

Chapter 15

The Interplay of Rationality and Identity in a Mathematical Group Work



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Abstract This contribution originates from a joint work aimed at networking theoretical tools and employ them to better understand teaching and learning episodes, with a special focus on mathematical group work. In a socio-cultural perspective, two theoretical lenses are combined: the construct of rational behavior, initially developed by Habermas and adapted in mathematics education, and that of identity. In this paper we propose a general description of our approach and present the main findings emerged after investigations in grade 6 (group work on negative numbers) and grade 4 (arithmetics problem solving). The networked analysis sheds light into mathematical group works: the students' mathematical identities turn into prevailing dimensions of rational behavior and the interplay of dimensions of rationality affects the participation into the group activity. Moreover, the teacher is shown to have a role in students' identifying process, affecting indirectly the students' participation.

Keywords Identity · Rationality · Group work

15.1 Introduction

Adopting a sociocultural perspective, we argue that the learning of mathematics takes place in a social context through interactions. Students' interaction in a small group is a complex process in which students are involved at many levels, not only at the cognitive one, mediated by culture and characterized by processes of objectification (alignment with culture) and subjectification (thinking and becoming process of being-with-others mediated by alterity) (Radford, 2011).

In this contribution, we discuss the issue of group work, combining two theoretical lenses: the construct of rational behavior, originally developed to better understand

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individual argumentation and proving processes, and the construct of identity, aimed at figuring out the effects of verbal and non-verbal acts concerning the participants on a teaching and learning activity. In the first part of the contribution we present a literature review on group work, we frame our research interest in a sociocultural perspective and we introduce the theoretical lenses we choose for our research. Afterwards, we briefly refer to the networking methodology and set up the research question in terms of the combined theoretical tools. Finally, we present the main findings of our networked analysis, illustrating them by means of relevant examples coming from our experiments.

15.2 Literature Background: Group Work

To be able to work in group in order to solve problems is considered nowadays one of the most important skills to develop at school in order to become able to work in team and contribute fruitfully to face challenges within a social context (OECD, 2013).

In the field of education, group work is not only aimed at developing social skills—in the traditional sense of becoming able to interact and work with other managing emotions and conflicts—but is also used as a resource for teaching, that is to say a method to make the students learn better and in a more significant and lasting way. The educational use of group works has been widely studied, in particular after the milestones of Johnson and Johnson (1980), Sharan (1980) and Slavin (1990). Those studies report a good effect of group work on students' learning and discuss some indicators of efficacy in group work. Group work, with different strategies such as cooperative learning, collaborative and peer tutoring, is valued for its fostering significant learning processes; however, group work is not always fruitful, as pointed out by the aforementioned researchers (and known by many teachers). Relevant required features are: positive interdependence (every student's contribution is necessary), promotive face-to-face interaction (ongoing interaction is necessary for success), individual accountability (the performance of every member should be taken in account seriously by the others), group processing (to manage conflicts, to give and receive assistance, to exchange information, to use critical thinking). We may note that the aforementioned features deal with social skills that are independent from the subject and the topic of the discussion, even if at a closer view the accountability of every single student may depend on the topic of discussion.

Group work at school has additional requirements, since in that case the social interaction happens within an institutional context that has its specific goals: it may happen that the desired (in the teacher's mind) goal of the group work may be different from the direction the natural evolution of the activity leads to. What happens in group work at school is not a mere question of negotiation of meaning but is a continuous interplay between the individual dimension, the group, the classroom, the teacher and, finally, the culture (Radford, 2011). Recently, some authors identified indicators

for the analysis and evaluation of the quality of group work. In particular, drawing from the work of Pons Parra et al. (2012), Spagnuolo (2017, p. 86) elaborated the following list of Likert indicators:

1. Being responsible of one's own role, respecting the assigned tasks.
2. Expressing the understanding of the activity, of the assignment and of the task during the approach, process and productions steps.
3. Asking (spontaneously or not) for explanations on the content (concepts, interpretations) and on the objectives of the assigned activity.
4. Providing (spontaneously or not) explanations on content (concepts, interpretations) and on the objective of the assigned activity.
5. Asking (spontaneously or not) for explanations on the different sides of the resolution process of the activity.
6. Providing (spontaneously or not) explanations on the different sides of the resolution process of the activity.
7. Asking (spontaneously or not) for explanations on the different sides of the obtained final result and of its possible implications.
8. Providing (spontaneously or not) explanations on the different sides of the obtained final result and of its possible implications.
9. Accepting and respecting the decision taken by the group on the obtained result.
10. Taking into consideration others' opinion.
11. Speaking critically, but constructively, on one's own and others' argumentation and considerations.
12. When speaking, attempt to reach an agreement point with the others, of reaching a point of shared knowledge.

Our interest is framed in this trend of research, with a specific focus on what may happen during mathematical classes.

15.3 Theoretical Framework

15.3.1 *Group Work in a Sociocultural Perspective*

The pioneering analysis carried out by Vygotskij (1978) concerning the crucial role of social activities mediated by signs and language in the development of mathematical thinking traced a path in which a whole thread of researchers placed their roots. The effects of students' interactions in classroom activities have been studied, described and interpreted in mathematics education since its origins, see Radford (2011) for an overview. We rely on a sociocultural perspective, according to which the learning of mathematics takes place in a social context through interactions and is deeply affected by culture (Radford, 2006, 2011). Radford (2006, p. 58) affirms:

Certainly, the students were actively engaged in what has been termed a "negotiation of meaning". But this term can be terribly misleading in that it may lead us to believe that the

attainment of the concept is a mere consensual question of classroom interaction. [...] meaning also has a cultural-historical dimension [...]. It is in fact this cultural object that shapes and explains the teacher's intervention [...] classroom interaction and the students' subjective meaning are pushed towards specific directions of conceptual development. Cultural conceptual objects are like lighthouses that orient navigators' sailing boats. They impress classroom interaction with a specific teleology.

Students are involved in a double-faced problem: they meet at the same time the culture and the others and have to find a place in both the cultural and the classroom discourses, that are related but not necessarily equal. Particularly, we will focus on the classroom discourse side. Even if the attainment of a concept is not a mere consensual question, the agreement between students is very important in mathematics group activities, as we will show. We refer to Radford's interpretation of cultural-historical activity theory (Roth, Radford, & Lacroix, 2012). This theory is rooted in Leont'ev and Vygotskij's dialectical psychological theories. The keyword *activity* is defined as a common place in which cognition and consciousness arise; through activity individuals relate not only to the world of objects but also to other individuals. Learning is the result of a shared common practice that involves students' subjectivities and in which subjectivities moves towards others and culture to find and transform themselves. In Radford (2008) it is pointed out that students' interaction in a small group is a complex process in which students are involved at many levels, not only at the cognitive one. The processes of objectification (students align their thoughts with culture) and subjectification (a *thinking and becoming* process of being-with-others mediated by alterity) that take place in the teamwork are mediated by culture.

15.3.2 Identity

We refer to the definition of identity by Sfard and Prusak (2005, p. 1): "*Identity is a set of reifying, significant, endorsable stories about a person*". Identity stories can talk about the way a person relates to the mathematics and so can influence the participation in the teamwork, the engagement, and definitively, success or failure in mathematics activities. Heyd-Metzuyanin (2009) adds non-verbal acts to the verbal stories in order to better explore the interaction dynamics. Then she classifies the acts, clarifying whether they are identifying processes or not. Identifying utterances (verbal or non-verbal) are "*those that signal that the identifier considers a given feature of the identified person as permanent and significant*" (Heyd-Metzuyanin, 2009, p. 2). The prototypical cases are exemplified in the quoted paper by Heyd-Metzuyanin (2009). In a further work, Heyd-Metzuyanin (2013) analyzes teacher-student interactions and argues that in some cases interaction is non-productive and turns into a co-construction of the student's identity of failure. Thus teachers play a role not only in the mathematizing process, but also in the identifying one.

15.3.3 Rationality

The construct of rationality was developed by Habermas (1998) in reference to discursive practice and later adapted to mathematical activity (see Morselli & Boero, 2009 for the special case of mathematical proving). According to Habermas, rational behavior may be seen as made up of three interrelated dimensions: epistemic dimension (related to the control of the propositions and their chaining), teleological dimension (related to the conscious choice of tools to achieve the goal of the activity) and communicative one (related to the conscious choice of suitable means of communication within a given community). In the case of mathematics, fostering students' approach to argumentation and proof as a rational behavior means promoting the integration between the epistemic aspect (conscious validation of statements according to shared premises and legitimate ways of reasoning), the communicative aspect (conscious adhering to rules that ensure both the possibility of communicating steps of reasoning, and the conformity of the products (proofs) to standards in a given mathematical culture), the teleological aspect (conscious choices to be made in order to obtain the aimed product). When dealing with peer interaction, the communicative dimension plays a crucial role, as well as the epistemic one, which is linked to the possibility of changing opinion:

Someone is irrational if she puts forward her beliefs dogmatically, clinging to them although she sees that she cannot justify them. In order to qualify a belief as rational, it is sufficient that it can be held to be true on the basis of good reasons in the relevant context of justification - that is, that it can be accepted rationally. The rationality of a judgment does not imply its truth but merely its justified acceptability in a given context. (Habermas, 1998, p. 310)

15.3.4 Networking Identity and Rationality

Mathematics education is a wide research field characterized by many different approaches, methodologies, problems and theoretical tools. In order to allow the researchers to collaborate using different research tools belonging to different theories, expert researchers carried out a meta-analysis about theories in mathematics education. This process of study is characterized by the search for possible connecting strategies and the theorization of the networking of theories (Prediger, Bikner-Ahsbals, & Arzarello, 2008). The choice among the possible connecting strategies, like comparing, contrasting, coordinating, combining, depends on the specific kind of analysis and on the compatibility of theories at different levels (principles, methodologies, research problems, research questions), as theorized by Radford (2008). In our case, we chose the combining strategy, which consists in "looking at the same phenomenon from different theoretical perspectives as a method for deepening insights on the phenomenon" (Prediger et al., 2008, p. 172). We decided to combine the lenses of rationality and identity in order to analyze in depth episodes of group interaction. The choice of combining the two lenses is linked to our aim of understanding how the individual participation and the students' interactions affect the group work in

the special case of a mathematical activity aimed at producing and presenting argued solutions; moreover, we are interested in understanding what dynamics may lead a student to renounce to participate or to go on contributing significantly to the mathematical activity conserving a personal point of view. Thus, we need analytical tools to take into account both the individual and the social processes.

In our view, the two lenses are functional to our focus on group work in mathematical activities. The lens of identity is used to study situations in which each student interacts with peers presenting himself as good or not in mathematics and is recognized by the peers as accountable or not in mathematics lessons. The lens of rationality supports the analysis of the search for a group agreement on a mathematical question, that is not only a social process, but also a relevant mathematical activity: students must provide answers and explanations that are correct from a mathematical point of view, but also useful according to the goal of the activity (the question posed by the teacher) and accepted by their peers, thus combining the different dimensions of rationality.

15.4 Purpose of the Research

Our research focuses on the degree at which the members accept the solution of the group (Branchetti & Morselli, 2016, 2017). In particular, we are interested in a specific case of mathematical activity: problem solving in which a solution has to be motivated and argumentation tasks. In those cases, reaching an agreement may be particularly complex since the students have to manage simultaneously their own processes and the others' processes and to deal with mathematical objects and culturally accepted ways of reasoning. Hence, we aim at studying the dynamics that lead a group of students to reach an agreement on the solution to present to the classroom within an institutional context. To this aim, we perform a double, simultaneous, analysis of what is happening to every student and what is happening in the interaction, taking in account both the problem-solving level and discussion level.

In order to describe the final state of the group in respect of the solution and to define precisely what we consider a group solution and an agreed solution we introduce some terms.

First, we define '*solution*' a couple composed by an answer and an explanation. It is possible—and yet frequent, as we will show—that the solution is not completely understood and accepted by all the members as couple composed by an answer and an explanation. For instance, a student could accept and agree with the answer but not with the explanation, or could have an intuition of the answer but may not be able to motivate it and thus could “borrow” the explanation of another group mate. It may then happen that the group converges towards a solution that is accepted as a group solution by the group, even if there is not a real individual convergence and understanding. We consider those situations relevant, since we are interested not only in the social skills but also in the disciplinary relevance of the activity for

every student and we assume that it is not good for the student to interrupt his/her own thinking process and to move towards a group solution that is not understood completely. Thus, we aim at understanding what kind of dynamics make the members of the group decide to present a solution as a group solution and what happens to the students who did not agree with or did not understand it. More specifically, we refer to specific situations when students, working in group, are asked to carry out a task and provide a group solution, made up of the answer and the explanation for the answer. After the end of the group work, each group is expected to present the group solution to the whole class. The group presentation is the starting point for a class discussion. We point out that such a setting, where students are asked to reach a group solution (that contains an explanation) and to present it to the whole class, makes working in group really central and related to the mathematical activity at issue. During the first part of the activity (group work), the students are expected to solve the task collaboratively. Two levels are necessarily intertwined: solving the problem and converge towards a unique, agreed solution, both in terms of answer and in terms of explanation. We call agreement the situation in which all the group members, at the end of the group work, accepts to present a given solution (answer A, explanation E) as a group solution, and non-agreement the situations in which the group does not come to a convergence and does not present a group solution. Of course, not all situations of agreement represent a real convergence. We call *solid agreement* the situation in which the group members not only accept, but also comprehend both the answer A and the explanation E. On the contrary, we call *forced agreement* all those situations in which the group presents a group solution, but there is not a real convergence and comprehension. Cases of forced agreement may take place when the group presents the solution (answer + explanation) as something accepted by all the members, but actually the group members converge to the answer but do not converge on the explanation, not all the members understand the explanation.

In Branchetti and Morselli (2017) we observed a group presenting the solution to the whole class and the teacher. By means of the combined framework we showed that a group solution may not be reached when the interaction is only at the communicative level and not at the epistemic one. A possible cause of the bad balance between dimensions was indeed the presence of different mathematical identities in the group that discouraged the groupmates to propose different explanations, contradicting the one who was more accounted by the teacher as good in mathematics. The dynamic emerged exactly during the group solution presentation: when the teacher proposed to the leader to change the answer—that had been accepted just because of the student's accountability—one of the classmates reacted saying that he had not understood the explanation but he had agreed anyway because he had a similar intuition about the answer. He could not accept to change the answer, while he had accepted not to understand the explanation and to propose the solution as a shared one. This example shows the existence of situations of forced agreement, and suggests the crucial role of the teacher in co-constructing mathematical identities.

In this contribution, we pursue our reflection on the cases of non-agreement and forced agreement. We wonder whether conflicting identities and/or the intertwining

of dimensions of rationality may act as obstacles for convergence. More specifically, we address the following research questions:

- RQ1: Whether and how dynamics within the group (read in terms of identity and rationality) may lead to a forced agreement rather than on a real agreement? Whether and how such dynamics may lead to a non-agreement?
- RQ2: What is the role of the teacher in leading to a non-agreement, forced or solid agreement?

In order to investigate these issues, we performed a networked analysis on a series of episodes issued from different teaching experiments. The episodes refer to the crucial phases of the group work (with or without the intervention of the teacher). In the following we report excerpts from two episodes and we exemplify the data analysis based on the research questions. Then we resume the main findings of our research, confirmed by the observations of different episodes.

15.5 Method

In order to tackle our research questions, we performed cycles of analysis and interpretation: we questioned the episodes with the aim of better understanding the dynamics of group work, and we drew from the analysis the first idea of *non-agreement* as a result of conflicting identities, acting on different dimensions of rationality (preliminary results, here further developed, may be found in Branchetti & Morselli, 2016, 2017).

We present here the analysis of two examples. The first excerpt comes from an experiment carried out in grade 6 concerning the concept of negative numbers. The work of a group of 5 students (age 12) is analyzed in detail. The second excerpt comes from an experiment concerning the evolution of interactions in a particular setting for teamwork in primary school (grade 4). We study the work of a group of 4 students who interact with the teachers. The example comes from a Master thesis in Psychology with particular attention to the role of the spontaneous representations (Monaco, 2015).

The experiments we refer to, although referring to different grades (grade 5, grade 8), share some crucial features: the students were used to work in group and provide group solutions (in terms of answer and explanation); the teacher was used to set up class discussions after the group work; argumentative activities were widespread and valued as fruitful occasions for meaning making.

We point out here that we used data coming from different experiments; the coherence of results is promising in terms of possible generalization. Data at disposal are the video recordings of the class sessions, the pictures of the whiteboards, the written notes of the students. For our analysis we mainly relied on the transcripts of the discussions (group discussions and/or class discussions). Hereunder we describe the method we used for the analysis of each episode.

Table 15.1 Prototypical cases and labels (Heyd-Metzuyanım, 2009)

	Type		Example	Identifying?	
Mathematizing	Mth			No	
Subjectifying	Pe: participation evaluating	Sp: related to a specific performance	Saying “I do not understand”	No	
		Ge: generalizing	Saying “I hate doing this”	If consistent with other data	
	Me: membership evaluating	Vb: verbal	Di: direct	Saying “I am a math person”	Yes
			Id: indirect	Changing the subject of discourse, which can be interpreted as “I do not want to talk about this”	Depends on the nature and frequency of the utterance
		Nv: non-verbal	Di: direct	Raising one’s hand, which may be interpreted as “I wish to speak about this”	Only if recurrent
			Id: indirect	Groaning at a given task, which may be interpreted as “I do not like this”	Only if recurrent

As already outlined, we combined the lenses of rationality and identity. At first, we analyzed the episode in terms of identity and subjectification, as derived from Heyd-Metzuyanım’s paper (2009). Some sentences from the transcript were interpreted following the criteria proposed by the author in Table 15.1 and labeled with the codes identity (I) and subjectification (S). Also, the specific codes for subjectification used by Heyd-Metzuyanım were used.

Afterwards, we added the analytical tool of rational behavior. We referred to the epistemic dimension when one sentence was linked to a mathematical fact, and we

spoke of lack at epistemic level if some assumption was taken per se, without the need for a justification. For instance, accompanying an answer by an explanation such as “Because the teacher told us”, without paying any effort towards a real understanding, was interpreted in terms of lacks in epistemic rationality. We referred to the teleological dimension when the action was clearly linked to a goal (and we reported a lack in teleological rationality when the reference to the final goal was missing). We referred to communicative rationality when a special care was paid to the organization of the discourse, so as to make the listener to understand. For instance, one student’s wide use of drawings and diagrams was linked to her effort in making her positions understandable to others, thus to a communicative dimension.

15.6 Data Analysis

15.6.1 Example 1

The episode comes from the experiment carried out in grade 6 concerning the concept of negative numbers. At first, students were asked to answer individually these questions: (1) *What is a number?* (2) *What is it possible to do with numbers?* Afterwards, they worked in group on a task to be solved on the Cartesian axes. The negative part of the axes was used by the students and such a solution was after institutionalized by the teacher. Finally, the students were asked to answer in group to the following questions: (3) *You said that numbers are [reference to their former individual answers][...] and with numbers you can do [reference to their former individual answers]... Do you confirm your opinions now?* (4) *Negative numbers are numbers in the sense you intended before?*

Here we confine the analysis to one episode referring to the group work on questions 3 and 4; we focus on the group made up by five students: Giu, Ari, Luc, Mar, Ale.

The transcript is analyzed in terms of identity and rationality. Concerning the analysis in terms of identity (performed according to the labels that were introduced in Table 15.1), we point out that some labels are assigned even if in the selected excerpt the utterances are not recurrent, because they are repeated many times in the whole transcript.

- 11 **Giu:** it must be for all the operations, and then -3 times -2 is equal to?
 12 **Ari:** just a minute, I wrote: This means that it is negative number, but anyway you can do $-2 + -3$ [in column] you get -5 .
 13 **Teacher:** and how do you get it?
 14 **Ari:** the sign means that it is a negative number, then I can do... that sign does not mean anything! It just means that it is a negative number, so...
 15 **Giu:** no, Ari, what you are saying is meaningless.
 16 **Ari:** yes.

- 17 **Giu:** –16:–4? [laughing].
 18 **Ari:** –16? You must do $16 - 4$ [after, she adds the minus before the numbers, in column].
 19 **Giu:** try and do the division.
 The division.
 20 **Ari:** ok.
 21 **Luc:** wait a minute! [she takes the pen].
 22 **Giu:** let me speak!
 23 **Luc** [she does -4 divided by -2 , and she writes -2]. Here you are! -4 divided by -2 is -2 .
 24 **Ari** [does again 16:4].

In sentence 11, Giu is mathematizing but also identifying as good in math. (Mth) (I) [Me Nv Di]. Giu provokes a change in the discourse generality: *If it is true, it has to be always true, for every kind of operation* (Mth). Giu wants to bring the discussion to an epistemic level: she suggests that, in order to have negative numbers as numbers, it must be possible to perform operations, and she asks for justification and meaning for this (epistemic dimension of rationality).

In sentence 12, Ari proposes to write down how to perform an addition of negative numbers. She is satisfied with the result and she is not looking for a justification or a deeper understanding of the meaning of negative numbers. Her intervention is less on the epistemic level and more on the teleological one, according to the final aim *reaching a solution to write down* (teleological dimension).

In sentence 13, Ari is mathematizing and suggesting a path from numbers as natural to numbers as positive and negative (Mth).

In sentence 15, Giu introduces the question of the sense of the operations with negative numbers. She also talks about Ari [Pe Sp], identifying Ari as a person not good in math. She did it for the whole discussion saying that as usually she does not understand [Me V Di]. This intervention, as the previous one, may be interpreted in terms of rationality. Giu has a clear position (negative numbers are not like positive numbers) and she wants to hold this position. To this aim (teleological dimension) she challenges the proposals of her mates. Her way of challenging is mainly at the epistemic level: she asks for further justification.

This changes the status of the following statements (epistemic dimension).

In sentence 17, Giu laughs when she does not agree with Ari's proposals, showing self-confidence in math (I) [Me Nv Di]. In order to challenge Ari's position (teleological dimension), Giu challenges her in terms of result and meaning of the operations with negative numbers (epistemic dimension).

In sentence 19, Giu is mathematizing but also identifying as good in math and saying to Ari what she has to do (I). [Me Nv Di].

In sentence 20, Ari interrupts her activity accepting Giu's request [Pe Nv Sp].

Also Luc (sentence 21) tries to find a place in the discussion [Me Nv Di].

Giu (sentence 22) intervenes claiming her right to participate [Me Nv Di].

Luc (sentence 23) tries to answer to Giu's provoking question but she does it in a very hasty way that does not satisfy Giu's request. She acts aiming at intervening

and proposing a solution for the group, but we may note that her solution is not accompanied by a justification (lack on the epistemic dimension). She is rather more interested in providing a solution and foster an agreement among the mates. She has a clear goal and acts accordingly (teleological dimension). To reach this aim, she puts an effort in communication (communicative dimension).

The analysis of the transcript shows that individuals' identities affect the students' participations to the group work; moreover, students have different aims (reaching a shared conclusion, see Luc and Ari, versus imposing his/her solution, see Giu, versus reaching understanding) and act accordingly (teleological rationality); this has effect in terms of prevailing epistemic (in the case of Giu) or communicative (in the case of Luc) dimensions and, consequently, on a fruitful participation to the group work. As a consequence, there is a final situation of non-agreement.

15.6.2 Example 2

The example comes from an experiment performed in grade 4. The group was composed by four primary school students. The students were asked to solve a problem together using a big paper and marker pens while the teacher and the other classmates were looking at them; then they were asked to present the solution to the classroom and to explain how they reached a common solution. This setting, conceived by the teacher-researcher as a means to foster the elicitation of explanation and problem solving strategies gives us information both on the group work and on the subsequent presentation of the group solution. The last request (to explain how they reached a common solution) is particularly important for our analysis since we observed in previous analyses that being asked to reach a common solution could lead the group to a forced agreement that is not based on statements proposed the epistemic level but rather on social dynamics influenced by the students' identities.

The teacher posed the following problem:

A snail wants to climb a wall 7 meters high. It starts from the bottom in the morning and goes 2 meters up until the sunset; then during the night it glides a meter down. It restarts again the day after and so on: during the day it goes 2 meters up, during the night it glides a meter down. How many days does it take to it to glide the wall?

Here we present the transcript and analysis of the group work, including the interaction with the teacher.

- 1. Sara:** Each centimeter is a meter: 1, 2, 3, 4, 5, 6, and 7. Now I trace all the lines. Let's say that the pink is the first day. Here I draw the snail and it does two meters and climbs down one meter. Then, starting from here, it does two more and goes down one. From here again it does two and goes down one. Then two more and one down. From here it does two more and goes down one. I did some mistake... wait... I got 6, but I noticed that there is this little piece, since it goes down one meter [she points to the trait between B and A, see Fig. 15.1].

Fig. 15.1 The sketch provided by the group



2. **Marco:** After it goes up, another day... it does in this way: one it goes up, and after it jumps on the wall.
3. **Sara:** But it cannot arrive on the air, eh...
4. **Raul:** If it already finished, why does it have to go down?
5. **Sara:** Anyway, I noticed this: six days, but I was not sure; because it does in this way: the first day, the second, the third, the fourth, but at the sixth it comes here [she points to point A] it does not come upper.
6. **Marco:** Then, it will take seven [days].
7. **Raul:** When it arrives, it has finished, has not it? What does it do, go down?
8. **Sara:** Anyway it is written that by night it slides down.
9. **Raul:** It slides when it is in the wall and not when it is arrived.
10. **Sara:** Then it results to me: it does 6 days, but after it does also this little piece to get up.

In sentence 1 we see an instance of Mathematizing (Mth): Sara explains to the group mates her reasoning showing a good self-confidence (I) [Me Vb Id]. She is efficient in representing the problem on the paper and describing her reasoning; she is also able to revise and correct herself during the explanation (communicative and epistemic dimension). In the last part of her intervention she imagines the snail reaching the top of the wall (7 m) at the end of the 6th day, but she claims that, during the subsequent night, the snail must climb one meter down, according to what is written in the text of problem. Hence, the problem is not yet solved, because there is still one “little piece” (one meter) to be climbed. Sara is working at epistemic level, accompanying all the steps of the solving process with a clear reference to the information provided by the text of the problem. She is led by a pure “mathematical” rationality, where the problem and the situation must conform to the “rules” given by the text.

In sentence 2, Marco tries to complete Sara's discourse providing an interpretation of what might happen in the little piece. Here again we see an instance of Mathematizing (Mth). Marco is led by the final goal of reaching a solution (teleological rationality).

In sentence 3, Sara expresses some doubts on Marco's conclusion: if the snail climbs up from 6 m, it must climb up 2 more meters, because the text says that each day the snail climbs 2 m up. Hence the snail should reach the level of 8 m. But the wall is only 7 m high, then the snail should, when climbing 2 m up, reach "the air". From one side, Sara wants to find a solution that respects the mathematical "rule" given by the text; from the other side, she does not feel comfortable with a hypothetical solution where the snail arrives "on the air". Sara is trying to respect the text rule and to match it with the sense of the problem. Her intervention is still at the epistemic level.

In sentence 4, Raul poses a question, intervening in the discussion with a personal mathematical point of view (Mth) (I) [Me Vb Id] and trying to interpret the solution modelling the concrete situation. Raul's intervention is at epistemic level, since he supports his intervention with his own understanding of the problem: the text gives a mathematical rule for the movement of the snail, but once the top of the wall is reached, the snail stays on the wall and does not need anymore to climb down. Differently from Sara, he relies on a "modeling" and contextualized rationality, that supports his going back and forth between mathematics and real life. Despite Raul's intervention, in sentence 5 Sara goes on affirming her point of view, confirming herself as good in mathematics (I) [Me Vb Id].

In sentence 6, Marco intervenes once again in order to complete Sara's reasoning. We note that Marco is efficient in providing quick solutions to Sara's doubts, according to his final aim of providing a final solution, possibly shared by all the mates (teleological and communicative dimension).

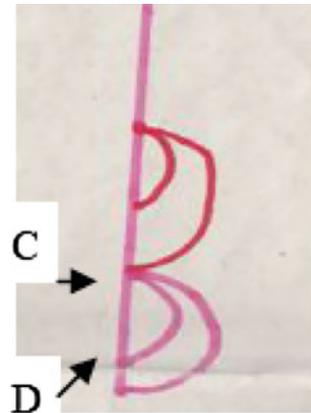
In sentence 7, Raul goes on keeping his personal point of view and confirming himself as good in mathematics (Mth)(I) [Me Vb Id] and posing again the same question. His intervention is still at epistemic level, but in reference to his modeling and contextualized rationality.

In sentence 8, Sara goes on affirming her point of view, confirming herself as good in mathematics (Mth) (I) [Me Vb Id]. Also Sara's intervention is at epistemic level, since she grounds her position on the text of the problem. She relies on a mathematical and non-contextualized rationality.

11. Martina: I would do the same as Sara. The snail climbs up of two in the morning and one down by night, because it slides down. The day after it restarts, but by night it slides down again. I did like Sara [she draws another sketch, see Fig. 15.2].

12. Sara: But Martina, in my mind your solution has something wrong. Sorry but if it slides until here [pointing D], now it restarts from here [pointing C]?

13. Marco: It must restart from here [pointing to point A on the previous sketch].

Fig. 15.2 Martina's sketch

14. Sara: But for me there is something wrong ... the snail employs 6 days and a little piece because this remains empty, it arrives there [pointing to A].

15. Marco: 6 days and a sunset.

Martina (sentence 11) tries to participate in the discussion reproducing Sara's reasoning and identifying herself as not good in math (she repeats twice "I did like Sara, I would do as Sara"). [Pe Vb Id]. We may note that Martina is not able to reconstruct Sara's reasoning, possibly because she did not completely understand it. Trying to reproduce rather than really understanding, and not realizing that the reproduced reasoning contains some mistakes are evidences of lack in epistemic rationality. She does not mathematize.

In sentence 12, Sara replies to Martina acting on the epistemic level and confirming herself as good in mathematics (Ep) (I) [Mb Vb Id].

In sentence 13, Marco rephrases Martina another time without adding something or proposing a point of view (communicative dimension).

Once again, in sentence 15 Marco proposes a quick solution to solve Sara's doubt (teleological dimension).

16. Teacher: So what is your solution?

17. Sara: For me it is 6 days and a bit.

18. Marco: 6 days and a half.

19. Raul: 6 days and a half?

20. Sara: Because it arrives here, and not here in 6 days.

21. Marco: Because when it slides it goes down.

22. Sara: And then, now, from here I cannot go 2 steps upper, because otherwise it goes in the air. It cannot fluctuate.

23. Marco: But here is the end of the wall and after it starts sliding down.

24. Raul: I do not agree with this. [...] I counted 1, 2, 3, 4, 5, 6 and then it stays there, if we come back it never arrives to the top.

- 25. Sara:** But I wanted to say that when it arrives it must slide down. The text says that by day it climbs two and by night it slides down one.
- 26. Teacher:** So what is your answer?
- 27. Raul:** It is 6.
- 28. Teacher:** Do you all consider the answer convincing?
- 29. Marco:** For me yes, because if we do what Sara suggested it does not work. Raul is right because if we make it slide down it remains here and never reaches the end.
- 30. Sara:** Now I'm convinced too, because in the text we are asked "How many days does it take?". This is the question. For me they are right, because in the end the snail is on a line and cannot go down. Then for me it is 6 days and stop.
- 31. Martina:** I think like Sara.

In sentence 16, the teacher intervenes in order to lead to the group to the end of the activity.

In sentence 17, Sara presents her answer.

In sentence 18, Marco rephrases Sara's solution.

Raul expresses his doubts about the proposed solution. Hence, in sentence 20 Sara adds an explanation for her solution.

In sentence 21 Marco adds something to Sara's explanation (in order to foster Raul's understanding of Sara's proposal). In sentence 22, Sara expresses again her doubt about the final part. We may note that Sara is acting at epistemic level (grounding her explanation on her understanding of the text) and communicative level (using the sketch to make Raul understand her explanation).

In sentence 23, Marco reinforces Sara's explanation. Anyway, Raul disagrees and proposes his own explanation to support a different answer. Raul is acting at epistemic level, since he accompanies his solution with an explanation, which is grounded on his understanding of the text.

In sentence 25, Sara replies going on her own path: since the text says it slides down by night it **MUST** slide down. She identifies again herself as good in mathematics (I) [Me Vb Id] and tries to found her explanation on solid bases (epistemic dimension).

In sentence 26, the teacher intervenes again to make the group converge to a unique solution.

After Raul's answer (sentence 27), the teacher accounts him as the one who gave the good answer even if it is evident that not all the members share his position.

After the teacher's intervention, Marco changes suddenly his answer (he no more supports Sara) and also the explanation (sentence 29). We may note that Marco's explanation, anyway, is different from Raul's one: it is based on the fact that adopting Sara's perspective it is not possible to reach the end. The teleological dimensions of rationality (it is necessary to reach a solution) is prevailing.

Due to the teacher's intervention, also Sara goes back to her solution and modifies it (sentence 30). Sara accepts Raul's answer but tries to explain it relying again on her personal epistemic approach: once reached the top of the wall, the snail should slide down, but the process is interrupted by a line. This explanation is very different

from Raul's one, because Raul bases his considerations on real life (almost thinking as a snail in the situation), while Sara de-personalizes the snail, that could also be a point on a line, and thus a further element is necessary to agree with the others: there must be an external constraint that impede to it to go on with the process "imposed" by the text. In other words, Sara adds a constraint to her interpretation of the text.

Finally, in sentence 31 agrees with Sara. Martina, as we observed before, renounces to her personal point of view, only acts at the teleological level (the final goal is to provide a solution to the teacher) and also confirms her lack of self-confidence in mathematics.

The analysis shows one case where the group solution is far from being shared at the same level by all the group members. Indirectly, Sara and Raul identify themselves as good in mathematics (see sentences 5, 7, 8, and 25). Martina does not really participate into the solving process, identifying as not good in mathematics and aligning herself to the position of Sara (see sentences 11 and 31). Her identity of not good in mathematics goes along with a lack in epistemic rationality. Marco puts a great effort in finding a mutual understanding: his final aim is to produce a group solution, and the prevailing teleological dimension leads him to act as mediator between Raul and Sara, rather than a facilitator that really helps mutual understanding (see sentences 6, 13, and 15).

Concerning Martina and Marco, we may say that the combined analysis helps us to better characterize their participation into the group work.

The interactions between Raul, Sara and the teacher, read in terms of the combined framework, give us new insights into the concept of *forced agreement*. First, we may observe that the initial lack of agreement between Sara and Raul is due to the presence of two strong "mathematical identities" and on the fact that each of them is relying on the epistemic dimension of rationality, but with a great difference: Raul employs a modeling and contextualized rationality (see sentences 4, 7, 24), while Sara relies on a mathematical and non-contextualized rationality (see sentences 1, 5, 8). The different rationalities cause a different approach to the problem and, finally, a different solution (see sentences 17 and 27). Only the teacher's intervention leads Sara to change their position. Sara's way to manage the "obliged change" (see sentence 30), without giving up on the epistemic level and trying to conserve her identity, is very interesting since it leads us to enlarge the set of possible behaviors due to the interactions between different identities and rationalities in group works. The effect of the interaction between different identities with the mediation of the teacher leads to a particular case of forced agreement that we never observed before: one of the students, Sara, aligned her position without renouncing to her identity, but rather forcing the suggested solution to be included in her previous strategy as a particular case. We may note that the presence of different forms of rationality (modeling and contextualized vs. non contextualized) could have become even more fruitful if it had been identified and stressed by the teacher.

15.7 Findings

In this section, we summarize the main findings of our analysis in reference to the research questions.

Finding 1: Identities and prevailing dimensions of rational behavior are intertwined and their mismatch may cause a lack of agreement

A crucial issue, when dealing with group works, is the students' participation into the activity. Identity may affect such a participation, at two levels: at the level of degree of involvement and at the level of nature of the interventions during the activity.

Concerning the first level, we may say that students that do not identify themselves as good in mathematics tend to avoid participation (see Martina in Example 2), while students who identify themselves as good in mathematics tend to involve into the activity (see Giu in Example 1, Sara and Raul in Example 2). The second level adds insight into the nature of such participation, which can be described by means of the dimensions of rationality. Concerning teleological rationality, individual students may act according to different (and sometimes conflicting) aims: imposing his/her own solution over all the group (see Giu in Example 1); refuting somebody else's proposal, independently of its correctness (see again Giu in Example 1); reaching a shared conclusion, providing a solution (any solution) to the teacher (see Luc, Example 1; Marco, Example 2). Accordingly, some students focus mainly on epistemic aspects (finding a correct answer and a sound explanation) (see Giu, Example 1; Sara and Raul, Example 2), while for other students the communicative dimension is prevailing (not only reaching a solution, but making it comprehensible for all the groupmates) (see Luc, Example 1; Marco, Example 2). Networking the two analyses, we may say that the mathematical identity turns into prevailing dimensions of rational behavior—epistemic in the case of good mathematical identity and communicative in the other cases—and that the interplay of dimensions of rationality affects the participation into the group activity and the quality of the interaction. The effectiveness of the group work, both in terms of solution production and of meaning making depends strongly on such factors.

Finding 2: Teachers' interventions may affect significantly interactions in group works

That teachers may have relevant effects on the ways students participate in group works and interact with the groupmates. More specifically, teachers' interventions during the group work may change the balance between students' identities, causing one student, identified by the teacher as good in mathematics, to impose his/her solution on the group-mates, and, conversely, other students, identified as less good in mathematics, to give up and accept passively a solution proposed by one mate. In Example 2 we showed a situation of initial non-agreement, caused by two strong mathematical identities that rely on different kinds of epistemic rationality, that progressively turns into a forced agreement, due to the intervention of the teacher that implicitly accepts one of the proposed solutions and forces the other students to accept it. We also observed that the students, according to their mathematical identities and their epistemic dimensions, react to this situation in different ways, and for

the student Sara the forced agreement is turned into a deeper understanding of the situation.

15.8 Conclusions

Our analysis shows that both social skills and disciplinary issues influence the students' ability to participate in a group work in a quite complex way. Particularly, we focused on the interplay between identity and the prevailing dimension of rationality of the students' discourses that we consider important and not independent factors influencing group works in mathematics. We argue that only a good "mathematical identity" (the student feels he/she is good in mathematics) may lead the students to participate in the group work moving to an epistemic level and thus to develop relevant social skills related to mathematics, like being able to pursue the convergence of a group work towards a common solution respecting an own personal point of view and checking by him/herself the validity of the explanation, in despite of the 'popularity' of the solution or of the accountability of the group mate who is proposing a solution.

In the analysis of Example 2, we also showed that the teacher's intervention caused a significant reaction in the group, changing the authentic interaction between the students and provoking a forced agreement. This confirms that it is a plurality of factors to affect, time after time, the students' "mathematical identities" (and identities in general) and the teachers may play a significant role. This also reinforces the general statement that the relation teacher-students involves not only the mathematizing but also the identifying process, leading in some cases to a co-construction of the student's identity of failure, as pointed out by Heyd-Metzuyanım (2013).

15.9 Further Developments and Implications for Practice

From our findings, the necessity of a deep reflection on the role of the teacher emerges: the teacher seems to play not only as a mediator between students' different positions, but also as—maybe indirect and unaware—identifier of his/her students as people who can have accountability in a mathematical group work or not. Hence, possible directions for further research are: analyzing teachers' awareness of their role during group work and solution presentation, and of the impact of their interventions on the evolution of social dynamics and on the students' prevailing dimensions of rationality; studying whether and how teachers may help students to manage the relation with the group and make the students more and more aware of the social dynamics underlying group work. Particularly, we are interested in understanding how far the teachers are able to distinguish between productive interventions, that make the students keep their own identities and encourage them to move to the epistemic level

of rationality, and interventions that lead the members to give up their own proposal and accept another one.

Moreover, we plan to organize professional development activities based on the findings of our research, in order to promote the evolution of the teachers' awareness on the issue of identity and rationality during group work. To this aim, we will provide them theoretical tools to analyze transcripts of their own interventions or of interventions of other teachers and we will organize sessions of joint analysis with them. In this way, our theoretical reflection on the interplay between identity and rationality will provide the ground for teacher professional development interventions (Tsamir, 2008).

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