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Homogeneously modified special relativity applications for UHECR and neutrino oscillations

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Abstract. One of the key open questions in fundamental physics concerns the supposed quantum structure of spacetime. New physical effects are expected as residual evidence of a more fundamental theory of nature. In this fundamental theory the underlying physical symmetries could be modified by the quantized structure of geometry. One of the most important symmetries in our description of Physics is Lorentz Invariance (LI). Nowadays LI is at the root of our understanding of nature and underlies its physical description. Even if there is no definitive evidence to sustain departures from this symmetry, there are consistent points indicating that Lorentz Invariance Violation (LIV) can be a consequence of quantum gravity. A critical issue concerns therefore the necessity of testing this fundamental symmetry validity. Ultra High Energy Cosmic Rays (UHECR) and neutrino flavor oscillation are promising physical processes of investigation for LIV, since the high energy involved and the interaction of different particle species.

1. Introduction

The investigation for Lorentz Invariance Violations is motivated by the necessity of searching for residual quantum gravity effects at the Planck-scale. One indeed expects that Planck-scale interactions could manifest themselves in a "low" energy scenario as tiny residual effects, that can modify standard physics. These modifications could emerge in a symmetry breaking fashion. Spontaneous symmetry breaking is indeed a concept, employed in many physical fields. Even in the high energy limit, the Planck scenario, quantum gravity could therefore manifests resorting to the same mechanism. The Lorentz symmetry would therefore be broken, introducing a class of privileged reference frames. The introduction of such privileged frames might not be a real problem, even if it might result conceptually difficult. Some studies, for example, attempt to correlate this privileged reference systems with the natural one, used for the description of the Cosmic Microwave Background Radiation (CMBR). But there are apparently not physical reasons that can justify any connection between a supposed quantum phenomenon of the Planck scale, with the CMBR classical physics description. In fact, nowadays, it is not clear how to introduce and justify these preferred inertial observers. In this context HMSR represents a new theoretical model that investigates the possibility to preserve spacetime isotropy in a LIV scenario. The new model is indeed characterized by its peculiar geometrical approach that allows the preservation of the isotropy with respect to rotations and boosts. Finally this model allows formulating some phenomenological predictions about LIV effects on Ultra High Energy Cosmic Rays (UHECR) propagation and the neutrino flavor oscillation phenomenon.

2. Homogeneously Modified Special Relativity

2.1. Spacetime geometry

In all LIV theories the effects of the supposed quantum structure of spacetime manifest itself modifying massive particles kinematics [1, 2, 3, 4, 5, 6, 7, 8]. This means that the dispersion relations result modified. Even in HMSR [9] the kinematical effects caused by LIV determine a modification of the dispersion relations:

$$E_i^2 - |\vec{p}_i|^2 \left(1 - f_i(p_i)\right) = m_i^2 \tag{1}$$

Every particle species presents a peculiar Modified Dispersion Relation (MDR), this means that every particle species presents a different Maximum Attainable Velocity (MAV). LIV is introduced by the non universal character of the particles MAV that differ from the light speed. In order to preserve the geometrical structure of the MDR, HMSR resorts to homogeneous perturbations of the dispersion relations:

$$F^2(p) = E^2 - \left(1 - f\left(\frac{|\vec{p}|}{E}\right)\right) |\vec{p}|^2 = m^2 \tag{2}$$

where $F^2(p)$ represents a Finsler pseudo-norm. This MDR definition allows to introduce a pseudo-Finsler geometric structure of momentum space with metric $\widetilde{g}^{\mu\nu}(p)$. Every particle is supposed to generate its own personal spacetime and has its own metric in coordinate space $g_{\mu\nu}(p)$. The correlation between the momentum space and the coordinate space is established via the *Legendre* transformation. In this way a "trivial" correspondence between the four momentum and the four velocity is established. The metric and co-metric are related through the relation:

$$\widetilde{g}^{\mu\alpha}(p)\, g_{\alpha\nu}(p) = \delta^\mu_\nu \tag{3}$$

and the MDR geometrical origin is established via the equation:

$$\widetilde{g}^{\mu\nu}(p)\, p_\mu\, p_\nu = m^2 \tag{4}$$

The resultant spacetime is an asymptotically flat pseudo-Finsler geometry [10, 11, 12, 13, 14, 15] and every massive particle feels a personal local parametrization of spacetime, depending on its momentum.

Every particle lives in a personal curved space-time and all the physical quantities are generalized, acquiring an explicit dependence on the momentum. It is necessary therefore to introduce a new mathematical formalism to conduct computations between physical quantities related to different interacting particles. This result is obtained constructing the generalized tetrad (vierbein) that presents an explicit dependence on the particle momentum and are defined in order to obtain:

$$\begin{aligned} g_{\mu\nu}(\dot{x}) &= e_\mu^a(p(\dot{x}))\, \eta_{ab}\, e_\nu^b(p(\dot{x})) \\ \widetilde{g}^{\mu\nu}(p) &= e_a^\mu(p)\, \eta^{ab}\, e_b^\nu(p) \end{aligned} \tag{5}$$

The elements of the tetrad can be used as projectors from the local curved space to a common support Minkowski spacetime (2.1).

The possibility to construct a modified form of the Lorentz group is an original feature of this model:

$$\Lambda_\mu^{\ \nu}(p) = e_\mu^a(\Lambda p)\, \Lambda_a^{\ b}\, e_\nu^b(p) \tag{6}$$

This means that HMSR preserves covariance even if in an amended formulation and the introduced modified Lorentz transformations are the isometries of the MDRs. Moreover every

$$(TM, \eta_{ab}, p) \xrightarrow{\quad \Lambda \quad} (TM, \eta_{ab}, p')$$

$$\downarrow e(p) \qquad\qquad \overline{e}(p') \downarrow$$

$$(T_x M, g_{\mu\nu}(p)) \xrightarrow[\overline{e} \circ \Lambda \circ e^{-1}]{} (T_x M, \overline{g}_{\mu\nu}(p'))$$

Figure 1. Generalied tetrad used as projectors to correlate different spacetimes modified by the particle's speed.

species presents its personal Modified Lorentz Transformations (MLTs), which are the isometries for the Modified Dispersion Relation (MDR) of the particle. Every particle species has its own metric, this implies that every particle has its own MAV. This model is therefore a generalization of Very Special Relativity (VSR) [16, 17], more precisely it admits VSR as an high energy limit. The ratio $\frac{|\vec{p}|}{E}$ has a finite limit $\frac{|\vec{p}|}{E} \to (1 + \delta)$ with $0 < \delta \ll 1$ for $E \to \infty$. As consequence, the perturbation function admits a finite limit $f\left(\frac{|\vec{p}|}{E}\right) \to \epsilon \ll 1$ and the MAV for every massive particle becomes:

$$c'(E) = 1 - \epsilon \tag{7}$$

2.2. Extension of the Standard Model of particles

In HMSR [9] every particle species has its own metric, with a personal MAV. The new physics, caused by LIV, emerges only in the interaction of two different particle species. That is every particle type physics is modified in a different way by the Lorentz symmetry violation. To analyze the interaction of two different particles, it is necessary therefore to determine how the reaction invariants - that is the Mandelstam relativistic invariants - are modified. Resorting to the *vierbein* to project the particles momenta on the Minkowski tangent space, it is possible to generalize the definition of internal product of the sum of two different particle species momenta as:

$$\langle p + q | p + q \rangle = (p_\mu \, e_a^\mu(p) + q_\mu \, \tilde{e}_a^\mu(q)) \, \eta^{ab} \, (p_\nu \, e_b^\nu(p) + q_\nu \, \tilde{e}_b^\nu(q)) \tag{8}$$

where e stands for the tetrad related to the first particle and $\tilde{e}$ represents the vierbein related to the second one. The Mandelstam variables s, t and u can be generalized using this internal product definition. If the interaction takes place between particles of the same species, the internal product and consequently the Mandelstam variables present no differences from standard Physics. If the interacting particles belong to different species, the internal product is amended, since it is necessary to correlate two different modified spacetimes. The interaction physics description is made resorting to formalism of the S matrix, which results to be a Mandelstam variables analytic function. Since the MLT preserve these quantities covariance, the concept of isotropy is restored and the necessity of introducing a privileged class of inertial observers disappears.

The geometry deformation modifies the Dirac equation form, the spinors and the correlated conserved currents. The amended Dirac matrices can be computed requiring that they satisfy the Clifford Algebra relation:

$$\{\Gamma_\mu, \Gamma_\nu\} = 2 \, g_{\mu\nu}(p) = 2 \, e_\mu^a(p) \, \eta_{ab} \, e_\nu^b(p) \tag{9}$$

from which it is simple to compute the modified Dirac matrices explicit form:

$$\Gamma^\mu = e_a^\mu(p) \, \gamma^a \tag{10}$$

Even the spinor fields result modified and it is possible to compute their normalization, requiring the plane wave expansion preservation:

$$\psi^+(x) = u_r(p)e^{-ip_\mu x^\mu}$$
$$\psi^-(x) = v_r(p)e^{ip_\mu x^\mu} \tag{11}$$

where the spinors $u_r(p)$ and $v_r(p)$ result modified compared to the usual definition. Finally it is possible to obtain an amended formulation of the SM of particles based on the principle that every field has an associated vierbein used to project the physical quantities from the modified personal curved spacetime to the common Minkowski space. The amended formulation of the SM can be obtained in the high energy limit, where the modified Dirac matrices can be approximated with their high energy constant limit. As an example the modified QED Lagrangian is reported:

$$\mathcal{L} = \sqrt{|\det[g]|}\;\overline{\psi}(i\Gamma^\mu\partial_\mu - m)\psi + e\sqrt{|\det[\widetilde{g}]|}\;\overline{\psi}\,\Gamma_\mu\,\psi\,\overline{e}^\mu_\nu\,A^\nu \tag{12}$$

where $\overline{e}$ is the *vierbein* associated with the gauge field and the index μ represents the Minkowski spacetime coordinates $(TM, \eta_{\mu\nu})$. The term multiplying the conserved current is a generalization of the analogous term borrowed from curved spacetime QFT. In the conserved currents the corrections for the fermions wave functions and the correction of the modified Dirac matrices compensate, instead the gauge bosons are supposed as Lorentz invariant (posing $\overline{e} = \mathbb{I}$.

Using as kinematical symmetry group the new amended Lorentz-Poincaré group, the Coleman-Mandula theorem results still valid. The SM internal gauge group $SU(3) \times SU(2) \times U(1)$ is preserved. This means that no exotic interactions or exotic particles are introduced. Therefore HMSR implies an isotropic minimal Standard Model of particles extension.

3. HMSR phenomenology
3.1. Ultra High Energy Cosmic Rays and GZK cut-off
Cosmic rays are bare nuclei of extraterrestrial origin that are revealed at soil on Earth. Because of the high energy they can reach and the cosmological distances over which they propagate, Ultra High Energy Cosmic Rays (UHECR) are one ideal environment where testing Lorentz Invariance. These charged particles are accelerated by astronomical objects and present an upper limit on their acceleration energy caused by the physical limits of the candidate sources. Moreover Universe is not transparent for the propagation of charged cosmic rays, since these particles interact with the Cosmic Microwave Background (CMB) and dissipate energy. Protons interact with the CMB via a photopion production:

$$p + \gamma \to \Delta \to p + \pi^0$$
$$p + \gamma \to \Delta \to n + \pi^+ \tag{13}$$

instead more complex nuclei interact through a photodissociation process:

$$A + \gamma \to (A-1) + n \tag{14}$$

where A represents the atomic number of the bare nucleus considered. This means that even Ultra High Energy Cosmic Rays (cosmic rays with energy that exceeds $10^19\,eV$) propagating for a long enough path in Universe thermalize with the CMB and can be detected only under a determined energy soil. This process is named GZK (from the name of the physicists Greizen Zatsepin and Kuzmin) cut-off of the cosmic rays detected spectrum. In this work only the propagation of light cosmic rays (protons)is analyzed. In particular the introduction of LIV in photopion production is analyzed. In this sector Lorentz violation causes a modification of the inelasticity of the photopion production and determines the propagating proton attenuation

length modification.

The *attenuation length* of a proton is defined as the average length of propagation that the particle must travel to reduce its energy by a factor of e^{-1} and is calculated integrating the probability of interaction of the Cosmic Microwave Background (CMB) with a proton propagating in the Universe:

$$\tau_{p\gamma} = \frac{1}{l_{p\gamma}} = \int_{E_{thr}}^{+\infty} dE \int_{-1}^{+1} d\mu \frac{1-\mu}{2} n(E) \sigma_{p\gamma}(s) K(s) \tag{15}$$

where $\mu = \cos(\theta)$, $\sigma_{p\gamma}$ represents the cross section of the photon-proton interaction, K is the inelasticity of the process and $n(E)$ is the Planck black body distribution of the CMB radiation. Finally it is possible to obtain the optical depth in the form [18]:

$$\tau_{p\gamma} = \frac{-KT}{2\pi^2\gamma^2} \int_{E_{thr}}^{+\infty} d\omega \, \sigma_{p\gamma}(\omega) K(\omega) \omega \ln(1 - e^{-\omega/2KT\gamma}) \tag{16}$$

The inelasticity of the reaction is defined as the energy fraction conserved by the proton after the photopion production. The previous computation must be conducted in a flat space, since the propagation of UHECR happens in an asymptotically flat spacetime. So the most evident effect on the photopion process is limited to the modification of the inelasticity function, that is a modification of the allowed phase space for this kind of reaction. The inelasticity evaluated in a standard physics scenario is given by the formula:

$$K(s) = \frac{1}{2}\left(1 + \frac{m_p^2 - m_\pi^2}{s}\right) \tag{17}$$

Instead the modified kinematics caused by the introduction of LIV [19, 20]changes the inelasticity computation and this function becomes:

$$(1 - k_\pi(\theta)) = \frac{1}{\sqrt{s}}\left(F(s) + \cos\theta\sqrt{F(s)^2 - m_p^2 + f_p p^2}\right) \tag{18}$$

where $F(s)$ is the function:

$$F(s) = \frac{s + m_p^2 - f_p p_p^{*2} - m_\pi^2 + f_\pi p_\pi^{*2}}{2\sqrt{s}} \tag{19}$$

where f_p and f_π represent respectively the proton and the pion related perturbations.

Now it is possible to obtain the inelasticity as a function of the collision angle θ. The quantity must then be averaged on the interval $\theta \in [0, \pi]$ to obtain the inelasticity useful for the computation:

$$k_\pi = \frac{1}{\pi}\int_0^\pi k_\pi(\theta) \, d\theta \tag{20}$$

The inelasticity computation is conducted in the laboratory frame reference, but the obtained result must be valid in every reference frame, since the LIV theory adopted is covariant and preserves isotropy with respect to generalized Lorentz/Poincaré transformations. The inelasticity depends on the difference between the magnitude of the perturbation correlated with the proton and the pion: $\delta f = f_\pi - f_p$. The following plots (Figure 3.1) report the standard inelasticity compared with the LIV case inelasticity, showing the dramatic change in the allowed phase space for the photopion production in the case of $\delta f = 6 \times 10^{-23}$ [20].

Finally in Figure 3.1 the attenuation length for protons is plotted without LIV and in the case of LIV for various values of the parameter δf [20].
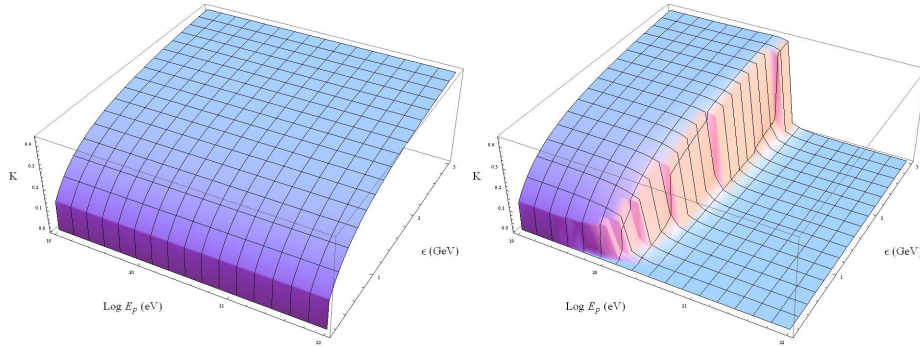
Figure 2. Standard Physics photopion production inelasticity and inelasticity in the case of LIV corrections and $\delta f = 6 \times 10^{-23}$.
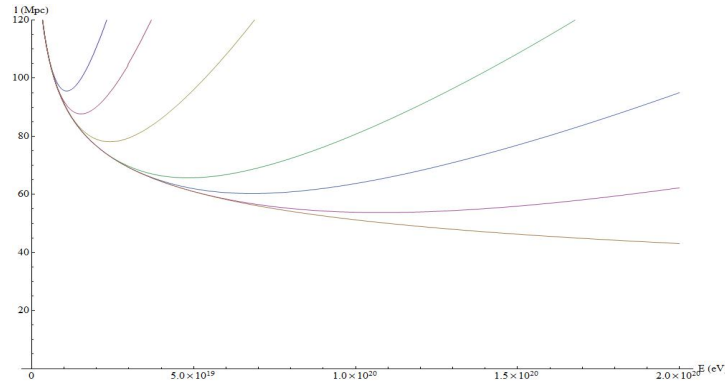

Figure 3. Attenuation length as a function of the energy of the proton and of the LIV parameter: 1) $\delta f = 9 \times 10^{-23}$, 2) $\delta f = 6 \times 10^{-23}$, 3) $\delta f = 3 \times 10^{-23}$, 4) $\delta = f = 9 \times 10^{-24}$, 5) $\delta f = 6 \times 10^{-24}$, 6) $\delta f = 3 \times 10^{-24}$, 7) $\delta f = 0$.

3.2. Neutrino flavor oscillations

Neutrino phenomenology is an ideal environment where conducting the search for new "exotic" physics, since the oscillation phenomenon is not included in the original formulation of the minimal Standard Model (SM) of particles. Since in oscillation phenomenon are involved three different particle species (three different neutrino mass eigenstates), it may be promising to test Lorentz Invariance in this particular sector.

A way to introduce LIV in flavor oscillations consists in modifying the neutrino dispersion relations in order to obtain a geometrical description for the effects of the particle interaction with the background [21, 22]. The neutrino mass eigenstates propagation in vacuum is described by the Schrödinger equation, whose solutions in the form of plane waves are:

$$e^{i(p_\mu x^\mu)} = e^{i(Et - \overrightarrow{p} \cdot \overrightarrow{x})} = e^{i\phi} \tag{21}$$

The MDR can be supposed as required by HMSR (1) and resorting to the approximation of ultrarelativistic particle $|\overrightarrow{p}| \simeq E$, one can obtain:

$$|\overrightarrow{p}| = \sqrt{|\overrightarrow{p}|^2 \left(1 - f\left(\frac{|\overrightarrow{p}|}{E}\right)\right) + m^2} \simeq E\left(1 - \frac{1}{2}f\left(\frac{|\overrightarrow{p}|}{E}\right)\right) + \frac{m^2}{2E} \tag{22}$$

It is therefore possible to compute the plane wave phase ϕ for every mass eigenstate, in natural measure units, i.e. $t = L$:

$$\phi = Et - EL + \frac{f}{2}EL - \frac{m^2}{2E}L = \left(fE - \frac{m^2}{E} \right)\frac{L}{2} . \tag{23}$$

The two mass neutrino eigenstates phase difference at given energy can be written as:

$$\Delta\phi_{kj} = \phi_j - \phi_k = \frac{(f_j - f_k)}{2}EL - \left(\frac{m_j^2}{2E} - \frac{m_k^2}{2E} \right)L = \left(\frac{\Delta m_{kj}^2}{2E} - \frac{\delta f_{kj}}{2}E \right)L \tag{24}$$

In addition to the usual 3×3 unitary matrix PMNS, the oscillation probability shows therefore a dependence on the phase differences $\Delta\phi_{kj}$. It is essential to notice that HMSR foresees oscillation effects caused by the difference of perturbations between different mass eigenstates. The reasonable physical assumption is that every mass state presents a different personal MAV. It is even important to underline that the form of LIV, predicted by HMSR necessitates resorting to masses to explain the neutrino oscillation. In fact, the LIV induced perturbation terms are proportional to the energy of the particle, in contrast with the evidences of neutrino oscillations for the general pattern.

In Figure 3.2 are plotted the standard Physics oscillation patterns compared with the LIV foreseen effects in the case of a neutrino beam with energy $E = 1\,GeV$ and phase differences $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-23}$ [21, 22]. For smaller δf_{ij} LIV parameters, oscillation corrections
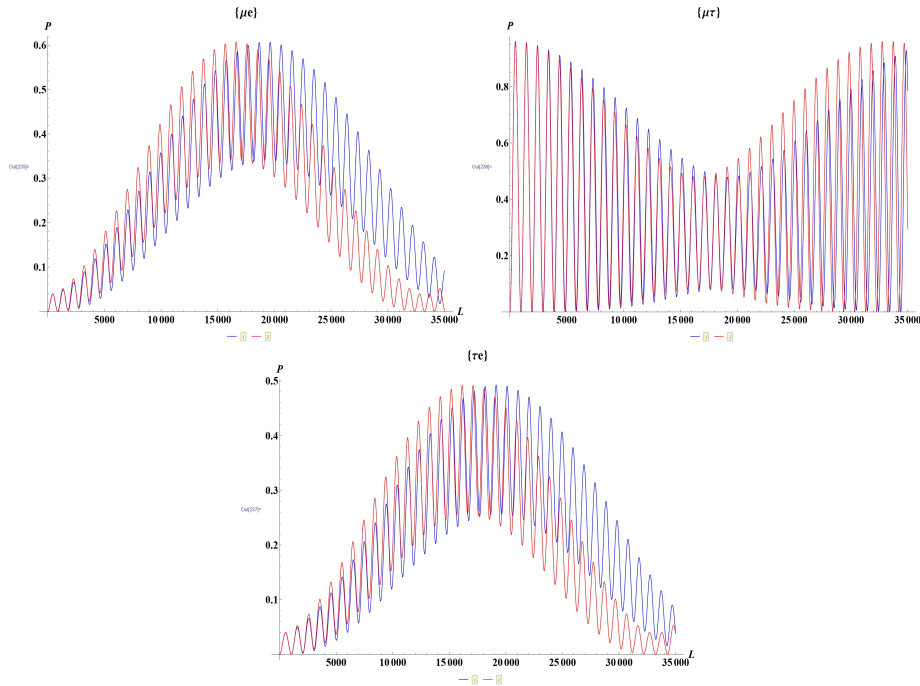

Figure 4. Oscillation probabilities, computed for neutrino energy $E = 1\,GeV$, standard theory" (red curve) and LIV (blue curve), for LIV parameters $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-23}$, as function of the baseline L. Baseline L in km

remain visible only for higher neutrino beams energy. In Figure 3.2 is plotted the same analysis, but in the case of beams with energy $E = 100\,GeV$, LIV parameters $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$ and longer baseline [21, 22].
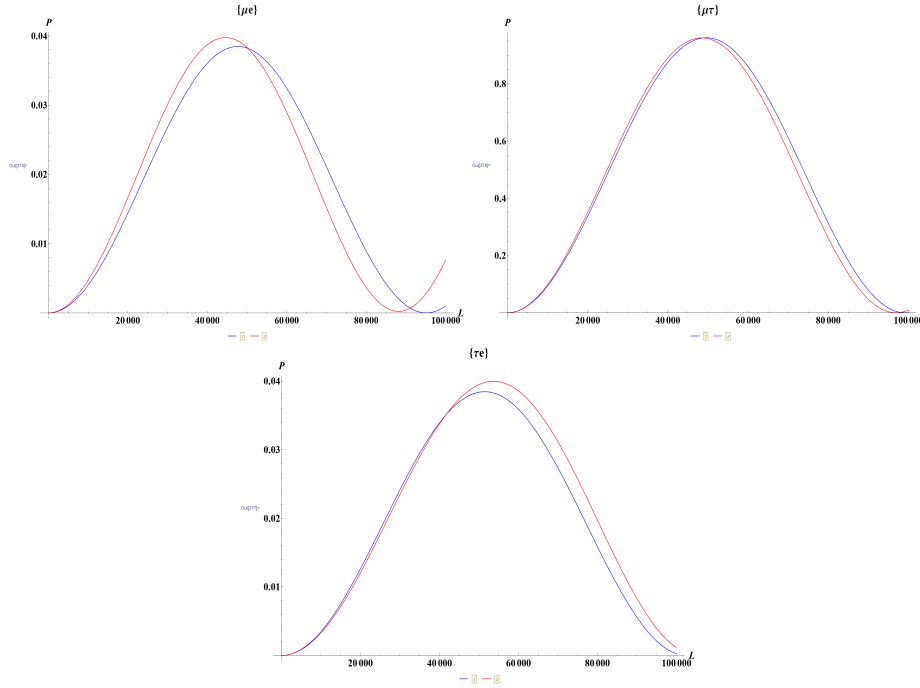
Figure 5. Same analysis as the previous case, "standard theory" (red curve) and LIV (blue curve), for LIV parameters $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$ and energy $E = 100\,GeV$, as function of the baseline L. Baseline L in km

4. Conclusion

Homogeneously Modified Special Relativity (HMSR) represents a new LIV theoretical model with the original feature of preserving the covariance with respect to rotations and boosts. This constitutes a great advantage for experiments, since the theory does not require the introduction of a preferred reference frame. The foreseen phenomenological effects are expected to be valid in every reference frame and the spacetime isotropy results preserved, even if with respect to an amended formulation of the kinematical symmetry group, i.e. the Lorentz/Poincaré group. Indeed the original mathematical formalism introduced in HMSR can be used to geometrically describe the supposed quantum structure of spacetime and can provide a way to formulate an effective field theory that presents the aspects of a fundamental physical theory. Moreover in the context of HMSR it results possible to predict phenomenological visible effects that can be employed to test Lorentz Invariance in a wide spectrum of physical processes. In this work the promising analysis for UHECR and for neutrino oscillation phenomenon are presented. In the case of UHECR, the predicted effect consists in a dilatation of the GZK opacity sphere. This phenomenon can be tested studying the candidate sources of ultra high energy protons. The presence of LIV in this sector can be established if the origin of these high energy particles is located outside the predicted opacity sphere. Another promising sector consists in studying the LIV effects on neutrino oscillations. The preliminary analysis seems to indicate that new under construction experiments with great sensitivities can be used to pose more precise constrains on LIV sector. Indeed a more extensive phenomenological analysis is pursued for cosmic neutrino detected by neutrino telescopes (IceCube, Auger) and high energy atmospheric neutrinos (JUNO experiment).

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