Comment on "Roughness of Interfacial Crack Fronts: Stress-Weighted Percolation in the Damage Zone"

A recent Letter [1], by Schmittbuhl, Hansen, and Batrouni (SHB) addresses the question of how interfacial cracks roughen in the presence of disorder. SHB explain this process by a stress induced gradient percolation model that takes into account the damage accumulated, and translates that into a self-affine crack front profile. In this Comment, we point out that the results presented in Ref. [1] do not prove self-affinity but rather support self-similarity of the crack fronts. This result, however, would be in disagreement with experiments [2].

In the model of SHB the strain gradient induces a damage profile and a crack front results. As the load is raised the width of the front $W$ increases approximately as a power law, and eventually saturates. As in gradient percolation [3], the saturated width $W_s$ scales with the gradient of the damage profile $1/\ell_s$ as $W_s \sim \ell_s^\alpha$ with $\alpha = \nu/(1 + \nu)$ where $\nu$ is the correlation exponent of the underlying percolation problem. Since in Ref. [1] $\ell_s \sim L_x$, where $L_x$ is the lattice size parallel to the front, SHB combine the initial dynamic scaling with that of the saturated width into a "Family-Vicsek"-like scaling form $W(L_x, t) = L_x^\alpha f(t/L_x^\alpha)$, and conclude that the fronts are self-affine interfaces. Such an attempt is misleading, since presenting data in such a form does not imply that the fronts are self-affine. In gradient percolation $\alpha$ cannot be interpreted as a roughness exponent [3]: the front is self-similar (i.e., the scaling is isotropic) up to a length scale $\xi \sim W$ [4] and it is trivially flat on scales beyond $\xi$. Self-affinity implies instead that on any length scale $l < \xi$ the system rescales anisotropically. Although strain induced correlations could change the values of the critical exponents from the standard percolation ones, the basic picture remains the same.

Figure 1 shows the data of the corresponding Fig. 1 from [1], displaying the broken springs. We also include the hull of the (damage) gradient percolation cluster and the corresponding solid-on-solid (SOS) interface. Comparing these two shows that the SOS presentation is just an artificial projection from the fractal perimeter of the damage zone which is not self-affine. In particular, we see that the size of overhangs is of the same order of the width. We have also studied an effective medium model in the spirit of Ref. [5] in which the strain profile is computed similarly to Ref. [1], but the damage is replaced by its average along the transverse direction [6]. This model is able to reproduce the features of the Family-Vicsek data collapse of SHB, but the fronts are obviously described by standard gradient percolation. From our simulations we find that the gradient $\ell_s$ depends on the elastic constants of the problem. In Ref. [1] the Green function $G_{ij}$ is normalized so that $\sum_{i,j} G_{ij}/(L_x L_y)$ is constant. Since $L_y$ is kept constant this amounts to rescaling the elastic constant by $L_x$, producing an effective dependence of $\ell_s$ on $L_x$.

In conclusion, a correct interpretation in the framework of gradient percolation of the data presented in Ref. [1] implies that fronts are self-similar rather than self-affine. Thus the model of Ref. [1] does not explain the roughness of planar cracks observed experimentally [2].

M. J. Alava$^{1,2}$ and S. Zapperi$^2$

$^1$Laboratory of Physics
Helsinki University of Technology
FIN-02015 HUT, Finland
$^2$SMC-INFM
Dipartimento di Fisica
Università "La Sapienza"
Piazzale le A. Moro 2 00185 Roma, Italy

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