Please note that the comments associated to green highlighted text are not corrections but ian ndication on how I suggest to add the enries in the index. Thus "Toni Malet" in footnote 1 is commented with "Antoni" because in the index his official name should be used.

I attach a word file in which I have listed all the entries marked in green,

Thank you

THE RECEPTION OF NEWTON'S METHOD OF SERIES AND FLUXIONS IN EIGHTEENTH-CENTURY EUROPE[1]

Niccolò Guicciardini

1 Some General Remarks

Newton's impact on eighteenth-century Continental calculus and mathematical physics is usually judged negatively in comparison with Leibniz's achievement. The paradox that it was thanks to the Leibnizian calculus that progress was made in the mathematization of gravitation theory is often mentioned as a clear sign of a crisis in the Newtonian camp. What was wrong with Newton's mathematical style? The conventional answer is conservatism, British chauvinism, lack of interest in algorithmic techniques, a preference for geometric demonstrations less powerful and less general in comparison with Leibnizian calculus. Morris Kline's negative judgment of eighteenth-century British mathematics is, in this respect, typical:

> excessive reverence for Newton's geometrical work in the *Principia*, reinforced by the enmity against the Continental mathematicians engendered by the dispute between Newton and Leibniz, caused the English mathematicians to persist in the geometrical development of the calculus. But their contributions were trivial compared to what the Continentals were able to achieve using the analytical approach.
>
> *Kline 1972, 392*

The received view is not altogether wrong, but, after the work of several historians of mathematics, it is in need of revision and better qualification (Bruneau 2011; Feigenbaum 1985; Gowing 1983; Grabiner 1997; Guicciardini 1989; Sageng 1989; Schneider 2006; Tweddle 1988; Tweddle 2007).

Above all, it is apparent that most of the work carried out by some of Newton's disciples was far from being purely geometrical or remote from Continental algorithmic style. Works such as Roger Cotes's 'Logometria' (1714) and *Harmonia mensurarum* (Harmony of measures, 1722); Brook Taylor's *Methodus incrementorum* (The method of increments, 1715); James Stirling's *Methodus differentialis* (Differential method, 1730), or Thomas Simpson's *Miscellaneous Tracts* (1757), provide examples of a way of

[1] Acknowledgements: I thank Guy Boistel, Natalia Ermolaeva, Toni Malet, Ivor Grattan-Guinness, Helge Kragh, Luis Saraiva and Ivo Schneider for their helpful advice.

doing mathematics well attuned with contemporary Continental work. Even Colin Maclaurin (1698–1746) – the great champion of geometrical rigour – devoted the second book of his *Treatise of Fluxions* (1742) to 'computations in the method of fluxions', stating that the value of the algorithmic approach, based on the algebra of inequalities, is that its demonstrations are 'independent of the [kinematic] notion of a fluxion' (Maclaurin 1742, 752; Grabiner 1997). The active presence of a group of 'analytical' fluxionists in Britain, who practised infinite series manipulations in a formal style of the kind usually attributed to Leonhard Euler, and who dealt with quadrature techniques that demanded considerable skill in manipulating symbols, should not be a surprise to those who know Newton's mathematical work. In fact, several of Newton's works, such as the 'Tractatus de quadratura curvarum' (Treatise on the quadrature of curves, 1704c) and the 'Methodus differentialis' (1711c) represented a triumph of algorithmic techniques. While in the *Principia* (1687) Newton gave prominence to the geometrical style with which he is usually associated, several of his mathematical tracts were written in symbolical style, and this heritage was pursued in Britain by men such as Cotes, Stirling, Abraham De Moivre, Taylor, Thomas Simpson and Maclaurin. These analytical fluxionists produced results that were noted and praised on the Continent. It is worthwhile briefly and incompletely to enumerate results such as Cotes' factorization theorem, the Stirling numbers, De Moivre's theorem, Taylor's expansion, or the Euler–Maclaurin summation formula, as counterexamples to the idea that the British remained mathematically isolated from the Continent. These results were part of the education and mathematical practice of many Continental mathematicians. The analytical fluxionists, it should be added with some emphasis, were often indebted – as they candidly declared – to Continental research. It is easy to find clear signs of influence of the work on integration and series of the Bernoullis in the researches of John Craig, David Gregory, Simpson, Maclaurin and De Moivre (Schneider 2006).

It also needs to be recognized that the Newtonian mathematical heritage was fragmented and complex, to a degree that its author would not have been keen to recognize. Newton not only excelled in algebra and calculus, he also practised projective geometry in the attempt to discover an ancient analysis that, he was convinced, was superior to the Cartesian method of the moderns. The Scots Robert Simson (1687–1768) and Matthew Stewart (1717–85) took this heritage as their agenda and produced works on porisms that were appreciated on the Continent by classicists and anti-modernists in Naples (Mazzotti 1998) and later in France, especially by Michel Chasles (1793–1880).

Once we have a clearer idea of the complexity of the Newtonian mathematical heritage and of the fact that it included algorithmic approaches which were attuned to the style of the Leibnizians on the Continent, the question nevertheless remains how one should evaluate the impact of the works of Newton and of his acolytes on European mathematicians.

Until the publication of John Wallis' *Algebra* (1685) and *Opera mathematica* (Mathematical works, 1693–1699), knowledge about Newton's series and fluxions was restricted to a close circle of acolytes, a network eminently coordinated by John Collins

(1626–83).[2] Newton decided to make his mathematical discoveries known via Collins, while keeping control over the circulation of manuscripts. Interested visitors, such as John Craig in 1685, or David Gregory and Nicolas Fatio de Duillier in the 1690s, appear to have been allowed access to Newton's rooms at Trinity and to have annotated results on series, quadratures and geometry. Some knowledge of Newton's fluxions spread in England, and even on the Continent, before the printing in 1704 of 'De quadratura', mainly as a result of Wallis' works and through correspondence. From Kragh's Chapter 4 in Volume 1 we learn that Harald Vallerius, professor of mathematics at Uppsala (1690–1712) referred to Newton's method of first and ultimate ratios of the *Principia* as early as 1694.[3] Leibniz himself was deeply interested in knowing more about what was going on in the British Isles, and thanks to his correspondence with Henry Oldenburg (*c.* 1619–77), was aware of Newton's work on series. He therefore requested and obtained information from Newton in two long letters – mostly devoted to the use of series in quadratures – which were sent via Oldenburg in 1676.[4] When the *Principia* appeared in 1687, it was clear however that Newton excelled not only in series expansion, but was in possession of a powerful method for 'squaring curves'. This method was not explained in the *Principia*, even though in Lemma 2, Book 2, something analogous to the calculus appeared to surface. Further, some results in the *Principia* (for example, Corollary 3, Proposition 41, Book 1; Corollary 2, Proposition 91, Book 1) were based on the assumption that a method to square curvilinear figures was granted. A handful of Continental mathematicians were anxious to know more. In the early 1690s, Christiaan Huygens began writing to Fatio de Duillier, who enjoyed Newton's intimacy, hoping to get more details from him. Meanwhile Leibniz's calculus was making progress in the public arena on the Continent, thanks to the work of Leibniz himself, Jacob and Johann Bernoulli and a group of French mathematicians, including the Marquis de l'Hospital (1661–1704), Pierre Varignon (1654–1722) and Pierre Rémond de Montmort (1678–1719).

Newton's tracts on fluxions themselves were available to interested Continentals too late in the day to exert an important impact on research. His early 'De analysi per aequationes numero terminorum infinitas' (On analysis by means of equations with an infinite number of terms, written in 1669) only saw the light of day in a collection of Newtonian tracts edited by William Jones (*c.* 1675–1749) in 1711 (Newton 1711b), which enjoyed a wide circulation in the wake of the priority dispute over the invention of the calculus. 'De analysi' was reprinted in the *Commercium epistolicum* ([Newton] 1713); in the Amsterdam edition of the *Principia* (1723), in Castiglione's edition of Newton's *Opuscula* (Newton 1744) and in Samuel Horsley's *Opera* (Newton 1779–1785). The 'De analysi' was purely of historical

[2] Paraphrases from Newton's letters of 1676 to Leibniz were printed by Wallis (Wallis 1685, 330–46). Wallis also later reviewed their content (Wallis 1693–1699, II, 368–96). There, Wallis' comments rely in part on material delivered by Newton himself (the prime theorem on quadratures, which formed part of a draft of 'De quadratura' may be found at Wallis 1693–1699, II, 390–96; the full text of the letters to Leibniz is reproduced in Wallis 1693–1699, III, 622–29, 634–45) (Guicciardini 2012).

[3] Vallerius lectured in 1715 on differential calculus and possibly also on the fluxion theory, taking as his source the mathematical essays appended to Newton's *Opticks* of 1704 (cf. Newton 1704a).

[4] See Breger's Chapter 30 in Volume 3 of this publication.

interest, however, and its publication had only been prompted by the forensic necessities of the priority dispute. The priority dispute, and the many hostilities that followed it, is of great interest here since it contributed to making Newton's method of fluxions better known on the Continent.[5] Certainly, the first full encounter of Continental readers with Newton's fluxions was necessitated by the dispute with Leibniz.

The other early masterpiece by Newton, the *De methodis serierum et fluxionum* (On the methods of series and fluxions, which had been written in the 1670s) appeared in print only in 1736, in an English annotated translation by the rather obscure Lucasian Professor, John Colson (1680–1759), as well as in a pirated edition of 1737 (Newton 1736 and 1737).[6] This wonderful treatise had by now only a pedagogical interest as it was made clear in the title chosen by Colson, which declared the work to be 'for the use Learners'. The *Method* was translated into French by Buffon in 1740. Two Latin translations appeared: one by the ~~Tucsan~~ mathematician, Giovanni Francesco Salvemini (known as Castiglione, Latinized as Castillioneus, also the French version Jean de Castillon is frequent) (1709–91) (Newton 1744); the other by the editor of Newton's collected works, Samuel Horsley (Newton 1779–1785).[7]

The more mature work, 'Tractatus de quadratura curvarum' (Treatise on the quadrature of curves), of which a first version had been completed in 1692–93, first appeared as an appendix to the *Opticks* (1704c). There is evidence that Pierre Rémond de Montmort financed a reprint of De quadratura in 1707 which circulated in Paris (Greenberg 1995, 232). 'De quadratura' also appeared in Jones' edition of Newton's mathematical tracts (Newton 1711a) and was likewise reprinted as part of the Amsterdam edition of the *Principia* (1723), as well as in Castiglione's *Opuscula* (1744) in the commented edition by Melanderhjelm (1762), and in Horsley's *Opera* (1779–1785). 'De quadratura' did contain some passages about integration that would have instructed even mathematicians of the calibre of Johann Bernoulli. Most European practitioners of integration who were active in the first decade of the eighteenth century were indeed keenly interested in, if somewhat overwhelmed by, the generality of Newton's second 'tables of curves', and by the developments of Newton's methods which resulted from Cotes's *Harmonia Mensurarum* (1722). The two minim friars, Thomas Le Seur (1703–70) and François Jacquier (1711–88), who promoted knowledge of the *Principia* with their detailed commentary to the so-called 'Jesuit' edition (Newton 1739–1742), devoted Chapters 4, 5, 6 and 7 of the first volume and Chapter 2 of the second volume of their *Elemens du calcul intégral* to a commentary on Newton's 'De quadratura' and 'De methodis' (Le Seur and Jacquier 1768, I, 135–544; II, 52–103).[8] Newton's methods of

[5] See Chapter 31 by Speiser ~~Bär~~ and Ó Mathúna in Volume 3 of this publication.

[6] Henry Pemberton claimed that he had obtained Newton's approval for a publication of the 'De methodis' (Pemberton 1728, preface).

[7] Castiglione's work was based on ~~Colosn's~~ English (Newton 1736) and, in part, on Buffon's French translation (Newton 1740), while Horsley had access to the manuscripts held at Hurstbourne Park (the home of Newton's heirs, the Portsmouth family, and the location of their collection of Newtoniana).

[8] Le Seur and Jacquier's edition was published in Geneva under the supervision of Jean-Louis Calandrini and with an extensive commentary. Part of the commentary, especially sections on advanced topic (e.g. Moon

quadrature, based on approximations via series expansion, were soon, however, superseded by integration techniques in finite form that were preferred by Continental mathematicians. At the end of the century, Jean Étienne Montucla (1725–99) summarized his evaluation of Newton's techniques of quadrature by noting that the 'geometers reserve Newton's method for hopeless cases'.[9]

Newton's version of the calculus in terms of limits was often contrasted with Leibniz's in terms of infinitesimals and judged to be more rigorous. Leibniz in his private notes had in fact elaborated an approach to his calculus in terms of limits (Knobloch 1989). The Leibnizian calculus, however, came to be known in the version offered in treatises such as l'Hospital's *Analyse des infiniment petits* (1696), which were based on infinitely little quantities. Newton's presentation of the basic concept of his method in terms of 'limits of first and last (or ultimate) ratios', at the start of both the *Principia* and the 'De quadratura', was a source of inspiration for many Continental mathematicians. The idea of founding the calculus on limits, rather than on infinitesimals, was considered many times during the eighteenth century, and Newton was invoked as a forerunner of such an approach.[10] Even though the adoption of limits did not imply an endorsement of Newton's fluxions, versions of the calculus in terms of limits helped to promote a sympathetic image of Newton. The main proponents of limits were Jean le Rond d'Alembert (1717–83), in his famous articles 'Différentiel' and 'Limite' (second part) in the *Encyclopédie*; the French mathematicians, Jacques Antoine Joseph Cousin (1739–1800) and Sylvestre François Lacroix (1765–1843) and the Genevan Simon-Antoine-Jean l'Huilier (1750–1840). These mathematicians, however, did not abandon the advantages of the Leibnizian notation. Such an approach occurred frequently on the Continent. Many Continental mathematicians were prepared to endorse a foundation for the calculus in terms of limits, which was perceived as 'Newtonian'. In doing so, they referred, after 1742, to Maclaurin's *Treatise of Fluxions* as providing the definitive answer to objections raised against the calculus by, for instance, George Berkeley's *The Analyst* (1734), but they nevertheless worked with Leibniz's notation and followed research trends that were locally determined by their immediate political, cultural and intellectual environment. The Portuguese Francisco de Borja Garção Stockler (1759–1829), by contrast, sided openly with Newton's foundation for the calculus (Stockler 1794 and 1797). He elaborated Newton's limits in algebraic terms and extended them to the treatment of transcendental functions (Saraiva 2001). Another Portuguese writer who critically considered Newton's theory of first and last ratios in a very creative way was José Anastácio da Cunha (1744–87) whose *Principios mathematicos* (1790) anticipated

theory and magnetism) is due to Calandrini. The so-called 'Jesuit edition' became the standard means of accessing the text of the *Principia*. It reappeared in several further editions: in 1760 in Geneva, and in 1822 and 1833 at Glasgow. An edition published in 1780–1785 in Prague contains innovative contributions by Johann Tessanek (Jan Tesánek).

[9] 'les Géomètres, réservant la méthode de Neuton pour les cas désespérés' (Montucla 1799–1802, III, 165).

[10] For example, d'Alembert in the second edition of his *Traité de dynamique* (1758) praised the Newtonian foundation of the calculus in terms of limits as presented by Maclaurin's *Treatise of fluxions* (Grabiner 1997, 397).

aspects of Cauchy's theory of limits.[11] A similar attitude can be discerned in work by the Russian Semion Emelianovich Gour'ev (1766–1813), who departed from the prevailing orientation towards the Leibnizian calculus that characterized Russian mathematics as a result of the influence of the eminent German and Swiss mathematicians who held positions at the academy in St. Petersburg (Jacob Hermann, Christian Goldbach, Daniel and Nicolaus II Bernoulli, Leonhard Euler) (Gour'ev 1815). Gour'ev's position remained critical in respect of Newton's adoption of kinematical concepts, but he preferred Newtonian limits to Leibnizian infinitesimals. His studies may have influenced the nineteenth-century Russian mathematician Pierre Rakhmanov, who wrote on the foundations of the calculus.

Mathematics in Germany and Switzerland were, predictably, dominated by the Leibnizian calculus. A more intriguing case was represented by the Netherlands, where the *Journal litéraire* was vocal in defence of Newton against Leibniz during the priority dispute. However, notwithstanding the sympathies for Newtonianism that characterized eighteenth-century Dutch natural philosophy, it seems that fluxions were not promoted there. A single case of a Spanish mathematician who endorsed Newton's calculus will be discussed below.[12]

After these general remarks, it is desirable to turn to more specific test cases in the reception of fluxions in France and Italy. There, there were examples of the endorsement of Newton's fluxional concepts (and even of his notation), which revealed a polemical stance against the prevailing Continental schools. The use of fluxions appears to have been a vehicle for criticism and self-criticism within the Continental mathematical elite.

2 The French Fluxionists

George-Louis Leclerc, Comte de Buffon (1707–88) provides an example of how an anglophile attitude could determine the desire of polemically voicing in favour of fluxions. As Hanks and Roger have both shown, Buffon grew up in the intellectual climate of Dijon, where a colony of English aristocrats resided (Hanks 1966; Roger 1997). His intellectual ties with Britain were numerous. He was a friend of Evelyn Pierrepont (1712–73), second Duke of Kingston-upon-Hull and of Martin Folkes (1690–1754), sometime President of both the Royal Society and the Society of Antiquaries. He collaborated with the English Catholic priest and microscopist, John Turberville Needham (1713–81) and was himself elected a fellow of the Royal Society in 1739 (Roger 1997, 41–43). Buffon was not alone among his French contemporaries in nurturing an admiration for English culture. Buffon had scientific ambitions, and in his youth he had formed the idea of devoting himself to mathematics. He included the Genevan professor

[11] See Malet's Chapter 6 in Volume 1 of this publication.
[12] Knowledge of infinitesimal analysis reached Finland in the eighteenth century (see Chapter 5 by Kallinen in Volume 1 of this publication), where a student of Martin Johan Wallenius at the University of Abo, Johan Lindqvist, discussed a thesis on differential equations in which, in a typically eclectic way, he mixed Newtonian and Leibnizan terminology. Lindqvist's thesis was entitled *De integratione fluxionum formae* and it employed Leibnizian notation.

of mathematics, Gabriel Cramer (1704–52) among his correspondents. Although Buffon later abandoned mathematics, taking up forestry and natural history as his fields, he nevertheless has a place in the history of French mathematics as the translator (albeit in a rather free and inaccurate manner) of Newton's *Method of fluxions*, taken from Colson's English version (Newton 1736; see Hanks 1966, 110–12).[13] The *Méthode des fluxions et des suites infinies* appeared in 1740, prefaced by a few 'incendiary' pages in which Buffon positioned himself against the use of the infinite and the infinitesimals in mathematics (Hanks 1966, 112). Buffon briefly dealt with the priority dispute, in which he sided with Newton against Leibniz – as Montucla and Lacroix would later regret (Montucla 1799–1802, III, 106; Lacroix 1797, I, xiv). He entered into the debate that had started with Berkeley's *Analyst*, taking up a position against one of Berkeley's respondents, the military engineer Benjamin Robins (1707–51), and endorsing enthusiastically James Jurin's reply to the Bishop of Cloyne. Although Buffon's translation was well received both in the *Mémoires de Trévoux* and in the *Journal des Sçavans*, his attack on Robins did not pass unnoticed (*Mémoires de Trévoux*, February 1741: 260; *Journal des Sçavans*, April 1742: 196–98). James Wilson, the editor of Robins' works, sharply protested against the position taken by the translator of the *Method of fluxions* (Robins 1761, II, 309–12). It also seems that Buffon had some connection with the Newtonian physician James Jurin (1674–1750), since, in 1749, Jurin published a *Lettre à M. de Buffon, [. . .] en réponse à quelques censures contenues dans le Traité du Coeur de Monsieur Senac* (Hanks 1966, 121). Buffon adopted the dotted, Newtonian notation in his translation from Colson's edition of Newton.

Buffon's position concerning the infinite, which in the preface to the *Méthode des fluxions* took the form of harsh criticism of Fontenelle's *Elemens,* resonated well with the methodological ideas that he also put forward in a polemic with Clairaut (1748–1749) and with the 'Premier Discours' of the first volume of his *Histoire naturelle* (Brunet 1931 and 1936; Dear 2005, 398–99). Buffon felt that there was a prevaling tendency to attribute real meaning to symbols, which he disliked. The defenders of the use of infinity in mathematics, most notably Fontenelle, had affirmed the reality of the mathematical infinite and infinitesimal, but, according to Buffon, the idea of infinity was an idea of privation, and did not represent a real object. 'Most of our errors in metaphysics', Buffon explained, 'derive from the reality that we attribute to ideas of privation.'[14] Similarly, he answered Clairaut, who was proposing to alter the law of gravitation by adding a term to the inverse square term, by pointing out that the algorithmic success of such a symbolic move was not a proof of its existence. The extra term – which violated the simplicity of nature – was only a calculational device. Buffon embedded his philosophy of mathematics into a Lockean theory of abstraction which favoured an empiricist, or even a constructivist, view of mathematics. As he wrote in the *Histoire naturelle*, 'People imagine they know because of having increased the number of symbolic expressions and learned phrases,

[13] Buffon was also responsible for the translation into French of Hales' *Vegetable Staticks* (1727).

[14] 'la plupart de nos erreurs de Métaphysique viennent de la réalité que nous donnons aux idées de privation' (Newton 1740, x).

and pay no attention to the fact that all these arts are nothing but scaffolding for achieving science, not science itself' (Dear 2005, 400). It is thus understandable why Newton's *Method of fluxions*, based on the kinematic representation of fluent and fluxions, could appeal to Buffon as an alternative to the increasingly algebraized calculus practised by the Parisian mathematicians based at the Académie.

Buffon's interest in British mathematics was not so exceptional in mid-eighteenth-century France (Greenberg 1986, 69). Most notably, a group of Jesuits who contributed to the *Mémoires de Trévoux* was instrumental in promoting British science in France. The leader of this group was Father Louis-Bertrand Castel (1688–1757), an arch-anti-Newtonian, who, nevertheless, showed appreciation for British mathematics (Greenberg 1986, 69 f.; Shank 2008, 194–209). In 1720 Castel, who was then editor of the *~~Journal~~ de Trévoux*, which had already played a role in the controversy over the new analysis that had inflamed the Académie des Sciences at the turn of the eighteenth century, began a campaign against Fontenelle and the abstract mathematics practised at the Académie that resonated with contemporary Newtonian polemical writings against Cartesian and Leibnizian mathematics, which had in turn been triggered by the priority dispute (Shank 2008 and Chapter 1 in Volume 1). In his reviews of Jean-Pierre de Crousaz's commentary on l'Hospital's *Analyse* (1696) and of Fontenelle's *Eléments de la géométrie de l' infini* (Elements of the geometry of the infinite, 1727), Castel expressed his criticisms of the mathematics of the 'récents' and claimed that infinitesimal analysis was plagued by paradoxes. Not so the fluxional method, which Castel praised again and again. In his review (1732) of Stone's *The Method of Fluxions both Direct and Inverse* (1730), Castel underlined the fact that the author was a self-taught mathematician, the son of a gardener working for John Campbell, second Duke of Argyll, who had approached his subject in a natural way, because of a 'pure and disinterested love for geometry'.[15] Well-paid academicians, by contrast, practised an abstract language which was unnatural and remote to the understanding of lay persons. The professionalization of mathematics that was under way in France appeared to this Jesuit to be the origin of a distorted way of using algebra and infinitesimal analysis. Castel's protracted attacks on the algebraic and infinitesimal mathematics of the academicians generated many skirmishes, in some of which Fontenelle was involved. His campaign against the Académie was carried out in terms which resembled those adopted by the British – for example, Maclaurin's excoriating criticisms of Fontenelle's *Elements* in the footnotes of the *Treatise of Fluxions* (Maclaurin 1742, 68). Castel's comments, however, were developed for reasons which were specific to the kind of scientific culture then favoured by the Jesuits in the early to mid-eighteenth century. Father Castel and his coreligionists campaigned against any 'idée de système', and were thus opposed to both Cartesianism and Newtonism. They adopted a moderate scepticism and an experimentalism which was consistent with the idea that human science allowed people only to 'conquer a few results, and make quite a number of errors' (Albertan 2002, 96, quoting *Mémoires de Trévoux* (1762)). The Jesuit endorsement of

[15] '[. . .] amour pur & désintéressé pour la Géométrie' (*Mémoires de Trévoux*, Janvier 1732: 113).

Newtonian mathematics was part of a larger polemic against the abstract and aggressive rationalism of the academicians of the French state (Shank 2008, 194–209).

The philo-Newtonian policy followed by Castel in mathematics led him to promote and supervise Rondet's French translation of the second volume of Edmund Stone's *The Method of Fluxions both Direct and Inverse* (1730). Castel prefaced this work with a long 'Discours préliminaire' concerning the history of the calculus, in which he took up a position in favour of Newton. L'Hospital had dealt only with the differential calculus, whereas Stone had provided his work with a second volume devoted to the squaring of curves. He gave some currency there to the methods of inverse fluxions pioneered by Cotes' *Harmonia mensurarum* (1722). In the first volume of Stone's English *Method*, Newton's notation using dots replaced l'Hospital's ds. In the French translation of Stone's work, by contrast, Rondet (and Castel) restored the Leibnizian notation. This choice of notation in a sense rendered the challenge from Britain more apparent, since the translation of Stone's treatise, presented as a work devoted to the 'calcul integral', appeared at a moment when there remained considerable need for a satisfactory Leibizian textbook on integration. The French version of Stone's work was badly reviewed by Johann Bernoulli, always jealous about maintaining his supremacy in matters concerning integration, who complained about the anglophile presentation by Castel (Bernoulli 1742, IV, 169–92; see also Bernoulli 1722).

Cotes' methods of integration, publicized by Stone, had indeed featured in the polemic between British and Continental mathematicians, especially Johann Bernoulli (Bernoulli 1742, II, 402–18; Gowing 1983, 75–79). However, Cotes' methods were also used and appreciated on the Continent, for example by Maupertuis in a paper read before the Parisian Académie Royale des Sciences in 1732–33 (Gowing 1983, 91–92, 133). The Benedictine prelate Charles Walmesley (1722–97), who before being elected vicar-apostolic of the Western district of England, lived in France at the English Benedictine College of St. Gregory at Douai and in his order's monastery of St. Edmund in Paris, where he published a French commentary on Cotes' *Harmonia mensurarum* in 1749 (a further edition appeared in 1753).[16] Cotes' theorem and its generalization by De Moivre also attracted the attention of Samuel Klingenstierna, professor of physics at the University of Uppsala since 1750, who contributed a paper, presented by Stirling, to the *Philosophical Transactions* for 1731. Klingenstierna was the first to point out errors in Newton's theories of refraction, and his mathematical analysis was used by John Dollond in making optical instruments. In a paper on the aberration of light that was published in the *Philosophical Transactions* for 1759, Klingenstierna also employed Leibniz's differential notation, a rare occurrence of continental notation in Britain before the times of Robert Woodhouse and after John Craig (Klingenstierna 1731; cf. 1759: 952, see also Kragh's Chapter 4 in Volume 1 of this publication). In this context, mention should be made of Daniel Melanderhjelm (1726–1810), a student of Klingenstierna at Uppsala,

[16] Further evidence of the impact of Cotes's work concerns his treatment of astronomical data. Most notably, Nicolas-Louis de Lacaille made use of Cotes's formulas in one section ('Aestimatio Errorum In Mixta Mathesi') of Cotes's *Harmonia Mensurarum* (1722). For this, see Lacaille 1741.

who defended in 1752 a thesis on the 'nature and truth of the method of fluxions'. He was appointed lecturer in physics and professor of astronomy at the same university. In 1782 he moved to Stockholm, where he became one of the leading scientists in Sweden, with a major worldwide correspondence. His extensive commentary to Newton's *De quadratura*, one of the best ever, was carried out in differential notation (Melanderhjelm 1752 and 1762).

It is interesting to note that another Jesuit who contributed to the *Mémoires de Trévoux* took a leading role in the promotion of British science in France and the advocacy of fluxions in particular. Esprit Pezenas (1692–1776), who was educated at the College of Avignon, and was appointed in 1749 to the Observatoire Ste. Croix in Marseille (a Jesuit establishment that later became the Observatoire Royal de la Marine), was a recognized expert on the mathematics of navigation. He was also responsible for several translations from English, for example Desaguliers's *Experimental Philosophy*, Baker's *Treatise on the Microscope*, Ward's *The Young Mathematician's Guide*, Smith's *Opticks* and Gardiner's logarithmic tables.[17] In 1749, he published a French translation of Maclaurin's *Treatise of Fluxions* (for an abridged translation, edited by Pierre-Charles Le Monnier, see Maclaurin 1765). One might think that Maclaurin's *Treatise* was merely a bulky tome which contained outdated foundational questions, but this was not the case (Sageng 1989; Grabiner 1997). Maclaurin's *Traité des fluxions* was in fact well known on the Continent and was read and praised by mathematicians of the calibre of Clairaut, Euler, d'Alembert and Lagrange (Grabiner 1997). The book was praised not only for its rigour but also because it contained important results, such as the Euler–Maclaurin summation formula and a treatment of ellipsoids of equilibrium which was known to Clairaut, Euler, Laplace, Lagrange, Legendre and Poisson. Maclaurin's treatment of elliptic integrals similarly inspired work by Euler, d'Alembert and Legendre. Lagrange, moreover, in the unpublished lectures on the calculus that he gave in Turin in the 1750s referred to Maclaurin's *Treatise* for a treatment of maxima and minima (Borgato and Pepe 1987). Lacroix in the historical introduction to the second edition of his *Traité* noticed the decisive contributions contained in Book II of Maclaurin's *Treatise* (Lacroix 1797, I, xxvii).[18]

Pezenas, in the 'Avertissement' to his translation of Maclaurin's *Treatise of Fluxions* praised the rigour with which Maclaurin avoided infinitesimals and distanced himself

[17] See Smith 1767; Ward 1756; Baker 1754; Desaguliers 1751. Pezenas in the 'Avertissement du traducteur' (Maclaurin 1749) also announced the translation of Maclaurin 1748, Newton 1745 and an augmented edition of Clarke 1730. To the best of my knowlege none of these projects materialized. According to Guy Boistel (personal communication, 17 October 2006) Pezenas had made a translation of some of Nicholas Saunderson's mathematical treatises, which he submitted to the Société Royale de Montpellier. These manuscripts have not been found.

[18] Maclaurin 1748 was eventually translated into French, supplemented by materials taken from Cramer and Euler, by Yves Le Cozic (c^r1718–90), Professor of Mathematics at the école d'artillerie de la Fère (Maclaurin 1753). Maclaurin's *Account of Sir Isaac Newton's Philosophical Discoveries* (London: for the Author's Children, sold by Millar, and others, 1748) appeared in French almost immediately in 1749, translated by Louis Anne Lavirotte: *Exposition des découvertes philosophiques de M. le chevalier Newton, par M. Mac-Laurin [. . .] ouvrage traduit de l'anglais par M. Lavirotte* (Paris: Durand e Pissot). All of which illustrates the considerable impact of Maclaurin's work on French culture of the mid-eighteenth century.

from the infinitesimal tradition that had been initiated in France by l'Hospital in the *Analyse des infiniment petits* (1696). The *Treatise of Fluxions* contained, among other things, extensive notes addressed against Fontenelle's *Elements de la géométrie de l'infini*, which were now made available to the French reader (Maclaurin 1749, xli–xliv). Pezenas in addition criticized existing French works on the integral calculus, in particular Carré's *Méthode* (1700) and Rondet's translation of Stone (1735). It seems therefore that Pezenas's enterprise was independent from that of Castel. In his translation he adhered, moreover, to Newton's dotted notation.

Pezenas also communicated his enthusiasm for Newtonian mathematics to a Spanish Jesuit, Tomàs Cerdà, who worked with him for three years at the Observatory at Marseille. In 1756 Cerdà was appointed to the Chair of Mathematics at the Jesuit College of Cordelles in Barcelona. He planned a translation of Thomas Simpson's *Doctrine and application of fluxions* (1750) and, after corresponding with Simpson, began his annotated translation, the manuscript of which survives in the archives of the Academia de la Historia in Madrid (see Malet's Chapter 6 in Volume 1 of this publication). Pedro Padilla, a military engineer, wrote a treatise in four volumes (Padilla 1753–1756). The last volume, entitled *De los cálculos diferencial e integral o méthodo de las fluxiones* (1756), was based on Pezenas' French translation of Maclaurin's Treatise.[19]

3 Fluxions in Italy

It is interesting to consider the case of the reception of Newton's fluxions in Italy not least because of its similarities with what happened in France. Luigi Pepe has thoroughly charted the diffusion of a Newtonian approach to the calculus in Italian eighteenth-century treatises (Pepe 1982 and 1988). He noticed that after a period (roughly until 1715) in which the Italian scene was dominated by the calculus imported by Leibniz himself during his *iter italicum* of 1689, and by the influence of Jacob Hermann and Nicolaus I Bernoulli, who held chairs of mathematics in Padua from 1707 to 1713 and from 1716 to 1719, respectively, a reaction against the Swiss 'oltramontani' and in favour of Newtonian mathematics was initiated mainly thanks to the group around Jacopo Riccati (1676–1754). Riccati's student Giuseppe Suzzi wrote the *Disquisitiones mathematicae* (1725), which was the first Italian treatise where some currency was given to 'evanescent quantities' and 'fluent and fluxions' (for example, Suzzi 1725, 9). Suzzi's little pamphlet was composed as a collection of letters between Riccati and his pupil. These dealt with some of Riccati's previous research and with the controversies (on second-order fluxions, central forces, or motion in resisting media) which had divided Leibnizians from Newtonians, and which had separated the position of Riccati from that of Johann Bernoulli in the early 1710s (Mazzone and Roero 1997). The approach of Suzzi and Riccati was clearly anti-Bernoullian: 'it is not Newton who makes mistakes, [. . .] but Bernoulli'.[20] They rejected the criticisms

[19] For further information, see Ausejo and ~~Mediano~~ Sánchez 2010.
[20] 'non fallitur ergo Newtonus [. . .] fallitur Bernoullius' (Suzzi 1725, 38).

formulated against Newton by Johann Bernoulli very much according to the manner that was canonical in philo-Newtonian writings, even though they employed more civilized terms than those adopted by John Keill.[21] In Tuscany, there had been an earlier endorsement of Newtonian mathematics by Guido Grandi (1671–1742), a gifted mathematician and professor at the University of Pisa. Grandi was an early admirer of the *Principia*, who used fluxions in his mathematical practice. He was elected FRS in 1709 and sided with the English in the priority dispute (see Chapter 7 by Mazzotti in Volume 1). Salvemini, the editor of Newton's *Opuscula* (1744), studied mathematics with Grandi, before moving to the Pays de Vaud as a Calvinist refugee.

In the highly influential *Institutiones analyticae* (1765–1767), composed by the Jesuit mathematician Vincenzo Riccati (1707–75) with the help of Girolamo Saladini (1731–1813), a definite preference was expressed for Newton's first and last ratios. In the preface to their second volume, the authors clearly stated that nobody would be able to inquire more deeply into foundational matters than Newton had in his treatment of limits in Section 1, Book 1, of the *Principia*. Their first chapter was devoted to the idea of infinitesimal quantities, and was squarely based on Lemma 1, Section 1, Book 1 of the *Principia*, which they cited verbatim.

Maria Gaetana Agnesi was in touch with and deeply influenced by Jacopo Riccati (see Chapter 38 by Cavazza in Volume 3 of this publication). In her *Instituzioni analitiche* (1748), she showed some indications of preference for a Newtonian foundation to the calculus. She opened the second part of her work with the following words: 'With the name "variable quantities", we wish to designate those quantities which are liable to augmentation and diminution, and which we conceive as fluents, or so to speak as generated by a continuous motion.'[22] No reader would have been able to miss the Newtonian orientation of these lines. Indeed, John Colson (1680–1759), a mathematician who was much concerned with the defence of Newton's method, is said to have learnt Italian in order to translate Agnesi's treatise (according to the introduction by John Hellins to Agnesi 1801). Colson's translation appeared, however, only in 1801. In 1749, the Parisian Académie des Sciences promoted a French translation of the second part of Agnesi's *Instituzioni*, which brought a touch of Newtonian fluents into France via this Italian treatise written by the first woman to be offered a university chair in mathematics (cf. Agnesi 1775).

It should nevertheless be stressed that while the Newtonian ideas of fluent, fluxion and first and last ratios were given some currency in these Italian treatises, the notation and the methods of solution used in them were wholly within the Continental framework (especially in matters concerning integration). It seems that these Italian authors did not want to renounce the advantages of notation in terms of differentials and integrals, while

[21] It would be also very interesting to know something about Stirling's stay in Italy. It is possible that because of his political views, he had to leave Oxford in 1716 without graduating. At some point after this, he arrived in Italy where he might have had contacts with the University of Padua. His stay in Italy ended in 1724. There is a letter from Stirling to Newton dated from Venice on 17 August 1719 (see Newton 1959–1977, VII, 53–54).

[22] 'Col nome di quantità variabili si vogliono significare quelle, che sono capaci di aumento, e di decremento, e si concepiscono come fluenti, e per così dire, generate da un moto continuo' (Agnesi 1748, 432).

still adopting a foundational outlook which remained open to some concepts typical of Newton, an eclecticism that is often evident in other countries, such as France, in the work, for example, of Mathurin-Georges Girault de Keroudou. Furthermore, in accounts of the development of mathematics (for instance in the historical introductions which open each volume of the work by Riccati and Saladini), Italian mathematicians reconstructed the development of the 'nuova analisi' in a manner which gave credit not only to Leibnizian but also to British contributions.

The reasons which induced this group of Italians to move towards positions that were autonomous from those of Johann Bernoulli and of Leibniz remain the object of scholarly scrutiny. Robinet sees a 'conflict' between the Italians and the Leibnizians, which he places in a broad political context, in which mathematical conflict represents an aspect of the confrontation between the Serenissima and Vienna (Robinet 1992). Be that as it may, it is clear that the vacancy of the chair of mathematics in Padua, which followed Hermann's departure in 1713, provoked a period of tension (Mazzone and Roero 1997). Over the next few years, nationalist jargon claiming that 'l'Italia farà da sé' ('Italy will do it by herself') began to surface, most notably in the invectives of Giuseppe Verzaglia (*c*. 1669–1730). Riccati, who had defended Hermann against Johann Bernoulli in an argument over the inverse problem of central forces, and others distanced themselves from such public campaigns to restore the name of Italy. Nonetheless, they opposed Leibniz's and Johann Bernoulli's manoeuvres to install another 'oltramontano' in the chair. This was despite the fact that the relations of Hermann with Italian mathematicians had been more than cordial, and regardless of the independence that Hermann had shown from the Basel school, which had in turn irritated both Christian Wolff and Johann Bernoulli (see Mazzone and Roero 1997).

Members and former members of Catholic religious orders were among those who promoted Newton's work. The enlightened Barnabite cleric regular and professor in Milan, Paolo Frisi (1728–84) played an important role in the reception of Newton in Italy. He was also a supporter of the method of fluxions, as is especially apparent from a manuscript entitled 'De Newtoniano [sic] methodo fluxionum et fluentium geometrica tractatio' (n.d.), as well as his published works (Di Sieno and Galuzzi 1990; see also Chapter 7 by Mazzotti in Volume 1 and Chapter 26 by Iliffe in Volume 2 of this publication). Another enthusiast Italian philofluxionist was the Oratorian, Tommaso Valperga di Caluso (1737–1815) whose support for Newton appeared in a paper that he delivered to the Royal Academy of Sciences in Turin in 1787 (Valperga di Caluso 1788; Panza 1987).

Much as in France, Italian Jesuits were also prominent in promoting Newton's fluxions. Vincenzo Riccati, whose work has been discussed above, belonged to the Society of Jesus. Pepe lists several Jesuits, such as Roger Boscovich (Ruđer Josip Bošković) and Francesco Luino, in whose works the Newtonian method of fluxions was considered with some sympathy (Pepe 1988; Homan 1993; Boscovich 1741; Luino 1769, which is a rather sloppy reply to Berkeley). Luino's tract was profusely concerned with Christian apologetic. In it, Newton was depicted as a pious scientist who did not deserve to be the butt of Berkeley's scathing remarks on the infidelity of mathematicians. It might even be surmised that the fortune of Newtonian mathematics amongst Jesuits in France, Italy and Spain might owe something to growing suspicion towards the lack of religiosity displayed by some of the French academicians.

4 Concluding Remarks

The story of the reception of Newton's method of fluxions in Europe is far from being straightforward. The plurality of Newton's own methods interacted with a fragmented mathematical culture, which even within the borders of one land presented diversities and contrasts. A general feature that can be discerned is that Newton's method was often viewed as being a more rigorous presentation than that of Leibniz, especially when Leibniz's calculus was interpreted in bold infinitesimalist terms in the manner of Fontenelle. In general, Newton's notation was ignored on the Continent. It was used in only a few works, in the case of Buffon or Pezenas for polemical reasons. The mathematical results on series and fluxions achieved by Newton and by some British mathematicians were, however, well received, especially when they were produced thanks to formal and symbolical methods that were well attuned to the increasingly algebraicized calculus in vogue in the Continental academies. By the middle of the eighteenth century, however, a deep transformation had come about in the mathematical scene in Europe. Multivariate calculus and the calculus of variations were predominantly a Continental invention. These more advanced techniques would penetrate into British mathematics only in the nineteenth century. The judgement of the Edinburgh professor John Playfair (1748–1819) is telling concerning the feeling of crisis that was shared by many British mathematicians at the beginning of the nineteenth century:

> In the list of the mathematicians and philosophers, to whom that science [physical astronomy], for the last sixty or seventy years, has been indebted for its improvements, hardly a name from Great Britain falls to be mentioned.
>
> *Playfair 1808, 279–80*

Such a verdict, although shared by men such as John Toplis or Robert Woodhouse, should not induce us to think that the divide between Britain and the Continent was in fact so clear-cut. The exchange of notations, mathematical results and more broadly of methodological ideas, fertilized both Britain and Europe in ways that are obscured by the programmatic reconstructions of early nineteenth-century British scientific reformers, who somewhat simplistically dubbed the Newtonian mathematical heritage as a 'Dot-Age'.