IDENTITY AND RATIONALITY IN CLASSROOM DISCUSSION: DEVELOPING AND TESTING AN ANALYTICAL TOOLKIT

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This contribution originates from a joint work aimed at networking theoretical tools and employ them to better understand teaching and learning episodes. In a former research we studied group works in mathematics classes adopting a socio-cultural perspective and combining two theoretical lenses: the construct of identity and that of rational behavior. In this contribution we turn to another phase of the teaching and learning process, namely when group work has to be shared with the school-mates and the teacher during a classroom discussion.

THEORETICAL BACKGROUND

Classroom interaction

We rely on a sociocultural perspective, according to which the learning of mathematics takes place in a social context through interactions. We also consider culture a decisive factor in the discussions, since it may orient individuals' interaction in the classroom (Radford, 2006; 2011), and in particular teachers' interventions (Radford, 2006).

Students' interaction in a small group is presented by Radford (2011) as a complex process in which students are involved at many levels, not only at the cognitive one. The processes of objectification (students align their thoughts with culture) and subjectification (a *thinking and becoming* process of being-with-others mediated by alterity) that take place in the teamwork are mediated by culture (Radford, 2008). In line with Radford we consider interactions as potential catalyzers or, conversely, obstacles, in the learning processes of the individuals involved in the discussion.

The role of the teacher in classroom discussions was deeply studied also by Bartolini Bussi (1996), who elaborated a theoretical framework to analyze different kind of discussions and strategies of interventions of teachers.

Identity

There are at least two macro-categories of approaches to the investigation of the role of identity in mathematics education: "local" studies, that investigate the evolution of a classroom discussion in stages or brief periods, and "longitudinal" studies, in which students' behaviors are investigated in longer periods. As Gómez-Chacón (2000) stresses,

> "it is not enough to observe and know the stages in the process of emotional shifts or changes during problem solving ("local affective dimension"). It is also not enough to

> detect cognitive processes associated with positive or negative emotions. We need to contextualize their emotional reactions within the social reality which gives rise to them. The "global affective dimension" is understood as a result of the paths followed by the individual in the local affective dimension. These paths are established with the cognitive system and they contribute to the construction of the general structures of one's self concept as well as beliefs about mathematics and the learning of mathematics. […] Identity is understood as a structured joining of elements which permits the individual to define himself/herself in a situation of interaction and to act as a social agent."

Although we agree with these important remarks and we keep in mind that the behaviors we observe depend on past experiences that structured the students' identities in the classroom, we will carry on the first kind of analysis, since we are interested more in the local effects of intertwined factors that influence rather than in exploring how in general identities end to be structured by the practice. In particular we investigate the limits and potentialities of group works and of discussion of group results with the peers and the teacher in order to find out factors that make them effective or not in the mathematics "social learning processes".

Heyd-Metzuyanim (2009) distinguishes the different ways of interacting of each student in terms of individuality, in particular in a mathematics group work, so as "*to point out how identity and emotional processes influence the effectiveness of learning. Subjectifying may help in mathematizing or obstruct it*" (Heyd-Metzuyanim, 2009, p. 2). The subjectification process is linked both theoretically and operationally to the identity construction process and to the mathematizing activity in group work. The author refers to this definition of identity by Sfard and Prusak (2005, p.1): "*Identity is a set of reifying, significant, endorsable stories about a person*." This definition is deeply related to the *commognitive perspective* (Sfard, 2008), whose cores are the notions of thinking and communicating. Since thinking is a form of human doing, it can only develop as a collective patterned activity: "*Thinking is an individualized version of (interpersonal) communicating*." (Sfard, 2008). Heyd-Metzuyanim frames also mathematizing and subjectifying in the commognitive perspective: mathematizing is communicating about mathematical objects, subjectifying is communicating about participants of the discourse. Identities stories can talk about the way in which a person relates to the mathematics and so can influence the participation in the teamwork, the engagement, and definitively, success or failure in mathematics activities. In her work, Heyd-Metzuyanim (2009) looks at verbal and non-verbal acts of subjectification, distinguishing participation and membership. Then she classifies the acts clarifying whether they are identifying processes or not. Identifying utterances (verbal or non-verbal) are "*those that signal that the identifier considers a given feature of the identified person as permanent and significant*." (Heyd-Metzuyanim, 2009, p. 2). The prototypical cases of different aspects of the relation between subjectifying, mathematizing and identifying are exemplified in the quoted paper by Heyd-Metzyuyanim (2009). In a further work, Heyd-Metzuyanim (2013) employs the commognitive framework to analyze teacher-individual interactions and argues that in some cases interaction is non-productive and turns into a co-construction of the student's identity of failure. The study sheds a new light on the role of the teacher in interaction with students, since he/she plays a role not only in the mathematizing process, but also in the identifying one. Moreover, the study brings to the fore the existence of different forms of participation to the mathematical discourse, namely acting "as if" she were participant into the discourse, pretending to mathematize but, in reality just pursuing the designated identity of participant.

These findings, concerning the crucial role of the teacher in co-constructing identity and the alternative forms of participation, will also help us to frame our reflection.

Rationality

The construct of rationality was developed by Habermas (1998) in reference to discoursive practice and later adapted to mathematical activity (see: Morselli & Boero, 2009 for the special case of mathematical proving; Boero et alii, 2010 for its integration with Toulmin's model and its use for classroom implementation). According to Habermas, rational behaviour may be seen as three interrelated dimensions: epistemic dimension (related to the control of the propositions and their chaining), teleological dimension (related to the conscious choice of tools to achieve the goal of the activity) and communicative one (related to the conscious choice of suitable means of communication within a given community). In the case of mathematics, fostering students'approach to argumentation and proof as a rational behavior means promoting the students' acquisition of basic content knowledge, but also the ability to manage (from a logical and linguistic point of view) the reasoning steps and their enchaining and the ability to communicate the arguments in an understandable way, thus taking into account three interrelated dimensions:

> "- an epistemic aspect, consisting in the conscious validation of statements according to shared premises and legitimate ways of reasoning […];
>
> - a teleological aspect, inherent in the problem solving character of proving, and the conscious choices to be made in order to obtain the aimed product;
>
> - a communicative aspect: the conscious adhering to rules that ensure both the possibility of communicating steps of reasoning, and the conformity of the products (proofs) to standards in a given mathematical culture". (Morselli & Boero, 2009, p. 100)

As outlined in our previous study (Branchetti & Morselli, in press), when dealing with peer interaction, communicative dimension plays a crucial role, as well as epistemic rationality, which is linked to the possibility of changing opinion:

> "Someone is irrational if she puts forward her beliefs dogmatically, clinging to them although she sees that she cannot justify them. In order to qualify a belief as rational, it is sufficient that it can be held to be true on the basis of good reasons in the relevant context of justification - that is, that it can be accepted rationally. The rationality of a judgment does not imply its truth but merely its justified acceptability in a given context" (Habermas, 1998, p. 310).

In the subsequent part we will briefly summarize our previous results concerning rationality and identity during group work as a special case of peer interaction.

Identity and rationality: our former study

In a former study (Branchetti & Morselli, in press) we analyzed a group of middle school students (grade 6) dealing with some questions concerning negative numbers. They worked in group so as to produce a shared answer to the question posed by the teacher.

The analysis in terms of subjectification and identity revealed some recurrent utterances and behaviors. The first observed phenomenon concerns students' different ways of participating (or non-participating) to the group work. This issue is crucial because participation affects personal concept development. Another issue concerns agreement (or lack of agreement) and the different reactions of students when their mates do not agree with them. Our working hypothesis was that dimensions of rationality may help to understand such phenomena. First of all, teleological rationality may refer to different goals; furthermore some interventions are clearly on communicative or epistemic level. Combining the two analysis, we suggested that

> "individual participation or resistance to participation and also membership or non-membership may be described in terms of dimensions of rationality: if individual interventions are on different levels (epistemic vs communicative), it seems very difficult to reach an agreement. If a dimension prevails, some students can avoid to participate. Moreover, individuals may have different aims and act accordingly (teleological rationality), may consider the epistemic dimension or not, and this may affect individual/collective conceptual change ".

Accordingly, we claimed the need of taking into account all the three dimensions of rationality and we proposed the mismatch between dimensions (different students focus on different dimensions) as a possible source of difficulty during group-work.

To sum up, students' interaction in group work (without the teacher's interventions) may be affected by social dynamics that lead students to look for a "forced agreement" that may cause the loss of constructed knowledge because of a negative interaction with the pairs due to identifying and/or subjectifying acts or because of a difference in Habermas' prevailing dimension in the discussion. The two potential causes may not be disjointed but rather interconnected.

In this contribution we turn to another kind of activity (the moment when a group presents the solution to the whole class) to test the transferability of such conclusions and also to refine and adapt the theoretical tools at disposal.

CONTEXT

The teaching experiment we refer to was carried out in a lower secondary school (grade 7) in the north-west of Italy. The teaching experiment is part of a bigger data collection concerning a joint research work on the development of argumentative competences (Levenson & Morselli, 2014).

The task sequence was inspired by a formative assessment unit of the MARS project (http://map.mathshell.org/materials/lessons.php). The mathematical content at issue was ratio as a way of comparing quantities. Here is the task proposed to the students:

> Guglielmo loves organizing parties with his friends. When he and his friends get together, Guglielmo makes a fizzy orange drink by mixing orange juice with soda. On Friday, Guglielmo makes 7 liters of fizzy orange by mixing 3 liters of orange juice with 4 liters of soda. On Saturday, Guglielmo makes 9 liters of fizzy orange by mixing 4 liters of orange juice with 5 liters of soda. Does the fizzy orange on Saturday taste the same as Friday's fizzy orange, or different? If you think it tastes the same, explain how you can tell. If you think it tastes different, does it taste more or less orange? Explain how you know.

The students worked individually, afterwards (and before any feedback by the teacher) they were asked to work in small groups (3-4 students), share their solutions and, if possible, to reach a common agreement. Aftwerwards, there was a balance discussion, where the students of each group had to report to all the classmates the solution and convince them of its validity.

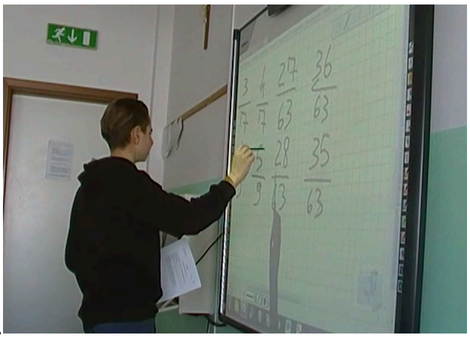
ANALYSIS OF SOME EXCERPTS

In this contribution we focus on the moment when a group of four middle-high level students (Francesco, Elena, Giacomo, Nicolò) report its solution[1]. Due to space constraints, we will propose only some excerpts. In the first one, Francesco goes to the Interactive Whiteboard and exposes the group solution to the whole class.

1. Francesco: Anyway the answer is yes, **for us** they taste the same, because in order to compare them, to see if they are the same or not, **I did** the least common multiple, then.. (*writing the solution to on the interactive whiteboard*)…

[1] Here are the individual solutions provided during the preliminiary work: **Francesco** had not provided a real argumentation (his worksheet contained only some calculations with fractions); **Nicolò** ("It tastes the same because the different quantity of the two liquids remains always at the same distance, that is to say the difference is always one"), **Elena** ("They taste the same, only the quantity of liters that are prepared changes" and **Giacomo** ("Yes because the quantity of soda is always greater than the quantity of orange juice").

2. Francesco. But if **we look** at this and this (*the fraction for saturday*) 1/63 is here (*points at the orange*) while the other 1/63 is here (points to the quantity of soda of friday). Then they balance […]



3. Teacher: because you say: on saturday we have 1/63 of orange more than on friday.
4. Francesco: yes.
5. Teacher: right?
6. Francesco: right.
7. Teacher: but orange or soda?
8. Francesco: well… on friday soda and on saturday orange.
9. Observer: then, do they taste the same or not?
10. Francesco: yes…
11. Teacher (*going to the interactive whiteboard*): then, think to your reasoning… you said that here (*she points at the orange of friday and saturday*) there is a difference of 1/63 and here (*she points to the soda of friday and saturday*)?
12. Francesco: the same.
13. Teacher: the same. So, now draw your conclusion from that point.
14. Francesco: then..
15. Teacher: I don't understand the conclusion. You said: here it is one more.
16. Nicolò: ah, teacher!
17. Francesco: I am not able to explain this, the difference is always the same.

In line 1, Francesco proposes the explanation in terms of fractions. In line 3, the teacher reformulates the explanation given by Francesco. The interactions between the teacher and the student Francesco reflect two needs for the teacher: involving all the students into the discussion, making as much clear as possible the explanation of the group of Francesco, and leading the students of the group to realize that the explanation does not work and needs to be amended. Some reformulations of the teacher are at first perceived by Francesco as requests to clarify the explanation (communicative level), while the final aim of the teacher is to bring to the fore that the method of fractions leads to the opposite conclusion (the taste is not the same) (epistemic level). In line 11 the teacher reformulates Francesco'explanation, with the aim (more and more explicitly at epistemic level) of making Francesco revise his reasoning. We may note that the teacher speaks to Francesco showing assurance. In lines 13 and 15, the teacher intervenes on the epistemic level. Francesco is still on his position, claiming he is not able to explain it in another way. We may note that in this part, the other students of the group and the other classmates seem to be out of the interaction. Nicolò (line 17) tries to get into the discussion, but nor the teacher and Francesco linsten at him.

If we look at the verbal acts, we may note that at the very beginning, when expressing the solution, Francesco uses the plural pronoun (**for us**). Anyway, when expressing the explanation for the solution, Franesco turns to the singular pronoun (**I did**). Also the teacher talks to him using the singular pronoun, even if Francesco is supposed to report a group solution.

Immediately after, Nicolò succeeds in intervening and reports the group explanation ("Yes, the taste is the same, because, indeed, if there it is one more, **for us** the taste is the same because anyway the 28 is added to the 35. Up (*he means on the row of saturday*) the 36 is added to 27 and… that is to say, you get the same"), as established during the group-work. His aim is to support the group from a communicative point of view (making the solution clear to the classmates). At this point both Francesco and Nicolò seem "lost" in a pure arithmetic game, where having the same result (the sum is 63) is perceived as a warrant for the fact of having the same taste. In terms of interaction, we may observe that it is the observer to encourage Nicolò to talk, while the teacher is still focused on the interaction with Francesco. Nicolò's intervention is not taken into consideration by Francesco and the teacher, who go on with their interaction. Immediately after, also Elena is invited to intervene, but she renounces to talk *("No, it is that… he explained it better and I gave up! (laughing)")*, identifying herself as less good in maths than Francesco.

Afterwards, thanks to the interventions (questions) of the teacher, Francesco finds out that something does not work and proposes a new solution. Elena rapidly changes her mind, grasping the new solution and succeeding also in re-explaining it to the mates. Nicolò, on the contrary, does not agree with the new solution and distances himself from Francesco and Elena. At first he expresses his doubts, but his explanation is disturbed by Elena, who makes gestures to signify that Nicolò is wasting time ("Teacher, I mean. The taste remains the same because… stop for a moment! (*speaking to Elena*) After the orange is added, then the taste remains the same"). We may say that Elena identifies Nicolò as less good in maths than Francesco, thus as not deserving the same attention than Francesco. The teacher seems to agree, or at least she does not reproach Elena. Elena tells again the new explanation, but Nicolò does not accept it.

18. Elena: in one case, on friday, there is 1/63 soda more, in comparison to saturday, and on saturday there is 1/63 more orange.
19. Nicolò: so, there is always a 1/63 difference. Nicolò: indeed I had not written this thing, I had written another thing, anyway…
20. Teacher: what did you write?
21. Nicolò: I had written as the others, that there was a difference of 1 liter (*he laughs; also Elena laughs*).
22. Teacher: and how did you convince of their…
23. Nicolò: because after I had seen… anyway… because at the beginning they had said that it (*the taste*) was the same then I had convinced myself…

Nicolò's interventions brings to the fore that the group solution was not a really agreed solution: the group had reached an agreement in terms of final answer (the taste is the same) but not in terms of explanation (difference of 1 liter versus fractions). Nicolò had accepted the explanation with fractions just because it initially led to the agreed solution (same taste). Now that the method leads to the opposite conclusion, he is no more ready to accept it. While Francesco and Elena changed their mind in order to accept the solution given by the trusted method of fraction, Nicolò refuses the method in order to keep the (intuitive) solution.

DISCUSSION AND PRELIMINARY CONCLUSIONS

The analysis carried out through the lens of identity, that we showed just reporting some sentences, highlights differences in students' identities from the point of view of the relationship with mathematics, the classmates and the teacher. The most of the data that we can categorize as identifying acts are verbal and indirect, but they are so recurrent to allow us to consider them as significant.

Francesco results the most considered as good in mathematics, both by the teacher and by the groupmates. We highlighted the way the teacher speaks to him, taking into account his answers, turning sometimes from the group to him. This has also an effect in terms of participation: Francesco is the most involved in the discussion. Also Elena recognizes his reliability. Nicolò is identified, directly and indirectly, as less influent in the group than Francesco, both by Elena and the teacher. The teacher identifies him indirectly and through non verbal acts, when quite she ignores his interventions and nods to Elena who's making fun of him, while Elena is more direct.

Looking at teachers' intervention we can also see identifying subjectification acts in terms of rationality. In the first part of the discussion we may see the intertwining of the epistemic and communicative dimensions. At first the teacher intervenes at the communicative level, but then the interventions become more and more epistemic. When the interventions turn to be clearly at epistemic level ("*Now draw your conclusion from that point*"; "*I don't understand the conclusion*"), Francesco is able to revise his explanation. Being ready to revise the explanation, when it does not work anymore, is connected to epistemic rationality.

Nicolò's interventions are very interesting form the point of view of rationality. His initial agreement with the mates, based on the final conclusion (but not on the common kind of explanation) may be read in terms of lacks at epistemic level, as well as his disagreement towards the new explanation and solution proposed by Francesco. When the method of fractions leads to a new conclusion (different tastes), which is against the former one, Nicolò refuses the method, rather than changing the final conclusion.

The different identities may have influenced the group work evolution, since, as Nicolò said, the group turned to Francesco's choice of using fractions without a deep comprehension of the method itself. The choice redirected all the groupmates' strategies towards an approach they didn't master very well. This social dynamic lead the group to present a solution that was not a group solution (Branchetti & Morselli, in press), rather a "forced agreement" based on Francesco's epistemic identity and, possibly, Elena's communicative one. Nicolò's claim (line 23) sounds very interesting in this sense. The internal dynamics that underlie the group work emerged during the class discussion, which confirms to be a crucial moment, not only for establishing a common class solution, but also for giving individual contributions and voices, that had disappeared during group work, to appear again. Without this discussion maybe Nicolò would have just reached a superficial understanding of the problem, and Francesco himself would have conserved a wrong idea, convinced of a wrong argumentation by the agreement of the group.

The analysis brings to the fore the crucial role of the teacher. The teachers' behavior may contribute to reinforce the students' identities through indirect or direct, verbal or not verbal acts, and so it may influence also students' future participation in group works and other mathematical activities. Furthermore, the rationality levels of teachers' intervention may depend on the identity of the students.

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