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Probing Transverse Coherence with the Heterodyne Speckle Approach: Overview and Perspectives

M. Manfredda^{a,b,*}, M.D. Alaimo^a, M. Giglio^a, M.A.C. Potenza^a

^a *Università degli Studi di Milano, via Festa del Perdono 7, Milan 20122, Italy*

^b *Elettra - Sincrotrone Trieste s.r.l., S.S. 14 - km 163.5 - 34149 Basovizza (TS), Italy*

Abstract

The properties of spatial coherence of radiation emitted by relativistic electrons is far from being trivial. Assessing the coherence of high-brilliance X-ray sources (3rd generation synchrotrons or free electron lasers) is of crucial importance for machine diagnostics, as well as in planning experiments exploiting coherent techniques. The Heterodyne Speckles method, firstly described by Alaimo et al. (2009), is a valuable alternative to standard methods (e.g. Young's interferometer) which 1) provides a direct measure of transverse coherence without any a-priori assumption, 2) provides a full 2D map of coherence, 3) is capable of one shot, time-resolved measures, 4) is scalable over a wide range of wavelengths. It relies upon the statistical analysis of radiation scattered by spherical particles randomly distributed and suspended in a fluid. Here we give an overview of this method, from the theoretical framework to the operating conditions to be adopted in order to obtain coherence measurements in several conditions.

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1. Introduction

The knowledge of spatial coherence plays an important role in the characterizing radiation sources, whatever they are exploited for experimental purposes or they are objects of investigation themselves. In some cases these topics are strongly related, as it is the case of fourth-generation Free Electron Laser (FEL) sources envisioned to produce highly coherent soft and hard X-rays. Traditionally, spatial coherence is measured from the visibility of interference fringes produced by some ad-hoc engineered device, Young's pinhole interferometry being the ancestor of these techniques. The information is usually expressed by one number, the transverse coherence length. Nevertheless, spatial coherence is ultimately described by a complex function that needs accessing to many length scales. This function is the Mutual Intensity Function (MIF):

$$J(\mathbf{r}, \Delta\mathbf{r}) = \langle E_0(\mathbf{r})E_0^*(\mathbf{r} + \Delta\mathbf{r}) \rangle \quad (1)$$

* M. Manfredda

E-mail address: Michele.Manfredda@gmail.com (M. Manfredda).

Here E_0 is the test field, \mathbf{r} is a position in the space, $\Delta\mathbf{r}$ a displacement vector, and $\langle \dots \rangle$ denotes the time average (or the ensemble average in the case of radiation produced by relativistic bunched particles). The normalized version of the MIF is the Complex Coherence Function (CCF), which accounts for the intensity changes across the beam (see below). Measuring the two-dimensional MIF by means of Young's interferometry is a formidable task, since it implies to explore all the possible pinhole displacements in all the directions to probe all the \mathbf{r} and $\Delta\mathbf{r}$. For this reason, a number of different solutions has been explored. The use of gratings, as referred by Pfeiffer et al. (2005) and Guigay et al. (2004), proved to ease the probing different scale lengths, i.e. different values $\Delta\mathbf{r}$, simply by changing the grating-to-detector distance. However, in such a way, real-time measures are not possible, since only one value of $\Delta\mathbf{r}$ is probed at once (and in some cases, in one direction only). A simultaneous measurement of J over a discrete set of $\Delta\mathbf{r}$ was obtained by means of cleverly designed array Lin et al. (2003), comprising a whole range of different Young's slit aperture. However, engineering and manufacturing is an obvious drawback of this approach, since the precise knowledge of the test plate structure enters the numerical process yielding the CCF. Moreover, the measure is possible along one direction only. Ultimately, the use of gratings or of more sophisticated test plates is somehow leaking in providing a full two-dimensional interferometric measure of spatial coherence.

Alaimo et al. (2009) first described the Heterodyne Speckle method which we consider advantageous with respect to others since it is capable of measuring the MIF over a continuous set of $\Delta\mathbf{r}$ in two dimension, thanks to an extremely simple setup yielding data of remarkable finesse.

It relies upon the statistical analysis of a speckle field obtained interfering the (weak) wavefronts scattered from a random phase sample illuminated by the test beam and the (strong) transmitted beam itself, thus producing the heterodyne speckle field as shown in Fig. 1(1). A complete theoretical study of these speckles can be found in Dainty (1984). This interference phenomenon can be alternatively described in terms of spherical waves in a way that recalls the Gabor's in-line holography scheme, that is particularly convenient for the case of interest here (further details are discussed by Potenza et al. (2010) for details). Indeed, in our method each particle generates a spherical wave which acts as a reference beam for mapping the phase profile of the incoming beam. Circular fringes then form, whose local visibility is revealing of the degree of spatial coherence between the incoming field at the particle position and at any other position in the region nearby, as shown in Fig. 1(b). This scheme is conceptually reminiscent of the point-diffraction interferometer described by Smartt and Steel (1975). Actually, in most cases the interference pattern of a single particle is not easily measurable due to the weakness of the interference signal. The captivating aspect of such in-line holographic layout lies in the fact that, when moving from the single particle-interferogram to the speckle field, the information about the degree of visibility of a certain fringe-order is translated in the value of the round mean square fluctuations around a certain spatial frequency. Such information can be recovered in the Fourier domain by taking the spatial power spectrum of the heterodyne field, thus yielding the modulus of the MIF (or CCF).

2. The technique

2.1. Getting the information about spatial coherence

Here below we enter in some more details describing the way HS field are used to recover the spatial coherence information. The heterodyne speckle intensity distribution is the superposition of a number of circular interference fringes randomly distributed across the imaging plane, as depicted in Fig. 1(a). Provided that some conditions are guaranteed (see below and Alaimo et al. (2009)), the power spectrum of the intensity distribution does contains the information yielding the averaged squared modulus of $\tilde{J}(\Delta\mathbf{r})$ over the entire range of displacements vectors $\Delta\mathbf{r}$. Here the notation \tilde{J} (or equivalently $\tilde{\mu}$) emphasizes that the power spectrum yields an averaged information over all the illuminated particles.

The Operating conditions of the technique can be listed as follows

1. particles are much smaller than the coherent size σ_c and scatter light at an angle ϑ_s , so that $\vartheta_s z \gg \sigma_c$, where z is the observation distance.
2. speckles satisfy the heterodyne condition (scattered intensity is about 1% of the incident one)
3. radiation is collected in the near field of the coherence area

The scattering centres are colloidal particles suspended in a liquid. Particles used both at visible and X-ray wavelengths are commercial (around 500 nm in diameter) and extremely cheap. Requirement 1 is easily fulfilled. Moreover, no accurate positioning is required for the sample, making this approach extremely simple to set up. An advantage particularly useful at the large scale facilities.

To cast things in a more quantitative way, let E_0 be the (almost unperturbed) transmitted field and E_S the field scattered by the s -th particle. The intensity at a position \mathbf{r} is then:

$$I = |E^2| + \sum_S^N 2\Re E_0^* E_S + \sum_{S,S'}^N 2\Re E_S^* E_{S'} \quad (2)$$

If the extinction efficiency of the sample is small enough, the last term (homodyne) can be neglected. Requirement 2 is then fulfilled. Assuming that $|E_0|^2$ can be subtracted, we directly obtain the power spectrum of the heterodyne term by numerical computation. If particles are randomly distributed, the summation of the cross terms gives a vanishing background, so that the power spectrum is simply:

$$I = \sum_S^N E_0 \star E_S \quad (3)$$

where \star is a convolution. The importance of being under the near field conditions (requirement 3) emerges here. In a forthcoming publication we will rigorously show that, due to the highly oscillatory nature of interference patterns, the convolution in Eq. (3) can be approximated to a product provided that $z \ll k\sigma_c^2$. Therefore, the power spectrum can be written as a product:

$$I(\mathbf{q}) = F(\mathbf{q})T(\mathbf{q})H(\mathbf{q})C(\mathbf{q}) + K(\mathbf{q}) \quad (4)$$

where F is the normalized particle form factor (constant over the q range of interest), T is the Talbot transfer function (well known in phase contrast methods, but with some changes including the dependence on detector size (see the work of Potenza et al. (2010)), H is the imaging system transfer function (dominated by the response of the scintillator in the X-ray case), K is the shot noise contribution and $C(q) = |\tilde{J}(\mathbf{q})|$ is the squared modulus of the MIF. Wavevectors q are mapped in the real space according to the scaling law:

$$\Delta \mathbf{r} = \mathbf{q}z/k \quad (5)$$

The direct relation between the real and Fourier spaces is a consequence of the fact that we use spherical waves as the reference, and light is collected in the near field of the scattered light. From Eq. (5) it follows that the accessible q range can be easily selected by fixing the observation distance z . A simulated power spectrum is shown in Fig. 1(c), with the characteristic oscillations due to Talbot transfer function. On top of the oscillating behaviour, the relevant information of 1(c) is the average value of the power spectrum, which shall be compared with fringe visibility of 1(b): notice that the power spectral density echoes the fringe visibility of Fig. 1(b), as predicted by the direct mapping relation of Eq. 5.

A delicate step in data analysis is the subtraction of the background from the raw data sets (speckle images). This is crucial because it brings to the pure heterodyne signal to be further analyzed to extract the quantitative information. Subtraction of the background is feasible since the particles are free to move within the sample, because of Brownian motions and/or liquid convection (see the work by Ferri et al. (2004)) Therefore the heterodyne term varies from shot to shot, while background does not. With synchrotron radiation (with which the technique has mainly been developed) this is straightforward, since synchrotron radiation facilities deliver stable beams. With FEL radiation this may be not exactly the case, because intrinsic fluctuations in the beam intensity distribution are expected.

2.2. Mutual Intensity Function and Complex Coherence Function

Since the visibility of a fringe system produced by Young's interferometer depends on the average intensity illuminating the two apertures, a normalized version of the MIF, the CCF, is often used in order to compare the coherence properties of beams exhibiting different intensity distributions. The CCF is historically defined as $\mu(\mathbf{r}, \Delta \mathbf{r}) =$

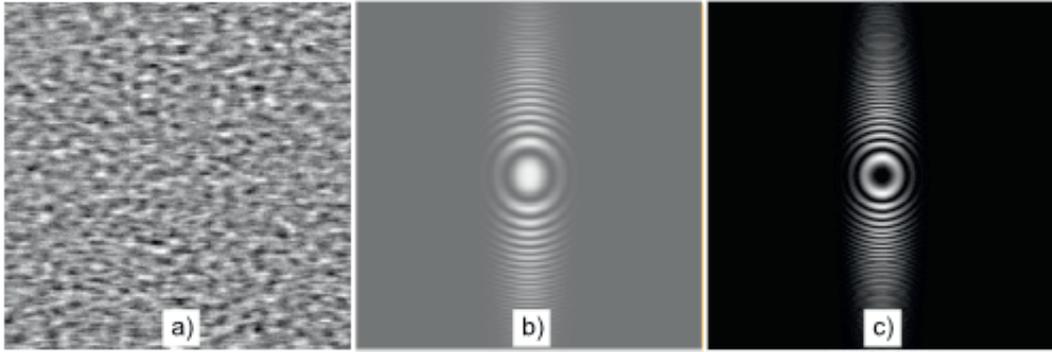


Fig. 1. (a) Heterodyne speckle intensity distribution produced by scattering from many particles; (b) Heterodyne interference pattern produced by a single particle; (c) Spatial power spectrum of the speckle distribution (a) or, equivalently, of the single fringe system in (b).

$J(\mathbf{r}, \Delta\mathbf{r})/I(\mathbf{r})I(\mathbf{r} + \Delta\mathbf{r})$, where $I(\mathbf{r})$ and $I(\mathbf{r} + \Delta\mathbf{r})$ are the local intensities at the apertures. Clearly, such a definition must be properly reexamined in the statistical approach we introduced, where the MIF $\tilde{J}(\Delta\mathbf{r})$ describes the *average* degree of coherence of the radiation, rather than the *mutual* degree of coherence between two points.

To this aim we have performed numerical simulations of a two-dimensional ensemble of scatterers illuminated by beams with different phase and amplitude distributions, setting in the conditions described in Sec. 2.1. We obtained numerical evidence that the power spectrum of the heterodyne speckles equals the intensity autocorrelation of the illuminating beam A_I . Thus, we can define the quantity:

$$\tilde{\mu}(\Delta\mathbf{r}) = \frac{\tilde{J}(\Delta\mathbf{r})}{\sqrt{A_I(\Delta\mathbf{r})}} \quad (6)$$

which depends on the spatial coherence properties of the test field only (not on the intensity distribution).

It shall be noticed that $\tilde{J}(\Delta r)$ and $\tilde{\mu}(\Delta r)$ do not appreciably differ when the beam intensity distribution is slowly varying over the transverse coherence length. This is usually the case of synchrotron radiation, where the beam size is of the order of a few mm and the coherent length rarely exceeds a few hundred microns. Conversely, the two quantities are significantly different whenever the test beam is endowed with a high degree of coherence, as it is the case of FEL sources. In such cases, measures shall be performed with the sensor collecting the whole beam intensity distribution, in order to numerically compute the intensity autocorrelation function. We stress that in other approaches, such as that described by Gutt et al. (2012), the transmitted beam is blocked by a beam-stopper, thus preventing the access to the beam intensity auto-correlation function. Conversely, the real-time computation of $A_I(\Delta\mathbf{r})$, and hence of $\tilde{\mu}(\Delta r)$, is always possible if heterodyne speckles are used, since the transmitted beam is accessible.

2.3. Limited Temporal Coherence

As any interferometric technique, this method is simultaneously sensitive to transverse (spatial) and longitudinal (temporal) coherence. For a detector of width L , the maximum path difference between two rays emerging from the sample at distance z is of the order of $\Delta\ell \approx L^2/(2z)$ (under paraxial approximation). Longitudinal effects are negligible provided that $\Delta\ell < \ell_c$, where $\ell_c = \Delta\lambda/2\lambda^2$ is the longitudinal coherence length.

In the case of synchrotron radiation, electrons within a bunch are uncorrelated emitters with characteristic emission times of the order of $10^{14} - 10^{17}$ sec. Hence, the ensemble average in Eq. 1 can be figured out to be computed over a huge number of field realization within the typical experimental exposure time (1 – 50ms). This process determines a contrast reduction in the speckle field, but does not affect its statistical properties (the speckle size does not change). This basically states that the measure just depends on the spectral bandwidth $\Delta\lambda$ (usually assigned by a monochromator) and is not affected by the exposure time.

The case of a single-pulse radiation (such a FEL) is slightly different, since electrons radiate almost coherently and only a few longitudinal modes hit the sample during the integration time. In the limiting case of longitudinal fully coherent radiation, the ensemble average in Eq. 1 is ill-defined, and the concept of “transverse coherence” may

need to be reconsidered. As discussed above, in this limiting case the power spectrum of heterodyne speckles equals the autocorrelation of the beam intensity profile, and radiation shall necessarily be regarded as transversally fully coherent. The 1st-order statistics of the intensity distribution can be related to the number of longitudinal modes. The same result can be obtained by means of 1st-order temporal statistics, which follows the well known gamma distribution. Notice that the number of modes obtained from the 1st order statistics is the product of the numbers of longitudinal and transverse modes $M = M_L M_T$. Heterodyne speckles directly provides M_T from $\mu(\Delta\mathbf{r})$, and therefore M_L can be obtained. This holds true for both SASE and seeded FEL radiation.

3. Results and Perspectives

This overview has been prepared on the basis of the results on a number of tests performed in different conditions and different machines. With synchrotron radiation delivered by an undulator source at ESRF (Grenoble, France), as well as with visible FEL radiation at SPARC (Frascati, Italy). Thanks to the wavelength scalability mentioned above, accurate testing of the procedures and data analysis schemes have been performed with optical radiation, a tool which allows for a complete control of the experimental conditions. Results show that, by means of a single acquisition, it is possible to obtain a full two-dimensional coherence map. Investigations with synchrotron radiation put to evidence non-trivial oscillating behavior in the modulus of the CCF, that could be attributed either to diffractive effects due to the presence of apertures, or to interferential effects due to imperfections in the focussing optics. Moreover, tests have been done to evaluate how the coherence degrades due to the presence of several materials along the beam path (e.g. graphite foil), or due to monochromator mechanical instabilities. Investigations with FEL radiation were mainly addressed to the assessment of the machine operating conditions. In that case, a gaussian-like coherent area was found, as predicted by theory, but with a coherence length ≈ 3 times smaller than the expected one. These results are presently described in Manfreda (2012-03-15) and will be presented in forthcoming publications.

The direct consequence of full 2D mapping is the possibility of performing time-resolved measures of the coherence. At ID06-ESRF the effects introduced by instabilities of the detector have been studied in detail. This is one of the expected breakthroughs of this technique, especially with FEL sources where single pulses are accessible. As a side result, the technique has been used to provide high-quality characterization of the transfer function of phosphorous scintillators used in X-ray imaging and related methods. This is possible even with a low degree of spatial coherence, provided that the beam is monochromatic.

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