Spin-dipole nuclear matrix element for the double beta decay of ^{76}Ge by the (^{3}He,t) charge-exchange reaction

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Abstract.
Nuclear matrix elements (NMEs) for double beta decays (DBDs) are crucial for studying the neutrino mass and other neutrino properties beyond the standard electro-weak model by measuring neutrino-less DBDs. The spin-dipole (SD) J^π=2$^-$ NME is one of the major components associated with the DBD NME. The SD NME for ^{76}Ge was derived for the first time by using the (^{3}He,t) charge-exchange reactions (CERs) on ^{74,76}Ge at RCNP Osaka. The obtained SD NME for the ^{76}Ge→^{76}As ground-state transition is $|M^-_{EXP}(SD)|$=1.5×10^{-3} in natural units. This is smaller by a coefficient around $k \approx 0.2$ with respect to the quasi-particle (QP) model NME $|M^-_{QP}(SD)|$. The impact of the reduced (quenched) SD NME on DBD neutrino studies is discussed.

Key words: Double beta decay of ^{76}Ge, charge-exchange reaction, spin-dipole matrix element, quenching of axial-vector coupling g_A^{eff}.

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Neutrino-less double beta decay (DBD) is a very sensitive and realistic probe for studying neutrino properties such as the Majorana nature, the absolute mass scale and the mass hierarchy, the lepton-sector CP phase and other properties beyond the standard electro-weak model (SM). The nuclear matrix element (NME) is a key element for extracting the effective neutrino mass and other properties of particle -physics interest beyond the SM from the experimental DBD rate, if the decay is observed. It is even crucial for the design of the DBD detector since the detector sensitivity depends very much on the NME (one needs to build an order of magnitude larger detector if the NME is smaller by a factor 2). These subjects are discussed in review articles and references therein [1, 2, 3, 4, 5]. The DBD NMEs are also discussed in DBD reviews [6, 7, 8, 9, 10, 11, 12, 13] and references therein. The single-β and DBD trasition operators and their NMEs are as given in [1, 2, 3, 14, 15].

The present letter aims to report for the first time an experimental study of the spin-dipole (SD) single-β NME $M^-(SD)$ for the ^{76}Ge→^{76}As ground-state transition by means of the (^{3}He,t) charge-exchange reaction (CER). Here the SD NME is one of the major components associated with the DBD NME, and ^{76}Ge is one of the key DBD nuclei in DBD experiments [3, 7, 8].

Actually, it is extremely difficult to accurately calculate the DBD NME since the NME is very sensitive to nucleonic and non-nucleonic correlations and nuclear medium effects, some of which are effectively incorporated in the effective axial-vector coupling g_A^{eff}. Consequently, calculated DBD NMEs strongly depend on nuclear models and nuclear parameters to be used for the NME calculation. Accordingly, the evaluated NMEs scatter over an order of magnitude [3, 9]. Thus experimental inputs are quite important to help evaluate the DBD NME [2, 3, 7, 9].

The neutrino-less DBD transition of ${}^A_Z$X↔${}^A_{Z-2}$X with A being the mass number and Z being the atomic number is considered. In case of the light Majorana-mass process, the decay process is expressed schematically as ${}^A_Z$X↔${}^A_{Z-1}$X↔${}^A_{Z-2}$X, where the virtual neutrino with the the light mass and the medium momentum of $q \approx 20$ - 300 MeV/c is exchanged in the intermediate nucleus ${}^A_{Z-1}$X via the neutrino potential [3, 9].

The neutrino-less DBD NME is associated with the τ^-- and τ^+- side single-β NMEs, $M^-(\alpha)$ for ${}^A_Z$X→${}^A_{Z-1}$X and $M^+(\alpha)$ for ${}^A_{Z-1}$X←${}^A_{Z-2}$X, with τ^- and τ^+ being the isospin-lowering and isospin-raising operators and α being the transition mode. The single-β NMEs associated with the DBD NME are $M^\pm(J^\pi)$ with J^π being the spin-parity of the intermediate state. The states with $J^\pi = 0^\pm$, $1^\pm$, $2^\pm$ and so on up to $J \approx$6 are involved. Among them, $M^\pm(2^-)$ plays an important role for the neutrino-less DBD NME, while $M^\pm(1^+)$ does for the two-neutrino DBD NME. The 2^- transition, which is mainly due to the spin dipole (SD) operator as we discuss later, is denoted as SD, while the 1^+ one due to the Gamow-Teller (GT) is denoted as GT.

So far, the single-β $M^\pm(GT)$ for DBD nuclei have been studied by using single-$\beta^\pm$ and EC rates for some DBD nuclei where the ground state is 1^+ and the half-life is measured. CERs on DBD nuclei have been used to measure $M^\pm(GT)$ for the low-lying GT(1^+) states in DBD nuclei, as given in [3]. Recently muon CERs have been shown to be a useful tool to study the $M^+(J^\pi)$ in wide energy and momentum ranges [16, 17, 18, 19]. Experimental studies of the DBD NMEs by using lepton, photon and nuclear probes are given in recent reviews [3, 2, 20]. The quenching of the axial-vector coupling g_A for the large momentum transfer, which is relevant to the neutrino-less DBD, is discussed in [21]. The GT response for ^{116}Sn has been studied recently [22]. Double CERs are interesting to study DBD NMEs [23, 24].

The CER cross section is expressed in terms of the nuclear response $B(\alpha)$ as [2, 3]

$$\frac{d\sigma(\alpha)}{d\Omega} = C(\alpha)B(\alpha), \tag{1}$$

$$B(\alpha) = (2J_i + 1)^{-1}|M(\alpha)|^2, \tag{2}$$

$$C(\alpha) = K(\alpha,\omega)F(\alpha,q,\omega)J(\alpha,\omega)^2, \tag{3}$$

where $K(\alpha,\omega)$ and $J(\alpha,\omega)$ are the kinematic factor and the volume integral of the α mode ^{3}He-n interaction, respectively, for the momentum q and energy ω transfers. In the present case of the even-even DBD nucleus with the initial state spin J_i=0, one gets $B(\alpha) = |M(\alpha)|^2$. The kinematic q,ω-dependence of $F(\alpha,q,\omega)$ is given by the DWBA (distorted wave Born approximation) calculation. The CER responses for $\alpha = 0^+ \rightarrow 0^+, 1^+$ and 2^- low-lying states in DBD nuclei are discussed in [2, 3, 25, 29].

The high energy-resolution (^{3}He,t) CERs for GT (α=GT) transitions have been applied extensively for decades at RCNP to study τ^--side GT responses in DBD and neighboring nuclei, where the GT responses ($B(GT)$) for the ground states in the mass region are known from the measured EC/β^+ rates. Thus, the coefficient $C(GT)$ to relate the GT CER cross-section to the GT response is known experimentally, as discussed in the reviews [2, 3, 13].

The SD NMEs $M^\pm(SD)$ for DBD nuclei, however, have not been known experimentally. The ground states in the intermediate nuclei for the DBD nuclei, except ^{76}Ge, are not the SD (2^-) state, thus no EC/β^+ data are available. In case of ^{76}Ge, the $M^+(SD)$ is known from the β^--decay rate, but the $M^-(SD)$ is not known because the EC rate is too small to be measured accurately.

Recently, SD states in DBD nuclei have been shown to be well excited by the (^{3}He,t) CERs at RCNP, and the SD cross-sections are compared with the FSQP (Fermi surface quasi particle) SD responses [25], but the SD NMEs are not derived since the coefficient $C(SD)$ to relate the CER cross section to the SD response is not known experimentally.

So, in the present work, we select ^{76}Ge, which has been extensively studied in DBD experiments by using high energy-resolution Ge detectors [26, 27], and we study experimentally the CER cross section for the ground SD state in ^{74}Ge, where the SD response $B(SD)$ is known from the SD EC rate (f_1t value), in order to derive the coefficient $C(SD)$. Then, using this $C(SD)$ in ^{74}Ge for ^{76}Ge and the cross section measured in the previous experiment for the SD state in ^{76}Ge [25, 28, 29], we get the response $B(SD)$

in ^{76}Ge.

The (^{3}He,t) CER on ^{74}Ge was studied by using the 420 MeV ^{3}He beam provided by the RCNP ring cyclotron. The beam was transported to the target via the WS beam line and the out-going triton (t) was momentum-analyzed by using the high energy-resolution spectrometer GRAND RAIDEN [30]. The experimental details are the same as those for the previous experiment [28]. The target used is a thin 0.25 mg/cm^2 Ge film enriched to 94 % in ^{74}Ge. The energy resolution of around 70 keV in FWHM was good enough to separate the 2^- ground state from the 1^+ and other excited states at around 200 keV [31].

The measured energy spectrum is shown in Figure 1. The 2^- ground state and the 0^+ 6.72 MeV state (isobaric analogue state, IAS) are clearly excited. Note that the 1^+ 0.21 MeV state is well separated from the ground state. The observed angular distribution for the 2^- state shows a typical distribution characteristic of the orbital angular-momentum transfer ΔL=1 in accordance with the DWBA distribution as shown in Figure 2.

Here, we consider two modes, α=SD for the 2^- ground state and α=F for the 0^+ IAS, respectively. The F state is strongly excited in the CER, and is conventionally used as a reference state with the full strength of $B(F) = N - Z$ where N and Z are the neutron and proton numbers of the target nucleus [2, 32, 33].

Then, using the F cross section and the F response as references, the SD cross section and the SD response are expressed as

$$\frac{d\sigma(SD)}{d\sigma(F)} = R(SD/F)\frac{B(SD)}{B(F)}, \quad R(SD/F) = \frac{C(SD)}{C(F)}, \tag{4}$$

where $d\sigma(SD)$ and $d\sigma(F)$ are, respectively, the differential cross-section for the SD transition at $\theta \approx 2.3°$ and that for the F (IAS) transition at $\theta \approx 0°$. Note the SD (ΔL=1) and F (ΔL=0) cross sections have maximum, respectively, at around 2.3° ($q \approx$ 0.3 fm^{-1}) and 0° ($q \approx 0$ fm^{-1}) in their angular (q) distributions for both ^{74}Ge and ^{76}Ge, as shown in Figure 2 and Figure 3 in [28], and thus $d\sigma(SD)$ and $d\sigma(F)$ are rather stable, being insensitive to the momentum transfer (angle), at $\theta \approx 2.3°$ and 0°, respectively.

$R(SD/F)$ is the ratio of the SD to F coefficients as given in eq. (4). The ratio of $R(SD/F)$ is expressed by a product of the three SD to F ratios of the kinematic factors of $K(\alpha,\omega)$, the distortion factors of $F(\alpha,q,\omega)$ and the volume integral squares of $J(\alpha,\omega)^2$ (see eq.(3)). The incident ^{3}He energy is 420 MeV and the out-going t energies are around 418 MeV and 411 MeV for the SD and F states in both ^{74}Ge and ^{76}Ge. The momentum transfers for both ^{74}Ge and ^{76}Ge are almost the same, the difference being 0.2 % and 0.005% for the SD and F transitions. Therefore, the kinematic conditions for the CERs on both nuclei are nearly the same. The present SD transition from the ground state (0^+) to the ground state (2^-) is given approximately by the simple well-bound QP transition of the ($1g_{9/2}$ quasi-neutron $\rightarrow$ $1f_{5/2}$ quasi-proton) for both ^{74}Ge and ^{76}Ge, where the neutron and proton binding energies are similar in both ^{74}Ge and ^{76}Ge [25, 34]. Accordingly, the ratios for the three factors should be nearly the same

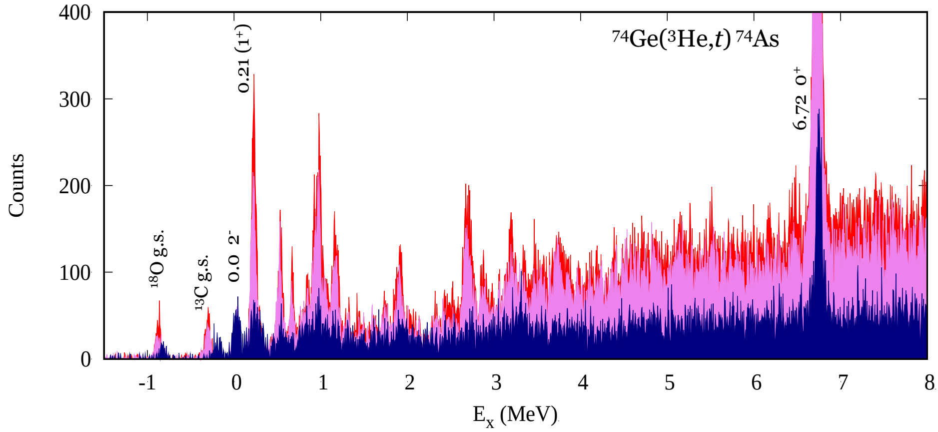

Figure 1. The energy spectra of the ^{74}Ge (^{3}He,t)^{74}As reaction. The spectra at $\theta \approx$0-1°(red), $\theta \approx$1-2° (pink) and $\theta \approx$2-3°(blue) are overlaid. The GT and F states with ΔL=0 show a peak (red) at the forward angles of $\theta \approx$0-1°, while the SD ground state with ΔL=1 shows a large yield (blue) at larger angles of $\theta \approx$2-3°

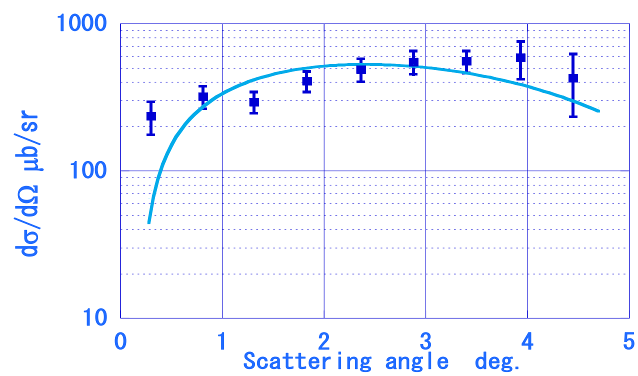

Figure 2. Measured (squares) and DWBA (solid line) angular distributions for the SD state in the ^{74}Ge (^{3}He,t)^{74}As. The solid line (mainly ΔL=1) includes a small component of ΔL=3 at the large angles [28].

for both ^{74}Ge and ^{76}Ge, and thus the ratio $R(SD/F)$ for the two isotopes remains the same within a few %. Actually, the coefficient $C(GT)$, which is a kind of the GT unit cross section, reflects the interaction integral associated with the GT transition and the value for $C(GT)$ has been determined experimentally by referring to the coefficient in the neighboring nuclei as used in the present SD case [2, 3, 35].

The measured SD to F cross section ratio is $d\sigma(SD)/d\sigma(F) = 0.0442 \pm 0.0031$ for the ^{74}Ge$\rightarrow$^{74}As ground state transition. The large error (7%) is mainly a statistical one since the systematic errors cancel out in the ratio. The SD to F response ratio is derived by using the $B(SD)$=2.82±0.34 in units of 10^{-6} (n.u. natural unit)^2 [34] from the EC rate [31] and the $B(F) = N - Z$=10 as $B(SD)/B(F)$=0.282 ±0.034 in units of 10^{-6}

(n.u.)^2. Then one gets the coefficient ratio as $R(SD/F)$=0.157 ±0.022 in units of 10^{-6} (n.u.)^2 for ^{74}Ge.

The cross-section ratio is derived as $d\sigma(SD)/d\sigma(F)$= 0.0295±0.0015 for ^{76}Ge from the CER data on ^{76}Ge [28]. By using this cross-section ratio and the coefficient ratio of $R(SD/F)$ for ^{74}Ge one gets the response ratio $B(SD)/B(F)$=0.188±0.039 in units of 10^{-6} (n.u.)^2 for ^{76}Ge. Here the SD response includes the systematic error of around 15% due to the small admixture of the spin-octupole contribution to the cross sections for ^{74,76}Ge at $\theta \approx$2.3°. Then, using the F response of $B(F) = N - Z$=12 for ^{76}Ge, the SD response is derived as $B(SD)$=2.26 ±0.46 in units of 10^{-6} (n.u.)^2, and the SD NME for ^{76}Ge is obtained as $|M^{-}_{EXP}(SD)|$=1.50 ±0.15 in units of 10^{-3} n.u.. Note that we used the F cross section at $\theta \approx$0 ° where the momentum transfer is around $q \approx$0.085/fm, and that the cross section extrapolated to q=0 fm^{-1} is larger by 7 % for both ^{74}Ge and ^{76}Ge. This effect cancels out in the present ratio of the $C(F)$ coefficients for both nuclei. The obtained SD NME for ^{76}Ge is similar to the NME of 1.68 in units of 10^{-3} n.u. for ^{74}Ge [34], but smaller than the value evaluated on the basis of the FSQP [25].

The experimental SD NME for ^{76}Ge derived from the present CER, together with the ones for the neighboring nuclei derived from EC/$\beta^{\pm}$ decay data [34], are all around $|M^{\pm}_{EXP}(SD)| \approx$ 1.9±0.5 in units of 10^{-3} n.u., as shown in Figure 3.

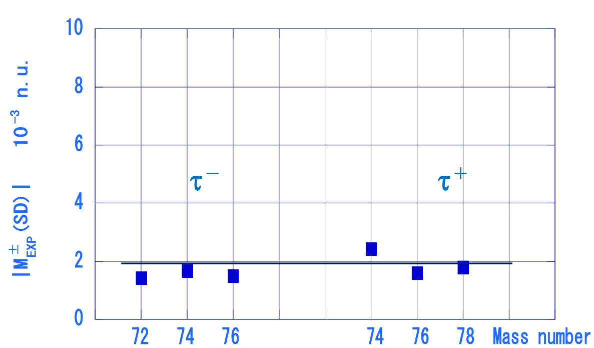

Figure 3. Experimental SD NMEs (squares) for the Ge ground-state to ground-state transitions, $|M^{\pm}_{EXP}(SD)|$, in units of 10^{-3} n.u. in the mass region of A=72-78. τ^- and τ^+ are for the τ^-- and τ^+-side NMEs. The errors of the experimental NMEs are within the size of the squares.

Now we briefly discuss the obtained SD NME in views of the axial-vector NMEs and DBD NMEs. Since the main component of the present SD transition is the spin-stretched QP transition of $(l+1/2)_n \leftrightarrow ((l-1)-1/2)_p$ with l=4, the transition operator in the present case of $q \approx 0.3$ fm^{-1} is given by the first-order SD (i.e. β-decay) one as

$$T(SD) = g\tau^{\pm}[i^1\sigma \times rY_1]_2, \tag{5}$$

where τ and σ are the isospin and spin operators, respectively, and g is the interaction constant [1, 2, 14, 15]. Note that the nuclear radius r (inverse of momentum q) is included in $M(SD)$, following the convention of β-γ NMEs, and thus the momentum q is included in $K(SD,\omega)$ [3, 25]. Here we follow the definition of the SD NME obtained from the SD f_1t as given in eq.(3) in [34], which differs from that in [14].

The SD NME for the QP transition is expressed as

$$M^{\pm}(SD) = k^{\pm}M^{\pm}_{QP}(SD), \tag{6}$$

where $M^{\pm}_{QP}(SD)$ is the SD NME in the QP model and $k^{\pm}$ is the re-normalization coefficient. The QP NMEs scatter around $M^{\pm}_{QP}(SD)$= 10-15 in units of 10^{-3} n.u. [34]. The experimental SD NMEs are indeed much reduced by the coefficient of $k^{\pm} \approx 0.2$ with respect to the single QP NMEs [1, 2, 34]. The reduction of the SD NMEs in Ge nuclei is in accordance with the reductions for the axial-vector β-γ NMEs in medium heavy nuclei as discussed in [1, 2, 3, 14, 15, 34, 36]. The reduction is considered to be due to various kinds of nuclear interactions as nucleonic multipole interactions, nucleonic $\tau\sigma$ interactions, non-nucleonic (mesons, isobars) interactions and other interactions, which are not in the simple QP model. Among them, the strong nucleonic $\tau\sigma$ interaction gives rise to the SD giant resonance at the high excitation region and plays a crucial role for the reduction of the axial-vector NMEs for low -lying states, as shown by using the $\tau\sigma$ multipole interaction and QRPA (quasi particle random phase approximation) in [37, 38] and in the reviews [1, 2, 3, 15].

The nucleonic $\tau\sigma$ interaction is somehow incorporated in the pn (proton-neutron) QRPA model [34]. Then, the NME is given as

$$M^{\pm}(\alpha) = k^{\pm}_{NM}M^{\pm}_{QR}(\alpha), \tag{7}$$

where $M^{\pm}_{QR}(\alpha)$ is the pnQRPA model NME and $k^{\pm}_{NM}$ stands for the re-normalization (quenching) coefficient due to non-nucleonic interactions and nuclear medium effects, which are not explicitly included in the pnQRPA model. It is noted that the re-normalization (quenching) coefficient $k^{\pm}_{NM}$ is conventionally expressed as g^{eff}_A/g_A where g^{eff}_A is the effective axial-weak coupling and g_A=1.27 is the coupling for a free nucleon in unit of the vector coupling g_V [1, 3]. The pnQRPA NME for the ^{74}Ge is $M^{-}_{QR}(SD)$=4.87 in units of 10^{-3} n.u. [34], and the re-normalization coefficient is $k^{\pm}_{NM}$ ≈0.35. This shows a severe re-normalization (reduction) effect for the SD NME. It is interesing to evaluate the SD NME for ^{76}Ge by using the pnQRPA.

The neutrino-less DBD NME is expressed by [3, 20]

$$M^{0\nu} = [M^{0\nu}(GT) + M^{0\nu}(T)] + (\frac{g_V}{g_A})^2 M^{0\nu}(F), \tag{8}$$

where $M^{0\nu}(GT)$, $M^{0\nu}(T)$ and $M^{0\nu}(F)$ are the axial-vector, tensor and vector DBD-NMEs, respectively, and g_V/g_A is the vector coupling in unit of the axial-vector coupling of g_A=1.27. Here g^2_A for the GT and T NMEs is included in the phase-space factor. The $M^{0\nu}(GT)$ involves the axial-vector NMEs with 2^-, 3^+ and others [3].

The axial-vector and tensor DBD-NMEs are considered to be reduced with respect to the QP and pnQRPA DBD-NMEs by the re-normalization (quenching) coefficients of $k^2 \approx 0.04$ and $k^2_{NM} \approx 0.2$ with respect to the simple QP and pnQRPA DBD-NMEs, respectively [3, 20]. Then the DBD NME $M^{0\nu}$ may be reduced by the re-normalization (quenching) coefficient of around 0.3, depending on the relative weight of the vector NME with respect to the sum of the axial-vector and tensor DBD-NMEs. The DBD isotope (detector) mass required for a given ν-mass sensitivity is inversely proportional to $(M^{0\nu})^4$ [3, 7]. Then, the detector mass gets two orders of magnitude more than the detector mass in case of the pnQRPA DBD-NME without the quenching [3, 20]. The reduction (quenching) of the DBD NMEs is discussed in [3, 20, 39].

In case of the two-neutrino $\beta\beta$ NMEs at the low momentum of $q \leq$ a few MeV/c, the matrix element consists of $M^-(GT) \times M^+(GT)/\Delta$ with Δ being the energy denominator [4, 7, 8]. Thus, the two-neutrino $\beta\beta$ NMEs are reduced by the coefficient $k \approx k^- \times k^+ \approx$0.04, with $k^\pm \approx$0.2 being the reduction coefficients for the $\tau^\pm$ GT NMEs, with respect to the QP two-neutrino NME [3, 40, 41].

On the other hand, in case of the neutrino-less DBD, the major components associated with the DBD-NME are the τ^--side and τ^+-side 2^- NMEs at the medium momentum of q=30-100 MeV/c via the neutrino potential. Then, it is worthwhile to evaluate the neutrino-less DBD-NME by using, for example, such pnQRPA model with the re-normalization coefficient that reproduces the experimental τ^-- and τ^+-SD NMEs for ^{76}Ge. Theoretical pnQRPA calculations on the $\tau^\pm$ SD NMEs and DBD-NMEs for ^{76}Ge are under progress to be presented elsewhere.

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