Dilution Effects, Population Growth and Economic Growth under Human Capital Accumulation and Endogenous Technological Change

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Abstract: The paper analyzes the impact of population growth on economic growth under endogenous technological change and human capital investment. The novelty of this chapter is the inclusion of a "dilution effect" of population growth on per capita human capital accumulation, which is not present in the original Uzawa-Lucas model. The present paper has shown that an increase in the population growth rate yields an ambiguous impact on the growth rate of per-capita income due to the relative contribution of two distinct effects of population growth: The direct dilution effect and the indirect ideas effect. This study revealed that the dilution effect has a central role in explaining the ambiguous impact of population growth on economic growth. When the dilution effect is sufficiently low, an unambiguously positive correlation between population growth and economic growth is obtained. When it is sufficiently high the correlation may be either positive or negative or neutral. Another result is that more population growth generates an indirect ideas effect on the rate of innovation and economic growth.

Keywords: Population Growth, Human Capital Accumulation, Dilution Effect, Endogenous Technological Change

JEL Classifications: J24, J29, O30, O41

1. Introduction

The Secretary-General of the United Nations (UN) Ban Ki-moon's annual speech for 2014 World Population Date holds key messages for a more sustainable future for generations to come. He calls attention to investing in young people for the economic prosperity of all countries and underlines the importance of giving priorities to the youth in development plans in order to increase the young involvements in every state of life and to strength the partnerships between young organizations and business. The reason why Ban Ki-moon draws such an attention mainly relies upon the reality in the world population that is captured by the UN itself. According to the last revision of the UN, it is a fact that *the world population reached to 7.2 billion in mid-2013 with 5.9 billion are living in less developed countries which is equal to 82.5 per cent of the world's total*, and *it is projected to reach 9.6 billion in 2050 by increasing more than 2.4 billion more than in 2013*[...] even under the assumption of decreasing fertility rates (UN, 2013a, pp.1-5). Additionally, World Population Policies 2013 report of the UN reveals that differentiated policies and programs at both national and international level are strongly needed because of the new population patterns and trends. According to this report "*In the past two decades, many governments in less developed regions have realized the importance of reducing high rates of population growth, while a growing number of governments in more developed regions have expressed concerns about low rates of population growth...*" (UN, 2013b, p. 1).

The question is that why some countries focus on reducing the rates of population growth while the others focus on raising it? The latter fact shows that the correlation between population and economic prosperity varies with the level of economic progress.

The relationship between population growth and economic growth has always been taken into account comprehensively by the economists and the policy makers. From the economic point of view, - starting from the Malthusian theory - studies on the impact of population growth on economic growth can be categorized as follows: Pessimistic views, optimistic views, and neutralist views. According to the pessimists, population growth has detrimental effects on economic growth. Simply they claim that economic resources (such as food supply) are fixed in the long run, and technological progress is also limited to increasing population.[1] Unlike

[1] For the main proponents of this group, see Malthus (1798), Coale and Hoover (1958), Ehrlich (1968). In the standard growth theory where savings rate and technological change are exogenous, population growth lowers income because of the (physical) capital dilution. For some empirical studies, see Mankiw *et al*. (1992), Ahituv (2001), Li and Zhang, (2007) and Herzer *et al*. (2012).

pessimists, optimists argue that population growth affects economic growth positively due to the endogenous technological progress and scale effects of larger populations.[2] Last group, neutralists claim that the impact of population growth on economic growth is so little (either positive or negative or non-existent) that can be negligible. [3]

Until now, the literature revealed that "[...] *population growth is not all good or all bad for economic growth*" as Kelly and Schmidt (1995, p. 554) argue in their paper. Instead of asking "what is the net impact of population growth on economic growth?", asking the question of "why does population growth affect countries' economic growth differently?" would be much more significant in order to get an accurate answer about the sign of the relationship between population growth and economic growth. Prettner (2013), Romero (2013) and Mierau and Turnovsky (2014) argue that the role of demography is another factor to evaluate the relation between population and economic growth. However; the nature of the demographic changes (mortality, fertility, and aging) is not at the focus of this paper. [4] Both theoretical and empirical studies in the (endogenous growth) literature have showed that "*...Whether population growth or population size foster or hamper economic growth strongly depends on the modeling framework...*" (Prettner and Prskawetz, 2010, p. 607).

In the light of this wide literature, this paper attempts to analyze the impact of population growth on economic growth under endogenous technological change and human capital investment by agents. The model we study in this paper is based on an endogenous growth model with expanding variety of products[5] where return to specialization is always positive. We consider a closed economy in which final output, intermediate and research sectors are vertically integrated, and there are three types of homogenous agents which are perfectly mobile and fully employed. In this economy governmental activity does not exist, population grows exogenously, and there is no external shock such as migration. Individuals are assumed to spend their time to work and invest in human capital.

The important novelty of this paper comes from a critic which is not presented in the original Uzawa (1965) and Lucas (1988) model. Lucas (1988) argues that newborns do not reduce the

[2] For some models (with endogenous technological change) in which population size (and/or growth) affects economic growth positively; see Kuznets (1967), Boserup (1981), Simon (1981), Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Kremer (1993), Jones (1995).
[3] For an example; see Srinivasan (1998).
[4] To address the question, Kelly (1988) provides an extensive review. For some recent substantial analyses, see also Prettner and Canning (2014) and Prettner *et al*. (2013). Lastly, Strulik *et al*. (2013)'s "*child quantity-quality trade off*", and Prettner (2014)'s *schooling intensity* approaches are important examples for showing the adverse effect of population growth on economic growth under the R&D-based growth models with human capital accumulation.
[5] For a detailed explanation of the model structure see Barro and Sala-i Martin (2004, Chp.6, p. 285).

current skill level of individuals hence; population growth does not exist in the formulation of human capital accumulation. Unlike Lucas, Bucci (2013, p. 2029), Strulik (2005 p. 137) and Dalgaard and Kreiner (2001, p. 190) illustrate that population growth decreases the average human capital level of an economy, and therefore, has a dilution effect on the accumulation of per-capita human capital. Additionally, we know that there are some empirical studies also concluding that the population growth has a direct and negative dilution effect on human capital investment.[6] Our explanation mainly rests on the inclusion of an explicit dilution effect of population growth on human capital accumulation. Then, we extend our benchmark formulation by introducing a parameter which measures the strength of this negative effect of population growth on per-capita human capital investment.

The objective of the present paper is therefore twofold. First, it answers the latter question by providing an alternative but complementary theoretical framework that explains why an increase in the population growth rate -regardless of the source of demographic change[7]- may yield an ambiguous (positive, negative or neutral) impact on the growth rate of per-capita income in the very long run. Second, it aims to evaluate that to what extent the dilution effect of population growth explains the different rates of economic growth across countries. The results have demonstrated that the strength of this dilution effect has a central role in accounting for the ambiguous impact of population growth on economic growth along the BGP equilibrium. Another result of the paper is that population growth has an indirect ideas effect on real per-capita income.

The remaining part of the paper is organized as follows. In Section 2 we lay out the benchmark model whose predictions are analyzed along the BGP in Section 3. Section 4 we demonstrate the relationship between population growth and per capita income growth under the BGP equilibrium. Finally, section 5 we conclude the paper and provide a ground for possible future extensions.

2. The Model

2.1 Production

Consider an environment in which three sectors of activity are vertically integrated. The research sector is characterized by free entry. Here, firms combine human capital and

[6] See Boikos *et al*. (2013, pp. 52-56). See also Coale and Hoover (1958) for the types of dilution effect of population growth.
[7] Notice that, Boucekkine *et al*. (2002) also follow a similar method of approach to investigate the effects of population growth on economic growth. The authors find a population growth rate which maximizes the growth rate.

(eventually) the existing number of ideas to engage in innovative activity that results in the invention of new blueprints for firms operating in the intermediate sector. The intermediate sector is composed of monopolistic competitive firms. There is a distinct firm producing each single variety of intermediates/durables and holding a perpetual monopoly power over its sale. In the competitive final output sector, atomistic firms produce a homogeneous consumption/ final good/output by employing human capital and all the available varieties of intermediate inputs. The representative firm producing final output has the following technology:[8]

$$Y_t = n_t^{\bar{\alpha}} H_{Yt}^{1-Z} \int_0^{n_t} (x_{it})^Z \, di\,, \qquad \bar{\alpha} \geq 0\,, \qquad 0 < Z < 1 \tag{1}$$

In Eq. (1) Y denotes the total production of the homogeneous final good (the *numeraire* in the model), x_i and H_Y are, respectively, the quantity of the *i-th* intermediate and human capital input employed in the sector. The number of ideas existing at a certain point in time (n_t) coincides with the number of intermediate-input varieties and represents the actual stock of *non-rival* knowledge capital available in the economy. Here, we assume that having a larger number of intermediate-input varieties do not lead any detrimental effect on aggregate productivity in the production process. As a whole, the aggregate production function (1) displays constant returns to scale to the two private and rival factor-inputs (H_Y and x_i), with $1-Z$ and Z corresponding to their shares in GDP.[9] When $Z \in (0;1)$, final output production takes place by using simultaneously human capital and intermediates.

The inverse demand function for the *i-th* intermediate reads as:

$$p_{it} = Z n^{\bar{\alpha}} H_{Yt}^{1-Z} (x_{it})^{Z-1} \tag{2}$$

Eq. (2) represents that *i-th* intermediate producer receives its own marginal product at time t, since the industry is competitive. In the absence of any strategic interaction across firms in the intermediate sector[10], the demand for the *i-th* durable has price elasticity (in absolute

[8] We follow Ethier (1982) and Romer (1987; 1990).

[9] Since final output is produced competitively under constant returns to scale to rival inputs, at equilibrium H_Y and x_i are rewarded according to their own marginal products. Hence, $(1-Z)$ is the share of Y going to human capital and Z is that accruing to intermediate inputs.

[10] That amounts to assuming that the number of intermediate firms (n) is so large that each of them produces only a very negligible share of the total supply of intermediates.

value) equal to $1/(1-Z)>1$, which coincides with the elasticity of substitution between any two generic varieties of capital goods in the final output production.

In the intermediate sector, firms engage in monopolistic competition. Each of them produces one (and only one) horizontally differentiated durable and must purchase a patented design before producing its own output. Thus, the price of the patent represents a fixed entry cost. Following Grossman and Helpman (1991, Ch. 3), we assume that local monopolists have access to the same one-to-one technology:

$$x_{it} = h_{it}, \qquad \forall i \in [0; n_t], \qquad n_t \in [0; \infty) \tag{3}$$

where h_i is the amount of skilled labor (human capital) required in the production of the *i-th* durable, whose output is x_i. For given n, Eq. (3) implies that the total amount of human capital used in the intermediate sector at time $t\left(H_{It}\right)$ is:

$$\int_0^{n_t} (x_{it})\, di = \int_0^{n_t} (h_{it})\, di \equiv H_{It} \tag{4}$$

By continuing to assume that there exists no strategic interaction across intermediate firms, and making use of Eq. (2), maximization of the generic *i-th* firm's instantaneous flow of profits leads to the usual *constant markup* rule:

$$p_{it} = \frac{1}{Z} w_{It} = \frac{1}{Z} w_t = p_t, \qquad \forall i \in [0; n_t], \qquad n_t \in [0; \infty) \tag{5}$$

Eq. (5) says that the price is the same for all intermediate goods i and is equal to a constant markup $\left(1/Z > 1\right)$ over the marginal cost of production $\left(w_I\right)$. In a moment it will be explained that in this economy the whole available stock of human capital $\left(H\right)$ is employed and spread across production of consumption goods $\left(H_Y\right)$, durables $\left(H_I\right)$, and new ideas $\left(H_n\right)$. Since it is assumed to be perfectly mobile across sectors, at equilibrium human capital will be rewarded according to the same wage rate $w_{Yt} = w_{It} = w_{nt} \equiv w_t$, with w_I denoting the wage paid to any generic unit of human capital employed in the intermediate sector. Under the hypothesis of symmetry – *i.e.*, p and x equal across i 's – Eq. (4) leads to:

$$x_{it} = H_{It} / n_t = x_t, \qquad \forall i \in [0; n_t] \tag{4'}$$

$$\pi_{it} = \left[Z(1-Z)H_{Yt}^{1-Z}H_{It}^{Z}\right]n^{\bar{\alpha}-Z} = \pi_t, \qquad \forall i \in [0;n_t] \tag{6}$$

Thus, each intermediate firm will decide at time t to produce the same quantity of output (x), to sell it at the same price (p), accruing the same instantaneous profit (π). The symmetry across durables is a direct consequence of the fact that each intermediate firm uses the same production technology (3) and faces the same demand function (see 2 and 5). Notice that, $Z \in (0;1)$ and the product within the square brackets is therefore, greater than zero. π_t would have been equal to zero if Z had been equal to one (instantaneous profit are zero in a perfectly-competitive market). Under symmetry, Eq. (1) can be recast as:

$$Y_t = \left(H_{Yt}^{1-Z}H_{It}^{Z}\right)n_t^{R}, \qquad R \equiv \bar{\alpha} + 1 - Z > 0 \tag{1'}$$

where R measures the degree of returns to specialization, that is "*[…]The degree to which society benefits from 'specializing' production between a larger number of intermediates*" (Benassy, 1998, p. 63). In the present paper, it is immediate to verify that R is always positive. The hypothesis $R > 0$ implies that the impact on aggregate productivity (Y) of having a larger number of intermediate-input varieties is always positive ($n > 0$) for any $H_I > 0$ and $H_Y > 0$ (see Eq. 1'). According to Eq. (1'), the aggregate production function exhibits constant returns to H_Y and H_I together, but either increasing $(R > 1)$, or decreasing $(0 < R < 1)$, or else constant $(R = 1)$ returns to an expansion of variety, while holding the quantity employed of each other input fixed. With respect to other settings, this article introduces important novelties. Unlike Devereux *et al*. (1996a; 1996b; 2000) where, if all intermediates are hired in the same quantity x the returns to specialization are either unambiguously increasing[11] or at most constant,[12] we allow for the possibility that the returns to specialization might also be decreasing. Unlike Bucci (2013), we explicitly rule out the possibility that the returns to specialization R are negative.[13]

[11] In Devereux *et al*. (1996a, p. 236, Eq. 1; 2000, p. 549, Eq. 1), under symmetry ($x_i = x, \forall i$) the aggregate production function reads as: $Y = xN^{1/\rho}, \rho \in (0;1)$. Therefore, the degree of returns to specialization equals $1/\rho$, a number clearly larger than one. This is the "*increasing returns to specialization case*" in Devereux *et al*. (1996b, p. 633, Eq. 4b, with $\lambda = 0$).

[12] See Devereux et al. (1996b, p. 633, Eq. 4b, with $\lambda = 1 - 1/\rho$).

[13] A negative R means that an increase in n would lead to some sort of '*inefficiency*' in the economy since, following a rise of the number of intermediate-good varieties, aggregate GDP (Y) would *ceteris paribus* decline in this case.

2.2 Research and Development (R&D)

There is a large number of small competitive firms undertaking R&D activity. These firms produce ideas indexed by zero through an upper bound $n \geq 0$. Ideas take the form of new varieties of intermediate inputs that are used in the production of final output. They are partially excludable, but nonrival. With access to the same stock of knowledge, n, a representative research–firm uses only human capital to develop new ideas:

$$\dot{n}_t = \psi_t H_{nt}, \qquad n(0) > 0 \tag{7}$$

In Eq. (7) is H_n the number of people attempting to discover new ideas, and ψ is the rate at which a single researcher can generate a new idea. Since the representative R&D-firm is small with respect to the whole sector, it takes ψ as given. Hence, Eq. (7) suggests that R&D-activity is conducted under constant returns to scale to the human capital input (H_n). We postulate that the arrival rate ψ has the following specification:

$$\psi_t = \frac{1}{\chi} \frac{H_{nt}^{\mu-1}}{H_t^{\Phi}} n_t^{\eta}, \qquad \chi > 0, \qquad \mu > 0, \qquad \Phi \gtreqless 0, \qquad \eta < 1 \tag{7'}$$

Using together (7) and (7'), the R&D-technology (the production-function of new ideas) reads as:

$$\dot{n}_t = \frac{1}{\chi} \frac{H_{nt}^{\mu}}{H_t^{\Phi}} n_t^{\eta}, \quad n(0) > 0, \quad \chi > 0, \quad \mu > 0, \quad \Phi \gtreqless 0, \quad \mu \neq \Phi, \quad \eta < 1 \tag{8}$$

In the equations above, χ is a strictly positive technological parameter and H is the aggregate amount of human capital available in the economy. The rate at which a researcher can generate a new idea (ψ) is related to three different effects. The parameter η measures the traditional *intertemporal spillover-effect* arising from the existing stock of knowledge, $n : \eta < 0$ reflects the case where the rate at which a new innovation arrives declines with the number of ideas already discovered ("*fishing-out effect*"); if $0 < \eta < 1$, previous discoveries raise the productivity of current research effort ("*standing-on-shoulders effect*"); $\eta = 0$ represents the situation in which the arrival rate of new ideas is independent of the available stock of knowledge.[14] The case $\eta = 1$ is ruled out from the analysis in order to avoid possible

[14] For a detailed discussion of the "*fishing out*" and "*standing on shoulders*" effects, see Jones (1995; 2005).

scale effects, whereby an increase in the level of available human capital may affect the rate at which new ideas are produced over time. The parameter μ captures the effect on the arrival rate of a new innovation of the actual size of the R&D process (measured by the number of units of skilled labor-input actually devoted to it). A value $\mu = 0$ would imply that H_n is not an input to R&D-activity (Eq. 8). We rule out this unrealistic case by assuming that research human capital is indispensable to the discovery of new designs and that its contribution to the production of new ideas is always positive (*i.e.*, $\mu > 0$). If $\mu = 1$, doubling the number of researchers H_n would not affect the arrival rate of a new idea in Eq. (7'), so leading to exactly double the production of new innovations per unit of time (Eqs. 7 and 8); if $\mu \in (0;1)$ due to the existence of congestion/duplication externalities (*"stepping-on-toes effect"*), increasing the number of researchers leads to a reduction of the rate at which each of them can discover a new idea (Eq. 7') and to a simultaneous increase (but less than proportional) in the total number of new innovations produced in the unit of time (Eq. 8).[15] In accordance with Jones (2005, p. 1074, Eq. 16), we keep our analysis as much general as possible and impose no upper-bound to μ. According to Eq. (8), inventing the latest idea requires a skilled-labor input equal to $H_n = (\chi H^{\Phi} / n^{\eta})^{1/\mu}$, which can change over time either because of the growth of *n* (*intertemporal knowledge-spillover effect*), or because of the growth of H. If Φ is positive, an increase in population size would *ceteris paribus* lead to a rise of H and, ultimately, to a decrease of research human capital productivity (an increase in H_n). The hypothesis that the productivity of human capital employed in research may fall due to an increase in population size can be justified by the fact that it becomes increasingly difficult to introduce successfully new varieties of (intermediate) goods in a more crowded market (*R&D-difficulty* grows also with the size of population, as suggested by Dinopoulos and Segerstrom, 1999, p. 459). In Eq. (8) a positive Φ measures the strength of this effect: all the rest being equal, the larger Φ and the bigger the decline in the R&D human capital productivity following an increase of population size. On the other side, negative shows that the productivity of human capital employed in research sector increases because of the fact that growing human capital stock leads to an increase in the ease of exchanging of ideas and expanding the possibilities for creating interactions between researchers. Notice that the

[15] Likewise, if $\mu > 1$, increasing the number of researchers would imply an increase (more than proportional) in the total number of new innovations produced in the unit of time (Eq. 8).

Jones' (2005) formulation of the R&D process does not take these important features of the inventive activity into account.[16]

The R&D sector is competitive and there is free-entry. A representative R&D firm has instantaneous profits equal to:

$$\text{R\&D firm profits} = \underbrace{\left(\frac{1}{\chi}\frac{H_{nt}^{\mu}}{H_{t}^{\Phi}}n^{\eta}\right)}_{\dot{n}}V_{nt} - w_{nt}H_{nt} \tag{9}$$

where:

$$V_{nt} = \int_{t}^{\infty}\pi_{i\tau}e^{-\int_{t}^{\tau}r(s)ds}d\tau\,, \qquad \tau > t \tag{10}$$

In the last two equations, V_n denotes the value of the generic *i-th* intermediate firm (the one that has got the exclusive right of producing the *i-th* variety of capital goods by employing the *i-th* blueprint), $\pi_{i\tau}$ is the flow of profits accruing to the same *i-th* intermediate firm at date τ, $\exp\left[-\int_{t}^{\tau}r(s)ds\right]$ is a present value factor which converts a unit of profit at time τ into an equivalent unit of profit at time t, r denotes the instantaneous interest rate (the real rate of return on households' asset holdings, to be introduced in a moment), and w_n is the wage rate going to one unit of research human capital. Eq. (9) says that profits of a representative R&D firm are equal to the difference between total R&D revenues (R&D output, $\dot{n}$, times the price of ideas, V_n) minus total R&D costs related to *rival* inputs (human capital employed in research, H_n, times the wage accruing to one unit of this input, w_n). Eq. (10), instead, reveals that the price of the generic *i-th* idea is equal to the present discounted value of the returns resulting from the production of the *i-th* variety of capital-goods by profit-making intermediate firm *i*.

Using Eq. (9), the zero-profit condition in the R&D sector implies:

[16] When $\Phi = 0$, Eq. (8) becomes: $\dot{n}_t = \frac{1}{\chi}H_{nt}^{\mu}n_t^{\eta}$, $\chi > 0$, $\mu > 0$, and $\eta < 1$. This specification coincides with Jones (2005, Eq. 16, p. 1074).

$$w_{nt} = \frac{1}{\chi}\frac{H_{nt}^{\mu-1}}{H_t^{\Phi}} n^{\eta} V_{nt} = \psi_t V_{nt} \quad (9')$$

2.3 Households

The economy is closed and consists of many structurally-identical households. Therefore, we focus on the choices of a single infinitely-lived family with perfect foresight whose size coincides with the size of the whole population (*L*) and that owns all the firms operating in the economy. Each member of the household can purposefully invest in human capital. Consequently, the aggregate stock of this factor-input ($H_t = h_t L_t$) can rise either because population grows at a constant and exogenously given rate $g_L > 0$, or because per capita human capital, h_t, endogenously increases over time. The household uses the income it does not consume to accumulate new assets that take the form of ownership claims on firms. Thus:

$$\dot{A}_t = (r_t A_t + w_t H_{Et}) - C_t, \qquad A(0) > 0 \quad (11)$$

where A and C denote, respectively, household's asset holdings and consumption and $H_E \equiv uH = H_Y + H_I + H_n$ is the fraction of the available human capital employed in production activities (namely, production of consumption goods and intermediate inputs, and discovery of new ideas).[17] Eq. (11) suggests that household's investment in assets (the left-hand side) equals household's savings (the right-hand side). Household's savings, in turn, are equal to the difference between household's total income (the sum of interest income, *rA*, and human capital income, wH_E) and household's consumption (C). Given Eq. (11), the law of motion of assets in per-capita terms ($a_t \equiv A_t / L_t$) reads as:

$$\dot{a}_t = (r_t - g_L) a_t + (u_t h_t) w_t - c_t, \qquad a(0) > 0 \quad (11')$$

Where $c_t \equiv C_t / L_t$ and $h_t \equiv H_t / L_t$ denote consumption and human capital per capita, respectively. The term $-g_L$ in (11') captures the *dilution* occurring in per-capita asset holdings accumulation due to population growth and reflects the 'cost' of bringing the amount of per-capita assets of the newcomers up to the average level of the existing

[17] As already mentioned, at equilibrium all human capital employed in production activities (H_E) is rewarded according to the same wage, *w*.

population. This formulation implies that *ceteris paribus*, population growth tends to slow down the investment in assets of the average individual in the population.

At each time $t \geq 0$, the household uses the remaining fraction $(1-u_t)$ of H_t in educational assignments. Human capital per capita accumulates as:

$$\dot{h}_t = [\sigma(1-u_t) - \xi g_L] h_t, \qquad \sigma > 0, \qquad \xi \geq 0, \qquad h(0) > 0 \tag{12}$$

where σ and ξ are parameters. The first measures the productivity of education, whereas the second reveals the strength, if any, of the negative effect of population growth on per-capita human capital investment. When $\xi = 1$, Eq. (12) shows the existence of a linear, one-to-one, dilution effect of population growth on per capita human capital accumulation (similar to that of Eq. 11'). A possible explanation of such effect would be that since newborns enter the world uneducated they naturally reduce, *ceteris paribus* and at a given point in time, the existing stock of human capital per capita. Indeed, this effect is not presented in the original Lucas' (1988, Eq. 13, p. 19) formulation. Lucas' assumption (newborns enter the work-force endowed with a skill-level proportional to the level already attained by older members of the family, so population growth *per se* does not reduce the current skill level of the representative worker) is based on the *social nature* of human capital accumulation, which has no counterpart in the accumulation of physical capital and of any other form of tangible assets. When $\xi = 0$ Eq. (12) is able to recover this idea (Lucas, 1988, p.19). A value of $\xi \in (0;1)$ represents an intermediate case between the previous two.

With a *Constant Intertemporal Elasticity of Substitution (CIES)* instantaneous felicity function, the problem faced by a representative infinitely-lived family seeking to maximize the utility (attained from consumption) of its members is:

$$\underset{\{c_t, u_t, a_t, h_t\}_{t=0}^{\infty}}{Max} U \equiv \int_0^{\infty} \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-(\rho - g_L)t} dt, \qquad \rho > g_L \geq 0, \qquad \theta > 0 \tag{13}$$

s.t.: $\dot{a}_t = (r_t - g_L) a_t + (u_t h_t) w_t - c_t, \qquad u_t \in [0;1], \ \forall t \geq 0; \qquad \dot{L}_t / L_t \equiv g_L > 0$

$\dot{h}_t = [\sigma(1-u_t) - \xi g_L] h_t, \qquad \sigma > 0; \qquad \xi \geq 0$

$a(0) > 0, \qquad h(0) > 0$ given.

In Eq. (13) population at time $0, L(0)$, has been normalized to one. The household chooses the optimal path of per-capita consumption (c) and the share of human capital to be devoted to production activities (u). The other symbols have the following meaning: U and $\left(\frac{c_t^{1-\theta}-1}{1-\theta}\right)$ are the household's intertemporal utility function and the instantaneous felicity function of each member of the dynasty. We indicate by $\rho > 0$ the pure rate of time-preference and by $1/\theta > 0$ the constant intertemporal elasticity of substitution in consumption. The hypothesis $\rho > g_L$ ensures that U is bounded away from infinity if c remains constant over time.

3. General Equilibrium and BGP Analysis

Since human capital is fully employed and there exists perfect mobility of this factor-input across sectors, the following equalities must hold at equilibrium:

$$H_E \equiv u_t H_t = H_{Yt} + H_{It} + H_{nt} \tag{14}$$

$$w_{It} = w_{nt} \tag{15}$$

$$w_{It} = w_{Yt} \tag{16}$$

Eq. (14) says that aggregate labor demand (the right-hand side) should equal the fraction of the available human capital stock employed in production and R&D activities (the left-hand side). Eqs. (15) and (16) together state that, for the previous equality to be checked, wages do adjust in such a way that the salary earned by one unit of skilled labor in the intermediate sector should be equal to the salary earned by the same unit of skilled labor if employed in research or in the production of final goods. Moreover, since household's asset holdings must equalize the aggregate value of firms, the following equation should also be met in equilibrium:

$$A_t = n_t V_{nt} \tag{17}$$

Where is given by Eq. (10) and satisfies the usual *no-arbitrage condition*:

$$\dot{V}_{nt} = r_t V_{nt} - \pi_t$$

In the model, the *i-th* idea allows the *i-th* intermediate firm to produce the *i-th* variety of durables. This explains why in Eq. (17) total assets (A) equal the number of profit-making intermediate firms (n) times the market value (V_n) of each of them (equal, in turn, to the price of the corresponding idea). On the other hand, the *no-arbitrage condition* suggests that the return on the value of the *i-th* intermediate firm ($r_t V_{nt}$) must be equal to the sum of the instantaneous monopoly profit accruing to the *i-th* intermediate input producer (π_t) and the capital gain/loss matured on V_{nt} during the time interval *dt*, $\dot{V}_{nt}$. We are now able to move to a formal definition and characterization of the model's BGP equilibrium.

DEFINITION: BGP EQUILIBRIUM

A BGP Equilibrium in this economy is a long-run equilibrium path along which:

(i) *All variables depending on time grow at constant (possibly positive) exponential rates;*

(ii) *The sectoral shares of human capital employment ($s_j = H_j / H$, j = Y, I, n) are constant.*

From this definition, Proposition 1 follows:

PROPOSITION 1

Along the BGP, the fraction of the aggregate stock of human capital employed in production activities is constant (that is, $u_t = u, \forall t \geq 0$). ■

Proof: Immediate from Eq. (12), and the fact that the growth rate of all time-dependant variables is constant along the BGP equilibrium. The following results do hold along the BGP (mathematical derivation can be found in the *Appendices, Appendix A*):

$$\frac{\dot{H}_{Yt}}{H_{Yt}} = \frac{\dot{H}_{It}}{H_{It}} = \frac{\dot{H}_{nt}}{H_{nt}} = \frac{\dot{H}_t}{H_t} \equiv \gamma_H = \frac{\left[(\sigma-\rho)-(\xi-1-\theta)g_L\right]}{\Upsilon R(\theta-1)+\theta} \tag{18}$$

$$\frac{\dot{n}_t}{n_t} \equiv \gamma_n = \frac{\Upsilon\left[(\sigma-\rho)-(\xi-1-\theta)g_L\right]}{\Upsilon R(\theta-1)+\theta} = \Upsilon\gamma_H \tag{19}$$

$$r=\frac{\sigma\theta+\Upsilon R\left(\sigma\theta-\rho\right)-\left\{\theta\left[\xi\left(1+\Upsilon R\right)-\left(1+2\Upsilon R\right)\right]\right\}g_L}{\Upsilon R\left(\theta-1\right)+\theta} \tag{20}$$

$$\gamma_a\equiv\frac{\dot{a}_t}{a_t}=\gamma_c\equiv\frac{\dot{c}_t}{c_t}=\frac{1}{\theta}\left(r-\rho\right) \tag{21}$$

$$\gamma_y\equiv\frac{\dot{y}_t}{y_t}=\gamma_a=\gamma_c=\frac{(1+\Upsilon R)\left(\sigma-\rho\right)-\left[\Upsilon R\left(\xi-2\right)+\left(\xi-1\right)\right]g_L}{\Upsilon R\left(\theta-1\right)+\theta} \tag{22}$$

$$u=1-\frac{\left(\sigma-\rho\right)-\left[\Upsilon R\left(1-\xi\right)\left(\theta-1\right)+\xi\left(1-\theta\right)-1\right]g_L}{\sigma\left[\Upsilon R\left(\theta-1\right)+\theta\right]} \tag{23}$$

$$s_n=\frac{Z\left(1-Z\right)\gamma_n}{\left[1-Z+Z^2\right]\left[r+\left(1-R\right)\gamma_n-\gamma_H\right]+Z\left(1-Z\right)\gamma_n}u \tag{24}$$

$$s_I=\left[\frac{Z^2}{1-Z+Z^2}\right]\left(u-s_n\right) \tag{25}$$

$$s_Y=\left[\frac{1-Z}{1-Z+Z^2}\right]\left(u-s_n\right) \tag{26}$$

$$\frac{H_t^{\mu-\Phi}}{n^{1-\eta}}=\frac{\chi}{s_n^{\mu}}\gamma_n \tag{27}$$

$$R\equiv\bar{\alpha}+1-Z \qquad \Upsilon\equiv\frac{\mu-\Phi}{1-\eta}$$

Eq. (18) gives the BGP-equilibrium growth rate of the economy's human capital stock (H), and of the human capital employment in final output, intermediate and research sectors. Eq. (19) gives the BGP-equilibrium growth rate of the economy's stock of knowledge (n). Eq. (20) provides the equilibrium real rate of return on asset holdings (r). According to Eqs. (21) and (22) per capita consumption (c), per capita asset holdings (a) and per capita real income ($y\equiv Y/L$) all grow at the same constant rate. Eq. (23) gives the allocation of the available stock of human capital between production and educational activities along the BGP. The equilibrium shares of the existing human capital stock devoted to production of ideas (s_n), production of intermediates (s_I) and production of consumption goods (s_Y) are reported in Eqs. (24), (25) and (26), respectively. Finally, Eq. (27) expresses the ratio of (some function of) the two state-variables in terms of the growth rate of the number of ideas (γ_n), and the share of the available human capital stock devoted to R&D-activity (s_n). It is evident from this equation that the restriction $\mu \neq \Phi$ prevents, *ceteris paribus*, γ_n to be independent of H_t.

Assumption A introduces constraints on the (relationship among) the feasible values of the model's parameters.

ASSUMPTION A. *Assume*

(*i*) $\Upsilon > 0$

(*ii*) $\sigma > 0$

(*iii*) $\theta > \text{Max}\left\{ 0;\ \frac{\Upsilon R}{1+\Upsilon R};\ \frac{\Upsilon R \rho}{\sigma(1+\Upsilon R) - \left[\xi(1+\Upsilon R) - (1+2\Upsilon R)\right] g_L};\ \right\}$

(*iv*) $(\sigma - \rho) > \text{Max}\left\{ (\xi - 1 - \theta) g_L; \frac{\left[\Upsilon R(\xi - 2) + (\xi - 1)\right] g_L}{1+\Upsilon R}; \left[\Upsilon R(1-\xi)(\theta - 1) + \xi(1-\theta) - 1\right] g_L \right\}$

The assumption $\Upsilon > 0$ comes directly from the assumptions $\mu > \Phi$ and $\eta < 1$. This also coincides with Jones (2005, p. 1074, Chap. 16, Eq.16).

If Assumption A is satisfied, then:

PROPOSITION 2

- γ_H *and* γ_n *are positive;*
- γ_y *is positive;*
- r *is positive;*
- $0 < u < 1$*;*
- $r > \gamma_H - (1-R)\gamma_n$. *Ceteris paribus, this condition allows* V_{nt} *to be positive at any time* $t \geq 0$ *along the BGP;*
- *The two transversality conditions:* $\lim_{t \to +\infty} \lambda_{at} a_t = 0$ *and* $\lim_{t \to +\infty} \lambda_{ht} h_t = 0$ *are simultaneously checked along the BGP.*

Proof: When (*i*) and (*iii*) in *Assumption A* are met, then the denominator of Eqs. (18), (19), (20) and (22) is positive, *i.e.* $\left[\Upsilon R(\theta-1)+\theta\right]>0$. Given this, and the fact that in the model $g_L>0$ and $\sigma>0$, we conclude that: (*i*)-(*ii*)-(*iii*) ensure $r>\gamma_H-(1-R)\gamma_n$, $r>0$, $u>0$, and the respect of the two transversality conditions; (*i*)-(*iii*)-(*iv*) ensure $\gamma_y>0, \gamma_H>0$ and $\gamma_n>0$. Finally, (*i*)-(*ii*)-(*iii*)-(*iv*) ensure $u<1$. ■

4. Population Growth and Economic Growth

The following theorem analyzes the interaction between population and economic growth rates in this economy.

THEOREM

Assume that parameter-restrictions (i) and (iii) of Assumption A are checked for $\xi\geq 0$ *and* $\Upsilon>0$. *Then;*

- *When the dilution effect of population growth on human capital investment is greater than one* $(\xi>1)$ *, the correlation between population and economic growth rates is ambiguous, i.e.* $\frac{\partial\gamma_y}{\partial g_L}>0, \quad \frac{\partial\gamma_y}{\partial g_L}<0, \quad \frac{\partial\gamma_y}{\partial g_L}=0.$
- *When* $0\leq\xi\leq 1$*, there exists an unambiguously positive correlation between population growth and economic growth, i.e.* $\frac{\partial\gamma_y}{\partial g_L}>0.$

Results are summarized in ***Table 1*** (mathematical derivation can be found in *Appendix B*).

When $\Upsilon R>0; \quad \xi\geq 0$	$\partial\gamma_y/\partial g_L$
$\xi=0$	$\frac{\partial\gamma_y}{\partial g_L}>0$
$\xi\leq 1$	$\frac{\partial\gamma_y}{\partial g_L}>0$
$1<\xi<\frac{1+2\Upsilon R}{1+\Upsilon R}$	$\frac{\partial\gamma_y}{\partial g_L}>0$

$\xi = \frac{1+2\Upsilon R}{1+\Upsilon R}$	$\frac{\partial \gamma_y}{\partial g_L} = 0$
$\xi > \frac{1+2\Upsilon R}{1+\Upsilon R}$	$\frac{\partial \gamma_y}{\partial g_L} < 0$

The intuition behind the results of Theorem is as follows. By using again the BGP-equilibrium relation:

$$\gamma_y = \gamma_H + R\gamma_n - g_L \tag{28}$$

One can observe that

$$\begin{aligned} \frac{\partial \gamma_y}{\partial g_L} &= \left(\frac{\partial \gamma_H}{\partial g_L} + R \frac{\partial \gamma_n}{\partial g_L} \right) - 1 \\ &= \left(\frac{1}{\Upsilon} \frac{\partial \gamma_n}{\partial g_L} + R \frac{\partial \gamma_n}{\partial g_L} \right) - 1 \end{aligned}$$

$$\frac{\partial \gamma_y}{\partial g_L} = \underbrace{\left(\frac{1+\Upsilon R}{\Upsilon} \right)}_{\substack{under\ Assumption\ A \\ > 0}} \underbrace{\frac{\partial \gamma_n}{\partial g_L}}_{\substack{ideas \\ effect}} - \underbrace{1}_{\substack{dilution \\ effect}} \tag{29}$$

According to Eq. (29), the impact of population growth on real per-capita income depends on the relative contribution of two distinct effects:

- The direct dilution effect: This effect is always negative since when newborns enter the world they reduce the existing per-capita stock of any reproducible factor–input. So, in order to equip every single member of the growing population with a given (per capita) amount of such input, some resources need to be explicitly devoted to this aim, which slows productivity growth down.
- The indirect ideas effect: This effect describes the impact that at a certain point in time an exogenous change of population size (due to a change of g_L) may have on the economy's growth rate of ideas (γ_n), and hence on *n*: "…*More people means more*

Isaac Newtons and therefore more ideas…" (Jones, 2003, p. 505). Unlike the previous one, this effect is always positive as long as $1+\theta-\xi>0$.[18]

According to *Table 1,* when $\xi\leq 1$ the impact of population growth on economic growth is always positive. However; when $\xi>1$, a threshold level of ξ has emerged. In particularly, when ξ is below the threshold, the ideas effect of population growth is positive and greater than the dilution effect. As a result of this, the impact of population growth on economic growth continues to be positive under a certain threshold level of the dilution effect. When ξ equals to the threshold, the dilution effect neutralizes the ideas effect which is still positive, and thus; the impact of population growth on per-capita income growth is neutral. And lastly when ξ is above the threshold, the dilution effect of population growth is quite strong that results a negative impact on economic growth. Note that If ξ is sufficiently high $(1+\theta-\xi<0)$; the ideas effect of population growth can also turn to negative.

5. Conclusion

Given the last population facts of the UN and all the results of the literature, it seems that population growth will hold the questions about its effects on economic prosperity. This paper, therefore, attempts to understand the sign of the relationship between population growth and per-capita income growth. While doing this, the paper provides an alternative but complementary theoretical framework explaining the impacts of population growth on economic growth under endogenous technological change and human capital investment by the economic agents along the BGP equilibrium.

The present paper has showed that an increase in the population growth (regardless of the source of demographic change such as fertility, mortality or aging) rate yields an ambiguous (positive, negative or else neutral) impact on the growth rate of per-capita income. This ambiguity comes from the relative contribution of two distinct effects of population growth: *The direct dilution effect* and *the indirect ideas effect.* The direct dilution effect mainly rests on the modification of Lucas (1988) formulation of human capital accumulation. The present paper has showed that (i) The dilution effect has a central role in explaining the ambiguous impact of population growth on economic growth. (ii) There exists a threshold level of the

[18] The ideas effect is given by $\frac{\partial\gamma_n}{\partial g_L}$. From Eq.(19), $\frac{\partial\gamma_n}{\partial g_L}=\frac{\Upsilon(1+\theta-\xi)}{\Upsilon R(\theta-1)+\theta}$, the ideas effect is positive as long as $\xi<1+\theta$.

dilution effect that the correlation between population and economic growth rates may be either positive or negative or neutral according to this threshold. (iii) When the dilution effect is sufficiently low, $0 \le \xi \le 1$, an unambiguously positive correlation between population growth and economic growth is obtained. Another result of the paper is that more population growth generates an indirect ideas effect (ambiguous) on the rate of innovation and economic growth.

Lastly, we believe that these findings shed new lights on the determinants of the ambiguous impacts of population growth on economic growth, and will help to introduce more realistic models to the literature of modern growth theory with human capital accumulation. We underline that further empirical research (e.g. panel data analyses) would be a good extension of this paper to verify the theoretical results. We leave the formal empirical investigation of this theory to future research.

References

Aghion, P., Howitt, P., (1992). "A Model of Growth through Creative Destruction", *Econometrica*, 60(2): pp. 323-351.

Ahituv, A., (2001). "Be Fruitful or Multiply: On the Interplay between Fertility and Economic Development", *Journal of Population Economics*, 14(1): pp. 51-71.

Barro, R.J., Lee J.W., (2013). "A New Data Set of Educational Attainment in the World, 1950-2010", *Journal of Development Economics*, 104, pp. 184-198.

Barro, R.J., Sala-i Martin, X., (2004). "*Economic Growth*" second ed. MIT Press, Cambridge, MA, Chapter 6, pp: 285-313.

Benassy, J.P., (1998). "Is There Always too Little Research in Endogenous Growth with Expanding Product Variety?", *European Economic Review*, 42(1), pp. 61-69.

Boikos, S., Bucci, A., Stengos, T., (2013). "Non-Monotonicity of Fertility in Human Capital Accumulation and Economic Growth", *Journal of Macroeconomics*, 38: pp. 52-56.

Boserup, E., (1981). "*Population and Technical Change: A Study of Long-Term Trends*", Chicago, University of Chicago Press.

Boucekkine,R., de la Croix, D., Licandro, O., (2002). "Vintage Human Capital, Demographic Trends, and Economic Growth", *Journal of Economic Theory*, 104(2): pp. 340-375.

Bucci, A., (2008). "Population Growth in a Model of Economic Growth with Human Capital Accumulation and Horizontal R&D", *Journal of Macroeconomics*, 30(3): pp. 1124-1147.

Bucci, A., (2013). "Returns to Specialization, Competition, Population and Growth", *Journal of Economic Dynamics and Control*, 37: pp. 2023-2040.

Coale, A.J, and Hoover, E., (1958). "*Population Growth and Economic Development in Low-Income Countries*", Princeton: Princeton University Press.

Dalgaard, C.J., Kreiner, A.C., (2001). "Is Declining Productivity Inevitable?", *Journal of Economic Growth*, 6(3), pp. 187-203.

Deverux, M.B., Head, A.C., and Lapham, B.J., (1996a). “Monopolistic Competition, Increasing Returns and the Effects of Government Spending”, *Journal of Money, Credit and Banking*, 28(2): pp. 233-254.

Deverux, M.B., Head, A.C., and Lapham, B.J., (1996b). “Aggregate Fluctuations with Increasing Returns to Specialization”, *Journal of Economic Dynamics and Control*, 20(4), pp. 627-656.

Deverux, M.B., Head, A.C., and Lapham, B.J., (2000). “Government Spending and Welfare with Returns to Specialization”, *Scandinavian Journal of Economics*, 102(4): pp. 547-561.

Dinopoulos, E., and Segerstrom P., (1999). “A Schumpeterian Model of Protection and Relative Wages”, *American Economic Review*, 89(3), pp. 450-472.

Ehrlich, P.R., (1968). “*The Population Bomb*” New York: Ballentine.

Ethier, W.J., (1982). “National and International Returns to Scale in the Modern Theory of International Trade”, *American Economic Review*, 72(3), pp. 389-405.

Grossman, G.M., and Helpman, E., (1991). “*Innovation and Growth in the Global Economy*”, Cambridge, MA: MIT Press.

Herzer, D., Strulik, H., Volmer, S., (2012). “The Long-run Determinants of Fertility: One Century of Demographic Change, 1900-1999”, *Journal of Economic Growth*, 17(4): pp. 357-385.

Jones, C.I., (1995). “R&D-Based Models of Economic Growth”, *Journal of Political Economy*, 103(4), pp. 759-784.

Jones, C.I., (2003). “Population and Ideas: A Theory of Endogenous Growth” in Aghion, P., Frydman, R., Stiglitz, J., Woodford, M., (eds.) *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press, Princeton, NJ, pp: 498-521.

Jones, C.I., (2005). “Growth and Ideas” in Aghion P., and Durlauf, S.N., (eds.), *Handbook of Economic Growth*, Amsterdam: Elsevier-North Holland, Chap.16: pp. 1063-1111.

Kelly, A.C., (1988). “Economic Consequences of Population Change in the Third World”, *Journal of Economic Literature*, 26(4), pp: 1685-1728.

Kelly, A.C., Schmidt, R.M., (1995). "Aggregate Population and Economic Growth Correlations: The Role of the Components of Demographic Change", *Demography*, 32(4), pp. 543-555.

Kremer, M., (1993). "Population Growth and Technological Change: One Million B.C. to 1990", *Quarterly Journal of Economics*, 108(3), pp. 681-716.

Kuznets, S., (1967). "Population and Economic Growth", *Proceedings of the American Philosophical Society*, 111(3), pp. 170-193.

Li, H., Zhang, J., (2007). "Do high birth rates hamper economic growth?" *Review of Economics and Statistics*, 89(1), pp. 110-117.

Lucas, R.E., (1988). "On the Mechanics of Economic Development", *Journal of Monetary Economics*, 22(1), pp. 3-42.

Malthus, T.R., (1798). "*An Essay on the Principle of Population: An Essay on the Principle of Population: As It Affects the Future Improvement of Society with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers*." London, Electronic Scholarly Publishing Project, (1998 Edition).

Mankiw, N.G., Romer, D., Weil, D.N., (1992). "A contribution to empirics of economic growth", *Quarterly Journal of Economics*, 107(2), pp. 407-437.

Mierau, J.O., Turnovsky, S.J., (2014). "Demography, Growth and Inequality", *Economic Theory*, 55(1), pp. 29-68.

Prettner, K., Prskawetz, A., (2010). "Demographic Change in Models of Endogenous Economic Growth: A Survey", *Central European Journal of Operations Research*, 18(4), pp. 593-608.

Prettner, K., (2013). "Population Aging and Endogenous Economic Growth", *Journal of Population Economics*, 26(2), pp. 811-834.

Prettner, K., Bloom, D.E., Strulik, H., (2013). "Declining Fertility and Economic Well-Being: Do Education and Health Ride to the Rescue?", *Labour Economics*, 22, pp. 70-79.

Prettner, K., (2014). "The Non-Monotonous Impact of Population Growth on Economic Prosperity", *Economics Letters*, 124(1), pp: 93-95.

Prettner, K., Canning D., (2014). "Increasing Life Expectancy and Optimal Retirement in General Equilibrium", *Economic Theory*, 56(1), pp. 191-217.

Romer, P.M., (1987). "Growth Based on Increasing Returns Due to Specialization", *American Economic Review*, 77(2), pp. 56-62.

Romer, P.M., (1990). "Endogenous Technological Change", *Journal of Political Economy*, 98(5), pp. S71-S102.

Romero M.S., (2013). "Role of Demography on Per Capita Output Growth and Saving Rates", *Journal of Population Economics*, 26(2), pp. 811-834.

Simon, J., (1981). "*The Ultimate Resource*", Princeton: Princeton University Press.

Srinivasan, T.N., (1988). "Population Growth and Economic Development", *Journal of Policy Modelling*, 10(1), pp. 7-28.

Strulik, H., (2005). "The Role of Human Capital and Population Growth in R&D-based Models of Economic Growth", *Review of International Economics*, 31(1), pp. 129-145.

Strulik, H., Prettner, K., Prskawetz, A., (2013). "The Past and Future of Knowledge-Based Growth", *Journal of Economic Growth*, 18(4), pp. 411-437.

United Nations (2013a). "World Population Prospects: The 2012 Revision, Key Findings and Advance Tables", Department of Economic and Social Affairs, Population Division, *Working Paper*, No: ESA/P/WP.227, pp. 1-5.

United Nations (2013b), "World Population Policies 2013", Department of Economic and Social Affairs, Population Division, *Report*, No: ST/ESA/SER.A/341, p. 1.

Uzawa, H., (1965), Optimum Technical Change in Aggregative Model of Economic Growth, *International Economic Review*, 6(1), pp. 18-31.

APPENDICES

APPENDIX A: EQS. (18) – (27)

The *Hamiltonian function* $\left(J_t\right)$ related to the intertemporal problem (13) in the body-text is:

$$J_t = \left(\frac{c_t^{1-\theta}-1}{1-\theta}\right)e^{-(\rho-g_L)t} + \lambda_{at}\left[\left(r_t - g_L\right)a_t + \left(u_t h_t\right)w_t - c_t\right] + \lambda_{ht}\left[\sigma\left(1-u_t\right) - \xi g_L\right]h_t,$$

Where λ_{at} and λ_{ht} are the *co-state variables* associated, respectively, to the *state variables* a_t and h_t. The necessary *FOC*s are:

(A1) $$\frac{\partial J_t}{\partial c_t} = 0 \quad \Leftrightarrow \quad \frac{e^{-(\rho-g_L)t}}{c_t^{\theta}} = \lambda_{at}$$

(A2) $$\frac{\partial J_t}{\partial u_t} = 0 \quad \Leftrightarrow \quad \lambda_{at} = \frac{\sigma}{w_t}\lambda_{ht}$$

(A3) $$\frac{\partial J_t}{\partial a_t} = -\dot{\lambda}_{at} \quad \Leftrightarrow \quad \lambda_{at}\left(r_t - g_L\right) = -\dot{\lambda}_{at}$$

(A4) $$\frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_{ht} \quad \Leftrightarrow \quad \lambda_{at}u_t w_t + \lambda_{ht}\left[\sigma\left(1-u_t\right) - \xi g_L\right] = -\dot{\lambda}_{ht}$$

along with the two transversality conditions:

$$\lim_{t\to+\infty}\lambda_{at}a_t = 0, \qquad \lim_{t\to+\infty}\lambda_{ht}h_t = 0,$$

and the initial conditions:

$$a(0) > 0, \qquad h(0) > 0.$$

Combining (A2) and (A4) yields:

(A5) $$\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = -\left(\sigma - \xi g_L\right)$$

Eqs. (A3) and (A2) imply, respectively:

(A6) $$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -\left(r_t - g_L\right)$$

(A7) $$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = \frac{\dot{\lambda}_{ht}}{\lambda_{ht}} - \frac{\dot{w}_t}{w_t}$$

The combination of (A5), (A6) and (A7) leads to:

(A8) $$r_t = \left(1-\xi\right) g_L + \sigma + \frac{\dot{w}_t}{w_t}$$

Since human capital is perfectly mobile across sectors, at equilibrium it will be rewarded according to the same wage: $w_{Yt} = w_{It} = w_{nt} \equiv w_t$. Moreover, along the BGP this common wage would grow at a constant exponential rate, implying that $\frac{\dot{w}_{Yt}}{w_{Yt}} = \frac{\dot{w}_{It}}{w_{It}} = \frac{\dot{w}_{nt}}{w_{nt}} \equiv \frac{\dot{w}_t}{w_t}$ is constant. Accordingly, in the BGP equilibrium the real rate of return on asset holdings, *r* will be constant (Eq. A8). With *r* constant, and making use of Eqs. (6) and (10) in the main text, we find that along the BGP:

(A9) $$V_{nt} = Z\left(1-Z\right)\left(\frac{H_{Yt}}{n_t}\right)^{1-Z}\left(\frac{H_{It}}{n_t}\right)^{Z}\frac{n_t^R}{\left[r+\left(1-R\right)\gamma_n - \gamma_H\right]}, \quad R \equiv \bar{\alpha} + 1 - Z, \quad \frac{\dot{n}_t}{n_t} \equiv \gamma_n, \quad \frac{\dot{H}_t}{H_t} \equiv \gamma_H$$

For any $0 < Z < 1, \quad H_Y > 0, \quad H_I > 0, \quad n > 0,$ and $R > 0, \quad V_{nt}$ is positive provided that:

(A9’) $$r > \gamma_H - \left(1-R\right)\gamma_n$$

Given V_{nt}, from Eq. (9’) in the main text:

(A10) $$w_{nt} = \frac{Z}{\chi}\left(1-Z\right) s_n^{\mu-1} H_t^{\mu-1-\Phi} n_t^{\eta}\left(\frac{H_{Yt}}{n_t}\right)^{1-Z}\left(\frac{H_{It}}{n_t}\right)^{Z}\frac{n_t^R}{\left[r+\left(1-R\right)\gamma_n - \gamma_H\right]}$$

where $s_n \equiv H_{nt}/H_t$ is constant along the BGP. We can now use Eqs. (5), (2) and (4’) in the main text, obtaining:

(A11) $$w_{It} = Z^2\left(\frac{H_{Yt}}{n_t}\right)^{1-Z}\left(\frac{H_{It}}{n_t}\right)^{Z-1} n_t^R$$

From Eq. (15) in the main text, by equalizing (A11) and (A10) in this appendix one gets:

$$\text{(A12)} \qquad s_I \equiv \frac{H_{It}}{H_t} = \frac{Z\chi}{(1-Z)} \frac{\left[r+(1-R)\gamma_n - \gamma_H\right]}{s_n^{\mu-1}} \frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Combining Eqs. (1) and (4') in the text:

$$\text{(A13)} \qquad w_{Yt} \equiv \frac{\partial Y_t}{\partial H_{Yt}} = (1-Z)\left(\frac{H_{Yt}}{n_t}\right)^{-Z} \left(\frac{H_{It}}{n_t}\right)^{Z} n_t^R$$

From (16) in the main text and (A12) above, equalization of Eqs. (A11) and (A13) in this appendix delivers:

$$\text{(A14)} \qquad s_Y \equiv \frac{H_{Yt}}{H_t} = \left(\frac{1-Z}{Z^2}\right) s_I = \frac{\chi}{Z} \frac{\left[r+(1-R)\gamma_n - \gamma_H\right]}{s_n^{\mu-1}} \frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Along the BGP all variables depending on time grow at constant rates and the sector shares of human capital employment are also constant. Therefore, from Eq. (8) in the main text:

$$\text{(A15)} \qquad \frac{\dot{n}_t}{n_t} \equiv \gamma_n = \left(\frac{\mu-\Phi}{1-\eta}\right)\gamma_H, \quad \gamma_H \equiv \frac{\dot{H}_t}{H_t}$$

If $\mu-\Phi=1-\eta$ we have a very special case of the model in which human and technological capital grow at the same rate $\gamma_n = \gamma_H \equiv \gamma$ along the BGP. We rule out this possibility and analyze the most general possible case: $\mu \neq \Phi \neq \Phi+1-\eta$.

Using Eqs. (A10), (A11), (A13) and (A15) we see that along the BGP wages grow at a common and constant rate:

$$\text{(A15')} \qquad \frac{\dot{w}_{nt}}{w_{nt}} = \frac{\dot{w}_{It}}{w_{It}} = \frac{\dot{w}_{Yt}}{w_{Yt}} \equiv \frac{\dot{w}_t}{w_t} = R\gamma_n$$

Combining Eqs. (A1) and (A6), the usual Euler equation follows:

$$\text{(A16)} \qquad \frac{\dot{c}_t}{c_t} \equiv \frac{1}{\theta}(r-\rho), \quad c \equiv \frac{C}{L}$$

From (17) in the text and (A9) in this appendix we conclude that along the BGP:

(A17) $\frac{\dot{a}_t}{a_t} \equiv \gamma_a = \gamma_H + R\gamma_n - g_L, \qquad a_t \equiv A_t/L_t, \quad g_L \equiv \dot{L}_t/L_t$

Merging (11') in the main text and (A6) in this appendix yields:

(A18) $\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -\gamma_a + u_t \frac{h_t w_t}{a_t} - \frac{c_t}{a_t}$, where $u_t = u, \forall t \geq 0$ along the BGP.

Instead, from the combination of (12) in the text and (A5) in this appendix we get:

(A19) $\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = -\gamma_h - \sigma u, \qquad h_t \equiv H_t/L_t, \quad \gamma_h \equiv \dot{h}_t/h_t$

Eqs. (A7), (A15'), (A17), (A18) and (A19) together lead to:

(A20) $\frac{c_t}{a_t} = u\left[\frac{h_t w_t}{a_t} + \sigma\right]$, where $\gamma_h = \gamma_H - g_L$ has been used.

Using (12) in the main text, Eqs. (A15'), (A17) and (A20) and the fact that $\gamma_H = \gamma_h + g_L = \sigma(1-u) + (1-\xi) g_L$ one obtains:

(A20') $\frac{c_t}{a_t} = u\left[\frac{h(0)\, w(0)}{a(0)} + \sigma\right]$,

where $h(0), w(0)$ and $a(0)$ are the initial values (*i.e.*, at $t = 0$) of h_t, w_t and a_t, respectively. With u constant, the just-mentioned initial values given, and $\sigma > 0$ the last equation implies:

(A21) $\gamma_c = \gamma_a$

This means that along the BGP a_t and c_t grow at the same rate. Using (A21) and equating (A16) and (A17) it is possible to get:

(A22) $r = \rho + \theta\left(\gamma_H + R\gamma_n - g_L\right)$

Next, by equalizing (A22) to (A8), and using (A15'):

(A23) $\gamma_n \equiv \frac{\dot{n}_t}{n_t} = \frac{\left[(\sigma-\rho) - (\xi - 1 - \theta) g_L - \theta\gamma_H\right]}{R(\theta - 1)}$

Equating (A23) to (A15), and solving for γ_H , we finally obtain:

(A23') $$\gamma_H \equiv \frac{\dot{H}_t}{H_t} = \frac{\left[(\sigma-\rho)-(\xi-1-\theta)g_L\right]}{\Upsilon R(\theta-1)+\theta}, \qquad \Upsilon \equiv \left[\frac{\mu-\Phi}{1-\eta}\right]$$

Given, it is possible to re-cast γ_n as:

(A23'') $$\gamma_n \equiv \frac{\dot{n}_t}{n_t} = \frac{\Upsilon\left[(\sigma-\rho)-(\xi-1-\theta)g_L\right]}{\Upsilon R(\theta-1)+\theta} = \Upsilon\gamma_H$$

Eqs. (A23') and (A23'') confirm that $\gamma_H = \gamma_n = \frac{\left[(\sigma-\rho)-(\xi-1-\theta)g_L\right]}{R(\theta-1)+\theta}$ in the special case $\Upsilon \equiv \left[\frac{\mu-\Phi}{1-\eta}\right] = 1$. The BGP equilibrium- value of *r* is obtained by combining (A22), (A23') and (A23''):

(A22') $$r = \frac{\sigma\theta + \Upsilon R(\sigma\theta-\rho) - \left\{\theta\left[\xi(1+\Upsilon R)-(1+2\Upsilon R)\right]\right\}g_L}{\Upsilon R(\theta-1)+\theta}$$

Eqs. (A21) and (A16) together imply:

(A21') $$\gamma_a \equiv \frac{\dot{a}_t}{a_t} = \gamma_c \equiv \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r-\rho),$$ where *r* is given by Eq. (A22').

After using Eq. (12) in the main text, the definition of $h \equiv H/L$ and the fact that $\dot{L}/L \equiv g_L$, we conclude:

(A24) $$\gamma_H \equiv \frac{\dot{H}}{H} = \sigma(1-u) + (1-\xi)g_L$$

Equalization of (A23') and (A24) allows obtaining the BGP equilibrium value of *u*:

(A25) $$u = 1 - \frac{(\sigma-\rho)-\left[\Upsilon R(1-\xi)(\theta-1)+\xi(1-\theta)-1\right]g_L}{\sigma\left[\Upsilon R(\theta-1)+\theta\right]}$$

From (1) in the text, (A22'), (A23') and (A23'') in this appendix, the hypothesis of symmetry (Eq. 4' in the text), the definitions of $y(\equiv Y/L)$, $R \equiv \bar{\alpha} + 1 - Z$ and $\dot{L}/L \equiv g_L$, we obtain the growth rate of real per-capita output along the BGP:

(A26) $$\gamma_y \equiv \frac{\dot{y}_t}{y_t} = \frac{(1+\Upsilon R)(\sigma-\rho) - \left[\Upsilon R(\xi-2) + (\xi-1)\right] g_L}{\Upsilon R(\theta-1)+\theta} = \gamma_a = \gamma_c = \frac{1}{\theta}(r-\rho)$$

We now compute the BGP-equilibrium values of s_n, s_I and s_Y. Eq. (14) in the main text suggests: $u = s_Y + s_I + s_n$

From (A14) in this appendix we use $s_Y = \left(\frac{1-Z}{Z^2}\right) s_I$ into the expression above and obtain:

(A27) $$s_I = \left[\frac{Z^2}{1-Z+Z^2}\right](u - s_n),$$ where u is given by (A25).

Hence:

(A28) $$s_Y = \left[\frac{1-Z}{1-Z+Z^2}\right](u - s_n)$$

According to (A14), however, it is also true that:

$$s_Y \equiv \frac{H_{Yt}}{H_t} = \frac{\chi}{Z} \frac{\left[r + (1-R)\gamma_n - \gamma_H\right]}{s_n^{\mu-1}} \frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Equating this expression to (A28) yields:

(A29) $$\frac{H_t^{\mu-\Phi}}{n_t^{1-\eta}} = \frac{\chi\left[1-Z+Z^2\right]}{Z(1-Z)} \frac{\left[r + (1-R)\gamma_n - \gamma_H\right]}{s_n^{\mu-1}(u - s_n)}$$

From Eq. (8) in the body-text:

(A30) $$\frac{H_t^{\mu-\Phi}}{n^{1-\eta}} = \frac{\chi}{s_n^{\mu}} \gamma_n$$

Equalization of (A29) and (A30) leads to:

(A31) $$s_n = \frac{Z(1-Z)\gamma_n}{\left[1-Z+Z^2\right]\left[r + (1-R)\gamma_n - \gamma_H\right] + Z(1-Z)\gamma_n} u$$

Given Eqs. (A22'), (A23'), (A23''), (A25) and (A31), it is possible to compute the BGP ratio $\frac{H_t^{\mu-\Phi}}{n^{1-\eta}}$ (by using either Eq. 29 or Eq. A30), along with s_I and s_Y (Eqs. A27 and A28).

Finally, by employing Eqs. (A6), (A7), (A15'), (A17) and the definition of $h \equiv H/L$, it can be showed that along the BGP the two transversality conditions $\lim_{t\to+\infty} \lambda_{at} a_t = 0, \quad \lim_{t\to+\infty} \lambda_{ht} h_t = 0$ are simultaneously checked when: $r > \gamma_H + R\gamma_n$

In turn, when the two transversality conditions are met, then the requirement (Eq. A9'): $r > \gamma_H - (1-R)\gamma_n$ is also met, for any positive. ■

APPENDIX B: TABLE 1

$$\frac{\partial \gamma_y}{\partial g_L} = \frac{-\left[\Upsilon R(\xi-2)+(\xi-1)\right]}{\Upsilon R(\theta-1)+\theta}$$

When (i) and (iii) in Assumption A in the main text are met, $\left[\Upsilon R(\theta-1)+\theta\right] > 0$ is always satisfied. With $\Upsilon > 0, \quad R > 0,$ and $\xi \geq 0$ we conclude:

- $$\frac{\partial \gamma_y}{\partial g_L} > 0 \quad \Rightarrow \quad -\Upsilon R(\xi-2)-(\xi-1) > 0 \quad \Rightarrow \quad \begin{array}{c} -\Upsilon R(\xi-2) > (\xi-1) \\ \Upsilon R\xi + \xi < 1+2\Upsilon R \\ \xi(1+\Upsilon R) < 1+2\Upsilon R \\ \xi < \dfrac{1+2\Upsilon R}{1+\Upsilon R} \end{array}$$

- $$\frac{\partial \gamma_y}{\partial g_L} < 0 \quad \Rightarrow \quad -\Upsilon R(\xi-2)-(\xi-1) < 0 \quad \Rightarrow \quad \xi > \frac{1+2\Upsilon R}{1+\Upsilon R}$$

- $$\frac{\partial \gamma_y}{\partial g_L} = 0 \quad \Rightarrow \quad -\Upsilon R(\xi-2)-(\xi-1) = 0 \quad \Rightarrow \quad \xi = \frac{1+2\Upsilon R}{1+\Upsilon R}$$

- $$if\, \xi = 1 \quad \Rightarrow \quad \frac{\partial \gamma_y}{\partial g_L} = \frac{-\left[\Upsilon R(-1)\right]}{\Upsilon R(\theta-1)+\theta} = \frac{\Upsilon R}{\Upsilon R(\theta-1)+\theta} \quad \Rightarrow \quad \frac{\partial \gamma_y}{\partial g_L} > 0$$

- $if\ \xi = 0 \quad \Rightarrow \quad \frac{\partial \gamma_y}{\partial g_L} = \frac{-\left[\Upsilon R(-2) + (-1)\right]}{\Upsilon R(\theta - 1) + \theta} = \frac{2\Upsilon R + 1}{\Upsilon R(\theta - 1) + \theta} \quad \Rightarrow \quad \frac{\partial \gamma_y}{\partial g_L} > 0$ ■