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Optimal Redistributive Income Taxation and Efficiency Wages*

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Abstract

In this paper, we integrate efficiency wage setting with the theory of optimal redistributive income taxation. In doing so, we use a model with two skill types, where efficiency wage setting characterizes the labor market faced by the low-skilled, whereas the high-skilled face a conventional, competitive labor market. We show that the marginal income tax implemented for the high-skilled is negative under plausible assumptions. The marginal income tax facing the low-skilled can be either positive or negative, in general. An increase in unemployment benefits contributes to a relaxation of the binding self-selection constraint, which makes this instrument particularly useful from the perspective of redistribution.

Keywords: Nonlinear income taxation; redistribution; unemployment benefits

JEL classification: *H*21; *H*53; *J*23; *J*31

I. Introduction

The modern theory of optimal redistributive taxation, as developed from the influential contribution of Mirrlees (1971), is largely based on models where the labor market is perfectly competitive. Although analytically convenient, such a description of the labor market is clearly at odds with most real-world developed economies, where unemployment has been an important social problem for a long time. Albeit small by comparison, a body of literature on optimal redistributive taxation under unemployment has gradually evolved during the latest decades, in which the incentives underlying an optimal tax policy partly differ from those following under perfect competition. The major mechanisms generating unemployment in these studies are trade-union wage formation (e.g., Aronsson and Sjögren,

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2003, 2004; Aronsson *et al.*, 2009; Hummel and Jacobs, 2018), minimum wages (Marceau and Boadway, 1994), and search frictions (Lehmann *et al.*, 2011). Although fundamentally different, a common denominator is a policy incentive to reduce the level of unemployment, which is likely to result in higher marginal tax rates than under perfect competition.

However, to our knowledge, there have been no earlier studies on optimal redistributive taxation in economies where the labor market is characterized by efficiency wages. The purpose of the present paper is to fill this gap by integrating efficiency wages in the self-selection approach to optimal taxation. Such an extension is interesting for several reasons. First, efficiency wage theory has played an important role in labor economics for a long time, by explaining involuntary unemployment as well as wage differentials across workers and sectors.[1] Second, it has been used in related areas of public economics as a framework for studying relationships between tax policy, wages, and unemployment in representative-agent models (e.g., Pisauro, 1991; Chang, 1995). Third, and compared with the related literature on trade-union behavior, efficiency wage theory does not rely on any (arbitrary) assumption of objective function for trade-unions,[2] because the wage rate is decided unilaterally by firms realizing that increased wages lead to higher productivity.[3]

Our study is based on a model with two skill types,[4] where efficiency wage setting characterizes the labor market faced by the low-skilled, whereas the high-skilled face a conventional, competitive labor market. The rationale for this assumption is that unemployment is typically higher and more persistent among low-skilled individuals than among high-skilled individuals. This suggests that it is more important to examine the mechanisms generating equilibrium unemployment in the context of agents

[1] See Katz (1986) for an overview of efficiency wage models. See also Shapiro and Stiglitz (1984).

[2] See Kaufman (2002) for an overview of models with trade-unionized labor markets.

[3] It is not easy to discriminate empirically between efficiency wage models and other labor market models that generate equilibrium unemployment. However, one specific feature of efficiency wage models is that workers employed in firms where shirking is difficult to detect, or where the supervisory intensity is low, will earn more, *ceteris paribus*. Empirical evidence pointing in this direction is presented in, for example, Kruse (1992), Rebitzer (1995), and Ewing and Payne (1999). See Neal (1993) for a study contradicting these results. Another specific feature is that the effort choice of workers is likely to depend on industry characteristics (because the organization of work, as well as of supervision and monitoring, is likely to vary across industries). Krueger and Summers (1988) examine wage differentials across equally skilled workers and find substantial differentials related to industry; a finding that is consistent with efficiency wage theory (albeit not likely to arise under competitive wage formation). See Moretti and Perloff (2002) for evidence suggesting that wage formation in the US agricultural labor market is consistent with efficiency wage theory.

[4] The discrete two-type version of the Mirrleesian optimal income tax problem was developed in its original form by Stern (1982) and Stiglitz (1982).

with relatively low productivity. The government uses a nonlinear income tax and an unemployment benefit to correct for the imperfection in the labor market and to redistribute income from high-skilled to low-skilled individuals. We assume that two types of jobs are available in the economy: low-demanding jobs, which can be carried out both by low-skilled and high-skilled individuals, and high-demanding jobs, which can only be carried out by high-skilled individuals. We also assume that effort, which is thought of as the "effort exerted per hour spent at the workplace", is a decision-variable in low-demanding jobs, meaning that individuals employed in this type of job have the option to shirk with an exogenous probability of detection (through imperfect monitoring).[5] As a consequence, there are two ways for a high-skilled individual to mimic the income of a low-skilled type: either by choosing a low-demanding job, or by reducing the hours of work when employed in a high-demanding job. In turn, the government must recognize both these options when solving the optimal tax and expenditure problem.

We show that the marginal income tax rate implemented for the high-skilled is negative under reasonable assumptions, while the marginal income tax rate implemented for the low-skilled can be either positive or negative. The intuition for the first result is that an increase in the hours of work by high-skilled individuals leads to increased demand for low-skilled labor, thus counteracting the unemployment problem. The policy rule for marginal income taxation of low-skilled individuals reflects a mixture of effects generated through the binding self-selection constraint and the possibility to affect wage formation and employment through tax policy. Whereas employment-related motives (and thus the ability to raise more tax revenue) typically push in the direction of increased marginal income taxation for low-skilled individuals, the self-selection constraint might affect the marginal tax policy in the opposite direction depending on which of the two strategies the mimicker prefers. If the mimicker adopts the strategy of reducing the hours of work when employed in a high-demanding job, as briefly discussed above, it is possible that a decrease in the marginal income tax rate implemented for low-skilled individuals contributes to relax the binding self-selection constraint (this is not possible under the other mimicking strategy of choosing a low-demanding job). We also find that

[5]Notice that imperfect monitoring in low-demanding jobs generates a second source of asymmetric information, between firms and workers employed in low-demanding jobs, on top of the standard asymmetric information problem between the government and the workers (regarding their skill type). In this sense, our paper also relates to a recent strand in the Mirrleesian literature (see Stantcheva, 2014; Bastani *et al.*, 2015; Cremer and Roeder, 2017) analyzing optimal tax policy in settings with two sources of asymmetric information. The difference is that whereas, in the aforementioned papers, the second source of asymmetric information manifested itself in an adverse selection problem, in our model it manifests itself in a moral hazard problem.

an increase in the unemployment benefit leads to a relaxation of the binding self-selection constraint, irrespective of which strategy the mimicker prefers. This suggests that the unemployment benefit is likely better targeted to low-skilled individuals than a corresponding benefit paid out to low-income earners in general.

The outline of the paper is as follows. In Section II, we present the model by describing the decision-problem and the behavior of individuals and firms, and the optimal tax and expenditure problem facing the government. The optimal public policy is characterized and discussed in Section III, while our conclusions are summarized in Section IV. All the proofs are presented in an Online Appendix.

II. The Model

The economy is populated by two skill types. High-skilled workers are paid the before-tax wage rate w^h, and low-skilled workers are paid the before-tax wage rate w^ℓ, where $w^h > w^\ell$. The total population is normalized to one and the fraction of individuals of type j is denoted π^j (for $j = \ell, h$).

Two different types of jobs are available in the economy: low-demanding jobs that can be carried out by all workers, and high-demanding jobs that are exclusive to high-skilled workers. Whereas the high-demanding jobs pay a competitive wage, the low-demanding jobs are characterized by a monitoring technology such that the employers pay a wage above the market clearing level in order to boost the effort level of the employees. This generates involuntary unemployment among low-skilled workers.

Individuals and Firms

Denoting consumption by c, hours spent at the workplace by L, and effort exerted at the workplace by $e \in [0, 1]$, individual preferences are represented by the concave and partially additive separable utility function

$$U = g(c) + v(L, e). \tag{1}$$

Individuals derive utility from consumption, and disutility from work hours and effort, respectively (i.e., $\partial g/\partial c > 0$, $\partial v/\partial L < 0$, and $\partial v/\partial e < 0$). To simplify the interpretation of the results, we also add the (quite realistic) assumption that $\partial^2 v/\partial e \partial L < 0$; that is, the marginal utility of leisure $(-\partial v/\partial L)$ increases with the effort exerted when working. Furthermore, we assume that $v(0, 0) = 0$.

For workers employed in high-demanding jobs, where there is no monitoring problem and competitive wages are paid, we assume that the effort exerted by workers is equal to 1, which can be interpreted to mean that workers in high-demanding jobs never shirk.[6] Instead, for workers employed in low-demanding jobs, effort is a choice variable and affects the probability of being fired. The probability of being monitored in low-demanding jobs is assumed to be exogenous, and a worker is fired if caught shirking.[7] Assuming that the firms monitor each worker in a low-demanding job with probability p, and interpreting $1 - e$ as the fraction of time that an individual spends shirking while on the job, the probability of being fired is given by $(1 - e)p \equiv \varphi(e)$, where $\partial\varphi(e)/\partial e < 0$ and $\partial^2\varphi(e)/(\partial e)^2 = 0$. If fired, low-skilled workers face two alternatives: they can either be hired by another firm or become unemployed.[8] The economy-wide unemployment rate is denoted by u.

In the tradition of the optimal income tax literature following Mirrlees (1971), we assume that an individual's skill type (as reflected in the before-tax wage rate) is private information, while the individual's income if employed, $I^j = w^j L^j$, is publicly observable. This rules out first-best type-specific lump-sum taxes but allows income to be taxed via a general, nonlinear tax schedule, $T(I^j)$. The other policy instrument at the disposal of the government is a transfer, b, paid to each unemployed individual. To characterize the set of (constrained) Pareto-efficient resource allocations, we derive an optimal revelation mechanism. For our purpose, a mechanism consists of a set of type-specific before-tax incomes I^j and disposable incomes B^j for individuals in employment, an unemployment benefit b, and the unemployment rate u. A complete solution to the optimal tax problem per se, such that I^j and B^j are determined via individual utility maximization, then requires the design of a general income tax function, $T(\cdot)$, such that $B^j = I^j - T(I^j)$. This is described in greater detail below.

We begin by characterizing the decision-problem and the behavior of individuals and firms, and we then continue with the optimal tax and expenditure problem. Each individual of any type j acts as an atomistic agent by treating the hourly wage rates, the unemployment benefit, the unemployment rate, and the parameters of the tax function (including the structure of marginal taxation) as exogenous.[9] Private consumption equals

[6]This is clearly a simplification – albeit a consequence of the assumption of a perfectly competitive market for this type of job. A possible interpretation is that the job characteristics make shirking impossible or uninteresting for those employed in high-demanding jobs.

[7]Assuming away turnover costs, this turns out to be the best strategy for a firm (see Shapiro and Stiglitz, 1984, p. 437).

[8]With all firms being identical, and under the assumption that agents are fully informed, the wage of low-skilled workers, as well as their choice of effort, will be the same in each firm.

[9]This just reflects the assumption that each individual is small relative to the economy as a whole. All these variables are, of course, endogenous in the model.

the disposable income B^j for an employed individual of skill type $j = \ell, h$, and equals b for an unemployed individual.

Starting with an individual of the low-skilled type, note that the individual makes two decisions: (i) an optimal effort choice (which influences the likelihood of becoming unemployed); (ii) a consumption–leisure choice conditional on being employed. For a given (I^ℓ, B^ℓ)-bundle, we can define the individual's indirect expected utility function as

$$EV^\ell(B^\ell, I^\ell, u, b; w^\ell) = \max_{e^\ell}[1 - u\varphi(e^\ell)]\left[g(B^\ell) + v\left(\frac{I^\ell}{w^\ell}, e^\ell\right)\right] + u\varphi(e^\ell)g(b),$$

where we have used $L^\ell = I^\ell/w^\ell$, and E denotes the expectations operator. The variable $u\varphi(e^\ell)$ can be interpreted as the probability that any low-skilled individual becomes unemployed (the overall unemployment rate among low-skilled workers times the probability of being fired when shirking).[10] To rule out the possibility that low-skilled workers are voluntarily unemployed, we assume that $g(B^\ell) + v(I^\ell/w^\ell, 0) > g(b)$. Then, in choosing the optimal level of effort, a low-skilled worker trades off the additional cost of effort against the gain in expected utility, because of the decreased probability of becoming unemployed. At an interior optimum,[11] the first-order condition for the effort choice is given by

$$[1 - u\varphi(e^\ell)]\frac{\partial v}{\partial e^\ell} = u\varphi'(e^\ell)\left[g(B^\ell) + v\left(\frac{I^\ell}{w^\ell}, e^\ell\right) - g(b)\right]. \qquad (2)$$

Equation (2) implicitly defines e^ℓ as a function of B^ℓ, I^ℓ/w^ℓ, b, and u, i.e.,

$$e^\ell = e^\ell\left(B^\ell, \frac{I^\ell}{w^\ell}, b, u\right). \qquad (3)$$

Based on equation (2), we can derive the following comparative statics properties of the effort function with respect to B^ℓ, I^ℓ, u, b, and w^ℓ:

[10]Note that there are three possible states of the world for a low-skilled worker: (i) tenure of employment in firm i, with probability $1 - \varphi(e)$; (ii) fired by firm i and hired by another firm, with probability $(1 - u)\varphi(e)$; (iii) fired by firm i and unemployed, with probability $u\varphi(e)$.

[11]Assuming that $\partial v/\partial e^\ell \longrightarrow 0$ when $e^\ell \longrightarrow 0$, we can rule out the possibility that $e^\ell = 0$ is optimal. Moreover, if p is sufficiently low, we can also rule out the possibility that $e^\ell = 1$ is optimal. To see this, notice that $e^\ell = 1$ would be suboptimal provided that

$$-v\left(\frac{I^\ell}{w^\ell}, 1\right) - \frac{\partial v[(I^\ell/w^\ell), e^\ell]/\partial e^\ell \mid_{e^\ell=1}}{up} > g(B^\ell) - g(b),$$

or, equivalently, denoting by $(\epsilon_{v,e}) \mid_{e^\ell=1}$ the elasticity of the v-function with respect to e evaluated at $e^\ell = 1$,

$$\frac{(\epsilon_{v,e}) \mid_{e^\ell=1}}{up} > -\frac{g(B^\ell) - g(b)}{v[(I^\ell/w^\ell), 1]} - 1.$$

$$\frac{\partial e^{\ell}}{\partial B^{\ell}} = \frac{u\varphi'(e^{\ell})(\partial g/\partial B^{\ell})}{\Phi} > 0, \tag{4}$$

$$\frac{\partial e^{\ell}}{\partial I^{\ell}} = -\frac{[1-u\varphi(e^{\ell})][\partial^2 v/(\partial e^{\ell}\partial L^{\ell})] - u\varphi'(e^{\ell})(\partial v/\partial L^{\ell})}{\Phi}\frac{1}{w^{\ell}} < 0, \tag{5}$$

$$\frac{\partial e^{\ell}}{\partial u} = \frac{\varphi(e^{\ell})(\partial v/\partial e^{\ell}) + [g(B^{\ell}) + v(I^{\ell}/w^{\ell}, e^{\ell}) - g(b)]\varphi'(e^{\ell})}{\Phi} > 0, \tag{6}$$

$$\frac{\partial e^{\ell}}{\partial b} = -\frac{u\varphi'(e^{\ell})(\partial g/\partial b)}{\Phi} < 0, \tag{7}$$

$$\frac{\partial e^{\ell}}{\partial w^{\ell}} = \frac{[1-u\varphi(e^{\ell})][\partial^2 v/(\partial e^{\ell}\partial L^{\ell})] - u\varphi'(e^{\ell})(\partial v/\partial L^{\ell})}{\Phi}\frac{L^{\ell}}{w^{\ell}} > 0, \tag{8}$$

where

$$\Phi \equiv [1-u\varphi(e^{\ell})]\frac{\partial^2 v}{\partial e^{\ell}\partial e^{\ell}} - 2u\varphi'(e^{\ell})\frac{\partial v}{\partial e^{\ell}} < 0.$$

These results are intuitive. An increase in the post-tax income in the employment state, B^{ℓ}, raises the effort as it increases the opportunity cost of shirking (by widening the gap between the utility if employed and the utility if unemployed); the opposite effect on effort is induced by an increase in pre-tax income I^{ℓ}.[12] An increase in the economy-wide unemployment rate increases the effort, in order to reduce the probability of ending up in unemployment, while an increase in the unemployment benefit leads to decreased effort as it reduces the income loss to the individual if becoming unemployed.

Noticing that employed, low-skilled workers behave as if they are maximizing $EV^{\ell}[I^{\ell} - T(I^{\ell}), I^{\ell}, u, b; w^{\ell}]$ with respect to I^{ℓ}, we can implicitly characterize the marginal income tax rate faced by low-skilled workers, based on the first-order condition, as

$$T'(I^{\ell}) = 1 + \frac{\partial EV^{\ell}/\partial I^{\ell}}{\partial EV^{\ell}/\partial B^{\ell}}. \tag{9}$$

Regarding high-skilled workers, remember that shirking is not an option in high-demanding jobs. Thus, provided that high-skilled workers have no incentive to leave their high-demanding jobs,[13] they will maximize

[12]In this case, the reduction in effort is explained both by a reduction in the opportunity cost of shirking (as an increase in I^{ℓ} lowers the gap between the utility if employed and the utility if unemployed) and by our assumption that the marginal disutility of effort is increasing, in absolute value, in hours of work (i.e., $\partial^2 v/\partial e \partial L < 0$).

[13]As discussed in the next section, the optimal tax policy chosen by the government will ensure that this is, indeed, the case.

$$V^h[I^h - T(I^h), I^h; w^h] = g[I^h - T(I^h)] + v\left(\frac{I^h}{w^h}, 1\right)$$

with respect to I^h. We can then implicitly characterize the marginal income tax rate facing a high-skilled individual as

$$T'(I^h) = 1 + \frac{\partial V^h/\partial I^h}{\partial V^h/\partial B^h} = 1 - \frac{1}{w^h}\frac{\partial v(I^h/w^h, 1)/\partial L^h}{\partial g(B^h)/\partial B^h}. \tag{10}$$

Turning to the production side of the economy, we assume that identical, competitive firms produce a homogeneous good by using labor as the only input. The production process is characterized by constant returns to scale. Because the number of firms is treated as exogenous, it is normalized to one for notational convenience. Let N^ℓ and N^h denote the number of workers employed in low-demanding and high-demanding jobs, respectively. Using the assumption that $e^h = 1$, the production function is given by

$$F(e^\ell L^\ell N^\ell, L^h N^h). \tag{11}$$

The production function is increasing in each argument, $F_1' \equiv \partial F/\partial(e^\ell L^\ell N^\ell) > 0$ and $F_2' \equiv \partial F/\partial(L^h N^h) > 0$, the marginal products are diminishing such that $F_{11}'' \equiv \partial^2 F/(\partial e^\ell L^\ell N^\ell)^2 < 0$ and $F_{22}'' \equiv \partial^2 F/(\partial L^h N^h)^2 < 0$, and the production factors are technical complements, i.e., $F_{12}'' = F_{21}'' \equiv \partial^2 F/\partial(e^\ell L^\ell N^\ell)\partial(L^h N^h) > 0$. The decision-problem facing the representative firm is given by

$$\max_{w^\ell, N^\ell, N^h} F\left[N^\ell \frac{I^\ell}{w^\ell} e^\ell\left(B^\ell, \frac{I^\ell}{w^\ell}, u, b\right), N^h \frac{I^h}{w^h}\right] - \sum_{i=\ell,h} N^i I^i.$$

Each individual firm treats the choices made by the government, as reflected in (I^ℓ, B^ℓ, I^h, B^h, b, u), and the wage rate paid to the high-skilled, w^h, as exogenous. By recognizing how the wage rate paid to workers in low-demanding jobs affects effort through equation (3), the representative firm chooses w^ℓ and N^ℓ based on the following first-order conditions:

$$\frac{\partial e^\ell}{\partial L^\ell}\frac{L^\ell}{e^\ell} = -1, \tag{12}$$

$$F_1' = \frac{w^\ell}{e^\ell}. \tag{13}$$

Equation (12) implicitly characterizes the optimal wage rate paid to workers employed in low-demanding jobs, and equation (13) characterizes the optimal number of agents employed in low-demanding jobs. With the disposable income in the employment state, B^ℓ, held constant, the wage rate enters the effort equation only through $I^\ell/w^\ell = L^\ell$. Therefore, equation (12) is just a variant of the standard condition for wage setting in an efficiency wage model, i.e., $(\partial e^{\ell}/\partial w^{\ell})w^{\ell}/e^{\ell} = 1$. Finally, the first-order condition for N^h is given by

$$F_2' = w^h, \tag{14}$$

which defines the equilibrium wage rate for workers in the high-demanding job, w^h.

For later purposes, we need to evaluate how N^{ℓ}, w^{ℓ}, and w^h vary in response to changes in I^{ℓ}, B^{ℓ}, I^h, B^h, u, and b. Denoting by σ the elasticity of substitution between the two labor inputs in production, we obtain the comparative statics results given in Table 1 (see the Online Appendix).

In particular, note that the elasticity of substitution, σ, plays a key role in how N^{ℓ} responds to variations in I^{ℓ}, B^{ℓ}, u, and b. If $\sigma > 1$ (which seems to be a plausible assumption based on empirical research),[14] we can see that $dN^{\ell}/dI^{\ell} < 0$, $dN^{\ell}/dB^{\ell} > 0$, $dN^{\ell}/du > 0$, and $dw^{\ell}/db < 0$. Regarding the w^{ℓ}-responses to variations in I^{ℓ}, B^{ℓ}, u, and b, note that the signs of dw^{ℓ}/dI^{ℓ}, dw^{ℓ}/dB^{ℓ}, and dw^{ℓ}/db are unambiguous (with $dw^{\ell}/dI^{\ell} > 0$, $dw^{\ell}/dB^{\ell} < 0$, and $dw^{\ell}/db > 0$),[15] whereas the sign of dw^{ℓ}/du is in principle ambiguous. However, under the realistic assumption that $\partial^2 e^{\ell}/\partial L^{\ell}\partial u > 0$,[16] we can establish that $dw^{\ell}/du < 0$. Some of these results are discussed in greater detail below, where they are used to characterize the optimal marginal tax and expenditure policy.

Social Decision-Problem

As in much of the earlier literature on optimal taxation, we assume that the government is the first mover, while the agents in the private sector (individuals and firms) are followers. Furthermore, we consider the general governmental objective of reaching a Pareto-efficient resource allocation. This is accomplished by maximizing the (expected) utility of the low-skilled subject to a minimum utility restriction for the high-skilled, as well as subject to the appropriate self-selection and resource constraints. We also assume that the government (or social planner) wants to redistribute from high-skilled to low-skilled types, which Stiglitz (1982) refers to as the "normal" case, meaning that the optimal resource allocation must

[14]For instance, as reported in Ottaviano and Peri (2012, p. 182), a value of around 2 represents a reasonable estimate of the elasticity of substitution between high-education and low-education groups.

[15]The denominator of the expression on the right-hand side of dw^{ℓ}/dB^{ℓ} in Table 1 is unambiguously positive from the second-order conditions of the firm's profit maximization problem with respect to w^{ℓ}.

[16]This assumption can be interpreted to mean that an increase in the unemployment rate counteracts the negative effect that L^{ℓ} has on e^{ℓ}, *ceteris paribus*.

Table 1. *Comparative statics results*

I^ℓ	$\dfrac{dN^\ell}{dI^\ell} = \dfrac{1-(1/\sigma)}{(e^\ell L^\ell)^2 F''_{11}} - \dfrac{N^\ell}{I^\ell}$ $\dfrac{dw^\ell}{dI^\ell} = \dfrac{1}{L^\ell} > 0$ $\dfrac{dw^h}{dI^\ell} = -\dfrac{N^\ell}{N^h L^h} < 0$
B^ℓ	$\dfrac{dN^\ell}{dB^\ell} = \dfrac{-(\partial e^\ell/\partial B^\ell)(w^\ell L^\ell/e^\ell)[1-(1/\sigma)]}{(e^\ell L^\ell)^2 F''_{11}}$ $\dfrac{dw^\ell}{dB^\ell} = \dfrac{-(1/L^\ell)(\partial e^\ell/\partial B^\ell)}{(L^\ell/w^\ell)\{[2e^\ell/(L^\ell)^2] - (\partial^2 e^\ell/\partial L^\ell \partial L^\ell)\}} < 0$ $\dfrac{dw^h}{dB^\ell} = \dfrac{N^\ell L^\ell(\partial e^\ell/\partial B^\ell)(w^\ell/e^\ell)}{N^h L^h} > 0$
I^h	$\dfrac{dN^\ell}{dI^h} = \dfrac{N^\ell}{w^h L^h} > 0$ $\dfrac{dw^\ell}{dI^h} = 0$ $\dfrac{dw^h}{dI^h} = 0$
B^h	$\dfrac{dN^\ell}{dB^h} = 0$ $\dfrac{dw^\ell}{dB^h} = 0$ $\dfrac{dw^h}{dB^h} = 0$
u	$\dfrac{dN^\ell}{du} = \dfrac{[(1/\sigma)-1](w^\ell/e^\ell)L^\ell(\partial e^\ell/\partial u)}{(e^\ell L^\ell)^2 F''_{11}}$ $\dfrac{dw^\ell}{du} = \dfrac{-(\partial^2 e^\ell/\partial L^\ell \partial u) - (1/L^\ell)(\partial e^\ell/\partial u)}{(L^\ell/w^\ell)\{[2e^\ell/(L^\ell)^2] - (\partial^2 e^\ell/\partial L^\ell \partial L^\ell)\}}$ $\dfrac{dw^h}{du} = \dfrac{w^\ell}{e^\ell}\dfrac{N^\ell L^\ell}{N^h L^h}\dfrac{\partial e^\ell}{\partial u} > 0$
b	$\dfrac{dN^\ell}{db} = \dfrac{[(1/\sigma)-1](w^\ell/e^\ell)L^\ell(\partial e^\ell/\partial b)}{(e^\ell L^\ell)^2 F''_{11}}$ $\dfrac{dw^\ell}{db} = \dfrac{-(1/L^\ell)(\partial e^\ell/\partial b)}{(L^\ell/w^\ell)[2e^\ell/(L^\ell)^2 - \partial^2 e^\ell/(\partial L^\ell)^2]} > 0$ $\dfrac{dw^h}{db} = \dfrac{w^\ell}{e^\ell}\dfrac{N^\ell L^\ell}{N^h L^h}\dfrac{\partial e^\ell}{\partial b} < 0$

be constrained to prevent high-skilled individuals from mimicking the low-skilled. As such, the optimal marginal tax and expenditure policies characterized below will satisfy any social welfare function, which is increasing in the utility of both skill types, if consistent with the assumed profile of redistribution.

To ensure that the high-skilled prefer the allocation intended for their type (I^h, B^h) over the before-tax and disposable income combination intended for the employed the low-skilled (I^ℓ, B^ℓ), we impose self-selection constraints designed to make mimicking unattractive. In our setting, mimicking can occur in two alternative ways. One possibility would be for high-skilled individuals to reduce their labor supply in high-demanding jobs to the extent required to earn I^ℓ instead of I^h. Because high-skilled individuals are more productive than low-skilled individuals, a high-skilled mimicker can reach the income level I^ℓ by supplying fewer hours of work than needed by a low-skilled individual. Another possibility for a high-skilled individual to act as a mimicker would be to take a low-demanding job, which also gives the before-tax income I^ℓ. With regards to the latter option, we assume that, for any given level of effort exerted in the workplace, the productivity of a high-skilled worker does not differ from the productivity of a low-skilled worker in a low-demanding job. A high-skilled individual working in a low-demanding job will thus be paid the wage rate w^ℓ (instead of the wage rate w^h). If choosing a low-demanding job, the effort exerted by a high-skilled individual will depend on the outside option if caught shirking and fired. Assuming that, if fired from a low-demanding job, a high-skilled individual can always find employment in a high-demanding job (i.e., no threat of becoming unemployed), the optimal effort choice of a mimicker in a low-demanding job would be zero. Note that the utility of a mimicker would, in this case, exceed the utility faced by the low-skilled type, as the effort provided by a high-skilled individual in a low-demanding job falls short of the effort chosen by low-skilled individuals.[17]

Even though both available mimicking strategies require a high-skilled mimicker to earn the same before-tax income as a low-skilled individual, for convenience hereafter we use the term "income-replication" strategy to

[17] By making alternative assumptions about the outside option, and thus the fallback utility, of a high-skilled individual caught shirking in a low-demanding job, we can relax the (admittedly extreme) assumption, leading to the result that the optimal effort chosen by a high-skilled individual in a low-demanding job is nil. The qualitative results we obtain would be unchanged as long as the effort provided by a high-skilled individual in a low-demanding job falls short of the effort provided by a low-skilled individual. This would indeed be the case unless we were to assume that a high-skilled individual caught shirking in a low-demanding job has a zero probability of being re-employed in a high-demanding job.

refer to the case when a high-skilled individual behaves as a mimicker and chooses a high-demanding job (working fewer hours than a low-skilled individual), whereas we use the expression "job-replication" strategy to refer to the case when a high-skilled individual behaves as a mimicker and chooses a low-demanding job (exerting less effort than a low-skilled individual).

Now, because the government can implement any desired combination of work hours and disposable income for each skill type subject to constraints, we follow the common practice in much of the earlier literature on optimal nonlinear taxation by writing the social decision-problem directly in terms of the before-tax and disposable incomes, instead of in terms of parameters of the tax function.

We introduce the following notation, pertaining to a high-skilled individual behaving as a mimicker by choosing, respectively, the income-replication strategy and the job-replication strategy,

$$\widehat{V}^{h,e=1}(B^\ell, I^\ell; w^h) \equiv g(B^\ell) + v\left(\frac{I^\ell}{w^h}, 1\right), \tag{15}$$

$$\widehat{V}^{h,e=0}(B^\ell, I^\ell; w^\ell) \equiv g(B^\ell) + v\left(\frac{I^\ell}{w^\ell}, 0\right). \tag{16}$$

Thus, the social-decision problem can be written as

$$\max_{I^\ell, B^\ell, I^h, B^h, b, u} EV^\ell(B^\ell, I^\ell, u, b; w^\ell),$$

subject to

$$(\delta) \qquad V^h(B^h, I^h; w^h) \geq \bar{V}^h, \tag{17}$$

$$(\lambda) \qquad V^h(B^h, I^h; w^h) \geq \widehat{V}^{h,e=1}(B^\ell, I^\ell; w^h), \tag{18}$$

$$(\zeta) \qquad V^h(B^h, I^h; w^h) \geq \widehat{V}^{h,e=0}(B^\ell, I^\ell; w^\ell), \tag{19}$$

$$(\mu) \qquad F\left(e^\ell N^\ell \frac{I^\ell}{w^\ell}, N^h \frac{I^h}{w^h}\right) \geq \pi^h B^h + N^\ell B^\ell + (\pi^\ell - N^\ell)b, \tag{20}$$

$$(\theta) \qquad u = 1 - \frac{N^\ell(B^\ell, I^\ell, u, b)}{\pi^\ell}, \tag{21}$$

where the Lagrange multipliers attached to the respective constraints are given in parentheses. The δ-constraint represents the minimum utility constraint, implying that the utility of each high-skilled individual must not fall short of $\bar{V}^h$. The λ-constraint and the ζ-constraint are two self-selection constraints that jointly ensure that a high-skilled individual has no incentive to act as a mimicker (i.e., has no incentive to earn the income level intended for the low-skilled type through the income-replication or job-replication strategy, respectively).[18,19] The μ-constraint is the economy-wide resource constraint, requiring that the aggregate output must not fall short of the aggregate consumption. Note also that, instead of substituting the expression for the unemployment rate into the objective and constraint functions, the unemployment rate is used as an additional (and artificial) control variable, which explains the θ-constraint. This formulation is convenient by allowing us to interpret the Lagrange-multiplier θ in terms of a (utility-based) shadow price of unemployment.

In our analysis, we restrict attention to the case where unemployment is an involuntary phenomenon for low-skilled individuals by assuming that $g(B^\ell) + v(I^\ell/w^\ell, e^\ell) > g(b)$ at a social optimum (as mentioned above). Note that this rules out the possibility that high-skilled individuals find it attractive to drop out of the labor market in order to live on unemployment benefits.[20]

The first-order conditions of the social decision-problem are presented in the Online Appendix. Next, we turn to the implications of these conditions for the optimal public policy.

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[18]We have already remarked how a high-skilled mimicker would optimally choose $e = 0$ under a job-replication strategy. Thus, given that workers in low-demanding jobs are monitored with exogenous probability p, it follows that such a mimicker would be caught shirking with probability p. Therefore, the condition that needs to be fulfilled in order to deter mimicking via a job-replication strategy is $g(B^h) + v(I^h/w^h, 1) \geq (1 - p)[g(B^\ell) + v(I^\ell/w^\ell, 0)] + p[g(B^h) + v(I^h/w^h, 1)]$, or, equivalently $g(B^h) + v(I^h/w^h, 1) \geq g(B^\ell) + v(I^\ell/w^\ell, 0)$, which is what is required by the ζ-constraint in the social decision-problem.

[19]Note that

$$\frac{d}{dI^\ell}[\widehat{V}^{h,e=1}(B^\ell, I^\ell; w^h) - \widehat{V}^{h,e=0}(B^\ell, I^\ell; w^\ell)] = \frac{d}{dI^\ell}\left[v\left(\frac{I^\ell}{w^h}, 1\right) - v\left(\frac{I^\ell}{w^\ell}, 0\right)\right]$$
$$= \frac{\partial v[(I^\ell/w^h), 1]}{\partial(I^\ell/w^h)}\left[1 + \frac{N^\ell I^\ell}{N^h I^h}\right]\frac{1}{w^h} < 0.$$

Therefore, there is only one value of I^ℓ at which a high-skilled mimicker is indifferent between the two available mimicking strategies. For sufficiently high values of I^ℓ, the job-replication strategy will be more attractive than the income-replication strategy, and vice versa.

[20]We can rewrite the two self-selection constraints as $V^h(B^h, I^h; w^h) \geq \max\{\widehat{V}^{h,e=1}(B^\ell, I^\ell; w^h); \widehat{V}^{h,e=0}(B^\ell, I^\ell; w^\ell)\}$, or equivalently (using equations (15) and (16)) as $V^h(B^h, I^h; w^h) \geq g(B^\ell) + \max\{v(I^\ell/w^h, 1); v(I^\ell/w^\ell, 0)\}$. If $\max\{v(I^\ell/w^h, 1); v(I^\ell/w^\ell, 0)\} = v(I^\ell/w^h, 1)$, we find that $v(I^\ell/w^h, 1) > v(I^\ell/w^\ell, 0) > v(I^\ell/w^\ell, e^\ell)$. From $g(B^\ell)+v(I^\ell/w^\ell, e^\ell) > g(b)$ it then follows that $g(B^\ell)+v(I^\ell/w^h, 1) > g(b)$; that is, high-skilled workers have no incentive to drop out of the labor market. If, instead, $\max\{v(I^\ell/w^h, 1); v(I^\ell/w^\ell, 0)\} = v(I^\ell/w^\ell, 0)$, we find that $v(I^\ell/w^\ell, 0) > v(I^\ell/w^\ell, e^\ell)$. Once again, from $g(B^\ell) + v(I^\ell/w^\ell, e^\ell) > g(b)$ it follows that $g(B^\ell) + v(I^\ell/w^\ell, 0) > g(b)$, implying that high-skilled workers have no incentive to drop out of the labor market.

III. Optimal Tax and Expenditure Policy

To simplify the presentation, we introduce the following short notation for marginal rates of substitution between the before-tax income and disposable income for a low-skilled individual and a mimicker, respectively, where we have distinguished between the income-replication and job-replication strategies for the mimicker:

$$\begin{aligned} MRS^{\ell}_{I,B} &= -\frac{\partial EV^{\ell}/\partial I^{\ell}}{\partial EV^{\ell}/\partial B^{\ell}} > 0; \\ \widehat{MRS}^{h,e=1}_{I,B} &= -\frac{\partial \widehat{V}^{h,e=1}/\partial I^{\ell}}{\partial \widehat{V}^{h,e=1}/\partial B^{\ell}} > 0; \\ \widehat{MRS}^{h,e=0}_{I,B} &= -\frac{\partial \widehat{V}^{h,e=0}/\partial I^{\ell}}{\partial \widehat{V}^{h,e=0}/\partial B^{\ell}} > 0. \end{aligned}$$

For notational convenience, we also define "utility compensated" wage and employment responses to an increase in the before-tax income, I^{ℓ}, such that

$$\frac{d\tilde{w}^{\ell}}{dI^{\ell}} = \frac{dw^{\ell}}{dI^{\ell}} + MRS^{\ell}_{I,B}\frac{dw^{\ell}}{dB^{\ell}}, \tag{22}$$

$$\frac{d\tilde{N}^{\ell}}{dI^{\ell}} = \frac{dN^{\ell}}{dI^{\ell}} + MRS^{\ell}_{I,B}\frac{dN^{\ell}}{dB^{\ell}}, \tag{23}$$

$$\frac{d\widetilde{w}^{h}}{dI^{\ell}} = \frac{dw^{h}}{dI^{\ell}} + MRS^{\ell}_{I,B}\frac{dw^{h}}{dB^{\ell}}. \tag{24}$$

Equations (22) and (23) measure how an increase in the before-tax income, I^{ℓ}, affects the wage rate, w^{ℓ}, and the number of employed individuals, N^{ℓ}, respectively, of the low-skilled agents, when they are compensated by an increase in the disposable income, B^{ℓ}, to remain at the initial (expected) utility level. Similarly, equation (24) shows how a marginal increase in I^{ℓ}, compensated by a change in B^{ℓ} to leave the (expected) utility of low-skilled individuals unchanged, affects the wage rate paid to workers in high-demanding jobs, w^{h}. We are now ready to characterize the marginal income tax rates implemented for the two skill types.

Proposition 1. *The marginal income tax rate faced by high-skilled workers is negative (positive) if* $(I^{\ell} - B^{\ell} + b)\pi^{\ell} + \theta/\mu > 0$ (< 0) *and given as follows:*

$$T'(I^{h}) = -\frac{1-u}{\pi^{h} I^{h}}\left[(I^{\ell} - B^{\ell} + b)\pi^{\ell} + \frac{\theta}{\mu}\right]. \tag{25}$$

Let $\widetilde{\epsilon}_{N^{\ell},I^{\ell}}$ *denote the compensated elasticity of* N^{ℓ} *with respect to* I^{ℓ}*, such that* $\widetilde{\epsilon}_{N^{\ell},I^{\ell}} = (d\tilde{N}^{\ell}/dI^{\ell})(I^{\ell}/N^{\ell})$*. The marginal income tax rate faced by low-skilled workers can then be written as*

$$\begin{aligned} T'(I^{\ell}) \quad = \quad & \frac{\lambda(\partial \widehat{V}^{h,e=1}/\partial B^{\ell})}{\mu(1-u)\pi^{\ell}}\left(MRS_{I,B}^{\ell} - \widehat{MRS}_{I,B}^{h,e=1}\right) \\ & + \frac{\zeta(\partial \widehat{V}^{h,e=0}/\partial B^{\ell})}{\mu(1-u)\pi^{\ell}}\left(MRS_{I,B}^{\ell} - \widehat{MRS}_{I,B}^{h,e=0}\right) \\ & + \frac{1}{\mu(1-u)\pi^{\ell}}\left(\zeta\frac{\partial \widehat{V}^{h,e=0}}{\partial w^{\ell}} - \frac{\partial EV^{\ell}}{\partial w^{\ell}}\right)\frac{d\tilde{w}^{\ell}}{dI^{\ell}} \\ & + \frac{1}{\mu(1-u)\pi^{\ell}}\left(\lambda\frac{\partial \widehat{V}^{h,e=1}}{\partial w^{h}} - \mu N^{h}L^{h}MRS_{I,B}^{h}\right)\frac{d\widetilde{w}^{h}}{dI^{\ell}} \\ & - \frac{1}{\pi^{\ell}I^{\ell}}\left[(I^{\ell} - B^{\ell} + b)\pi^{\ell} + \frac{\theta}{\mu}\right]\widetilde{\epsilon}_{N^{\ell},I^{\ell}}. \end{aligned} \tag{26}$$

Proof: See the Online Appendix. □

According to equation (25), high-skilled individuals should face a negative marginal income tax rate under realistic assumptions, implying that their labor supply is distorted upwards. This finding is reminiscent of a result derived by Stiglitz (1982) in a model with competitive labor markets, where a negative marginal income tax rate for high-skilled individuals works as a device to reduce the wage differential between the skill types.[21] In our setting, a negative marginal income tax rate for high-skilled individuals is justified as a mechanism through which the government can stimulate the demand for low-skilled labor. This is, in turn, socially beneficial for two reasons: first, by increasing the net tax revenue (provided that the transfer paid to the unemployed is larger than the transfer paid to low-skilled workers), and, second, by reducing the unemployment rate, which is socially desirable whenever the social value of decreased unemployment measured in terms of public funds, θ/μ, is positive (see Proposition 2).

However, the fact that a distortion in the labor supply behavior of high-skilled workers is useful as a device for boosting the demand for low-skilled workers should not be interpreted as implying that general equilibrium wage effects play no role in equation (25). The reason is that, even though a change in I^h or B^h per se has no direct effects on w^{ℓ} and w^h (as we can see from Table 1), a variation in I^h triggers a change in the number of workers N^{ℓ} that each firm is willing to hire in low-demanding jobs.[22] At

[21] See also Pirttilä and Tuomala (2001), and Gahvari (2014a, 2014b).

[22] Notice that a variation in B^h has no effect on N^{ℓ}, w^{ℓ}, and w^h as B^h does not enter the system of equations (12)–(14). The variable I^h, however, enters the system of equations (12)–(14) through equations (13) and (14). Nonetheless, a variation in I^h does not affect the equilibrium wage rates w^{ℓ} and w^h because its direct effect on wages is exactly offset by the indirect effect on wages that works through the induced change in N^{ℓ}.

the economy-wide level, this will have an effect on the unemployment rate and, as a consequence, on the level of effort e^{ℓ} that low-skilled workers will choose. Finally, this change in the incentives to provide effort affects the equilibrium wage rates (through equations (12) and (14)), which in turn has an effect on the self-selection constraints faced by the government. The fact that the general equilibrium wage effects of a variation in I^h are indirect and mediated by unemployment-rate effects explains why, in equation (25), they are "concealed" in the term θ/μ, which captures the social value of decreased unemployment. As we discuss at length when presenting Proposition 2, most of the policy incentives determining the structure of θ/μ are, indeed, related to general equilibrium wage effects.

Turning to marginal income taxation of low-skilled workers in equation (26), the first two terms on the right-hand side tell us how taxation should distort the labor supply of low-skilled workers if the wage rates and employment are fixed. Both terms depend on self-selection considerations and would vanish in a first-best environment where individual skill is observable, in which case $\lambda = \zeta = 0$. If the self-selection constraint given in equation (18) – that is, the constraint associated with the income-replication strategy – is binding ($\lambda > 0$), then the first term on the right-hand side of equation (26) can be either positive or negative. This is interesting in itself: in a standard optimal income tax model with competitive labor markets, the corresponding term would be unambiguously positive because of the assumption that the marginal rates of substitution satisfy the condition $\partial MRS_{I,B}/\partial w < 0$.[23] In the standard model, normality of consumption is a sufficient condition to guarantee that the single-crossing condition holds. In our setting, however, this is not enough. When comparing a low-skilled individual with a high-skilled mimicker adopting the income-replication strategy, it is still true that a high-skilled individual needs fewer hours to earn the same before-tax income as a low-skilled individual (as $w^h > w^{\ell}$). At the same time, however, a high-skilled individual employed in a high-demanding job exerts more effort than a low-skilled individual employed in a low-demanding job. Therefore, because $\partial MRS^j_{I,B}/\partial e^j > 0$, it is, in principle, possible that the first term on the right-hand side of equation (26) takes a negative sign.[24]

[23]This is typically referred to as the agent-monotonicity condition (see Seade, 1982).

[24]The intuition for $\partial MRS^j_{I,B}/\partial e^j > 0$ is that increased on-the-job effort makes leisure outside the workplace more valuable at the margin, *ceteris paribus* (i.e., for given values of I, B, and w). Formally, as $MRS^j_{I,B} = -\partial v/\partial L^j/(\partial g/\partial B)w^j$, we find that $sign(\partial MRS^j_{I,B}/\partial e^j) = sign(\partial^2 v/\partial e \partial L)$. Because our initial assumption was that $\partial^2 v/\partial e \partial L < 0$, it follows that $\partial MRS^j_{I,B}/\partial e^j > 0$. Notice that the sign of the first term on the right-hand side of equation (26) remains ambiguous also if we drop the possibility of high-skilled workers performing the jobs of low-skilled workers.

Instead, if the binding self-selection constraint given by equation (19) – that is, the constraint associated with the job-replication strategy is binding ($\zeta > 0$) – then the second term on the right-hand side of equation (26) will be unambiguously positive, thus contributing to a higher marginal income tax rate for low-skilled workers. The reason is that a low-skilled individual exerts a positive level of effort, whereas a high-skilled mimicker adopting the job-replication strategy exerts lower effort (and, in particular, no effort at all under our assumptions about the outside option for a high-skilled individual caught shirking in a low-demanding job). This implies that a low-skilled individual attaches a higher marginal value to leisure outside the workplace than a mimicker adopting the job-replication strategy (despite the fact that they are paid the same wage rate, w^ℓ). An increase in the marginal income tax rate, which induces low-skilled workers to reduce their hours of work, will thus hurt the mimicker more than it hurts low-skilled workers, meaning that this policy opens up for more redistribution through a relaxation of the job-replication self-selection constraint.

The remaining components in equation (26) reflect wage- and employment-responses to a change in the income-consumption bundle for low-skilled workers. The responses in N^ℓ and w^ℓ arise as a consequence of efficiency wage setting for low-skilled workers; in turn, these induce a spill-over effect on w^h as a result of the imperfect substitutability between factors in the production function.

Starting with the marginal tax response to an induced change in the hourly wage rate paid to low-skilled workers, w^ℓ, note first that

$$\frac{\partial EV^\ell}{\partial w^\ell} = -[1 - u\varphi(e^\ell)]\frac{L^\ell}{w^\ell}\frac{\partial v(I^\ell/w^\ell, e^\ell)}{\partial L^\ell} > 0.$$

Here, we have used the fact that the indirect effects of w^ℓ, arising via the individuals' labor supply and effort choices, vanish as a consequence of optimality (i.e., by the envelope theorem). Note that $\partial EV^\ell/\partial w^\ell$ can be interpreted as a direct expected benefit faced by each low-skilled individual following an increase in their wage rate. If the self-selection constraint associated with the job-replication strategy (i.e., equation (19)) is binding, such a wage increase would also make mimicking more attractive, which explains the first term in parentheses in the third row of equation (26).[25] We can then interpret the difference in parentheses in the third row of equation (26) as reflecting the net social cost of an increase in w^ℓ for a given unemployment rate; if this difference is positive (negative), an increase in

[25] We have

$$\zeta\frac{\partial \widehat{V}^{h,e=0}}{\partial w^\ell} = -\zeta\frac{I^\ell}{(w^\ell)^2}\frac{\partial v(I^\ell/w^\ell, 0)}{\partial L^\ell} > 0$$

.

w^ℓ would lead to lower (higher) welfare, *ceteris paribus*. As we show in the Online Appendix (see equation (C12)), we would expect that $d\tilde{w}^\ell/dI^\ell > 0$ (such that a combined increase in the before-tax and disposable income for low-skilled individuals pushes up their hourly wage rate). Thus, there will be an incentive for the government to increase (decrease) the labor supply of low-skilled workers via a lower (higher) marginal tax rate if

$$\zeta \frac{\partial \widehat{V}^{h,e=0}}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell} = \left\{ [1 - u\varphi(e^\ell)] \frac{\partial v(I^\ell/w^\ell, e^\ell)}{\partial L^\ell} - \zeta \frac{\partial v(I^\ell/w^\ell, 0)}{\partial L^\ell} \right\} \frac{L^\ell}{w^\ell} < 0 \qquad (> 0). \tag{27}$$

For instance, the third row of equation (26) will provide an incentive to lower the marginal tax rate faced by low-skilled agents in the case when the constraint associated with the job-replication strategy is non-binding ($\zeta = 0$).[26]

Consider now the marginal tax response to an induced change in the hourly wage rate paid to high-skilled individuals, w^h. Note first that an increase in w^h would be costly for the government by tightening the λ-constraint, making it more attractive for high-skilled individuals to become mimickers by choosing the income-replication strategy.[27] However, an increase in w^h would also allow the government to raise additional tax revenue from high-skilled individuals without violating their minimum utility restriction (as prescribed by the δ-constraint).[28] We can then interpret the difference $\lambda \partial \widehat{V}^{h,e=1}/\partial w^h - \mu N^h L^h MRS^h_{I,B}$, appearing in the fourth row

[26]More generally, recalling that $\partial^2 v/\partial e \partial L < 0$ by assumption, the sign of equation (27) is more likely to be negative the higher the effort level e^ℓ, the lower the unemployment rate u, and the lower the value of the Lagrange multiplier ζ.

[27]Denoting by $\widehat{L}^{h,e=1}$ the labor supply of a high-skilled individual choosing the income-replication mimicking strategy, we have

$$\frac{\partial \widehat{V}^{h,e=1}}{\partial w^h} = -\frac{I^\ell}{(w^h)^2} \frac{\partial v(I^\ell/w^h, 1)}{\partial \widehat{L}^{h,e=1}} > 0.$$

[28]Because

$$\frac{\partial V^h}{\partial w^h} = -\frac{L^h}{w^h} \frac{\partial v(I^h/w^h, 1)}{\partial L^h} = -L^h \frac{\partial v(I^h/w^h, 1)}{\partial I^h} = L^h MRS^h_{I,B} \frac{\partial g(B^h)}{\partial B^h},$$

we obtain

$$\frac{\partial V^h}{\partial w^h} + \frac{\partial V^h}{\partial B^h} dB^h = 0$$

for

$$dB^h = -\frac{\partial V^h/\partial w^h}{\partial V^h/\partial B^h} = -L^h MRS^h_{I,B}.$$

of equation (26), as capturing the net social cost of a marginal increase in w^h (for a given unemployment rate). Because $d\widetilde{w}^h/dI^\ell < 0$ (see equation (C11) in the Online Appendix), it follows that wage-response considerations pertaining to w^h call for increasing (decreasing) the labor supply of low-skilled individuals via a lower (higher) marginal tax rate if

$$\frac{\lambda}{\mu}\frac{\partial \widehat{V}^{h,e=1}}{\partial w^h} > (<) \, N^h L^h MRS^h_{I,B}.$$

Finally, a direct employment effect of marginal taxation, showing how a compensated (for low-skilled individuals) increase in I^ℓ affects N^ℓ, is captured by the last term in equation (26). Note, first, that the sign of $\widetilde{\epsilon}_{N^\ell,I^\ell}$ is, in principle, ambiguous. For the purpose of interpretation, we focus on the case where $d\tilde{N}^\ell/dI^\ell < 0$. As shown in the Online Appendix (see equation (C10)), a sufficient condition for $d\tilde{N}^\ell/dI^\ell < 0$ is that $\sigma \geq 1$, which is a realistic assumption based on available empirical evidence (see footnote 14).[29] In this case, the government has an incentive to distort the labor supply of low-skilled workers downwards through higher marginal taxation for two reasons. First, when the transfer paid to the unemployed is larger than the transfer paid to the employed low-skilled workers ($I^\ell - B^\ell + b > 0$), an increase in N^ℓ will increase the net tax revenue. Second, an increase in N^ℓ leads to a lower unemployment rate, which is socially beneficial when the social value of decreased unemployment measured in terms of public funds, θ/μ, is positive. Employment-related motives for higher marginal tax rates are also found in earlier research on optimal redistributive taxation under equilibrium unemployment based on models of trade-unionized labor markets or search models.[30]

Taking into account the fact that the number of high-skilled individuals is $\pi^h = N^h$, a marginal increase in w^h allows the government to extract a total amount $N^h L^h MRS^h_{I,B}$ of additional tax revenue from the group of high-skilled individuals.

[29]For instance, this condition is satisfied under under a Cobb–Douglas production function where $\sigma = 1$. In this case, we can use equation (C10) to derive $d\tilde{N}^\ell/dI^\ell = -N^\ell/I^\ell$, which implies $\widetilde{\epsilon}_{N^\ell,I^\ell} = -1$.

[30]A more detailed comparison is complicated by the fact that studies differ in several important ways; not just in terms of how the labor market is modeled. Aronsson and Sjögren (2003) analyze optimal redistributive income and commodity taxation in an economy with two skill types and monopoly-union wage setting, and they find employment motives for marginal tax policy reminiscent of those derived here. Aronsson and Sjögren (2004) consider a model with wage bargaining between trade unions and firms (a right-to-manage framework), and they show that the optimal tax policy will, in certain cases, implement a full-employment equilibrium, in which the marginal income tax rate is zero. However, if such an equilibrium cannot be reached (which, in their study, depends on restrictions in the use of profit taxation), the marginal income tax policy entails employment effects reminiscent of those described above. These two studies assume that both the number of employed people and the hours of work per employee are endogenous, as we do. In the context of a search model, Hungerbuhler *et al.* (2006) show that the average tax

Before moving to the analysis of the factors determining θ/μ, three remarks are in order. First, note that the final row of equation (26) is the exact counterpart of the expression on the right-hand side of equation (25).[31] Thus, albeit implicitly, the final row of equation (26) captures general equilibrium wage effects of a (compensated) variation in I^ℓ. In particular, it captures those indirect effects that are mediated by unemployment rate variations.

Second, the reason why the terms appearing in the third and fourth rows of equation (26) have no counterpart in equation (25) is that, whereas changes in I^ℓ or B^ℓ exert effects on w^ℓ and w^h, which are defined for a given unemployment rate, a change in the bundle intended for high-skilled individuals only affects w^ℓ and w^h indirectly via the unemployment rate.

Third, by using the short notation

$$\widetilde{\epsilon}_{w^h,I^\ell} \equiv \frac{d\widetilde{w}^h}{dI^\ell}\frac{I^\ell}{w^h} < 0 \qquad \text{and} \qquad \widetilde{\epsilon}_{w^\ell,I^\ell} \equiv \frac{d\widetilde{w}^\ell}{dI^\ell}\frac{I^\ell}{w^\ell} > 0,$$

rates are increasing and the marginal tax rates are positive. The intuition is that higher marginal tax rates lead to lower wage claims, and thus to lower unemployment among each skill type. See also Lehmann *et al*. (2011) for a generalization to a case where the marginal tax rates might be negative at the lower end of the skill distribution. In the latter two studies, the hours of work per employee (of various types) are fixed, which makes the policy incentives somewhat different from those examined here.

[31] In both cases, the structure is the same. Given that $dN^\ell/dI^h = N^\ell/w^h L^h$ and $dN^\ell/dB^h = 0$, we have $d\widetilde{N}^\ell/dI^h = N^\ell/w^h L^h$. Therefore, we can rewrite the right-hand side of equation (25) as

$$-\frac{1-u}{\pi^h I^h}\frac{w^h L^h}{N^\ell}\left[\left(I^\ell - B^\ell + b\right)\pi^\ell + \frac{\theta}{\mu}\right]\frac{d\widetilde{N}^\ell}{dI^h}$$

or, equivalently,

$$-\frac{1}{\pi^h}\left[I^\ell - B^\ell + b + \frac{\theta}{\mu\pi^\ell}\right]\frac{d\widetilde{N}^\ell}{dI^h}.$$

The final row of equation (26) can be rewritten as

$$-\frac{1}{\pi^\ell I^\ell}\frac{I^\ell}{N^\ell}\left[\left(I^\ell - B^\ell + b\right)\pi^\ell + \frac{\theta}{\mu}\right]\frac{d\tilde{N}^\ell}{dI^\ell},$$

or, equivalently,

$$-\frac{1}{N^\ell}\left[I^\ell - B^\ell + b + \frac{\theta}{\mu\pi^\ell}\right]\frac{d\tilde{N}^\ell}{dI^\ell}.$$

equation (26) could be equivalently restated as

$$\begin{aligned} T'(I^\ell) &= \frac{\lambda(\partial \widehat{V}^{h,e=1}/\partial B^\ell)}{\mu(1-u)\pi^\ell}\left[MRS^\ell_{I,B} - \widehat{MRS}^{h,e=1}_{I,B}(1-\widetilde{\epsilon}_{w^h,I^\ell})\right] \\ &+\frac{\zeta(\partial \widehat{V}^{h,e=0}/\partial B^\ell)}{\mu(1-u)\pi^\ell}\left[MRS^\ell_{I,B} - \widehat{MRS}^{h,e=0}_{I,B}(1-\widetilde{\epsilon}_{w^\ell,I^\ell})\right] \\ &-\frac{1}{\mu(1-u)\pi^\ell}\frac{\partial EV^\ell}{\partial w^\ell}\frac{d\tilde{w}^\ell}{dI^\ell} - \frac{1}{(1-u)\pi^\ell}N^h L^h MRS^h_{I,B}\frac{d\widetilde{w}^h}{dI^\ell} \\ &-\frac{1}{\pi^\ell I^\ell}\left[(I^\ell - B^\ell + b)\pi^\ell + \frac{\theta}{\mu}\right]\widetilde{\epsilon}_{N^\ell,I^\ell}. \end{aligned}$$

Note that the policy incentives created by each self-selection constraint are now merged into a singe term given by the first and second rows, respectively. Because $\widetilde{\epsilon}_{w^h,I^\ell} < 0$ and $\widetilde{\epsilon}_{w^\ell,I^\ell} > 0$, and by accounting for the general equilibrium effects that operate via the self-selection constraints, we can see that the incentive term related to the discouragement of the job-replication strategy is still positive and becomes even larger (the second row). However, it becomes more likely that the incentive term related to the discouragement of the income-replication strategy calls for a downward adjustment in the marginal tax rate implemented for low-skilled individuals (the first row).

We can now turn to the factors determining the social value of decreased unemployment, θ/μ. For this purpose, let

$$MRS^\ell_{u,b} = -\frac{\partial EV^\ell/\partial u}{\partial EV^\ell/\partial b} = \frac{g(B^\ell) + v(I^\ell/w^\ell, e^\ell) - g(b)}{u\partial g(b)/\partial b} > 0 \qquad (28)$$

denote the marginal rate of substitution between the unemployment rate and the unemployment benefit for a low-skilled individual, and define the utility compensated wage and employment responses to an increase in the unemployment rate such that

$$\frac{d\tilde{w}^\ell}{du} = \frac{dw^\ell}{du} + MRS^\ell_{u,b}\frac{dw^\ell}{db}, \qquad (29)$$

$$\frac{d\tilde{N}^\ell}{du} = \frac{dN^\ell}{du} + MRS^\ell_{u,b}\frac{dN^\ell}{db}, \qquad (30)$$

$$\frac{d\tilde{w}^h}{du} = \frac{dw^h}{du} + MRS^\ell_{u,b}\frac{dw^h}{db}. \qquad (31)$$

Proposition 2 gives an expression for the social value of decreased unemployment at a second-best optimum.

Proposition 2. *The social value of decreased unemployment can be expressed as*

$$\frac{\theta}{\mu} = \frac{\pi^{\ell}}{\Gamma}\left\{uMRS^{\ell}_{u,b} + (I^{\ell} - B^{\ell} + b)\widetilde{\epsilon}_{N^{\ell},1-u} + \frac{N^h}{N^{\ell}}MRS^{h}_{I,B}I^{h}\widetilde{\epsilon}_{w^h,1-u}\right\}$$
$$+\frac{\lambda}{\mu\Gamma}\frac{\partial\widehat{V}^{h,e=1}}{\partial w^h}\frac{d\widetilde{w}^h}{du} + \frac{1}{\mu\Gamma}\left(\zeta\frac{\partial\widehat{V}^{h,e=0}}{\partial w^{\ell}} - \frac{\partial EV^{\ell}}{\partial w^{\ell}}\right)\frac{d\widetilde{w}^{\ell}}{du}, \tag{32}$$

where

$$\Gamma \equiv 1 + \frac{1}{\pi^{\ell}}\frac{d\tilde{N}^{\ell}}{du}, \quad \widetilde{\epsilon}_{N^{\ell},1-u} \equiv -\frac{1-u}{N^{\ell}}\frac{d\widetilde{N}^{\ell}}{du}, \quad \text{and} \quad \widetilde{\epsilon}_{w^h,1-u} \equiv -\frac{1-u}{w^h}\frac{d\widetilde{w}^h}{du}.$$

Proof: See the Online Appendix. □

Note, first, that Γ in the denominator on the righ-hand side of equation (32) represents a feedback effect, which is reminiscent of the corresponding effect entering in expressions for the social shadow price of a consumption externality (e.g., Sandmo, 1980; Pirttilä and Tuomala, 1997). As shown by Sandmo (1980), stability requires that the feedback effect is positive. Therefore, we base our discussion of the result in Proposition 2 on the assumption that $\Gamma > 0$. In our case, where we have used the social first-order conditions for u and b to derive equation (32), it follows that the feedback effect depends on how the number of employed low-skilled individuals is influenced by both the unemployment rate and the unemployment benefit. The term $d\tilde{N}^{\ell}/du$ in the expression for Γ can be thought of as the employment response to a utility compensated increase in the unemployment rate, where the compensation is measured in terms of the unemployment benefit.

We can interpret $\pi^{\ell} u\, MRS^{\ell}_{u,b}$ in the first row of equation (32) as the sum of the marginal willingness to pay to avoid unemployment among the unemployed, where $\pi^{\ell} u$ represents the number of unemployed persons. This component resembles the sum of the marginal willingness to pay to avoid a public bad, with the modification that it is only measured over the relevant part of the population.

The remaining two terms in the first row of equation (32) can be interpreted as public revenue effects. A direct public revenue effect of a variation in the unemployment rate is captured by the second term in curly brackets, which reflects that the net tax revenue increases by $I^{\ell} - B^{\ell} + b = T(I^{\ell}) + b$ when a low-skilled individual switches from unemployment to employment. The increased tax revenue is, in turn, multiplied by $\widetilde{\epsilon}_{N^{\ell},1-u}$, measuring the extent to which each firm's demand for low-skilled workers is affected by a change in the economy-wide unemployment rate. By using the comparative statics results provided in Table 1, we can re-express equation (30) as

$$\frac{d\widetilde{N}^\ell}{du} = \frac{[(1/\sigma) - 1](w^\ell/e^\ell)L^\ell}{(e^\ell L^\ell)^2 F_{11}''}\frac{\partial \widetilde{e}^\ell}{\partial u}.$$

By using equations (6) and (7) to derive

$$\frac{\partial \widetilde{e}^\ell}{\partial u} = \frac{\partial e^\ell}{\partial u} + \frac{\partial e^\ell}{\partial b} MRS_{u,b}^\ell = \frac{\varphi(e^\ell)(\partial v/\partial e^\ell)}{\Phi} > 0, \qquad (33)$$

we can see that the sign of the compensated response of N^ℓ to a marginal increase in u depends on the elasticity of substitution between the labor inputs. Therefore, this public revenue effect vanishes in the special case where $\sigma = 1$, in which $d\widetilde{N}^\ell/du = 0$. Instead, in the empirically plausible case where $\sigma > 1$, we have $d\widetilde{N}^\ell/du > 0$ and, therefore, $\widetilde{\epsilon}_{N^\ell,1-u} < 0$, which implies that public revenue considerations lead to a decrease in θ/μ.

The third term in curly brackets in the first row of equation (32) captures another public revenue effect, descending from the fact that a variation in the unemployment rate among low-skilled individuals affects the equilibrium wage rate paid in high-demanding jobs. Exploiting equation (33), from the comparative statics results provided in Table 1 we can see that

$$\frac{d\widetilde{w}^h}{du} = \frac{w^\ell}{e^\ell}\frac{N^\ell L^\ell}{N^h L^h}\frac{\partial \widetilde{e}^\ell}{\partial u} > 0. \qquad (34)$$

Therefore, an increase in the unemployment rate will lead to an increase in the wage rate paid to high-skilled workers, (i) because an increase in the unemployment rate leads to higher effort among the low-skilled, and (ii) because of the assumed technical complementarity between skilled and unskilled labor in terms of the production function. In turn, this opens up the possibility of raising additional tax revenue from high-skilled individuals without violating the minimum utility restriction. As a consequence, the final term in curly brackets is also negative and contributes to a further downward adjustment in θ/μ.[32]

The self-selection constraints directly affect the social value of decreased unemployment through the second row of equation (32). The first term captures the effect of a variation in w^h, induced by a change in the unemployment rate, on the self-selection constraint associated with the income-replication strategy. With $d\widetilde{w}^h/du > 0$, an increase in the unemployment rate makes this mimicking strategy more attractive, which, in turn, contributes to raising the social value of decreased unemployment. Finally, the second term appears because an increase in the unemployment rate affects the wage rate paid in low-demanding jobs, which leads to an

[32] Notice that $d\widetilde{w}^h/du > 0 \Longrightarrow \widetilde{\epsilon}_{w^h,1-u} < 0$.

additional social cost or benefit for reasons similar to those illustrated when discussing equation (26). In particular, $d\tilde{w}^\ell/du$ can be thought of as a utility compensated wage response to an increase in the unemployment rate, where the compensation appears in the form of an increase in the unemployment benefit, while $\zeta\partial\widehat{V}^{h,e=0}/\partial w^\ell - \partial EV^\ell/\partial w^\ell$ can be interpreted as the net social cost of an increase in the wage paid to the low-skilled (as explained above in the context of marginal income tax policy). As we show in the Online Appendix that $d\widetilde{w}^\ell/du < 0$, the final term on the right-hand side of equation (32) contributes to an increase in the social value of decreased unemployment whenever $\zeta\partial\widehat{V}^{h,e=0}/\partial w^\ell - \partial EV^\ell/\partial w^\ell < 0$ (as would be the case, for instance, if the ζ-constraint is non-binding at a second-best optimum). The intuition is, in this case, that reduced unemployment would also imply an indirect welfare benefit through an effect on the wage rate paid in low-demanding jobs. The opposite policy incentive arises if $\zeta\partial\widehat{V}^{h,e=0}/\partial w^\ell - \partial EV^\ell/\partial w^\ell > 0$.

To complete the characterization of the optimal second-best policy, we now turn to the policy rule for the unemployment benefit. For this purpose, let

$$MRS_{b,B}^\ell = \frac{\partial EV^\ell/\partial b}{\partial EV^\ell/\partial B} = \frac{u\varphi\left(e^\ell\right)\partial g\left(b\right)/\partial b}{\left[1 - u\varphi\left(e^\ell\right)\right]\partial g\left(B^\ell\right)/\partial B^\ell} > 0$$

denote the marginal rate of substitution between the unemployment benefit and the disposable income in the employed state for a low-skilled individual. Also, define the utility compensated wage and employment responses to an increase in the unemployment benefit, b, such that

$$\frac{d\tilde{N}^\ell}{db} = \frac{dN^\ell}{db} - MRS_{b,B}^\ell \frac{dN^\ell}{dB^\ell}, \tag{35}$$

$$\frac{d\tilde{w}^\ell}{db} = \frac{dw^\ell}{db} - MRS_{b,B}^\ell \frac{dw^\ell}{dB^\ell}, \tag{36}$$

$$\frac{d\tilde{w}^h}{db} = \frac{dw^h}{db} - MRS_{b,B}^\ell \frac{dw^h}{dB^\ell}. \tag{37}$$

Proposition 3 characterizes the efficient level of the unemployment benefit at a second-best optimum.

Proposition 3. *The optimal unemployment benefit abides by the following policy rule:*

$$
\begin{aligned}
(1-u)\pi^{\ell} MRS_{b,B}^{\ell} \quad = \quad & u\pi^{\ell} - \left(I^{\ell} - B^{\ell} + b + \frac{\theta}{\mu\pi^{\ell}} \right) \frac{d\widetilde{N}^{\ell}}{db} \\
& -\pi^{h} L^{h} MRS_{I,B}^{h} \frac{d\widetilde{w}^{h}}{db} - \frac{MRS_{b,B}^{\ell}}{\mu} \\
& \times \left(\lambda \frac{\partial \widehat{V}^{h,e=1}}{\partial B^{\ell}} + \zeta \frac{\partial \widehat{V}^{h,e=0}}{\partial B^{\ell}} \right) + \frac{\lambda}{\mu} \frac{\partial \widehat{V}^{h,e=1}}{\partial w^{h}} \frac{d\widetilde{w}^{h}}{db} \\
& + \frac{1}{\mu} \left(\zeta \frac{\partial \widehat{V}^{h,e=0}}{\partial w^{\ell}} - \frac{\partial EV^{\ell}}{\partial w^{\ell}} \right) \frac{d\widetilde{w}^{\ell}}{db}. \qquad (38)
\end{aligned}
$$

Proof: See the Online Appendix. □

The left-hand side of equation (38) can be interpreted as the sum of the marginal willingness to pay for a higher unemployment benefit, measured among the employed individuals of the low-skilled type. In particular, $MRS_{b,B}^{\ell}$ reflects the amount of income that each low-skilled individual would be willing to forego, when employed, in order to marginally raise their consumption in the event of becoming unemployed. Thus, $(1-u)\pi^{\ell} MRS_{b,B}^{\ell}$ can be interpreted as measuring the aggregate insurance benefit for low-skilled individuals of a marginal increase in the consumption available if becoming unemployed.[33]

Turning to the right-hand side, the direct public budget cost of a marginal increase in b (which is paid to the $u\pi^{\ell}$ workers who are unemployed) is measured by the first term, while the second and third terms are employment and public revenue effects reminiscent of those described in the context of Proposition 1. More specifically, the second term captures the net social gain of the employment effect induced by a compensated marginal increase in b, where the compensation is measured in terms of B^{ℓ}. By using the comparative statics results provided in Table 1, we can rewrite equation (35) as

$$
\begin{aligned}
\frac{d\tilde{N}^{\ell}}{db} \quad & = \quad \frac{[(1/\sigma)-1][(w^{\ell}L^{\ell})/e^{\ell}]}{(e^{\ell}L^{\ell})^2 F_{11}''} \left(\frac{\partial e^{\ell}}{\partial b} - MRS_{b,B}^{\ell} \frac{\partial e^{\ell}}{\partial B^{\ell}} \right) \\
& = \quad \frac{[(1/\sigma)-1][(w^{\ell}L^{\ell})/e^{\ell}]}{(e^{\ell}L^{\ell})^2 F_{11}''} \frac{\partial \tilde{e}^{\ell}}{\partial b}.
\end{aligned}
$$

[33] $(1-u)\pi^{\ell} MRS_{b,B}^{\ell}$ can also be interpreted as the additional income tax revenue that the government can collect if marginally raising the unemployment benefit in a compensated way; that is, raising b while at the same time adjusting $T(I^{\ell})$ upwards to leave the expected utility unchanged for low-skilled individuals.

From our discussion in Section II, we find that $\partial e^\ell / \partial b < 0$ and $\partial e^\ell / \partial B^\ell > 0$, implying $\partial \widetilde{e^\ell} / \partial b < 0$. Thus, as long as $\sigma > 1$ (which, as we pointed out before, is the empirically most plausible scenario), we have $d\tilde{N}^\ell / db < 0$. Therefore, with $I^\ell - B^\ell + b + \theta / \mu \pi^\ell > 0$, the employment effects of a compensated increase in b calls for an upward adjustment in the net resource cost of raising the unemployment benefit. This contributes to decrease the unemployment benefit, *ceteris paribus*. Similarly, the third term on the right-hand side of equation (38) (i.e., the first term in the second row) captures a public budget effect resulting from the fact that a variation in the unemployment benefit affects the wage rate facing workers in high-demanding jobs. From the comparative statics results provided in Table 1, we can derive

$$\frac{d\widetilde{w}^h}{db} = \frac{dw^h}{db} - MRS^\ell_{b,B} \frac{dw^h}{dB^\ell} = \frac{(w^\ell / e^\ell) N^\ell L^\ell}{N^h L^h} \frac{\partial \widetilde{e^\ell}}{\partial b} < 0, \qquad (39)$$

which implies that an increase in the unemployment benefit reduces the before-tax wage rate faced by high-skilled individuals. In turn, a lower wage necessitates a lower income tax payment by high-skilled individuals in order to leave their utility unchanged. This indirect public budget effect leads to an increase in the net marginal cost of raising the unemployment benefit.

The fourth and fifth terms on the right-hand side of equation (38) and the first term in the third row reflect the fact that a compensated marginal increase in b (an increase in b accompanied by a utility compensated upward adjustment in $T(I^\ell)$) contributes to relax the self-selection constraint. First, as low-skilled workers face the risk of becoming unemployed whereas a potential mimicker does not, a compensated (for low-skilled agents) increase in the unemployment benefit makes mimicking less attractive, irrespective of which strategy the mimicker chooses. This effect is summarized by the fourth term on the right-hand side of equation (38), where the first component in brackets is negative under the income-replication strategy ($\lambda > 0$ and $\zeta = 0$), while the second component is negative under the job-replication strategy ($\lambda = 0$ and $\zeta > 0$). Intuitively, a transfer paid to the unemployed is an instrument better targeted to low-skilled individuals than a transfer to low-income earners in general. This targeting advantage, lowers the net resource cost of raising the generosity of the unemployment benefit system. The fifth term captures instead the effect of an induced change in w^h on the utility of a high-skilled mimicker adopting the income-replication strategy. Because $d\widetilde{w}^h / db < 0$ (see equation (39)), it follows that a marginal increase in b has the further advantage of lowering the utility associated with this type of mimicking strategy, lowering the net resource cost of making the unemployment benefit more generous.

The final term on the right-hand side of equation (38) captures the net social cost of a change in w^ℓ induced by a compensated marginal increase in b. In general, this component can be either positive or negative. To go further, we use the comparative statics results provided in Table 1 to calculate an expression for $d\widetilde{w}^\ell/db$ as follows:

$$\begin{aligned}\frac{d\widetilde{w}^\ell}{db} &= \frac{dw^\ell}{db} - MRS^\ell_{b,B}\frac{dw^\ell}{dB^\ell}\\ &= -\frac{1}{L^\ell}\frac{\partial\widetilde{e}^\ell/\partial b}{(L^\ell/w^\ell)[2e^\ell/(L^\ell)^2 - \partial^2 e^\ell/(\partial L^\ell)^2]} > 0.\end{aligned}$$

By inducing firms to raise the wage rate paid to low-skilled workers, a compensated increase in b would produce a benefit (in terms of higher utility due to reduced work hours) both for employed low-skilled agents and for high-skilled mimickers choosing the job-replication strategy. When the former (latter) effect dominates, such that $\zeta\partial\widehat{V}^{h,e=0}/\partial w^\ell - \partial EV^\ell/\partial w^\ell < 0$ (> 0), the wage response to a compensated marginal increase in b is socially beneficial (detrimental), thus lowering (raising) the net resource cost of making the unemployment benefit more generous.

IV. Concluding Remarks

In this paper, we have integrated efficiency wage setting with the theory of optimal redistributive income taxation. In doing so, we have used a model with two skill types, where efficiency wage setting characterizes the labor market faced by the low-skilled, while the high-skilled face a conventional, competitive labor market. Furthermore, there are two types of jobs in this economy: low-demanding jobs, which can be carried out by all individuals, and high-demanding jobs, which can only be carried out by high-skilled individuals. High-demanding jobs require maximum effort per hour spent at the workplace, whereas effort per work hour is a decision-variable for individuals employed in low-demanding jobs, such that individuals employed in this type of job have the option to shirk with an exogenous probability of detection. The government uses a nonlinear income tax and an unemployment benefit to redistribute income from high-skilled individuals to low-skilled individuals, and to correct for imperfect competition in the labor market. As such, the government must also recognize that high-skilled individuals have two different options of mimicking the income of the low-skilled: either by reducing the hours of work when employed in high-demanding jobs (referred to as the income-replication strategy) or by choosing low-demanding jobs (referred to as the job-replication strategy).

We would like to emphasize four results. First, the marginal income tax rate implemented for high-skilled individuals is likely to be negative. Albeit reminiscent of a result derived by Stiglitz (1982), the underlying mechanism is different: the negative marginal income tax rate implemented for high-skilled individuals provides a mechanism for increasing the demand for low-skilled labor. As such, it contributes to increase the net tax revenue and reduce the unemployment rate, both of which are socially desirable under plausible assumptions.

Second, the marginal income tax rate faced by low-skilled individuals is not necessarily positive (as it would be in a standard model with competitive labor markets and no extensive margin of labor supply).[34] Although employment-related motives behind the tax policy (i.e., the incentive to increase employment among low-skilled individuals) are likely to push up this marginal tax rate, its sign might also depend on which of the two self-selection constraints is binding. Whereas the self-selection constraint designed to prevent the job-replication strategy works to increase the marginal tax rate of low-skilled individuals, the qualitative effect of the self-selection constraint designed to prevent the income-replication strategy is ambiguous. The intuition is that, although the income-replication strategy allows high-skilled individuals to spend more time on leisure than low-skilled individuals, a high-skilled mimicker employed in a high-demanding job still exerts more effort per work hour than the mimicked, low-skilled individual under the income-replication strategy.

Third, the social value of decreased unemployment takes a form reminiscent of the shadow prices of a public bad in the sense of depending on (i) the sum of the marginal willingness to pay to avoid unemployment among the unemployed, (ii) the effects induced by the self-selection constraint, and (iii) the tax revenue effects created by varying the unemployment rate. Note also that the social value of decreased unemployment directly affects the marginal income tax rates facing both skill types at the second-best optimum, despite the fact that unemployment can only arise among low-skilled individuals in our model.

Fourth, an increase in the unemployment benefit typically leads to a relaxation of the relevant self-selection constraint, irrespective of whether potential mimickers adopt an income-replication or job-replication strategy. As such, this effect contributes to reduce the social resource cost of the unemployment benefit.

[34]The result that the optimal marginal tax rate faced by low-skilled workers is not necessarily positive when firms pay efficiency wages in low-demanding jobs is reminiscent of a similar finding obtained by da Costa and Maestri (2019). By modifying the canonical Mirrleesian model to accommodate the assumption that firms have market power in the labor market, they show that almost all workers face negative marginal tax rates.

Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Online Appendix

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