Up-Frequency Conversion in a Two-Resonant-Wave High-Gain Free-Electron-Laser Amplifier

N. Piovella, V. Petrillo, C. Maroli, and R. Bonifacio
Dipartimento di Fisica dell'Università di Milano, Via Celoria 16, 20133 Milano, Italy
(Received 15 July 1993)

A free-electron laser is able to resonate at two different frequencies, both in free space and in a waveguide. The two waves have positive and negative slippage. We describe the nonlinear interaction between the two waves by a set of partial differential equations which in free space do not require the slowly varying envelope approximation (SVEA). In a waveguide a less restrictive SVEA is applied to each wave. By injecting a small signal at the low frequency, a strong signal and bunching are produced at the high frequency. This effect suggests a new method of generating short wavelength radiation.

PACS numbers: 41.60.Cr, 42.60.Jf

The possibility of controlling the slippage between the radiation and the electron beam in a free-electron laser (FEL) by means of a waveguide with narrowly spaced parallel plates has been first described in Refs. [1,2] and [3,4], and has stimulated in the past few years considerable theoretical [5,6] and experimental [7–10] work. The waveguide slippage control is particularly important in the generation of far-infrared and microwave radiation by means of short electron pulses, of the order of picoseconds, as provided by rf linear accelerators.

Another important feature of the use of a waveguide is the existence of two different resonant frequencies [4] at which the FEL can operate, both lower than the usual free-space resonant frequency and with positive slippage for the higher frequency and negative for the lower one. Previous papers [5,6,11] have studied the waveguide operation in the low-gain regime. Sternbach and Ghalila [12] have suggested that, in free space, the incoherent, low-frequency radiation could lower the length of the undulator necessary to achieve large values of high-frequency power.

In this Letter, we extend the analysis of waveguide operation to the high-gain regime and, above all, we study the nonlinear space and time interaction between the two resonant waves, which was not taken into account in Ref. [12]. The novel and unexpected result that comes out from our analysis is that, by injecting a signal at the lower frequency, without any input signal at the upper frequency, a strong signal and strong electron bunching at the higher frequency are obtained, suggesting a new and powerful method for the generation of short wavelength radiation.

The dispersion relation for off-axis propagation is given by $\omega = c\sqrt{k^2 + k_\perp^2}$, where k ($k_\perp$) is the longitudinal (transverse) radiation wave number. In particular, for the TE_{01} mode in a rectangular waveguide, $k_\perp = \pi/b$, where b is the smaller transverse dimension of the waveguide. The resonance condition on the axial velocity $v_\parallel = c\beta$ of the electron beam requires that the electron travels a wiggler period $\lambda_w = 2\pi/k_w$ in the same time that the wave travels a distance $\lambda_w + \lambda$ ($\lambda = 2\pi/k$), so that $\lambda_w/v_\parallel = (\lambda + \lambda_w)/v_{\rm ph}$, where $v_{\rm ph} = \omega/k$ is the radiation phase velocity. The previous equality can also be written in the form $\omega = c\beta(k_w + k)$. By equating this last formula to the dispersion relation, one obtains, for $k_\perp \le \beta\gamma_\parallel k_w$ [where $\gamma_\parallel = (1-\beta^2)^{-1/2}$], the expression for the two resonant frequencies [4,5],

$$\omega_{1,2} = \frac{\omega_s}{1+\beta}\left[1 \pm \beta\sqrt{1-X}\right], \qquad (1)$$

with wave numbers $k_{1,2} = (\omega_s/c)[\beta \pm \sqrt{1-X}]/(1+\beta)$, where $\omega_s = c\beta k_w/(1-\beta)$ is the free-space resonant frequency and $X = (k_\perp/\beta\gamma_\parallel k_w)^2$ is the waveguide parameter [1] $0 \le X \le 1$. Furthermore, the group velocities relative to the resonant frequencies are $v_{g1,2} = c^2(k_{1,2}/\omega_{1,2})$, so that the slippage lengths $\ell_{s1,2}$ between the electron beam and the two waves after a wiggler length $L_w = \lambda_w N_w$ are $\ell_{s1,2} = (v_{g1,2} - v_\parallel)L_w/v_\parallel = \pm\sqrt{1-X}(2\pi c/\omega_{1,2})N_w$. We also introduce the frequency ratio parameter $\alpha = \omega_1/\omega_2 = (1+\beta\sqrt{1-X})/(1-\beta\sqrt{1-X})$, with $1 \le \alpha \le (1+\beta)/(1-\beta)$; for high relativistic electron beam ($\gamma_\parallel \gg 1$), $\omega_1 \simeq \alpha\omega_s/(1+\alpha)$ and $\ell_{s1} = -\ell_{s2}/\alpha \simeq \lambda_s N_w(\alpha-1)/\alpha$, with $\lambda_s = 2\pi c/\omega_s$.

The FEL resonance condition is obviously satisfied at two different frequencies also without waveguide [12] ($X = 0$); their expressions are $\omega_1^{(0)} = \omega_s \sim 2ck_w\gamma_\parallel^2$ and $\omega_2^{(0)} = ck_w\beta/(1+\beta) \sim ck_w/2$. The wave at $\omega_2^{(0)}$ is counterpropagating with respect to the electron beam and, since the ratio $\alpha = \omega_1^{(0)}/\omega_2^{(0)} \simeq 4\gamma_\parallel^2$, the frequencies $\omega_1^{(0)}$ and $\omega_2^{(0)}$ are widely different. In the waveguide, the upper frequency ω_1 always corresponds to a forward wave with positive slippage. The lower frequency ω_2, on the other hand, corresponds to a backward wave when $X < \gamma_\parallel^{-2}$ (or $b > \lambda_w/2$), and to a forward wave with negative slippage, in the opposite condition. For $X = 1$ ("zero-slippage condition") $\omega_1 = \omega_2 = \omega_s/(1+\beta)$; i.e., only one resonant frequency exists, which is approximately half the frequency ω_s in free space and corresponds to a wave whose group velocity is equal to the electron beam velocity.

A linear analysis of the spectrum in the low-gain regime [5,6] shows that the gain has two maxima around the resonant frequencies (1). In the high-gain regime [13], the gain grows exponentially near the two resonant frequencies, as $\mathcal{G}(z,\omega) \propto \exp\{2|\mathrm{Im}\lambda|g(\omega)z\}$; here, λ is the complex root of the cubic equation $\lambda^3 - \delta(\omega)\lambda^2 + 1 = 0$ and $\delta(\omega) = -(\omega-\omega_1)(\omega-\omega_2)/2\omega^2\rho(\omega)$ is the detuning parameter, so that the gain curve exhibits maxima at the zeros of $\delta(\omega)$, which occur at the two frequencies (1); $g(\omega) = 2k_w(\omega/\omega_s)\rho(\omega)$ is the gain coefficient and $\rho(\omega) = \rho_0 F(\xi)^{2/3}(\omega_s/\omega)^{2/3}$ is the mode-dependent FEL parameter, with $F(\xi) = J_0(\xi) - J_1(\xi)$, where J_n is the nth-order Bessel function of the first kind and $\xi = (\omega/\omega_s)\xi_0$, with $\xi_0 = a_w^2/2(1+a_w^2)$; the other quantities are as follows: $\rho_0 = \gamma_0^{-1}(a_w\omega_p/4ck_w)^{2/3}$, free space, fundamental FEL parameter [14], with $m_ec^2\gamma_0$, average initial electron energy, $\omega_p = (4\pi e^2 n_e/m_e)^{1/2}$, plasma frequency, $n_e = (2/ab)(I/ec)$ beam density, I electron current, $ab/2$ effective area of the TE_{01} transverse mode in the rectangular waveguide, whose short (long) dimension is b (a); $a_w = eB_w/\sqrt{2}m_ec^2k_w$, wiggler parameter; $\omega_s = 2ck_w\gamma_0^2/(1+a_w^2)$, free-space resonant frequency.

In Fig. 1 we summarize the most important features of the gain function versus the dimensionless frequency ω/ω_s, both in the low-gain regime ($z = \ell_{g0}$, with $\ell_{g0} = \lambda_w/4\pi\rho_0$, dashed line) and in the high-gain regime ($z = 5\ell_{g0}$, continuous line), for four values of the waveguide parameter: (a) $X = 0.75$ ($\alpha = 3$), (b) $X = 0.9$ ($\alpha = 1.92$), (c) $X = 0.96$ ($\alpha = 1.5$), and (d) $X = 1$ ($\alpha = 1$, "zero-slip condition"). We observe that, in the high-gain regime, the gain itself is always positive and that, by increasing the waveguide parameter toward the zero-slip condition $X = 1$, the two maxima of the gain curve move toward one another, until they give rise to a single broad curve. When $X \sim 1$, the gain bandwidth is approximately given by $\Delta\omega/\omega \sim 2^{4/3}\sqrt{\rho_0}$ [see Fig. 1(d)]. By requiring that the frequency separation between the two maxima be larger than the gain bandwidth, we conclude that they are well separated when $\alpha - 1 \gg \sqrt{\rho_0}$, or in terms of the waveguide parameter, when $1 - X \gg \rho_0$.

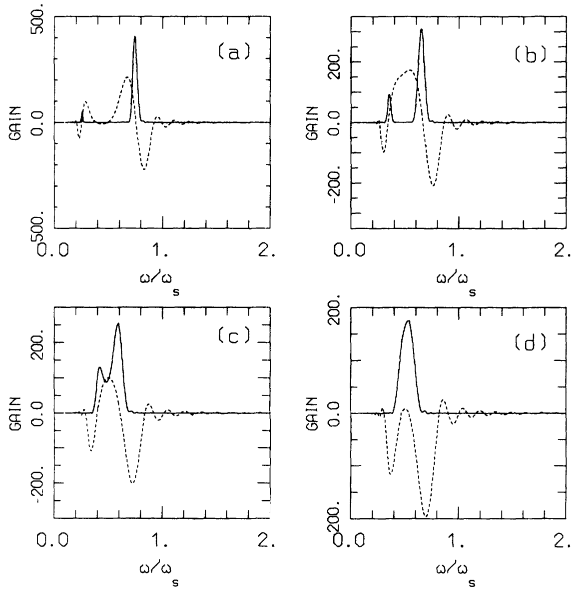

FIG. 1. Gain versus the dimensionless frequency ω/ω_s, both in the low-gain ($z = \ell_{g0}$, dashed line) and in the high-gain regime ($z = 5\ell_{g0}$, continuous line), for $\rho_0 = 0.01$ and $a_w = 1$, at a fixed position z inside the wiggler and for (a) $X = 0.75$; (b) $X = 0.9$; (c) $X = 0.96$; (d) $X = 1$; the values relative to the low-gain regime (dashed line) have been multiplied by a factor of 500.

The existence of two resonant frequencies raises the question of whether coupling can exist between the two waves as a result of the nonlinear interaction. In order to study this nonlinear interaction in the limit $\alpha - 1 \gg \sqrt{\rho_0}$, i.e., when interference effects between the two waves can be ignored, we derived the following set of partial differential equations:

$$\frac{\partial^2\theta_{2j}}{\partial\overline{z}^2} = -\left(fA_1e^{i\alpha\theta_{2j}} + A_2e^{i\theta_{2j}} + \text{c.c.}\right), \quad (2)$$

$$\left[\frac{\partial}{\partial\overline{z}} + s_1\frac{\partial}{\partial\overline{z}_1}\right]A_1 = f\left\langle e^{-i\alpha\theta_2}\right\rangle, \quad (3)$$

$$\left[\frac{\partial}{\partial\overline{z}} + s_2\frac{\partial}{\partial\overline{z}_1}\right]A_2 = \epsilon\left\langle e^{-i\theta_2}\right\rangle, \quad (4)$$

where $j = 1,\ldots,N$ and $\langle(\cdots)\rangle = N^{-1}\sum_{j=1}^{N}(\cdots)$. In deriving these equations, we assumed a transverse mode TE_{01} inside a rectangular waveguide, whose short (long) dimension is b (a), parallel (orthogonal) to the wiggler field $\mathbf{B}_w = -\hat{\mathbf{y}}B_w\sin(k_wz)$, with the following vector potential:

$$\mathbf{A}(\mathbf{x},t) = -(i/\sqrt{2})\hat{\mathbf{x}}\sin(k_\perp y) \times\left[\sum_{m=1}^{2}(cE_{01}^{(m)}/\omega_m)e^{i\psi_m} - \text{c.c.}\right], \quad (5)$$

where $\psi_m = k_mz - \omega_mt$ and ω_m (k_m) are the resonant frequencies (wave numbers) defined in (1), with $m = 1,2$; $E_{01}^{(m)}(z,t)$ are slowly varying functions of their arguments and $\theta_2 = k_wz + \psi_2$ is the electron phase obtained as a combination of the wiggler and the lower frequency mode phases. We have introduced dimensionless variables as follows: $\overline{z} = z/\ell_{g2}$, scaled coordinate along the wiggler, with $\ell_{g2} = (\lambda_w/4\pi\rho_2)(1+\alpha)$, gain length; $\overline{z}_1 = 2k_2\rho_2(z - v_\parallel t)$, scaled time in the frame moving with the average electron velocity $v_\parallel$; $\rho_2 = \rho(\omega_2) = \rho_0F_2^{2/3}(1+\alpha)^{2/3}$ and $f = F_1/F_2$, with $F_{1,2} = F(\xi_{1,2})$ and $\xi_1 = \alpha\xi_2 = \alpha\xi_0/(1+\alpha)$; $A_m = E_{01}^{(m)}/\sqrt{4\pi m_ec^2\gamma_0n_e\rho_2}$, dimensionless wave amplitude; $\epsilon = v_{g1}/v_{g2}$, ratio between the group velocities of the two waves, that can be set equal to one for highly relativistic electrons; $s_1 = (\alpha-1)/\epsilon\alpha$ and $s_2 = 1 - \alpha$, slippage coefficients. Equations (2)–(4) are derived neglecting space-charge effects, in the Compton limit $|\gamma_j - \langle\gamma_0\rangle| \ll \langle\gamma_0\rangle$. Equations (3) and (4) describe the evolution of the two resonant waves separately and have been derived assuming the resonant frequency ratio α to be a rational number, m_1/m_2. With this assumption, a temporal average over the interval $2\pi m_2/\omega_2$ removes the fast-oscillating terms proportional to $\exp(2i\psi_{1,2})$ and $\exp[i(\psi_1 \pm \psi_2)]$ in the wave equations.

This approximation requires an electron beam several low-frequency periods $2\pi/\omega_2$ long. Equations (2)–(4) are formally similar to those describing the excitation of the harmonics in a planar wiggler [15,16], where A_1 appears as a "pseudoharmonic" field of the "fundamental" A_2. However, they differ from those of Ref. [16] in three main aspects: first, α is not an odd number and can be varied continuously by changing the waveguide height; second, the slippage depends on α and is different for the two waves; third, the Bessel factor f in Eqs. (2) and (3) differs from the usual factor for the odd harmonics in the planar wiggler [15], $f_h = J_{(h-1)/2}(h\xi_0) - J_{(h+1)/2}(h\xi_0)$ (h odd), and tends to $F(\xi_0)$ for large α.

The equations have been integrated along the characteristics with boundary conditions for the fields on the trailing $[A_1(\overline{z},0) = A_{10}]$ and on the leading $[A_2(\overline{z},\overline{\ell}_b) = A_{20}]$ edges of the electron bunch, where $\overline{\ell}_b = 2k_2\rho_2\ell_b$ and ℓ_b is the electron bunch length. Furthermore, we have assumed $A_1(\overline{z}=0) = A_{10}$, $A_2(\overline{z}=0) = A_{20}$, $(\partial\theta_{2j}/\partial\overline{z})(\overline{z}=0) = 0$ $(j = 1,\ldots,N)$ and the distribution of θ_{2j} at $\overline{z} = 0$ such that both bunching parameters $b_1 = \langle\exp(-i\alpha\theta_2)\rangle$ and $b_2 = \langle\exp(-i\theta_2)\rangle$ are zero, with $\alpha = m_1/m_2$.

The numerical integration shows that if a wave at the upper frequency ω_1 is injected into the wiggler, no appreciable deviations from the customary results obtained with the one-wave model in the slowly varying envelope approximation (SVEA) appear. In fact, A_1 exhibits the usual behavior, while A_2 and the bunching parameter b_2 on the lower frequency remain almost zero. Much more interesting and unexpected is the opposite situation, when a low-frequency wave is injected into the wiggler. In fact, in this case, an intense signal and strong bunching grow on the upper frequency ω_1. The intensity and the characteristics of this emission depend on the parameter α. In Fig. 2, for example, $|A_1|^2$ and $|A_2|^2$ as well as $|b_1|$ and $|b_2|$ versus $\overline{z}$ are shown for $\alpha = 10$, $\overline{z}_1 = 2.9$, $\overline{\ell}_b = 4$, and $|A_{20}|^2 = 2.5\times10^{-3}$. As can be seen, the first maximum of $|A_1|^2$ reaches a value of about 0.08 and appears at the same position in the wiggler as the first maximum of $|A_2|^2$, but a maximum of larger amplitude ($|A_1|^2 \sim 0.3$) develops farther down the wiggler. As regards the shape of the bunchings, one can see that they attain comparable values ($|b_1| \sim 0.5$ and $|b_2| \sim 0.7$). Summarizing, we are able to generate a strong bunching and a signal approximately 2 orders of magnitude more intense than the injected signal and at a frequency 10 times higher.

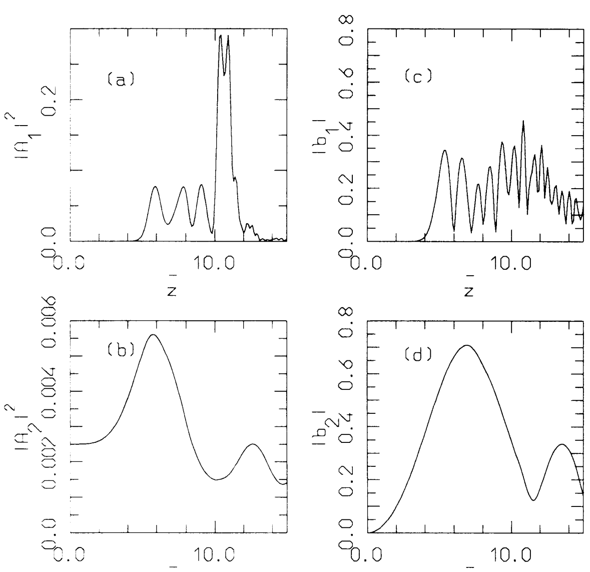

FIG. 2. Emitted intensity and bunching at the lower and upper frequency, as a function of $\overline{z}$, when a small signal at the lower frequency is injected, $|A_{20}|^2 = 2.5\times10^{-3}$, for $\overline{z}_1 = 2.9$, $\overline{\ell}_b = 4$, $a_w = 1$, and $\alpha = 10$; (a) $|A_1|^2$; (b) $|A_2|^2$; (c) $|b_1|$; (d) $|b_2|$.

The dependence of the emission on the parameter α has also been studied. The results of this analysis are summarized in Fig. 3 which gives the first maximum of $|A_1|$ and the corresponding value of $|b_1|$ at $\overline{z}_1 = 2$ as a function of α. As can be seen, $|A_1|$ and $|b_1|$ develop a series of maxima for integer values of α, which decrease rather slowly with a scaling low approximately given, for $\alpha = n$, by $|A_1|_{\max} \propto n^{-1/2}$ and $|b_1|_{\max} \propto n^{-1/3}$ (the dashed lines in the figure). For noninteger α, the values attained by $|A_1|$ are smaller, but slightly increasing with α, so that the difference between the cases α integer and α noninteger tends to diminish. Furthermore, cases performed with noninteger α show that emission comparable to that obtained with integer α can be achieved, provided the length of the wiggler be suitably increased. For instance, for $\alpha = 2.66$, $|A_1| \sim 0.5$ at $\overline{z}_1 = 3$ and $\overline{z} = 18$.

A simple physical interpretation of all preceding results can be given in the following terms: strong bunching on a wavelength λ_2 also gives rise to an equally strong bunching on a wavelength $\lambda_1 = \lambda_2/n$, since the electrons as seen in the short wavelength field are packed together every n wavelength λ_1.

It is also possible to have an analytical evidence that, for integer α, a strong bunching on the upper frequency is generated by nonlinear coupling with the lower-frequency field, when a small signal at the lower fre-

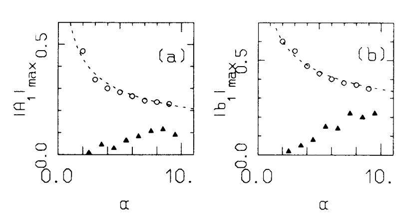

FIG. 3. Dependence of the maximum of $|A_1|$ (a), and $|b_1|$ (b), on the frequency ratio α, at $\overline{z}_1 = 2$ and for $\overline{\ell}_b = 4$, $\overline{z}_{\max} = 15$, $A_{10} = 0$, and $a_w = 1$; open circles refer to simulations with integer α and $A_{20} = 0.05$; black triangles refer to simulations with half-integer α and $A_{20} = 0.01$; dashed lines are numerical fits, $|A_1|_{\max} = 0.58\times\alpha^{-0.43}$ and $|b_1|_{\max} = 0.76\times\alpha^{-0.34}$.

quency is injected. In fact, following the method discussed in Ref. [17] for an optical klystron configuration, it is easy to show that, solving Eq. (2) for θ_2 in the absence of the upper frequency field A_1, the high-frequency bunching b_1, induced by the low-frequency emission when $\alpha = n$, is $b_1 = e^{in\phi_2}J_n[2nB_2]$, where $B_2 e^{i\phi_2} \equiv \int_0^{\bar{z}} d\bar{z}'(\bar{z}-\bar{z}')A_2(\bar{z}',\bar{z}_1)$, whereas it vanishes for noninteger values of α. Neglecting the slippage, a direct calculation of the linear solution gives $\phi_2 \sim -2\pi/3+\bar{z}/2$ and $B_2 \sim (A_{20}/3)\exp[\sqrt{3}\bar{z}/2]$. Since $J_n(x)$ behaves as x^n for small values of its argument, the growth rate of the upper frequency bunching b_1 is n times larger than the growth rate of the lower-frequency bunching. Moreover, since the maximum value of J_n decreases slowly with n, the bunching b_1 is quite large also for large values of the frequency ratio n. With similar calculation, it is possible to demonstrate that the low-frequency bunching induced by the growth of the upper-frequency field A_1 is zero for any values of α. This demonstrates that the upper-frequency radiation cannot produce radiation at the lower frequency.

Equations (2)–(4) have been derived assuming the SVEA separately for the two wave packets A_1 and A_2, whereas the usual waveguide FEL model [1] takes only the forward wave A_1 into account. It can be shown that a set of partial differential equations similar to Eqs. (2)–(4) can be obtained for free-space propagation, with a helical wiggler and a circularly polarized electromagnetic field, without using the SVEA. In fact, assuming $\mathbf{E}(z,t) = E_0(z,t)\hat{\mathbf{e}} + \text{c.c.}$ and $\mathbf{B}(z,t) = -i[B_0(z,t)\hat{\mathbf{e}} - \text{c.c.}]$, with $\hat{\mathbf{e}} = (\hat{\mathbf{x}} + i\hat{\mathbf{y}})/2$, and a wiggler field $\mathbf{B}_w = -(m_e c^2 k_w/e)a_w[\hat{\mathbf{e}}\exp(-ik_w z)+\text{c.c.}]$, the set of Eqs. (2)–(4) are indeed obtained, provided that $s_1 = 1$, $s_2 = \alpha$, $f = 1$, and $\epsilon = -1$. In addition, the quantities appearing in Eqs. (2)–(4) must be defined as follows:

$$A_{1,2} = \frac{E_0 \pm B_0}{2\sqrt{4\pi m_e c^2 \gamma_0 n_e \rho_2^{(0)}}} \exp[\pm i\omega_{1,2}^{(0)}(t \mp z/c)], \quad (6)$$

$\bar{z} = (2k_w\rho_2^{(0)}/\alpha)z$ and $\bar{z}_1 = -2\omega_2^{(0)}\rho_2^{(0)}(t - z/v_\parallel)$, where $\rho_2^{(0)} = \rho_0\alpha^{2/3}$, $\omega_{1,2}^{(0)}$ have been defined previously and $\alpha = 4\gamma_0^2/(1+a_w^2)$.

In free-space propagation, A_2 is a counterpropagating wave of central frequency $\omega_2^{(0)}$ and is usually neglected in the SVEA. It is important to note that the transformation (6) reduces in an exact way the second-order wave equation into two first-order partial differential equations for the waves A_1 and A_2 separately, which are coupled only via the bunching factor. The main difference between the propagation in waveguide and in free space is that in this last case A_2 is a backward wave, and that different boundary conditions on $\bar{z}$ must be properly assumed. With a waveguide, the longitudinal field component couples the two transverse components; the use of the SVEA is therefore necessary to separate the second-order wave equation in two first-order equations for each resonant frequency. As discussed in Ref. [18], the limit of validity of the SVEA is that the radiation pulse length ℓ_p must be much larger than $\lambda(1-\beta) \sim \lambda/\gamma_\parallel^2$.

In conclusion, we have studied the nonlinear interaction between the two resonant waves in a FEL, one with higher frequency and positive slippage and the other one, which is usually neglected in the SVEA, with lower frequency and negative slippage. We have shown that by injecting a small signal at the lower frequency, strong bunching and signal at the upper frequency can be obtained. This up-frequency conversion process is maximum when the frequency ratio is an integer number. This method can be of most practical interest in the generation of microwave or infrared radiation, when conventional input sources are not easily available. We envision the following set of parameters for a proof-of-principle experiment to generate radiation at 430 GHz with an input source at 43 GHz: $\lambda_w = 10$ cm, $N_w = 100$, $a_w = 2$, waveguide dimensions, 10×50 mm^2, electron beam energy, 9.6 MeV, $I = 100$ A, beam duration, 100 ps, input power at 43 GHz, 150 kW, output power at 430 GHz, 18 MW, efficiency, 2%. An extension to higher frequency is in principle possible but requires longer wigglers and beams.

[1] R. Bonifacio and L. De Salvo Souza, Nucl. Instrum. Methods Phys. Res., Sect. A **289**, 394 (1989).

[2] R. Bonifacio *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **289**, 1 (1990).

[3] S.S. Yu, W.M. Sharp, W.M. Fawley, and E.T. Scharlemann, Nucl. Instrum. Methods Phys. Res., Sect. A **259**, 219 (1987).

[4] S.K. Ride, R.M. Pantell, and J. Feinstein, Appl. Phys. Lett. **57**, 1283 (1990).

[5] A. Doria, G.P. Gallerano, and A. Renieri, Opt. Commun. **80**, 417 (1991).

[6] W.J. Golightly and S.K. Ride, IEEE J. Quantum Electron. **27**, 2656 (1991).

[7] F. Ciocci *et al.*, Phys. Rev. Lett. **70**, 928 (1993).

[8] Li-Yi Lin and T.C. Marshall, Phys. Rev. Lett. **70**, 2403 (1993).

[9] R.J. Harvey, F.A. Dolezal and D.J. Gregoire, Nucl. Instrum. Methods Phys. Res., Sect. A **318**, 736 (1992).

[10] Y. Kawamura *et al.*, Phys. Rev. E **47**, 4399 (1993).

[11] P.E. Latham and B. Levush, IEEE Trans. Plasma Sci. **18**, 472 (1990).

[12] E.J. Sternbach and H. Ghalila, Nucl. Instrum. Methods Phys. Res., Sect. A **304**, 691 (1991).

[13] N. Piovella, V. Petrillo, C. Maroli, and R. Bonifacio, in Proceedings of the 15th International Free Electron Laser Conference, The Hague, The Netherlands, 23–27 August 1993 (to be published).

[14] R. Bonifacio, C. Pellegrini, and L. Narducci, Opt. Commun. **50**, 373 (1984).

[15] W.B. Colson, Phys. Rev. A **24**, 639 (1981).

[16] J.B. Murphy, C. Pellegrini, and R. Bonifacio, Opt. Commun. **53**, 197 (1985).

[17] R. Bonifacio, R. Corsini, and P. Pierini, Phys. Rev. A **45**, 4091 (1992).

[18] R. Bonifacio, R.M. Caloi, and C. Maroli, Opt. Commun. **101**, 185 (1993).