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Poster

Visual assessments of functional maps

S. Melzi[1], R. Marin[1], P. Musoni[2], U. Castellani[1], M. Tarini [2]

[1] Università degli Studi di Verona, Italy [2] Università degli Studi di Milano, Italy

Abstract

*Shape-matching is one central topic in Geometry Processing, with numerous important applications in Computer Graphics and shape analysis, such as shape registration, shape interpolation, modeling, information transfer and many others. A recent and successful class of shape-matching methods is based on the functional maps framework [OBCS*12] where the correspondences between the two surfaces is described in terms of a mapping between functions. Several effective approaches have been proposed to produce accurate and reliable functional maps, leading to need for a way to assess the quality of a given solution. In particular, standard quantitative evaluation methods focus mainly on the* global *matching error disregarding the annoying effects of wrong correspondences along the surface details. Therefore, in this context, it is very important to pair quantitative numeric evaluations with a* visual, *qualitative assessment. Although this is usually not recognized as a problem, the latter task is not trivial, and we argue that the commonly employed solutions suffer from important limitations. In this work, we offer a new visual evaluation method which is based on the transfer of the* object-space normals *across the two spaces and then visualize the resulting lighting. In spite of its simplicity, this method produces readable images that allow subtleties of the mapping to be discerned, and improve direct comparability of alternative results.*

CCS Concepts
• ***Mathematics of computing*** → ***Functional analysis;*** • ***Computing methodologies*** → *Shape modeling; Shape analysis;*

1. Background: functional maps

Given a pair of shapes $\mathcal{M}$ and $\mathcal{N}$ embedded in $\mathbb{R}^3$ and a point-to-point map $\Pi : \mathcal{N} \mapsto \mathcal{M}$, we consider the *functional map* T induced by Π:

$$T(f) = f \circ \Pi, \ \forall f : \mathcal{M} \to \mathbb{R}. \qquad (1)$$

In the most common discrete settings T is encoded as a small matrix C and both the functional spaces, defined on $\mathcal{M}$ and $\mathcal{N}$, are represented with a truncated subset of the Fourier basis. The approach proposed in [OBCS*12] consists of optimizing for the matrix C under certain suitable constraints. Once C is available it maps the spectral embedding of $\mathcal{M}$ into the space of the spectral embedding of $\mathcal{N}$. Point-to-point map Π can then be reconstructed as the nearest neighbor assignment in the spectral embedding. This approach has been studied intensively, with several recent work improving the estimation of the mapping C and its conversion into a point-to-point map. Within this context, it is very important the task of assessing and comparing the quality of either a given point-to-point mapping between surfaces, or the functional map which produced it. This foresees a need of an effective *visual* assessment and comparisons. Unfortunately, this task is not trivial, and we argue that existing solutions, which are commonly employed in this specific field, suffer from important limitations (Sec. 2).

2. Commonly employed visual assessment techniques

Smooth colors. This method consists in transferring a smoothed color functions, for gradient between red and blue defined over the entire surface, (see Figure1 top). This can be effective to visually reveal large mismatches, but it is not optimal:
Limitation 1: the color function contains only low frequencies; therefore, it completely hides an important feature of the inspected functional map, namely its ability (or inability) to transfer the high frequency components of a function;
Limitation 2: this method excessively relies on the observer's ability to discern between similar colors.

Texture transfer. In spite of its name, this method does not transfer a "texture" over $\mathcal{N}$ but rather a 2D regular grid orthogonally projected over $\mathcal{M}$. This is analogous of what is done in the field of surface parametrization when a 2D regular grid is over-imposed over the flat texture domain and then lifted onto the surface. The analogy is, however, imperfect:
Limitation 1: in contrast to Surface Parametrization, the 2D grid is already distorted by the orthogonal projection over $\mathcal{M}$. It is virtually impossible, especially from a single rendering, to distinguish between artefacts originating from the projections of the grid over $\mathcal{M}$, and from the trasfrerrment over $\mathcal{N}$.
Limitation 2: in the space between grid-lines, the mapping cannot be discerned at all.

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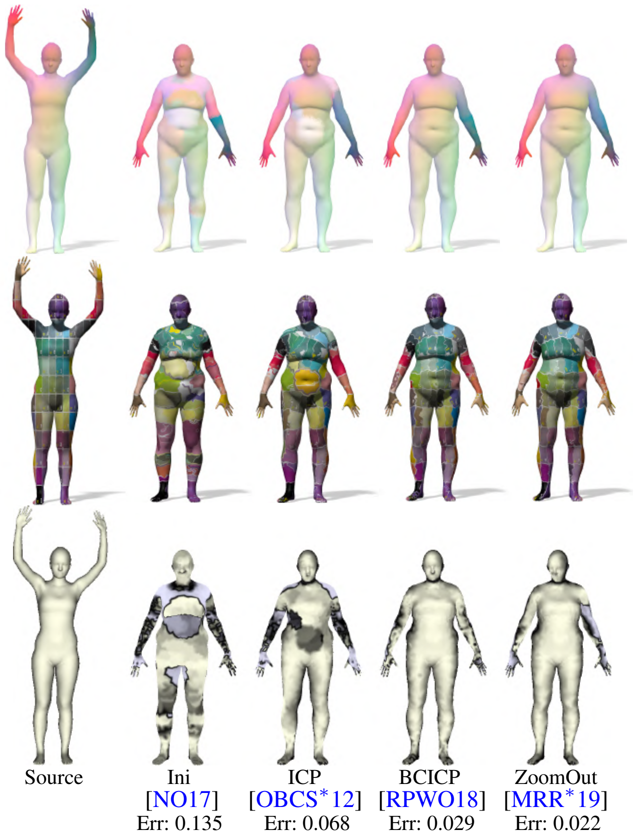

Figure 1: *A comparison of visualization techniques (see Figure 7 from [MRR*19]): smooth colors gradients (top), texture transfer (middle), and the proposed normal-based visual assessment (bottom). The average geodesic error proposed in [MRR*19] is reported as a quantitative evaluation.*

3. New approach: normal-based visual assessment

Method. Per-vertex, object-space normals are computed over $\mathcal{M}$ and then directly transferred to $\mathcal{N}$ using a point-to-point mapping extracted from the function map to be evaluated. Lighting is then applied on $\mathcal{N}$ (using lights environment of $\mathcal{N}$). Note that this makes little sense from a realistic rendering point of view, as the "normals" used to light $\mathcal{N}$ are defined independently of its shape and are therefore unrelated to (and potentially far from) the geometric normals of $\mathcal{N}$ (e.g., they are not orthogonal to its surface). The objective, however, is visual assessment, and this approach is particularly effective in showing imprecise matching caused by wrong high frequency transfer among shapes. Our inspiration comes from the similar need of evaluating the quality of surface 3D to 2D parametrizations, where often object space normals are displayed, and lit, over the 2D texture domain (in spite of the latter being flat, i.e. having constant normals).

Examples. In Figures 1 four alternative functional maps (constructed with four competing recently proposed techniques) are visually compared using three visualization techniques: i) smooth colors gradients (top), ii) texture transfer (middle), and iii) the proposed normal transfer (bottom). Figure 2 offers a close up around the face area. The *normal transfer* visualization suggests that the third functional map is more effective in correctly preserving the geometric features, especially around the head (see close up) and pubis area. This does not emerge as clearly from the two alternatives visualization methods.

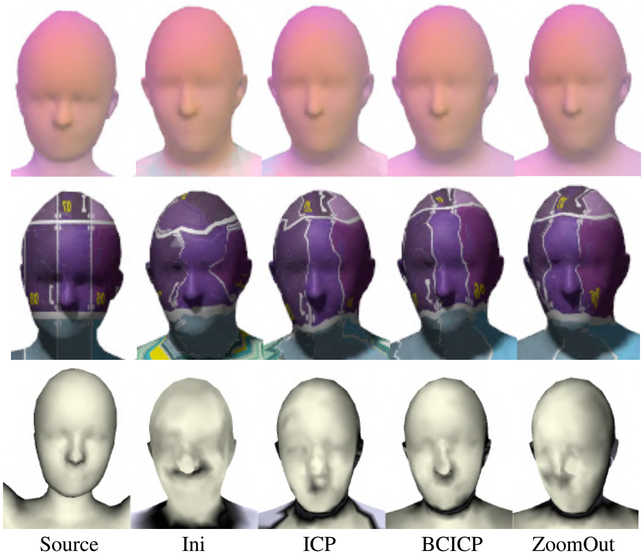

Figure 2: *A close-up of Fig. 1. Our visual assessment provides more information about the error also when the comparison with the ground truth is missed.*

4. Conclusions

We focus on the importance of visual assessment for shape matching and functional maps. Preliminary experiments show that, in spite of its simplicity, the proposed normal transfer method provides a more revealing, and intuitive to read, visual assessments of the imperfections of the inspected mappings. We think that this method can be a good complementary or alternative solution for visually comparing functional maps, and, by extensions, the methods proposed to construct them.

Naturally, visual assessment cannot be intended as a replacement for numerical quantitative quality measurements.

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