(-15+66z-93z^2+40z^3))H(2,y)^2 + H(1,z)(-432(-1+y)^2yz(-4(-1+z)^3z^2(1-z+z^2)+y^5(-4+48z \\
&- 87z^2+40z^3)+4y(-1+z)^2z(-3-5z+27z^2-31z^3+13z^4)+y^4(8-128z+407z^2-465z^3+172z^4) \\
&+ y^3(-8+124z-552z^2+1046z^3-889z^4+276z^5)+y^2(4-28z+244z^2-760z^3+1079z^4-735z^5+196z^6)) \\
&- 864(-1+y)^2y(-1+z)^2z(y+z)^2(2-28z+69z^2-67z^3+24z^4+y^3(-1+4z)+3y^2(1-7z+8z^2)+y(-4 \\
&+ 36z-63z^2+28z^3))H(2,y) + 864(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3 \\
&+ y^2(-19+12z)+2y(7-10z+6z^2))H(3,y)) + 864(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2
\end{aligned}
$$

$$+ 2y(-2+z) - 2z + z^2)H(0,0,y) + 864(-1+y)^2y(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y$$
$$+ z)^2(-1+4z)H(0,0,z) + 864(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z) - 2z$$
$$+ z^2)H(0,1,z) + 864(-1+y)^2y(-1+z)^2z(y+z)^2(4-18z+33z^2-27z^3+8z^4+y^3(-2+8z)+6y^2(1-4z$$
$$+ 2z^2) + y(-8+36z-45z^2+20z^3))H(0,2,y) + 864(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z$$
$$- 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7-10z+6z^2))H(1,1,z) + 864(-1+y)^2y(-1+z)^2z(y+z)^2(4-18z$$
$$+ 33z^2 - 27z^3 + 8z^4 + y^3(-2+8z) + 6y^2(1-4z+2z^2) + y(-8+36z-45z^2+20z^3))H(2,0,y) + 1728(-1$$
$$+ y)^2y(-1+z)^2z(y+z)^2(5+4y^4-39z+87z^2-81z^3+28z^4+y^3(-15+16z)+3y^2(7-17z+12z^2)$$
$$+ y(-15+66z-93z^2+40z^3))H(2,2,y)) - 2376(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z$$
$$- 19z^2 + 8z^3 + y^2(-19+12z) + 2y(7-10z+6z^2))H(3,2,y) + H(0,y)(1188y^2(-1+z)^2z^2(-1+y+z)(y$$
$$+ z)^2(8y^3 + y(30-27z) + 12(-1+z) + 2y^2(-13+6z)) - 432(-1+y)^2y(y^2+2y(-1+z)+2(-1+z)^2)(-1$$
$$+ z)^2z(-1+y+z)(y+z)^2(-1+4z)H(0,z)^2 - 864(-1+y)^2y(-1+z)^2z(y+z)^2(5+28y^4-15z+21z^2$$
$$- 15z^3 + 4z^4 + y^3(-81+40z) + y^2(87-93z+36z^2) + y(-39+66z-51z^2+16z^3))H(1,z)^2 - 432y(-1$$
$$+ z)^2z(y^7(-4+52z) + 4y^6(4-57z+49z^2) - 4z^2(-1+2z-2z^2+z^3) + y^5(-28+408z-735z^2+276z^3)$$
$$+ 4yz(-3-7z+31z^2-32z^3+12z^4) + y^4(28-360z+1079z^2-889z^3+172z^4) + y^2(4+4z+244z^2-552z^3$$
$$+ 407z^4 - 87z^5) + y^3(-16+136z-760z^2+1046z^3-465z^4+40z^5))H(2,y) - 864(-1+y)^2y(-1+z)^2z(y$$
$$+ z)^2(4y^4 + (-1+z)^3(-1+4z) + y^3(-13+8z) + 3y^2(5-7z+4z^2) + y(-7+18z-21z^2+8z^3))H(2,y)^2$$
$$+ H(0,z)(-864(-1+y)^2y(-1+z)^2z(y+z)^2(4+8y^4-8z+6z^2-2z^3+y^3(-27+20z)+3y^2(11-15z$$
$$+ 4z^2) + 2y(-9+18z-12z^2+4z^3))H(1,z) - 864(-1+y)^2y(-1+z)^2z(y+z)^2(4-18z+33z^2-27z^3+8z^4$$
$$+ y^3(-2+8z) + 6y^2(1-4z+2z^2) + y(-8+36z-45z^2+20z^3))H(2,y)) + H(1,z)(2376(-1+y)^2y(-1+4y)(-1$$
$$+ z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2) - 864(-1+y)^2y(-1+z)^2z(y+z)^2(2+24y^4-4z$$
$$+ 3z^2 - z^3 + y^3(-67+28z) + 3y^2(23-21z+8z^2) + y(-28+36z-21z^2+4z^3))H(2,y) + 864(-1+y)^2y(-1$$
$$+ z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z+6z^2))H(3,y)) + 864(-1$$
$$+ y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(0,0,y) + 864(-1+y)^2y(y^2$$
$$+ 2y(-1+z) + 2(-1+z)^2)(-1+z)^2z(-1+y+z)(y+z)^2(-1+4z)H(0,0,z) + 864(-1+y)^2y(-1+z)^2z(y$$
$$+ z)^2(4+8y^4-8z+6z^2-2z^3+y^3(-27+20z)+3y^2(11-15z+4z^2)+2y(-9+18z-12z^2$$
$$+ 4z^3))H(0,1,z) - 864(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z$$
$$+ z^2)H(0,2,y) + 864(-1+y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z$$
$$+ z^2)H(1,0,y) + 864(-1+y)^2y(-1+z)^2z(y+z)^2(4+8y^4-8z+6z^2-2z^3+y^3(-27+20z)+3y^2(11-15z$$
$$+ 4z^2) + 2y(-9+18z-12z^2+4z^3))H(1,0,z) + 1728(-1+y)^2y(-1+z)^2z(y+z)^2(5+28y^4-15z+21z^2$$
$$- 15z^3 + 4z^4 + y^3(-81+40z) + y^2(87-93z+36z^2) + y(-39+66z-51z^2+16z^3))H(1,1,z) - 864(-1$$
$$+ y)^2y(-1+4y)(-1+z)^2z(-1+y+z)(y+z)^2(2+2y^2+2y(-2+z)-2z+z^2)H(2,0,y) + 864(-1+y)^2y(-1$$
$$+ z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z+6z^2))H(2,2,y)$$
$$+ 864(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z$$
$$+ 6z^2))H(3,2,y)) + H(1,z)(1188(-1+y)^2y(-1+z)^2z(-1+y+z)(y^4(-3+20z) - 3z^2(2-2z+z^2) + y^3(6$$
$$- 50z + 60z^2) + 2yz(4+17z-25z^2+10z^3) + y^2(-6+34z-94z^2+60z^3)) + 108(-1+y)^2y(-1+z)^2z(y$$
$$+ z)^2(124 + 1240y^4 - 1510z + 4011z^2 - 3865z^3 + 1240z^4 + y^3(-3865+2972z) + y^2(4011-6609z+4008z^2) + y(-1510$$
$$+ 4752z - 6609z^2 + 2972z^3))H(2,y)^2 - 2376(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2$$
$$+ 8z^3 + y^2(-19+12z) + 2y(7-10z+6z^2))H(3,y) + 1728(-1+y)^2y(-1+z)^2z(y+z)^2(4+12y^4-20z+39z^2$$
$$- 35z^3 + 12z^4 + 5y^3(-7+4z) + 3y^2(13-17z+8z^2) + y(-20+48z-51z^2+20z^3))H(2,y)H(3,y) + 3456(-1$$
$$+ y)^2y(-1+z)^2z(y+z)^2(2+10y^4-4z+3z^2-z^3+2y^3(-15+8z)+3y^2(11-12z+4z^2)+y(-15+24z$$
$$- 15z^2 + 4z^3))H(0,0,y) + 864(-1+y)^2y(-1+z)^2z(y+z)^2(2-28z+69z^2-67z^3+24z^4+y^3(-1+4z)$$
$$+ 3y^2(1-7z+8z^2) + y(-4+36z-63z^2+28z^3))H(0,0,z) + 1728(-1+y)^2y(-1+z)^2z(y+z)^2(2+12y^4$$
$$- 4z + 3z^2 - z^3 + 2y^3(-17+8z) + 12y^2(3-3z+z^2) + y(-16+24z-15z^2+4z^3))H(0,1,z) + 864(-1$$
$$+ y)^2y(-1+z)^2z(y+z)^2(2+24y^4-4z+3z^2-z^3+y^3(-67+28z)+3y^2(23-21z+8z^2)+y(-28+36z$$
$$- 21z^2 + 4z^3))H(0,2,y) - 864(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19$$
$$+ 12z) + 2y(7-10z+6z^2))H(0,3,y) - 864(-1+y)^2y(-1+z)^2z(y+z)^2(2+y^2(3-9z)-4z+3z^2-z^3$$
$$+ y^3(-1+4z) + y(-4+12z-9z^2+4z^3))H(1,0,z) + 1728(-1+y)^2y(-1+z)^2z(y+z)^2(4+12y^4-20z+39z^2$$
$$- 35z^3 + 12z^4 + 5y^3(-7+4z) + 3y^2(13-17z+8z^2) + y(-20+48z-51z^2+20z^3))H(1,1,z) + 864(-1$$
$$+ y)^2y(-1+z)^2z(y+z)^2(2+24y^4-4z+3z^2-z^3+y^3(-67+28z)+3y^2(23-21z+8z^2)+y(-28+36z$$
$$- 21z^2 + 4z^3))H(2,0,y) - 216(-1+y)^2y(-1+z)^2z(y+z)^2(124+1240y^4-1510z+4011z^2-3865z^3+1240z^4$$

$$+ y^3(-3865 + 2972z) + y^2(4011 - 6609z + 4008z^2) + y(-1510 + 4752z - 6609z^2 + 2972z^3))H(2, 2, y) - 1728(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(4 + 12y^4 - 20z + 39z^2 - 35z^3 + 12z^4 + 5y^3(-7 + 4z) + 3y^2(13 - 17z + 8z^2)$$
$$+ y(-20 + 48z - 51z^2 + 20z^3))H(2, 3, y) - 864(-1 + y)^2y(-1 + z)^2z(-1 + y + z)(y + z)^2(-4 + 8y^3 + 14z - 19z^2$$
$$+ 8z^3 + y^2(-19 + 12z) + 2y(7 - 10z + 6z^2))H(3, 0, y) - 1728(-1 + y)^2y(-1 + z)^2z(y + z)^2(4 + 12y^4 - 20z$$
$$+ 39z^2 - 35z^3 + 12z^4 + 5y^3(-7 + 4z) + 3y^2(13 - 17z + 8z^2) + y(-20 + 48z - 51z^2 + 20z^3))H(3, 2, y))$$
$$+ H(2, y)(1188(-1 + y)^2y(-1 + z)^2z(-1 + y + z)(y^4(-3 + 20z) - 3z^2(2 - 2z + z^2) + y^3(6 - 50z + 60z^2)$$
$$+ 2yz(4 + 17z - 25z^2 + 10z^3) + y^2(-6 + 34z - 94z^2 + 60z^3)) + 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 24y^4$$
$$- 4z + 3z^2 - z^3 + y^3(-67 + 28z) + 3y^2(23 - 21z + 8z^2) + y(-28 + 36z - 21z^2 + 4z^3))H(0, 0, y) + 3456(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(2 - 15z + 33z^2 - 30z^3 + 10z^4 + y^3(-1 + 4z) + 4y(-1 + z)^2(-1 + 4z) + 3y^2(1 - 5z$$
$$+ 4z^2))H(0, 0, z) + 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 - 28z + 69z^2 - 67z^3 + 24z^4 + y^3(-1 + 4z) + 3y^2(1$$
$$- 7z + 8z^2) + y(-4 + 36z - 63z^2 + 28z^3))H(0, 1, z) - 1728(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 12y^4 - 4z + 3z^2$$
$$- z^3 + 2y^3(-17 + 8z) + 12y^2(3 - 3z + z^2) + y(-16 + 24z - 15z^2 + 4z^3))H(0, 2, y) + 864(-1 + y)^2y(-1$$
$$+ z)^2z(y + z)^2(2 + 24y^4 - 4z + 3z^2 - z^3 + y^3(-67 + 28z) + 3y^2(23 - 21z + 8z^2) + y(-28 + 36z - 21z^2$$
$$+ 4z^3))H(1, 0, y) + 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 - 28z + 69z^2 - 67z^3 + 24z^4 + y^3(-1 + 4z) + 3y^2(1$$
$$- 7z + 8z^2) + y(-4 + 36z - 63z^2 + 28z^3))H(1, 0, z) - 216(-1 + y)^2y(-1 + z)^2z(y + z)^2(124 + 1240y^4 - 1510z$$
$$+ 4011z^2 - 3865z^3 + 1240z^4 + y^3(-3865 + 2972z) + y^2(4011 - 6609z + 4008z^2) + y(-1510 + 4752z - 6609z^2$$
$$+ 2972z^3))H(1, 1, z) - 1728(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 12y^4 - 4z + 3z^2 - z^3 + 2y^3(-17 + 8z) + 12y^2(3$$
$$- 3z + z^2) + y(-16 + 24z - 15z^2 + 4z^3))H(2, 0, y) + 1728(-1 + y)^2y(-1 + z)^2z(y + z)^2(4 + 12y^4 - 20z + 39z^2$$
$$- 35z^3 + 12z^4 + 5y^3(-7 + 4z) + 3y^2(13 - 17z + 8z^2) + y(-20 + 48z - 51z^2 + 20z^3))H(2, 2, y) + 1728(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(4 + 12y^4 - 20z + 39z^2 - 35z^3 + 12z^4 + 5y^3(-7 + 4z) + 3y^2(13 - 17z + 8z^2)$$
$$+ y(-20 + 48z - 51z^2 + 20z^3))H(3, 2, y)) - 216(-1 + y)^2y(-1 + z)^2z(y + z)^2(296 + 3224y^4 - 518z + 453z^2$$
$$- 431z^3 + 200z^4 + y^3(-8315 + 5764z) + 3y^2(2475 - 3233z + 984z^2) + y(-2630 + 4728z - 2919z^2$$
$$+ 820z^3))H(0, 0, 0, y) - 216(-1 + y)^2y(-1 + z)^2z(y + z)^2(296 + 200y^4 - 2630z + 7425z^2 - 8315z^3 + 3224z^4$$
$$+ y^3(-431 + 820z) + 3y^2(151 - 973z + 984z^2) + y(-518 + 4728z - 9699z^2 + 5764z^3))H(0, 0, 0, z) - 864(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(2 + 16y^4 - 4z + 3z^2 - z^3 + y^3(-51 + 28z) + 3y^2(19 - 21z + 8z^2) + y(-24 + 36z$$
$$- 21z^2 + 4z^3))H(0, 0, 1, z) + 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(16y^4 + y^3(-53 + 36z) + 3y^2(21 - 27z + 8z^2)$$
$$- 3(-2 + 4z - 3z^2 + z^3) + y(-32 + 60z - 39z^2 + 12z^3))H(0, 0, 2, y) - 864(-1 + y)^2y(-1 + 4y)(-1 + z)^2z(-1 + y$$
$$+ z)(y + z)^2(2 + 2y^2 + 2y(-2 + z) - 2z + z^2)H(0, 1, 0, y) - 864(-1 + y)^2y^2(-1 + z)^2z(y + z)^2(8y^3 + y^2(-25$$
$$+ 12z) - 2(5 - 6z + 3z^2) + 3y(9 - 9z + 4z^2))H(0, 1, 0, z) - 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(48y^4 + y^3(-137$$
$$+ 68z) + 3y^2(49 - 51z + 16z^2) - 5(-2 + 4z - 3z^2 + z^3) + y(-68 + 108z - 69z^2 + 20z^3))H(0, 1, 1, z) + 864(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(16y^4 + y^3(-53 + 36z) + 3y^2(21 - 27z + 8z^2) - 3(-2 + 4z - 3z^2 + z^3) + y(-32 + 60z$$
$$- 39z^2 + 12z^3))H(0, 2, 0, y) + 2592(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 16y^4 - 4z + 3z^2 - z^3 + 5y^3(-9 + 4z)$$
$$+ y^2(47 - 45z + 16z^2) + y(-20 + 28z - 17z^2 + 4z^3))H(0, 2, 2, y) - 864(-1 + y)^2y(-1 + z)^2z(-1 + y + z)(y$$
$$+ z)^2(-4 + 8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19 + 12z) + 2y(7 - 10z + 6z^2))H(0, 3, 2, y) - 1728(-1 + y)^2y(-1$$
$$+ 4y)(-1 + z)^2z(-1 + y + z)(y + z)^2(2 + 2y^2 + 2y(-2 + z) - 2z + z^2)H(1, 0, 0, y) + 864(-1 + y)^2y(-1 + z)^2z(y$$
$$+ z)^2(2 + y^2(3 - 9z) - 4z + 3z^2 - z^3 + y^3(-1 + 4z) + y(-4 + 12z - 9z^2 + 4z^3))H(1, 0, 0, z) - 1728(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(2 + 12y^4 - 4z + 3z^2 - z^3 + 2y^3(-17 + 8z) + 12y^2(3 - 3z + z^2) + y(-16 + 24z$$
$$- 15z^2 + 4z^3))H(1, 0, 1, z) - 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 24y^4 - 4z + 3z^2 - z^3 + y^3(-67 + 28z)$$
$$+ 3y^2(23 - 21z + 8z^2) + y(-28 + 36z - 21z^2 + 4z^3))H(1, 0, 2, y) + 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + y^2(3$$
$$- 9z) - 4z + 3z^2 - z^3 + y^3(-1 + 4z) + y(-4 + 12z - 9z^2 + 4z^3))H(1, 1, 0, z) - 216(-1 + y)^2y(-1 + z)^2z(y$$
$$+ z)^2(156 + 1336y^4 - 1670z + 4323z^2 - 4145z^3 + 1336z^4 + y^3(-4145 + 3132z) + y^2(4323 - 7017z + 4200z^2) + y(-1670$$
$$+ 5136z - 7017z^2 + 3132z^3))H(1, 1, 1, z) - 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 24y^4 - 4z + 3z^2 - z^3 + y^3(-67$$
$$+ 28z) + 3y^2(23 - 21z + 8z^2) + y(-28 + 36z - 21z^2 + 4z^3))H(1, 2, 0, y) + 864(-1 + y)^2y(-1 + z)^2z(y$$
$$+ z)^2(16y^4 + y^3(-53 + 36z) + 3y^2(21 - 27z + 8z^2) - 3(-2 + 4z - 3z^2 + z^3) + y(-32 + 60z - 39z^2$$
$$+ 12z^3))H(2, 0, 0, y) + 2592(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 16y^4 - 4z + 3z^2 - z^3 + 5y^3(-9 + 4z) + y^2(47$$
$$- 45z + 16z^2) + y(-20 + 28z - 17z^2 + 4z^3))H(2, 0, 2, y) - 864(-1 + y)^2y(-1 + z)^2z(y + z)^2(2 + 24y^4 - 4z$$
$$+ 3z^2 - z^3 + y^3(-67 + 28z) + 3y^2(23 - 21z + 8z^2) + y(-28 + 36z - 21z^2 + 4z^3))H(2, 1, 0, y) + 2592(-1$$
$$+ y)^2y(-1 + z)^2z(y + z)^2(2 + 16y^4 - 4z + 3z^2 - z^3 + 5y^3(-9 + 4z) + y^2(47 - 45z + 16z^2) + y(-20 + 28z$$
$$- 17z^2 + 4z^3))H(2, 2, 0, y) - 216(-1 + y)^2y(-1 + z)^2z(y + z)^2(156 + 1336y^4 - 1670z + 4323z^2 - 4145z^3$$
$$+ 1336z^4 + y^3(-4145 + 3132z) + y^2(4323 - 7017z + 4200z^2) + y(-1670 + 5136z - 7017z^2 + 3132z^3))H(2, 2, 2, y)$$

$$
\begin{aligned}
&- 1728(-1+y)^2y(-1+z)^2z(y+z)^2(4+12y^4-20z+39z^2-35z^3+12z^4+5y^3(-7+4z)+3y^2(13-17z\\
&+8z^2)+y(-20+48z-51z^2+20z^3))H(2,3,2,y)-864(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2(-4+8y^3\\
&+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z+6z^2))H(3,0,2,y)-864(-1+y)^2y(-1+z)^2z(-1+y+z)(y\\
&+z)^2(-4+8y^3+14z-19z^2+8z^3+y^2(-19+12z)+2y(7-10z+6z^2))H(3,2,0,y)-3456(-1+y)^2y(-1\\
&+z)^2z(y+z)^2(4+12y^4-20z+39z^2-35z^3+12z^4+5y^3(-7+4z)+3y^2(13-17z+8z^2)+y(-20+48z\\
&-51z^2+20z^3))H(3,2,2,y)\Big\}\Big/\Big(864(-1+y)^2y^2(-1+z)^2z^2(-1+y+z)(y+z)^2\Big);
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{3;C_A{}^2n_f} = \Big\{ &- 8(-1+y)(-1+z)(y+z)(103y^6(-1+4z)+6y^5(47-272z+170z^2)+z^3(206-385z+282z^2-103z^3)+y^4(-385\\
&+2703z-4017z^2+1000z^3)+3y^2z(206-1022z+1843z^2-1339z^3+340z^4)+yz^2(618-2620z+2703z^2-1632z^3\\
&+412z^4)+y^3(206-2620z+5529z^2-4976z^3+1000z^4))+9(-1+y)(-1+z)(y+z)^4(-56+72y^4+529z-1569z^2\\
&+1834z^3-744z^4-4y^3(31+12z)-3y^2(5-182z+216z^2)+y(105-1020z+2160z^2-1336z^3))H(0,y)^2-216(-1\\
&+y)(-1+z)z(-1+y+z)(y+z)^4(3-12z+10z^2+y(-3+6z))H(0,y)^3-9(-1+y)(-1+z)(y+z)^4(56+744y^4\\
&-105z+15z^2+124z^3-72z^4+2y^3(-917+668z)+3y^2(523-720z+216z^2)+y(-529+1020z-546z^2\\
&+48z^3))H(0,z)^2-216(-1+y)y(3+10y^2+6y(-2+z)-3z)(-1+z)(-1+y+z)(y+z)^4H(0,z)^3+1080(-1+y)(-1\\
&+z)(-1+y+z)(y+z)^4(10y^3+6y^2(-2+z)+y(3-6z+6z^2)+z(3-12z+10z^2))H(1,z)^3-72(-1+y)(-1\\
&+z)(358y^8+14y^7(-63+152z)+y^6(732-4821z+5640z^2)+y^5(-223+3798z-11757z^2+9136z^3)+z^4(14-223z\\
&+732z^2-882z^3+358z^4)+yz^3(128-1175z+3798z^2-4821z^3+2128z^4)+2y^3z(64-1109z+5394z^2-8670z^3\\
&+4568z^4)+y^2z^2(84-2218z+8460z^2-11757z^3+5640z^4)+y^4(14-1175z+8460z^2-17340z^3\\
&+10532z^4))H(2,y)^2+1080(-1+y)(-1+z)(-1+y+z)(y+z)^4(10y^3+6y^2(-2+z)+y(3-6z+6z^2)+z(3-12z\\
&+10z^2))H(2,y)^3+H(0,y)(18(-1+z)(y+z)^4(y^4(-31+220z)+y^3(124-607z+96z^2)+31(-2+4z-3z^2+z^3)\\
&-31y(-6+16z-12z^2+5z^3)+y^2(-217+759z-387z^2+124z^3))+216(-1+y)(-1+z)(y+z)^4(2+y^2(3-9z)\\
&-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z-9z^2+4z^3))H(0,z)+216(-1+y)(-1+z)(y+z)^4(2+16y^4-4z\\
&+3z^2-z^3+2y^3(-21+8z)+6y^2(7-6z+2z^2)+y(-18+24z-15z^2+4z^3))H(1,z)+288(-1+y)y(-1\\
&+z)z(-5y^3+3y^4+yz^2+y^2(6+z-6z^2)+z^2(6-5z+3z^2))H(2,y))+H(1,z)^2(-72(-1+y)(-1+z)(358y^8\\
&+14y^7(-63+152z)+y^6(732-4821z+5640z^2)+y^5(-223+3798z-11757z^2+9136z^3)+z^4(14-223z+732z^2\\
&-882z^3+358z^4)+yz^3(128-1175z+3798z^2-4821z^3+2128z^4)+2y^3z(64-1109z+5394z^2-8670z^3+4568z^4)\\
&+y^2z^2(84-2218z+8460z^2-11757z^3+5640z^4)+y^4(14-1175z+8460z^2-17340z^3+10532z^4))+3240(-1+y)(-1\\
&+z)(-1+y+z)(y+z)^4(10y^3+6y^2(-2+z)+y(3-6z+6z^2)+z(3-12z+10z^2))H(2,y))+H(0,z)(18(-1+y)(y\\
&+z)^4(-31(-1+z)^2(2-2z+z^2)+31y^3(1-5z+4z^2)+y^2(-93+372z-387z^2+96z^3)+y(124-496z+759z^2\\
&-607z^3+220z^4))+288(-1+y)y(-1+z)z(-5y^3+3y^4+yz^2+y^2(6+z-6z^2)+z^2(6-5z+3z^2))H(1,z)\\
&+216(-1+y)(-1+z)(y+z)^4(y^3(-1+4z)+4y(-1+z)^2(-1+4z)+3y^2(1-5z+4z^2)+2(1-9z+21z^2-21z^3\\
&+8z^4))H(2,y))-18(-1+y)(-1+z)(y+z)^4(-56+72y^4+529z-1569z^2+1834z^3-744z^4-4y^3(31+12z)\\
&-3y^2(5-182z+216z^2)+y(105-1020z+2160z^2-1336z^3))H(0,0,y)+18(-1+y)(-1+z)(y+z)^4(56+744y^4\\
&-105z+15z^2+124z^3-72z^4+2y^3(-917+668z)+3y^2(523-720z+216z^2)+y(-529+1020z-546z^2\\
&+48z^3))H(0,0,z)-72(-1+y)(-1+z)(48y^8+6y^7(-21+40z)+6y^6(21-102z+86z^2)-3z^4(-2+4z-3z^2\\
&+z^3)+3y^5(-18+196z-411z^2+212z^3)+yz^3(48-122z+120z^2-57z^3+12z^4)+2y^2z^2(18-142z+234z^2\\
&-153z^3+42z^4)+4y^3z(12-92z+237z^2-210z^3+66z^4)+y^4(6-248z+1053z^2-1335z^3+504z^4))H(0,1,z)\\
&+72(-1+y)(-1+z)(48y^8+6y^7(-21+40z)+6y^6(21-102z+86z^2)-3z^4(-2+4z-3z^2+z^3)+yz^4(-82\\
&+96z-57z^2+12z^3)+4y^3z^2(-94+249z-210z^2+66z^3)+3y^5(-18+188z-411z^2+212z^3)+2y^2z^2(18\\
&-146z+234z^2-153z^3+42z^4)+y^4(6-208z+1053z^2-1335z^3+504z^4))H(0,2,y)-216(-1+y)y(-1+z)(y\\
&+z)^4(16y^3+y^2(-41+12z)-2(7-6z+3z^2)+3y(13-9z+4z^2))H(1,0,y)+72(-1+y)(-1+z)(3y^7(-1+4z)\\
&+y^6(9-39z+48z^2)-3z^4(-2+4z-3z^2+z^3)+yz^4(-40+60z-39z^2+12z^3)+3y^5(-4+20z-51z^2\\
&+28z^3)+y^3z^2(-124+312z-285z^2+84z^3)+y^2z^2(36-124z+207z^2-153z^3+48z^4)+y^4(6-40z+207z^2\\
&-285z^3+96z^4))H(1,0,z)+144(-1+y)(-1+z)(358y^8+14y^7(-63+152z)+y^6(732-4821z+5640z^2)+y^5(-223\\
&+3798z-11757z^2+9136z^3)+z^4(14-223z+732z^2-882z^3+358z^4)+yz^3(128-1175z+3798z^2-4821z^3\\
&+2128z^4)+2y^3z(64-1109z+5394z^2-8670z^3+4568z^4)+y^2z^2(84-2218z+8460z^2-11757z^3+5640z^4)\\
&+y^4(14-1175z+8460z^2-17340z^3+10532z^4))H(1,1,z)+H(2,y)(36(-1+y)(-1+z)(y^7(-9+84z)+3y^6(9-83z\\
&+144z^2)-9z^4(-2+4z-3z^2+z^3)+3y^5(-12+70z-373z^2+324z^3)+yz^3(-168-26z+210z^2-249z^3\\
&+84z^4)+y^2z^2(-60-314z+1053z^2-1119z^3+432z^4)+y^3z(-168-314z+1740z^2-2175z^3+972z^4)+y^4(18
\end{aligned}
$$

$$
- 26z + 1053z^2 - 2175z^3 + 1248z^4)) - 6480(-1+y)(-1+z)(-1+y+z)(y+z)^4(10y^3 + 6y^2(-2+z) + y(3 - 6z
+ 6z^2) + z(3 - 12z + 10z^2))H(1,1,z)) + 72(-1+y)(-1+z)(48y^8 + 6y^7(-21 + 40z) + 6y^6(21 - 102z + 86z^2)
- 3z^4(-2 + 4z - 3z^2 + z^3) + yz^4(-82 + 96z - 57z^2 + 12z^3) + 4y^3z^2(-94 + 249z - 210z^2 + 66z^3) + 3y^5(-18
+ 188z - 411z^2 + 212z^3) + 2y^2z^2(18 - 146z + 234z^2 - 153z^3 + 42z^4) + y^4(6 - 208z + 1053z^2 - 1335z^3
+ 504z^4))H(2,0,y) + 144(-1+y)(-1+z)(358y^8 + 14y^7(-63 + 152z) + y^6(732 - 4821z + 5640z^2) + y^5(-223 + 3798z
- 11757z^2 + 9136z^3) + z^4(14 - 223z + 732z^2 - 882z^3 + 358z^4) + yz^3(128 - 1175z + 3798z^2 - 4821z^3 + 2128z^4)
+ 2y^3z(64 - 1109z + 5394z^2 - 8670z^3 + 4568z^4) + y^2z^2(84 - 2218z + 8460z^2 - 11757z^3 + 5640z^4) + y^4(14 - 1175z
+ 8460z^2 - 17340z^3 + 10532z^4))H(2,2,y) + H(1,z)(36(-1+y)(-1+z)(y^7(-9 + 84z) + 3y^6(9 - 83z + 144z^2)
- 9z^4(-2 + 4z - 3z^2 + z^3) + 3y^5(-12 + 70z - 373z^2 + 324z^3) + yz^3(-168 - 26z + 210z^2 - 249z^3 + 84z^4)
+ y^2z^2(-60 - 314z + 1053z^2 - 1119z^3 + 432z^4) + y^3z(-168 - 314z + 1740z^2 - 2175z^3 + 972z^4) + y^4(18 - 26z
+ 1053z^2 - 2175z^3 + 1248z^4)) + 3240(-1+y)(-1+z)(-1+y+z)(y+z)^4(10y^3 + 6y^2(-2+z) + y(3 - 6z + 6z^2)
+ z(3 - 12z + 10z^2))H(2,y)^2 - 216(-1+y)(-1+z)(y+z)^4(4 + 16y^4 - 22z + 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z)
+ 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,y) - 6480(-1+y)(-1+z)(-1+y+z)(y+z)^4(10y^3
+ 6y^2(-2+z) + y(3 - 6z + 6z^2) + z(3 - 12z + 10z^2))H(2,2,y)) - 216(-1+y)(-1+z)(y+z)^4(4 + 16y^4 - 22z
+ 45z^2 - 43z^3 + 16z^4 + y^3(-43 + 20z) + 3y^2(15 - 17z + 8z^2) + y(-22 + 48z - 51z^2 + 20z^3))H(3,2,y) + 1296(-1
+ y)(-1+z)z(-1+y+z)(y+z)^4(3 - 12z + 10z^2 + y(-3 + 6z))H(0,0,0,y) + 1296(-1+y)y(3 + 10y^2 + 6y(-2+z)
- 3z)(-1+z)(-1+y+z)(y+z)^4H(0,0,0,z) - 6480(-1+y)(-1+z)(-1+y+z)(y+z)^4(10y^3 + 6y^2(-2+z) + y(3
- 6z + 6z^2) + z(3 - 12z + 10z^2))H(1,1,1,z) - 6480(-1+y)(-1+z)(-1+y+z)(y+z)^4(10y^3 + 6y^2(-2+z) + y(3
- 6z + 6z^2) + z(3 - 12z + 10z^2))H(2,2,2,y)\Big\}\Big/\Big(864(-1+y)y(-1+z)z(-1+y+z)(y+z)^4\Big);
$$

$$
\mathcal{A}^{(2)}_{3;C_F n_f} = \Big\{(-1+y)(-1+z)((-1+z)^2z^2(490 - 463z + 245z^2) + y^6(245 - 982z + 656z^2) + y^5(-953 + 4142z - 4366z^2
+ 1096z^3) + y^4(1661 - 8739z + 14467z^2 - 8188z^3 + 880z^4) + yz(980 - 4869z + 9468z^2 - 8739z^3 + 4142z^4 - 982z^5)
+ y^3(-1443 + 9468z - 21992z^2 + 21140z^3 - 8188z^4 + 1096z^5) + y^2(490 - 4869z + 15614z^2 - 21992z^3 + 14467z^4
- 4366z^5 + 656z^6)) + 9(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2 + 2y^3 + 4z - 3z^2 + z^3 + y^2(-6 + 4z) + y(6 - 8z
+ 3z^2))H(0,y)^2 - 9(1-y)(-1+y)(y^3 + 3y^2(-1+z) + 4y(-1+z)^2 + 2(-1+z)^3)(-1+z)^2(y+z)^2(-1
+ 4z)H(0,z)^2 + 63(1-y)(-1+y)(-1+z)^2(y+z)^2(4 + 8y^4 - 18z + 33z^2 - 27z^3 + 8z^4 + y^3(-27 + 20z) + 3y^2(11
- 17z + 8z^2) + y(-18 + 48z - 51z^2 + 20z^3))H(1,z)^2 + H(0,y)(-27y(-1+z)^2z(-1+y+z)(y+z)^2(8y^3 + y(30
- 27z) + 12(-1+z) + 2y^2(-13 + 6z)) - 54(-1+y)^2(-1+4y)(-1+z)^2(-1+y+z)(y+z)^2(2 + 2y^2 + 2y(-2+z)
- 2z + z^2)H(1,z)) + 27(1-y)(-1+y)(-1+z)^2(y^5(-3 + 20z) + y^4(9 - 73z + 80z^2) - 3z^2(-2 + 4z - 3z^2 + z^3)
+ 6y^3(-2 + 15z - 34z^2 + 20z^3) + yz(-8 - 32z + 90z^2 - 73z^3 + 20z^4) + 2y^2(3 - 16z + 81z^2 - 102z^3
+ 40z^4))H(2,y) + 63(1-y)(-1+y)(-1+z)^2(y+z)^2(4 + 8y^4 - 18z + 33z^2 - 27z^3 + 8z^4 + y^3(-27 + 20z)
+ 3y^2(11 - 17z + 8z^2) + y(-18 + 48z - 51z^2 + 20z^3))H(2,y)^2 + H(0,z)(-27(-1+y)^2yz(-1+y+z)(y+z)^2(-12
+ 30z - 26z^2 + 8z^3 + 3y(4 - 9z + 4z^2)) - 54(-1+y)^2(y^2 + 2y(-1+z) + 2(-1+z)^2)(-1+z)^2(-1+y+z)(y
+ z)^2(-1+4z)H(2,y)) + H(1,z)(27(1-y)(-1+y)(-1+z)^2(-1+y+z)(y^4(-3 + 20z) - 3z^2(2 - 2z + z^2) + y^3(6
- 50z + 60z^2) + 2yz(4 + 17z - 25z^2 + 10z^3) + y^2(-6 + 34z - 94z^2 + 60z^3)) + 54(-1+y)^2(-1+z)^2(-1+y+z)(y
+ z)^2(-4 + 8y^3 + 14z - 19z^2 + 8z^3 + y^2(-19 + 12z) + 2y(7 - 10z + 6z^2))H(3,y)) + 18(1-y)(1-5y+4y^2)(-1
+ z)^2(y+z)(2y^4 + 6y^3(-1+z) + y^2(6 - 14z + 7z^2) + z(-2 + 4z - 3z^2 + z^3) + y(-2 + 10z - 11z^2
+ 4z^3))H(0,0,y) + 18(1-y)(-1+y)(-1+z)(y+z)(1-5z+4z^2)(y^4 + 2(-1+z)^3z + 2y(-1+z)^2(-1+3z)
+ y^3(-3 + 4z) + y^2(4 - 11z + 7z^2))H(0,0,z) + 54(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2 + 2y^3 + 4z - 3z^2 + z^3
+ y^2(-6 + 4z) + y(6 - 8z + 3z^2))H(0,1,z) - 54(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2 + 2y^3 + 4z - 3z^2 + z^3
+ y^2(-6 + 4z) + y(6 - 8z + 3z^2))H(0,2,y) + 54(-1+y)^2(-1+4y)(-1+z)^2(y+z)^2(-2 + 2y^3 + 4z - 3z^2 + z^3
+ y^2(-6 + 4z) + y(6 - 8z + 3z^2))H(1,0,y) + 126(-1+y)^2(-1+z)^2(y+z)^2(4 + 8y^4 - 18z + 33z^2 - 27z^3 + 8z^4
+ y^3(-27 + 20z) + 3y^2(11 - 17z + 8z^2) + y(-18 + 48z - 51z^2 + 20z^3))H(1,1,z) - 54(-1+y)^2(-1+4y)(-1+z)^2(y
+ z)^2(-2 + 2y^3 + 4z - 3z^2 + z^3 + y^2(-6 + 4z) + y(6 - 8z + 3z^2))H(2,0,y) + 126(-1+y)^2(-1+z)^2(y+z)^2(4
+ 8y^4 - 18z + 33z^2 - 27z^3 + 8z^4 + y^3(-27 + 20z) + 3y^2(11 - 17z + 8z^2) + y(-18 + 48z - 51z^2 + 20z^3))H(2,2,y)
+ 54(-1+y)^2(-1+z)^2(y+z)^2(4 + 8y^4 - 18z + 33z^2 - 27z^3 + 8z^4 + y^3(-27 + 20z) + 3y^2(11 - 17z + 8z^2)
+ y(-18 + 48z - 51z^2 + 20z^3))H(3,2,y)\Big\}\Big/\Big(108(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^2\Big);
$$

$$
\mathcal{A}^{(2)}_{4;C_A^2} = \frac{39}{20}\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{4;C_F^2} = -\frac{44}{5}\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{4;n_f^2} = 0; \qquad \mathcal{A}^{(2)}_{4;C_A C_F} = \frac{93}{10}\,\mathcal{A}_0; \qquad \mathcal{A}^{(2)}_{4;C_A n_f} = 0; \qquad \mathcal{A}^{(2)}_{4;C_F n_f} = 0;
$$

$$\mathcal{A}^{(2)}_{5;C_A^2} = \Big\{6048y^9z + (-1+z)^5(814 - 814z + 83z^2) + 216y^8z(-177 + 74z - 12z^2 + 3z^3) + y(-1+z)^4(4884 - 4916z - 10029z^2 + 20360z^3 - 14040z^4 + 6048z^5) + y^7(83 + 112808z - 124552z^2 + 79000z^3 - 39863z^4 + 8812z^5) + y^2(-1+z)^3(12293 - 2060z - 65043z^2 + 119980z^3 - 76600z^4 + 15984z^5) - y^3(-1+z)^2(-16695 - 44834z + 223663z^2 - 364320z^3 + 263636z^4 - 73816z^5 + 2592z^6) + y^6(-1229 - 199901z + 397732z^2 - 413860z^3 + 281873z^4 - 103639z^5 + 14832z^6) + y^5(5714 + 219568z - 670767z^2 + 965408z^3 - 836204z^4 + 417156z^5 - 103639z^6 + 8812z^7) + y^4(-13040 - 131106z + 629609z^2 - 1215939z^3 + 1324022z^4 - 836204z^5 + 281873z^6 - 39863z^7 + 648z^8) + 36(-1+y)^4(-1+z)^4(2 + y^2(3 - 9z) - 4z + 3z^2 - z^3 + y^3(-1+4z) + y(-4 + 12z - 9z^2 + 4z^3))H(0,y) + 36(-1+y)^4(-1+z)^4(2 + y^2(3 - 9z) - 4z + 3z^2 - z^3 + y^3(-1+4z) + y(-4 + 12z - 9z^2 + 4z^3))H(0,z) - 36(-1+y)^4(-1+z)^4(220y^4 + y^3(-443 + 324z) + 3y^2(81 - 117z + 32z^2) + y(-8 + 24z + 6z^2 - 24z^3) + 6(-2 + 4z - 3z^2 + z^3))H(1,y) + 36(-1+y)^4(-1+z)^4(220y^4 + 4y^3(-111 + 82z) + 6y^2(41 - 60z + 16z^2) + 5(-2 + 4z - 3z^2 + z^3) - y(12 - 36z + 3z^2 + 20z^3))H(1,z) + 72(-1+y)^4(-1+z)^4(-11 + 110y^4 + 6z + 114z^2 - 219z^3 + 110z^4 + y^3(-219 + 152z) + 3y^2(38 - 59z + 32z^2) + y(6 + 30z - 177z^2 + 152z^3))H(2,y)\Big\} \Big/ \Big(144(-1+y)^4y(-1+z)^4z(-1+y+z)\Big);$$

$$\mathcal{A}^{(2)}_{5;C_F^2} = \Big\{ - 432y^9z - 6(-1+z)^5(31 - 30z + 20z^2) - 12y^8z(-261 + 147z - 40z^2 + 10z^3) - 6y(-1+z)^4(185 - 354z + 224z^2 + 102z^3 - 234z^4 + 72z^5) + y^7(-120 - 8820z + 8139z^2 - 1414z^3 - 733z^4 + 356z^5) - 3y^2(-1+z)^3(960 - 2620z + 2922z^2 - 754z^3 - 949z^4 + 588z^5) + y^3(-1+z)^2(-4260 + 13428z - 23904z^2 + 19624z^3 - 6031z^4 - 454z^5 + 480z^6) + y^6(780 + 11256z - 11571z^2 - 4643z^3 + 9499z^4 - 4121z^5 + 528z^6) + y^5(-2286 - 2220z - 5247z^2 + 31232z^3 - 36206z^4 + 18060z^5 - 4121z^6 + 356z^7) + y^4(3930 - 13818z + 38097z^2 - 69183z^3 + 68534z^4 - 36206z^5 + 9499z^6 - 733z^7 - 120z^8) - 12(-1+y)^4(-1+z)^4(14y^4 + 6y^2(-3+z)z + y^3(-23+28z) + y(13 - 18z + 15z^2 - 8z^3) + 2(-2 + 4z - 3z^2 + z^3))H(1,y) + 12(-1+y)^4(-1+z)^4(2 + 14y^4 - 4z + 3z^2 - z^3 + y^3(-26+40z) + y^2(9 - 45z + 6z^2) + y(1 + 18z - 12z^2 + 4z^3))H(1,z) + 12(-1+y)^4(-1+z)^4(-2 + 14y^4 + 9z + 3z^2 - 24z^3 + 14z^4 + 8y^3(-3+4z) + 3y^2(1 - 10z + 4z^2) + y(9 - 30z^2 + 32z^3))H(2,y)\Big\} \Big/ \Big(6(-1+y)^4y(-1+z)^4z(-1+y+z)\Big);$$

$$\mathcal{A}^{(2)}_{5;n_f^2} = 0;$$

$$\mathcal{A}^{(2)}_{5;C_AC_F} = \Big\{3456y^9z + (-1+z)^5(2470 - 2290z + 1613z^2) + 72y^8z(-351 + 207z - 64z^2 + 16z^3) + y^7(1613 + 64136z - 47008z^2 - 12968z^3 + 21871z^4 - 6908z^5) + y(-1+z)^4(14640 - 32276z + 25053z^2 - 2392z^3 - 11448z^4 + 3456z^5) + y^2(-1+z)^3(37763 - 128708z + 163755z^2 - 83348z^3 - 2296z^4 + 14904z^5) - y^3(-1+z)^2(-55665 + 243490z - 455855z^2 + 399648z^3 - 166612z^4 + 22184z^5 + 4608z^6) - y^6(10355 + 47891z + 31748z^2 - 206372z^3 + 191809z^4 - 69095z^5 + 7488z^6) + y^5(30050 - 97592z + 392007z^2 - 755056z^3 + 679888z^4 - 308028z^5 + 69095z^6 - 6908z^7) + y^4(-51380 + 292182z - 867721z^2 + 1421763z^3 - 1305946z^4 + 679888z^5 - 191809z^6 + 21871z^7 + 1152z^8) + 36(-1+y)^4(-1+z)^4(64y^4 + 18y^2z(-5+2z) + 4y^3(-27+34z) + y(66 - 96z + 81z^2 - 44z^3) + 11(-2 + 4z - 3z^2 + z^3))H(1,y) - 36(-1+y)^4(-1+z)^4(4 + 64y^4 - 8z + 6z^2 - 2z^3 + y^3(-121+188z) + 3y^2(13 - 69z + 12z^2) + 2y(7 + 30z - 18z^2 + 4z^3))H(1,z) - 72(-1+y)^4(-1+z)^4(-9 + 32y^4 + 29z + 3z^2 - 55z^3 + 32z^4 + y^3(-55 + 72z) + y^2(3 - 63z + 36z^2) + y(29 - 18z - 63z^2 + 72z^3))H(2,y)\Big\} \Big/ \Big(72(-1+y)^4y(-1+z)^4z(-1+y+z)\Big);$$

$$\mathcal{A}^{(2)}_{5;C_An_f} = \Big\{ - 1296y^9z - 37(-1+z)^5(2 - 2z + z^2) - 18y^8z(-459 + 177z - 8z^2 + 2z^3) - y^7(37 + 23464z - 21866z^2 + 9620z^3 - 4555z^4 + 1076z^5) - y(-1+z)^4(444 - 388z - 1653z^2 + 3376z^3 - 3078z^4 + 1296z^5) - y^2(-1+z)^3(1147 + 878z - 10719z^2 + 18518z^3 - 12308z^4 + 3186z^5) + y^6(259 + 38809z - 65000z^2 + 56576z^3 - 38287z^4 + 15203z^5 - 2376z^6) + y^3(-1+z)^2(-1665 - 9214z + 35399z^2 - 55404z^3 + 37768z^4 - 9332z^5 + 144z^6) - y^5(814 + 40088z - 106383z^2 + 140272z^3 - 121660z^4 + 62292z^5 - 15203z^6 + 1076z^7) + y^4(1480 + 24504z - 100897z^2 + 183975z^3 - 196954z^4 + 121660z^5 - 38287z^6 + 4555z^7 - 36z^8) + 108(-1+y)^4y(-1+z)^4(10y^3 - 3(-1+z)^2 + 2y^2(-11+8z) + 3y(5 - 7z + 2z^2))H(1,y) - 108(-1+y)^4y(-1+z)^4(10y^3 - 3(-1+z)^2 + 2y^2(-11+8z) + 3y(5 - 7z + 2z^2))H(1,z) - 108(-1+y)^4(-1+z)^4(-1+y+z)(10y^3 + 6y^2(-2+z) + y(3 - 6z + 6z^2) + z(3 - 12z + 10z^2))H(2,y)\Big\} \Big/ \Big(72(-1+y)^4y(-1+z)^4z(-1+y+z)\Big);$$

$$\mathcal{A}^{(2)}_{5;C_Fn_f} = -\frac{1}{9}\,\mathcal{A}_0;$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{6;C_A^2} = \Big\{ & 8(-1+y)(-1+z)(y+z)(-1008y^{12}z + 4(-1+z)^4z^5(10-z+5z^2) + 4y^{11}(5+622z-51z^2-1631z^3+551z^4) \\
& - y(-1+z)^3z^4(200-96z+1182z^2-2503z^3+536z^4+1008z^5) + y^{10}(-84+1087z-12868z^2+50961z^3-50902z^4 \\
& + 13822z^5) - y^2(-1+z)^2z^3(-400+3298z-10431z^2+24380z^3-31260z^4+13276z^5+204z^6) + y^9(176-9291z \\
& + 57608z^2-186114z^3+279822z^4-174455z^5+38302z^6) + y^8(-264+11687z-100176z^2+374314z^3-739622z^4 \\
& + 736539z^5-343702z^6+61224z^7) + y^7(276-6537z+90451z^2-436329z^3+1100987z^4-1543684z^5+1154814z^6 \\
& - 427250z^7+61224z^8) + y^3z^2(400-4252z+32055z^2-129393z^3+304882z^4-436329z^5+374314z^6-186114z^7 \\
& + 50961z^8-6524z^9) + 2y^6(-82+1035z-24270z^2+152441z^3-484929z^4+903470z^5-973380z^6+577407z^7 \\
& - 171851z^8+19151z^9) + y^4z(200-4098z+32055z^2-165612z^3+515832z^4-969858z^5+1100987z^6-739622z^7 \\
& + 279822z^8-50902z^9+2204z^{10}) + y^5(40-696z+17427z^2-129393z^3+515832z^4-1240356z^5+1806940z^6 \\
& - 1543684z^7+736539z^8-174455z^9+13822z^{10})) - 9(-1+y)^4(-1+z)^4(y+z)^6(496y^4+y^3(-1129+1244z) \\
& + 3y^2(213-557z+284z^2)+y(62+444z-870z^2+416z^3)+4(-17-46z+234z^2-301z^3+130z^4))H(0,y)^2 \\
& - 9(-1+y)^4(-1+z)^4(y+z)^6(-68+520y^4+62z+639z^2-1129z^3+496z^4+4y^3(-301+104z)+y^2(936-870z \\
& + 852z^2)+y(-184+444z-1671z^2+1244z^3))H(0,z)^2 + 9(-1+y)^4(-1+z)^4(y+z)^6(-104+992y^4+246z+861z^2 \\
& - 1995z^3+992z^4+5y^3(-399+436z)+3y^2(287-881z+440z^2)+y(246+696z-2643z^2+2180z^3))H(1,z)^2 \\
& + H(1,y)(-48(-1+y)^3(-1+z)^4(y+z)^6(-98+160y^5+247z-279z^2+96z^3+y^4(-697+232z)+y^3(1221-871z \\
& + 318z^2)+y(503-856z+768z^2-192z^3)+3y^2(-363+416z-270z^2+32z^3)) + 144(-1+y)^4(-1+z)^4(y \\
& + z)^6(84y^4+y^3(-193+76z)+3y^2(51-39z+16z^2)+2(-2+4z-3z^2+z^3)-2y(20-12z+3z^2 \\
& + 4z^3))H(1,z)) - 12(672y^{15}z-(-1+z)^5z^6(-306+1094z-1417z^2+640z^3) - 8y^{14}(80+211z-246z^2-284z^3 \\
& + 71z^4) - 3y^{13}(-1539+3624z-2740z^2+6256z^3-6117z^4+1188z^5) + y^{12}(-14579+71063z-127704z^2+152312z^3 \\
& - 144029z^4+67593z^5-10032z^6) + y(-1+z)^4z^5(-1644+10824z-23462z^2+24135z^3-10904z^4+1000z^5+672z^6) \\
& + 6y^2(-1+z)^3z^4(947-7217z+22516z^2-36737z^3+33102z^4-15206z^5+2354z^6+328z^7) - 2y^{11}(-13173+94377z \\
& - 256362z^2+378172z^3-374314z^4+238281z^5-76309z^6+8656z^7) - 2y^{10}(14920-147049z+552045z^2-1083308z^3 \\
& + 1307983z^4-1034540z^5+496204z^6-121179z^7+10892z^8) + y^9(21725-293156z+1483434z^2-3858986z^3+5967465z^4 \\
& - 5942620z^5+3826296z^6-1476766z^7+296968z^8-23016z^9) + 2y^3(-1+z)^2z^3(-3384+44062z-198605z^2+462415z^3 \\
& - 632873z^4+523576z^5-249468z^6+60796z^7-7112z^8+1136z^9) + y^8(-9947+189503z-1308468z^2+4503474z^3 \\
& - 8996757z^4+11383863z^5-9480686z^6+5112368z^7-1673910z^8+296968z^9-21784z^{10}) - 2y^7(-1312+38311z \\
& - 380649z^2+1756308z^3-4551510z^4+7339866z^5-7783882z^6+5514200z^7-2556184z^8+738383z^9-121179z^{10} \\
& + 8656z^{11}) + y^5z(-1644+60348z-580226z^2+2711300z^3-7384344z^4+12777180z^5-14679732z^6+11383863z^7 \\
& - 5942620z^8+2069080z^9-476562z^{10}+67593z^{11}-3564z^{12}) + y^4z^2(-5682+101660z-722026z^2+2711300z^3 \\
& - 6165396z^4+9103020z^5-8996757z^6+5967465z^7-2615966z^8+748628z^9-144029z^{10}+18351z^{11}-568z^{12}) \\
& - 2y^6(153-8700z+141024z^2-903687z^3+3082698z^4-6388590z^5+8608206z^6-7783882z^7+4740343z^8-1913148z^9 \\
& + 496204z^{10}-76309z^{11}+5016z^{12}))H(2,y) + 9(-1+y)^4(-1+z)^4(y+z)^6(-104+992y^4+246z+861z^2-1995z^3 \\
& + 992z^4+5y^3(-399+436z)+3y^2(287-881z+440z^2)+y(246+696z-2643z^2+2180z^3))H(2,y)^2 + H(0,y)(-12(-1 \\
& + y)^3(y+z)^6(672y^6z+49(-1+z)^5(2-2z+z^2)+24y^5z(-93+74z-12z^2+3z^3)-y(-1+z)^4(-306+1096z \\
& - 1680z^2+1205z^3)+y^3(-1+z)^2(304-3453z+4062z^2-2983z^3+984z^4)+y^2(-1+z)^3(415-2620z+4200z^2 \\
& - 3661z^3+1156z^4)+y^4(-97+3488z-6112z^2+3784z^3-1283z^4+220z^5)) - 432(-1+y)^4(-1+z)^4(y+z)^6(2 \\
& + y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z-9z^2+4z^3))H(0,z) + 144(-1+y)^4(-1+z)^4(y \\
& + z)^6(22y^3+4y^4-6y^2(10-9z+4z^2)+y(40-60z+39z^2-12z^3)+3(-2+4z-3z^2+z^3))H(1,z) + 144(-1 \\
& + y)^4(-1+z)^4(y+z)^6(2+y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z-9z^2+4z^3))H(2,y)) \\
& + H(0,z)(-12(-1+z)^3(y+z)^6(-((-1+z)^2(98-110z+97z^2))+y^7(49-1205z+1156z^2)+y^6(-343+6500z \\
& - 7129z^2+984z^3)+y^5(1078-15046z+18651z^2-4951z^3+220z^4)+y^3(2205-15725z+24536z^2-14560z^3+3784z^4 \\
& - 288z^5)+y^4(-1960+19590z-27359z^2+11012z^3-1283z^4+72z^5)+y^2(-1519+7900z-13305z^2+11272z^3 \\
& - 6112z^4+1776z^5)+y(588-2320z+3865z^2-4061z^3+3488z^4-2232z^5+672z^6)) + 144(-1+y)^4(-1+z)^4(y \\
& + z)^6(88y^4+y^3(-201+124z)+3y^2(49-57z+20z^2)+2(-2+4z-3z^2+z^3)-2y(15-18z+6z^2 \\
& + 4z^3))H(1,y) + 144(-1+y)^4(-1+z)^4(y+z)^6(2+y^2(3-9z)-4z+3z^2-z^3+y^3(-1+4z)+y(-4+12z-9z^2 \\
& + 4z^3))H(1,z) - 288(-1+y)^4(-1+z)^4(y+z)^6(44y^4+34y^3(-3+2z)-(-1+z)^2(-1+14z+2z^2)+3y^2(26 \\
& - 35z+14z^2)-y(21-48z+33z^2+4z^3))H(2,y)) + H(1,z)(-12(672y^{15}z-(-1+z)^5z^6(86-134z+91z^2) \\
& - 8y^{14}(80+211z-246z^2-284z^3+71z^4)-3y^{13}(-1411+2984z-1460z^2+4976z^3-5477z^4+1060z^5) \\
& + y^{12}(-11927+54659z-85020z^2+92588z^3-96701z^4+47481z^5-6456z^6)+y(-1+z)^4z^5(708-1452z+1870z^2 \\
& - 2253z^3+2456z^4-1560z^5+672z^6)-2y^{11}(-9295+64627z-158940z^2+201252z^3-181596z^4+112239z^5-30455z^6
\end{aligned}
$$

$$
+ 1496z^7) + 2y^2(-1+z)^3z^4(-99 + 1107z - 3248z^2 + 8087z^3 - 15336z^4 + 17678z^5 - 10838z^6 + 2904z^7) + 2y^{10}(-8632
+ 86311z - 304059z^2 + 516984z^3 - 511401z^4 + 325416z^5 - 105440z^6 - 739z^7 + 5592z^8) - 2y^3(-1+z)^2z^3(-536 - 852z
+ 7455z^2 - 903z^3 - 46165z^4 + 106706z^5 - 114712z^6 + 62368z^7 - 14048z^8 + 144z^9) + 3y^9(3127 - 46340z + 231586z^2
- 539722z^3 + 671983z^4 - 471956z^5 + 148964z^6 + 36522z^7 - 42364z^8 + 8648z^9) + y^8(-2527 + 64339z - 489624z^2
+ 1601594z^3 - 2644553z^4 + 2271207z^5 - 738570z^6 - 439680z^7 + 556890z^8 - 212212z^9 + 27760z^{10}) + 2y^7(34 - 6481z
+ 102519z^2 - 502456z^3 + 1134338z^4 - 1263358z^5 + 486190z^6 + 442468z^7 - 684158z^8 + 375311z^9 - 94127z^{10} + 8712z^{11})
+ y^4z^2(198 + 2720z - 54406z^2 + 215036z^3 - 276280z^4 - 222736z^5 + 1183523z^6 - 1711791z^7 + 1308226z^8 - 546976z^9
+ 107843z^{10} - 5429z^{11} + 72z^{12}) + y^5z(708 + 2880z - 75318z^2 + 390360z^3 - 862488z^4 + 683692z^5 + 668588z^6
- 2089833z^7 + 2146884z^8 - 1135548z^9 + 302226z^{10} - 32683z^{11} + 1204z^{12}) + 2y^6(43 - 562z - 22170z^2 + 191307z^3
- 617668z^4 + 934096z^5 - 486604z^6 - 520444z^7 + 1071647z^8 - 795420z^9 + 294048z^{10} - 50805z^{11} + 3204z^{12})) - 144(-1
+ y)^4(-1+z)^4(y+z)^6(100y^4 + 2y^3(-127 + 60z) + 6y^2(38 - 41z + 24z^2) + z(-38 + 153z - 203z^2 + 88z^3) + y(-74
+ 144z - 231z^2 + 132z^3))H(2,y) - 144(-1+y)^4(-1+z)^4(y+z)^6(-4 + 36y^4 - 4z + 51z^2 - 79z^3 + 36z^4 + y^3(-79
+ 52z) + y^2(51 - 75z + 48z^2) + y(-4 + 24z - 75z^2 + 52z^3))H(3,y)) + 18(-1+y)^4(-1+z)^4(y+z)^6(496y^4
+ y^3(-1129 + 1244z) + 3y^2(213 - 557z + 284z^2) + y(62 + 444z - 870z^2 + 416z^3) + 4(-17 - 46z + 234z^2 - 301z^3
+ 130z^4))H(0,0,y) + 18(-1+y)^4(-1+z)^4(y+z)^6(-68 + 520y^4 + 62z + 639z^2 - 1129z^3 + 496z^4 + 4y^3(-301
+ 104z) + y^2(936 - 870z + 852z^2) + y(-184 + 444z - 1671z^2 + 1244z^3))H(0,0,z) - 288(-1+y)^4(-1+z)^4(y+z)^6(2
+ 10y^4 - 4z + 3z^2 - z^3 + 7y^3(-3 + 4z) + 6y^2(2 - 6z + z^2) + y(-3 + 18z - 12z^2 + 4z^3))H(0,1,z) + 144(-1
+ y)^4(-1+z)^4(y+z)^6(-8 + 36y^4 + 2z + 27z^2 - 37z^3 + 16z^4 + 11y^3(-7 + 4z) + y^2(45 - 51z + 36z^2) - y(-4
+ 12z + 3z^2 + 4z^3))H(0,2,y) - 144(-1+y)^4y(-1+z)^4(y+z)^6(4y^3 + y^2(19 + 12z) + 4(7 - 6z + 3z^2) - 3y(17
- 9z + 8z^2))H(1,0,y) - 144(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(88y^3 + 13y^2(-9 + 4z) - 2(2 - 2z + z^2) + y(42
- 38z + 8z^2))H(1,0,z) + 288(-1+y)^4y(-1+z)^4(y+z)^6(5 + 2y^3 + 6z - 3z^2 + 4y^2(-1 + 6z) + 3y(-1 - 9z
+ 2z^2))H(1,1,y) - 18(-1+y)^4(-1+z)^4(y+z)^6(-72 + 1792y^4 + 182z + 909z^2 - 2011z^3 + 992z^4 + y^3(-4043
+ 3204z) + 3y^2(911 - 1505z + 664z^2) + y(-410 + 1560z - 3123z^2 + 2244z^3))H(1,1,z) + 144(-1+y)^4(-1+z)^4(y
+ z)^6(84y^4 + y^3(-193 + 76z) + 3y^2(51 - 39z + 16z^2) + 2(-2 + 4z - 3z^2 + z^3) - 2y(20 - 12z + 3z^2
+ 4z^3))H(1,2,y) + 144(-1+y)^4(-1+z)^4(y+z)^6(-4 + 4y^4 + 46z - 159z^2 + 205z^3 - 88z^4 + y^3(21 + 4z) + y^2(-57
+ 75z - 84z^2) + y(36 - 108z + 219z^2 - 140z^3))H(2,0,y) - 144(-1+y)^4(-1+z)^4(y+z)^6(20y^4 + y^3(-49 + 60z)
+ 3y^2(11 - 35z + 24z^2) + 4z(-10 + 27z - 25z^2 + 8z^3) + 4y(-1 + 18z - 30z^2 + 12z^3))H(2,1,y) - 18(-1+y)^4(-1
+ z)^4(y+z)^6(-104 + 1536y^4 - 26z + 1821z^2 - 3227z^3 + 1536z^4 + y^3(-3227 + 2756z) + 3y^2(607 - 1217z + 632z^2)
+ y(-26 + 1272z - 3651z^2 + 2756z^3))H(2,2,y) - 144(-1+y)^4(-1+z)^4(y+z)^6(-4 + 36y^4 - 4z + 51z^2 - 79z^3
+ 36z^4 + y^3(-79 + 52z) + y^2(51 - 75z + 48z^2) + y(-4 + 24z - 75z^2 + 52z^3))H(3,2,y)\Big\}\Big/\Big(576(-1+y)^4y(-1
+ z)^4z(-1+y+z)(y+z)^6\Big);
$$

$$
\mathcal{A}^{(2)}_{6;C_F^2} = \Big\{4(-1+y)(-1+z)(y+z)(288y^{13}z - 3(-1+z)^5z^5(90 - 90z + 43z^2) + y^{12}(-129 - 1327z + 1720z^2 + 488z^3
- 176z^4) + y^{11}(915 + 342z - 6679z^2 + 3734z^3 + 1168z^4 - 56z^5) + y(-1+z)^4z^4(1350 - 4968z + 5525z^2 - 2086z^3
- 175z^4 + 288z^5) + y^2(-1+z)^3z^3(-2700 + 15672z - 38367z^2 + 41471z^3 - 16563z^4 - 1519z^5 + 1720z^6) + y^{10}(-2910
+ 11667z - 6846z^2 - 9291z^3 + 9388z^4 - 7000z^5 + 3264z^6) + y^9(5340 - 38596z + 84883z^2 - 66628z^3 + 1089z^4 + 39320z^5
- 36152z^6 + 10744z^7) + y^3(-1+z)^2z^2(2700 - 29448z + 105101z^2 - 188876z^3 + 176750z^4 - 72056z^5 - 359z^6 + 4710z^7
+ 488z^8) + y^8(-6045 + 62541z - 210950z^2 + 320503z^3 - 225267z^4 + 21438z^5 + 93980z^6 - 69672z^7 + 15200z^8) + y^7(4179
- 59394z + 271749z^2 - 614432z^3 + 753510z^4 - 469026z^5 + 49426z^6 + 123492z^7 - 69672z^8 + 10744z^9) + y^6(-1620 + 33497z
- 206288z^2 + 659603z^3 - 1187244z^4 + 1201898z^5 - 610940z^6 + 49426z^7 + 93980z^8 - 36152z^9 + 3264z^{10}) + y^4z(1350
- 23772z + 166697z^2 - 565574z^3 + 1068831z^4 - 1187244z^5 + 753510z^6 - 225267z^7 + 1089z^8 + 9388z^9 + 1168z^{10}
- 176z^{11}) + y^5(270 - 10368z + 93483z^2 - 428526z^3 + 1068831z^4 - 1510552z^5 + 1201898z^6 - 469026z^7 + 21438z^8
+ 39320z^9 - 7000z^{10} - 56z^{11})) - 3(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(46 + 704y^3 - 112z + 254z^2 - 192z^3
+ y^2(-717 + 404z) - 2y(-21 + 11z + 88z^2))H(0,y)^2 + 3(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(-46 + 192y^3 - 42z
+ 717z^2 - 704z^3 + 2y^2(-127 + 88z) + y(112 + 22z - 404z^2))H(0,z)^2 + 9(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(4
+ 56y^3 + 34z - 101z^2 + 56z^3 + y^2(-101 + 84z) + y(34 - 76z + 84z^2))H(1,z)^2 + H(1,y)(-8(-1+y)^2(-1+z)^4(-1+y
+ z)(y+z)^6(150 + 136y^6 - 406z + 411z^2 - 155z^3 + y^5(-761 + 460z) + y^4(1819 - 2033z + 456z^2) + y^2(1859 - 3607z
+ 2367z^2 - 465z^3) + y^3(-2401 + 3730z - 1665z^2 + 164z^3) + y(-802 + 1856z - 1569z^2 + 474z^3)) - 96(-1+y)^4y(-1
+ z)^4(-1+y+z)^2(y+z)^6(3 + 14y^2 - 3z + 3y(-5 + 2z))H(1,z)) + H(0,y)(8(-1+y)^2y(-1+y+z)^2(y
$$

$$+ z)^6(144y^5z + 12(-1+z)^3z(7-6z+z^2) + 4y^4z(-153+111z-40z^2+10z^3) - 3y(-1+z)^2(-4-162z+220z^2$$
$$- 108z^3+11z^4) + y^3(12+1112z-1955z^2+1610z^3-799z^4+164z^5) + 2y^2(-12-511z+1395z^2-1516z^3+831z^4$$
$$- 189z^5+2z^6)) + 144(-1+y)^4y(-1+z)^4(-1+y+z)^2(y+z)^6(4+12y^2-4z+y(-15+8z))H(1,z)) + 8(-1+y$$
$$+ z)(144y^{15}z + (-1+z)^5z^6(-132+340z-341z^2+136z^3) + 4y^{14}(34-397z+567z^2-176z^3+44z^4) + y^{13}(-1021$$
$$+ 8884z - 22137z^2 + 20586z^3 - 8560z^4 + 1816z^5) + y(-1+z)^4z^5(468-3576z+7726z^2-7779z^3+3972z^4-1012z^5$$
$$+ 144z^6) + y^{12}(3405-30315z+100809z^2-152893z^3+116054z^4-46660z^5+8448z^6) + 3y^2(-1+z)^3z^4(-748+5012z$$
$$- 15992z^2+27958z^3-27830z^4+16002z^5-5111z^6+756z^7) + y^{11}(-6602+66866z-275775z^2+575010z^3-652809z^4$$
$$+ 420138z^5-149708z^6+23168z^7) + y^{10}(8150-98054z+493695z^2-1321709z^3+2031803z^4-1859997z^5+1019924z^6$$
$$- 313380z^7+41296z^8) - y^3(-1+z)^2z^3(-1416+28792z-145538z^2+374050z^3-574998z^4+551544z^5-328166z^6$$
$$+ 113833z^7-19178z^8+704z^9) + y^9(-6561+96216z-598074z^2+2006252z^3-3969623z^4+4828260z^5-3670950z^6$$
$$+ 1713820z^7-448924z^8+49872z^9) + y^8(3361-62011z+494076z^2-2075590z^3+5157019z^4-8018097z^5+8000348z^6$$
$$- 5125514z^7+2032884z^8-448924z^9+41296z^{10}) + 2y^7(-500+12419z-137577z^2+734318z^3-2271456z^4+4429489z^5$$
$$- 5635698z^6+4719742z^7-2562757z^8+856910z^9-156690z^{10}+11584z^{11}) + y^4z^2(2244-31624z+242798z^2$$
$$- 1036852z^3+2692286z^4-4542912z^5+5157019z^6-3969623z^7+2031803z^8-652809z^9+116054z^{10}-8560z^{11}+176z^{12})$$
$$+ y^5z(468-21768z+204538z^2-1036852z^3+3244920z^4-6575600z^5+8858978z^6-8018097z^7+4828260z^8-1859997z^9$$
$$+ 420138z^{10}-46660z^{11}+1816z^{12}) + 2y^6(66-2724z+49908z^2-346959z^3+1346143z^4-3287800z^5+5273177z^6$$
$$- 5635698z^7+4000174z^8-1835475z^9+509962z^{10}-74854z^{11}+4224z^{12}))H(2,y) + 9(-1+y)^4(-1+z)^4(-1+y+z)^2(y$$
$$+ z)^6(4+56y^3+34z-101z^2+56z^3+y^2(-101+84z)+y(34-76z+84z^2))H(2,y)^2 + H(0,z)(8(-1+z)^2z(-1+y$$
$$+ z)^2(y+z)^6(12(-1+z)^2z+y^6(12-33z+4z^2)+2y^5(-54+195z-189z^2+82z^3)+y^4(336-1341z+1662z^2$$
$$- 799z^3+40z^4) - 2y^3(240-1065z+1516z^2-805z^3+80z^4) + y^2(324-1620z+2790z^2-1955z^3+444z^4)$$
$$+ 2y(-42+231z-511z^2+556z^3-306z^4+72z^5)) + 48(-1+y)^4y(-1+z)^4(-1+y+z)^2(y+z)^6(6+8y^2-6z$$
$$+ 3y(-5+4z))H(1,y) - 48(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(8y^3+3y^2(-5+4z)+6y(1+z-4z^2)-3z(4$$
$$- 15z+12z^2))H(2,y)) + H(1,z)(8(-1+y+z)(144y^{15}z+6(-1+z)^5z^6(3-2z+2z^2)+4y^{14}(34-397z+567z^2$$
$$- 176z^3+44z^4) + y^{13}(-866+8100z-20569z^2+19018z^3-7767z^4+1652z^5) + 2y(-1+z)^4z^5(-216+221z+222z^2$$
$$- 456z^3+370z^4-234z^5+72z^6) + y^{12}(2374-23858z+83917z^2-129245z^3+97321z^4-38669z^5+7008z^6) + y^2(-1$$
$$+ z)^3z^4(6-780z-2588z^2+13184z^3-18942z^4+14294z^5-6479z^6+1452z^7) + y^{11}(-3622+43744z-199491z^2$$
$$+ 436042z^3-501379z^4+321120z^5-113638z^6+17512z^7) + y^{10}(3290-50918z+300333z^2-878721z^3+1407853z^4$$
$$- 1304275z^5+713376z^6-217490z^7+28280z^8) - y^3(-1+z)^2z^3(1584-2338z-14788z^2+72720z^3-149820z^4$$
$$+ 171732z^5-117066z^6+45939z^7-8478z^8+160z^9) + 2y^9(-863+18146z-145902z^2+567991z^3-1216836z^4$$
$$+ 1534256z^5-1178239z^6+547769z^7-141466z^8+15288z^9) + 2y^8(213-6663z+89049z^2-480126z^3+1360821z^4$$
$$- 2265678z^5+2330148z^6-1502619z^7+590049z^8-126886z^9+11116z^{10}) + 2y^7(3+76z-31201z^2+259264z^3$$
$$- 985699z^4+2156109z^5-2918981z^6+2511740z^7-1366047z^8+447153z^9-77957z^{10}+5324z^{11}) + y^4z^2(-6+5026z$$
$$+ 3058z^2-165362z^3+688552z^4-1444642z^5+1850424z^6-1531142z^7+816755z^8-264717z^9+44437z^{10}-2423z^{11}$$
$$+ 40z^{12}) + y^5z(-432-66z+18020z^2-196646z^3+926280z^4-2346594z^5+3574134z^6-3448086z^7+2127944z^8$$
$$- 811205z^9+172740z^{10}-16485z^{11}+540z^{12}) + 2y^6(-9+845z+4861z^2-81055z^3+437656z^4-1299435z^5$$
$$+ 2341652z^6-2665679z^7+1940166z^8-884963z^9+236428z^{10}-31919z^{11}+1596z^{12})) + 48(-1+y)^4(-1+z)^4(-1+y$$
$$+ z)^2(y+z)^6(-4+28y^3+18z-21z^2+8z^3+4y^2(-8+5z)+2y(5-10z+2z^2))H(2,y) - 144(-1+y)^4(-1$$
$$+ z)^4(-1+y+z)^2(y+z)^6(12y^3+y^2(-15+8z)+y(4-8z+8z^2)+z(4-15z+12z^2))H(3,y)) + 6(-1+y)^4(-1$$
$$+ z)^4(-1+y+z)^2(y+z)^6(46+704y^3-112z+254z^2-192z^3+y^2(-717+404z)-2y(-21+11z+88z^2))H(0,0,y)$$
$$- 6(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(-46+192y^3-42z+717z^2-704z^3+2y^2(-127+88z)+y(112+22z$$
$$- 404z^2))H(0,0,z) - 96(-1+y)^4(-1+4y)(-1+z)^4(-1+y+z)^2(y+z)^6(2+2y^2+2y(-2+z)-2z$$
$$+ z^2)H(0,1,y) - 144(-1+y)^4y(-1+z)^4(-1+y+z)^2(y+z)^6(4+12y^2-4z+y(-15+8z))H(0,1,z) + 48(-1$$
$$+ y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(52y^3+y^2(-81+40z)-2(2-2z+z^2)+4y(9-8z+2z^2))H(0,2,y)$$
$$- 144(-1+y)^4y(-1+z)^4(-1+y+z)^2(y+z)^6(4+12y^2-4z+y(-15+8z))H(1,0,y) - 48(-1+y)^4y(-1+z)^4(-1$$
$$+ y+z)^2(y+z)^6(6+8y^2-6z+3y(-5+4z))H(1,0,z) + 48(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(20y^3$$
$$+ y^2(-9+8z)+2(2-2z+z^2)-4y(3-2z+2z^2))H(1,1,y) + 6(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(56y^3$$
$$+ y^2(63-156z)-18y(3-10z+14z^2)-3(4+34z-101z^2+56z^3))H(1,1,z) - 96(-1+y)^4y(-1+z)^4(-1+y$$
$$+ z)^2(y+z)^6(3+14y^2-3z+3y(-5+2z))H(1,2,y) + 48(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(36y^3$$
$$+ 3y^2(-15+8z)+z(-6+15z-8z^2)-6y(-2+z+2z^2))H(2,0,y) - 48(-1+y)^4(-1+z)^4(-1+y+z)^2(y$$
$$+ z)^6(36y^3+16y(-1+z)^2+4(-1+z)^2(-1+4z)+y^2(-47+32z))H(2,1,y) + 6(-1+y)^4(-1+z)^4(-1+y+z)^2(y$$

$$+ z)^6(-76 + 184y^3 + 170z - 241z^2 + 184z^3 + y^2(-241 + 36z) + 2y(85 - 94z + 18z^2))H(2,2,y) - 144(-1 + y)^4(-1$$
$$+ z)^4(-1 + y + z)^2(y + z)^6(12y^3 + y^2(-15 + 8z) + y(4 - 8z + 8z^2) + z(4 - 15z + 12z^2))H(3,2,y)\Big\}\Big/\Big(48(-1$$
$$+ y)^4y(-1 + z)^4z(-1 + y + z)^2(y + z)^6\Big);$$

$$\mathcal{A}^{(2)}_{6;n_f^2} = \frac{1}{18}\,\mathcal{A}_0;$$

$$\mathcal{A}^{(2)}_{6;C_AC_F} = \Big\{ - 2(-1 + y)(-1 + z)(y + z)(1152y^{13}z - 9(-1 + z)^5z^5(90 - 94z + 41z^2) + 3y^{12}(-123 - 2483z + 4229z^2$$
$$- 1283z^3 + 428z^4) + 3y(-1 + z)^4z^4(1350 - 5784z + 5943z^2 - 1088z^3 - 947z^4 + 384z^5) + 3y^{11}(897 + 5004z$$
$$- 25436z^2 + 30556z^3 - 16597z^4 + 4808z^5) + 3y^2(-1 + z)^3z^3(-2700 + 13740z - 36361z^2 + 35633z^3 - 2149z^4$$
$$- 12749z^5 + 4229z^6) + y^{10}(-8730 + 9231z + 146355z^2 - 412685z^3 + 451131z^4 - 253782z^5 + 61568z^6) + 2y^9(8100$$
$$- 47868z - 594z^2 + 355100z^3 - 742645z^4 + 700596z^5 - 341189z^6 + 68500z^7) - y^3(-1 + z)^2z^2(-8100 + 99768z$$
$$- 310125z^2 + 455122z^3 - 249510z^4 - 144438z^5 + 240896z^6 - 83970z^7 + 3849z^8) + y^8(-18405 + 190647z - 410874z^2$$
$$- 280262z^3 + 2163621z^4 - 3406317z^5 + 2683478z^6 - 1092120z^7 + 177144z^8) + y^7(12699 - 194892z + 695613z^2$$
$$- 809704z^3 - 883105z^4 + 3818004z^5 - 4987759z^6 + 3306568z^7 - 1092120z^8 + 137000z^9) + y^6(-4896 + 111537z$$
$$- 565908z^2 + 1469879z^3 - 1595836z^4 - 916967z^5 + 4424978z^6 - 4987759z^7 + 2683478z^8 - 682378z^9 + 61568z^{10})$$
$$+ y^4z(4050 - 65520z + 517761z^2 - 1698158z^3 + 2639853z^4 - 1595836z^5 - 883105z^6 + 2163621z^7 - 1485290z^8$$
$$+ 451131z^9 - 49791z^{10} + 1284z^{11}) + y^5(810 - 33552z + 257043z^2 - 1175140z^3 + 2639853z^4 - 2346720z^5 - 916967z^6$$
$$+ 3818004z^7 - 3406317z^8 + 1401192z^9 - 253782z^{10} + 14424z^{11})) + 9(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(-27$$
$$+ 612y^4 + 7z - 18z^2 + 130z^3 - 92z^4 + 6y^3(-219 + 206z) + 6y^2(127 - 251z + 78z^2) - 3y(11 - 102z + 26z^2$$
$$+ 52z^3))H(0,y)^2 - 9(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(27 + 92y^4 + 33z - 762z^2 + 1314z^3 - 612z^4$$
$$+ 26y^3(-5 + 6z) + y^2(18 + 78z - 468z^2) - y(7 + 306z - 1506z^2 + 1236z^3))H(0,z)^2 - 9(-1 + y)^4(-1 + z)^4(-1$$
$$+ y + z)(y + z)^6(-58 + 80y^4 + 136z + 27z^2 - 185z^3 + 80z^4 + 5y^3(-37 + 68z) + 3y^2(9 - 139z + 112z^2) + y(136$$
$$- 12z - 417z^2 + 340z^3))H(1,z)^2 + H(0,y)(-12(-1 + y)^2y(-1 + y + z)(y + z)^6(192y^6z + 4y^5z(-255 + 207z$$
$$- 64z^2 + 16z^3) - 6(-1 + z)^4(1 + 18z - 47z^2 + 32z^3) + 3y(-1 + z)^3(3 + 6z + 242z^2 - 344z^3 + 134z^4)$$
$$- y^2(-1 + z)^2(-63 - 1120z + 692z^2 + 644z^3 - 824z^4 + 208z^5) + y^4(27 + 2224z - 3883z^2 + 2826z^3 - 1250z^4$$
$$+ 248z^5) - y^3(75 + 2315z - 5945z^2 + 5649z^3 - 2530z^4 + 376z^5 + 60z^6)) - 72(-1 + y)^4(-1 + z)^4(-1 + y + z)(y$$
$$+ z)^6(56y^4 - 3y^2(-7 + 9z) + y^3(-91 + 60z) + y(20 - 36z + 27z^2 - 12z^3) + 3(-2 + 4z - 3z^2 + z^3))H(1,z))$$
$$+ H(1,y)(12(-1 + y)^3(-1 + z)^4(-1 + y + z)(y + z)^6(-406 + 388y^5 + 1020z - 1029z^2 + 383z^3 + y^4(-1877 + 1092z)$$
$$+ y^3(3592 - 3873z + 1116z^2) + y(1814 - 3714z + 3018z^2 - 859z^3) + y^2(-3511 + 5475z - 3111z^2 + 476z^3)) + 72(-1$$
$$+ y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(2 + 24y^4 - 4z + 3z^2 - z^3 + 4y^3(-7 + 10z) - 6y^2(2 + 3z + 2z^2) + y(14$$
$$- 3z^2 + 4z^3))H(1,z)) - 6(-1 + y + z)(384y^{15}z + (-1 + z)^5z^6(-726 + 1882z - 2105z^2 + 776z^3) + 8y^{14}(97$$
$$- 739z + 1077z^2 - 452z^3 + 113z^4) + y^{13}(-5985 + 40168z - 92560z^2 + 89128z^3 - 40267z^4 + 8364z^5) + y(-1$$
$$+ z)^4z^5(2964 - 19836z + 41602z^2 - 41559z^3 + 20360z^4 - 4376z^5 + 384z^6) + y^{12}(20167 - 150791z + 451180z^2$$
$$- 659196z^3 + 502581z^4 - 202437z^5 + 35424z^6) + 2y^2(-1 + z)^3z^4(-6765 + 42879z - 127932z^2 + 212727z^3$$
$$- 204354z^4 + 112598z^5 - 33356z^6 + 4308z^7) + 2y^{11}(-19473 + 173943z - 646524z^2 + 1262820z^3 - 1387944z^4$$
$$+ 876471z^5 - 304141z^6 + 45232z^7) + 2y^{10}(23690 - 260707z + 1196939z^2 - 2976262z^3 + 4350991z^4 - 3849912z^5$$
$$+ 2048644z^6 - 608211z^7 + 77132z^8) - 2y^3(-1 + z)^2z^3(-4252 + 81656z - 390703z^2 + 946461z^3 - 1380495z^4$$
$$+ 1270200z^5 - 730084z^6 + 245894z^7 - 40948z^8 + 1808z^9) + y^9(-37381 + 518516z - 2983546z^2 + 9301958z^3$$
$$- 17356961z^4 + 20206452z^5 - 14838192z^6 + 6708682z^7 - 1702288z^8 + 183528z^9) + y^8(18775 - 338839z + 2538420z^2$$
$$- 9955302z^3 + 23235465z^4 - 34337223z^5 + 32931038z^6 - 20432684z^7 + 7885302z^8 - 1702288z^9 + 154264z^{10})$$
$$+ 2y^7(-2756 + 69365z - 731925z^2 + 3664120z^3 - 10625532z^4 + 19591238z^5 - 23845782z^6 + 19305676z^7 - 10216342z^8$$
$$+ 3354341z^9 - 608211z^{10} + 45232z^{11}) + y^4z^2(13530 - 180320z + 1309202z^2 - 5314260z^3 + 13155096z^4 - 21251064z^5$$
$$+ 23235465z^6 - 17356961z^7 + 8701982z^8 - 2775888z^9 + 502581z^{10} - 40267z^{11} + 904z^{12}) + y^5z(2964 - 126348z$$
$$+ 1116534z^2 - 5314260z^3 + 15703800z^4 - 30293056z^5 + 39182476z^6 - 34337223z^7 + 20206452z^8 - 7699824z^9$$
$$+ 1752942z^{10} - 202437z^{11} + 8364z^{12}) + 2y^6(363 - 15846z + 276864z^2 - 1809523z^3 + 6577548z^4 - 15146528z^5$$
$$+ 23154650z^6 - 23845782z^7 + 16465519z^8 - 7419096z^9 + 2048644z^{10} - 304141z^{11} + 17712z^{12}))H(2,y) - 9(-1 + y)^4(-1$$
$$+ z)^4(-1 + y + z)(y + z)^6(-58 + 80y^4 + 136z + 27z^2 - 185z^3 + 80z^4 + 5y^3(-37 + 68z) + 3y^2(9 - 139z$$
$$+ 112z^2) + y(136 - 12z - 417z^2 + 340z^3))H(2,y)^2 + H(0,z)(12(-1 + z)^2z(-1 + y + z)(y + z)^6(-3(-1 + z)^3(2$$
$$+ 9z) + 2y^7(96 - 201z + 104z^2) + 2y^6(-525 + 1119z - 620z^2 + 30z^3) - 4y^5(-597 + 1257z - 625z^2 - 94z^3$$

$$
\begin{aligned}
&+ 62z^4) - y(-1+z)^2(-84 - 159z + 760z^2 - 636z^3 + 192z^4) + y^2(-678 + 699z + 2869z^2 - 5945z^3 + 3883z^4 \\
&- 828z^5) - 2y^4(1443 - 2829z + 710z^2 + 1265z^3 - 625z^4 + 32z^5) + y^3(1944 - 3165z - 1860z^2 + 5649z^3 - 2826z^4 \\
&+ 256z^5)) - 72(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(-2 + 32y^4 + 4z - 3z^2 + z^3 + y^3(-93 + 68z) + 3y^2(29 \\
&- 39z + 16z^2) - y(24 - 36z + 15z^2 + 4z^3))H(1,y) + 288(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(1 + 8y^4 - 4z \\
&- 6z^2 + 23z^3 - 14z^4 + 4y^3(-6+5z) + 12y^2(2 - 3z + z^2) + y(-9 + 18z + 3z^2 - 16z^3))H(2,y)) \\
&+ H(1,z)(-6(-1+y+z)(384y^{15}z + (-1+z)^5z^6(86 - 122z + 97z^2) + 8y^{14}(97 - 739z + 1077z^2 - 452z^3 \\
&+ 113z^4) + y^{13}(-5219 + 36152z - 84156z^2 + 80352z^3 - 35693z^4 + 7412z^5) + y(-1+z)^4z^5(-1908 + 2348z - 286z^2 \\
&- 1305z^3 + 1608z^4 - 1272z^5 + 384z^6) + y^{12}(15045 - 117921z + 362964z^2 - 532604z^3 + 400231z^4 - 158267z^5 \\
&+ 27480z^6) + 2y^2(-1+z)^3z^4(-675 - 195z - 1902z^2 + 12975z^3 - 20676z^4 + 17018z^5 - 8414z^6 + 1980z^7) \\
&+ 2y^{11}(-12039 + 115335z - 450060z^2 + 899424z^3 - 987042z^4 + 612507z^5 - 208045z^6 + 30304z^7) + 2y^{10}(11498 \\
&- 141313z + 701873z^2 - 1830608z^3 + 2724693z^4 - 2395960z^5 + 1248128z^6 - 359939z^7 + 43932z^8) - 2y^3(-1 \\
&+ z)^2z^3(3868 - 2404z - 34723z^2 + 117005z^3 - 200991z^4 + 214578z^5 - 145420z^6 + 58736z^7 - 11368z^8 + 256z^9) \\
&+ y^9(-12895 + 213812z - 1415134z^2 + 4813534z^3 - 9392031z^4 + 11059916z^5 - 8020592z^6 + 3525722z^7 - 858756z^8 \\
&+ 87192z^9) + y^8(3685 - 89137z + 912288z^2 - 4186350z^3 + 10584771z^4 - 16190181z^5 + 15575102z^6 - 9479600z^7 \\
&+ 3525138z^8 - 718540z^9 + 59752z^{10}) + 2y^7(-112 + 5193z - 179561z^2 + 1190012z^3 - 3904772z^4 + 7681554z^5 \\
&- 9620890z^6 + 7790928z^7 - 4023410z^8 + 1256653z^9 - 210091z^{10} + 13920z^{11}) + y^4z^2(1350 + 16400z + 21642z^2 \\
&- 611924z^3 + 2286652z^4 - 4384532z^5 + 5218711z^6 - 4097291z^7 + 2122526z^8 - 683628z^9 + 116791z^{10} - 6825z^{11} \\
&+ 128z^{12}) + y^5z(-1908 - 10644z + 125622z^2 - 840744z^3 + 3311496z^4 - 7624456z^5 + 10838140z^6 - 9884853z^7 \\
&+ 5831340z^8 - 2153460z^9 + 452070z^{10} - 43743z^{11} + 1524z^{12}) + 2y^6(-43 + 3050z + 40106z^2 - 411629z^3 + 1786766z^4 \\
&- 4630398z^5 + 7634856z^6 - 8152400z^7 + 5636279z^8 - 2461502z^9 + 635000z^{10} - 83925z^{11} + 4224z^{12})) - 288(-1 \\
&+ y)^4(-1+z)^4(-1+y+z)(y+z)^6(4y^3 + 4y^4 + 3(-1+z)z^2 - 3y^2(6 - 7z + 6z^2) - 2y(-5 + 9z - 12z^2 \\
&+ 6z^3))H(2,y) + 432(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(12y^3 + y^2(-15 + 8z) + y(4 - 8z + 8z^2) + z(4 \\
&- 15z + 12z^2))H(3,y)) - 18(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(-27 + 612y^4 + 7z - 18z^2 + 130z^3 - 92z^4 \\
&+ 6y^3(-219 + 206z) + 6y^2(127 - 251z + 78z^2) - 3y(11 - 102z + 26z^2 + 52z^3))H(0,0,y) + 18(-1+y)^4(-1 \\
&+ z)^4(-1+y+z)(y+z)^6(27 + 92y^4 + 33z - 762z^2 + 1314z^3 - 612z^4 + 26y^3(-5+6z) + y^2(18 + 78z - 468z^2) \\
&- y(7 + 306z - 1506z^2 + 1236z^3))H(0,0,z) + 144(-1+y)^4y(-1+z)^4(-1+y+z)(y+z)^6(16y^3 + y^2(-41 + 12z) \\
&- 2(7 - 6z + 3z^2) + 3y(13 - 9z + 4z^2))H(0,1,y) + 72(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(2 + 64y^3 - 2z \\
&+ z^2 + 8y^2(-9+5z) - 2y(-6 + 7z + 2z^2))H(0,1,z) - 72(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(104y^4 + 2(-1 \\
&+ z)^3(-1+4z) + y^3(-245 + 148z) + 3y^2(65 - 77z + 28z^2) + 8y(-7 + 12z - 9z^2 + 2z^3))H(0,2,y) + 72(-1 \\
&+ y)^4(-1+z)^4(-1+y+z)(y+z)^6(56y^4 - 3y^2(-7+9z) + y^3(-91 + 60z) + y(20 - 36z + 27z^2 - 12z^3) + 3(-2 \\
&+ 4z - 3z^2 + z^3))H(1,0,y) + 72(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(-2 + 32y^4 + 4z - 3z^2 + z^3 + y^3(-93 \\
&+ 68z) + 3y^2(29 - 39z + 16z^2) - y(24 - 36z + 15z^2 + 4z^3))H(1,0,z) - 216(-1+y)^4y(-1+z)^4(-1+y+z)(y \\
&+ z)^6(6 + 8y^3 + 4z - 2z^2 + y^2(-13 + 28z) + y(-1 - 27z + 4z^2))H(1,1,y) + 18(-1+y)^4(-1+z)^4(-1+y+z)(y \\
&+ z)^6(-50 + 16y^4 + 120z + 39z^2 - 189z^3 + 80z^4 + y^3(-253 + 356z) + y^2(327 - 741z + 528z^2) + y(-40 + 228z \\
&- 549z^2 + 356z^3))H(1,1,z) + 72(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(2 + 24y^4 - 4z + 3z^2 - z^3 + 4y^3(-7 \\
&+ 10z) - 6y^2(2 + 3z + 2z^2) + y(14 - 3z^2 + 4z^3))H(1,2,y) - 288(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(-1 \\
&+ 14y^4 + 9z - 24z^2 + 24z^3 - 8z^4 + y^3(-23 + 16z) - 3y^2(-2 + z + 4z^2) + y(4 - 18z + 36z^2 - 20z^3))H(2,0,y) \\
&+ 144(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(4 + 32y^4 - 38z + 87z^2 - 77z^3 + 24z^4 + y^3(-75 + 68z) + 3y^2(19 \\
&- 39z + 20z^2) + y(-18 + 84z - 111z^2 + 44z^3))H(2,1,y) - 18(-1+y)^4(-1+z)^4(-1+y+z)(y+z)^6(90 + 176y^4 \\
&- 280z + 381z^2 - 367z^3 + 176z^4 + y^3(-367 + 12z) - 3y^2(-127 + 45z + 48z^2) + y(-280 + 396z - 135z^2 \\
&+ 12z^3))H(2,2,y) + 432(-1+y)^4(-1+z)^4(-1+y+z)^2(y+z)^6(12y^3 + y^2(-15 + 8z) + y(4 - 8z + 8z^2) + z(4 \\
&- 15z + 12z^2))H(3,2,y)\Big\}\Big/\Big(144(-1+y)^4y(-1+z)^4z(-1+y+z)^2(y+z)^6\Big);
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{6;C_A n_f} = &\Big\{(-1+y)(-1+z)(y+z)(864y^{12}z + 7(-1+z)^4z^5(2 - 2z + z^2) + y^{11}(7 - 3181z + 2877z^2 + 3053z^3 - 1028z^4) \\
&+ y(-1+z)^3z^4(-70 + 138z + 447z^2 - 697z^3 - 589z^4 + 864z^5) - 2y^{10}(21 - 1831z + 3460z^2 + 8127z^3 - 12227z^4 \\
&+ 3314z^5) + y^9(112 - 93z - 3293z^2 + 54841z^3 - 123339z^4 + 85304z^5 - 18716z^6) + y^2(-1+z)^2z^3(140 - 770z \\
&- 1605z^2 + 8180z^3 - 8502z^4 - 1166z^5 + 2877z^6) - y^8(168 + 2705z - 24018z^2 + 117638z^3 - 307100z^4 + 350637z^5 \\
&- 170234z^6 + 30204z^7) + y^7(147 + 1554z - 26467z^2 + 142908z^3 - 438373z^4 + 710452z^5 - 567495z^6 + 212662z^7 \\
&- 30204z^8) + y^6(-70 + 177z + 10620z^2 - 97217z^3 + 373368z^4 - 794957z^5 + 925784z^6 - 567495z^7 + 170234z^8 - 18716z^9)
\end{aligned}
$$

$$+ y^3z^2(140 + 376z - 7503z^2 + 37294z^3 - 97217z^4 + 142908z^5 - 117638z^6 + 54841z^7 - 16254z^8 + 3053z^9) + y^5(14 - 348z + 75z^2 + 37294z^3 - 197115z^4 + 514818z^5 - 794957z^6 + 710452z^7 - 350637z^8 + 85304z^9 - 6628z^{10}) + y^4z(70 - 1050z - 7503z^2 + 62552z^3 - 197115z^4 + 373368z^5 - 438373z^6 + 307100z^7 - 123339z^8 + 24454z^9 - 1028z^{10})) + 12(-1 + y)^4(y + z)^6(72y^5z - (-1 + z)^5(2 - 2z + z^2) + y^4z(-171 + 177z - 8z^2 + 2z^3) + y^3(-1 + z)^2(-1 + 150z - 33z^2 + 12z^3) + 3y^2(-1 + z)^3(-1 + 25z - 33z^2 + 22z^3) + y(-1 + z)^4(-4 + 30z - 81z^2 + 64z^3))H(0, y) + 27(-1 + y)^4(-1 + z)^4z(-1 + y + z)(y + z)^6(3 - 12z + 10z^2 + y(-3 + 6z))H(0, y)^2 + 27(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(0, z)^2 - 108(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(10y^3 + 6y^2(-2 + z) + y(3 - 6z + 6z^2) + z(3 - 12z + 10z^2))H(1, z)^2 + H(1, y)(12(-1 + y)^4(-1 + z)^4(y + z)^6(22y^4 + z(-1 + 3z) + 2y^3(-29 + 8z) - 3y(7 - 6z + 3z^2) + 3y^2(19 - 13z + 6z^2)) - 72(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(1, z)) + 6(144y^{15}z - (-1 + z)^5z^6(-2 + 44z - 73z^2 + 44z^3) - 2y^{14}(22 + 371z - 477z^2 - 80z^3 + 20z^4) + y^{13}(293 + 1168z - 5070z^2 + 2496z^3 + 821z^4 - 140z^5) + y(-1 + z)^4z^5(-24 + 332z - 904z^2 + 1005z^3 - 360z^4 - 166z^5 + 144z^6) + y^{12}(-849 + 873z + 9576z^2 - 16678z^3 + 5423z^4 + 311z^5 + 192z^6) + 6y^2(-1 + z)^3z^4(5 - 169z + 700z^2 - 1162z^3 + 833z^4 + 15z^5 - 368z^6 + 159z^7) + 2y^{11}(696 - 3138z - 1425z^2 + 18410z^3 - 21716z^4 + 10176z^5 - 3835z^6 + 976z^7) + 2y^{10}(-700 + 5626z - 9744z^2 - 13652z^3 + 51737z^4 - 54675z^5 + 33289z^6 - 13437z^7 + 2420z^8) + 2y^3(-1 + z)^2z^3(-168 + 1335z - 5622z^2 + 12199z^3 - 12840z^4 + 3543z^5 + 5796z^6 - 5603z^7 + 1408z^8 + 80z^9) + y^9(869 - 11156z + 40020z^2 - 28260z^3 - 100851z^4 + 231456z^5 - 227034z^6 + 134608z^7 - 45724z^8 + 6360z^9) + y^8(-313 + 6709z - 39528z^2 + 82844z^3 - 7079z^4 - 224019z^5 + 385714z^6 - 333886z^7 + 169290z^8 - 45724z^9 + 4840z^{10}) + 2y^7(27 - 1188z + 11322z^2 - 42860z^3 + 61782z^4 + 15764z^5 - 162079z^6 + 229116z^7 - 166943z^8 + 67304z^9 - 13437z^{10} + 976z^{11}) + y^5z(-24 + 1104z - 16920z^2 + 80478z^3 - 172200z^4 + 157568z^5 + 31528z^6 - 224019z^7 + 231456z^8 - 109350z^9 + 20352z^{10} + 311z^{11} - 140z^{12}) + y^4z^2(-30 + 3342z - 24582z^2 + 80478z^3 - 141088z^4 + 123564z^5 - 7079z^6 - 100851z^7 + 103474z^8 - 43432z^9 + 5423z^{10} + 821z^{11} - 40z^{12}) + 2y^6(-1 + 214z - 3666z^2 + 24778z^3 - 70544z^4 + 78784z^5 + 23768z^6 - 162079z^7 + 192857z^8 - 113517z^9 + 33289z^{10} - 3835z^{11} + 96z^{12}))H(2, y) - 108(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(10y^3 + 6y^2(-2 + z) + y(3 - 6z + 6z^2) + z(3 - 12z + 10z^2))H(2, y)^2 + H(0, z)(12(-1 + z)^4(y + z)^6(2 - 4z + 3z^2 - z^3 + y^7(-1 + 64z) + y^6(7 - 337z + 66z^2) + y^5(-22 + 738z - 297z^2 + 12z^3) + y^3(-45 + 584z - 591z^2 + 228z^3 - 8z^4) + y^4(40 - 866z + 570z^2 - 57z^3 + 2z^4) + y^2(31 - 225z + 333z^2 - 334z^3 + 177z^4) + y(-12 + 46z - 84z^2 + 152z^3 - 171z^4 + 72z^5)) - 72(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(1, y) + 72(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(2, y)) + H(1, z)(6(144y^{15}z + (-1 + z)^5z^6(2 - 2z + z^2) - 2y^{14}(22 + 371z - 477z^2 - 80z^3 + 20z^4) + y^{13}(293 + 1168z - 5070z^2 + 2496z^3 + 821z^4 - 140z^5) + y(-1 + z)^4z^5(-24 + 78z - 16z^2 - 225z^3 + 400z^4 - 342z^5 + 144z^6) + y^{12}(-843 + 831z + 9720z^2 - 16954z^3 + 5717z^4 + 149z^5 + 228z^6) + 2y^2(-1 + z)^3z^4(15 - 186z + 305z^2 + 667z^3 - 2514z^4 + 3319z^5 - 2176z^6 + 609z^7) + 2y^{11}(683 - 3014z - 1956z^2 + 19678z^3 - 23503z^4 + 11652z^5 - 4496z^6 + 1100z^7) + 2y^{10}(-678 + 5313z - 8022z^2 - 18773z^3 + 60879z^4 - 64750z^5 + 39999z^6 - 15912z^7 + 2808z^8) - 2y^3(-1 + z)^2z^3(168 - 900z + 2040z^2 + 85z^3 - 10206z^4 + 22176z^5 - 23052z^6 + 12257z^7 - 2704z^8 + 8z^9) + y^9(833 - 10292z + 33702z^2 - 4660z^3 - 153421z^4 + 305336z^5 - 293084z^6 + 170960z^7 - 56910z^8 + 7824z^9) + y^8(-299 + 6023z - 32496z^2 + 49134z^3 + 86751z^4 - 389079z^5 + 574814z^6 - 474284z^7 + 234114z^8 - 62510z^9 + 6680z^{10}) + 2y^7(26 - 1040z + 8961z^2 - 27672z^3 + 8010z^4 + 134028z^5 - 332223z^6 + 391604z^7 - 268514z^8 + 106908z^9 - 22068z^{10} + 1764z^{11}) + y^4z^2(-30 + 2672z - 15902z^2 + 33612z^3 + 1816z^4 - 152124z^5 + 346211z^6 - 406001z^7 + 279114z^8 - 108142z^9 + 19425z^{10} - 655z^{11} + 4z^{12}) + y^5z(-24 + 822z - 11768z^2 + 45164z^3 - 41376z^4 - 142688z^5 + 485144z^6 - 685563z^7 + 546704z^8 - 249740z^9 + 58512z^{10} - 5199z^{11} + 156z^{12}) + 2y^6(-1 + 187z - 2784z^2 + 16395z^3 - 31152z^4 - 31320z^5 + 222248z^6 - 400783z^7 + 385277z^8 - 215282z^9 + 66695z^{10} - 9876z^{11} + 540z^{12})) + 72(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(10y^3 + 6y^2(-2 + z) + y(3 - 6z + 6z^2) + z(3 - 12z + 10z^2))H(2, y)) - 54(-1 + y)^4(-1 + z)^4z(-1 + y + z)(y + z)^6(3 - 12z + 10z^2 + y(-3 + 6z))H(0, 0, y) - 54(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(0, 0, z) + 72(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(1, 0, z) + 72(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(40y^3 + 24y^2(-2 + z) + 3y(4 - 7z + 6z^2) + 3z(3 - 12z + 10z^2))H(1, 1, z) - 72(-1 + y)^4y(3 + 10y^2 + 6y(-2 + z) - 3z)(-1 + z)^4(-1 + y + z)(y + z)^6H(1, 2, y) + 72(-1 + y)^4(-1 + z)^4z(-1 + y + z)(y + z)^6(3 - 12z + 10z^2 + y(-3 + 6z))H(2, 0, y) + 288(-1 + y)^4(-1 + z)^4(-1 + y + z)(y + z)^6(10y^3 + 6y^2(-2 + z) + y(3 - 6z + 6z^2) + z(3 - 12z + 10z^2))H(2, 2, y)\Big\}\Big/\Big(144(-1 + y)^4y(-1 + z)^4z(-1 + y + z)(y + z)^6\Big);$$

$$\mathcal{A}^{(2)}_{6;C_F n_f} = \Big\{45(-1 + z)^3z^6(2 - 2z + z^2) + 3y^{11}(15 - 6z - 39z^2 + 28z^3) - 3y(-1 + z)^2z^5(180 - 528z + 506z^2 - 167z^3 + 6z^4) + 3y^{10}(-75 + 179z + 81z^2 - 417z^3 + 224z^4) + y^9(495 - 2538z + 2541z^2 + 3020z^3 - 5974z^4 + 2432z^5)$$

$$+ y^8(-585 + 5121z - 10623z^2 + 3201z^3 + 13112z^4 - 15226z^5 + 5024z^6) - 3y^2z^4(450 - 2184z + 4776z^2 - 5694z^3 + 3541z^4 - 847z^5 - 81z^6 + 39z^7) + 2y^7(180 - 2613z + 8541z^2 - 11754z^3 + 2089z^4 + 12163z^5 - 11756z^6 + 3180z^7) + y^3z^3(-1800 + 13656z - 34518z^2 + 41110z^3 - 23508z^4 + 3201z^5 + 3020z^6 - 1251z^7 + 84z^8) + 2y^4z^2(-675 + 6828z - 24831z^2 + 39456z^3 - 26784z^4 + 2089z^5 + 6556z^6 - 2987z^7 + 336z^8) + 2y^5z(-270 + 3276z - 17259z^2 + 39456z^3 - 38370z^4 + 7389z^5 + 12163z^6 - 7613z^7 + 1216z^8) + 2y^6(-45 + 1332z - 7164z^2 + 20555z^3 - 26784z^4 + 7389z^5 + 13973z^6 - 11756z^7 + 2512z^8) - 12(-1+y)^2(-1+4y)(-1+z)^2(y+z)^6(-2 + 2y^3 + 4z - 3z^2 + z^3 + y^2(-6+4z) + y(6 - 8z + 3z^2))H(1,y) + 3(-1+y)^2(-1+z)^2(32y^{10} + y^9(-101 + 244z) + 3y^8(37 - 241z + 280z^2) - z^6(-2 + 4z - 3z^2 + z^3) + 2y^7(-22 + 351z - 1125z^2 + 856z^3) + yz^5(-36 + 4z + 54z^2 - 39z^3 + 4z^4) + 6y^2z^4(69 - 146z + 118z^2 - 55z^3 + 12z^4) + 2y^3z^3(-348 - 98z + 825z^2 - 659z^3 + 200z^4) + 6y^5z(-6 - 206z + 527z^2 - 726z^3 + 332z^4) + 2y^4z^2(207 - 198z + 1263z^2 - 1518z^3 + 572z^4) + 2y^6(1 - 98z + 1110z^2 - 1987z^3 + 1132z^4))H(1,z) + 3(-1+y)^2(-1+z)^2(32y^{10} + 5y^9(-21 + 52z) + 3y^8(41 - 269z + 328z^2) + 6y^7(-10 + 145z - 469z^2 + 384z^3) + z^6(10 - 60z + 123z^2 - 105z^3 + 32z^4) + 6y^2z^4(89 - 302z + 516z^2 - 469z^3 + 164z^4) + yz^5(12 - 348z + 870z^2 - 807z^3 + 260z^4) + 6y^5z(2 - 302z + 927z^2 - 1400z^3 + 724z^4) + 2y^3z^3(-268 - 778z + 2781z^2 - 2961z^3 + 1152z^4) + 2y^4z^2(267 - 778z + 3213z^2 - 4200z^3 + 1860z^4) + 2y^6(5 - 174z + 1548z^2 - 2961z^3 + 1860z^4))H(2,y)\Big\}\Big/\Big(36(-1+y)^2y(-1+z)^2z(-1+y+z)(y+z)^6\Big).$$

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The Hierarchy problem and new dimensions at a millimeter*, *Phys.Lett.* **B429** (1998) 263–272, [hep-ph/9803315].

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys.Lett.* **B436** (1998) 257–263, [hep-ph/9804398].

[3] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D59** (1999) 086004, [hep-ph/9807344].

[4] L. Randall and R. Sundrum, *A Large mass hierarchy from a small extra dimension*, *Phys.Rev.Lett.* **83** (1999) 3370–3373, [hep-ph/9905221].

[5] P. Mathews, V. Ravindran, K. Sridhar and W. van Neerven, *Next-to-leading order QCD corrections to the Drell-Yan cross section in models of TeV-scale gravity*, *Nucl.Phys.* **B713** (2005) 333–377, [hep-ph/0411018].

[6] M. Kumar, P. Mathews and V. Ravindran, *PDF and scale uncertainties of various DY distributions in ADD and RS models at hadron colliders*, *Eur.Phys.J.* **C49** (2007) 599–611, [hep-ph/0604135].

[7] P. Mathews and V. Ravindran, *Angular distribution of Drell-Yan process at hadron colliders to NLO-QCD in models of TeV scale gravity*, *Nucl.Phys.* **B753** (2006) 1–15, [hep-ph/0507250].

[8] M. Kumar, P. Mathews, V. Ravindran and A. Tripathi, *Diphoton signals in theories with large extra dimensions to NLO QCD at hadron colliders*, *Phys.Lett.* **B672** (2009) 45–50, [0811.1670].

[9] N. Agarwal, V. Ravindran, V. Tiwari and A. Tripathi, *Z boson pair production at the LHC to O(alpha(s)) in TeV scale gravity models*, *Nucl.Phys.* **B830** (2010) 248–270, [0909.2651].

[10] N. Agarwal, V. Ravindran, V. K. Tiwari and A. Tripathi, *W^+W^- production in Large extra dimension model at next-to-leading order in QCD at the LHC*, *Phys.Rev.* **D82** (2010) 036001, [1003.5450].

[11] P. Mathews, V. Ravindran and K. Sridhar, *NLO-QCD corrections to dilepton production in the Randall-Sundrum model*, *JHEP* **0510** (2005) 031, [hep-ph/0506158].

[12] M. Kumar, P. Mathews, V. Ravindran and A. Tripathi, *Direct photon pair production at the LHC to order α_s in TeV scale gravity models*, *Nucl.Phys.* **B818** (2009) 28–51, [0902.4894].

[13] N. Agarwal, V. Ravindran, V. K. Tiwari and A. Tripathi, *Next-to-leading order QCD corrections to the Z boson pair production at the LHC in Randall Sundrum model*, *Phys.Lett.* **B686** (2010) 244–248, [0910.1551].

[14] N. Agarwal, V. Ravindran, V. K. Tiwari and A. Tripathi, *Next-to-leading order QCD corrections to W^+W^- production at the LHC in Randall Sundrum model*, *Phys.Lett.* **B690** (2010) 390–395, [1003.5445].

[15] S. A. Li, C. S. Li, H. T. Li and J. Gao, *Constraints on Randall-Sundrum model from the events of dijet production with QCD next-to-leading order accuracy at the LHC*, 1408.2762.

[16] R. Frederix, M. K. Mandal, P. Mathews, V. Ravindran, S. Seth et al., *Diphoton production in the ADD model to NLO+parton shower accuracy at the LHC*, *JHEP* **1212** (2012) 102, [1209.6527].

[17] R. Frederix, M. Mandal, P. Mathews, V. Ravindran and S. Seth, *Drell-Yan, ZZ, W^+W^- production in SM & ADD model to NLO+PS accuracy at the LHC*, *Eur.Phys.J.* **C74** (2014) 2745, [1307.7013].

[18] G. Das, P. Mathews, V. Ravindran and S. Seth, *RS resonance in di-final state production at the LHC to NLO+PS accuracy*, *JHEP* **10** (2014) 188, [1408.3970].

[19] M. Kumar, P. Mathews, V. Ravindran and S. Seth, *Neutral triple electroweak gauge boson production in the large extra-dimension model at the LHC*, *Phys.Rev.* **D85** (2012) 094507, [1111.7063].

[20] L. Xiao-Zhou, D. Peng-Fei, M. Wen-Gan, Z. Ren-You and G. Lei, *WWZ/γ production in large extra dimensions model at LHC and ILC*, *Phys.Rev.* **D86** (2012) 095008, [1209.6401].

[21] L. Xiao-Zhou, M. Wen-Gan, Z. Ren-You and G. Lei, *$WW\gamma/Z$ production in the Randall-Sundrum model at LHC and CLIC*, *Phys.Rev.* **D87** (2013) 056008, [1303.2307].

[22] C. Chong, G. Lei, M. Wen-Gan, Z. Ren-You, L. Xiao-Zhou et al., *ZZW production at the LHC within large extra dimensions model in next-to-leading order QCD*, 1401.4765.

[23] G. Das and P. Mathews, *Neutral Triple Vector Boson Production in Randall-Sundrum Model at the LHC*, *Phys. Rev.* **D92** (2015) 094034, [1507.08857].

[24] G. Das, C. Degrande, V. Hirschi, F. Maltoni and H.-S. Shao, *NLO predictions for the production of a (750 GeV) spin-two particle at the LHC*, 1605.09359.

[25] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer et al., *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079, [1405.0301].

[26] D. de Florian, M. Mahakhud, P. Mathews, J. Mazzitelli and V. Ravindran, *Quark and gluon spin-2 form factors to two-loops in QCD*, *JHEP* **02** (2014) 035, [1312.6528].

[27] T. Ahmed, G. Das, P. Mathews, N. Rana and V. Ravindran, *Spin-2 Form Factors at Three Loop in QCD*, *JHEP* **12** (2015) 084, [1508.05043].

[28] D. de Florian, M. Mahakhud, P. Mathews, J. Mazzitelli and V. Ravindran, *Next-to-Next-to-Leading Order QCD Corrections in Models of TeV-Scale Gravity*, *JHEP* **1404** (2014) 028, [1312.7173].

[29] T. Ahmed, P. Banerjee, P. K. Dhani, M. C. Kumar, P. Mathews, N. Rana et al., *NNLO QCD Corrections to the Drell-Yan Cross Section in Models of TeV-Scale Gravity*, 1606.08454.

[30] D. Abercrombie et al., *Dark Matter Benchmark Models for Early LHC Run-2 Searches: Report of the ATLAS/CMS Dark Matter Forum*, 1507.00966.

[31] S. Karg, M. Kramer, Q. Li and D. Zeppenfeld, *NLO QCD corrections to graviton production at hadron colliders*, *Phys. Rev.* **D81** (2010) 094036, [0911.5095].

[32] T. Ahmed, M. Mahakhud, P. Mathews, N. Rana and V. Ravindran, *Two-Loop QCD Correction to massive spin-2 resonance → 3 gluons*, *JHEP* **1405** (2014) 107, [1404.0028].

[33] T. Gehrmann and E. Remiddi, *Analytic continuation of massless two loop four point functions*, *Nucl. Phys.* **B640** (2002) 379–411, [hep-ph/0207020].

[34] S. Catani, *The Singular behavior of QCD amplitudes at two loop order*, *Phys. Lett.* **B427** (1998) 161–171, [hep-ph/9802439].

[35] F. V. Tkachov, *A Theorem on Analytical Calculability of Four Loop Renormalization Group Functions*, *Phys. Lett.* **B100** (1981) 65–68.

[36] K. G. Chetyrkin and F. V. Tkachov, *Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops*, *Nucl. Phys.* **B192** (1981) 159–204.

[37] T. Gehrmann and E. Remiddi, *Differential equations for two loop four point functions*, *Nucl. Phys.* **B580** (2000) 485–518, [hep-ph/9912329].

[38] T. Binoth and G. Heinrich, *An automatized algorithm to compute infrared divergent multiloop integrals*, *Nucl. Phys.* **B585** (2000) 741–759, [hep-ph/0004013].

[39] V. A. Smirnov, *Analytical result for dimensionally regularized massless master double box with one leg off-shell*, *Phys. Lett.* **B491** (2000) 130–136, [hep-ph/0007032].

[40] V. A. Smirnov, *Analytical result for dimensionally regularized massless master nonplanar double box with one leg off-shell*, *Phys. Lett.* **B500** (2001) 330–337, [hep-ph/0011056].

[41] T. Gehrmann and E. Remiddi, *Two loop master integrals for gamma* —¿ 3 jets: The Planar topologies*, *Nucl. Phys.* **B601** (2001) 248–286, [hep-ph/0008287].

[42] T. Gehrmann and E. Remiddi, *Two loop master integrals for gamma* –¿ 3 jets: The Nonplanar topologies*, *Nucl. Phys.* **B601** (2001) 287–317, [hep-ph/0101124].

[43] G. F. Sterman and M. E. Tejeda-Yeomans, *Multiloop amplitudes and resummation*, *Phys. Lett.* **B552** (2003) 48–56, [hep-ph/0210130].

[44] T. Han, J. D. Lykken and R.-J. Zhang, *On Kaluza-Klein states from large extra dimensions*, *Phys.Rev.* **D59** (1999) 105006, [hep-ph/9811350].

[45] G. F. Giudice, R. Rattazzi and J. D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, *Nucl.Phys.* **B544** (1999) 3–38, [hep-ph/9811291].

[46] E. Remiddi and J. A. M. Vermaseren, *Harmonic polylogarithms*, *Int. J. Mod. Phys.* **A15** (2000) 725–754, [hep-ph/9905237].

[47] S. M. Aybat, L. J. Dixon and G. F. Sterman, *The Two-loop anomalous dimension matrix for soft gluon exchange*, *Phys. Rev. Lett.* **97** (2006) 072001, [hep-ph/0606254].

[48] S. M. Aybat, L. J. Dixon and G. F. Sterman, *The Two-loop soft anomalous dimension matrix and resummation at next-to-next-to leading pole*, *Phys. Rev.* **D74** (2006) 074004, [hep-ph/0607309].

[49] V. Ravindran, J. Smith and W. L. van Neerven, *Two-loop corrections to Higgs boson production*, *Nucl. Phys.* **B704** (2005) 332–348, [hep-ph/0408315].

[50] S. Moch, J. A. M. Vermaseren and A. Vogt, *Three-loop results for quark and gluon form-factors*, *Phys. Lett.* **B625** (2005) 245–252, [hep-ph/0508055].

[51] T. Becher and M. Neubert, *Infrared singularities of scattering amplitudes in perturbative QCD*, *Phys. Rev. Lett.* **102** (2009) 162001, [0901.0722].

[52] E. Gardi and L. Magnea, *Factorization constraints for soft anomalous dimensions in QCD scattering amplitudes*, *JHEP* **03** (2009) 079, [0901.1091].

[53] J. A. M. Vermaseren, *New features of FORM*, math-ph/0010025.

[54] A. von Manteuffel and C. Studerus, *Reduze 2 - Distributed Feynman Integral Reduction*, 1201.4330.

[55] R. N. Lee, *Presenting LiteRed: a tool for the Loop InTEgrals REDuction*, 1212.2685.

[56] R. N. Lee, *LiteRed 1.4: a powerful tool for reduction of multiloop integrals*, *J. Phys. Conf. Ser.* **523** (2014) 012059, [1310.1145].

[57] P. Nogueira, *Automatic Feynman graph generation*, *J. Comput. Phys.* **105** (1993) 279–289.

[58] P. Mathews, V. Ravindran and K. Sridhar, *NLO - QCD corrections to e+ e- —¿ hadrons in models of TeV-scale gravity*, *JHEP* **08** (2004) 048, [hep-ph/0405292].

[59] R. N. Lee, *Group structure of the integration-by-part identities and its application to the reduction of multiloop integrals*, *JHEP* **07** (2008) 031, [0804.3008].

[60] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations*, *Int. J. Mod. Phys.* **A15** (2000) 5087–5159, [hep-ph/0102033].

[61] P. Nason, *MINT: A Computer program for adaptive Monte Carlo integration and generation of unweighted distributions*, 0709.2085.

[62] R. N. Lee and A. A. Pomeransky, *Critical points and number of master integrals*, *JHEP* **11** (2013) 165, [1308.6676].

[63] T. Ahmed, M. Mahakhud, P. Mathews, N. Rana and V. Ravindran, *Two-loop QCD corrections to Higgs* $\to b + \bar{b} + g$ *amplitude*, *JHEP* **08** (2014) 075, [1405.2324].