NOTES AND RECORDS

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DAVID GREGORY'S MANUSCRIPT 'ISAACI NEUTONI METHODUS FLUXIONUM' (1694): A STUDY ON THE EARLY PUBLICATION OF NEWTON'S DISCOVERIES ON CALCULUS

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David Gregory's manuscript 'Isaaci Neutoni methodus fluxionum' is the first systematic presentation of the method of fluxions written by somebody other than Newton. It was penned in 1694, when Gregory was the Savilian Professor of Astronomy at Oxford. I provide information about its content, sources and circulation. This short treatise reveals what Newton allowed to be known about his method in the mid-1690s. Further, it sheds light upon Gregory's views on how Newton's mathematical innovations related to the work of other mathematicians, both British and Continental. This paper demonstrates two things. First, it proves that Newton, far from being—as often stated—wholly isolated and reluctant to publish the method of fluxions, belonged to a network of mathematicians who were made aware of his discoveries. Second, it shows that Gregory—very much as other Scottish mathematicians such as George Cheyne and John Craig—received Newton's fluxional method within a tradition that was independent from England and that, before getting in touch with Newton, had assimilated elements of the calculi developed on the Continent.

Keywords: David Gregory; Isaac Newton; fluxions; manuscript circulation

THE ITEM IN QUESTION

This paper is devoted to David Gregory's 44-page holograph manuscript 'Isaaci Neutoni Methodus Fluxionum; ubi Calculus differentialis Liebnitij, et Methodus tangentium Barrovij explicantur et exemplis quam plurimis, omnis generis, illustrantur. Auctore D: Gregory. A.P.S.' (hereafter the *Methodus fluxionum*), which is held at the University of St Andrews Library (MS 31011) as part of the 'Papers of James Gregorie the elder (1638–1675), David Gregory (1661–1708) and David Gregory (1712–1765)' (see Figure 1). As it appears from the dates penned by Gregory himself, it was written, maybe in its entirety,

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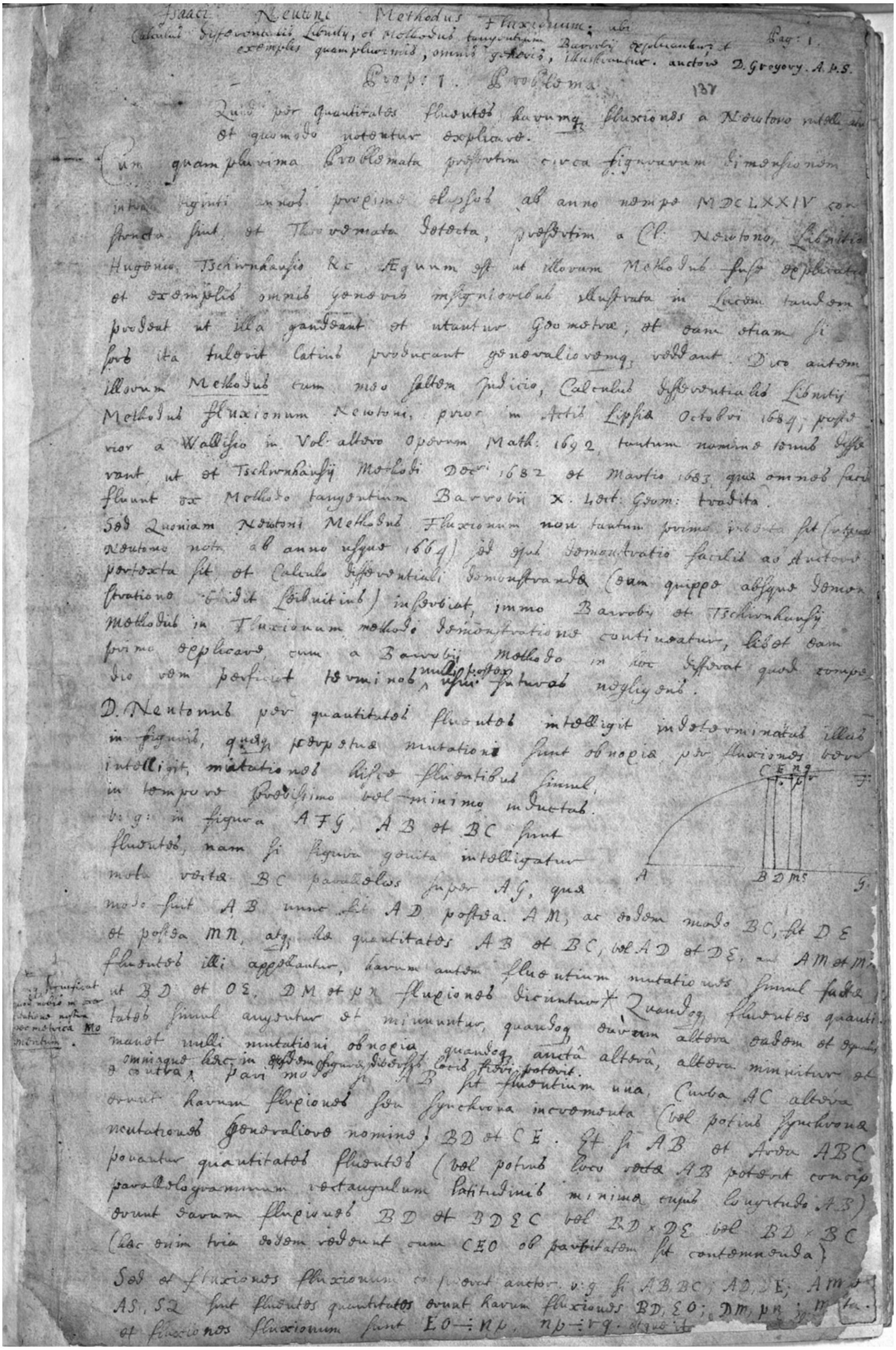


Isaaci Newtoni Methodus Fluxionum; ubi Calculus Differentialis Leibnitij, et Methodus tangentium Barrovij explicantur, et exemplis quamplurimis, omnis generis, illustrantur. auctore D. Gregory. A. P. S. Pag: 1.

Prop: 1. Problema 137

Quid per Quantitates fluentes, harumque fluxiones a Newtono intelligatur et quomodo notentur explicare.

Cum quamplurima Problemata praesertim circa figurarum dimensionem intra viginti annos proxime elapsos (ab anno nempe MDCLXXIV) confecta sint, et Theoremata detecta, praesertim a Cl. Newtono, Leibnitio Hugenio, Tschirnhausio &c. Aequum est ut illorum Methodus fusè explicata et exemplis omnis generis insignioribus illustrata in Lucem tandem prodeat ut illa gaudeant et utantur Geometrae, et eam etiam si fors ita tulerit latius producant generalioremque reddant. Dico autem illorum Methodus cum meo saltem judicio, Calculus differentialis Leibnitij Methodus fluxionum Newtoni, prior in Actis Lipsiae Octobri 1684, posterior a Wallisio in Vol. altero Operum Math: 1692, tantum nomine tenus differant, ut et Tschirnhausij Methodi Dec^ri 1682 et Martio 1683 quae omnes fere fluunt ex Methodo tangentium Barrovii X. Lect: Geom: tradita.

Sed Quoniam Newtoni Methodus Fluxionum non tantum prima inventa sit (utpote Newtono nota ab anno usque 1664) sed ejus demonstratio facilis ab Auctore pertexta sit, et Calculo differentiali demonstranda (eam quippe absque demonstratione tradit Leibnitius) inserviat, immo Barrovij et Tschirnhausij Methodus in Fluxionum methodi demonstratione continentur, libet eam primo explicare, cum a Barrovij Methodo in hoc differat quod compendio rem perficiat terminos multiplices usui futuros negligens.

D. Newtonus per quantitates fluentes intelligit indeterminatas illas in figuris, quaeque perpetuae mutationi sunt obnoxiae per fluxiones vero intelligit mutationes hisce fluentibus simul in tempore brevissimo vel minimo inductas. v: g: in figura AFG, AB et BC sunt fluentes, nam si figura genita intelligatur motu recta BC parallelos super AG, quae modo fuit AB nunc fit AD postea AM, ac eodem modo BC, fit DE et postea MN, atque hae quantitates AB et BC, vel AD et DE, aut AM et MN fluentes illi appellantur, harum autem fluentium mutationes simul factae ut BD et OE, DM et pn fluxiones dicuntur.* Quandoque fluentes quantitates simul augentur et minuuntur, quandoque earum altera eadem et aequalis manet nulli mutationi obnoxia, quandoque aucta altera, altera minuitur et e contra, omniaque haec in eadem figura diversis locis fieri potuit. Pari modo si AB sit fluentium una, Curva AC altera erunt harum fluxiones seu synchrona incrementa (vel potius synchronae mutationes generaliore nomine) BD et CE. Et si AB et Area ABC ponantur quantitates fluentes (vel potius loco recta AB potest concipi parallelogrammum rectangulum latitudinis minimae cujus longitudo AB) erunt earum fluxiones BD et BDEC vel BD × DE vel BD × BC (haec enim tria eodem redeunt cum CEO ob parvitatem sit contemnenda).

Sed et fluxiones fluxionum considerat auctor. v: g: si AB, BC; AD, DE; AM et AS, SΩ sint fluentes quantitates erunt harum fluxiones BD, EO; DM, pn; [illegible] et fluxiones fluxionum sunt EO ÷ np, np ÷ rq. atque ita [illegible]

* [illegible] significat quod nobis in demonstratione nostra Geometrica Momentum.

Figure 1. First page of Gregory's *Methodus fluxionum*. The hand is David Gregory's. (University of St Andrews Library, MS 31011, p. 1. Courtesy of the University of St Andrews Library.)

in Oxford in October and November 1694.[1] The *Methodus fluxionum* is particularly interesting, since it is the first systematic presentation of the method of fluxions written by somebody other than Newton.

A full transcription is provided by MS 131, ff. 193r–216v, Christ Church Library, Oxford (see figure 2). This is part of a scribal copy of three other works attributed to David Gregory: namely, 'Notæ in Newtoni Principia Mathematica' (ff. 1r–150v), 'Addenda Notis et suis locis reponenda' (ff. 151r–155r), and 'Geometria de Motu Pars prima. Definitiones' (ff. 157r–191v).[2] A note in Gregory's hand identifies the scribe as James Canarys (1653/4–1698) and the date on which this transcription was made as 1695.[3] The hand changes on ff. 151r–155r and, just near the end of the *Methodus fluxionum*, from the bottom of f. 210v up to the desinit of the manuscript. The hand in the above-mentioned folios is unmistakably Gregory's.

There is evidence that Gregory's *Methodus fluxionum* was not only copied, presumably at the author's request, just one year after its composition, but that in subsequent years it circulated among Newton's acolytes too. Two copies are in the Macclesfield Collection (MS Add. 9597/9/3 and MS Add. 9597/9/4, Cambridge University Library, Cambridge). Both are incomplete (they lack the last 9 and 10 propositions respectively out of 47) and might have belonged to the papers in William Jones's collection. The second is written in a hand that appears immature: it might be an exercise by the young George Parker (1697–1764), whom Jones tutored in mathematics after around 1708 (see figure 3).[4] In Cambridge, there is another copy, lacking the final propositions 46 and 47, among John Keill's letters and papers (MS O.XIV.278.13, Cambridge University Library). It is inscribed: 'For Dr James Keile at Mr J[oh]n Bannisters Apothecary in Oxon'. This places the date, at least of the transfer of the copy, between 1698 and 1703, since James Keill, John's brother, arrived in Oxford in 1698 and left for Northampton in 1703. Another copy is in the University of St Andrews Library (MS QA 33G8/D12). This is a complete scribal copy, lacking the title on f. 1r. Interestingly, this copy looks very much like the mock-up of a printed book, since all figures, carefully drawn by hand, are gathered at the end in six folded plates (see figure 4).[5] One might therefore infer that a printed publication was at

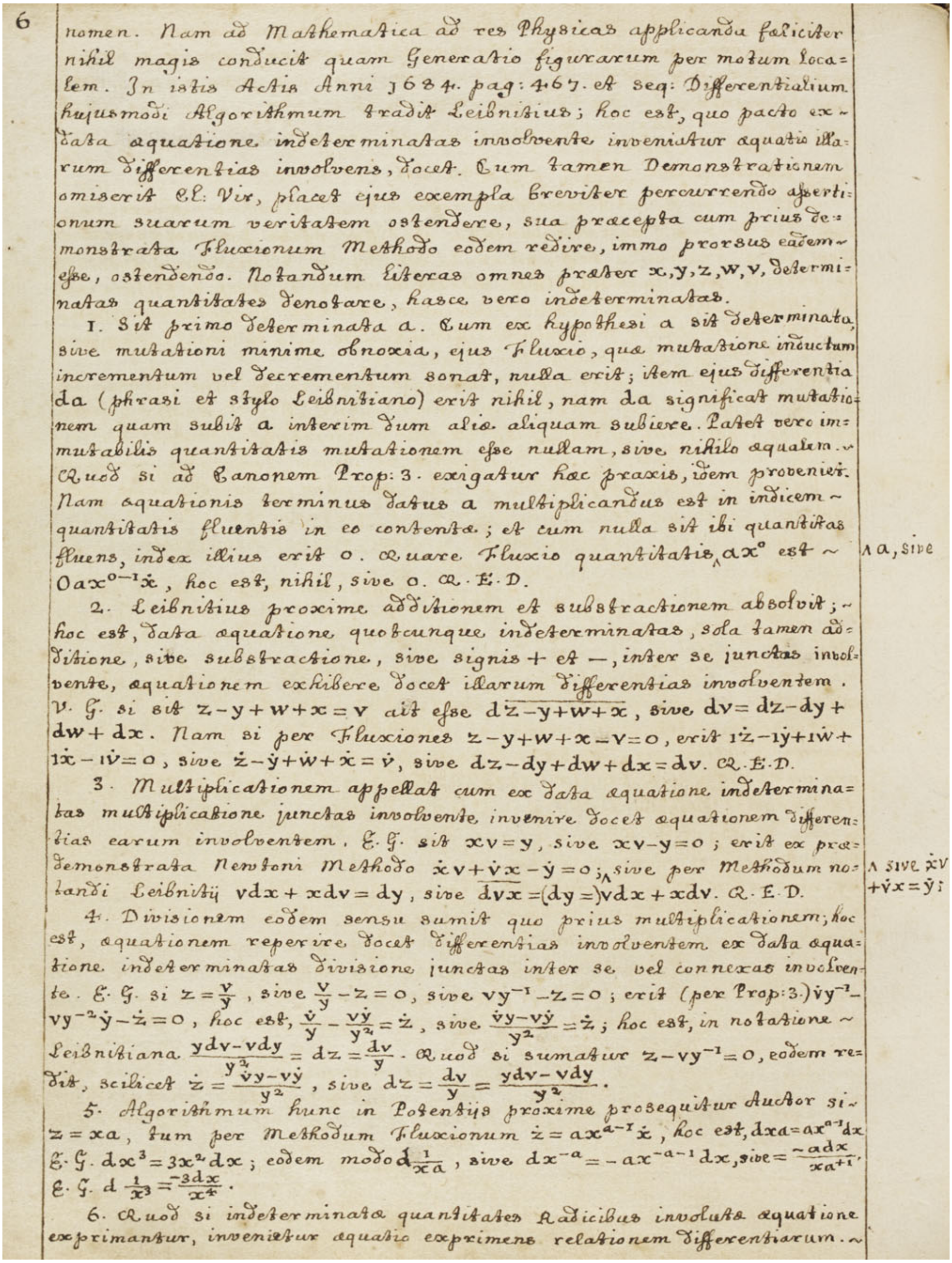
1 The manuscript is paginated from p. 1 and foliated from f. 132. Dates occur on p. 12 ('Oxoniæ 23 October 1694'), p. 16 ('Oxoniæ October 29 1694'), p. 20 ('7 November 1694 Oxoniæ') and p. 24 ('Oxoniæ die S. Ceciliæ 1694', i.e. 22 November 1694).

2 The four items are bound in reversed calf over pasteboards and are written on quires of six and eight folios measuring 231×185 mm. The detailed description available from the Bodleian Digital Library explains that MS 131 'was bequeathed to Christ Church Library with many other volumes by Gregory's son, David Gregory (1696–1767), first Professor of Modern History in the University of Oxford, and Dean of Christ Church from 1756–1767)'. See https://digital.bodleian.ox.ac.uk/objects/9f161887-ec8e-40fe-b417-c11e5845d91d/surfaces/e29e96f5-05ab-4846-8909-e1bbf79758a9/ (accessed 8 February 2021).

3 As explained in ibid., 'A note written vertically near the fore-edge of the inside of the upper board and signed by David Gregory explains its origins: "This book was written by James Canarys Doctor in Divinity, first Parson of Selkirk in Scotland, afterwards Vicar of Abingdon in Berkshire in England, in the year MDCXCV."'

4 Both MS Add. 9597/9/3 and MS Add. 9597/9/4 are written on paper bearing the Arms of Parker. The hand in MS Add. 9597/9/4 has some resemblance with George Parker's. On fol. 1r, one might note a funny mistake ('Prop 1 Prop' for 'Prop 1 Prob'), some corrections of the Latin, the use of ruling, and the rigidity of the hand as features that suggest that the scribe, certainly not a professional, was a student.

5 The copy is undated, unfortunately. Further work, possible only by a close study of watermarks, might help in approximately dating it. However, MS QA 33G8/D12 seems to be penned mostly, except the final tables, on a notebook bought by a stationer, which makes the dating even more difficult. It is likely that it was produced before 1704, when the publication of Newton's *Tractatus de quadratura curvarum* (in [Isaac Newton], *Opticks: Or, A Treatise of the Reflexions, Refractions, Inflexions and Colours of Light, Also Two Treatises of the Species and Magnitude of Curvilinear Figures*, pp. 163–211 (Sam. Smith and Benj. Walford, London, 1704)) and other printed works on fluxions made Gregory's *Methodus fluxionum* less informative. For some early printed treatments of fluxions, see John Harris, *A New Short Treatise of Algebra: with the Geometrical Construction of Equations as far as the Fourth Power or Dimension. Together with a Specimen of the*



6 nomen. Nam ad Mathematica ad res Physicas applicanda feliciter nihil magis conducit quam Generatio figurarum per motum localem. In istis Actis Anni 1684. pag: 467. et seq: Differentialium hujusmodi Algorithmum tradit Leibnitius; hoc est, quo pacto ex data aequatione indeterminatas involvente inveniatur aequatio illarum differentias involvens, docet. Cum tamen Demonstrationem omiserit Cl: Vir, placet ejus exempla breviter percurrendo assertionum suarum veritatem ostendere, sua praecepta cum prius demonstrata Fluxionum Methodo eodem redire, immo prorsus eadem esse, ostendendo. Notandum literas omnes praeter x,y,z,w,v, determinatas quantitates denotare, hasce vero indeterminatas.

1. Sit primo determinata a. Cum ex hypothesi a sit determinata, sive mutationi minime obnoxia, ejus Fluxio, quae mutatione inductum incrementum vel decrementum sonat, nulla erit; item ejus differentia da (phrasi et stylo Leibnitiano) erit nihil, nam da significat mutationem quam subit a interim dum alia aliquam subiere. Patet vero immutabilis quantitatis mutationem esse nullam, sive nihilo aequalem. Quod si ad Canonem Prop: 3. exigatur hac praxis, idem provenier. Nam aequationis terminus datus a multiplicandus est in indicem quantitatis fluentis in eo contentae; et cum nulla sit ibi quantitas fluens, index illius erit 0. Quare Fluxio quantitatis ∧ ax^0 est $0ax^{0-1}\dot{x}$, hoc est, nihil, sive 0. Q.E.D. ∧ a, sive

2. Leibnitius proxime additionem et substractionem absolvit; hoc est, data aequatione quotcunque indeterminatas, sola tamen additione, sive substractione, sive signis + et —, inter se junctas involvente, aequationem exhibere docet illarum differentias involventem. v.g. si sit $z-y+w+x=v$ ait esse $d\overline{z-y+w+x}$, sive $dv=dz-dy+dw+dx$. Nam si per Fluxiones $z-y+w+x-v=0$, erit $1\dot{z}-1\dot{y}+1\dot{w}+1\dot{x}-1\dot{v}=0$, sive $\dot{z}-\dot{y}+\dot{w}+\dot{x}=\dot{v}$, sive $dz-dy+dw+dx=dv$. Q.E.D.

3. Multiplicationem appellat cum ex data aequatione indeterminatas multiplicatione junctas involvente, invenire docet aequationem differentias earum involventem. E.g. sit $xv=y$, sive $xv-y=0$; erit ex praedemonstrata Newtoni Methodo $\dot{x}v+\dot{v}x-\dot{y}=0$; ∧ sive per Methodum notandi Leibnitij $vdx+xdv=dy$, sive $d\overline{vx}=(dy=)\,vdx+xdv$. Q.E.D. ∧ sive $\dot{x}v+\dot{v}x=\dot{y}$;

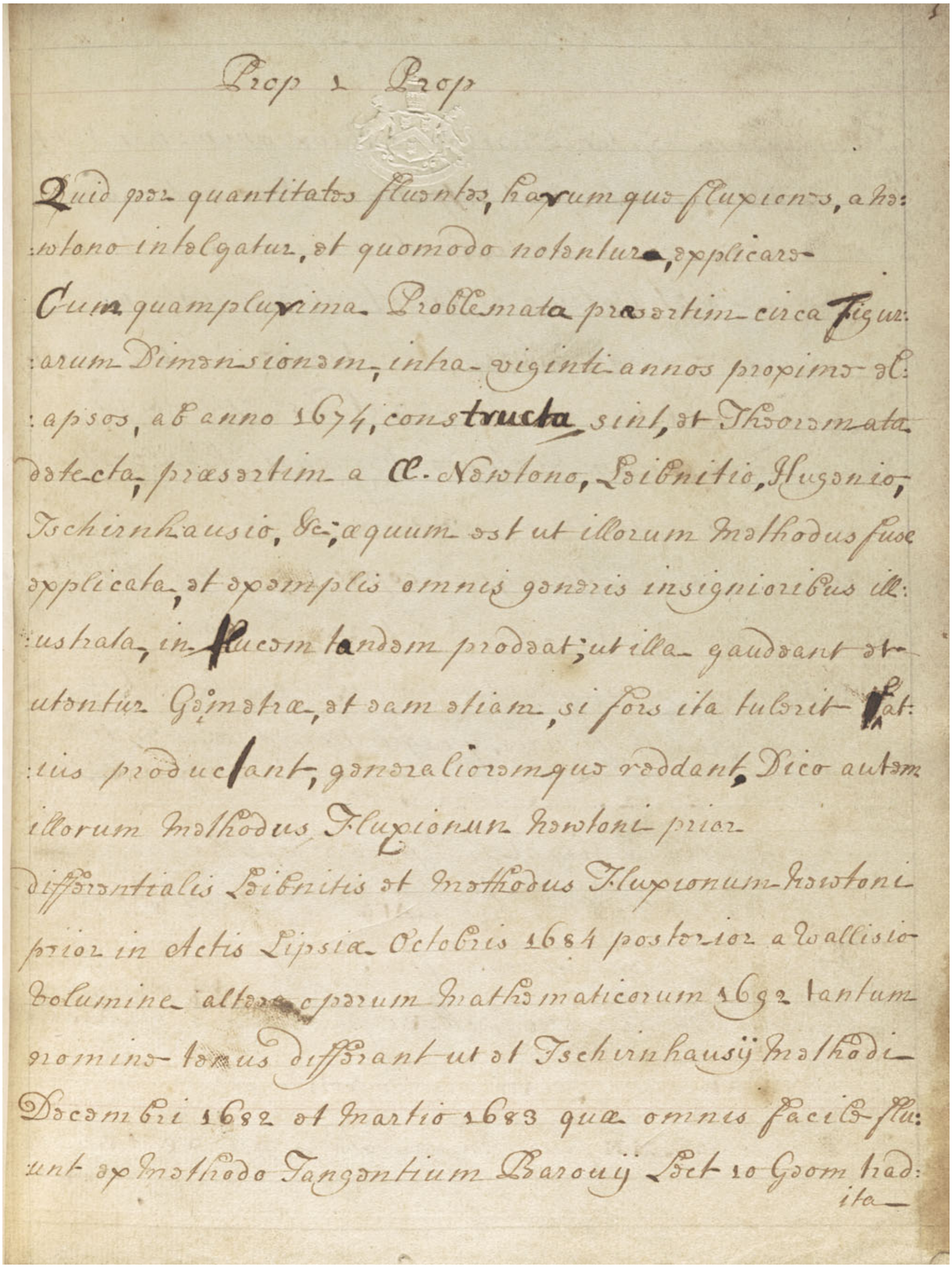
4. Divisionem eodem sensu sumit quo prius multiplicationem; hoc est, aequationem reperire docet differentias involventem ex data aequatione indeterminatas divisione junctas inter se vel connexas involvente. E.g. si $z=\frac{v}{y}$, sive $\frac{v}{y}-z=0$, sive $vy^{-1}-z=0$; erit (per Prop:3.) $\dot{v}y^{-1}-vy^{-2}\dot{y}-\dot{z}=0$, hoc est, $\frac{\dot{v}}{y}-\frac{v\dot{y}}{y^2}=\dot{z}$, sive $\frac{\dot{v}y-v\dot{y}}{y^2}=\dot{z}$; hoc est, in notatione Leibnitiana $\frac{ydv-vdy}{y^2}=dz=\frac{dv}{y}$. Quod si sumatur $z-vy^{-1}=0$, eodem redit, scilicet $\dot{z}=\frac{\dot{v}y-v\dot{y}}{y^2}$, sive $dz=\frac{dv}{y}=\frac{ydv-vdy}{y^2}$.

5. Algorithmum hunc in Potentijs proxime prosequitur Auctor si $z=x^a$, tum per Methodum Fluxionum $\dot{z}=ax^{a-1}\dot{x}$, hoc est, $dx^a=ax^{a-1}dx$. E.g. $dx^3=3x^2dx$; eodem modo $d\frac{1}{x^a}$, sive $dx^{-a}=-ax^{-a-1}dx$, sive $=\frac{-adx}{x^{a+1}}$. E.g. $d\frac{1}{x^3}=\frac{-3dx}{x^4}$.

6. Quod si indeterminatae quantitates Radicibus involutae aequatione exprimantur, invenietur aequatio exprimens relationem differentiarum.

Figure 2. The rules of the differential calculus demonstrated in terms of Newton's method and notation. (Christ Church Library, Oxford, MS 131, f. 195v (p. 6). Reproduced by kind permission of the Governing Body of Christ Church, Oxford.) (Online version in colour.)

Nature and Algorithm of Fluxions (J. M. for D. Midwinter and T. Leigh, London, 1702); George Cheyne, *Fluxionum Methodus Inversa: sive Quantitatum Fluentium Leges Generaliores* (J. Matthew, sold by R. Smith, London, 1703); Charles Hayes, *A Treatise of Fluxions: or, an*



Prop 1 Prop

Quid per quantitates fluentes, harumque fluxiones, a Ne:
:wtono intelligatur, et quomodo notentur, explicare.

Cum quamplurima Problemata praesertim circa Figur:
:arum Dimensionem, intra viginti annos proxime el:
:apsos, ab anno 1674, constructa sint, et Theoremata
detecta, praesertim a D. Newtono, Leibnitio, Hugenio,
Tschirnhausio, &c; aequum est ut illorum Methodus fuse
explicata, et exemplis omnis generis insignioribus ill:
:ustrata, in lucem tandem prodeat; ut illa gaudeant et
utentur Geometrae, et eam etiam, si fors ita tulerit, lat:
:ius producant, generalioremque reddant. Dico autem
illorum Methodus Fluxionum Newtoni prior
differentialis Leibnitis et Methodus Fluxionum Newtoni
prior in Actis Lipsiae Octobris 1684 posterior a Wallisio
volumine altero operum Mathematicorum 1692 tantum
nomine tenus differant ut et Tschirnhausii Methodi
Decembri 1682 et Martio 1683 quae omnes facile flu:
:unt ex Methodo Tangentium Barowii Lect 10 Geom hac:
ita—

Figure 3. Transcription of Proposition 1. Gregory cites the works by Leibniz, Tschirnhaus, Barrow and Newton. Formerly in the collection of the Earl of Macclesfield. (Cambridge University Library, Cambridge, MS Add. 9597/9/4, f. 1r. Reproduced by kind permission of the Syndics of Cambridge University Library.) (Online version in colour.)

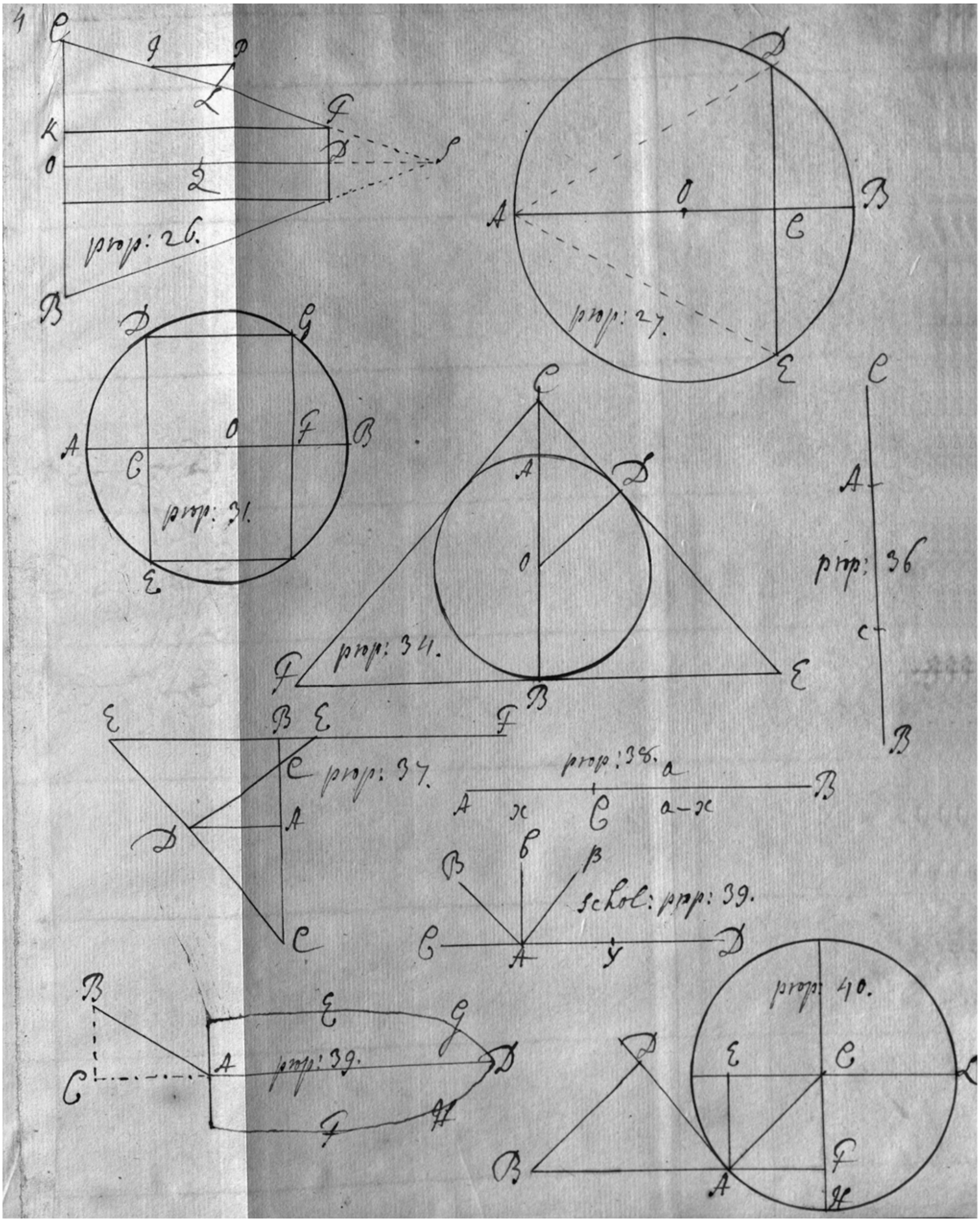


Figure 4. Folded plate (no. 4) of a scribal copy of David Gregory's *Methodus fluxionum*. Note the diagram for Proposition 26 on the cone frustrum of least resistance and the diagram for Proposition 39 on the optimal position of the rudder of a ship. (University of St Andrews Library, MS QA 33G8/D12. Courtesy of the University of St Andrews Library.)

Introduction to Mathematical Philosophy; Containing a Full Explication of that Method by Which the Most Celebrated Geometers of the Present Age Have Made Such Vast Advances in Mechanical Philosophy. A Work Very Useful for Those That Would Know How to Apply Mathematicks to Nature (E. Midwinter, for D. Midwinter and T. Leigh, London, 1704); William Jones, *Synopsis Palmariorum Matheseos; Or, a New Introduction to the Mathematics: Containing the Principles of Arithmetic & Geometry Demonstrated, in a Short and Easie* this point considered. But our understanding of the practices of production and circulation of mathematical manuscripts is still too tentative to allow this conclusion to be drawn. This way of presenting the diagrams in the manuscript might simply represent a further stage of scribal publication.

David Gregory, editor of James Gregorie and Isaac Newton

David Gregory was born in Upper Kirkgate, Aberdeen, on 3 June 1659. His was a family of Episcopalians which was steeped in the tradition of medicine and mathematics (not least through its relationship with the Andersons).[6] He was the nephew of one of the greatest mathematicians of the seventeenth century, James Gregorie (1638–1675).[7] After his move to England in 1691 David became one of Newton's closest mathematical correspondents. After studying at Marischal College, enrolling in it—as was customary in Scotland—at the very young age of 12, in 1675 he moved to the family estate of Kinnairdy, where he found his uncle's papers and correspondence. It was by reading the family archives that he became a proficient mathematician. Most notably, Gregory familiarized himself with cutting-edge research on quadratures via infinite series. In 1683, he was elected Professor of Mathematics at Edinburgh, the chair once occupied by his uncle. In the meanwhile, he had established contacts with a group of Scottish physicians and mathematicians which included Archibald Pitcairne, George Cheyne, John Craig, John Keill and Colin Campbell.[8]

Gregory's first publication, the *Exercitatio Geometrica de Dimensione Figurarum* (1684), is a treatise on the 'dimension of figures': that is, on the calculation of the areas and volumes of curvilinear figures, based on his uncle's discoveries, especially on the series expansion of

Method, with Their Applications to the Most Useful Parts Thereof… Design'd for the Benefit, and Adapted to the Capacities of Beginners (J. Matthews, for J. Wale, London, 1706).

6 'James's mother is reckoned to be a "close" (how close, it cannot be said) relative of the mathematician, Alexander Anderson (ca. 1582–after 1619), a student of Viète and editor of some of his posthumous works. Tradition has it that Janet taught James his first mathematical notions.' Antoni Malet, 'Studies on James Gregorie (1638–1675)', PhD thesis, Princeton University (1989), p. 17.

7 David Anglicized the spelling of his surname after moving to Oxford in 1691.

8 Archibald Pitcairne (1652–1713), Professor of Medicine at the University of Edinburgh, was a leading iatro-mechanical physician who had taught briefly but influentially at Leiden in the early 1690s. Born into an Episcopalian family in Aberdeenshire, George Cheyne (1671/2?–1743) distinguished himself as one of the staunchest defenders of Pitcairne in the latter's dispute with the Edinburgh College of Physicians over the nature of fevers. The Scottish iatro-mechanists' aim was to apply a 'Newtonian' matter theory, based on attractive and repulsive forces, to medicine. Cheyne had a brief career as a mathematician in the early 1700s (see Cheyne, *op. cit.* (note 5)) before devoting himself entirely to medicine and theology. John Craig (1663–1731) was a Scottish mathematician who was deeply influenced by Continental calculus. In 1685 he published a short treatise, written in Leibniz's notation, on the quadrature of curvilinear figures, the *Methodus Figurarum Lineis Rectis & Curvis Comprehensarum Quadraturas Determinandi* (M. Pitt, London, 1685). He continued to use the differential and integral notation in another treatise published in 1693, the *Tractatus Mathematicus de Figurarum Curvilinearum Quadraturis et Locis Geometricis* (Sam. Smith & Benj. Walford, London, 1693), and in papers published in the *Philosophical Transactions* up until 1708. John Keill (1671–1721), a friend of Gregory, moved from Scotland to Oxford in 1694 and began to lecture on Newtonian philosophy. In 1699 he was employed by Thomas Millington as deputy Professor of Natural Philosophy. His course was published as *Introductio ad Veram Physicam: seu Lectiones Physicae, Habitae in Schola Naturalis Philosophiae Academiae Oxoniensis, Quibus accedunt C.Hugenii theoremata de vi centrifuga et motu circulari demonstrata* (e Theatro Sheldoniano, Oxford, 1702). In 1712 he was elected Savilian Professor of Astronomy and lectured regularly until his death. Colin Campbell (1644–1728), who earned his living as minister of Ardchattan on Scotland's west coast, was highly regarded as a mathematician. He was consulted by Pitcairne, Craig and Gregory on mathematical matters related to the calculus and Newton's *Principia*. On Campbell's mathematical correspondence, see Philip Beeley, '"There are great alterations in the geometry of late": the rise of Isaac Newton's early Scottish circle', *Brit. J. Hist. Math.* **35**, 3–24 (2020), at pp. 12–14 (https://doi.org/10.1080/26375451.2019.1701862).

$y = (1 \pm x^2)^{(\pm 1/2)}$.[9] The fact that most of the results in the *Exercitatio* were due to James Gregorie should not induce one to think that David's was a trivial achievement. Reading through James's papers and correspondence and making sense of a brand-new mathematical topic was no mean feat, and both Gregory and his readers certainly understood this well enough. After all, commenting on the (mostly manuscript) output of an innovative thinker is something that allowed many scholars—such as Descartes's and Viète's mathematical editor Frans van Schooten in the United Provinces—to reach a more than respectable position in the Republic of Letters.[10] The fact that the *Exercitatio* showcased James Gregorie's mathematical discoveries was of course a very apt way to celebrate the prestige of the newly acquired chair in Edinburgh.

When the book reached Newton in Cambridge it elicited a less than cordial reaction. According to the extant letters, the Lucasian Professor of Mathematics did not even reply to the author, who had sent him a complimentary copy.[11] Rather, Newton set himself the task of writing a work—never published—for the purpose of claiming his priority and superiority over James Gregorie in matters concerning infinite series.[12] Indeed, as is apparent from the extant exchanges between John Collins, James Gregorie and the young Newton in the early 1670s (letters to which David had access and which he had used in the *Exercitatio*), the Scottish mathematician had obtained many results contained in Newton's short treatise *De analysi per æquationes numero termonorum infinitas* that in July 1669 Isaac Barrow had communicated to John Collins. Gregory's relationship with Newton, then, did not begin in a very auspicious way.

Gregory was one of the first to appreciate the importance of Newton's *Principia*. On 2 September 1687, he wrote an enthusiastic letter to the author.[13] There is evidence that he introduced some of his students to gravitation theory, even though he did not include Newtonian science in his lectures.[14] He also began drafting a detailed commentary on the *Principia*, entitled 'Notæ in Newtoni Principia mathematica philosophiæ naturalis', on which he continued to work until the end of his life.[15] At some point he cherished the idea of acting as editor of a second edition of the *magnum opus*, since it was known that, as early as the 1690s, Newton was thinking of publishing a revised edition. It was then that Gregory shifted from being a disseminator of his uncle's mathematical discoveries to

9 David Gregory, *Exercitatio Geometrica de Dimensione Figurarum: sive Specimen Methodi Generalis Dimetiendi Quasvis Figuras* (James Kniblo, Joshua van Solingen, and John Colmar, Edinburgh, 1684).

10 Frans van Schooten (1615–1660) was Professor of Mathematics at the University of Leiden. A friend and correspondent of Descartes, he established a school which included Johannes Hudde, Johan de Witt, Henrik van Heuraet and Christiaan Huygens. His annotated Latin translation of Descartes's *Géométrie* was highly influential. Van Schooten was not only a mathematical editor and teacher but also a very creative mathematician.

11 'Sr I perceive by severall letters from Mr Collins to my Uncle, from whose remains this is for ye most parte taken, that your selfe have of a long time cultivate this methode, and that ye world have long expected your discoveries therein.' David Gregory, Letter to Isaac Newton, 9 June 1684, in H. W. Turnbull, J. F. Scott, A. Rupert Hall and Laura Tilling (eds), *The correspondence of Isaac Newton*, 7 vols (Cambridge University Press, Cambridge, 1959–1977), vol. 2, p. 396.

12 See 'Matheseos universalis specimina' and 'De computo serierum', MS Add. 3964.3, fols 7r-20v, MS Add. 9597/2/6 and MS Add. 9597/2/7, Cambridge University Library, Cambridge (hereafter CUL), transcribed in D. T. Whiteside (ed.), *The mathematical papers of Isaac Newton*, 8 vols (Cambridge University Press, Cambridge, 1967–1981), vol. 4, pp. 526–45, 590–605.

13 Turnbull et al., *op. cit.* (note 11), vol. 2, p. 484.

14 Christina M. Eagles, 'David Gregory and Newtonian science', *Brit. J. Hist. Sci.* **10**, 216–25 (1977) (https://doi.org/10.1017/S0007087400015661).

15 The autograph is MS 210, Royal Society Library, London (hereafter RSL). Three further copies are known: MS 131, ff. 1–155, Christ Church Library, Oxford (hereafter CCL); MS Dc.4.35, Edinburgh University Library, Edinburgh (hereafter EUL); and MS 465, Aberdeen University Library, Aberdeen. On Gregory's career in Edinburgh, see Christina M. Eagles, 'The mathematical work of David Gregory, 1659–1708', PhD thesis, University of Edinburgh (1977).

being a would-be editor of Newtonian mathematical works.[16] Indeed, he based his most important work, the *Astronomiæ Physicæ & Geometricæ Elementa* (1702), on Newtonian science.[17] The *Astronomiæ* was very much part of Gregory's publication plan for Newtoniana, as he included in this work a version of the so-called 'Classical Scholia'—the ruminations on the *prisca sapientia* that Newton planned to include in a second edition of the *Principia*—and a Latin translation of Newton's *Theory of the Moon*.[18] By then Gregory had moved to Oxford as Savilian Professor of Astronomy, a position that he gained in December 1691, partly thanks to Newton's recommendation, probably due to the Lucasian professor's desire to secure a Newtonian outpost in Oxford—as Philip Beeley suggests.[19] Gregory's move was for political reasons and he was followed by a group of Episcopalians who, after 1689, found a more hospitable climate in the English university. He remained in Oxford until the end of his life in 1708. He was the main representative of a Scottish diaspora whose members pursued iatro-mechanics, in the spirit of Pitcairne, and mathematical natural philosophy in the wake of Newton. They were dubbed the 'Tory Newtonians' in a seminal paper that Anita Guerrini published 35 years ago.[20] This group held some political and religious views in common, were united at least in part by kinship, and shared the common fate of having felt compelled to leave Presbyterian Scotland. For the Tory Newtonians, mathematized Newtonianism and medicine were deeply intertwined enterprises: for example, George Cheyne pursued them both, while the brothers James and John Keill devoted themselves to iatro-mechanics and to Newtonian natural philosophy and astronomy respectively.[21]

Gregory discussed the possibility of using the 'Notæ' as a commentary for a second edition of the *Principia* with Newton himself, when they first met in Cambridge in early May 1694.[22]

16 It is interesting that in the 'Notæ' Gregory follows the editorial conventions of Frans van Schooten's commentary in his annotated Latin translation (1649 and 1659–1661) of Descartes' *Géométrie* (1637). See *Geometria à Renato Des Cartes Anno 1637 Gallicè Edita; Nunc Autem cum Notis Florimondi de Beaune in Curiâ Blæsensi Consiliarii Regii, in Linguam Latinam Versa, & Commentariis Illustrata, Operâ atque Studio Francisci à Schooten, Leydensis, in Academia? Lugduno-Batava?, Matheseos Professoris, Belgicè Docentis* (Maire, Leiden, 1649) and *Geometria à Renato Des Cartes Anno 1637 Gallicè Edita, Postea Autem Unà cum Notis Florimondi de Beaune … in Latinam Linguam Versa, & Commentariis Illustrata*, 2 vols (Ludovicum & Danielem Elzevirios, Amsterdam, 1659). It is fascinating to note that two heavily annotated copies of the first edition of the *Principia,* currently housed at the University of Glasgow Library (Sp Coll Hunterian Ec.1.15 and Sp Coll Ea7-b.10) circulated in Pitcairne's circle. A study of the hands and of the annotations would shed much light upon the early reception of the *Principia* in Scotland, since they seem to have been annotated when both Pitcairne and Gregory were active in Edinburgh. The copy at Moscow University Library (4.E.n.1) is likely to have annotations in Gregory's hand. See Vladimir S. Kirsanov, 'The earliest copy in Russia of Newton's *Principia*: is it David Gregory's annotated copy?', *Notes Rec. R. Soc. Lond.* **46**, 203–218 (1992) (https://doi.org/10.1098/rsnr.1992.0022). See Mordechai Feingold and Andrej Svorenčík, 'A preliminary census of copies of the first edition of Newton's *Principia* (1687)', *Ann. Sci.* **77**, 253–348 (2020), at p. 263 (https://doi.org/10.1080/00033790.2020.1808700).

17 David Gregory, *Astronomiæ Physicæ & Geometricæ Elementa* (e Theatro Sheldoniano, Oxford, 1702).

18 See Paolo Casini, 'Newton, the classical scholia', *Hist. Sci.* **22**, 1–58 (1984) (https://doi.org/10.1177/007327538402200101); and I. B. Cohen (ed.), *Isaac Newton's 'Theory of the Moon's motion' (1702): with a bibliographical and historical introduction* (Wm. Dawson and Sons, Folkestone, 1975).

19 See Beeley, *op. cit.* (note 8), p. 14.

20 Anita Guerrini, 'The Tory Newtonians: Gregory, Pitcairne, and their circle', *J. Brit. Stud.* **26**, 288–311 (1986) (https://doi.org/10.1086/385866). See also John Friesen, 'Archibald Pitcairne, David Gregory and the Scottish origins of English Tory Newtonianism, 1688–1715', *Hist. Sci.* **41**, 163–191 (2003) (https://doi.org/10.1177/007327530304100203).

21 On the iatro-mechanical agenda of the Tory Newtonians, see Anita Guerrini, 'Archibald Pitcairne and Newtonian medicine', *Med. Hist.* **31**, 70–83 (1987) (https://doi.org/10.1017/S0025727300046329).

22 'If my Notes on the Newtonian Philosophy are published (as indeed I heartily wish and expect, and made the proposal to the author himself on 8 May 1694) a great deal that serves to detect slips or even mistakes of Newton and is contained in my notes is to be omitted if a new edition of that work is made by the author: otherwise the notes are to be inserted in their proper places.' Translation from Latin by H. W. Turnbull in Turnbull et al., *op. cit.* (note 11), vol. 3, p. 386. Turnbull translates from a memorandum by Gregory: 'In editione nova Philos: Newtoniana hæc ab Auctore fient, May 1694', MS Dc.1.61, Folio C [42], EUL.

This five-day-long meeting had momentous consequences for Gregory. For some reasons, Newton gave him free access to his archive, showing the Savilian professor some of his most important scientific and theological manuscripts. In a way, Newton's openness towards Gregory is unexpected since the two had another reason to be on unfriendly terms, in addition to the priority issue raised by the publication of the *Exercitatio*. In 1688 Pitcairne had published a theorem on quadratures, attributing it to David Gregory, in a book entitled *Solutio Problematis de Historicis, seu Inventoribus*.[23] On 7 November 1691, just weeks before his election as Savilian Professor of Astronomy at Oxford, Gregory wrote a letter to Newton, dated from London, in which he tried obliquely to assert his independence in the discovery of this important result.[24] As a matter of fact, this theorem had been communicated to Leibniz in Newton's so-called *epistola posterior*, dated 24 October 1676.[25] The theorem had later been passed to Craig, who had visited Newton in his rooms at Trinity in 1685. It seems likely that it was through Craig that this rule for the quadrature of curves came into Gregory's hands, as Newton politely, yet firmly, replied to Gregory: 'your fellow-countryman Craig also, when he stayed with us [in Cambridge] for quite a long time six years ago, examined my manuscripts' and 'then he sent to you my squaring of that curve'.[26] However, one should not exclude the possibility that Gregory indeed achieved this result by relying on the forces at his own disposal, or after receiving just a few hints.

Be that as it may, on the occasion of Gregory's visit in May 1694, Newton put all dissent aside and revealed his well-guarded mathematical manuscripts on geometry, fluxions and the *Principia* to the Savilian professor. It was during this meeting that Gregory acquired most of the information that allowed him to devise the plan to write a short treatise on the still unpublished method of fluxions.

The extraordinary openness of Newton towards Gregory is, I must confess, not wholly clear to me. On the occasion of Gregory's visit, Newton not only opened up his mathematical archive covering fluxions, quadratures, optics and the *Principia*, but he let Gregory read and annotate his manuscripts on the *prisca sapientia* and religion too. One might surmise that Newton was particularly well disposed since Gregory's visit was solicited by Nicolas Fatio de Duillier, as Christina Eagles convincingly suggests.[27] Maybe, when Gregory visited Cambridge, he knocked on the door of the Lucasian professor, fashioning himself as a member of Newton's close network of trustworthy acolytes. Yet, at this juncture I must add that I cannot provide a convincing explanation. Further work is necessary in order to clarify this important aspect of Newton's life. Indeed, Gregory's memoranda are the main source of information at our disposal on Newton's work in the period 1694–1708.

23 Archibald Pitcairne, *Solutio Problematis de Historicis, seu Inventoribus* (J. Reid, Edinburgh, 1688).

24 Gregory, Letter to Newton, 7 November 1691, in Turnbull et al., *op. cit.* (note 11), vol. 3, pp. 172–176.

25 Isaac Newton, Letter to Henry Oldenburg, 24 October 1676, in *ibid.*, vol. 2, pp. 110–129. The so-called *theorema primum* is at pp. 115–117.

26 *Ibid.*, vol. 3, pp. 181–182. Translation by H. W. Turnbull at p. 183. The whole affair between Newton, Craig and Gregory is reconstructed by D. T. Whiteside in Whiteside, *op. cit.* (note 12), vol. 7, pp. 3–13. Gregory's method of quadrature was also printed in John Wallis, *Opera Mathematica*, 3 vols (e Theatro Sheldoniano, Oxford, 1693–1699), vol. 2, pp. 377–380. Newton's version is corroborated by Craig who, in 1718, detailed how he passed Newton's theorem to Gregory. See 'Præfatio ad lectorem', in John Craig, *De Calculo Fluentium Libri Duo: Quibus Subjunguntur Libri Duo de Optica Analytica* (Pearsons, London, 1718), sig. b3–b4.

27 Eagles, *op. cit.* (note 15), at pp. 398–399.

Gregory's *Methodus fluxionum*

Planning a tract on Newton's method of fluxions

After his momentous encounter with Newton in May 1694, Gregory wrote extensive memoranda in which he recorded what had been revealed to him about the Lucasian professor's intellectual output. The memoranda, which have only been partially published, are a treasure trove of information about Newton's ideas on scientific, mathematical and religious subjects.[28] According to Tom Whiteside's careful study of the mathematical memoranda[29], in May Gregory was shown parts of Newton's early treatise on fluxions known as *De methodis serierum et fluxionum*, [30] the masterpiece that was begun in 1670 and that was printed only in 1736 in John Colson's English translation.[31] Gregory also had the opportunity to look at, among many other things, Newton's working notes related to the demonstration of some propositions of the *Principia*. Thus, the memoranda provide an invaluable source of information about the methods employed by Newton in writing his *magnum opus*. From a memorandum dated 7 September 1694, we learn that by then Gregory was already planning to write the *Methodus fluxionum*. The memorandum is entitled 'Describenda et Chartis consignanda Mense Septembri MDCXCIV' and was listed in Gregory's later manuscript catalogue as 'Adumbratio nostræ de fluxionibus methodi'.[32]

Gregory's plan occupies one page and consists of a list of 11 points: it appears that in the composition of the *Methodus fluxionum*, which, as we know, took place in October and November, Gregory followed closely the project sketched in early September. Indeed, the first point clearly summarizes the content and purpose of the *Methodus*: '1. Newton's method of fluxions, as it is contained in Wallis's works is to be written out and illustrated more fully, followed by what he [Newton] understands by Fluxions, in its broadest sense.'[33] As is well known, up until then very little of the method of fluxions had been printed or circulated. Therefore, Gregory certainly appreciated the opportunity that his recently acquired access to Newton's personal mathematical archive offered him to be the first to 'write out' (*describere*) and 'illustrate' the new method. The method's notation and terminology had been recently and succinctly presented—as Gregory notes in the above quotation—in six folio pages of the second volume of Wallis's *Opera* (1693), while the binomial theorem and excerpts from Newton's 1676 letters to Henry Oldenburg for

28 A complete edition of the memoranda by Gregory, kept in the libraries of Christ Church, Oxford, Edinburgh University Library, and the Royal Society Library, is lacking. Some excerpts can be found in W. G. Hiscock (ed.), *David Gregory, Isaac Newton and their circle: extracts from David Gregory's memoranda 1677–1708* (printed for the editor, Oxford, 1937); and in Turnbull et al., *op. cit.* (note 11), vols 3 and 4.

29 Whiteside carefully reads the 'Adnotata Math: ex Neutono. 1694. Maio' (MS 247, Royal Society Library, London, p. 68) and the *memorandum*, datable to July 1694, MS Dc.1.61, Folio C [42], Edinburgh University Library, Edinburgh, translated into English by H. W. Turnbull in *op. cit.* (note 11), vol. 3, pp. 384-6. See, Whiteside's comments in *op. cit.* (note 12), vol. 7, p. 197.

30 MS Add. 3960.14, CUL.

31 Isaac Newton, *The Method of Fluxions and Infinite Series*, trans. and annotated by John Colson (H. Woodfall for J. Nourse, London, 1736).

32 MS 247, p. 64, RSL, edited in Turnbull et al., *op. cit.* (note 11), vol. 4, pp. 15–16. From the boxed number 79, it appears that this was originally part of Folio C of the Gregory papers in Edinburgh.

33 'Methodus fluxionum Newtoni prout in Wallisij operibus continetur plenius describenda et illustranda, deinde quid ille per Fluxiones late sumptas intelligat'. MS 247, p. 64, RSL.

Leibniz had already been included in the English *Algebra* (1685).[34] In the 1670s John Collins had circulated some of Newton's mathematical discoveries via correspondence, as did Nicolas Fatio de Duillier in the early 1690s.[35] Yet, in 1694 nobody, with the possible exception of Fatio and Wallis, had such an extensive knowledge and competence in Newtonian fluxions as Gregory. The newly appointed Savilian Professor of Astronomy clearly understood that what he had seen in May was of utmost importance. He set to work with alacrity and in about a couple of months, most probably by November 1694, he completed his planned treatise on the method of fluxions, to which we now turn.

Contents and sources of the Methodus fluxionum

The title of the *Methodus fluxionum* makes it clear that Gregory's intention was to present not only Newton's method of fluxions, but also Leibniz's differential calculus and Barrow's method of tangents, as provided in Lecture X of the *Lectiones geometricæ* (1670). Indeed, as we shall see, this short treatise is not simply a transcription of the papers on fluxions by Newton that Gregory had the opportunity to study in May 1694. Gregory aimed to offer a treatment of Newton's method and its relationship with previous and contemporary works, including the one by his uncle and by David himself. He relied upon a variety of sources that included both British and Continental work.

As can be inferred from a note datable to around 1685, while still in Edinburgh, Gregory had read excerpts from Newton's *De methodis serierum et fluxionum* from annotations brought to Scotland by Craig.[36] In 1694, of course, he became acquainted with Newton's dotted notation and the methods of quadrature published in Wallis's Latin *Algebra* (1693). It is also known that from the mid-1680s Gregory began a close reading of the papers on calculus published in the *Acta Eruditorum*: he was particularly interested in the work presented there by Ehrenfried Walther von Tschirnhaus, Leibniz and Jacob Bernoulli.[37] Not surprisingly, Gregory's sources were very similar to the ones used by Craig in his treatises from 1685 and 1693.[38] The memoranda kept at Christ Church also provide evidence of Gregory's study of the results on calculus and algebra published by the Continental mathematicians in the *Acta*.[39] Thus, when Gregory set himself the task of writing the *Methodus fluxionum*, he had already spent about eight years digesting the most

34 Wallis included paraphrases from the two famous 1676 *epistolæ* to Leibniz in his English (1685) and Latin (1693) *Algebra*. Here the reader could acquire some knowledge of Newton's methods of series and fluxions. Most notably, new material provided by Newton himself, which described the dotted notation for fluxions and some methods for the 'squaring' of curves, was included in Wallis, *op. cit.* (note 26), vol. 2 (1693), pp. 390–396. For details, see Niccolò Guicciardini, 'John Wallis as editor of Newton's mathematical work', *Notes Rec. R. Soc. Lond.* **66**, 3–17 (2012) (https://doi.org/10.1098/rsnr.2011.0051).

35 The classic reference is A. Rupert Hall, *Philosophers at war: the quarrel between Newton and Leibniz* (Cambridge University Press, Cambridge, 1980).

36 MS Dk.1.2, Quarto A [56(1)], EUL.

37 Gregory's first encounter with the works by Tschirnhaus and Leibniz is documented, as Philip Beeley has shown, in his correspondence with Colin Campbell dated to between 1685 and 1687. See Beeley, *op. cit.* (note 8), pp. 6–8. Ehrenfried Walther von Tschirnhaus (1651–1708) was a mathematician, physician and philosopher. He was a correspondent of Leibniz and contributed to the discovery of the calculus. As a mathematician he is known for the Tschirnhaus transformation. Jacob Bernoulli (1655–1705) was an eminent mathematician based in Basel. With his younger brother Johann (1667–1748), he was one of the first to improve and apply Leibniz's calculus.

38 Craig, *op. cit.* (note 8).

39 MS 346, esp. pp. 1–86, CCL. See also, MS Dk.1.2, Quarto A [81] and Quarto A [91] (notes on Leibniz's method for maxima and minima); MS Dc.1.61, Folio C [32] (notes on Tschirnhaus's method for maxima and minima); and MS Dc.1.61, Folio C [35] (a reading of some papers on calculus from the *Acta* produced between 1681 and 1692), EUL.

up-to-date mathematical methods, which he explained by way of exemplary 'problems' in 47 propositions.

The first proposition is purely definitory. It reads: 'Prop. I. Problem: To explain what Newton understands by fluent quantities, and their fluxions, and how they are denoted.'[40] In the lines immediately following the above statement, Gregory claims that after 1674 several problems and theorems concerning the 'dimension of figures' had been found by 'Newton, Leibniz, Huygens and Tschirnhaus', and that his aim is to explain their methods by way of examples, so that other mathematicians can apply them to more cases and make them more general.[41] Far from being a presentation of Newton's fluxions, Gregory's treatise is framed as a comparison between different notations and methods devised by mathematicians active in the preceding 20 years.

Gregory opens the *Methodus fluxionum* by assigning priority to Barrow's 'method of tangents' and by stating the equivalence between Leibniz's 'calculus' and Newton's 'method'. The 'former first published in the *Acta* in 1684, the latter in Wallis's second volume of the *Opera* in 1692 [*sic*] … slightly differ only by their name'.[42] Newton's method, according to Gregory, is not the first, since it dates from 1664, as it appears from an annotation by Newton.[43] All existing methods 'easily follow' from Barrow's method of tangents published in the tenth lecture of the *Lectiones geometricæ*.[44] Gregory rightly assumes that Barrow's method predates Newton's early work on fluxions.[45] A few pages below, Gregory reports the evaluation by Jacob Bernoulli published in the *Acta Eruditorum* in 1691, according to which 'Leibniz's differential calculus is not different from Barrow's', if one excludes the notation and the 'compendious way' in which the operations are carried out.[46]

40 'Quid per Quantitates fluentes harumque fluxiones a Newtono intelligatur et quomodo notentur explicare.' MS 31011, p. 1, University of St Andrews Library, St Andrews (hereafter USAL).

41 The date 1674 was probably inferred from the correspondence between James Gregory and Collins: see, e.g., Collins's reference to Tschirnhaus's 'new methods for quadrature of curvilinear figures and straightening of Curves' in a letter dated 3 August 1675, in H. W. Turnbull (ed.), *James Gregory tercentenary memorial volume*, pp. 314–319 (Bell & Sons for the Royal Society of Edinburgh, London, 1939), at p. 315. The reference to Huygens is probably due to Gregory's personal encounter with the Dutch polymath in May and June 1693. See Steffen Ducheyne, 'Adriaen Verwer (1654/5–1717) and the first edition of Isaac Newton's *Principia* in the Dutch Republic', *Notes Rec. R. Soc. Lond.* **74**, 479–505 (2020) (https://doi.org/10.1098/rsnr.2019.0008), at p. 491, n. 108. Huygens was deeply interested in the new methods of quadrature and he discussed Newton's findings on this topic with Gregory. See R. Vermij and J. van Maanen, 'An unpublished autograph by Christaan Huygens: his letter to David Gregory of 19 January 1694', *Ann. Sci.* **49**, 507–523 (1992) (https://doi.org/10.1080/00033799200200431). John Collins had met Tschirnhaus in London in the summer of 1675. He informed James Gregory about their discussions in letters dated between 3 August and 21 September 1675. The works by Tschirnhaus that Gregory cites at the incipit of the *Methodus* are 'Inventa nova exhibita Parisiis Societati Regiæ Scientiarum', *Acta Eruditorum*, 364–365 (1682), and 'Methodus datæ figuræ, rectis lineis & Curva Geometrica terminatæ, aut Quadraturam, aut impossibilitatem ejusdem Quadraturæ determinandi', *Acta Eruditorum*, 433–437 (1683).

42 'since, at least according to my judgement, Leibniz's differential calculus and Newton's method of fluxions … slightly differ only by their name' ('cum meo saltem judicio, Calculus differentialis Leibnizij [et] Methodus fluxionum Newtoni … tantum nomine tenue differant'). MS 31011, p. 1, USAL.

43 The date 1664 probably derives from the fact that Gregory was shown an early manuscript dated by Newton. One might surmise that Gregory was shown either the Waste Book (MS Add. 4004, CUL) or the Mathematical Notebook (MS Add. 4000, CUL), in which such early dates can be found.

44 Isaac Barrow, *Lectiones Geometricæ: in Quibus (præsertim) Generalia Curvarum Linearum Symptomata Declarantur* (Typis Gulielmi Godbid & prostant venales apud Johannem Dunmore & Octavianum Pulleyn Juniorem, London, 1670).

45 On the dating of Barrow's method of tangents and its influence on Newton, see Mordechai Feingold, 'Newton, Leibniz, and Barrow too: an attempt at a reinterpretation', *Isis* **84**, 310–338 (1993) at pp. 316–317 (https://doi.org/10.1086/356464). A manuscript in Barrow's hand on the method of tangents is in MS Add. 9597/11/50, CUL.

46 'Sed et a J Bernoullio agnitum est Calculum Differentialem Leibnizij a Methodo Barroviana præterquam *in differentialium notatione et operationis aliquo compendio non differre* in Actis Lipsiæ Mens: Januarij MDCXCI. pag:14.' MS 31011, pp. 4–5,

Newton's method, however, has the advantage of being based on an 'easier demonstration': through it one can demonstrate not only Leibniz's calculus, which was published 'without a demonstration', but also Barrow's and Tschirnhaus's methods.[47] As will be clear from later propositions, Gregory also mentions René François de Sluse's method: in this case, too, he provides a demonstration in terms of the method of fluxions.[48]

Gregory often repeats that Newton's method is easier and more demonstrative compared to the others. Another advantage of the method of fluxions is that the Newtonian conception of magnitudes as generated by motion is more suitable to be applied to 'physical matters' than Leibniz's conception of the magnitudes as constituted by infinitesimal components.[49] Throughout the *Methodus fluxionum*, Gregory stresses that the two methods are equivalent. He concludes the first proposition with the basic definitions of fluent and fluxion and the distinctive dotted notation, which Newton had invented in the early 1690s. These first definitions and notations had been already published in Wallis's Latin *Algebra* (1693): the 'fluents' are 'indeterminates' which vary when geometrical 'figures' are generated by motions; 'fluxions' are the 'changes' which the fluents undergo 'in very small or minimal times'.[50] This definition of fluxion allows Gregory to add that the concept is equivalent to that of 'moment' as employed in his *Exercitatio geometrica*. Gregory continues his presentation of the method of fluxions with three propositions (numbers 2, 3 and 4) in which he shows how to determine the tangent to a plane curve and how to calculate first- and higher-order fluxions. The first four propositions thus introduce the reader to Newton's 'direct' method of fluxions: that is, the method that allows the calculation of the (ratio of the) fluxions of fluent quantities occurring in an equation (in Leibnizian terms, this is the 'differential calculus', opposed to its inverse, the 'integral calculus').[51]

As announced in the incipit of the *Methodus fluxionum*, Gregory proceeds to show how one can 'deduce and demonstrate' Barrow's method of tangents (Propositions 5, 9 and 10), Leibniz's calculus (Propositions 6 and 7) and Sluse's 'canon' (Proposition 8) from Newton's method. As a matter of fact, Proposition 5 consists merely of a series of

USAL. Gregory is here citing *ad litteram* Jacob Bernoulli, 'Specimen Calculi Differentialis in dimensione Parabolæ helicoidis, ubi de flexuris curvarum in genere, earundem evolutionibus, aliisque', *Acta Eruditorum*, 13–23 (1691), at p. 14.

47 'quæ omnes facile fluunt ex Methodo tangentium Barrovij X. Lect. Geom: tradita. Sed quoniam Newtoni Methodus Fluxionum non tantum primo inventa sit (utpote Neutono nota ab anno usque 1664) sed ejus demonstratio facilis ab Auctore pertexta sit et Calculo differentiali demonstranda (cum quippe absque demonstratione edidit Leibnizius) inserviat, immo Barrovij et Tschirnhausij Methodus in Fluxionum methodi demonstratione contineantur, libet eam primo explicare.' MS 31011, p. 1, USAL.

48 René François de Sluse (1622–1685) was a Walloon mathematician who was deeply influenced by the Dutch school of Cartesian mathematicians led by Frans van Schooten. As we shall see below, in 1672 he contributed a paper on the drawing of tangents that was noted by the English mathematicians, most importantly by John Collins and Isaac Newton.

49 'Nam ad Mathematica, ad res physicas applicanda feliciter nihil magis conducit quam generatio figurarum per motum localem.' MS 31011, p. 5, USAL.

50 'D. Newtonus per quantitates fluentes intelligit indeterminatas illas in figuris, quæque perpetuæ mutationi sunt obnoxiæ. Per Fluxiones vero intelligit mutationes hisce fluentibus simul in tempore brevissimo vel minimo inductas'. MS 31011, p. 1, USAL. Newton's definitions in the text he sent for inclusion in Wallis's *Algebra* differ slightly, but significantly. Most notably, Newton defines 'fluxions' as the velocities of increment or decrement (*celeritates incrementi vel decrementi*) of the fluents and adds that the concepts of 'moment', 'minimal part' and 'infinitely small difference' are less natural and easy. See Wallis, *op. cit.* (note 26), vol. 2, p. 391.

51 Propositions 1 and 3 are lifted almost *verbatim* from Wallis, *op. cit.* (note 26), vol. 2, pp. 391–393, where they occur in the section in which Wallis added material provided by Newton for inclusion in his Latin *Algebra* (pp. 391–396). In these two propositions one finds the definitions and notation for fluents and fluxions, the definition of the direct method ('Data æquatione fluentes quotcunque quantitates involvente invenire Fluxiones', p. 391) and two examples of application of the method (pp. 392–293). Proposition 2, on the application of the method of fluxions to the drawing of tangents to plane curves appears to have been lifted from the so-called *De methodis serierum et fluxionum*. See Add. 3960.14, p. 43, CUL.

statements, rather than a demonstration, concerning the equivalence of Barrow's method of tangents, presented after the 'advice of a friend' at the end of the tenth lecture of *Lectiones geometricæ*,[52] with the methods for calculating tangents published by Tschirnhaus in 1682 and by Leibniz in 1684. Gregory observes that these equivalences had already been recognized by Jacob Bernoulli in papers published in the *Acta Eruditorum* for 1691.[53] He is still citing from Wallis's *Algebra*. Indeed, at the end of the six-page-long presentation of the method of fluxions, Wallis writes:

> Analogous to this [Newton's] method is the differential method of Leibniz and that other method, older than either, which Barrow expounded in his *Geometrical Lectures*; and this is acknowledged in the Leipzig Transactions (January 1691) by a writer [Jacob Bernoulli] making use of a method similar to that of Leibniz.[54]

Bernoulli had gone so far as to state that Leibniz's 'discoveries' were 'based on' Barrow's 'earlier discovery'. As Whiteside explains in his learned reconstruction of this episode, Leibniz was not slow in offering his own version of how he discovered the calculus during his celebrated stay in Paris (1672–1676), independently of both Barrow's and James Gregorie's works, in a paper published in a subsequent issue of the *Acta*. A few months later, again in the *Acta*, Jacob Bernoulli rectified his statement, no doubt in order to smooth his relationships with Leibniz.[55]

Gregory's attribution of the 'method of tangents' to Barrow, and its influence on both Newton and Leibniz, should not be understood as an attempt to disparage Newton. Barrow's prominent role in inspiring Newton and Leibniz became the official Newtonian narrative during the priority dispute, so much so that Barrow's priority in the invention of the method of tangents is put into relief in the *Commercium epistolicum*, the pamphlet circulated in early 1713 by the Royal Society, in the context of the priority dispute over the invention of the calculus.[56] Newton also maintained that his method of fluxions, as well as Leibniz's differential method, could be applied to equations in which the unknown occurs in a denominator of a fraction or under a root. In his opinion, both he and Leibniz had improved on Barrow's method in this respect.[57] What mattered most to Newton was to see others acknowledge his priority in momentous advancements in the inverse method of fluxions, or method of 'quadratures'. Newton and his

52 Barrow, *op. cit.* (note 44), pp. 80–84.

53 'Yet, to speak frankly, whoever has understood Barrow's calculus (which he outlined ten years earlier in his *Lectiones Geometricæ*; and of which the whole of that medley of the propositions contained in it constitutes examples), will hardly fail to know the other discoveries of Mr. Leibniz, considering that they were based on that earlier discovery, and do not differ from it, except perhaps in the notation of the differentials and in some abridgement of the operation of it' ('Quanquam, ut verum fatear, qui calculum Barrovianum (quem decennio ante in Lectionibus suis Geometricis adumbravit *Auctor*, cujusque specimina sunt tota illa propositionum inibi contentarum farrago) intellexerit, alterum a Dn. L. inventum ignorare vix poterit; utpote qui in priori illo fundatus est, & nisi forte in differentialium notatione, & operationis aliquo compendio ab eo non differt'). Bernoulli, *op. cit.* (note 46), p. 14, translation by J. F. Scott in Turnbull et al., *op. cit.* (note 11), vol. 4, p. 10, n. 2. A similar statement concerning Tschirnhaus's method is in Jacob Bernoulli, 'Specimen Alterum Calculi Differentialis in dimetienda Spirali Logarithmica, Loxodromiis Nautarum, &; Areis Triangulorum Sphæricorum: una cum Additamento quodam ad Problema Funicularium, aliisque, *Acta Eruditorum*, 282–290 (1691), at p. 290.

54 Wallis, *op. cit* (note 26), vol. 2, p. 396. Translation from Hall, *op. cit.* (note 35), p. 97.

55 See Whiteside's commentary in Whiteside, *op. cit.* (note 12), vol. 8, p. 585, n. 83. On Leibniz's discovery of the calculus in Paris, see Joseph E. Hofmann, *Leibniz in Paris 1672–1676: his growth to mathematical maturity* (Cambridge University Press, Cambridge, 1974).

56 See Niccolò Guicciardini, *Isaac Newton on mathematical certainty and method* (MIT Press, Cambridge, MA, 2009), pp. 372–381.

57 'An Account of the Book Entituled Commercium Epistolicum Collinii Et Aliorum, De Analysi Promota', *Phil. Trans. R. Soc. Lond.* **342**, 173–224 (1715), at pp. 194–197 (https://doi.org/10.1098/rstl.1714.0021).

acolytes were primarily interested in defending his priority in the 'inverse' method of fluxions, the method applied to solve, most notably, problems in 'quadratures', 'rectifications' and the 'inverse method of fluxions' (in Leibnizian terms, the 'integral calculus' applied to the solution of differential equations). The 'method of tangents'—that is, the 'differential calculus'—was not top of the agenda for early eighteenth-century mathematicians, so it was not difficult for Newton to attribute its invention to his mentor.[58] Indeed, Gregory ends Proposition 5 with the following qualification: 'While both Newton and Leibniz applied their methods to the determination of maxima and minima, Newton did so also to the measure of spaces encompassed by straight and curve lines, as it will appear in what follows.'[59] It is, of course, absurd to claim that Leibniz did not apply the calculus to quadratures (namely integration), but—strange as it might seem—this was the view defended by Newton and his acolytes.

In Proposition 6, Gregory aims to show how Leibniz's rules of the differential calculus, as published in the famous paper in the *Acta* of 1684, can be easily deduced from Newton's method, since the differential calculus deals with 'indeterminates' that 'are the very same thing as the fluents of Newton', and with 'differences' that are 'the fluxions of Newton'.[60] Gregory introduces the differential notation and demonstrates the rules of the calculus (for example, the rules for the differential of a product and the differential of a quotient) by using both the Leibnizian and the Newtonian notations (see Figure 2). He opines that:

> Therefore, from all this it is sufficiently manifest that Leibniz's differential calculus differs only by name from Newton's method of fluxions, and that at the same time the Leibnizian algorithm published in October 1684 in the *Acta* of Leipzig can be derived from it.[61]

Gregory aims to show this equivalence by a brief calculation of the tangent to parabolas (Proposition 7). The same equivalence holds true for Sluse's method as published in the *Philosophical Transactions* of 1672 (Proposition 8).[62] Interestingly, Gregory claims that, by deducing Leibniz's and Sluse's rules in terms of fluxions, he is providing a 'demonstration' of the two 'algorithms', since, in his opinion, Newton had given a demonstration of the rules.[63] When he considers Barrow's method, he shifts his emphasis: his aim is not so much to show the equivalence between Barrow's method and Newton's one, but the priority of the former, which is given pride of place: 'But we hasten to other examples, in the first place those by which the most learned Barrow has illustrated his

58 I have defended this thesis in Guicciardini, *op. cit.* (note 56), pp. 372–381. See also the beautiful essay Antoni Malet, 'Newton's mathematics', in *The Oxford handbook of Newton* (ed. Eric Schliesser and Chris Smeenk), pp. 1–28 (Oxford University Press, Oxford, online July 2020).

59 'Cum tamen Neutonus et Leibnitius suas Methodos ad quælibet maxima et minima determinanda adhibeant, Newtonus etiam ad Spatia Curvis et rectis comprehensa mensuranda ut in decursu patebit.' MS 31011, p. 5, USAL.

60 'Patet jam indeterminatas hasce esse ipsissimas fluentes Neutoni; et differentias Leibnizij esse fluxiones Neutoni.' MS 31011, p. 5, USAL.

61 'Ergo ex omnibus hisce satis patet Calculum differentialem Leibnizij nomine tantum diversam esse a fluxionum Methodo Neutoniana et simul Algorithmum Leibnizianum Octobri A MDCLXXXIV Act: Lipsiæ consignatum exinde derivari.' MS 31011, p. 7, USAL.

62 Employing fluxional notation, Gregory reproduces the main results in R-F., de Sluse, 'An extract of a letter from the excellent Renatus Franciscus Slusius, Canon of Liege and Counsellor to his Electoral Highness of Collen, written to the publisher in order to be communicated to the R. Society; concerning his short and easie method of drawing tangents to all Geometrical curves without any labour of calculation: Here inserted in the same language, in which it was written', *Phil. Trans. R. Soc. Lond.* **7**, 5143–5147 (1672) (https://doi.org/10.1098/rstl.1672.0061).

63 In 1691, Gregory received from some 'Amsterdamers' a five-page manuscript with 'demonstrations and improvements' of Sluse's method attributed to Burchard De Volder. MS Dk.1.2, Quarto A [39]; and MS Dc.1.61, Folio C [81], EUL. Cited in Gregory, Letter to Newton, 27 August 1691, in Turnbull et al., *op. cit.* (note 11), vol. 3, p. 166 (and p. 167, n. 10). For Gregory's interest in Sluse's method, see also MS Dk.1.2, Quarto A [72]; MS Dc.1.61, Folio C [23]; and MS Dc.1.61, Folio C [26], EUL.

Method of Tangents, upon which both the Method of Fluxions and the differential calculus are built.'[64] Gregory refers to the tenth lecture of the *Lectiones geometricæ*, at the end of which Barrow had enunciated his method of tangents and applied it to five curves.[65] Gregory reproduces the first three examples, devoting particular attention to the third, the so-called Descartes's folium (which Gregory, following Barrow, calls 'la Galande').

In Proposition 10, Gregory offers a treatment of this curve, which he expresses in Cartesian coordinates as $x^3+y^3=axy$. The study of the graph of the folium occupies several pages and is carried out in Newton's notation. However, the source is not Newton but rather Johannes Hudde, one of the authors of the mathematical appendices to the second edition of the Latin translation of Descartes's *Géométrie* (1659–1661) edited by Frans van Schooten. Gregory mentions that in 1693 he was able to take notes from Hudde's papers when he visited him in Amsterdam.[66] He ends this proposition by informing the reader that he postpones a presentation of Guillaume de L'Hospital's quadrature of the folium to another occasion.[67] In Proposition 10, the influence of Continental mathematics on Gregory is once again evident. As Sandra Bella details in her dissertation, L'Hospital and Huygens had discussed the quadrature of the folium in correspondence running from 22 October 1692 to 18 January 1694.[68] The French mathematician took this opportunity to defend the versatility and power of Leibniz's calculus and advocate its use. Gregory acquainted himself with L'Hospital's quadrature of the folium when he visited Huygens at Hofwijk on 16 and 17 May 1693.[69]

Before moving on to problems concerning quadrature, Gregory completes the treatment of the direct method with a proposition devoted to the calculation of inflection points to the conchoid (Proposition 11), probably lifted from Newton's *De methodis serierum et fluxionum*.[70] Quadrature methods are introduced in Proposition 12, where Gregory refers to his *Exercitatio Geometrica de Dimensione Figurarum*. As often occurs in the 'Notæ', he does not miss the opportunity to underline the importance of his contributions to the advancement of methods of quadrature.[71] It is at this juncture that he begins to tackle more complex problems.

The following set of problems reveals, yet again, the influence on Gregory's mathematical culture of the Continental school—in this case, specifically of Leibniz and Jacob Bernoulli. As Philip Beeley has shown, from the correspondence with Colin Campbell one gathers

64 'Sed ad alia exempla properemus ea imprimis quibus Doctiss: Barrovius suam Methodum tangentium, cui utraque hæc et Fluxionum et Calculi differentialis superstruitur, illustravit.' MS 31011, p. 8, USAL.

65 Barrow, *op. cit.* (note 44), pp. 80–84.

66 'Præcedens Curva ipsissima est quam Galli *la Galande* dixere, quamque primus quod sciam consideravit Ampliss: Jo Huddenius Consul Reipubl: … Atque hanc ejus figuram ipse Huddenius detexit ut ex Schedis ejus An: 1693 Amstelodami per ipsum Ampliss: virum mihi ostensis adnotavi.' MS 31011, p. 9, USAL.

67 'Quantum ad spatij [sic] hujus tam folio AGC comprehensi quam interminati AKHk dimensionem et quadraturam ab Illustri Hospitalio Gallo noviter exhibitam, eam suo loco modoque inventam in medium proferemus.' MS 31011, p. 11, USAL.

68 Sandra Bella, 'De la géométrie et du calcul des infiniment petits: les réceptions de l'algorithme leibnizien en France (1690–1706)', PhD thesis, Université de Nantes (2018), pp. 442–452.

69 MS Dk.1.2, Quarto A [8], EUL. Gregory visited Holland in May and June 1693. His annotations from Hofwijk dated 30 June 1693 are in Turnbull et al., *op. cit.* (note 11), vol. 3, pp. 272–273. See also Gregory's letter to Huygens, 12 August 1693, in *ibid.*, vol. 3, pp. 275–276.

70 Gregory's procedures appear to be inspired by Newton: see MS Add. 3960.14, p. 45, CUL, edited in Whiteside, *op. cit.* (note 12), vol. 3, p. 126. In 1685, Gregory had received extracts of this work via Craig. A possible influence from Sluse's *Miscellanea* cannot be excluded. See René-François de Sluse, *Mesolabum seu Duæ Mediæ Proportionales inter Extremas Datas per Circulum et per Infinitas Hyperbolas vel Ellipses et per Quamlibet Exhibitæ … Accessit Pars Altera de Analysi et Miscellanea* (Streel, Liège, 1668), pp. 117–130.

71 MS 31011, p. 14, USAL. The proposition concerns the quadrature of equations with the form $y = mx^{p/r}$.

that Gregory began studying the works by Tschirnhaus and Leibniz in *Acta Eruditorum* from the beginning of 1685.[72] He even faithfully transcribed for Campbell both Sluse's method of tangents as printed in the 1672 *Philosophical Transactions* and Leibniz's 'Nova methodus' as printed in the 1684 *Acta Eruditorum*.[73] Gregory's memoranda kept in Christ Church, Oxford, reveal that he continued to study in detail the papers on calculus published in the Leipzig journal.[74] Proposition 13 concerns a typical example of an 'inverse tangent problem', the so-called Florimond De Beaune's problem—that is, the determination of the curve whose 'subtangent' is constant. As Leibniz had shown in the closing lines of 'Nova methodus', this is a 'logarithmic curve'.[75] Gregory, however, refers to the solution provided by Barrow in the *Lectiones geomericæ*.[76] The more advanced problem of the rectification of the logarithmic curve is broached later on, in Proposition 41.[77] In 1692 Guillaume de L'Hospital had set the challenge of the calculation of the length of De Beaune's curve.[78] It might be surmised that in this proposition Gregory draws inspiration from Hendrik van Heuraet's rectification method, which he had read about in a manuscript transmitted to him in Amsterdam in 1693.[79] Proposition 14, proposed to the reader in 'order to verify the strength of the method (of fluxions) in the solution of some physical problems', is devoted to the determination, by application of a Fermatian minimum time principle, of the path followed by a light-ray passing through two media with different refraction indexes.[80] Just as in the previous proposition, Gregory's source is most probably Leibniz's 1684 seminal paper on the differential calculus.[81]

The following propositions, down to Proposition 24, are rewritings, in fluxional notation, of papers published in the *Acta Eruditorum* for 1691. Propositions 15–20 are based on a paper by Jacob Bernoulli: they concern the tangent, area, inflexion point and quadrature of the 'parabola helicoidis' and the calculation of evolutes.[82] Proposition 21 consists of a treatment of the area and rectification of the logarithmic spiral, the *spira mirabilis*, whose properties had been studied in depth by Bernoulli. In this, and in the two subsequent propositions, Gregory reframes in fluxional notation another paper by Bernoulli, again published in *Acta Eruditorum* in 1691.[83] However, he refers to James Gregorie's, Wallis's and Barrow's works too.[84] He then moves on, in Proposition 22, to investigate the rectification and the area subtended by the loxodrome (which, as was already known at the

72 See the letters by Gregory to Campbell dated 5 March 1685 and 25 February 1686, MS 3099.11, no. 14 and no. 17, EUL, cited in Beeley, *op. cit.* (note 8), pp. 6–7. See also MS Dc.1.61, Folio C [32], EUL.

73 Sluse, *op. cit.* (note 62); and G. W. Leibniz, 'Nova methodus pro maximis et minimis, itemque tangentibus, quæ nec fractas nec irrationales quantitates moratur, & singulare pro illis calculi genus', *Acta Eruditorum*, 467–473 (1684).

74 MS 346, CCL. See also Gregory's list of mathematical papers in the *Acta* in the period 1681–1692, MS Dc.1.61, Folio C [35], EUL.

75 Leibniz, *op. cit.* (note 73), p. 473. The subtangent is defined as the segment of the x-axis lying between the x-coordinate of the point at which a tangent is drawn to a curve and the intercept of the tangent with the x-axis.

76 Barrow, *op. cit.* (note 44), p. 123; MS 31011, p. 14, USAL.

77 MS 31011, pp. 37–38, USAL.

78 Guillaume de L'Hospital, 'Solution du problème que M. De Beaune proposa autrefois à M. Descartes, et que l'on trouve dans la 79. de ses lettres, tome 3', *Journal des Sçavants* **3**, 401–403 (1692).

79 'Excerpta de Codico M.S. Heuratii communicata a [illegible], 27 May 1693', MS Dk.1.2, Quarto A [19], EUL.

80 'ut hujus Methodi vires experiamur in Physicis quibusdam quæstionibus solvendis'. MS 31011, p. 15, USAL.

81 See Leibniz, *op. cit.* (note 73), pp. 471–472.

82 The equation of the 'parabola helicoidis' in polar coordinates is $r = \pm c\sqrt{\theta}$. Gregory lifts these calculations from Bernoulli, *op. cit.* (note 46).

83 Bernoulli, *op. cit.* (note 53).

84 Namely, James Gregorie, *Geometriæ pars universalis: inserviens quantitatum curvarum transmutationi & mensuræ* (Typis Heredum Pauli Frambotti, Padua, 1668), Prop. 16; John Wallis, *Tractatus duo. Prior, de cycloide et corporibus inde genitis. Posterior,*

time, is related to the logarithmic spiral by a projection). In Proposition 23, he applies this result to the calculation of the area of spherical triangles. These results, too, are lifted from Bernoulli's paper just cited above.

The *Methodus fluxionum* continues with several propositions tackling problems of maxima and minima in plane and spherical geometry. These propositions seem to play a didactic role. They solve very simple problems, such as the determination of the right circular cone of least volume circumscribed about a given sphere (Proposition 34).[85] Proposition 24 is slightly more advanced, being devoted to the calculation of the tangent of the quadratrix. In this case, too, it seems that Gregory has a didactic purpose in mind. This proposition leads him to devote a scholium to the properties of 'transcendental' curves. It is notable that he deploys Leibnizian terminology for the curves that Newton, following Descartes, would have called 'mechanical'.

Most of the last pages of the *Methodus fluxionum* originate from conversations with Newton. Proposition 39 is devoted to the determination of the 'most convenient place where a rudder should be placed in order make a vessel turn' (see figure 4).[86] Gregory's treatment of this problem is purely qualitative: it is, of course, a 'problem of maximum' but, not surprisingly, no mathematical 'function' (to use modern terminology), whose maximum might be sought by application of the fluxional method, is proposed. The problem of understanding the effect of the rudder in directing a ship was often broached during the Renaissance, most notably in commentaries on the 'nautical questions' occurring in the pseudo-Aristotelian *Mechanical problems*.[87] Gregory cites the works by Stephanus Gradius, Jacob Bernoulli and Paul Hoste on this issue.[88] It appears from his memoranda that he discussed Hoste's approach with Newton in May 1694.[89] The conversation between Newton and Gregory on the manoeuvring of ships reveals that the interest in 'mixed mathematics' that Newton had entertained in the 1670s was still alive. The fact that Gregory and Newton discussed issues concerning shipbuilding and the crewing of ships sheds some interesting light on Newton's engagement, even after the publication of the *Principia*, with what nowadays we would call 'applied mathematics'. Proposition 40 is related to another 'maximum problem' in mechanics that had apparently been proposed by Edmond Halley: the determination, given the intensity and direction of a

epistolaris; in qua agitur, de cissoide, et corporibus inde genitis: et de curvarum, tum linearum …, tum superficierum … (Typis Academicis Lichfieldianis, Oxford, 1659); Barrow, *op. cit.* (note 44), p. 124.

85 Propositions 28–38 and 43–45.

86 'Maxime commodum Gubernaculi situm ad navigium circumducendum determinare'. MS 31011, p. 34, USAL.

87 See e.g., Bernardino Baldi, 'Quæstio V. Dubitatur: cur parvum existens gubernaculum, & in extremo navigio tantas habeat vires, ut ab exiguo temone, & ab hominis unius viribus alioqui modice utentis magnæ navigiorum moveantur moles?', in *Mechanica Aristotelis Problemata Exercitationes: Adiecta Succincta Narratione de Autoris Vita et Scriptis* (typis et sumptibus viduæ Ioannis Albini, Maintz, 1621), p. 41. I thank Elio Nenci for suggesting this reference.

88 Stephanus Gradius (Stjepan Gradić), 'Dissertatio I de directione navis ope gubernaculi', in *Dissertationes Physico-mathematicæ Quatuor* (apud Danielem Elsevirium, Amsterdam, 1680), 1–21; Jacob Bernoulli, *Dissertatio de Gravitate Ætheris* (apud Henr. Wetstenium, Amsterdam, 1683); Paul Hoste, *Recueil des Traités de Mathématique, qui Peuvent Être Necessaires à un Gentil-homme pour Servir par Mer, ou par Terre*, vol. 3 (J. Anisson, Paris, 1692).

89 See 'Extrait de Manouvre [sic] des Vaisseaux … sequuntur … de Newtoni cogitatio, 1694', MS Dc.1.61, Folio C [57], EUL. Interestingly, both Newton's and Gregory's hands can be found on this page of jottings on the manoeuvring of vessels, most probably a note taken during Gregory's May 1694 encounter with Newton. In 1697 Gregory studied Tourville, Anne Hilarion de Cotentin, *Exercice en Général de Toutes les Manoeuvres qui se font à la Mer en Toutes les Occasions qui se Peuvent Presenter* (Jacques Hubault, Le Havre, 1693). See MS Dk.1.2, Quarto A [2], EUL.

force, of the point in a wheel or sphere to which the given force must be applied in order to cause the maximum speed of rotation.[90]

Proposition 42 consists in a fluxional solution to the determination of the solid of least resistance (the Scholium to Proposition 35 of the first edition of the *Principia*). Newton communicated it to Gregory on 14 July 1694.[91] Gregory inserted it not only in the *Methodus fluxionum* but also in his 'Notæ'.[92] Indeed, this proposition (with Proposition 26 on the cone frustrum of least resistance; see figure 4) is very much part of Gregory's planned editorial work for a second edition of the *Principia*.[93] He was particularly interested in the higher fluxional methods that, as was obvious to the few competent readers of the *magnum opus*, its author had used in some of the demonstrations.[94] Newton's fluxional treatment of the solid of least resistance, in the form communicated to Gregory, was eventually printed as an appendix to Andrew Motte's English translation of the *Principia* (1729), a fact that might indicate that either the *Methodus fluxionum* or the 'Notæ' was circulating among English mathematicians.[95]

Gregory and the publication of Newton's method

Newton the mathematician is notorious for his reluctance to publish his discoveries. There is more than a grain of truth in this image: the Lucasian Professor of Mathematics had a fraught relationship with the public sphere, at least after the traumatic critical reception of his great 1672 paper on the composition of white light.[96] However, by looking more closely at his correspondence, one easily discovers that throughout his lifetime he was part of various networks of mathematicians active in Britain. Newton took his first steps as a mathematician in Cambridge, under the guidance of his predecessor in the Lucasian chair,

90 See 'De gyratione Globorum de collisione mutua Probl: Halleianum 3, c January 1695', MS Dk.1.2, Quarto A [7]; 'Scheda D.G. de rotatione globi de percussione orta, 12 February 1695', MS Dk.1.2, Quarto A [25], EUL.

91 The draft of Newton's letter to Gregory has survived and is edited in Turnbull et al., *op. cit.* (note 11), vol. 3, pp. 380–382. D. T. Whiteside provides, with his usual thoroughness, the details of Newton's correspondence with Gregory on this matter, in Whiteside, *op. cit.* (note 12), vol. 6, pp. 470–480 (esp. p. 470, n. 1). Whiteside has edited all the drafts in Newton's hand of the material on the cone frustrum and the solid of least resistance written in order to provide Gregory with all the necessary information.

92 See MS 31011, pp. 39–40, USAL; MS 210, p. 88, RSL, where the pasted sheet with the fluxional analysis is missing: its transcription can be found in MS 131, pp. 135–136, CCL. In the collection of Gregory's papers at the Royal Society Library, one finds an analysis of the cone frustrum of least resistance in Newton's hand. See Turnbull et al., *op. cit.* (note 11), vol. 3, p. 323, and Whiteside, *op. cit.* (note 12), vol. 6, pp. 470–471. Gregory incorporated it as Proposition 26 of the *Methodus fluxionum*, and as a commentary on the Scholium to Proposition 35, Book 2, in the 'Notæ'. MS 31011, pp. 27–28, USAL; MS 210, p. 87, RSL.

93 The same holds true for the concluding Propositions 46 and 47 on the resistance exerted by a 'rare and elastic medium' to the motion of a solid whose cross-section is a square and a circular segment.

94 See 'In editione nova Philos: Newtoniana hæc ab Auctore fient, May 1694', MS Dc.1.61, Folio C [42], EUL, in part translated by H. W. Turnbull in Turnbull et al., *op. cit.* (note 11), vol. 3, pp. 384–386. Most significantly, Gregory notes that Newton had a plan to add an appendix on the quadratures of curves to a second edition of the *Principia* and that 'on these [quadratures] depend certain more abstruse parts in his philosophy as hitherto published'. ibid. p. 386: 'Tractatus Methodum suam Quadraturarum continebit quæ rem istam mire augebit et promovebit … innituntur quædam abstrusiora in Philosophia sua hactenus edita.'

95 In the Appendix there is a fluxional treatment of the gravitational attraction of a homogeneous ellipsoid of revolution upon a mass point situated on the prolongation of the axis (Cor. 2, Prop. 91, Book 1 of the *Principia*), and the fluxional treatment of the cone frustrum and the solid of least resistance. See 'Appendix. Among the Explications (given by a Friend,) of some Propositions in this Book, not demonstrated by the Author, the Editor finding these following, has thought it proper to annex them', in Isaac Newton, *The Mathematical Principles of Natural Philosophy, Translated into English by Andrew Motte, to Which are Added, the Laws of the Moon's Motion, According to Gravity, By John Machin* (printed for B. Motte, London, 1729), vol. 2, pp. i–viii.

96 Hall, *op. cit.* (note 35), p. 25, writes: 'What he [Newton] had done—even what he was now doing—was for his own satisfaction and not for applause. Through all the past years, though ready enough to share his results when a rare opportunity (like Halley's visit) offered itself, his attempts to address the public at large had been highly tentative. In 1685 none of Newton's mathematical work (amounting in modern form to four very stout volumes) had yet been printed, and little enough was known to any one at all.'

Isaac Barrow. Newton's mathematical concepts and techniques resemble Barrow's, so much so that he acknowledged his indebtedness to Barrow until the last years of his life. Even though we know very little about the transactions between the two, it is highly probable that Barrow lent Newton some books.[97]

What is certain is that it is through Barrow that Newton entered into contact with John Collins, and via Collins with the bustling world of the London mathematical practitioners.[98] Newton's binomial theorem was soon put into practice in fields such as the determination of volumes of barrels for the Excise Office, while his innovative algebraic methods were deemed useful for the writing of mathematical tables. It seems that Newton found these topics quite congenial to his interests and that his replies to gaugers and table makers such as Michael Dary and John Smith were solicitous. We often depict Newton as endowed with a 'mind forever voyaging through strange seas of thought, alone', as the genius who 'discovered the calculus', a highly abstract field of pure mathematics. Yet Newton might also have had practical purposes in mind when he formulated the method of series and fluxions.[99]

The *Methodus fluxionum* was conceived during a later stage in Newton's life. In 1694 Newton was the celebrated author of the *Principia*, a major contribution to mathematicized natural philosophy, and as a member of the Convention Parliament he had acquired a fair political standing too. Gregory, Nicolas Fatio de Duillier and before them Wallis had all addressed Newton in the hope of gaining information about his cutting-edge mathematical techniques. It was clear that he had made use of higher quadrature techniques and infinite series in writing his *magnum opus*. It is unsurprising that Fatio and Gregory approached Newton in order to learn the quadrature methods that were deployed, and in some cases just hinted at, in the *Principia*.[100] Quadrature methods, the inverse method of fluxions and the 'integral calculus' were the open problems which busied the minds of the best European mathematicians of Newton's generation. In the early 1690s, Newton himself was at work writing a treatise on this topic, which was to become the *Tractatus de quadratura curvarum*, published as an appendix to the *Opticks* in 1704. Indeed, Gregory met Newton when the Lucasian professor was pursuing two related publication projects: a second edition of the *Principia* and a systematic treatise on quadratures. These projects influenced Gregory when he composed the *Methodus fluxionum*.

As we have seen, the direct method is explicitly attributed to Barrow. Newton's method of fluxions is praised, on the one hand, because it allows an easier demonstration of Barrow's method of tangents and, on the other, because it can be applied to higher quadratures. Some of Newton's quadratures—such as those necessary to solve the solid of least resistance problem—are featured in the most mathematically advanced parts of the *Principia*. The fact that, in Gregory's conversations with Newton, the above problem was mentioned alongside the very practical one of determining the best position of the rudder

97 See Feingold, *op. cit.* (note 45).

98 See Philip Beeley, 'Practical mathematicians and mathematical practice in later seventeenth-century London', *Brit. J. Hist. Sci.* **52**(2), 225–248 (2019) (https://doi.org/10.1017/S0007087419000207).

99 I thank Derrick Mosley for his precious information on this issue.

100 Nicolas Fatio de Duillier (1664–1753) was a Swiss-born mathematician, who befriended Huygens and Newton. His contacts with Newton were especially intense in the early 1690s, just before Gregory's visit to Newton in 1694. Fatio and Newton worked together not only on mathematics and the theory of gravitation, but also on alchemy. See Robert Iliffe, 'Servant of two masters? Fatio de Duillier, Isaac Newton and Christiaan Huygens', in *Newton and the Netherlands: how Isaac Newton was fashioned in the Dutch Republic* (ed. Eric Jorink and Ad Maas), pp. 67–91 (Leiden University Press, Leiden, 2012); Scott Mandelbrote, 'The heterodox career of Nicolas Fatio de Duillier', in *Heterodoxy in early modern science and religion* (ed. John Brooke and Ian MacLean), pp. 263–297 (Oxford University Press, Oxford, 2005).

of a ship suggests that the hands-on approach of Collins's young correspondent was still part of the agenda for the author of the *Principia*.

As we have seen, after meeting Newton in May 1694, Gregory evolved from being an editor of his uncle's mathematical works to proposing himself as an editor of Newton's mathematical discoveries: the mathematization of gravitation offered in the *Principia* and the method of fluxions. It would be reductive, however, to define Gregory's mathematical culture as 'Newtonian', notwithstanding the impact that his May 1694 encounter with Newton's manuscripts must have had. Gregory was an attentive reader of Continental works, especially those on differential equations and quadratures by Tschirnhaus, Leibniz and Jacob Bernoulli. He had also gained information on the works of Hudde and Sluse. Therefore, he incorporated the Continental methods in his presentation of Newton's method of fluxions. Very much like other Scottish mathematicians in Pitcairne's circle, most notably John Craig, he merged Continental methods and notations with British ones. And very much like Fatio de Duillier, he deployed his contacts with savants in the United Provinces, most notably Huygens, in order to establish himself as a mathematician who could play a prominent role in the Continental arena. Further, in the early 1690s, it seems that he was proposing himself as an intermediary between Newton and the Dutch.[101] Consequently, the *Methodus fluxionum* informed its readers about topics that polarized the attention of mathematicians belonging to Leibniz's and Bernoulli's circles, such as the logarithmic curve, the loxodrome and the logarithmic spiral.

The *Methodus fluxionum*, then, was not only the first systematic treatise on series and fluxions written by somebody other than Newton: it was also an attempt to integrate Newton's method into a narrative extending from Hudde and Sluse to Bernoulli. Gregory did not miss the opportunity to underline the importance of his uncle's discoveries, and to some extent his own, in the long historical overview of the development of the calculus provided in the *Methodus*. This small treatise was also conceived as a way to affirm the role that the Scottish group had played in this cutting-edge research field, and the potential of the Scottish mathematicians, mostly based in Oxford, to promote Newton's discoveries, especially ones helpful in proving the most 'abstruse parts' of the *Principia*. As Gregory wrote in one of the memoranda about his May 1694 visit to Newton:

> The second treatise [a draft of *De quadratura*] will contain his [Newton's] Method of Quadratures which greatly extends and improves this matter … To this he will subjoin [tables on quadratures] upon which depend certain more abstruse parts in his philosophy as hitherto published.[102]

Concluding remarks

As I have detailed above, in writing the *Methodus fluxionum* Gregory made use not only of the material Newton shared with him, but also of manuscript and printed works on the calculus authored by Scottish, English and Continental mathematicians. His aim was often to compare the different methods, with an eye to proving their 'equivalence'. Most notably, quoting similar remarks by Jacob Bernoulli, Gregory claimed that both Newton's method of fluxions

101 This clearly emerges from study of Gregory's mathematical correspondence with Fatio, Newton and Huygens. See Vermij and van Maanen, *op. cit.* (note 41).

102 See note 94. It is interesting to note that in the Macclesfield Collection there is a partial copy of Newton's *De quadratura*. MS Add. 9597/2/18, ff. 83r–88r, CUL.

and Leibniz's differential calculus were equivalent to Barrow's method of tangents. This raises an interesting historiographical question that often emerges in studies devoted to the history of the priority dispute between Newton and Leibniz. Did Barrow, Newton and Leibniz discover the same thing? To what extent can we attribute priority to one of them? Or, perhaps, should we say that the calculus was discovered by Blaise Pascal, or maybe Evangelista Torricelli? As scholars trained in present-day mathematics, we all run the risk of sliding towards poor historiography, precisely because, as mathematicians, we are trained to detect equivalences. In a way, they all discovered the same thing: we can prove it by translating each of these past actors' language into present-day calculus! Yet, as historians we both need to acknowledge the continuity between the past and the present, and to appreciate the alterity of the past. What we have to appreciate is that there is not a 'calculus' that was discovered, or created, as a single act of invention that occurred to a single mind in a single eureka moment. The discovery of the calculus was a slow process, to which several generations of mathematicians contributed. The historical actors who participated in this process could not have the benefit of hindsight that characterizes our view of the past: they saw events 'from within', without a clear picture of where the (largely contingent) development of mathematics would have led them.

As historians, rather than adjudicating equivalences and priorities, we must recapture our predecessors' views as best as we can. This has been done in an admirable way by several historians of mathematics. Two examples can be put forward. Nico Bertoloni Meli has shown that Newton and Leibniz had different views of what was at stake in the priority dispute over the calculus: when they quarrelled about who had discovered the calculus first, they meant different things.[103] The problematic nature of the notions of 'equivalence' and 'priority' in the history of mathematics has been brought into relief by Catherine Goldstein's account of the reception of an elementary theorem formulated both by Fermat and by Bernard Frénicle de Bessy. As she shows, the meaning and equivalence of different presentations of this theorem cannot be given a priori by the historian, who must instead regard the network of relations between readers as the condition which lends meaning to a text and makes 'equivalence' the result of a contingent historical development.[104] Our reading of the *Methodus fluxionum* invites a similar historiographical viewpoint, since Gregory's text teaches us a great deal about how its author saw equivalences and which aspects of the method of fluxions he considered most important.

The *Methodus fluxionum* was never printed; yet, as the three copies in the Cambridge University Library show, at some point—presumably in the first decades of the eighteenth century—it enjoyed some circulation. The amanuensis copy in Christ Church reveals that a printed version, or some form of systematic scribal circulation of the work, was envisaged in 1695 (see figure 2). The hands of the extant copies suggest that the circulation occurred within the limits of a small network of Newton's trusted acolytes, such as John Keill and William Jones. As I noted at the beginning, the copy in St Andrews was probably meant for print publication or scribal circulation, as the concluding folded plates of figures suggest (see figure 4).[105] The printed publication of the Newtonian methods of series and fluxions had to await the *placet* of the mighty author, who, worried by the circulation of unauthorized versions of his discoveries such as George Cheyne's *Fluxionum Methodus*

103 Domenico Bertoloni Meli, *Equivalence and priority: Newton versus Leibniz* (Clarendon Press, Oxford, 1993).
104 Catherine Goldstein, *Un théorème de Fermat et ses lecteurs* (Presses Universitaires de Vincennes, St-Denis, 1995).
105 MS QA 33G8/D12, USAL.

Inversa (1703), ventured into print, first in 1704 with the mathematical appendices to the *Opticks*, and then in 1711 with a small pamphlet edited by William Jones.[106]

Gregory's *Methodus fluxionum* is significant for several reasons. Most notably, it shows that Newton, far from being a mathematician wholly isolated in an ivory tower, circulated knowledge of his fluxional method in the mid-1690s. There is still a story to be told about the ways in which he disseminated his mathematical discoveries, first in the 1670s via Barrow and Collins, then in the 1680s via Craig and Wallis, in the 1690s via Wallis, Fatio and Gregory, and finally in the 1700s via William Whiston, Roger Cotes, Joseph Raphson, John Keill, William Jones and Henry Pemberton, among others. The *Methodus fluxionum* also provides evidence that Newton's method was received by acolytes who were not passive defenders of an uncontested master. Gregory, as well as other Scottish mathematicians, such as Craig and Cheyne, received the fluxional method within a tradition that was proudly independent from England and that had already assimilated elements of the calculi by mathematicians such as Hudde, Sluse, Tschirnhaus, Leibniz and Jacob Bernoulli. It seems that one might even surmise that Gregory, as well as Craig, considered himself a mathematician whose results and methods could be received as original and of import by the Continentals.[107] Last but not least, this short treatise would not have been possible outside a mathematical culture in which information was shared via manuscript circulation, correspondence and personal encounters. Gregory received information about mathematical discoveries in the calculus not only from Newton, but also from many other mathematicians with whom he corresponded and whom he met, such as Craig, Campbell, Pitcairne, Hudde, Sluse, Fatio, Huygens and Halley. A study of the *Methodus fluxionum* has much to teach us about the scribal circulation of mathematical knowledge at the turn of the eighteenth century.

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106 Cheyne, *op. cit.* (note 5); Newton, *op. cit.* (note 5); [Isaac Newton], *Analysis per Quantitatum Series, Fluxiones, ac Differentias, cum Enumeratione Linearum Tertii Ordinis* (ex Officina Pearsoniana, London, 1711). On the publication of Cheyne's fluxional treatises and Newton's reaction, see Niccolò Guicciardini and Scott Mandelbrote, 'Collaboration and rivalry in the publishing of Newton's mathematics: a study of Russell Library, St Patrick's College, Maynooth, shelfmark: Sc. 22. 3', in *Mathematical books' histories* (ed. P. Beeley and C. Mac an Bhaird) (Birkhäuser, Basel, forthcoming) and Beeley, *op. cit.* (note 8), pp. 18–21.

107 The study of the sources of Gregory's *Methodus* and of the manuscript exchanges that made it possible should be placed into a broader context. Most notably, David McOmish has shown that that manuscript culture at Edinburgh was the main conduit through which mathematical and scientific culture from Europe was used for instruction at Edinburgh up to and including Pitcairne's time at the university. See David McOmish, 'The Scientific Revolution in Scotland revisited', *Hist. Universities* **31**, 153–172 (2018), at pp. 159–164 (https://doi.org/10.1093/oso/9780198835509.003.0006).