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SYMMETRY-ART: A STEAM TRAINING WORKSHOP FOR PRIMARY SCHOOL TEACHERS

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The aim of this paper is to analyse a symmetry and art workshop from a STEAM perspective. The theoretical framework of the Meta-Didactical Transposition is taken as a reference. The sample consists of seven Primary School teachers. A qualitative methodology is followed that is developed in four phases: learning, planning, implementation and reflection. The results show that the teachers are not flexible in dealing with the different conceptions of symmetry and the creative aspect of the workshop. In general, there is a positive attitude towards the interdisciplinary character of the workshop, despite the fact that they were not able to connect both disciplines in a balanced way.

INTRODUCTION

Recently, the ‘A’ of art has been included in the acronym STEM (Science, Technology, Engineering, Mathematics). The main goal of STEAM education is to make the students grasp the connections between different pieces of knowledge incorporating an artistic vision into the activities from a creative and emotional point of view (Henricksen, 2014; Yakman & Lee, 2012).

In particular, what is the relationship between Visual Arts Education and Mathematics? One reason for asking this question is that “on the one hand, mathematics is art, and on the other hand, working in art has a mathematical basis” (Hickman and Huckstep, 2003, p.1). Mathematics and art are two disciplines that have a close relationship since immemorial times. In order to motivate students to study mathematics, the connections between art and mathematics, in particular geometry, have been exploited in many works in mathematics education (Fenyvesi, K. & Lähdesmäki, T., 2017; Lavizca, Z. et. al., 2018; Portaankorva-Koivisto, P. & Havinga, M., 2019) showing them that these have been used for aesthetic reasons in the history and modern art.

Recently, the recommendations for including the arts and creativity in the teaching of mathematics significantly increased all over the world along with demands to move from paradigms of teaching concepts and methods in a purely disciplinary way to an interdisciplinary and integrated education that shows

connections, is based on complex problems and promotes critical and creative thinking (Council of the European Union, 2018). These recommendations come, in general, from outside the school. In particular, from EU and other transnational institutions and from labour market. That recommendations oblige the curriculum developer who wants to meet such promising but ambitious goals to take the issue of teacher training education seriously. Indeed, in order to make this new approach become a structural innovation in schools, a change of perspective would be necessary, first of all in teacher education: the teachers need to be prepared to carry out properly the classroom activities, becoming aware of their non-renounceable features and pursuing their goals with their more traditional ones in the complexity of the real classrooms.

In this paper a STEAM training workshop for Primary School teachers is analysed, emphasizing the disciplines of mathematics and art. The aims are to attend how the teachers react to the activities proposed and how they implement them in the classroom. Moreover, the process of personal transformation of the proposal made by some teachers is observed.

RESEARCH FRAMEWORK

The framework of the Meta-Didactical Transposition (MDT) (Aldon et al., 2013; Chevallard, 1999) is considered as a main reference. In particular, in this paper, the construct of praxeology is used. “The praxis or ‘to know how’ includes different kinds of problems to be studied as well as techniques available to solve them; and the logos or ‘knowledge’ includes the discourses that describe, explain and justify the techniques used and even produce new techniques” (Garcia et al, 2006, p.226). Within the MDT approach, the praxis is didactical and the logos not only concerns the knowledge of the discipline, but also of didactical and pedagogical research results. On one hand, in a teacher training activity, researchers’ and teachers’ praxeologies meet each other and members of two communities of practice have to find a common ground in order to allow the teachers to appropriate of the researchers’ proposals and effectively modify their praxeologies.

The transition from individual to shared praxologies is very delicate and requires the action of a ‘broker’, a subject that is a hybrid between the two communities who acts as a hinge between the two fields, the school itself and the academic. The broker has the difficult role of creating new connections and encouraging creations of meaning and learning (Rasmussen et al., 2009).

To analyse the teachers’ choices, when they plan and implement the activities of the symmetry-art workshop, the goal-oriented decision-making theory by Schoenfeld (2010) is relied on. This framework deals in particular with choices of the teachers in real-time. As Schoenfeld (2010) stated clearly, when the teachers move from the design to the implementation, something that changes even completely the goals of the designed activities often happens. Indeed, they

are only partially aware of their resources, goals and orientations, and these might remain invisible in the design phases, but appear clearly in the way they react to students' questions or unexpected happenings. Tensions appear between the planned and the implicit goals and orientations (Liljedal et al., 2015) and oblige the teachers to make real-time decisions according to their priorities. This point is crucial: a deep innovation requires the teachers to become aware of their knowledge and assumptions and seriously reconsider in a conscious way their goals and priorities.

RESEARCH METHODOLOGY

The training symmetry-art workshop was designed for Primary School teachers and was carried out in two Italian cities. In this paper, a sample of seven Primary School teachers is analysed. The objective is to answer the following research question: what is the general impact of the symmetry-art workshop on the teacher's design and implementation in their classrooms?

The research methodology is qualitative and from a STEAM perspective involves working the two disciplines together in a balanced way, both in terms of concepts procedures and procedures and attitudes. It was organized in four phases that are described below: (i) learning; (ii) planning; (iii) implementation; (iv) reflection.

(i) Learning phase. In this phase, the researchers present the STEAM methodology. Then, the teachers carry out the different workshops by interacting with the researchers. In accordance with the MDT, a PhD student graduate in Primary Education Sciences took on the role of broker, mediating the delicate passage of the interweaving of the praxeologies of the teachers with those ones of the researchers.

(ii). Planning phase. The objective is that teachers develop this proposal to the classroom, after a careful co-design shared between teachers and researchers. To this end, they should decide which tasks they are going to implement, whether and how they want to modify them, in which order, the time they are going to use for each task, the links with their curricular teaching plan and the methodology they are going to carry out (group or individual work, classroom discussions and the educational environment where the students would do the activities).

(iii). Implementation phase. In this phase, the teachers implement the symmetry-art workshop tasks as they have designed them in the previous phase. The aim of the research is to compare the decisions taken in the planning phase and the teachers' actual praxeologies in the classroom.

(iv). Reflection phase. Here, both researchers and teachers reflect on the entire instructional process. In this way, following the theoretical framework,

researchers' praxeologies should change interacting with the teachers to make the proposal more suitable from the cognitive and institutional points of view.

To collect the data the following instruments were used. In the planning phase, individual and group interviews with teachers were recorded. In addition, they were given a grid to fill in different sections regarding the organization of the tasks. In the implementation phase, video recordings were made of the observations of teachers and students in the classroom. Moreover, an observation tool was also designed which comprehends thirteen items. Within these items, special attention was given to those that refer to, among others, the good use of mathematical vocabulary, the mastery of the artistic techniques and the methodology carried out in class.

The tasks that were carried out in the STEAM training workshop are described below.

Description of the Tasks

Training Symmetry-Art workshop is made up of four tasks to carry out in two sessions of two hours. The tasks of this workshop are aimed at Primary School students (six to twelve years old). In mathematics education, the difficulties in the learning of this topic have been investigated in many studies (Bulf, 2011; Chesnais, A. & Munier, V., 2013, Bohorquez et. al., 2009), and it has been shown to be more complex as it might seem. These difficulties might affect the teachers' resources, both on the side of disciplinary knowledge and of the anticipation of students' difficulties. Within this proposal, a balance is sought between the two subjects of mathematics and art. Following a STEAM perspective, the objective is to work these two subjects in an equal way, that is, these tasks form a cycle starting from art (task 1) and coming back to art (task 4), with a renewed conceptualization of the everyday conception of symmetry (Chesnais, 2012) triggered by the artistic work and supported by research-based mathematical tasks (2 and 3).

Task 1: Artistic folding paper

This activity is designed with the intent to create a symmetrical artwork from the blank paper and without mentioning the concept of symmetry. The aim is to bring students closer to the study of symmetry and its elements, starting from the original artistic creation of each of them through the manipulation of different resources, in this case, thread, tempera and sheets. The contents that are worked on in this task are the concept of symmetry, the axis of symmetry, the types of lines, the equidistance, the concept of shape and dimension, the horizontal and vertical meaning, the manual work and the use of colour and its possible mixtures.

Task 2: TEPs.

Following to D'Amore and Maier (2003), the objective is, for each student, to create a TEP (Textual Eigen Production), which is an autonomous textual production, in this case, of the concept of symmetry and its characteristics based on the artistic work and the discussion carried out in the previous task. The contents worked on here are the use of the mathematical vocabulary to elaborate the definition, the written expression and, again, the concept of symmetry with some of its elements as the axis of symmetry, the equidistance of each point to that axis and the concept of form and dimension.

Task 3: Schematization

This task consists of drawing, on the grid sheet, the figure that the students made in the task 1. The aim is to make them work on symmetry and its characteristics through the elaboration of a scheme with drawing instruments as the rule or the compass. The students also work on the reproduction of a figure to scale, since at the moment of drawing the figure in a schematic way, they are transferring the figure to the grid sheet, taking the little square as a unit.

Task 4: Symmetrical figures with coloured threads

The last task is designed to finish the proposal with an artistic activity that gathers everything learned in the previous tasks. The activity consists of recreating, with coloured threads and pins, the figure made in task 1, and then outlined in the task 3. By stretching the threads and tightening them, the students create another artistic work in a different format in which the main theme is symmetry.

RESULTS AND DISCUSSION

The results are presented according to the aims set, derived from the research question presented in the previous section: to observe how the teachers react to the activities proposed in the symmetry-art workshop and how they implement them in the classroom.

Teachers Reactions

In terms of STEAM methodology, the teachers initially stated that they dealt with mathematics and art topics always separated. Although they had already dealt with the topics proposed in their classes, they did not realize that they could make an interdisciplinary lesson by drawing inspiration from artistic creations to get to the formalization of mathematical concepts. Moreover, it could be observed that the reactions of some teachers consisted on not considering the STEAM activities truly mathematical didactical activities, since the contents and the kind of tasks were different from the text-books exercises, that are their institutional reference. Some teachers perceived these activities as extracurricular motivation, since they emphasize their artistic character and gave importance only to the aesthetic aspect, that is, they did not consider them 'mathematical' (learning phase).

For most of the teachers, the tasks seemed to be not so far from their usual practice and the mathematical contents and artistic skills were considered easy. However, some of them did not feel confident to carry out the activities in the classroom observed by researchers and, in many cases, they had some difficulties to pursue the planned goals in the implementations. For example, a teacher somewhat insecure, asked “how I should start the lesson? Are we going to carry out the activity together?” (planning phase).

In the implementation phase, two of the seven teachers said “Do we have to carry out the lesson? But we can’t do it, we don’t know how to do it”, revealing to be unsure at the beginning of the class. Another teacher renounced to lead the activity and asked the researchers to do it. Part of the problem could be due to the presence of the researchers in the classroom or to the insecurity of applying the STEAM methodology.

Implementation in the Classroom

Of the seven teachers who planned to carry out the art and symmetry workshop in the classroom, six did so. Of those six, four implemented it autonomously while the other two needed further assistance from the researchers. Although the planning phase allowed them to modify and adapt the proposal to their classroom and students, only one of the teachers changed the order of the tasks and dedicated more time to the discussion that is carried out in task 1.

Paying attention to the mathematical aspect of the workshop, several facts are considered important. When the students commented on their TEPs for the rest of the class (task 2), the teachers corrected those who talked about important aspects of symmetry such as distance to the symmetry axis, because they identify the term symmetry only with the definition they know, which is the same one that appears in the textbook. Therefore, their goals were far from ours and were influenced by the textbook definition in a negative way for the students' mathematical processes.

For some teachers there is a total identification between the concept of symmetry and the fact that half of a figure could be superposed to its other half folding a piece of paper containing the picture; the paper folding activity helped them to feel comfortable but in some cases the symmetry-art workshop was not effective in enriching their concepts moving from the everyday to the mathematical concept. In some cases, the teachers did not take care properly of the students' spontaneous mathematical processes and interrupted the students who were carrying out their own reasonings in terms of symmetry. For instance, many students interpreted correctly the request of explaining with their words how to draw a ‘symmetric figure’ that is, a figure admitting (at least) one axis of symmetry while their teacher expected the students to use formal words and define the symmetry in the way the teacher had suggested and started limiting them without helping them in their developmental zone. This may be due to

teachers' lack of flexibility in conducting a group discussion with students on the concept of the symmetry (ibid., 2012). On the other hand, in many cases the teachers declared that their insecurities were due to unexpected difficulties with the mathematical contents, and emerged when the students were working and proposing their ideas in a manner that was different than the usual (reflection phase).

Focusing on the artistic part, it should be pointed out that it was the main aspect that motivates the teachers to implement the mathematics and art workshop. However, initially, most of them limited the creativity of the students, especially in task 1. This limitation could be due to the fact that the teachers showed a perfectionist attitude when they performed the workshop by themselves (learning phase) and wanted their students to obtain similar results to theirs, imposing some criteria like the colours they should use or indicating that the artwork should be 'beautiful' and 'well done' (implementation phase). Between these two phases, it could be seen that teachers' praxeologies (Schoefeld, 2010) changed, since they were forced to make decisions just in time. For example, because of the motivation students to do this workshop, many of the teachers spent more time experimenting with more colours and creating more artworks. In addition, some of them left the students total freedom when performing the schematization (task 3) allowing them to use different colours and shapes.

CONCLUSIONS

Taking into account one of the aims of this paper, it could be observed that teachers' reactions to the proposed STEAM workshop were positive. In the reflection phase, all teachers valued the importance of proposing activities with an interdisciplinary character. Adding the planning phase was intended to give teachers flexibility and creativity in implementing the workshop in their classrooms. However, the changes that were observed were very specific and only one of the seven teachers modified the tasks by adapting them to her classroom context. In this case, the intersection between the teacher's and the researcher's praxeologies was obviously no longer empty.

On the other hand, the tasks of the workshop have an intrinsic complexity that makes students act in unpredictable ways. Although many of the teachers stated that the schematization (task 3), specifically, was very difficult, the students performed it very effectively obtaining great results. In some cases, however, teachers were not flexible to adapt the activities to just-in-time happenings.

The fact that more than one teacher has declared that they want to continue experimenting with mathematics and art workshops means that some practices have changed and that the symmetry-art workshop has been successful. It is therefore desirable that a dynamic process of professional evolution has been triggered in which some components external to the teachers praxeologies, such

as the use of interdisciplinary teaching through appropriate tasks, become internal as an effect of the process of meta-didactic transposition. The meta-didactic transposition, in our case, has its strength in the use of innovative tasks and the adoption of interdisciplinary teaching. Therefore, we propose to continue carrying out workshops and to focus on the relationship between mathematics and art encouraging a balance between these two disciplines.

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