Financial Development and Economic Growth: Long Run Equilibrium and Transitional Dynamics*

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Abstract

We analyze the impact of financial development on economic growth. Differently from previous studies that focus mainly on balanced growth path outcomes, we also analyze the transitional dynamics of our model economy by using a finance-extended Uzawa-Lucas framework where financial intermediation affects both human and physical capital accumulation. We show that, under certain rather general conditions, economic growth may turn out to be non-monotonically related to financial development (as suggested by the most recent empirical evidence) and that too much finance may be detrimental to growth. We also show that the degree of financial development may affect the speed of convergence, which suggests that finance may play a crucial role in determining the length of the recovery process associated with exogenous shocks. Moreover, in a special case of the model, we observe that, under a realistic set of parameters, social welfare decreases with financial development, meaning that even when finance positively affects economic growth the short term costs associated with financial activities more than compensate their long run benefits.

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1 Introduction

Finance affects the real economy in several ways, hence understanding the possible mechanisms through which it may impact on economic growth is essential in order to derive sound policy recommendations (Levine, 2005). Numerous works emphasize that there may be a nonlinear (Deidda and Fattouha, 2002), and ultimately non-monotonic (Allen et al., 2014; Law and Singh, 2014), relationship between the degree of financial development and economic performance (i.e., long run economic growth). Specifically, according to some studies, too much finance may be harmful for economic growth, while at the same time too little finance may be suboptimal. While most of the works in the field adopt an empirical approach, the number of those relying on a theoretical methodology is limited. Pagano (1993) was among the first to emphasize the existence of several channels through which finance might affect economic growth in a simple Solow-type AK growth model. The main pathways discussed in his work are related to three fundamental activities generally run by financial intermediaries, namely funneling savings to firms, improving the allocation of capital, and affecting an economy's whole saving rate. After this seminal work, over the last two decades a number of studies has focused on the effects of financial intermediaries on human (De Gregorio, 1996;

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De Gregorio and Kim, 2000) as well as technological (Morales, 2003; Trew, 2008) capital accumulation, while only recently some step further has been made towards considering the role of financial intermediaries in channeling savings to the most efficient uses[1] (Trew, 2014). Most of the extant theoretical works on finance and growth suffer from two major limitations, namely they are unable to capture the apparently nonlinear, and possibly non-monotonic, relationship between finance and economic growth and, moreover, they do not analyze the transitional effects associated with financial development, as they mainly focus on balanced growth path (BGP) outcomes. In order to fill these gaps in the literature, we develop a simple extension of the two-sector Uzawa (1965)-Lucas (1988) growth model to account for the role and the effects of financial intermediation. We focus on the Lucas-Uzawa model because this is among the most celebrated and studied endogenous growth models, because it has frequently been extended along several different directions[2] and, more importantly, because it is the simplest two-sector endogenous growth model capable of yielding transitional dynamics in a very intuitive way. Its versatility allows us to obtain another view on the potential implications that different degrees of financial development may have on economic performance in the short as well as in the long run, via both physical and human capital accumulation.

Specifically, in our finance-extended Uzawa-Lucas model, financial development affects physical capital by altering the amount of resources that can be potentially allocated to investment purposes (savings funneling channel), while it also affects human capital via both a productivity and a depreciation channel. We first analyze the steady-state of our model economy in the ratio-variables (hence, we characterize its balanced growth solution in terms of level-variables), and then we discuss the transitional dynamics effects associated with changes in the degree of the economy's financial development. We show that along a BGP there may exist, consistently with the latest empirical evidence, a non-monotonic (possibly inverted-U-shaped) relationship between finance and economic growth: indeed, under some rather general conditions, such a link can be positive only up to a level of financial development, after which the relation turns negative. We also analyze how financial development affects the transitional dynamics of the model by impacting on the economy's speed of convergence. This analysis is important because it allows us to better understand why, after the same global financial crisis, the US seems to display a different speed of recovery in comparison with, for instance, some EU countries. Finally, following Xie (1994), we focus on a special case of our model with the aim of obtaining an analytical expression for the evolution of consumption over time and, therefore, of further investigating how financial development may affect the behavior of the economy during the transition toward its balanced growth path. In this regard, we observe that, for a realistic set of parameters, welfare is a monotonically decreasing function of financial development, which suggests that even if finance may be growth-enhancing at low levels of financial development, its short term (negative) impact on physical capital accumulation more than offsets its long run (positive) effect on human capital formation.

This paper proceeds as follows. Section 2 briefly reviews the huge body of empirical works on the finance-growth nexus in order to emphasize the variety of results obtained in applied works over the last decades. Section 3 introduces our finance-extended Lucas-Uzawa model, in which financial development influences both the investment in physical and human capital. The steady-state of our model, namely its BGP equilibrium, and its transitional dynamics are fully characterized in section 4. We show that along the BGP there may exist a non-monotonic relationship between economic growth and financial development, and that the speed of convergence is crucially dependent on the degree of financial development. In section

[1] The literature on financial development and economic growth is quickly becoming very extensive (see, among others, Trew, 2006; Ang, 2008; Arestis et al., 2015, and Valickova et al., 2015, for insightful surveys), and relatively varied in terms of results obtained, aims pursued, variables employed, and methodological approaches used. In section 2 we review, as compactly as possible, the main conclusions reached by the latest available empirical literature, along with some of the possible explanations that the rather few existing theoretical works on the topic have put forward in order to explain the sign of the effects that finance may have on economic growth.

[2] For example, Bucci and Segre (2011), and Marsiglio and La Torre (2012) analyze, respectively, the growth effects of culture and demographic change within a two sector Lucas-Uzawa model; La Torre and Marsiglio (2010) propose a three-sector extension of such a model to allow for endogenous technological progress.

5 a special case of the model is developed with the purpose of assessing the impact of financial development on social welfare. Section 6 briefly discusses to what extent our model can be useful in explaining some of the similarities/differences between the Great Depression following the 1929 stock-market crash and the recent global financial crises of 2007–2008. Section 7 concludes and proposes some possible directions for future research. In appendix A we present an alternative assessment of welfare effects under a different specification of some relevant functional forms.

2 Empirical Evidence on Finance and Growth

In this section we present a brief review of the main conclusions reached by the available empirical literature with the objective of grasping which kind of theoretical predictions our model should be able to yield in order to be consistent with real life experiences. In so doing, we also emphasize the main theoretical channels put forward up to now to explain the sign of the possible effects that finance may have on economic growth. According to Levine (2005), the main functions of a financial system can be grouped into the following five categories: (i) easing the exchange of goods and services, (ii) pooling and mobilizing savings, (iii) producing ex-ante information on investment possibilities, (iv) exerting corporate governance after providing finance, and (v) facilitating risk management.

Economists have long debated about the effects of financial development on economic growth, and today we can identify alternative views claiming that finance affects positively, negatively or not at all economic growth (see Ang, 2008, for a recent surveys; or also Stolbov, 2013, who puts the debate on the finance-growth nexus, from its origins up to the 1990s, into historical perspective). The positive relationship between these two variables, at least until a point, is generally explained (especially by endogenous growth models) through two possible and non-mutually exclusive channels, namely the reduction in informational frictions and the consequent improvement in resource allocation efficiency made possible by the existence of financial intermediaries. Thus, according to this view, the positive link between financial development and economic growth is mainly driven by financial intermediation reducing the costs of information and therefore increasing the efficiency, as opposed to the volume, of investment. The negative effect that financial development may have on economic growth (especially when finance becomes excessive) is, instead, explained, among other possible reasons, by the fact that too much finance incubates economic booms and asset prices bubbles that result in financial crises leading to low rates of economic growth for rather long periods of time. In keeping with this view, those economies that in the past have showed particularly high levels of credit to the private sector as a share of GDP (i.e., a measure of financial development) tended to experience costly banking crises in 2007-2008 and sharp downturns in 2007-2009. In other words, the proponents of this argument claim that the recent financial crisis episodes are a clear proof that too large financial systems directly and/or indirectly contribute to increase volatility, waste resources, discourage savings and promote speculation that, in turn, all lead to under-investment, misallocation of available resources and, ultimately, to lower economic growth rates. Finally, not all researchers are convinced about the importance (either in positive or in negative terms) of finance. For instance, Lucas (1988) stresses that there is a tendency to overemphasize the role of financial factors in the process of long run economic growth.

The huge empirics today existing on the finance-growth nexus can be broadly categorized into three groups: cross-country, time-series, and panel studies[3]. Given the bulk of mixed results now available in each of these three different groups of works, it is not possible to draw from them any clear-cut, definitive, and unambiguous conclusion regarding the role played by financial development on economic growth.[4] Just to

[3]Ang (2008) surveys in detail the three above mentioned different branches of the empirical literature on finance and growth and, for each of them, also discusses limitations and shortcomings (see also Stolbov, 2017).

[4]Exactly because the empirical literature on the finance-growth nexus is now rather extensive, quite heterogeneous, and highly sensitive to the use of different financial development indicators, observation periods, and econometric methods, recently a growing tendency to quantitatively assess this huge body of works through meta-analyses techniques has emerged (see Valickova

give a broad idea about the enormous degree of heterogeneity that we now face in the reported empirical estimates, at this stage it is worth mentioning a recent paper by Stolbov (2017) that investigates the causality issue between the ratio of domestic private credit to GDP and growth in real GDP per capita in a country-by-country time-series framework for 24 OECD economies over the period 1980-2013. It shows that only 12 out of the 24 sample-countries yield uniform results in terms of causality presence (or absence), and direction. Still, in those few countries where robust causal links are found, credit depth causes economic growth, not viceversa. Finally, the aggregate empirical results obtained by Stolbov (2017) appear also consistent with the view that there may be a threshold level of financial development (in his paper the ratio of private credit to GDP) beyond which the link between finance and growth becomes negative, or vanishes altogether. In another work Stolbov (2015) finds that excessive credit depth is among the major determinants of a financial crisis: a too much high level of the ratio of private bank credit to GDP tends indeed to make an economy more vulnerable to a banking crisis, and therefore more susceptible to future decreases in its economic growth rate.

The latest available empirical research on the finance-economic growth nexus recognizes the presence of nonlinearities in the relation, and it now acknowledges not only that financial systems play a fundamental role in determining variations in economic growth, but also that financial development has a positive effect on economic growth and that such effect does not last forever (as sooner or later it vanishes and/or becomes negative, especially if finance turns out to be excessive). This line of research is well represented by Arcand et al.'s (2015) paper according to which the vanishing effect found in earlier studies is not driven by a change in the fundamental relationship between finance and economic growth (which, instead, appears to be quite stable), but rather by the fact that standard empirical models do not allow for a non-monotonic relationship between financial development and economic growth. After allowing for this non-monotonicity, they find a positive marginal effect of financial depth on economic growth in economies where the level of credit to the private sector falls below a threshold of about 80-100% of aggregate GDP. Above this threshold, the relationship becomes definitely negative. These findings are showed to be robust to different types of data and estimators, and to controlling for macroeconomic volatility, banking crises, and institutional quality. An inverted-U-shaped relation between financial depth and economic growth is also found by Cecchetti and Kharroubi (2012), and Law and Singh (2014). Based on a sample of developed and emerging economies, Cecchetti and Kharroubi (2012) observe that when private credit exceeds 100% of GDP, or financial sectors share of total employment exceeds 4%, then economic growth starts deteriorating. The threshold above which private sector credit as a share of GDP negatively affects economic growth is found to be 88% in Law and Singh (2014), who conclude that their inverted-U result is robust to different measures of financial development indicators, additional explanatory variables, sub-samples of countries, as well as estimation procedures. Finally, the conjecture that financial development may have a nonlinear effect on output growth – that is, it promotes growth only up to a point – finds further support in the sectoral analysis provided by Aizenman et al. (2015).

Overall, the policy implication coming from this up-to-date and apparently rather robust empirical literature is that policymakers should focus less on increasing at all costs the size/depth of the financial sector and more on determining the optimal amount of financial resources to be channeled, through financial intermediaries, toward production activities. As we shall show later, our theoretical model can explain the eventual existence of an inverted-U relationship between financial development and economic growth. Unlike the scant theoretical literature on the topic, our explanation is based, though, on the relative importance of two simultaneous but opposing effects (which we shall refer to as productivity and depreciation effects, respectively) regulating human-capital-based growth at different degrees of financial development.

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et al., 2015; Arestis et al., 2015). These meta-regression studies suggest *"...the existence of a statistically significant and economically meaningful positive genuine effect from financial development to economic growth"* (Arestis et al., 2015).

3 The Model

The framework is an extended Uzawa-Lucas (1988) two-sector endogenous growth model that allows for a role of financial intermediation. We abstract from population growth and the population size is normalized to unity for the sake of simplicity; we thus state the problem directly in per capita terms. The economy produces a unique consumption good, y_t, by combining physical capital, k_t, and the amount of human capital allocated to productive activities, $u_t h_t$, where h_t is the human capital stock and $0 \leq u_t \leq 1$ is the share of the existing human capital devoted to production. In order to manufacture the homogeneous consumption good, the economy employs a Cobb-Douglas technology: $y_t = Ak_t^\alpha (u_t h_t)^{1-\alpha}$, where $0 < \alpha < 1$ is the physical capital share and $A > 0$ is a technological parameter. The social planner seeks to maximize the social welfare subject to the physical and human capital accumulation constraints, by choosing consumption, c_t, and the share of human capital to employ in production, u_t. Social welfare is the infinite discounted ($\rho > 0$ is the pure rate of time preference) sum of the instantaneous utilities; the utility function is assumed to be iso-elastic, $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}$, with $\sigma > 0$ representing the inverse of the intertemporal elasticity of substitution. Physical capital accumulation is given by the difference between production of the unique final good (net of financial intermediation and depreciation costs) and consumption activity: $\dot{k}_t = [1 - \xi(\phi)]y_t - \delta_k k_t - c_t$, where $0 < \delta_k < 1$ is the depreciation rate of physical capital, and $0 < \xi(\phi) < 1$ is the share of output lost in the process of financial intermediation which depends upon the degree of development of the financial sector, $\phi > 0$. Human capital accumulation coincides with the net (of depreciation) production of new human capital: $\dot{h}_t = \theta(\phi)(1 - u_t)h_t - \delta_h(\phi)h_t$, where $\theta(\phi) > 0$ measures the efficiency of the human capital creation process which is a function of financial development, $0 < \delta_h(\phi) < 1$ is the depreciation of human capital also depending on the degree of financial development, and $1 - u_t$ is the share of human capital devoted to the production of new human capital. In order not to impose a priori any limit to human capital accumulation we assume that $\theta(\phi) > \delta_h(\phi)$; indeed, if this were not the case human capital would be meant to either decrease or, at most, remain constant for any $0 \leq u_t \leq 1$. Given the initial conditions for physical and human capital, $k_0 > 0$ and $h_0 > 0$ respectively, the planner's problem reads as follows:

$$\max_{c_t, u_t} \quad W = \int_0^\infty \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{1}$$

$$s.t. \quad \dot{k}_t = [1 - \xi(\phi)]Ak_t^\alpha (u_t h_t)^{1-\alpha} - \delta_k k_t - c_t \tag{2}$$

$$\dot{h}_t = \theta(\phi)(1 - u_t)h_t - \delta_h(\phi)h_t \tag{3}$$

From the problem (1), (2) and (3), it is clear that financial intermediation affects the economy through two different channels, namely physical and human capital accumulation. Indeed, on the one hand, financial intermediation subtracts resources to physical capital investments (Pagano, 1993), while, on the other hand, it also affects human capital formation (de Gregorio and Kim, 2000). More specifically, financial intermediaries absorb a share of income equal to $\xi(\phi)$ that otherwise would go to physical capital accumulation. This absorption of resources is primarily a reward for the supply of financial services and may also reflect the X-inefficiency related to the market power of financial intermediaries. How intermediaries affect this term is not clear a priori, but it is reasonable to believe that the more the financial sector is developed (the larger is ϕ), the less resources are wasted in the process of intermediation, that is $\xi' < 0$. This is consistent with the view (Goldsmith, 1969; Bencivenga and Smith, 1991, among others) that, ceteris paribus, if a positive relationship between financial development and economic growth does exist, it may well be driven by financial intermediaries increasing the productivity of gross capital investment. Concerning the human capital accumulation equation, the impact of financial intermediation is twofold. First, we assume that financial intermediaries do also affect the total factor productivity of the education sector through the term $\theta(\phi)$: by relaxing borrowing constraints, they allow the best (most productive) workers to invest more in human capital, thus generating a positive effect on the process of acquiring new skills. This means that in our framework $\theta' > 0$. Second, we also postulate that financial intermediaries can influence human capital investment via a change in the rate at which the existing stock of human capital depreciates. In our formulation this occurs through the term $\delta_h(\phi)$. In this respect, it is well-recognized that one important task of the financial sector in mature economies consists of channeling resources towards the most promising R&D projects, which increases the rate of technological progress. A faster technical change, in turn, indirectly contributes to depreciate more rapidly individual abilities embodied in human capital[5] (for a formal example of this idea, see among many others Galor and Moav, 2002), implying that $\delta_h' > 0$.

Apart from the already mentioned research line that analyzes the link between economic growth and finance, similar issues have been also frequently analyzed, even if from a different perspective and with different objectives, by the real business cycle literature. In our model, as above outlined, in order to analyze how financial development may affect economic performance in the long run we take an aggregative approach. This is consistent with earlier works in the real business cycle literature addressing the implications of financial intermediation on output fluctuations, and is also partly consistent with the most recent works arising after the recent financial crisis. Several earlier papers have introduced credit and financial frictions in dynamic general equilibrium macroeconomic models showing that frictions represent a "financial accelerator" since they can effectively transmit and amplify the macroeconomic fluctuations induced by exogenous shocks (Kiyotaki and Moore, 1997; Carlstrom and Fuerst, 1997; Bernanke et al., 1999; Iacoviello, 2005). These works on the financial accelerator do not explicitly model a financial sector, exactly as in our setting where we aim to disentangle the channels through which financial activities affect physical and human capital accumulation without providing a microfoundation for the financial sector. After the recent financial crisis several works have tried to microfound the role of financial intermediaries, and a variety of different approaches to do so have been proposed (for recent surveys, see Quadrini, 2011; Brunnermeier et al., 2013; Beck et al., 2014).[6] A typical assumption in these works is that the financial sector is composed by profit/utility-maximizing agents who affect economic activities by attempting to achieve their self-interest (see, among others, Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Quadrini, 2017). Clearly, a similar characterization of financial intermediaries is completely absent in our setup, which however captures such a degree of market power by modeling the amount of resources that intermediation diverts from physical capital accumulation. Such a different point of view allows us to stress in a simplified setup the complicated role that financial activities (and financial development in particular) play in determining economic growth both in the short and long run via the effects induced on physical and human capital. Since in the real business cycle literature the main focus is placed on understanding the causes of fluctuations and how monetary and fiscal policies can be used to reduce such short run fluctuations, to the best of our knowledge, no paper has ever accounted for the effects of financial activities on human capital nor has ever analyzed their long run implications on economic performance, which are instead the main goals of our analysis. We believe thus that our approach can complement well that adopted by the real business cycle literature.

Necessary and sufficient conditions for the maximization problem (1), (2) and (3) yield the Euler equations for consumption and the share of human capital devoted to output production:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left\{\alpha[1-\xi(\phi)]Ak_t^{\alpha-1}(u_t h_t)^{1-\alpha} - \delta_k - \rho\right\} \quad (4)$$

$$\frac{\dot{u}_t}{u_t} = \frac{1-\alpha}{\alpha}[\theta(\phi) - \delta_h(\phi) + \delta_k] + \theta(\phi)u_t - \frac{c_t}{k_t} \quad (5)$$

Equation (4) is the standard Ramsey-Keynes rule for consumption and states that consumption growth

[5] Bucci (2008) proposes a growth model with purposive R&D activity and human capital investment in which technological progress affects per capita human capital depreciation, which is therefore endogenous. In the present paper we do not model the direct effect that financial development has on R&D activity (a recent work that analyzes this issue is Trew, 2014), but we move one step further by postulating that a higher degree of financial development, by accelerating the rate of technological progress, is indirectly able to speed up the rate of obsolescence of an individual's human capital.

[6] Another line of research following the Great Recession consists of introducing financial frictions in standard DGSE frameworks in order to incorporate into large scale monetary policy models the lessons learnt from the crisis (see, among others, Brzoza–Brzezina et al., 2013; Kolasa and Rubaszek, 2015; and Galvao et al., 2016).

increases with the average product of physical capital ($\frac{y_t}{k_t}$), the physical capital share and the share of output non-lost in the process of financial intermediation, while decreases with the rate of time preference, the depreciation rate of physical capital and the inverse of the intertemporal elasticity of substitution. Equation (5) states that the growth rate of the share of human capital allocated to production activities increases with the productivity of human capital in generating new human capital, the level of the human capital share allocated to production activities and the depreciation rate of physical capital, while it decreases with the depreciation of human capital and the consumption to capital ratio ($\frac{c_t}{k_t}$).

4 BGP Equilibrium and Transitional Dynamics Analysis

We are interested in characterizing a long run (BGP) equilibrium in which all variables grow at an exponential, constant (and possibly positive) rate. Under this definition, simple inspection of equations (2), (3), (4) and (5) reveals that along a BGP, consumption, physical capital and human capital must grow at the same rate (i.e., $\gamma_c = \gamma_k = \gamma_h$), and that the allocation of human capital between production and educational activities (u_t) must also remain constant. In light of this, the shape of the production function suggests that the output to capital ratio must be constant as well along a BGP. Hence, in the long run equilibrium, we would ultimately observe: $\gamma \equiv \gamma_c = \gamma_k = \gamma_h = \gamma_y$, along with a constant $u_t = u$. In order to study the transitional dynamics of the model it is convenient to introduce the following two intensive variables: $\chi_t = \frac{c_t}{k_t}$ and $\psi_t = \frac{y_t}{k_t} = Ak_t^{\alpha-1}(u_t h_t)^{1-\alpha}$, representing the consumption to capital ratio and the average product of physical capital, respectively. By using these two variables, it is possible to represent our economy through the following three-dimensional system of differential equations:

$$\frac{\dot{\chi}_t}{\chi_t} = \chi_t - \frac{\sigma-\alpha}{\sigma}[1-\xi(\phi)]\psi_t - \frac{\rho}{\sigma} + \frac{\sigma-1}{\sigma}\delta_k \tag{6}$$

$$\frac{\dot{\psi}_t}{\psi_t} = \frac{1-\alpha}{\alpha}[\theta(\phi) - \delta_h(\phi) + \delta_k] - (1-\alpha)[1-\xi(\phi)]\psi_t \tag{7}$$

$$\frac{\dot{u}_t}{u_t} = \frac{1-\alpha}{\alpha}[\theta(\phi) - \delta_h(\phi) + \delta_k] + \theta(\phi)u_t - \chi_t \tag{8}$$

The steady state of the system above, representing our BGP equilibrium, can be straightforwardly obtained by setting the previous equations equal to zero, which yields:

$$\overline{\chi} = \frac{(\sigma-\alpha)[\theta(\phi) - \delta_h(\phi)] + \alpha\rho + \sigma(1-\alpha)\delta_k}{\alpha\sigma} \tag{9}$$

$$\overline{\psi} = \frac{\theta(\phi) - \delta_h(\phi) + \delta_k}{[1-\xi(\phi)]\alpha} \tag{10}$$

$$\overline{u} = \frac{\rho + (\sigma-1)[\theta(\phi) - \delta_h(\phi)]}{\sigma\theta(\phi)}. \tag{11}$$

Sufficient conditions for the above expressions to be strictly positive and the share of human capital employed in production activities to be also smaller than one require that $(1-\alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ and $\sigma \geq \alpha$. By plugging (10) into (4) it is immediate to obtain the common BGP growth rate of the economy as: $\gamma = \frac{\theta(\phi)-\delta_h(\phi)-\rho}{\sigma}$, which under the above conditions turns out to be strictly positive too. In order to study the transitional dynamics of the system (6), (7) and (8), we can proceed via linearization, by obtaining the Jacobian matrix, which evaluated at steady state reads as:

$$J(\overline{\chi}, \overline{\psi}, \overline{u}) = \begin{bmatrix} \overline{\chi} & -\frac{\sigma-\alpha}{\sigma}[1-\xi(\phi)]\overline{\chi} & 0 \\ 0 & -(1-\alpha)[1-\xi(\phi)]\overline{\psi} & 0 \\ -\overline{u} & 0 & \theta(\phi)\overline{u} \end{bmatrix}$$

It is straightforward to note that its eigenvalues, ϑ, coincide with the elements on the main diagonal. Specifically, two of them are positive, $\vartheta_1 = \overline{\chi}$ and $\vartheta_3 = \theta(\phi)\overline{u}$, and one is negative, $\vartheta_2 = -(1-\alpha)[1-\xi(\phi)]\overline{\psi}$.

This means that the steady state $(\overline{\chi}, \overline{\psi}, \overline{u})$, and thus the BGP equilibrium, is saddle-point stable. We can thus summarize these results as follows:

Proposition 1. *Assume* $(1-\alpha)[\theta(\phi)-\delta_h(\phi)] < \rho < \theta(\phi)-\delta_h(\phi)$ *and* $\sigma \geq \alpha$*; then along the BGP equilibrium the common economic growth rate is given by:*

$$\gamma \equiv \gamma_c = \gamma_k = \gamma_h = \gamma_y = \frac{\theta(\phi) - \delta_h(\phi) - \rho}{\sigma} > 0 \tag{12}$$

while the consumption to physical capital ratio, $\overline{\chi} > 0$*, the average product of capital,* $\overline{\psi} > 0$*, and the share of human capital allocated to final goods production,* $\overline{u} \in (0, 1)$*, are respectively given by (9), (10) and (11). Moreover, the BGP equilibrium is saddle-point stable.*

The results in Proposition 1 are pretty standard in a Lucas-Uzawa framework. As usual, some technical conditions, namely $(1 - \alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ and $\sigma \geq \alpha$, are needed in order to ensure that the growth rate and the other endogenous variables are well defined. These conditions along with our model's assumption that $\theta(\phi) > \delta_h(\phi)$ jointly guarantee that the economic growth rate is strictly positive. This implies that in order for the BGP to be properly characterized, the degree of financial development, ϕ, cannot take arbitrarily large or small values but it necessarily needs to be bounded (from either above or below). Our following discussion is based on the assumption that ϕ falls in the required range; note, however, that since the degree of financial development can be thought of as an index number, our conclusions will hold with no loss of generality. Provided that these conditions are met, the steady state $(\overline{\chi}, \overline{\psi}, \overline{u})$, and thus the BGP equilibrium, is saddle-point stable. This means that our economy converges towards its BGP equilibrium along a saddle-path: given the initial conditions k_0 and h_0, a unique trajectory (c_0, u_0) ensures such a converging behavior. By investigating the characteristics of our BGP, we can see that financial development affects both the economic growth rate and the share of human capital employed in output production. Because our main goal in this paper is a better understanding of the growth-finance nexus, we will focus on this relationship in the remainder of the paper.

From (12) it is clear that the only engine of economic growth in the model is human capital investment. This result is due to the fact that our framework is an extension of the Lucas-Uzawa growth model. Unlike this model, however, finance plays a role in our setting. In particular, note that the ultimate effect that financial development has on BGP growth crucially depends on how a more developed financial system can eventually (and simultaneously) influence the two parameters governing the technology of skill acquisition. This is more formally suggested by the following derivative:

$$\frac{\partial \gamma}{\partial \phi} = \frac{\theta'(\phi) - \delta_h'(\phi)}{\sigma} \tag{13}$$

Equation (13) shows that two terms control the whole impact of financial development on economic growth. The former term, $\frac{\theta'(\phi)}{\sigma}$, which we refer to as the *"productivity effect"*, implies that a more developed financial sector tends to increase the productivity of human capital investment, and thus economic growth. The latter term, $-\frac{\delta_h'(\phi)}{\sigma}$, which we refer to as the *"depreciation effect"*, reveals instead that a more developed financial sector, by encouraging R&D activities and hence technical progress, may make at the same time the existing stock of human capital more subject to depreciation via faster obsolescence, which harms economic growth. Depending on which of these two opposing effects eventually prevails, in the very long run economic growth would be either positively, or negatively, or else not related at all to the degree of financial development of the economy. In this regard, it is straightforward to claim the following:

Proposition 2. *Along the BGP there may be a non-monotonic relationship between economic growth and financial development. Whenever the productivity effect is larger than the depreciation effect* $(\theta'(\phi) > \delta_h'(\phi))$*, economic growth increases with financial development; otherwise either financial development reduces* $(\theta'(\phi) < \delta_h'(\phi))$ *or has no impact at all* $(\theta'(\phi) = \delta_h'(\phi))$ *on economic performance.*

Proposition 2 deals with the sign that the finance-growth nexus can take along the BGP in our model economy. In order to make the economic implications of the Proposition more clear, consider the following simple explicit examples which help illustrating more concretely some of the possible effects that a more developed financial system may have on economic growth depending on the specific shape of the functions $\theta(\cdot)$ and $\delta_h(\cdot)$. Recall that in order for our model to make full sense we need to restrict our analysis to situations in which $\theta(\cdot) > \delta_h(\cdot)$, imposing thus some constraint on the relative size of the two functions.

Example 1. Assume that $\theta(\phi)$ and $\delta_h(\phi)$ are both linear functions of ϕ:

$$\theta(\phi) = \theta\phi \quad \text{and} \quad \delta_h(\phi) = \delta_h\phi,$$

with $\theta > \delta_h > 0$, consistently with our model's requirements. In this case, the productivity effect prevails over the depreciation effect. Thus, the relation between financial development and economic growth is monotonic and always positive, $\frac{\partial\gamma}{\partial\phi} > 0$.[7]

Example 2. Assume that $\theta(\phi)$ and $\delta_h(\phi)$ are exponential functions of ϕ, respectively:

$$\theta(\phi) = \theta e^{\phi} \quad \text{and} \quad \delta_h(\phi) = \delta_h\phi e^{\phi},$$

with $\theta > \delta_h > 0$. Our model's specification in this case requires that $\phi < \frac{\theta}{\delta_h}$, which does not impose a priori any restriction on the relative size of the productivity and depreciation effects. This implies that two possibilities can arise here. If $0 < \phi < \frac{\theta-\delta_h}{\delta_h}$, then the productivity effect prevails over the depreciation effect and $\frac{\partial\gamma}{\partial\phi} > 0$; otherwise, whenever $\frac{\theta-\delta_h}{\delta_h} < \phi < \frac{\theta}{\delta_h}$, the depreciation effect prevails over the productivity effect and $\frac{\partial\gamma}{\partial\phi} < 0$. As a whole, the relationship between financial development and economic growth is inverted-U-shaped, with a threshold of the degree of financial development at $\phi = \frac{\theta-\delta_h}{\delta_h} > 0$.

Example 3. Assume that $\theta(\phi)$ and $\delta_h(\phi)$ are linear and quadratic functions of ϕ, respectively:

$$\theta(\phi) = \theta\phi \quad \text{and} \quad \delta_h(\phi) = \delta_h\phi^2,$$

with $\theta > 0$ and $\delta_h > 0$. Our model's specification also in this case requires that $\phi < \frac{\theta}{\delta_h}$, which again does not impose a priori any restriction on the relative size of the productivity and depreciation effects. This implies that two possibilities can occur. If $0 < \phi < \frac{\theta}{2\delta_h}$, then the productivity effect prevails over the depreciation effect and $\frac{\partial\gamma}{\partial\phi} > 0$; otherwise, whenever $\frac{\theta}{2\delta_h} < \phi < \frac{\theta}{\delta_h}$, the depreciation effect prevails over the productivity effect and $\frac{\partial\gamma}{\partial\phi} < 0$. As in Example 2, the relationship between financial development and economic growth is inverted-U shaped, with a threshold of the degree of financial development at $\phi = \frac{\theta}{2\delta_h} > 0$ in this case.

As the examples above illustrate, the relationship between financial development and economic growth crucially depends on the shape of the productivity and depreciation functions, which determine whether this link is ultimately monotonic or non-monotonic in sign. Example 1 shows what happens when the degree of financial development affects linearly both the productivity and the depreciation terms in the human capital accumulation equation. In this case, since the productivity effect prevails, the relationship between financial development and economic growth is monotonically positive. Example 2 and 3, instead, exemplify what happens when financial development affects nonlinearly $\theta(\phi)$, $\delta_h(\phi)$, or both. While in Example 2

[7] The following trivial examples show that in case of a monotonic relationship between growth and finance, this does not necessarily need to be positive. Indeed, if $\theta(\phi) = \frac{\theta}{\phi}$ and $\delta_h(\phi) = \delta_h\phi$, with $\theta > \delta_h > 0$, our model requires that $\phi \leq 1$; it is straightforward to verify that over this range the relation between financial development and economic growth is always negative, $\frac{\partial\gamma}{\partial\phi} < 0$. If, instead, $\theta(\phi) = \theta + \phi$ and $\delta_h(\phi) = \delta_h + \phi$, with $\theta > \delta_h > 0$, the relation between financial development and economic growth is completely absent, that is $\frac{\partial\gamma}{\partial\phi} = 0$.

the nonlinear effect of ϕ concerns both terms, Example 3 suggests that in order to have a non-monotonic relationship between finance and growth it is not necessary to postulate a nonlinear impact of ϕ on both $\theta(\phi)$ and $\delta_h(\phi)$ simultaneously. The presence of an inverted-U relation between the degree of financial development and long run economic growth that we observe in Examples 2 and 3 is consistent with the most recent empirical evidence (Arcand et al., 2015), and can be explained by our model as follows. Ceteris paribus, for sufficiently low levels of financial development, a marginal increase in ϕ, by loosening agents' borrowing constraints, makes their investment in skill acquisition easier, while keeping the economy's human capital stock and the rate of technical change reasonably small. Thus, when ϕ is low enough, increases in financial development are likely to raise the productivity of the human capital accumulation process more rapidly than the depreciation costs related to the available amount of skills, and this leads to a higher economic growth rate. Above a given threshold, however, more financial development, by entailing a larger available stock of human capital in the economy (and, consequently, a faster rate of technological progress, too), ultimately contributes to increase the depreciation-costs of the existing amount of skills faster than the corresponding productivity-benefits related to further human capital acquisition, and this is at the heart of the emergence of a negative relation between financial development and economic growth after a point.

One of the main results attained thus far by our model suggests that, depending on the relative intensity of two effects governing the production of new human capital (the productivity and the depreciation effect, respectively), the long run (BGP) relationship between financial development and economic growth is potentially ambiguous (i.e., it can be non-monotonic/inverted-U; monotonically positive/negative; non-existent at all). Before discussing in more depth the short run transitional effects associated with financial development (in section 5 we will look at the behavior of the model also during the transition towards the BGP), we present another interesting implication of our model which deals with the speed of convergence. Indeed, the absolute value of the (unique, see Proposition 1) negative eigenvalue, ϑ_2, represents in this context the speed of convergence of the economy towards its BGP equilibrium. It is straightforward to show that this depends (possibly) non-monotonically on the degree of financial development, as clearly shown in the following derivative:

$$\frac{\partial|\vartheta_2|}{\partial\phi} = \frac{(1-\alpha)[\theta'(\phi) - \delta_h'(\phi)]}{\alpha} \tag{14}$$

According to whether the productivity or the depreciation effect dominates, the speed of convergence will rise or fall with financial development.

Proposition 3. *The speed of convergence may non-monotonically depend on the degree of financial development. It will increase when the relationship between finance and growth is positive (productivity effect larger than depreciation effect), and will decrease when the relationship between finance and growth is negative (productivity effect smaller than depreciation effect). In the absence of any long run relationship between finance and growth (productivity effect equal to depreciation effect), the speed of convergence is unaffected by changes in the degree of financial development.*

This proposition says that in our framework the degree of financial development can affect not only the BGP growth rate of the economy, but also its speed of convergence to a new BGP equilibrium following an economic or financial shock. To the best of our knowledge, this result is new in literature as in general it is found (see Aghion et al. 2005 for a notable example) that financial development can speed up the convergence to the steady-state but has ultimately a very limited effect on long run economic growth.[8] According to our results, in the wake of a shock, the economies that recover faster in terms of greater speed

[8] Aghion et al. (2005) develop a model of technological change that predicts that countries with levels of financial development above a critical threshold will converge in growth rates. Among these countries, financial development positively affects the rate of convergence, so financial development exerts a positive but diminishing influence on steady-state levels of real per capita output. The authors find empirical support for their model's predictions, as financial development seems to explain in their empirical analysis: (i) whether there is convergence or not, and (ii) the rate of convergence (when there is convergence). Unlike

of convergence to the new BGP are also those in which financial development plays a positive role on long term economic growth. In other words, the degree of financial development affects simultaneously long run (economic growth) and short run (transitional dynamics) outcomes. This finding is consistent with the recent analysis presented by Reinhart and Rogoff (2014), according to which out of the 12 countries that have experienced the financial crisis started in 2007–2008, only two (Germany and the US) have already reached their pre-crisis peak in per capita GDP. For all other countries, using IMF (2013) estimates, their projections (see Reinhart and Rogoff, 2014) suggest that even by 2018 the full recovery to pre-crisis GDP will not be completed yet. In terms of the ongoing debate on how to improve the contribution of the financial sector to avoid future long-lasting financial crises, our paper suggests that policymakers should be aware of the true sign of the finance-growth relationship as well as of the way (i.e., the specific channels) in which this relation takes place before implementing any specific public policy.

5 On Financial Development and Welfare: the Special Case $\sigma = \alpha$

After showing that our theoretical findings are, under certain conditions, consistent with the most recent empirical evidence on the long run finance-growth nexus, we now turn to the analysis of the transitional dynamics of our model in order to quantify the welfare effects associated with financial development. Since (6), (7) and (8) form a simultaneous system of differential equations, analyzing its transitional behavior is evidently not possible in analytical terms. However, by focusing on a specific case it is possible to decouple some of these equations and thus fully solve the system, which will finally allow us to analyze the welfare effects of financial development. Such a decoupling is possible only when $\sigma = \alpha$, that is whenever the inverse of the intertemporal elasticity of substitution coincides with the physical capital share[9] (Xie, 1994). In this case it is possible to show that the following result holds.

Proposition 4. *Assume* $(1-\alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ *and* $\sigma = \alpha$*; then, the optimal paths of the control (*c_t *and* u_t*) and state (*k_t *and* h_t*) variables in the maximization problem (1), (2) and (3) are given by the following expressions for all* $t = 0, .., \infty$*:*

$$c_t = \frac{\rho + (1-\alpha)\delta_k}{\alpha} k_t \tag{15}$$

$$u_t = \frac{\rho - (1-\alpha)[\theta(\phi) - \delta_h(\phi)]}{\alpha\theta(\phi)} = \overline{u} \tag{16}$$

$$k_t = \left\{ \left(k_0^{1-\alpha} - \frac{[1-\xi(\phi)]\alpha A \overline{u}^{1-\alpha} h_0^{1-\alpha}}{\theta(\phi) - \delta_h(\phi) + \delta_k} \right) e^{-\frac{(1-\alpha)(\rho+\delta_k)}{\alpha}t} + \frac{[1-\xi(\phi)]\alpha A \overline{u}^{1-\alpha} h_0^{1-\alpha}}{\theta(\phi) - \delta_h(\phi) + \delta_k} e^{(1-\alpha)\gamma t} \right\}^{\frac{1}{1-\alpha}} \tag{17}$$

$$h_t = h_0 e^{\gamma t}, \tag{18}$$

where the (strictly positive) economic growth rate is given by:

$$\gamma = \frac{\theta(\phi) - \delta_h(\phi) - \rho}{\alpha} \tag{19}$$

Since the restriction $\sigma = \alpha$ represents a particular case already considered in the previous section (see Proposition 1), the technical condition that ensures that the BGP equilibrium is well defined (positive growth

our model, however, in Aghion et al. (2005) financial development does not exert a direct effect on steady-state (or long run) economic growth. In a more recent paper, Aghion et al. (2010) have also showed how financial development may help reducing the growth-cost of economic fluctuations.

[9]This is a typical assumption in the growth literature aiming at disentangling the short run vs long run effects associated with alternative economic policies (Smith, 2006). Some related works in a Lucas-Uzawa framework include Bucci et al. (2011), Marsiglio and La Torre (2012). We are perfectly aware that such an assumption is barely consistent with empirical evidence, but nevertheless, as it will become more clear later, this will be extremely useful to achieve (even if in a simplified setup) our main goal in this section.

rate and positive but smaller than unity share of human capital devoted to output production) coincides with what presented in section 4. From Proposition 4 we can see that financial development affects the whole dynamic evolution of all the variables, and for all $t = 0, ..., \infty$, consumption is proportional to physical capital, the share of human capital devoted to output production is constant, the human capital growth rate coincides with the economic growth rate γ, while physical capital will grow at this same rate γ only asymptotically (whenever the first term inside the curly brackets vanishes). Despite the potential criticism towards the condition ($\sigma = \alpha$) required to derive Proposition 4, the above expressions, (15), (16), (17) and (18), result extremely useful in light of our final goal to assess welfare effects associated with financial development. Indeed, since the share of human capital allocated to the final sector is constant, as it will become more clear in a while, we can clearly isolate the two different channels through which finance may affect consumption, namely physical and human capital accumulation. The fact that human capital is from time zero growing at its long run rate (we have already extensively commented in the previous section on the effects of finance along the BGP) permits us to focus more precisely on the transitional effects of ϕ driven by physical capital accumulation, through the $\xi(\phi)$ term (on which we have not said much thus far). It should be clear from the following expression that financial development affects the evolution of consumption in a strong nonlinear way. In fact, plugging (17) into (15) yields:

$$c_t = \Omega \left\{ \left(k_0^{1-\alpha} - \frac{[1-\xi(\phi)]\alpha A \overline{u}^{1-\alpha} h_0^{1-\alpha}}{\theta(\phi) - \delta_h(\phi) + \delta_k} \right) e^{-\frac{(1-\alpha)(\rho+\delta_k)}{\alpha}t} + \frac{[1-\xi(\phi)]\alpha A \overline{u}^{1-\alpha} h_0^{1-\alpha}}{\theta(\phi) - \delta_h(\phi) + \delta_k} e^{(1-\alpha)\gamma t} \right\}^{\frac{1}{1-\alpha}} \quad (20)$$

where $\Omega = \frac{\rho+(1-\alpha)\delta_k}{\alpha}$. As anticipated above, this clearly shows that finance interacts with consumption activities thanks to both the human and physical capital accumulation channels. Thanks to human capital formation, it affects consumption through three terms, $\theta(\phi) - \delta_h(\phi)$, $\overline{u} = \frac{\rho-(1-\alpha)[\theta(\phi)-\delta_h(\phi)]}{\alpha\theta(\phi)}$ and $\gamma = \frac{\theta(\phi)-\delta_h(\phi)-\rho}{\alpha}$, contributing to uniquely determine the initial consumption level needed to address the economy along its BGP from time zero; this makes sure that eventual adjustments due to human capital accumulation during the transition towards the BGP are completely ruled out. This also implies that finance impacts on consumption through physical capital accumulation only through the $\xi(\phi)$ term, quantifying the amount of resources that the financial sector subtracts to capital investments; this does not play any role on long run growth rates but it does determine the consumption level at any point in time, and it can thus crucially affect social welfare. Even if, thanks to our assumption $\sigma = \alpha$, we can disregard the transitory adjustment effects associated with human capital accumulation (and its optimal intersectoral allocation), the above expression for consumption results to be particularly cumbersome, and assessing analytically welfare and the welfare effects associated with financial development may not be possible. However, in order to shed some light on this issue we turn to a numerical assessment. More specifically, we now run a numerical simulation with the objective of illustrating the impact of financial development on the dynamics of consumption, share of human capital allocated to production activities, physical and human capital. In order to proceed with our simulations, we need to explicitly specify the functions $\theta(\cdot)$, $\delta_h(\cdot)$ and $\xi(\cdot)$. The former two, consistently with the findings of the most recent empirical evidence (namely, Arcand et al., 2015), are assumed to take the same form of the previous Example 3, while the latter is assumed to take the following form:

$$\xi(\phi) = \frac{\xi}{\xi + \phi},$$

with $\xi > 0$ such that $\xi(0) = 1$, $\xi(\infty) = 0$ and $\xi' < 0$. The parameter values we employ in our simulation are shown in Table 1, which (apart from the intertemporal elasticity of substitution, that in our framework is restricted to be equal to the physical capital share) are consistent with the values in Mullingan and Sala–i–Martin (1993). The value of ξ is arbitrarily set to be equal to 0.1 in order to make figures more clear. From a qualitative point of view, our arbitrary choice of functional forms does not substantially affect the results, which are robust with respect to different parametrizations. Only the choice of the functional from for $\xi(\cdot)$ is likely to sensibly affect our quantitative conclusions about welfare effects (see appendix A for further details).

$\sigma = \alpha$	ρ	A	δ_k	θ	δ_h	ξ	$k_0 = h_0$
0.33	0.04	1	0.05	0.1	0.05	0.1	1

Table 1: Parameter values employed in our simulation.

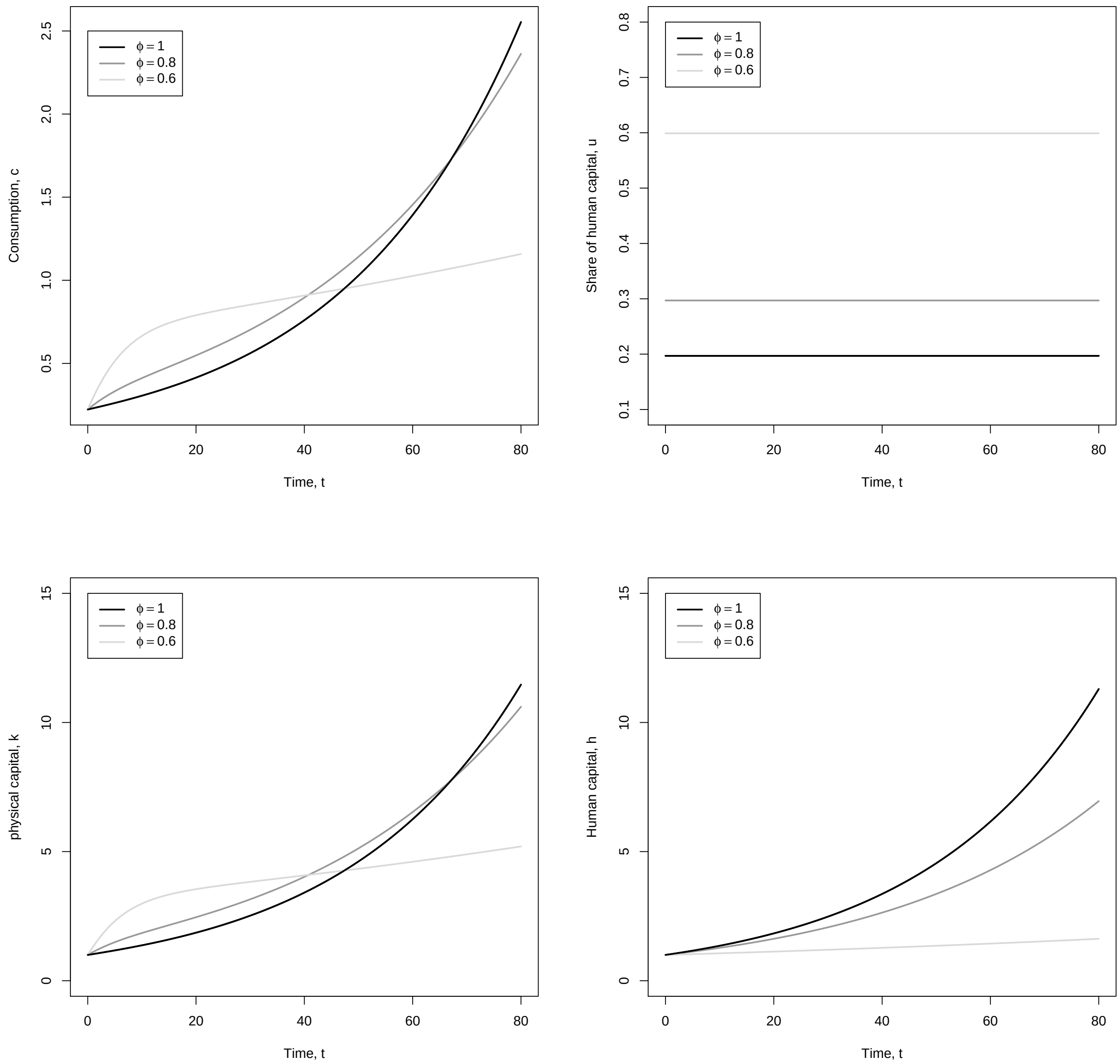

Figure 1: Evolution over time of consumption, share of human capital allocated to production, physical and human capital for values of $\phi \leq 1$.

Figure 1 and 2 present the results of our simulation exercise for values of ϕ respecting the technical conditions given in Proposition 4, that is values falling in the range $\phi \in (0.6, 1.4)$. Since, given the specified parameter values, $\phi = 1$ represents the threshold below and above which financial development affects economic growth with a different sign (see Example 3), we illustrate the two cases separately. Specifically, Figure 1 presents the case in which $\phi \leq 1$ (that is the case in which $\theta'(\phi) \geq \delta_h'(\phi)$) while Figure 2 presents that in which $\phi \geq 1$ (that is the case in which $\theta'(\phi) \leq \delta_h'(\phi)$).

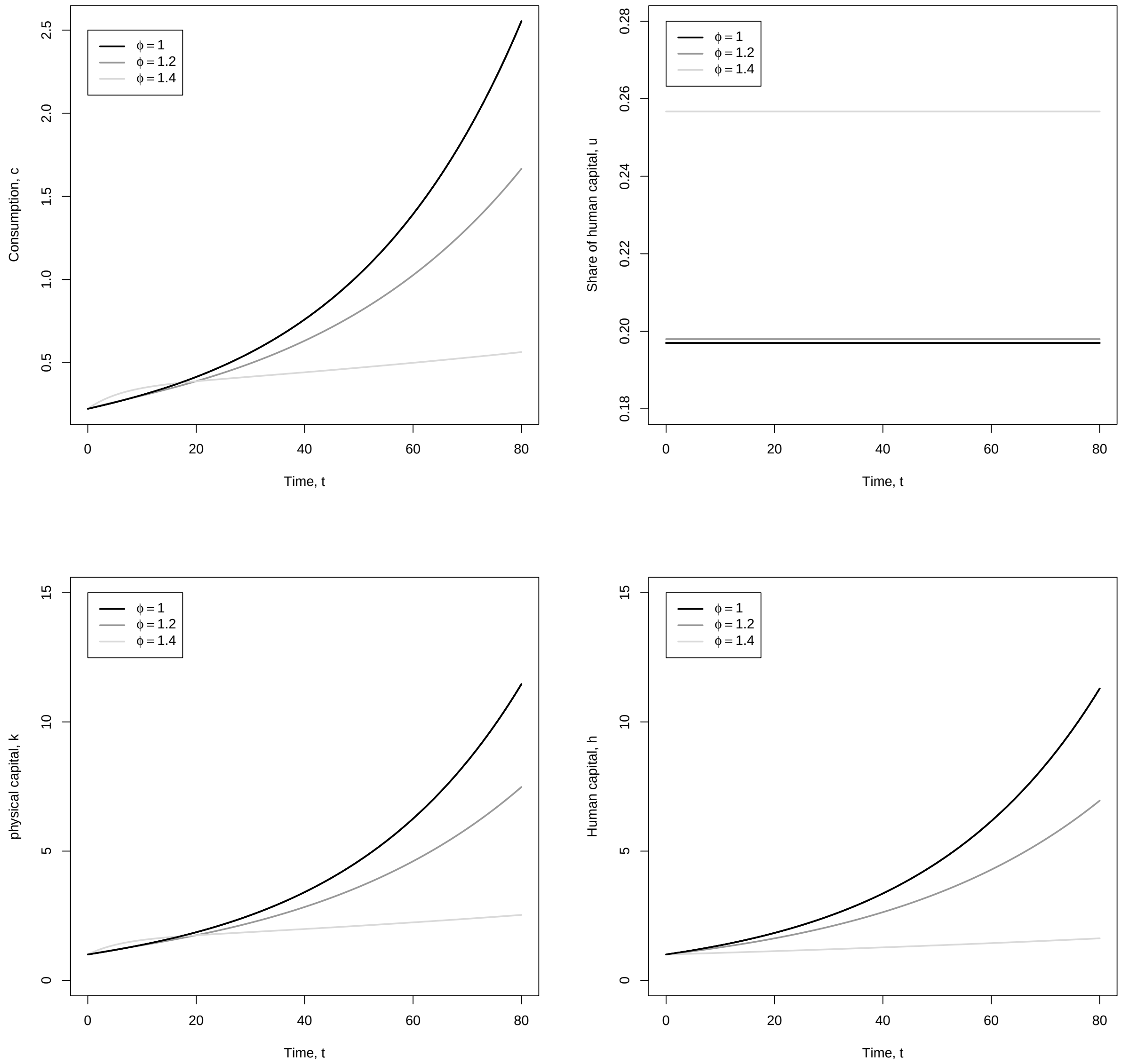

Figure 2: Evolution over time of consumption, share of human capital allocated to production, physical and human capital for values of $\phi \geq 1$.

By comparing Figures 1 and 2 it is possible to understand how financial development affects the dynamic evolution of consumption, share of human capital allocated to production, physical and human capital. Whenever financial development is lower than the threshold value $\phi = 1$ (Figure 1), increases in ϕ will decrease both consumption and physical capital in the short run and will tend to increase them in the long run. Such an increase is due to the fact that a higher degree of financial development tends to decrease the share of human capital allocated to production activities, thus increasing the rate of growth of human capital. As a result, since in this case the productivity effect is larger than the depreciation effect, rises in ϕ will increase the speed of convergence towards the BGP equilibrium, so lowering the length of the short run transitionary period (see Proposition 3). Whenever financial development is higher than the threshold value $\phi = 1$ (Figure 2), the results are qualitatively the opposite. Indeed, increases in ϕ will increase both consumption and physical capital in the short run and will tend to decrease them in the long run, and this is due to the fact that a higher degree of financial development tends to increase the share of human capital allocated to production activities, thus decreasing the rate of growth of human capital. As a result, since in this case the productivity effect is smaller than the depreciation effect, rises in ϕ will decrease the speed of convergence towards the BGP equilibrium, raising thus the length of the short run transitionary period (see Proposition 3)

Overall, we can conclude that the behavior of consumption is exactly the opposite in the two cases and thus it crucially depends upon the sign of the relationship between growth and finance. If such a relationship is negative ($\phi > 1$) then consumption will increase in the short run but will decrease in the long run; if instead it is positive ($\phi < 1$) then consumption will decrease in the short run but will increase in the long run. This suggests that independently of the nature of the finance and growth relationship, we will always observe an intertemporal trade off between short run and long run consumption. Such a trade off clearly reflects how the degree of financial development affects the allocation of human capital between the production and education sectors: a higher allocation in the production sector allows to increase short term consumption but, by lowering human capital accumulation, this comes at the cost of reducing long term consumption. The existence of such different effects on consumption according to the sign of the growth and finance relation reinforces our previous conclusion that before implementing policies aiming to promote or discourage financial development, a clear understanding of the sign of the finance and growth nexus is needed.

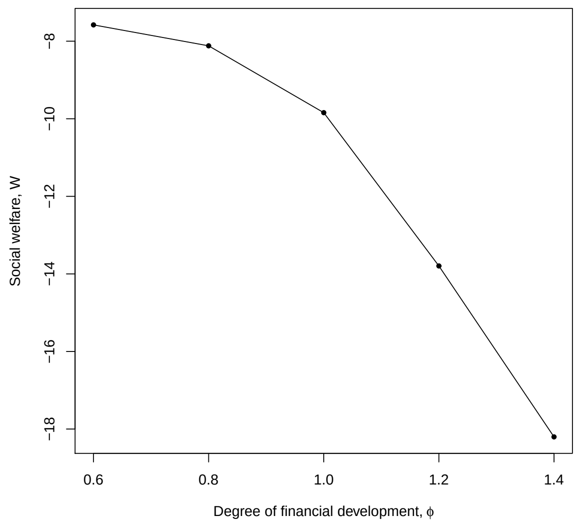

Figure 3: Relationship between social welfare and degree of financial development, ϕ.

The existence of such an intertemporal trade off characterizing how consumption responds to financial development further suggests that the only possible way to compare short and long run costs and benefits associated with financial development consists of assessing social welfare. Figure 3 analyzes this more specifically by showing how social welfare changes with ϕ. It clearly reveals that, for the functional forms and the parameter values employed in our simulations, welfare is always a decreasing function of ϕ. This means that irrespective of the fact that finance may or may not positively affect growth, the overall welfare effect is negative. Moreover, we can also see that the rate at which social welfare falls with financial development is faster whenever the relationship between finance and growth is negative ($\phi > 1$). Hence, under our model's parametrization it would be socially desirable to discourage financial development at all, and to focus on other possible ultimate determinants of long run growth in order to raise welfare. Such a final result needs to be taken with some grain of salt since it may be driven by our choice of some relevant functional forms (see appendix A for an example of how a different functional form for $\xi(\cdot)$ may lead, instead, to a bell-shaped relationship between social welfare and financial development). In order to obtain more rigorous conclusions about finance-induced welfare effects, more accurate empirical studies will be needed to more clearly understand what might be the true shape of the relevant functional forms.

6 Discussion

In this paper we have developed a stylized, finance-extended model of endogenous growth which (under certain conditions) turns out to be consistent with the recent empirical evidence on the shape of the finance-growth nexus. In this section we briefly discuss to what extent the model put forward can also help us to understand some important features of the recent global financial crisis. Needless to say, such a discussion cannot ignore the analysis of the main (dis-)similarities between the two most important crises ever occurred, namely the Great Depression and the more recent Great Recession. The 1929 stock-market crash and the subsequent Great Recession can, indeed, be considered as the largest economic and financial crises that the world has ever experienced. For this reason, understanding the peculiarities of the Great Depression and its immediate links with the global financial crisis can provide us with further insights on the growth-finance relation. Why has a financial downturn turned into a deep depression in 1929? Which have been the basic hallmarks of that crisis? Is there any analogy (e.g., in terms of fundamental causes, length and depth) between the 1929 and the 2007-2008 crises? Trying to answer these questions is nowadays a priority for both academics and policymakers, and this can also allow us to understand whether our model can say anything at all about the ultimate traits of the recent financial crash and its economic consequences.

According to economic historians, there are two main analogies between the two crises. Specifically, both have been preceded by an extended period of sustained economic growth and, at the same time, both have been characterized by speculative bubbles arising from the flow of easy credit to households and firms, thus fueling both property-based and stock market-based excesses. During 1928, the Times Industrials (a pre-cursor of the actual Dow Jones) had gained a huge 35%, pushing many speculators (in an attempt to maximize their risky profits to finance their own purchase of stocks through borrowed money) to buy on average one-thousand dollars of stock by putting down just one-hundred dollars (Canterbery, 2011). Similarly, the immediate cause of the recent global crisis can be traced to a *"...rapid credit expansion and financial innovation that led to high leverage"*, according to Helbling (2009). As Bernanke (2010) has recently pointed out, although the most prominent reason behind the 2007-2008 economic crash was the prospect of losses on the sub-prime market induced by excessive leverage on the part of households, businesses and financial firms, other determinant factors did include the overreliance of banks on short term wholesale funding, deficiencies in private sector risk management, an overreliance on ratings agencies, and a failure of existing regulatory procedures worldwide.

Economic historians, however, believe that there are also substantial differences between the two crises, both in terms of what we can now label as "degree of financialization" and in terms of speed of recovery. The most salient difference is probably represented by the fact that in the time passed by between the Great Depression and the recent global financial crisis the nature of the capitalist system has changed in a fundamental way moving from a model of productive industrialization towards a model of financial capitalization (Canterbery, 2011). In this regard, some economists (see Wade, 2008, among others) notice that the world as a whole has undergone, starting from the early 1990s, a massive process of financial liberalization which has finally led to a huge shift from production to financial services. The main consequence of such a process of financial liberalization has been the fact that the financial sector has rapidly become an important part of the national economy: for example, between 1978 and 2005 the financial sector as a whole has grown from 3.5% to 5.9% in the US economy in GDP terms. Moreover, in the same economy while from the 1930s to around 1980 the growth rates of the financial and non-financial sectors have been roughly the same, from 1980 to 2005 the financial sectors profits have grown by 800% whereas those of the non-financial sector have grown by a more modest 250% (Canterbery, 2011). It is under this (completely new) mode of capitalism that the recent global financial crisis has taken place and propagated across the entire world. In addition, in the US another side-effect of the financial liberalization process mentioned above was also represented by the strong deregulation occurred in the sub-prime mortgage market which has rapidly become the least regulated part of the entire American mortgage market: it is estimated that while in 2000 the $130 billion of sub-prime lending was backed up with $55 billion of mortgage bonds, by 2005 those figures jumped to $625 billion sub-prime loans backed by $500 billion in securitized bonds (McNally, 2010). The creation of innovative financial instruments in the form of credit default swaps and other debt securities could only exacerbate the situation further: by 2006 the credit default swaps on mortgage bonds became eight times the value of the bonds themselves, allowing the associated wealth to be quickly wiped out with the crisis (McNally, 2010). In brief, the ability of financial firms to generate all kinds of conceivable financial innovations due to market-liberalization and deregulation seems to have been a distinctive trait of the recent economic crisis, as opposed to the 1929 crash.[10] Another significant difference between the two episodes of crisis, no matter whether the metric is represented by global industrial production, or global trade volumes, or else global equity valuations, resides in the relative speed of recovery following the crash in the two cases (see Figures 1, 2, and 3 in Eichengreen and O'Rourke, 2012; see also Reinhart and Rogoff, 2014). Indeed, while most economists now agree that the Great Depression has lasted for over ten years (Reinhart and Rogoff, 2014), Eichengreen and O'Rourke (2012) emphasize the relatively fast recovery during the most recent global financial crisis since its peak in April 2008, although the picture is not the same everywhere.[11]

In the light of this discussion, we believe that our model can capture in a very simple and intuitive way the two distinctive features of the latest 2007-2008 financial crisis as opposed to the 1929 crash. In this respect, the story that our model tells goes as follows. The increasing degree of financialization of capitalism occurred in the second half of the XXth century (which means having heavily relied upon debt as a major means of making risky profits) has finally led to a rising degree of financial innovation, taking the form of a fast proliferation of new, more complex and more varied forms of financial instruments (i.e., to more financial development, ϕ, in our model's words). The increase in the degree of financial development has, in turn, affected a country's speed of convergence, substantially reducing the length of recovery from an adverse financial shock in some cases (i.e., in those countries in which, according to our setting, the relationship between finance and growth is positive), but not in others (i.e., in those countries characterized by a non-positive relationship between finance and growth).[12] When read under this light, what our model ultimately does is to study the consequences of financial development (and, eventually, of the recent financial crisis) in terms of long-run economic growth, speed of recovery after a shock, and welfare. Despite the relative simplicity of our approach, to the best of our knowledge no other paper is able to provide a formal explanation about the nonlinear, and eventually non-monotonic, relationship between financial development and economic growth. Such a general lack of theoretical explanations calls for the need of identifying other

[10] *"...Liquidity and funding problems have played a key role in the financial sector transmission in both episodes. Concerns about the net worth and solvency of financial intermediaries were at the root of both crises, although the specific mechanics differed given the financial systems evolution"* (Helbling, 2009).

[11] *"Examining the evolution of real per capita GDP around 100 systemic banking crises reveals that a significant part of the costs of these crises lies in the protracted and halting nature of the recovery. On average it takes about eight years to reach the pre-crisis level of income; the median is about 6.5 years. Five to six years after the onset of the current crisis only Germany and the United States (out of 12 systemic crisis cases) have reached their 2007-2008 peaks in per capita income"* (Reinhart and Rogoff, 2014).

[12] The fact that financial development can either deter or spur economic recovery after a crisis (basically depending on a country/region's extant financial structure) is also supported by some empirical analyses. For example, Langfield and Pagano (2016) show that the "bank-bias" especially prevailing in European financial structure is certainly detrimental to the economic growth prospects of this portion of the world. Indeed, after documenting the extraordinarily fast expansion, particularly since the 1990s, of the banking sector in Europe (as compared to the corresponding US and Japan's experiences), Langfield and Pagano (2016) present compelling support to the hypothesis that Europe's bank-biased financial structure (namely, the fact that in this region the size of banks dwarfs that of the stock and private bond markets) is associated with greater systemic risk and worse economic growth performance than if its structure were more balanced.

possible channels (in addition to those we have already isolated and characterized here) through which finance and the real economy can be linked to each other in the short, as well as in the long run.

7 Conclusion

Finance and financial intermediation do play an important role in modern economies. Despite the huge body of empirical research that tries to assess the nature and the sign of the finance-growth nexus, existing theoretical works on the topic are more limited in number and in general do not explain why the relation between finance and growth might be nonlinear, and possibly non-monotonic (inverted-U), as recent evidence seems mostly to suggest. Moreover, the majority of the existing theoretical works focuses mainly on BGP outcomes, so neglecting the implications of financial development for short run transitional dynamics. Our paper represents a first attempt at filling these two important gaps in literature. By analyzing the relationship between finance and economic growth in an extended version of the Lucas-Uzawa model, we postulate that financial development affects physical capital accumulation by altering the amount of resources that can potentially be used for investment purposes (a more development financial system wastes less resources in the process of financial intermediation), and human capital accumulation via both a productivity and a depreciation channel. Thus, while on the one hand a higher degree of financial development eases and makes human capital investment more productive via a relaxation of agents' borrowing constraints, on the other hand, by fostering technological progress, it also causes the existing stock of human capital to be more subject to depreciation via faster obsolescence. We study under which conditions, along a BGP equilibrium, there may exist a non-monotonic relationship between growth and financial development, and find that this crucially depends upon the relative intensity of the productivity and depreciation effects at different levels of financial depth. In particular, we show that finance turns out to be harmful for economic growth whenever the productivity effect is smaller than the depreciation effect, which is eventually more likely to occur at higher levels of financial development. We also analyze the impact of financial development along the transition to the BGP equilibrium, and show that the speed of convergence depends upon the degree of financial development, as well. In particular we observe that finance reduces the length of the transitional process associated with economic growth whenever the productivity effect is larger than the depreciation effect. Thus, the different role played by finance in different economies might explain why the speed of recovery from the common, recent, and global financial crisis varies so largely from country to country. By considering a special case of our model, that is a framework in which the inverse of the intertemporal elasticity of substitution equals the physical capital share of output, we finally analyze the welfare effects associated with financial development. Under a realistic set of the model's parameters, we show that social welfare decreases with the degree of financial development, suggesting that, despite the fact that finance may be growth enhancing, in order to improve living standards it might be convenient to focus on other possible sources of long run growth.

We believe that, in its simplicity, our model is capable of capturing many of the fundamental hallmarks of the relation between finance and economic growth. However, everything comes at a cost. Indeed, while on the one hand we adopt an aggregative approach in order to maintain the model sufficiently simple and tractable and to have a neat picture of how (i.e., through which possible channels) financial intermediation can affect BGP growth and transitional dynamics, on the other hand the same aggregative approach does not allow to take into account the possible microeconomic mechanisms that may ultimately drive the impact of finance on economic growth. Extending our current analysis through a better understanding of such microeconomic mechanisms, by which financial intermediation is likely to simultaneously influence investment decisions in capital accumulation and education by individual agents along with those in research and development by innovative firms, might definitely contribute to shed a new light on the whole implications of financial development on the process of modern economic growth. We leave this challenging task to future research.

A Alternative Finance-Induced Welfare Effects

As mentioned in section 5, our discussion about the welfare effects may be largely driven by the specific functional forms used in our numerical exercise. In particular, we briefly discuss here how our results may change under a different specification of the $\xi(\phi)$ function, measuring how financial development subtracts resources to physical capital investments. Note that this affects only the transitional dynamics of consumption and eventually welfare, since this is completely irrelevant for long run growth rates. Assume that this function takes the following form:

$$\xi(\phi) = \frac{\xi}{\xi e^{\phi}},$$

with $\xi > 0$ such that $\xi(0) = 1$, $\xi(\infty) = 0$ and $\xi' < 0$. It takes thus exactly the same shape of the function discussed in the main text, and the only difference between the two specifications is related to the extent to which finance diverts resources from physical capital accumulation. We proceed as in the main text with a numerical simulation to illustrate the effects of financial development both in the short and long run. The parameters are assumed to take exactly the same values as in Table 1, and the results of our numerical simulations are shown in Figures 4 and 5.

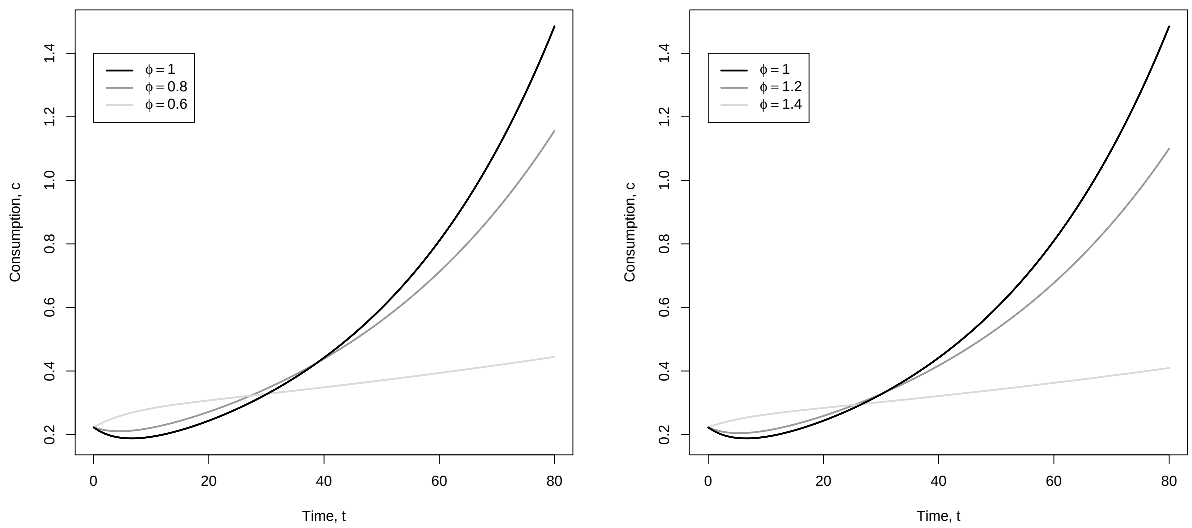

Figure 4: Evolution over time of consumption for values of $\phi \leq 1$ (left) and $\phi \geq 1$ (right).

Figure 4 shows that the dynamic evolution of consumption is qualitatively identical to what discussed in section 5. An intertemporal trade off is clearly present in both the $\phi \leq 1$ and $\phi \geq 1$ cases (note that since the specification of the $\theta(\cdot)$ and $\delta_h(\cdot)$ functions have not changed, the unity still represents the threshold value below and above which the relationship between finance and growth turns out to be positive and negative, respectively).

Even if the behavior of consumption is exactly the same, the size of the changes in consumption levels associated with financial development is not. Indeed, the welfare effects illustrated in Figure 5 are substantially different from those discussed in section 5. Under such a new specification of the $\xi(\cdot)$ function, for values of ϕ smaller than unity, welfare rises with the degree of financial development. This means that the nature of the finance and growth relationship does not determine only long run growth rates but also our conclusions about welfare effects. While financial development is not desirable if the relationship between growth and finance is negative ($\phi \geq 1$), it instead is whenever the relationship is positive ($\phi \leq 1$). The

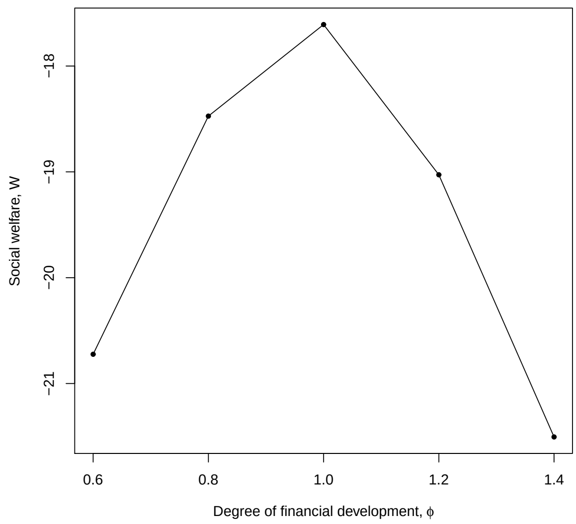

Figure 5: Relationship between social welfare and degree of financial development, ϕ.

eventual existence of bell-shaped welfare effects is an interesting point with potential policy implications. However, such a strong difference with the results discussed in the main text, which are exclusively due to the specific functional form of $\xi(\cdot)$, make it impossible to understand which of the two alternative scenarios is more plausible. This also suggests that more (empirical) work is needed in order to quantify the exact size of the total resources diverted from capital investments by financial intermediation, and thus which of two formulations is more consistent with real world observations.

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