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Neutrino capture cross sections for ^{40}Ar and β-decay of ^{40}Ti

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Abstract

Shell-model calculations of solar neutrino absorption cross sections for ^{40}Ar, the proposed component of the ICARUS detector, are presented. It is found that low-lying Gamow–Teller transitions lead to a significant enhancement of the absorption rate over that expected from the Fermi transition between the isobaric analog states, leading to an overall absorption cross section for ^{8}B neutrinos of $(11.5 \pm 0.7) \times 10^{-43}$ cm^2 or a total expected rate of 6.7 ± 2.5 SNU. We also note that the pertinent Gamow–Teller transitions in ^{40}Ar are experimentally accessible from the β-decay of the mirror nucleus ^{40}Ti. Predictions for the branching ratios to states in ^{40}Sc are presented, and the theoretical halflife of 55 ± 5 ms is found to be in good agreement with the experimental value of 56^{+18}_{-12} ms.

Keywords: Nuclear structure; Solar neutrino spectroscopy

The basic feature of solar neutrino astronomy is to provide a tool that permits the direct examination of processes that occur in the interior of the Sun. Neutrinos interact only via the weak interaction, and have essentially an infinite mean-free path in a normal stellar medium. Therefore, neutrinos observed on Earth probe solar processes that are occurring in the present, as opposed to photons, which emerge after $\approx 10^4$ yr.

Considerable interest has been generated by four solar neutrino experiments [1–4] that yield results that are different from those expected from the combined predictions of the standard solar model and the standard electroweak theory with zero neutrino masses. In essence, there seems to be a significant suppression in the neutrino flux over that predicted from the combined standard models. Two primary questions then remain: (1) do these experiments require new physics beyond the standard model for electroweak interactions, or (2) is the standard solar model at fault? A recent study [5] of over 1000 precise solar models concludes that it is not possible to simultaneously describe all four experiments within the framework of standard solar models, and suggests that physics beyond the standard electroweak theory is required. However, before definitive conclusions on the presence of new physics can be reached, further experimental verification is warranted. Indeed, three of the four experiments were sensitive to different parts of the solar neutrino spectrum, and it remains to be decided if the fluxes from all neutrino sources are suppressed, or if some mechanism sup-

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Table 1
Results of the shell-model calculation for the neutrino capture on ^{40}Ar and for the β-decay of ^{40}Ti

i	E_i(th) (MeV)	E_i(exp) (MeV)	$\lvert \mathcal{M}_{o\to i}\rvert^2$	$f_{o\to i}$	$t^i_{1/2}$ (s)	BR_i(th) (%)	BR_i(exp) (%)	Σ_i (10^{-46} cm^2)	$\mathscr{R}_i(^{40}$Ar) (SNU)
1	2.684	2.290	0.006	34850	29.5	0.19	4	20.21	0.012
2	2.971	2.730	1.195	26979	0.191	28.94	20	3262.28	1.892
3	3.291	3.110	0.946	21386	0.305	18.16	3	2075.09	1.204
4	3.622	3.146	0.101	20909	2.92	1.90	–	218.54	0.127
5	4.308	3.739	0.034	14190	12.4	0.45	–	50.68	0.029
6	[illegible].521	3.798	1.119	13629	0.405	13.69	–	1643.01	0.953
IAS	–	4.384	4	9125	0.169	32.76	16	3847.32	2.23
7	4.801	4.789	0.062	6561	11.5	0.48	–	57.34	0.033
8	5.282	–	0.239	4360	5.92	0.93	–	106.13	0.062
9	5.642	–	0.030	3161	65.0	0.09	–	9.12	0.005
10	5.823	–	0.000	2667	∞	0	–	0	0
11	5.922	–	0.383	2424	6.64	0.83	–	81.35	0.047
12	6.151	–	0.023	1929	139.0	0.04	–	3.76	0.002
13	6.428	–	0.698	1441	6.13	0.92	–	76.15	0.044
14	6.480	–	0.343	1362	13.2	0.42	–	36.75	0.021
15	6.683	–	0.052	1084	109.4	0.05	–	4.11	0.002
16	6.876	–	0.017	864	420.0	0.01	–	0.97	0.001
17	7.087	–	0.121	663	76.8	0.07	–	4.77	0.003
18	7.123	–	0.002	636	4853	0	–	0.07	0
19	7.368	–	0.233	459	58.4	0.09	–	5.02	0.003

The available experimental values for E_i and BR_i, and the parameters for the Fermi transition (IAS) are also reported. The excitation energies are reported for ^{40}K.

presses higher-energy neutrinos, such as those from ^{8}B, so that the Ga experiments (GALLEX [3] and SAGE [4]) may be interpreted as detecting the full flux from the pp-chain.

One proposal to examine higher energy solar neutrinos is the Imaging of Cosmic and Rare Underground Signals (ICARUS) [6,7] experiment in which liquid argon (primarily ^{40}Ar) is the detector medium. The primary reaction of interest is

$$\nu_e + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{K}^* + e^- , \qquad (1)$$

where, in general, the transition occurs to excited states in ^{40}K, which subsequently de-excite via γ-decay emission. As applied to solar neutrinos, ICARUS expects to measure the flux of solar ^{8}B neutrinos by both elastic scattering and absorption. In addition, by measuring the ratio between absorption events and elastic scattering events, which can be induced by all neutrino types, it is possible to deduce the probability of oscillations of electron neutrinos into μ and τ neutrinos independently of solar models [7]. Clearly, accurate knowledge of the neutrino absorption cross section is needed.

The reaction defined by Eq. (1) is composed of two distinct processes: (1) Fermi transitions to the isobaric analog state (IAS) ($J^\pi = 0^+, T = 2$) in ^{40}K at an excitation energy of 4.384 MeV and (2) Gamow–Teller transitions to low-lying $J^\pi = 1^+$, $T = 1$ states in ^{40}K. The location of these levels in ^{40}K is illustrated in Table 1, where the experimental excitation energies [9] are given in the second column. In the original ICARUS proposal [6,8], the feasibility of ^{40}Ar as a detector medium was assessed by assuming that neutrino absorption is dominated by Fermi transitions to the isobaric IAS. With this assumption (and the imposition of a 5 MeV cutoff on the minimum energy of the emitted electron), the solar neutrino absorption cross section is $\Sigma_{tot}|_{IAS} = 3.8 \times 10^{-43}$ cm^2, corresponding to a capture rate on ^{40}Ar nuclei of $\mathscr{R}(^{40}\mathrm{Ar})|_{IAS} = 2.2$ SNU. From the compilation of nuclear levels, however, there are at least six $J^\pi = 1^+$, $T = 1$ levels in ^{40}K with excitation energies lower than the isobaric analog state. Given the dependence in the neutrino absorption cross section on the square of the energy of the emitted electron, these low-lying levels may contribute significantly to the overall neutrino absorption cross section.

Another branch of nuclear physics of great interest is the study of exotic nuclei, in which radioactive beams are increasingly used to study the properties of nuclei along the proton and neutron drip lines. Of particular interest to the ICARUS experiment is the β^+-decay of ^{40}Ti (the mirror of ^{40}Ar), i.e.

$$^{40}\mathrm{Ti} \rightarrow {}^{40}\mathrm{Sc}^* + e^+ + \nu_e . \quad (2)$$

Since isospin is a nearly conserved quantity, the Gamow–Teller matrix elements pertinent to ^{40}Ar neutrino absorption can be deduced from experimental branching ratios of the β-decay of ^{40}Ti to levels in ^{40}Sc (the mirror of ^{40}K). In addition, because of the large Q-value ($Q_{EC} = 11.678(160)$ MeV [10]), all the transitions of interest in ^{40}Ar can be studied directly. The main difficulties that need to be overcome in this experiment are the short lifetime, measured to be $\tau_{1/2} = 56^{+18}_{-12}$ ms [11], and the fact that the β-decay occurs to states that are proton unbound in ^{40}Sc. This requires an experimental apparatus that observes the γ-decays in the final nucleus, ^{39}Ca, that occur after the delayed proton emission, as was done in Ref. [12] for the β-decay of ^{37}Ca. The current experimental situation for ^{40}Ti is that branching ratios have only been measured in which the decay proceeds via proton emission to the ground state of ^{39}Ca. Already at this stage, however, it can be deduced that Gamow–Teller transitions play an important role as the expected halflife for ^{40}Ti in the limit of a pure Fermi transition is 169 ms. In addition, $20 \pm 4\%$ of all decays are found experimentally [11] to proceeded via β-decay to the second $J^\pi = 1^+$ state in ^{40}Sc (at 2.7 MeV) followed by proton emission to the ground state of ^{39}Ca.

In this paper, we present the results of a shell-model calculation for the Gamow–Teller transitions between the $J^\pi = 0^+$, $T = 2$ ground state of ^{40}Ar(Ti) to $J^\pi = 1^+$, $T = 1$ states in ^{40}K(Sc). We find that these low-lying Gamow–Teller transitions significantly enhance the solar neutrino absorption cross section, increasing the cross section over that from the isobaric analog transition by nearly a factor of three, namely $\Sigma_{tot} = (11.5 \pm 0.7) \times 10^{-43}$ cm^2 and correspondingly, the absorption rate of solar ^{8}B neutrinos is $\mathcal{R}(^{40}\mathrm{Ar}) = 6.7 \pm 2.5$ SNU. For the purpose of comparison with future experiments, the branching ratios for the β-decay of ^{40}Ti are also presented. We note that the deduced half-life of 55 ± 5 ms is in very good agreement with the experimental value of 56^{+18}_{-12} ms.

Nuclei lying across major shells, such as ^{40}Ar, pose a serious challenge to the shell model approach as two major oscillator shells must be included in the calculation, e.g. the $0d_{5/2}1s_{1/2}0d_{3/2}$ (sd) and the $0f_{7/2}1p_{3/2}1p_{1/2}0f_{5/2}$ (fp) shells. Perhaps the most significant problem in performing calculations within this model space is the large number of configuration accessible. Indeed, it is not possible to carry out an unrestricted calculation for anything but the lightest nucleus in this model space. For this reason, a truncation on the model space must be imposed. Towards this end, we impose an $n\hbar\omega$ truncation, in which $n\hbar\omega$ denotes the excitation of n particles outside of the lower oscillator shell (in this case the sd-shell). A severe limitation, even within this approach, is the so-called "$n\hbar\omega$ truncation catastrophe". Since the expansion of the shell-model wave functions in an $n\hbar\omega$ model space converges slowly and the dimensions increase rapidly with n, one is often forced to use an effective interaction developed for use in the $0\hbar\omega$ space, which accounts for the gross properties of the rest of the series in an approximate way. Because of the slow convergence in n, the $n\hbar\omega$ catastrophe occurs even for $n = 2$. In essence, for mixed $(0+2)\hbar\omega$ calculations, the very strong interaction between the low-lying $0\hbar\omega$ and $2\hbar\omega$ states with similar symmetries causes the $0\hbar\omega$ states to be pushed considerably lower in energy, leaving an unrealistic gap in energy. In addition, not only are the binding energies grossly in error, but also the mixing between the 0 and $2\hbar\omega$ states is incorrect because it is dependent on the perturbed energies. Naturally, if dimensional considerations are not a concern, then one could at least partially solve the problem of the $2\hbar\omega$ states with the inclusion of $4\hbar\omega$ states.

Because of the problems inherent in an $n\hbar\omega$ truncation, we follow the example of Ref. [13] and diagonalize the $0\hbar\omega$ and $2\hbar\omega$ spaces separately. In this light, the low-lying positive-parity $T = 1$ and $T = 2$ states in $A = 40$ are purely $2\hbar\omega$ states, while the ground state of ^{40}Ca is the only $0\hbar\omega$ state. In addition, since the $2\hbar\omega$ states are not constructed by the excitation of particles of the same type out of the sd-shell, there are no spurious center-of-mass states in the calculation. The Hamiltonian used here is that of Ref. [13], which consists of the Wildenthal matrix elements for the sd-shell [14], McGrory's (0f, 1p) shell Hamiltonian for the fp-shell matrix elements [15], and a modification of the Millener–Kurath potential for the cross-shell interaction

[16]. For 25 nuclei with $Z=13$–20 this interaction reproduced the ground-state binding energies with an rms deviation of 305 keV. In this work, the wave functions were computed using the shell-model program OXBASH [17] on VAX-4000/60 computers.

After diagonalizing the resulting Hamiltonian, and obtaining the wave functions, we calculate the neutrino capture cross section for the reaction

$$\nu_e + {}^{40}\mathrm{Ar} \rightarrow {}^{40}\mathrm{K}^* + e^- , \qquad (3)$$

for all states in ^{40}K with excitation energy up to the particle-decay threshold at 7.58 MeV [9]. The cross section for absorbing a neutrino with energy E_ν from the ground state of ^{40}Ar to the ith excited state in ^{40}K is given by

$$\sigma_i(E_\nu) = \frac{G_V^2}{\pi c^3 \hbar^4} |\mathcal{M}_{o\rightarrow i}|^2 E_e^i k_e^i F(Z, E_e^i) . \qquad (4)$$

Here $F(Z, E_e^i)$ is the Fermi function associated with Coulomb correction factor appropriate for the charge density of the daughter nucleus, G_V is the vector coupling constant for nuclear weak processes, while k_e^i and E_e^i are the electron momentum and energy, respectively. They are given by

$$E_e^i = E_\nu - Q_i + m_e c^2 ,$$
$$(ck_e^i)^2 = (E_e^i)^2 - m_e^2 c^4 , \qquad (5)$$

with the Q-value being determined by the difference in binding energy for the initial and final states,

$$Q_i = E_i - E_o + m_e c^2 . \qquad (6)$$

The square of the transition matrix element $|\mathcal{M}_{o\rightarrow i}|^2$ is written as

$$|\mathcal{M}_{o\rightarrow i}|^2 = [B(\mathrm{F})_{o\rightarrow i} + B(\mathrm{GT})_{o\rightarrow i}] , \qquad (7)$$

where, in the long-wavelength limit, the Fermi and Gamow–Teller reduced transition probabilities are given by

$$B(\mathrm{F})_{o\rightarrow i} = \frac{1}{2J_o + 1} |\langle J_i \| t_\pm \| J_o \rangle|^2$$
$$= [T_o(T_i + 1) - T_{zo}T_{zi}]\, \delta_{o,i} , \qquad (8)$$

and

$$B(\mathrm{GT})_{o\rightarrow i} = \frac{1}{2J_o + 1} \left(\frac{g_A}{g_V}\right)^2 |\langle J_i \| (\sigma t_\pm)_{\mathrm{eff}} \| J_o \rangle|^2 , \qquad (9)$$

where the Fermi transition is between isobaric analog states (IAS) only, and the quantity $g_A/g_V = 1.2606 \pm 0.0075$ [18] is the ratio of the axial and vector weak coupling constants. In addition, we note that experimental $B(\mathrm{GT})$ values are generally quenched relative to theoretical estimates, and, therefore, we have renormalized the free-nucleon Gamow–Teller operators by the factor 0.775 [19].

In point of fact, Eq. (8) is valid only in the limit that the nuclear Hamiltonian is isoscalar in nature, i.e. there are no isospin-nonconserving (INC) interactions. With the introduction of INC interactions, such as the Coulomb force, the IAS can be mixed with nearby states of the same angular momentum and different isospin. The level of the mixing is dictated by the amplitude of the mixing matrix element (generally of the order 30–50 keV, and rarely larger than 100 keV (cf. Ref. [20])), and the energy separation between the IAS and the mixed state. If the IAS is separated by much more than the mixing matrix elements, then $B(\mathrm{F})$ in Eq. (8) is renormalized by the factor $(1-\delta_C)$ [21], where δ_C is of the order 0.3% and, to first order, can be ignored. In the limit where the IAS is imbedded among several other states, the Fermi strength will be distributed across the mixed states. It is important to note that the full Fermi strength is still present. However, isospin mixing can, in principle at least, affect the total neutrino absorption cross section (or the β-decay lifetime) because of the strong dependence on the end-point energy.

At first glance, isospin-mixing appears to complicate our analysis considerably, especially in a case such as ^{40}K (and ^{40}Sc below), where the IAS in the final nucleus is at approximately 4.5 MeV of excitation, and can mix with a background of $J=0$, $T=1$ states. Strong mixing, however, can occur only over a relatively small energy range, i.e. of the order of the mixing matrix element itself ≈ 100–200 keV, and we will show by the use of a two-level model that the effects of isospin-mixing on the Fermi component of either neutrino absorption or β-decay is negligible. In addition, we note that the experimental observation of the IAS in ^{40}K [22] seems to indicate that the IAS is not strongly mixed, hence, the energy separation is indeed most likely greater than the mixing matrix element. For these reasons, we first present the results obtained under the assumption of isospin invariance and then discuss the effect isospin-mixing can have on our calculations in terms of the maximal effect expected; in particular for the β-decay of ^{40}Ti where very little experimental data on the IAS in ^{40}Sc exists.

The total absorption cross section for solar neutrinos is obtained by folding the cross section defined in Eq. (2) with the normalized solar neutrino flux $\Phi(E_\nu)$, and summing contributions due to excited states, i.e.

$$\Sigma_{tot} = \sum_i \Sigma_i = \sum_i \int_{Q_i+E_{cut}}^{\infty} \Phi(E_\nu)\sigma_i(E_\nu)dE_\nu . \quad (10)$$

The quantity $E_{cut} = 4.489$ MeV is the minimum electron kinetic energy observable in the ICARUS detector, while the function $\Phi(E_\nu)$ associated with the ^{8}B spectrum was taken from Ref. [23]. Multiplying Σ_{tot} by the total integrated neutrino flux $\mathcal{F}(^8\mathrm{B}) = (5.8 \pm 2.1) \times 10^6\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, one obtains the neutrino capture rate on ^{40}Ar

$$\begin{aligned}\mathcal{R}(^{40}\mathrm{Ar}) &= \sum_i \mathcal{R}_i(^{40}\mathrm{Ar}) \\ &= \Sigma_{tot}(^{40}\mathrm{Ar})\mathcal{F}(^8\mathrm{B}) \\ &= 6.7 \pm 2.5\ \mathrm{SNU} . \end{aligned} \quad (11)$$

For the β-decay of ^{40}Ti

$$^{40}\mathrm{Ti} \to {}^{40}\mathrm{Sc}^* + e^+ + \nu_e , \quad (12)$$

the partial halflife for the decay to the ith state in ^{40}Sc is given by

$$t^i_{1/2} = \frac{K}{G_V^2 |\mathcal{M}_{o\to i}|^2 f_{o\to i}} , \quad (13)$$

where $K = 2\pi^3(\ln 2)\hbar^7/(m_e^5c^4)$, and we use the value $K/G_V^2 = 6170 \pm 4$ s [24]. For the statistical rate function $f_{o\to i}$, we use the formalism of Ref. [25], which is expected to be accurate to within 0.5%, namely

$$\begin{aligned} f_{o\to i} &= (1 + \delta_R + \delta_\beta^{Z\alpha^2} + \delta_\beta^{Z^2\alpha^3}) \\ &\times \int_1^{W_0} dW\, pW(W_0 - W)^2 F_0(Z, W) L_0(Z, W) \\ &\times C(Z, W) R(W) , \end{aligned} \quad (14)$$

where p and W are the electron momentum and energy, respectively, in units of m_ec^2, W_0 being the endpoint, and $F_0(Z, W)$, $L_0(Z, W)$, $C(Z, W)$, and $R(W)$ are parameterized correction factors given in Ref. [25]. The radiative correction factor δ_R is given by Ref. [26]

$$\delta_R = \frac{\alpha}{2\pi} \frac{\int dW\, pW(W_0 - W)^2 g(W, W_0)}{\int dW\, pW(W_0 - W)^2} , \quad (15)$$

where $g(W, W_0)$ is given by Eq. (III-21) in Ref. [26], while the higher-order radiative corrections are parameterized according to $\delta_\beta^{Z\alpha^2} = 4.0 \times 10^{-4} |Z|$ and $\delta_\beta^{Z^2\alpha^3} = 3.6 \times 10^{-6} Z^2$ [25]. The total halflife is then given by the sum of decay rates, i.e.

$$\frac{1}{t_{1/2}} = \sum_i \frac{1}{t^i_{1/2}} , \quad (16)$$

while the branching ratio to the ith state is given by

$$\mathrm{BR}_i = \frac{t_{1/2}}{t^i_{1/2}} . \quad (17)$$

Shown in Table 1 are the explicit values for transitions to each of the $J^\pi = 1^+$, $T = 1$ levels in ^{40}K (^{40}Sc for the β-decay of ^{40}Ti). For the excitation energies, theoretical values are tabulated, as well as the experimental values for the first seven $J^\pi = 1^+$ levels as determined from the ^{40}K spectrum. One sees that there is a one-to-one correspondence between the theoretical and experimental levels, with the theoretical levels having an excitation energy of approximately 0.5–1.0 MeV higher than experiment. For the purpose of computing the neutrino cross sections and the β-decay partial halflives, the experimental energies were used whenever possible. In addition, the excitation energy of the IAS in ^{40}Sc was taken to be 4.363(10) MeV as deduced from the isobaric-mass-multiplet equation (IMME) and the analog states in ^{40}Ar, ^{40}K, and ^{40}Ca [9]. Also presented in Table 1 are the theoretical branching ratios for the β-decay of ^{40}Ti and their experimental values as deduced in Ref. [11]. Note that the experimental values are lower limits as they involve the decay process in which the proton-unbound excited state of ^{40}Sc decays directly to the ground state of ^{39}Ca. For completeness, Table 1 also reports the parameters for the Fermi transition to the isobaric analog state.

The primary conclusion of the shell-model calculation is that there is significant low-lying Gamow–Teller strength that leads to an overall enhancement of the neutrino absorption cross section by about a factor three over that expected from the Fermi transition alone. This conclusion is also supported by existing experimental data, where we find good agreement between the theoretical halflife $t_{1/2}=55$ ms and the experimental value $\tau_{1/2}=56^{+18}_{-12}$ ms [11]. In addition, the limited branching-ratio data indicates that most of the Gamow–Teller strength is to the second $J^\pi=1^+$ state ($\geq 20\%$) as is predicted by theory.

As was mentioned before, INC interactions can mix the $J=0$, $T=2$ IAS in ^{40}Sc with the background $J=0$, $T=1$ excited states. Indeed, the shell-model interaction used in this work predicts that the second $J=0$, $T=1$ state is essentially degenerate with the IAS. The exact location of this state, however, is unknown, and the theoretical estimate has an uncertainty of approximately 400 keV. The other parameter detailing the level of isospin mixing in the IAS is the INC matrix element, which, in principal, can be computed using the methods outlined in Refs. [20,27]. This, however, would require a complicated fitting procedure over a range of nuclei, and would not affect the uncertainty caused by the location of the background state. In addition, as was pointed above, the experimental data on the analog nucleus ^{40}K [22] do not indicate that the $T=2$ state is strongly mixed. Nonetheless, we present here an estimate of the maximum effect that could be expected from isospin-mixing on the β-decay lifetime of ^{40}Ti.

To account for isospin mixing in the IAS we use a two-level approximation, where the nuclear wave functions are obtained by diagonalizing the matrix

$$\begin{pmatrix} E_1 & H \\ H & E_2 \end{pmatrix},$$

where E_1 and E_2 are the energies of the pure isospin states (including the diagonal part of the INC interaction), and H is the mixing matrix element. The eigenvalues are

$$E_\pm = \frac{E_1+E_2}{2} \pm \sqrt{\left(\frac{E_1-E_2}{2}\right)^2 + H^2}\,, \tag{18}$$

while the eigenvectors are given by

$$|\psi_-\rangle = N|\psi_1\rangle + \alpha|\psi_2\rangle\,, \tag{19}$$

$$|\psi_+\rangle = -\alpha|\psi_1\rangle + N|\psi_2\rangle\,, \tag{20}$$

where $N=\sqrt{(1-\alpha^2)}$. Without loss of generality, we assume $E_1 \leq E_2$, giving

$$\alpha^2 = 1 - \frac{2H^2/\Delta E^2}{1-(1-4H^2/\Delta E^2)^{1/2}}\,, \tag{21}$$

where the sign of α is given by the sign of H, and $\Delta E = E_+ - E_-$ is the energy difference of the final states, and has a minimum value of $2|H|$.

For the current analysis, we identify the $T=1$ and $T=2$ states with $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. The location of the unmixed $T=2$ state in ^{40}Sc, i.e. E_2, can be found using the isobaric mass multiplet equation and the experimentally known analog states, and is predicted to have an excitation energy of 4.363(10) MeV [9]. The Fermi decay rate for the two states is

$$\Gamma_- = K|M_{F0}|^2\alpha^2 f(W_-)\,, \tag{22}$$

$$\Gamma_+ = K|M_{F0}|^2 N^2 f(W_+) \tag{23}$$

where $|M_{F0}|^2$ is the square of the Fermi matrix element for the pure $T=2$ state, $W_\pm$ are the end-points for the eigenstates of the isospin-mixing Hamiltonian, which we now express in terms of the shift relative to the unmixed $T=2$ state as $W_- = W_2 - \Delta W_1$ and $W_+ = W_2 + \Delta W_2$. In the limit that the end point is large, we have $f(W) \propto W^5/30$, and

$$f(W_2 \pm \Delta W_i) \approx f(W_2)\left[1 \pm 5\Delta W_i/W_2 + 10(\Delta W_i/W_2)^2\right]\,. \tag{24}$$

Since the decay rates sum, the total Fermi decay rate is given by

$$\Gamma_{\rm Fermi} = K|M_{F0}|^2 f(W_2) \times\left(1 + 5\frac{\Delta W_2}{W_2} - \alpha^2 5\frac{\Delta W_2 + \Delta W_1}{W_2} + 10\frac{N^2\Delta W_2^2 + \alpha^2\Delta W_1^2}{W_2^2}\right). \tag{25}$$

Noting that the maximal mixing occurs when $W_1 = W_2$ (i.e. when $\Delta W = (W_+ - W_-)/2 = |H|$), and that $W_\pm = W_2 \pm (\Delta W/W_2)W_2 \pm |H/m_e|$, the *maximum* value for the total Fermi decay rate becomes

$$\Gamma_{\rm Fermi} = K|M_{F0}|^2 f(W_2)\left[1 + 10\left(\frac{\Delta W}{W_2}\right)^2\right]. \tag{26}$$

That is, the effect on the lifetime due to shift in the endpoints and the mixing between the two states cancel to first order. Taking a rather large value of 100 keV for the mixing matrix element H, and combining it with the β end-point energy of 6.4 MeV, gives a change in the Fermi-decay rate of $\pm 0.2\%$, which has a negligible effect on the total β-decay lifetime. For the most part, isospin mixing of the Fermi component has little effect on either the total β-decay life time or the total neutrino absorption cross section, even in the limit of strong mixing. This is due to the fact that mixing matrix element is fairly small, and strong mixing can occur only over a narrow energy range. The primary effect due to isospin mixing can have, therefore, is to cause spreading of the Fermi strength over an energy window of the order of the mixing matrix element, namely a few hundred keV at most.

One difficulty associated with a theoretical calculation is predicting the magnitude of the uncertainty in the results. For the case at hand, this entails the distribution of the Gamow–Teller strength. This is evident in Table 1, where the shell-model underpredicts the branching ratio to the first $J^{\pi}=1^{+}$ state. To be noted, however, is that even in complete shell-model calculations (i.e. without truncation and with a more sophisticated treatment of the quenching factors) the strengths obtained for the weakest states can show large deviations from their experimental values, while the strengths of the strongest states are usually more reliable. Generally speaking, the theoretical uncertainty in the $B(\mathrm{GT})$ values is of the order 5–10% [19]. Assuming a 10% uncertainty in the individual $B(\mathrm{GT})$ values leads to an uncertainty in the β-decay halflife for ^{40}Ti of ± 2 ms. In addition, the uncertainty of 160 keV in the Q-value leads gives an additional uncertainty of $\approx \pm 5$ ms to the halflife. Therefore, the predicted halflife for ^{40}Ti is 55 ± 5 ms, which is in excellent agreement with the experimental value of 56^{+18}_{-12} ms [11]. Whereas, the 10% uncertainty in each $B(\mathrm{GT})$ value leads to an uncertainty in the neutrino absorption cross section of $\approx \pm 0.7 \times 10^{-43}$ cm^2.

Aside from the theoretical uncertainties, and keeping with the fact that one is trying to obtain reliable results to calibrate a high-energy neutrino detector, the most preferable course would be to determine the $B(\mathrm{GT})$ values experimentally. In general, this is not possible since the nucleus of interest is usually stable. Instead, $B(\mathrm{GT})$ values must be extracted from (n, p) or (p, n) reaction studies, or from the β-decay of the mirror nucleus by exploiting analog symmetry. In fact, in the absence of the Coulomb potential (as well as the smaller nuclear charge-dependent interaction) and second-class weak currents, $B(\mathrm{GT})$ values for β^- and β^+ decays of mirror nuclei in the same isospin multiplet are identical. For the most part, in comparing the ft values of mirror analogs, the largest correction that needs to be applied is the that due to a reduction in the radial overlap due to different binding energies in the two analogs (c.f. Ref. [26], page 109), which is of the order of 5%.

Because of the large Q-value for the β^+-decay of ^{40}Ti, it is in principle possible to measure all the $B(\mathrm{GT})$ values of interest for neutrino absorption on ^{40}Ar directly. The primary difficulty in making this measurement is that ^{40}Ti β-decays into levels of ^{40}Sc that are proton unbound. Experimental methods exist, however, that overcome this difficulty, as in Ref. [12], and it should be possible to measure the $B(\mathrm{GT})$ values in ^{40}Ti β-decay. The current situation is expressed in Ref. [11], where the actual goal of the experiment was to observe direct two-proton radioactivity, and only transitions involving proton emission to the ground state of ^{39}Ca were analyzed. As a result, not enough information exists to deduce experimental $B(\mathrm{GT})$ values. Therefore, we strongly suggest that the experiment be repeated with a new focus. Namely, the *full* detection of β^+-transitions in ^{40}Ti to ^{40}Sc and their corresponding proton decays to ^{39}Ca, so that the branching ratios relevant to the β^+-decay of ^{40}Ti may be obtained. This course of action would, in point of fact, be of crucial importance for the ICARUS experiment in the context of the study of higher energy solar neutrinos, allowing an accurate calibration of the associated detector.

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References

[1] R. Davis Jr., in: Proc. Seventh Workshop on Grand Unification (Toyama, Japan, 1986), ed. J. Arafune (World Scientific, Singapore, 1987) p. 237;
R. Davis Jr., K. Lande, C.K. Lee, P. Wildenhain, A. Weinberger, T. Daily, B. Cleveland and J. Ulman, in: Proc. 21st Intern. Cosmic Ray Conf. (Adelaide, Australia, 1990), ed. R.J. Protheroe (Graphic Services, Northfield, South Australia, 1990);
J.K. Rowley, B.T. Cleveland and R. Davis Jr., in: Proc. Conf. on Solar Neutrinos and Neutrino Astronomy (Lead, SD, 1984), eds. M.L. Cherry, W.A. Fowler and K. Lande, AIP Conf. proc. No. 126 (AIP, New York, 1985) p. 1.

[2] K.S. Hirata et al., Phys. Rev. Lett. 63 (1989) 16; 65 (1990) 1297.

[3] P. Anselman et al., Phys. Lett. B 285 (1992) 376; B 285 (1992) 390.

[4] A.I. Abazov et al., Phys. Rev. Lett. 67 (1991) 332.

[5] J.N. Bachall and H.A. Bethe, Phys. Rev. D 47 (1993) 1298.

[6] CERN–Harvard–Milano–Padova–Rome–Tokyo–Wisconsin Collab., ICARUS, A proposal for the Gran Sasso Laboratory, preprint INFN-AE-85-7 (September 1985);
C. Rubbia, CERN Internal Report 77-78;
W.A. Huffman, J.M. Lo Secco and C. Rubbia, IEEE Trans. Nucl. Sci. NS-26 (1979) 64;
E. Aprile, K. Giboni and C. Rubbia, Nucl. Instrum. Methods 241 (1985) 62;
M. Baldo-Ceolin, Symp. on Photon and Lepton Interactions (Kyoto, 1985) p. 432.

[7] ICARUS Collabo., Proposal, ICARUS II, A second-generation proton decay experiment and neutrino observatory at Gran Sasso Laboratory.

[8] J.N. Bachall, M. Baldo-Ceolin, C.B. Cline and C. Rubbia, Phys. Lett. B 178 (1986) 324.

[9] P.M. Endt, Nucl. Phys. A 521 (1990) 1.

[10] G. Audi and A.H. Wapstra, Nucl. Phys. A 565 (1993) 1.

[11] C. Detraz et al., Nucl. Phys. A 519 (1990) 529.

[12] A. Garcia et al., Phys. Rev. Lett. 67 (1991) 3654.

[13] E.K. Wharburton, J.A. Becker and B.A. Brown, Phys. Rev. C 41 (1990) 1147.

[14] B.H. Wildenthal, Progress in Particle and Nuclear Physics, ed. D.H. Wilkinson, Vol. 11 (Pergamon, Oxford, 1984) p. 5.

[15] J.B. McGrory, Phys. Rev. C 8 (1973) 693.

[16] D.J. Millener and D. Kurath, Nucl. Phys. A 255 (1975) 315.

[17] B.A. Brown, A. Etchegoyen and W.D.M. Rae, OXBASH, The Oxford–Buenos Aires–MSU shell model code, Michigan State University Cyclotron Laboratory Report No. 524 (1985).

[18] D.H. Wilkinson, Nucl. Phys. A 377 (1982) 474.

[19] B.A. Brown and B.H. Wildenthal, At. Data Nucl. Data Tables 33 (1985) 347.

[20] W.E. Ormand, Ph.D. thesis, Michigan State University (1985).

[21] W.E. Ormand and B.A. Brown, Phys. Rev. Lett. 62 (1989) 866;
I.S. Towner, J.C. Hardy and M. Harvey, Nucl. Phys. A 284 (1977) 269.

[22] W.A. Sterrenburg, G. van Middelkoop and J.A.G. De Raedt, Nucl. Phys. A 290 (1977) 200.

[23] J.N. Bachall, Neutrino Astrophysics (Cambridge U. P., Cambridge, 1989).

[24] D.H. Wilkinson, A. Gallman and D.E. Alburger, Phys. Rev. C 18 (1978) 401.

[25] D.H. Wilkinson, Ions lourds et mesons en physique nucleaire/ Nuclear physics with heavy ions and mesons, Les Houches, Session XXX (1977), eds. R. Balian et al. (North Holland, Amsterdam, 1978) p. 877.

[26] R.J. Blin-Stoyle, Fundamental interactions and the nucleus (North Holland, Amsterdam, 1973).

[27] W.E. Ormand and B.A. Brown, Nucl. Phys. A 491 (1989) 1.