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4 An overview of a highly versatile forward and stable
5 inverse algorithm for airborne, ground-based and borehole
6 electromagnetic and electric data

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26 ABSTRACT

27 Geophysical methods for high resolution studies of the near surface have
28 become a powerful tool for many different modern applications of widely varying
29 scope and scale. Often geologists, engineers or hydrologists use a custom
30 combination of different geophysical techniques for their particular purpose
31 involving dedicated inversion algorithms. This is inconvenient and makes direct
32 result comparison difficult due to different model formulations, approximations,
33 regularization, lack of uncertainty estimates etc. We present an overview of a
34 mature, robust and general algorithm providing a single framework for the
35 inversion of most electromagnetic and electrical data types and instrument
36 geometries. The implementation mainly uses a 1D earth formulation for
37 electromagnetics and magnetic resonance sounding responses, while the
38 geoelectric responses are both 1D and 2D and the sheets response models a
39 3D conductive sheet in a conductive host with an overburden of varying
40 thickness and resistivity. In all cases focus is put on delivering full system
41 forward modeling across all supported types of data. Our implementation is
42 modular, meaning that the bulk of the algorithm is independent of data type,
43 making it easy to add support for new types of data. Having implemented
44 forward response routines and file I/O for a given data type provides access to
45 a robust and general inversion engine. This engine includes support for mixed
46 data types, arbitrary model parameter constraints, integration of prior
47 information and calculation of both model parameter sensitivity analysis and
48 depth of investigation. We present a review of our implementation and
49 methodology and show four different examples illustrating the versatility of the

50 algorithm. The first example is a Laterally Constrained joint Inversion (LCI) of
51 surface time domain induced polarization (TDIP) data and borehole TDIP data,
52 the second example shows a spatially constrained inversion (SCI) of airborne
53 transient electromagnetic (AEM) data, the third example is an inversion and
54 sensitivity analysis of Magnetic Resonance Sounding (MRS) data, where the
55 electrical structure is constrained with AEM data. The fourth example is an
56 inversion of AEM data, where the model is described by a 3D sheet in a layered
57 conductive host.

58 INTRODUCTION

59 A wide range of near surface electric and electromagnetic geophysical
60 measurement techniques are routinely employed for vastly different purposes
61 within a variety of disciplines in modern science and engineering. Combining
62 advanced forward modeling with sophisticated inversion schemes allows for
63 obtaining accurate information on electric resistivity properties of geological
64 layers, however, such algorithms are typically limited to the modeling of a single
65 or a few types of data. Examples of this include

- 66 • 1D inversion of frequency domain ground conductivity meter data
67 (Santos 2004),
- 68 • airborne frequency domain data in 1D (Sengpiel and Siemon 2000),
- 69 • versatile inversion of frequency domain EM data in 1D (Oldenburg and
70 Jones 2011b),
- 71 • 1D holistic inversion of airborne frequency- (Brodie and Sambridge
72 2006a) and time domain EM data (Brodie and Sambridge 2006b),

- 73 • 1D inversion of frequency and time domain data (Christensen and
74 Auken 1992),
- 75 • 1D approximate inversion of time domain data (Tartaras et al. 2000;
76 Christensen 2002; Macnae et al. 1991),
- 77 • 1D inversion of multiple types of time domain data (Oldenburg and
78 Jones 2011c),
- 79 • 3D inversion of airborne time- and frequency domain data (Newman
80 and Alumbaugh 1997;Cox, Wilson and Zhdanov 2010, Oldenburg,
81 Haber and Shekhtman 2013),
- 82 • 2D inversion of resistivity (ERT) and induced polarization data (IP)
83 (Loke and Barker 1996;Loke, Chambers and Ogilvy 2006; Oldenburg
84 and Li 1994, Fiandaca et al. 2013b),
- 85 • 3D inversion of ERT data (Günther, Rücker and Spitzer 2006;Rücker,
86 Günther and Spitzer 2006),
- 87 • 3D inversion of ERT data with IP (Yoshioka and Zhdanov 2005;Loke
88 2011; Oldenburg and Jones 2011a),
- 89 • 3D inversion of magnetotellurics data (Gribenko, Wan and Zhdanov
90 2010),
- 91 • inversion of Magnetic Resonance Soundings (MRS) recovering water
92 content (Mohnke and Yaramanci 2002;Mueller-Petke and Yaramanci
93 2010;Hertrich, Braun and Yaramanci 2005)

94 Implementing just the base of such inversion algorithms can be a
95 tremendous task in itself and often resources will limit the stage of development
96 that can be reached. For this paper we present a more unusual research
97 algorithm, AarhusInv, in the sense that it supports a broad spectrum of data

98 types. AarhusInv has further been allowed to reach a point of production
99 maturity, while at the same time being very actively developed for various
100 research projects. Reaching this stage has to a large degree been made
101 possible by support from the ongoing Danish national mapping of groundwater
102 resources (Thomsen, Søndergaard and Sørensen 2004;Møller et al. 2009).
103 This project was initiated in the late 1990's and has provided stable funding and
104 extensive production use of the algorithm, including inversion of ERT data,
105 PACES (Sørensen et al. 2005), groundbased TEM and airborne SkyTEM data
106 (Sørensen and Auken 2004). Furthermore, there have been many contributions
107 on various parts of the algorithm, hence the long list of co-authors on this paper.

108 The main distinction to other algorithms reported in the literature is how
109 different high accuracy forward models, spanning multiple data types, are
110 brought together in a common inversion framework. This framework is not only
111 optimized for production but also flexible for research, by supporting arbitrary
112 spatial constraints and full integration of a priori information on any model
113 parameter.

114 AarhusInv is freely available for university research institutions. Over
115 time it has been used for the inversion of an extensive amount of vastly different
116 datasets, collected over all continents of the world. Some of the most prominent
117 include an 18,000 line km VTEM survey conducted over the Okavango delta,
118 Botswana (Podgorski et al. 2013), a 34,000 line km SkyTEM survey in the
119 Broken Hill Area of Australia in 2009, 14,000 line km in India in 2013 and a
120 1,000 line km TEMPEST survey on Eyre peninsula of South Australia (Auken
121 et al. 2009a). For the Danish groundwater mapping project it has been used for
122 inversion of approximately 40,000 groundbased TEM soundings, 25,000 line

123 kilometers of SkyTEM and several thousand ERT profiles. We estimate that the
124 algorithm has inverted more than 400,000 line kilometers of airborne data since
125 2005, including data from the Resolve system (Fugro Inc.), DigHEM (Fugro
126 Inc.), SkyTEM (Sørensen and Auken 2004), VTEM (Witherly, Irvine and
127 Morrison 2004), AeroTEM (Balch, Boyko and Paterson 2003) and TEMPEST
128 (Lane et al. 2000).

129 In this paper we provide a review of the numerical implementation and
130 demonstrate its flexibility and capabilities through examples. A core component
131 is a constrained model framework which has proven successful for many
132 different geologies, ranging from mapping of paleo channels in sedimentary
133 environments to mapping of perched aquifers in a weathered volcanic geology
134 (Jørgensen and Sandersen 2006;d'Ozouville et al. 2008;Danielsen et al.
135 2003;Reid, Munday and Fitzpatrick 2007;Siemon et al. 2002). We begin by
136 describing the design of the algorithm along with a list of supported data types
137 and instrument geometries. This is followed by a review of the forward and
138 inverse modeling schemes. Finally four field data examples illustrate the
139 algorithms versatility to operate on data collected on the ground, in the ground
140 and in the air.

141 METHODOLOGY

142 ***Modular algorithm layout***

143 Since the origin of AarhusInv in 1995 - 1997 its feature set has evolved
144 significantly, but in its simplest form the functionality is still to invert a set of
145 geophysical soundings for a set of layered 1D models connected through
146 constraints. The algorithm was formerly known as em1dinv, but with the

147 introduction of multidimensional responses we changed the name to AarhusInv
148 in 2012. The code base is in Fortran 95/77 and the program itself is a command
149 line application with no user interface and with all input/output conducted
150 through ASCII files. This structure is flexible and very well suited for functioning
151 as an integrated inversion engine, as in the case of the front end GUI
152 applications Aarhus Workbench (Auken et al. 2009c), EMMA (Auken et al.
153 2002) and SPIA (www.hgg.au.dk). The algorithm is implemented in a general
154 modular manner as conceptually outlined in Figure 1. This figure is
155 supplemented by Table 1, providing full details on supported data types,
156 measurement geometries, and key references.

157 Starting from the top in Figure 1 a general model input file of flexible
158 format is first read. This model file contains a starting model definition, specifies
159 arbitrary regularization constraints and also holds prior information. Following
160 model file input is the reading of data files in the data file input module. These
161 files are encoded in data type specific formats, which are transparently handled
162 by the modules internal branching to the relevant sub modules. Note that mixing
163 of any number of different data types is supported, allowing for unrestricted joint
164 inversions. Having filled the internal data structures from input files these
165 structures are passed on to the general inversion module which starts the
166 iterative inversion process. During the iterative course of minimizing the
167 residual the inversion module calls the forward response module (see Appendix
168 A) for the calculation of forward response updates and derivatives. The forward
169 response module also handles data type specific branching transparently,
170 simply performing the requested calculations and returning the result to the
171 inversion module. Internally, the forward response module makes data type

172 dependent calls to a library of common routines. Having iteratively solved the
173 inverse problem (see next paragraph and Appendix B) the result is sent to the
174 general model analysis module, which calculates a model parameter sensitivity
175 analysis. Finally the depth of investigation (DOI) is calculated based on the
176 actual model output from the inversion process and a recalculated sensitivity
177 (Jacobian) matrix (Christiansen and Auken 2012). Following the DOI calculation
178 a result file is finally written in a general file format.

179 ***Forward modeling***

180 From a forward modeling point of view AarhusInv provides a wide range
181 of options for simulating a given measurement system. This is particularly
182 important for TEM data, as neglecting the influence of system parameters such
183 as geometry, waveform or filters can lead to serious modeling error as
184 described for TEM by (Christiansen, Auken and Viezzoli and Fiandaca et al.
185 (2012) for DC/IP. The extent to which the algorithm allows for full system
186 forward modeling is relatively unique and makes it possible to compare model
187 results from different data types as directly as possible. Key references for the
188 forward modeling are given in the caption of Table 1. It is outside the scope of
189 this paper to describe all these different solutions and therefore we have in
190 Appendix A chosen to give descriptive details on the 1D EM solutions which are
191 also the most used parts of AarhusInv.

192 The forward modeling is an impended parallel using OpenMP and scales
193 close to linearly running on 1 – 64 processors. All vectors and matrices are
194 furthermore implemented sparsely making the memory consumption relatively
195 low even for very large inverse problems. It is out of the scope of this paper to

196 describe the details of this implementation and we therefore refer to Kirkegaard
197 and Auken (2014) for all details.

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199 ***Inverse modeling***

200 The mathematical formalism of the inverse modeling scheme follows the
201 established practice by Menke (1989a). Details are provided by Auken et al.
202 (2005) and Appendix B gives in-depth details to the implementation of the
203 inversion. Here, we focus on presenting the flexibility and capabilities of the
204 algorithm. Essentially, we use a Gauss-Newton style minimization scheme with
205 a Marquart modification (Marquart 1963) to find the set of model parameters
206 that minimize the L_2 misfit with respect to observed data, regularizing and prior
207 information. All data sets are inverted simultaneously, minimizing a common
208 object function. Consequently, the output models are balanced between the
209 constraints, the physics and the data. Model parameters that have little
210 influence on the data will be controlled by the constraints, and vice versa. The
211 use of, for example, lateral or spatial constraints on a collection of models
212 allows for information to propagate across a survey area.

213 Model constraints are applied between any two parameters of the same
214 type by specifying the variance of the difference between the two and the
215 uncertainty of prior information is similarly specified by the variance of the given
216 prior value. The AarhusInv implementation itself is unbiased in terms of how
217 constraints should be defined and thus leaves this to be specified by the user
218 in the input model file. This format allows for arbitrary constraints linking any
219 model parameters, i.e. a set of 1D models can be linked by 3D constraints in
220 any way the user sees fit. Typically, we apply constraints in the form of 1D

221 laterally constrained inversion (LCI, Auken et al. 2005) and 1D spatially
222 constrained inversion (SCI, Viezzoli et al. 2008) for producing quasi 2D and
223 quasi 3D models, respectively. The constraining formulation is flexible and
224 allows for constraints and prior information on both primary and secondary
225 model parameters. We define primary model parameters as the actual
226 parameters of the layered model, e.g. resistivities and thicknesses, whereas
227 secondary model parameters are any parameters that can be derived from
228 linear combinations of the primary parameters, e.g. depths and elevations
229 (Auken et al. 2005). This can be utilized for example when integrating prior
230 information from a borehole, where the known parameter is typically the depth
231 of a layer interface and not a layer thickness.

232 For the inversion of airborne data we allow including the instrument
233 altitude as a model parameter, as will be discussed in more detail in the airborne
234 example. In the case of helicopter data the pilot attempts to follow the terrain
235 topography and maintains a relatively constant altitude, whereas in fixed wing
236 surveys the aircraft typically operate at almost constant elevation. In this case
237 it is desirable to supply lateral constraints linking the modeled instrument
238 elevation of neighboring models, instead of linking the primary model
239 parameters in the form of height above terrain. Table 2 shows which additional
240 model parameters are included in the inversion for certain airborne systems.
241 These parameters are all in excess of the primary and secondary parameters
242 of resistivity, thickness and depth.

243 In conjunction with any inversion result a model parameter analysis can
244 be calculated, obtaining estimates of the resulting model uncertainty. This is
245 done by calculating the covariance matrix of the resulting model of the inversion

246 (Tarantola and Valette 1982a, Auken and Christiansen 2004), which provides
247 a linearized error estimate. Since we perform our inversion in logarithmic model
248 space for numerical stability, the values obtained from such a sensitivity
249 analysis can conveniently be regarded relative measures of uncertainty.
250 Absolute error estimates from logarithmic space are translated into a standard
251 deviation factor in linear space, such that 1.0 is equal to perfect resolution and
252 1.1 corresponds to a standard deviation of approximately 10%. We find that
253 when such linearized error estimates become much larger than unity they are
254 best viewed as guidelines, however. The use of such error estimates will be
255 illustrated in the following examples.

256 We finalize the inverse modeling procedure by calculating the DOI for
257 the resulting output models, an operation which relies on a reweighting of the
258 Jacobian matrix. The computations employ a global and absolute sensitivity
259 threshold value, which has been tuned for operating in logarithmic model/data
260 space (Christiansen and Auken 2012). For a given model, the DOI calculations
261 consider only the parts of the Jacobian related to observed data, implying that
262 the effect of lateral, spatial or vertical model parameter constraints and a priori
263 information is not included. With this type of DOI estimate it is possible to judge
264 when the information in a model is driven by data or heavily dependent on the
265 starting model or specifics of the regularization.

266 ***Limitations due to dimensionality of the forward response and inversion***

267 The functionalities described so far illustrate the general capabilities of
268 the algorithm at its current stage of development, but obviously its inherent
269 limitations should also be considered.

270 First of all the algorithm base began as a 1D implementation and the
271 whole suite of EM methods are still limited to 1D forward modeling with the
272 exception of 3D sheets. For DC and IP the forward algorithms are present in
273 both 1D and 2D versions and a 3D implementation is close to being complete.
274 Solving the physical problem in 1D can be a limitation for applications such as
275 mineral exploration, whereas studies of sedimentary settings fits well within a
276 1D modeling envelope (eg. Auken et al. 2008, Viezzoli et al. 2010).

277 Secondly, while being implemented as general as possible, our
278 regularization and inversion schemes are fundamentally based on a static
279 formulation. This means that it is not possible to experiment with e.g. Tikhonov-
280 style variable regularization parameters. However, the least squares (L2-norm)
281 solution can be changed to use the L1-norm or the sharp boundary formalism
282 by Zhdanov, Fang and Hursán (2000) and Vignoli et al. (2013).

283 EXAMPLES

284 In the following we show four examples demonstrating the capabilities
285 and versatility of the AarhusInv algorithm. In the examples we show inversions
286 of data collected on the ground, in the ground and in the air. We also illustrate
287 the use of spatial-, lateral- and vertical constraints. The examples are chosen
288 to illustrate widely varying scales of interest, ranging small scale engineering
289 type of problems to large regional scale surveys. We further illustrate the use
290 of prior information and provide a synthetic example of joint inversion of mixed
291 type data. Finally we provide a synthetic example inverting an airborne data set
292 with a 3D sheet model.

293 ***On and in the ground: DC and time domain induced polarization***

294 For our first example we present a novel application of joint 1D LCI
295 inversion of surface time domain induced polarization (TDIP) data and borehole
296 TDIP data. The TDIP method is a natural and efficient extension of standard
297 DC multi electrode profiling by simply adding the measurement of the time
298 dependence of discharge after an injection current is turned off. Modeling the
299 complete time decay of each data point in the Cole-Cole formulation (Pelton et
300 al. 1978) allows for extracting additional independent model parameters, often
301 making it possible to discriminate earth structures with an otherwise identical
302 signature. Apart from resistivity, each layer of an IP model includes the
303 parameters τ , C and M_0 . Here m_0 is the magnitude of the chargeability when the
304 injection current is turned off at $t=0$, τ is the decay time constant and C is a
305 constant controlling the frequency dependence of the response. The TDIP
306 implementation is described in full detail by Fiandaca et al. (2012) and includes
307 modeling of the full injection current waveform and low pass filters, which can
308 be a cause of serious modeling error when neglected. Compared to the
309 methodology presented by Fiandaca et al. (2012) we add in this example
310 borehole TDIP data for a joint inversion of two data types sharing a common
311 model space, in order to demonstrate the versatility of the algorithm.

312 The investigated area is a former Danish landfill active from the 1950's
313 to the 1980's. During this period the landfill has been almost complete
314 unregulated and it was further established without any kind of membranes,
315 leachate capture or isolation systems. Previous studies show that oils and
316 chemical waste from the landfill is contaminating the area by water seepage
317 through the landfill, but the extent of the pollution is largely unknown. Additional

318 details of the survey area are provided by Gazoty et al. (2012). Figure 2 show
319 the dataset consisting of DC/IP data collected using a gradient array protocol
320 and an electrode spacing of 5 meters, as well as DC/IP data from a pole-pole
321 EI-log (Sørensen and Larsen 1999) collected in a borehole around the center
322 of the profile. For the logging a measurement was made for every half meter
323 down to 24 meters. This combined dataset was 1D LCI inverted for models of
324 21 layers, discretized in fixed boundaries and utilizing lateral and vertical
325 smoothness constraints.

326 On the model sections in Figure 2 the position of the borehole is
327 indicated near the center. In the right part of the sections further results for 2
328 drillings are superimposed for comparison. From these boreholes it is tempting
329 to conclude that the waste and pollution is likely to be localized to the resistive
330 structure in the vicinity of the boreholes. However, when including the
331 chargeability (m_0) section in the interpretation it becomes clear that the extent
332 of the waste body can possibly be much greater. In fact the results of the full
333 investigation of 13 profiles and 15 boreholes show a very strong correlation
334 between buried waste and chargeability, which is undetectable in terms of
335 resistivity alone. Including log data in a joint inversion contributes
336 complementary information due its perpendicular plane of sampling, introducing
337 a completely different frequency domain kernel since the electrodes are placed
338 in the ground. The result of including this complementary and very detailed
339 vertical information is for all model parameters to become well determined at
340 this position as seen in the right part of Figure 2. In turn the entire profile benefits
341 from this added information, since the use of LCI constraints enable lateral
342 migration of information.

343 ***In the air: Transient electromagnetic***

344 The vast majority of data being inverted by AarhusInv is airborne data from
345 helicopter and fixed wing systems. To illustrate this key application we show
346 result of an SCI inversion of SkyTEM data (Sørensen and Auken 2004). The
347 SkyTEM system uses dual transmitter moments to provide unbiased time
348 gates from as early as 6 μs up to 10 ms (Schamper, Auken and Sørensen
349 2014) . This allows for accurate forward modeling, including the effects of the
350 waveforms of the dual transmitter moments, low pass filters, front gate and
351 finite width gates. For the inversion we use measurements of the vertical
352 component of the secondary field only and include instrument altitude as a
353 model parameter. In the more general case of fixed wing type geometry the
354 algorithm allows for inversion of all field components, including receiver pitch
355 and roll as model parameters (Auken et al. 2009b). We have also
356 implemented inversion for receiver coil response model parameter, allowing
357 for the use of data from time gates as early as 6 μs (Schamper, Auken and
358 Sørensen 2013)

359 Our example dataset of 3250 line km was collected in 2009 covering an
360 area of approximately 820 km^2 intersected by the Danish/German border,
361 having virtually no topography and covering the coast to the North Sea. The
362 survey was conducted to provide a better overall understanding of the
363 hydrogeological setting in the area and to map the fresh/salt water interface in
364 the costal region. Following data acquisition, all couplings due to man-made
365 installations were removed from the raw data set and further processed using
366 the methodology of Auken et al. (2009c). The result is a data set of stacked

367 soundings for each transmitter moment approximately every 25 meters. This
368 data set was jointly inverted for both moments using 19 layer 1D models with
369 fixed layer boundaries and constrained using the SCI methodology. We show
370 the results of the inversion in Figure 3; sub figure (a) shows the flight lines of
371 the dataset, (b) the depth of investigation, (c) the modeled resistivity at a
372 depth of 74-88 meters and in (d) the model parameter analysis of (c). From
373 resistivity maps (c) it is possible to identify important features of both
374 regional- and more local scale if zooming into the details. This is illustrated by
375 marking the extent of two buried valley structures and a large area of salt
376 water intrusion. The geological interpretation of the dataset including cross
377 sections and average resistivity maps is discussed in detail by Jørgensen et
378 al. (2012).

379 ***Mixed data types: Magnetic Resonance Sounding and Airborne TEM***

380 For this last example we show a feasibility study involving forward
381 modeling, inversion and sensitivity analysis of Magnetic Resonance Sounding
382 (MRS) data in combination with airborne TEM data. The example is intended to
383 illustrate the capabilities of the algorithm on mixed type datasets, utilizing
384 multiple modules of the algorithm simultaneously.

385 The synthetic model of our study is outlined in Table 3 and represents a
386 20 meter thick aquifer situated at a depth of 50 meters, covered by dry sands,
387 a till layer and defined at depth by a thick layer of clay. For this synthetic model
388 we consider two types of synthetic data: an airborne TEM sounding and a
389 ground-based MRS sounding. The TEM sounding simulates a SkyTEM system
390 utilizing a 300 m² transmitter loop situated at an altitude of 30 meters, acquiring
391 data for time gates ranging from 10 μ s to 1 ms. For the inversion the simulated

392 data is assigned a uniform noise level of 3%, but no actual perturbation is
393 performed. In case of the MRS part of the dataset we consider a simulation
394 utilizing a 100 m x 100 m square loop. We further set the magnetic field of the
395 earth to 49,300 nT at an inclination of 70 degrees and a declination of +2
396 degrees and utilize pulse moment values ranging from 0.11 to 15.0 As. The
397 synthetically generated sounding was assigned and perturbed for a base noise
398 level of 20 nV, with an additional uniform noise contribution of 3%.

399 Traditionally MRS data has been inverted in what can be referred to as
400 step-wise inversion schemes (e.g., Günther et al. 2011; Mueller-Petke and
401 Yaramanci 2010; Behroozmand et al. 2012b). This type of inversion starts by
402 inverting a TEM or DC sounding for a resistivity model, which is then assumed
403 to be the true model in a subsequent inversion for MRS model parameters
404 (water content ratio W and relaxation time T_2^*). In other words the MRS kernel
405 function, which is a function of the resistivity model, is assumed fixed.
406 Behroozmand et al. (2012a) proposes a joint inversion methodology where the
407 resistivity model parameters are free in the inversion, implying that the MRS
408 kernel has to be updated for each iteration of the inversion requiring a fast
409 numerical implementation of the MRS kernel. In the following we compare the
410 resolution capabilities of these two methods.

411 In Figure 4 the model results of a traditional step-wise inversion of the
412 simulated data is shown. The figure features dashed blue lines for indicating
413 68% confidence intervals, as obtained from a sensitivity analysis. It is clear that
414 the upper- and lower-most boundaries of the resistivity model are very well
415 resolved, whereas the layering in between these boundaries is completely
416 undetermined. If a noise perturbation had been applied to the TEM data, the

417 obtained model result would have been very far from the true model for the two
418 highly resistive layers. Turning to the MRS model we see that the water content
419 of the synthetic model is well resolved for the top two layers, but underestimated
420 for the lower lying water bearing sand layer with the true value out of range of
421 the models 68% confidence intervals. This mismatch is caused by the slightly
422 wrong layer thicknesses fixed in the resistivity model, which in turn forces the
423 modeled water content to compensate for the MRS kernel function being
424 slightly off. For the bottom layer the inverted water content is found to be even
425 further from the true model, but in this case the deviation can to a larger degree
426 be attributed to a high degree of uncertainty as it is closer to the depth of
427 investigation. In general, $T2^*$ values are found to be reasonably well resolved
428 for all layers.

429 In Figure 5 the result of the joint inversion scheme is shown. The first
430 thing to note is how all layers of the resistivity model have become well
431 resolved, also in the case of the two resistive layers. Determination of layer
432 thicknesses has improved in particular, due to the combined, complementary
433 information provided by the SkyTEM and MRS signals. In the case of MRS
434 model parameters this point can be seen from the water content model being
435 very accurately reproduced for the top three layers, while the improvement for
436 the deepest lying layer is less pronounced as it is closer to the depth of
437 investigation. For the $T2^*$ model the impact of the coupled scheme is very little.

438 ***Airborne TEM and sheets inversion***

439 In the AarhusInv code, the forward modeling of thin sheets is based on
440 the algorithm developed by Zhou (1989) with the mathematical formulation from
441 Weidelt (1983). This algorithm calculates the response from a number of

442 arbitrarily located thin sheets in a conductive layered host. The sheets have
443 both inductive and channeling current modes. If several sheets are used they
444 will be inductively coupled. To speed up the computation the algorithm uses
445 OpenMP to take advantage of multi-processor computer architecture, and an
446 efficient solver has been added to compute the scattered fields for inversion of
447 airborne datasets where many source-receiver locations need to be computed.
448 Furthermore, the algorithm has been extended to include both dipole and finite
449 loop sources, and it calculates responses both in the time and frequency
450 domain as stated in Table 1 and Table 2.

451 We will illustrate the use of this part of AarhusInv on synthetic transient
452 AEM data as shown in Figure 6. The model has a 20 m conductive overburden
453 of 50 Ωm , covering a resistive halfspace of 1000 Ωm . The sheet is in the
454 resistive host and has a conductance of 50 S, a top depth of 30 m, a maximum
455 depth of 175 m, and both its dip and strike angles are 45° (Figure 6a). The
456 simulated AEM dataset has a line spacing of 100 m with a sounding spacing of
457 20 m along the lines (Figure 6b) and z-component data only. The flight altitude
458 is 30 m and recorded times span from about 10 μs to a few ms depending on
459 the noise. Random noise at a slope of $t^{-1/2}$ and a noise level of 1 nV/m² at 1 ms
460 was added to the data (Schamper et al. 2014). Figure 6c illustrates the fit
461 obtained after inversion with some of the recorded times along the central
462 profile. The starting model parameters are indicated in the figure. The
463 parameters in the inversion are sheet conductance, dip, strike, layered model
464 parameters and flight altitude. In the present case, the sheet is well determined
465 with the TEM method and the final estimated model is well recovered and

466 almost identical to the true model as is also indicated by the parameter
467 determination (not shown here).

468 CONCLUSIONS

469 We have presented an overview of a proven, versatile and flexible
470 production inversion algorithm, capable of inverting most electric and
471 electromagnetic data types and supporting most instruments normally used
472 within the field of near surface geophysics. The algorithm is freely available for
473 academic use and provides the user with freedom to specify arbitrary 3D
474 regularization constraints, include any amount of prior information and invert for
475 mixed type data sets. Our implementation focuses on consistently accurate
476 system forward modeling independent of data type and instrument, which not
477 only ensures accuracy, but can also be regarded a prerequisite for joint
478 inversion of data collected using instruments of different transfer function. The
479 implementation is modular, meaning that the bulk of the algorithm is
480 independent of data type, making it very easy to add support for new types of
481 data. Calculation of both model parameter sensitivity analysis and depth of
482 investigation is a key feature and it is handled automatically regardless of
483 configuration and data types.

484 The versatility of the algorithm is illustrated by four different examples.
485 First, we showed the joint inversion of a resistivity/time-domain IP data set with
486 electrodes both located in the ground and on the surface. This example
487 provides an illustration of how including Cole-Cole parameters in the model
488 greatly improves the delineation of a former landfill. Second, we have shown
489 results of a large scale airborne TEM survey where the combined information
490 from maps of average resistivity, depth-of-investigation and model parameter

491 analysis improves the understanding of the underlying geology. Thirdly, we
492 have shown a synthetic example where we jointly invert Airborne TEM data with
493 ground-based MRS data. These two model-spaces are only vaguely related,
494 but we show how the mutual information significantly improves the resolution of
495 both the resistivity- and the water content model. In the fourth example we
496 showed a synthetic example inverting Airborne TEM data with a 3D sheets
497 model.

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511 APPENDIX A – 1D EM FORWARD CALCULATION

512 The type of problem we consider consist of a given instrument source, receiver
513 and layered half space for which to solve Maxwell's equations. We do this in
514 order to calculate some or all of the electromagnetic field components, or their
515 time derivatives, at the position of the receiver. This type of calculation is
516 conveniently performed in the frequency domain and can be formulated to
517 exploit symmetry by choosing the gauge leading to the Schelkunoff potential
518 formulation. In the following we closely follow Ward and Hohmann (1988). In
519 the Schelkunoff formalism we operate on the electric and magnetic vector
520 potentials \mathbf{A} and \mathbf{F} from which the actual electromagnetic fields can be
521 determined by means of differentiation. This formulation is desirable since the
522 vector potentials point in the direction of a source with a convenient geometry,
523 allowing for calculation of all EM field components from just a single non-zero
524 component of either \mathbf{A} or \mathbf{F} . In cases of more complicated geometry the problem
525 can be reduced to solving multiple simple problems by means of mode
526 decomposition.

527 In equation A1 we show the characteristic type of equation to be solved
528 in order to calculate the non-zero component of the vector potentials, here in
529 the case of a Z directed magnetic source.

$$530 \quad \mathbf{F}(x, y, z) \propto \hat{\mathbf{z}} \iint_{-\infty}^{\infty} f(x, y, z, k_x^2 + k_y^2) e^{-i(k_x x + k_y y)} dk_x dk_y \quad (\text{A1})$$

531 where $\hat{\mathbf{z}} = i\mu\omega$, μ is the magnetic permeability ω is the angular frequency, k_x and
532 k_y is the wave number in the x and y direction.

533 We recognize this double integral as an inverse 2D Fourier transform, which
534 can be reduced to a single integral by applying an inverse Hankel transform
535 instead. This is possible since f is cylinder symmetric given its dependence on
536 $k_x^2 + k_y^2$ and allows for the equation to be rewritten in simpler form:

$$537 \quad F(\rho, z) \propto \hat{z} \int_0^\infty f(\lambda, z) \cdot \lambda \cdot J_0(\lambda\rho) d\lambda$$

538 (A2)

539 where λ is the space frequency, and J_0 is the 0th order Bessel function. This
540 type of inverse Hankel transform integral can be evaluated very efficiently using
541 digital filter methods as described by Johansen and Sørensen (1979) and
542 Christensen (1990). Given the ability to efficiently solve inverse Hankel
543 transform type integrals allows for fast calculation of any EM field component,
544 by simple means of differentiation of the vector potential equations. In the
545 calculation of the type of integral in equation A2 the most time consuming part
546 becomes the evaluation of the integrand function f . This function includes a
547 frequency dependent reflection coefficient, essentially accounting for the
548 amplitudes of up- and down- going damped waves within the layered halfspace.
549 This is calculated from a recurrence relation going up through the layering which
550 becomes the effective bottlenecking term, making the calculation time scale
551 linearly with the number of layers in the half space. In the case of frequency
552 domain data these are simply the transmitted frequencies of the system,
553 whereas the frequency space needs to be more systematically sampled for time
554 domain problems. In this latter case we solve for a data type specific number of
555 logarithmically spaced frequencies per decade and transform into the time
556 domain using an inverse sine/cosine digital filter transform (Johansen and
557 Sørensen (1979) , Newman, Hohmann and Anderson 1986). This approach is

558 much more efficient than a standard inverse fast Fourier transform and for late
559 times we also find it more stable and accurate than the faster Gaver-Stehfest
560 inverse Laplace transform (Knight and Raiche 1982). To get to the final time
561 domain result from the discretely transformed frequency domain data we apply
562 an interpolating bicubic spline to obtain results for the particular points in time
563 of interest.

564 For the accuracy of the resulting responses we note that this is
565 determined mainly by the sampling density of the digital filters. Higher density
566 implies more computations, meaning that a performance/accuracy compromise
567 has to be made. We allow for the user to manually tune this trade off, but provide
568 default settings for an error of around 0.3% for frequencies up to 100 – 200 kHz
569 and for a time domain error of less than 1% for times as early as 4 μ s. These
570 frequencies and times are somehow dependent on the overall conductivity of
571 the ground and thus high frequencies and early times are more inaccurate for
572 very high resistivities than for low resistivities.

573 Ever since the codebase was started more than 15 years ago, all
574 responses and common routines have been routinely validated against both
575 analytical expressions and other forward modeling algorithms.

576 ***Modeling the system transfer function for TEM systems***

577 An important aspect of modeling a complete TEM system is accounting for the
578 transmitter waveform. In the case of a step function this is done directly by
579 integration in the frequency domain (Bracewell 1986), while the general case of
580 arbitrary waveform is handled by convolution in the time domain using the
581 approach of Fitterman and Anderson (1987). These authors show that an
582 arbitrary waveform can be accounted for by a simple rewrite of the convolution

583 so it is expressed as a sum over a numerical differentiation of a piece wise
584 linear waveform.

585 Of equal importance is accounting for the receiver frequency characteristics.
586 Not only does a receiver coil itself have a frequency characteristic, but further
587 low-pass filters are typically also applied and both can have a significant
588 influence on the measured signal over the dynamic range of interest. Effersø,
589 Auken and Sørensen (1999) describes how low and high pass filtering are
590 implemented in the frequency domain before transforming to the time domain.
591 This is done by a simple multiplication of a filter response function, here in the
592 form of a Butterworth filter, with the same approach used for modeling of low
593 pass filters for time domain induced polarization (TDIP) responses. The effect
594 of finite gate width is included in the modeling by integrating cubic spline
595 functions over the width of the gate. If finite gates are not used the response at
596 the gate center times are calculated from a local interpolation using cubic
597 splines. Some instruments further utilize a special gate right after the receiver
598 coil. This gate serves to prevent strong primary fields from saturating the
599 receiver amplifiers during transmitter turn on and it is modeled by convolving a
600 shifted heavy side function with the step response.

601 Using SkyTEM as an example the full procedure for forward modeling
602 the response becomes: (1) the filter effect of the receiver coil is modeled in the
603 frequency domain as a simple product, (2) the frequency domain response is
604 transformed to the time domain using a Hankel transform, (3) the front gate is
605 convolved with the step response, (4) low pass filters in the receiver instrument
606 are applied as a new convolution, (5) a piece wise linear waveform is applied

607 by numerical differentiation and finally (6) gates are calculated by integrating
608 the response over the length of the gates.

609

610

APPENDIX B – LEAST SQUARES INVERSION

611

612 The inversion is performed iteratively, by following the established
 613 practice of linearized approximation of the non-linear forward mapping of the
 614 model to the data space, by the first term of the Taylor expansion (Menke
 615 1989b, Auken et al. 2005a). The $n+1^{\text{th}}$ update of the model vector \mathbf{m}_{n+1} is
 616 obtained by:

$$617 \quad \mathbf{m}_{n+1} = \mathbf{m}_n + [\mathbf{G}'_n{}^T \mathbf{C}'^{-1} \mathbf{G}'_n + \lambda_n \mathbf{I}]^{-1} \cdot [\mathbf{G}'_n{}^T \mathbf{C}'^{-1} \delta \mathbf{d}'_n] \quad (\text{B1})$$

618 Where the Jacobian \mathbf{G}'_n , the data vector update $\delta \mathbf{d}'_n$ and the covariance
 619 matrix \mathbf{C}' incorporate both the a priori and the roughness constraints and are
 620 defined as:

$$621 \quad \mathbf{G}'_n = \begin{bmatrix} \mathbf{G}_n \\ \mathbf{P} \\ \mathbf{R} \end{bmatrix} \quad (\text{B2})$$

$$622 \quad \delta \mathbf{d}'_n = \begin{bmatrix} \delta \mathbf{d}_n \\ \delta \mathbf{m}_n \\ \delta \mathbf{r}_n \end{bmatrix} = \begin{bmatrix} \mathbf{d}_n - \mathbf{d}_{obs} \\ \mathbf{m}_n - \mathbf{m}_{prior} \\ -\mathbf{Rm}_n \end{bmatrix} \quad (\text{B3})$$

$$623 \quad \mathbf{C}' = \begin{bmatrix} \mathbf{C}_{obs} & 0 & 0 \\ 0 & \mathbf{C}_{prior} & 0 \\ 0 & 0 & \mathbf{C}_R \end{bmatrix} \quad (\text{B4})$$

624 In equation B2 \mathbf{G}_n represents the Jacobian of the forward mapping. For
 625 1D LCI/SCI solutions the matrix \mathbf{G}_n is computed by differentiation of forward
 626 responses \mathbf{F}_{1D} : $\mathbf{G}_n^{i,j} = (\mathbf{F}_{1D}^i(\mathbf{m} + \Delta \mathbf{m}_j) - \mathbf{F}_{1D}^i(\mathbf{m})) / (\Delta \mathbf{m}_j)$, for datum i and
 627 model parameter j . On the contrary, in the 2D DC/IP implementation the
 628 Jacobian is computed with the adjoint method approach (Fiandaca et al.
 629 2013b). \mathbf{P} is the matrix with dimension $N_m \times N_m$ (N_m being the number of model
 630 parameters), necessary to impose the constraints on the *a priori* values. \mathbf{R} is
 631 the roughness matrix, in which each row represents one roughness constraint

632 (each row is zero everywhere except for the two elements corresponding to
 633 constrained parameters, elements being equal to 1 and -1).

634 The constraints also appear as extra terms in the definition of the data
 635 vector update $\delta \mathbf{d}'_n$ (equation B3), that is composed of the distance $\delta \mathbf{d}_n$
 636 between the n^{th} forward response \mathbf{d}_n and the observed data \mathbf{d}_{obs} , the distance
 637 $\delta \mathbf{m}_n$ between the n^{th} model vector \mathbf{m}_n and the a priori model vector \mathbf{m}_{prior}
 638 (used also as starting model for the iterative procedure) and the roughness of
 639 the n^{th} model vector $\delta \mathbf{r}_n = -\mathbf{R}\mathbf{m}_n$.

640 The covariance matrix \mathbf{C}' of equation B4 is defined in terms of the
 641 covariance on the observed data \mathbf{C}_{obs} , the covariance on the a priori information
 642 \mathbf{C}_{prior} and the covariance on the roughness constraints \mathbf{C}_R . All three matrices
 643 are considered diagonal; the elements of \mathbf{C}_{prior} and \mathbf{C}_R control the strength of
 644 the constraints, while the elements of \mathbf{C}_{obs} reflect the noise content of the data.

645 The matrices \mathbf{P} , \mathbf{R} , \mathbf{C}_{prior} and \mathbf{C}_R , as well as the vectors $\delta \mathbf{m}_n$ and $\delta \mathbf{r}_n$
 646 are split in two parts in order to account for prior/roughness constraints on both
 647 the primary parameters (e.g. resistivity and thickness) and on depths. In fact,
 648 the depths are not included directly in the model space, and a special
 649 formulation of the matrices is needed to include the depth prior/roughness
 650 constraints in equation B1 (see Auken et al. 2005a for details)

651 In equation B1 the parameter λ_n is the Marquart damping parameter
 652 (Marquart 1963), iteratively updated to stabilize the inversion process through
 653 an adaptive algorithm that damps the step size. This algorithm uses as damping
 654 value λ_n the maximum diagonal value of the $\mathbf{G}'_n \mathbf{C}'^{-1} \mathbf{G}'_n$ matrix, reduced by an
 655 iterative-dependent scaling factor f_n

$$656 \quad \lambda_n = \max \text{diag}(\mathbf{G}'_n \mathbf{C}'^{-1} \mathbf{G}'_n) \cdot f_n \quad (\text{B5})$$

657 The damping is used to stabilize the solution of the linear system of
658 equations (B1), but a constraint on the “step size” of the iteration, i.e. the size
659 of the model update $\|\mathbf{m}_{n+1} - \mathbf{m}_n\|$, is also imposed through f_n (the step size is
660 reduced by increasing f_n). When increasing the iteration number n , the scaling
661 factor f_n is decreased and the step size is increased ensuring a robust and
662 efficient damping scheme.

663 The object function minimized by equation B1 is expressed by

$$664 \quad Q = \left(\frac{[\delta \mathbf{d}'^T \mathbf{C}'^{-1} \delta \mathbf{d}']}{N_d + N_m + N_R} \right)^{\frac{1}{2}} \quad (\text{B6})$$

665 In which N_d , N_m , N_R represent the number of data points, the number of model
666 parameters and the number of constraints. The output models are then
667 balanced between the data (through the forward response, i.e. the physics), the
668 *a priori* constraints and the roughness constraints.

669 Finally the covariance of the estimator error \mathbf{C}_{est} (Tarantola and Valette
670 1982b) is used in AarhusInv to estimate the resolution of the inverted model by
671 using its expression for linear mappings on the last iteration N_{ite} :

$$672 \quad \mathbf{C}_{est} = \left(\mathbf{G}'^T_{N_{ite}} \mathbf{C}'^{-1} \mathbf{G}'_{N_{ite}} \right)^{-1} \quad (\text{B7})$$

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923 **Tables**

924

Source group	Source type	Receiver type	Source position	Receiver position	Domain
Dipole Reference 1,2,3	Vertical magnetic dipole	Dipole: Any E and H field component	Anywhere	Anywhere	Frequency and time
	Vertical electric dipole	Dipole: Any E and H field component	Anywhere	Anywhere	Frequency and time
	Horizontal electric dipole	Dipole: Any E and H field component	Ground surface or in the air	Anywhere	Frequency and time
	Horizontal magnetic dipole	Dipole: Any E and H field component	Ground surface or in the air	Ground surface or in the air	Frequency and time
Loop Reference 1,2,3	Rectangular loop	Dipole: Any E and H field component	Ground surface or in the air	Anywhere	Frequency and time
	Arbitrary segmented loop	Dipole: Any E and H field component	Ground surface or in the air	Anywhere	Frequency and time
	Circular center	Dipole: H_z	Ground surface or in the air	Anywhere on the axis	Frequency and time
	Circular offset	Dipole: E_x, E_y, H_x, H_y, H_z	Ground surface or in the air	Anywhere	Frequency and time
	Circular in-loop	H_z	Ground surface or in the air	Ground surface or in the air	Frequency and time
Wire Reference 1,2,3	Finite length grounded x-directed dipole	Dipole: E_x, E_z, H_y, H_z (or B)	Ground surface	Anywhere	Frequency and time
	Finite length grounded y-directed dipole	Dipole: E_y, E_z, H_x, H_z (or B)	Ground surface	Anywhere	Frequency and time
	Infinite x-directed line source	Dipole: E_x, H_y, H_z (or B)	Ground surface	Anywhere	Frequency and time
	Infinite y-directed line source	Dipole: E_y, H_x, H_z (or B)	Ground surface	Anywhere	Frequency and time
DC Reference 4,5, 6	Schlumberger	Apparent resistivity or potentials	Ground surface	Ground surface	
	Wenner	Apparent resistivity or potentials	Ground surface	Ground surface	
	Pole-pole	Apparent resistivity or potentials	Ground surface	Ground surface	

	Arbitrary quad pole	Apparent resistivity or potentials	Ground surface	Ground surface	
	Arbitrary quad pole in the ground	Apparent resistivity or potentials	Electrodes at the surface or in the ground	Electrodes at the surface or in the ground	
	Arbitrary quad pole in cylinder symmetry	Apparent resistivity or potentials	Electrodes in the ground	Electrodes in the ground	
DC/IP Reference 7, 8	Arbitrary quad pole	Apparent resistivity/chargeability or potentials	Ground surface	Ground surface	Time domain
	Arbitrary quad pole in the ground	Apparent resistivity/chargeability or potentials	Electrodes at the surface or in the ground	Electrodes at the surface or in the ground	Time domain
Magnetotelluric Reference 1		Apparent resistivity or dipole E_x , E_y , H_x , H_y		Ground surface or in the ground	
MRS Reference 12, 13	Arbitrary segmented loop		Ground surface	Ground surface	
Surface wave dispersion Reference 14			Ground surface	Ground surface	
Supported 2D and 3D responses					
Source group	Source type	Receiver type	Source position	Receiver position	Domain
DC/IP (2D) Reference 7, 8	Arbitrary quad poles	Apparent resistivity/chargeability or potentials	Ground surface	Ground surface	Time
Loop – thin sheets (3D) Reference 9, 10, 11	Arbitrary segmented loop	Dipole: Any E and H field component	Ground surface or in the air	Anywhere	Frequency and time

925 Table 1: Overview of supported data types and configurations. All time domain responses can
926 be calculated as step or impulse response or with an arbitrary waveform. Abbreviations: 1D,
927 2D and 3D are the dimensionality of the response, E is electric field, H is magnetic field, E_x ,
928 E_y , E_z , H_x , H_y , H_z where x,y and z are the principal components, DC is direct current
929 (geoelectric), DC/IP is DC with time domain induced polarization, MRS is Magnetic
930 Resonance Sounding. Key references for the basic electromagnetic calculations include
931 ¹Ward and Hohmann (1988), ²Wannamaker, Hohmann and SanFilipo (1984), ³Xiong (1989).

932 DC/IP responses in 1D are described by ⁴Zhdanov and Keller (1994) and ⁵Fiandaca et al.
 933 (2012) with the special case of electrodes in the layered halfspace which follows ⁶Sato (2000).
 934 2D DC/IP uses a generalization of the approach described by ⁷Kemna et al. (2000) and is
 935 reported by ⁸Fiandaca et al. (2013). Sheets follows the theory of ⁹Weidelt (1983) and the
 936 implementation by ¹⁰Zhou (1989) with an additional time transform as discussed in ¹¹
 937 Schamper, Auken and Kirkegaard (2013) . Magnetic resonance soundings (MRS) follows the
 938 equations in ¹²Weichman, Lavelly and Ritzwoller (2000) and is reported in ¹³Behroozmand et
 939 al. (2012b). Surface wave seismics is reported in ¹⁴Wisén and Christiansen (2005).

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Optional Inversion parameters		
System type	Additional Parameters	Purpose
All airborne TEM system	Receiver altitude/elevation	Correction for inaccurate determination of altitude
All airborne TEM systems	Data shift	Correction of data level
Fixed wing TEM systems	Receiver pitch roll and position	Correction for bird pitch, roll and position
MRS	Water content, T ₂ * and stretch parameter (T ₂ * distribution)	Additional parameters in model space

DC/IP	Chargeability, c and time constant (Cole-Cole parameters)	Additional parameters in model space
Sheets	Sheet position, size, strike and dip	Additional parameters in model space

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959 Table 2: Additional inversion model parameters for relevant system types. The base model

960 parameterization includes resistivity, thickness and depth

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Layer	Lithology	Resistivity (ohmm)	Thickness (m)	Water content (m ³ /m ³)	T2* (s)	C
1	Till, some water	40	30	0.1	0.1	1
2	Dry sand	300	20	0.02	0.2	1
3	Saturated sand	80	20	0.4	0.2	1
4	Clay	5		0.4	0.02	1

963

964 Table 3: The synthetic model used for mixed modeling of MRS and airborne TEM data

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966

967 Figure Captions

968 Figure 1: Illustration of the modular algorithm design. All modules are implemented in a
969 general manner to support any data type, except for the data file input and forward response
970 modules which contains data type specific branches.

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973 Figure 2: Joint surface and borehole DC/IP LCI inversion result. The model sections on the
974 left side are faded below the depth on investigation and the right side of the figure shows the
975 corresponding standard deviation sections. The position and depth of the logged borehole
976 included in the inversion is marked by a black square and results from 2 boreholes is
977 superimposed on the left hand figures.

978

979 Figure 3: SkyTEM SCI inversion results. (a) shows the flight lines of the dataset, (b) is the
980 depth of investigation, (c) the modeled resistivity at a depth of 74-88 meters and (d) is the
981 model parameter analysis of (c).

982

983 Figure 4: Results of MRS and Airborne TEM step-wise inversion. The green lines shows the
984 true model and the black lines are the inverted model results. The dashed blue lines indicate

985 68% confidence intervals, as obtained from model parameter analysis, with a red line
986 superimposed to indicate uncertainty so large that it goes out of the scale of the figure. We
987 note that for the resistivity the sensitivity analysis is based on TEM data alone.

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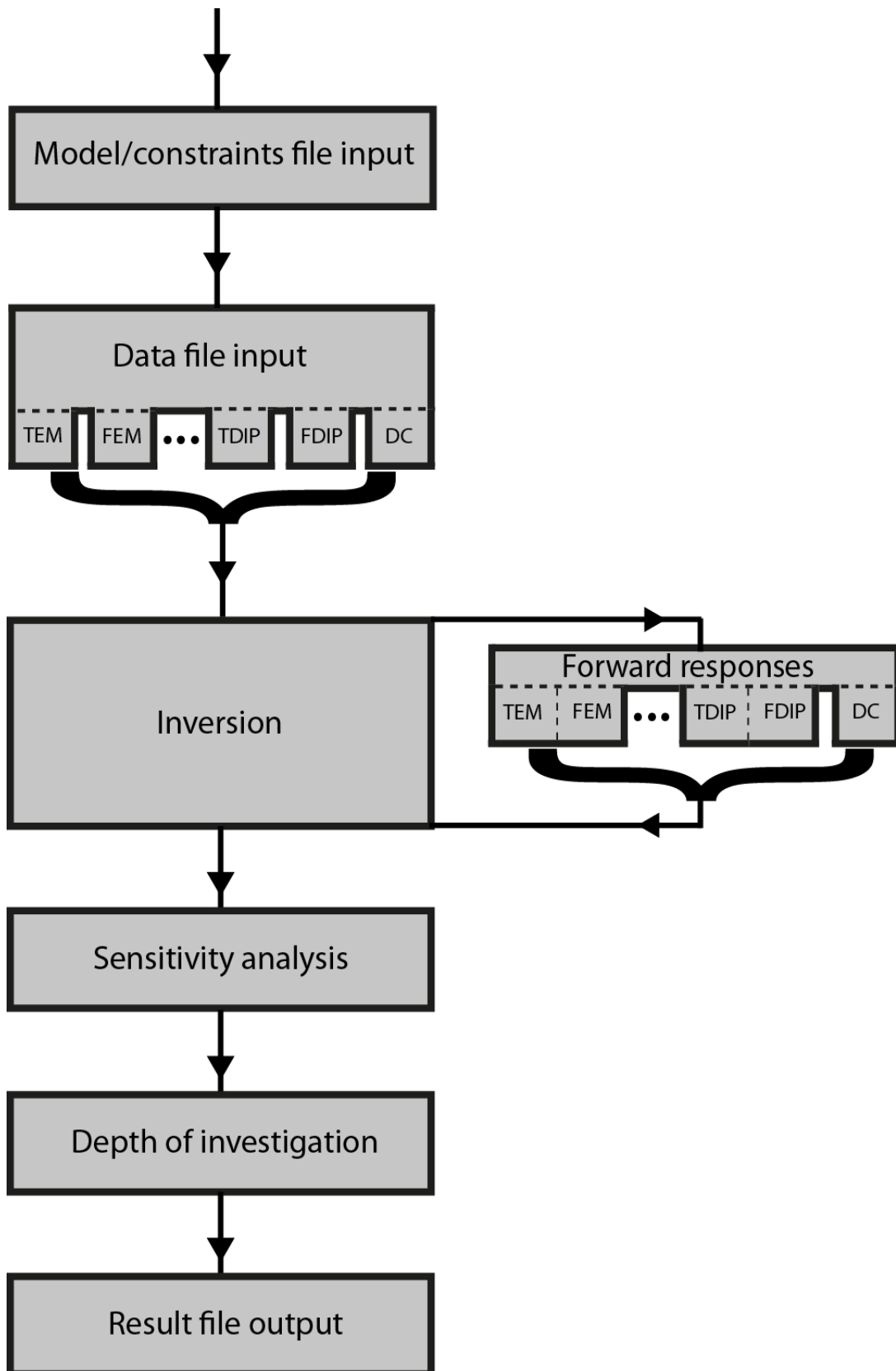
990 Figure 5: MRS and Airborne TEM full joint inversion results. The green lines show the true
991 model parameters and the black lines the inverted parameters. The dashed blue lines indicate
992 68% confidence intervals, as obtained from model parameter analysis.

993

994 Figure 6: Thin sheet inversion results where the sheet is buried under a conductive
995 overburden. (a) 3D view of the thin sheets with the starting, true and final estimated models
996 (those 2 last ones are confounded), the red dots corresponds to the airborne TEM sounding
997 locations; (b) 2D top view of the same sheet models; (c) Data profile with the synthetic
998 generated data (noise has been added at late times), the response of the starting model, and
999 the response of the final model.

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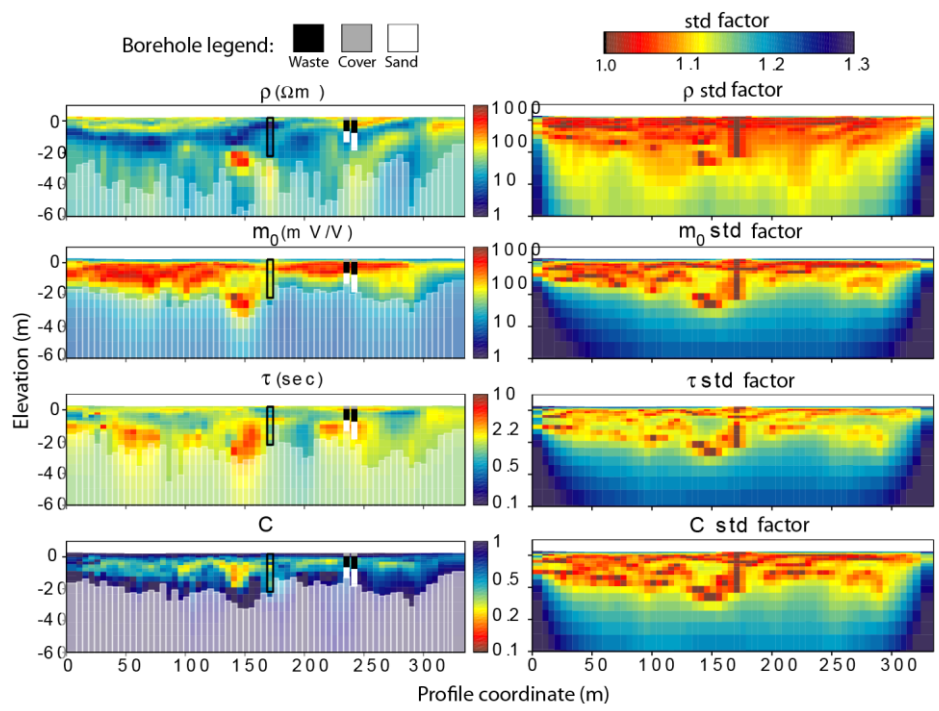
1001 Figure 1



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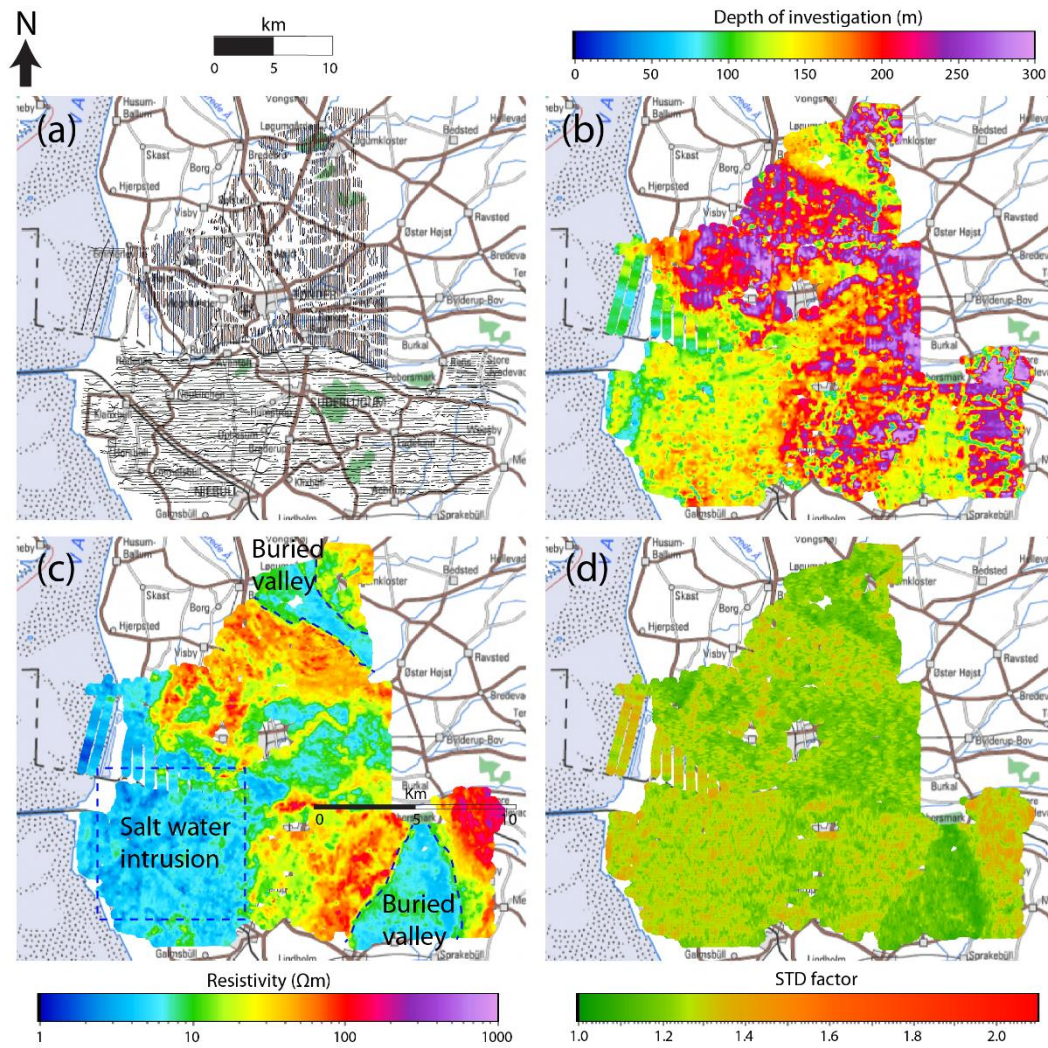
1004 Figure 2



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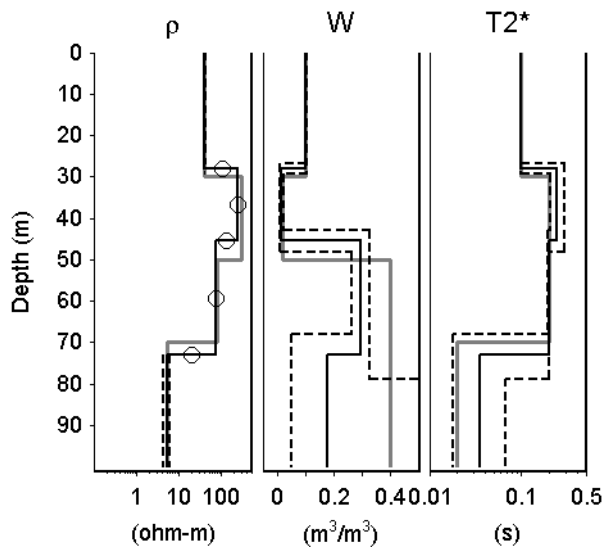
1007 Figure 3



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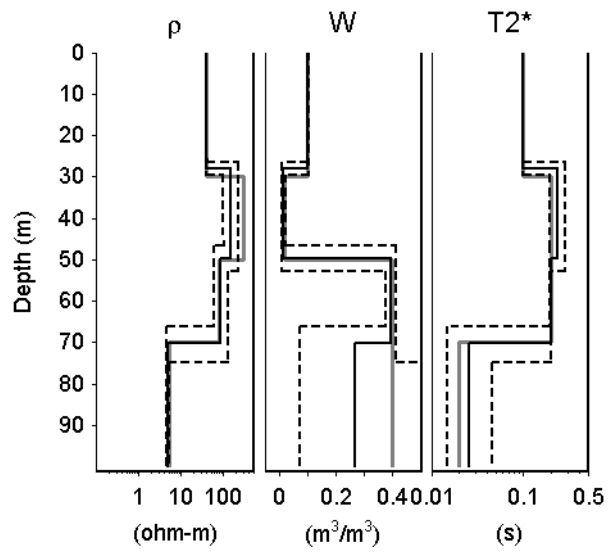
1010 Figure 4



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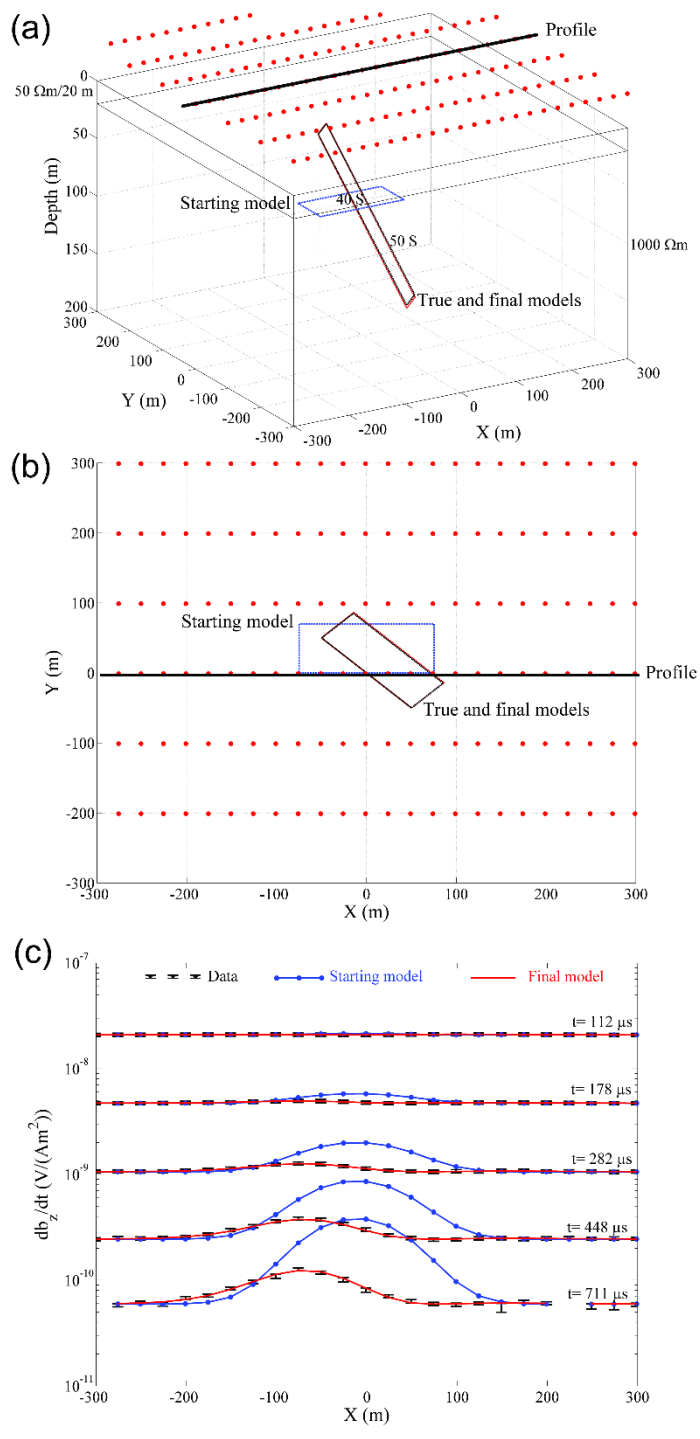
1013 Figure 5



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1016 Figure 6



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