Gaining insights on chemistry from the analysis of the charge density

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Giornate del CrisDi
The Role of Crystallography in Drug Science and Biology

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Outline

1. The Quantum Theory of Atoms in Molecules

2. Applications
   2.1 The problem of electron delocalization
   2.2 A tool for ‘seeing’ Non-Covalent Interactions in the real space
Outline

1. The **Quantum Theory of Atoms in Molecules**

2. Applications
   
   2.1 The problem of electron delocalization

   2.2 A tool for ‘seeing’ Non-Covalent Interactions in the real space
Quantum mechanics: the wavefunction $\psi(r,R)$ contains all the conceivable information of a given quantum system in a stationary state.

But: molecular orbitals are usually delocalized over a significant part of the nuclear ensemble.

The quantum description, despite being formally correct, often lacks of interpretability from the viewpoint of the synthetic chemist.
Do atoms in molecules even exist?

Synthetic chemist: yes

Physicist: What really matters is the total wavefunction \( \Psi(r, R) \)

Which, however, it is NOT an experimental observable.
The central role of the electron density

In the real space, chemistry is governed by the electron density:

\[ \rho(\mathbf{r}) = N \int_{-\infty}^{+\infty} \psi^*(\mathbf{r}_1...\mathbf{r}_N, \mathbf{R}_1...\mathbf{R}_M)\psi(\mathbf{r}_1...\mathbf{r}_N, \mathbf{R}_1...\mathbf{R}_M) \, ds_1 \, d^4\mathbf{r}_2...d^4\mathbf{r}_N \, d^4\mathbf{R}_1...d^4\mathbf{R}_M \]

It expresses the probability density (per unit volume) of finding an electron at point \( \mathbf{r} \), independent from its spin, and from spins and positions of all the other electrons and nuclei. By definition, it is always positive.

It is a continuous and differentiable scalar field in \( \mathbb{R}^3 \), with maxima (usually) at nuclei and minima either at \( r \to \infty \) or at the centre of closed atom cages.
The central role of the charge density

\[ \rho(r) = \frac{1}{V} \sum_{hkl=-\infty}^{\infty} F_{hkl} e^{-2\pi i (hx+ky+lz)} \]

\[ F_{hkl} = \int_{V} \rho(r) e^{2\pi i (hx+ky+lz)} d^3 r \]

\[ \rho(r) = N \int_{-\infty}^{+\infty} \psi^* \psi d r' d R \]

\[ -\frac{\hbar^2}{2m} \nabla^2 \psi + V(r) \psi = E \psi \]

Accessible from the experiment

Accessible from quantum simulations

Chemical bond
Molecular properties
Non Covalent Interactions
To define atoms in molecules, thus reconciling the quantum perspective with the chemist’s one, choosing a partitioning criterion is mandatory.
Quantum Theory of Atoms in Molecules

R. F. W. Bader defines an ‘atom’ through the topology of the $\rho(r)$ scalar field (Atoms in Molecules, AIM, approach)

**Topological atom**: a nucleus + the region of space enclosed by surfaces $S$ which are never crossed by the gradient lines of $\rho(r)$, i.e.

$$\int_S \nabla \rho(r) \hat{n} dS = 0$$

The topological atom

Gradient vector field

Charge density curves
The topological atom

Zero-flux surface

\[ \oint_S \nabla \rho(r) \hat{n} dS = 0 \]

Charge density curves

Gradient vector field
The topological atom

Zero-flux surface

\[ \int_S \nabla \rho(r) \hat{n} dS = 0 \]

Atomic basin \( \Omega \)

Gradient vector field

Charge density curves

Atomic basin \( \Omega \)
Critical points

\[ \nabla \rho (r_{cp}) = 0 \]  
A point \( r_{cp} \) is called a critical point if the gradient at that point is null.

Different critical points are characterized by the eigenvalues of the Hessian matrix, which quantifies the principal curvatures \( \lambda_1 \), \( \lambda_2 \) and \( \lambda_3 \) of \( \rho \) at that point.

\[
\begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{bmatrix}
\]

By convention, \( \lambda_1 < \lambda_2 < \lambda_3 \).
Critical points

Maximum (nuclear positions)

$\lambda_1 < \lambda_2 < \lambda_3 < 0$
Critical points

\[ \lambda_1 < \lambda_2 < 0 < \lambda_3 \]

**Bond critical point** (saddle): representative of chemical interactions

**Maximum** (nuclear positions)

\[ \lambda_1 < \lambda_2 < \lambda_3 < 0 \]
Critical points

\[ \lambda_1 < \lambda_2 < 0 < \lambda_3 \]

**Bond critical point** (saddle): representative of chemical interactions

A bond critical point is always associated to an **atomic interaction line**, i.e. it marks pairs of interacting atoms

**Maximum** (nuclear positions)

\[ \lambda_1 < \lambda_2 < \lambda_3 < 0 \]
The topological atom

Zero-flux surface

\[ \oint_S \nabla \rho(r) \hat{n} dS = 0 \]

Atomic basin \( \Omega \)

Charge density curves

Gradient vector field

Atomic basin \( \Omega \)
The topological atom

Zero-flux surface

\[ \int_{S} \nabla \rho(r) \hat{n} dS = 0 \]

Atomic basin \( \Omega \)

Critical points

\[ \nabla \rho(r) = 0 \]

Gradient vector field

Charge density curves

Atomic basin \( \Omega \)
The topological atom

Zero-flux surface

\[ \int_S \nabla \rho(r) \hat{n} dS = 0 \]

Atomic basin \( \Omega \)

Critical points

\[ \nabla \rho(r) = 0 \]

Gradient vector field

Maxima Bond

Charge density curves
The topological atom

Zero-flux surface

$$\int_S \nabla \rho(r) \hat{n} dS = 0$$

Atomic basin $$\Omega$$

Critical points

$$\nabla \rho(r) = 0$$

Maxima

Bond

Charge density curves

Atomic Interaction Line

A line defining the maximum charge density path between two nuclei, representative of atomic interactions

Gradient vector field
Laplacian of the charge density

\[ \nabla^2 \rho(r) = \lambda_1 + \lambda_2 + \lambda_3 \]
Laplacian of the charge density

Following the Gauss theorem:

\[ \int_V \nabla^2 \rho(r) \, dr = \int_S \hat{n} \nabla \rho(r) \, dS \]

More gradient lines enter the volume than the exiting ones: **charge is being accumulated** in the neighborhood of the critical point.

\[ \nabla^2 \rho(r) < 0 \]

Less gradient lines enter the volume than the exiting ones: **charge is being depleted** in the neighborhood of the critical point.

\[ \nabla^2 \rho(r) > 0 \]
Laplacian of ED

\[ \nabla^2 \rho(r_{cp}) = \lambda_1 + \lambda_2 + \lambda_3 \]

\[ \nabla^2 \rho(r) > 0 \quad \text{dashed lines): electron-poor regions} \]
\[ \nabla^2 \rho(r) < 0 \quad \text{full lines): electron-rich regions} \]

GLYCINE: 10835 measured intensities at T = 23 K
Laplacian of ED

\[ \nabla^2 \rho(r_{cp}) = \lambda_1 + \lambda_2 + \lambda_3 \]

\( \nabla^2 \rho(r) > 0 \) (dashed lines): electron-poor regions

\( \nabla^2 \rho(r) < 0 \) (full lines): electron-rich regions

GLYCINE: 10835 measured intensities at \( T = 23 \) K

Shared interaction (C=O covalent bond): \( \nabla^2 \rho < 0 \)

Closed shell interaction (OH\( \cdot \cdot \cdot \cdot \)O hydrogen bond): \( \nabla^2 \rho > 0 \)
Laplacian of ED

$\nabla^2 \rho = 0$ isosurfaces enclose regions of **charge concentration**

Co(CO)$_6$

FeCr$_2$S$_4$

6741 measured intensities at $T = 23$ K

Areas of charge concentration match areas of charge depletion

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Local form of the virial theorem

It relates the kinetic energy density, $G(r)$ (always $>0$), to the potential energy density $V(r)$ (always $<0$) at $r$ by means of the Laplacian of the charge density.

$$\frac{\hbar^2}{4m} \nabla^2 \rho(r) = V(r) + 2G(r)$$

**Laplacian of ED**

If $G(r)$ is known, for example by means of the Abramov’s functional or quantum calculations, $V(r)$ can be deduced from $\nabla^2 \rho$

$\nabla^2 \rho(r) < 0$  Prevailing $V(r)$, bound state

$\nabla^2 \rho(r) > 0$  Prevailing $G(r)$, unbound state

→ The strength of a given interaction can be estimated
Atomic properties

3D representation of atomic basins, each bounded by its zero-flux surface, for the AH series, A being any 2\textsuperscript{nd}-period element from Li to F.

Topological atoms are (quite well) transferable among different chemical systems.

Relative electronegativity determines the relative volumes.
Atomic properties

Atomic volume: \[ V(\Omega) = \int_{\Omega} d\mathbf{r} \]

Atomic charge: \[ q_\Omega = Z - \int_{\Omega} \rho(\mathbf{r}) d\mathbf{r} \]

Atomic energy: \[ E(\Omega) = -T(\Omega) \]

Atomic dipole moments:
\[ \mu_{x,\Omega} = \int_{V=\Omega} \rho(\mathbf{r}) \cdot r_x \cdot d\mathbf{r} \]
\[ \mu_{y,\Omega} = \int_{V=\Omega} \rho(\mathbf{r}) \cdot r_y \cdot d\mathbf{r} \]
\[ \mu_{z,\Omega} = \int_{V=\Omega} \rho(\mathbf{r}) \cdot r_z \cdot d\mathbf{r} \]
Molecular properties

Molecular volume: \[ V_{Mol} = \sum_{\Omega} V(\Omega) \]

Molecular charge: \[ q_{Mol} = \sum_{\Omega} q_{\Omega} \]

Molecular energy: \[ E_{Mol} = \sum_{\Omega} E(\Omega) \]

Molecular dipole moments:

\[ \mu_x = \sum_{\Omega} \left[ \mu_{\Omega,x} + x_{\Omega} q_{\Omega} \right] \]

\[ \mu_y = \sum_{\Omega} \left[ \mu_{\Omega,y} + y_{\Omega} q_{\Omega} \right] \]

\[ \mu_z = \sum_{\Omega} \left[ \mu_{\Omega,z} + z_{\Omega} q_{\Omega} \right] \]

Intrinsic atomic polarization

Charge transfer
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The Source Function


$$\rho(\mathbf{r}) = \int_{all \ space} LS(\mathbf{r}, \mathbf{r}') \cdot d\mathbf{r}'$$
The Source Function

\[ \rho(r) = \int_{all\ space} LS(r, r') \cdot dr' \]

\[ LS(r, r') = -\frac{1}{4\pi} \frac{1}{|r - r'|} \nabla^2 \rho(r') \]

Local Source
The Source Function

\[ \rho(r) = \int_{all \ space} \text{LS}(r, r') \cdot dr' \]

\[ \text{LS}(r, r') = -\frac{1}{4\pi} \frac{1}{|r - r'|} \nabla^2 \rho(r') \]

Green function (influence function). It represents the effectiveness of how the cause (\(\nabla^2 \rho(r')\)) contributes to the effect \(\rho(r)\)
The Source Function

\[
\rho(r) = \int_{all\ space} LS(r, r') \cdot dr' = \sum_{\Omega} \int LS(r, r') \cdot dr' = \sum_{\Omega} S(r, \Omega)
\]

\[
LS(r, r') = -\frac{1}{4\pi} \frac{1}{|r - r'|} \nabla^2 \rho(r')
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Green function (influence function). It represents the effectiveness of how the cause (\(\nabla^2 \rho(r')\)) contributes to the effect \(\rho(r)\)
The Source Function

\[ \rho(\mathbf{r}) = \int_{all \ space} \mathbf{LS}(\mathbf{r}, \mathbf{r}') \cdot d\mathbf{r}' = \sum_{\Omega} \int_{\Omega} \mathbf{LS}(\mathbf{r}, \mathbf{r}') \cdot d\mathbf{r}' = \sum_{\Omega} S(\mathbf{r}, \Omega) \]

\[ \mathbf{LS}(\mathbf{r}, \mathbf{r}') = -\frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|^2} \nabla^2 \rho(\mathbf{r}') \]

**Source Function** from the basin \( \Omega \) to the point \( \mathbf{r} \)

**Local Source**

**Green function** (*influence* function). It represents the effectiveness of how the *cause* \( (\nabla^2 \rho(\mathbf{r}')) \) contributes to the *effect* \( \rho(\mathbf{r}) \)
The charge density $\rho(r)$ at any point $r$ is *intrinsically* influenced by all the atomic basins in the system: this is well-known in DFT theory, as $\rho(r)$ is an unique function of the exchange-correlation potential

$$\rho(r) = \sum_\Omega S(r, \Omega)$$

*The Source Function accounts for this non-locality*, allowing one to investigate which is the *relative influence* of an atom, or a group of atoms, in determining the *local properties* of the electron distribution at distant points.
The Source Function (SF) can be used to study **electron conjugation** (and **aromaticity**) in the **real space**.

These concepts **lack rigorous definitions**, as they are not directly associated to quantum-mechanical observables.

The SF **focuses on the charge density observable**. It quantifies the different influence that distant atoms provide to a given **reference point** as a function of (i) the distance from the $\pi$ nodal plane and (ii) the kind of the insaturation.

**Bigger spheres** = larger % SF contribution to the point. **Yellow spheres** mark negative SF contributions.


$z = 0 \text{ a.u.}$
The Source Function (SF) can be used to study **electron conjugation** (and **aromaticity**) in the **real space**.

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**Yellow spheres** mark negative SF contributions.

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$z = 0$ a.u.

$z = 2$ a.u.
The Source Function as a descriptor of local aromaticity


Source Function Local Aromaticity Index (SFLAI)

\[
\text{SFLAI} = 1 - \frac{c}{6} \sqrt[6]{\sum_{\Omega=1}^{6} \left( \sum_{b=1}^{6} \text{SF}_{\Omega b} \right)^2}
\]

Normalization constant, to have SFLAI = 0 in cyclohexane

SF % contributions to the \( b \)th C-C bond critical point in the polycyclic hydrocarbon

SF % contributions to any C-C bond critical point in the benzene

More benzene-like

Less benzene-like

Ring I

Ring II

SFLAI 0.712 0.678

More benzene-like

Less benzene-like

The SFLAI results agree well with the outcomes of other quantum local aromaticity descriptors and also with the chemical sense, as positions 9 and 10 are known to be reactive in anthracene.
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Reduced density gradient

In DFT theory, the Reduced Density Gradient (RDG) is defined as

\[
s(r) = \frac{\left| \nabla \rho(r) \right|}{2 \left(3 \pi^2 \right)^{1/3} \rho(r)^{4/3}}
\]

RDG describes *local deviations of the electron density* (ED) *from a homogeneous electron gas*, that has \(s(r) = 0\) everywhere.

\(s(r)\) is *large* in regions *far from the nuclei*, where the total \(\rho \to 0\) exponentially and the denominator approaches zero faster than the numerator.

*Its lowest bound is 0*, which is achieved every time \(\nabla \rho(r) = 0\).
RDG properties

Generalizing…

$$\nabla \rho (\mathbf{r} \rightarrow \mathbf{r}_{cp}) \approx 0 \quad \Rightarrow \quad s(\mathbf{r} \rightarrow \mathbf{r}_{cp}) \approx 0$$

Low RDG values might be related to atom-atom interactions


If molecules in the solid state are analyzed, sharp spikes in the RDG vs ED plot appear in the low ED region → These are the signature of non-covalent interactions.

\[ s(r) = \frac{\left| \nabla \rho(r) \right|}{2 \left( 3 \pi^2 \right)^{1/3} \rho(r)^{4/3}} \]


RDG properties

The idea of Johnson et al.: plotting $\text{sign}(\lambda_2) \cdot \rho(r)$ onto low-values RDG isosurfaces

As ED is always positive, $\text{sign}(\lambda_2) \rho(r) > 0$ values should be related to allegedly repulsive interactions, while $\text{sign}(\lambda_2) \rho(r) < 0$ ones should highlight zones of ED buildup associated to specific NCI patterns

The idea of Johnson et al.: plotting $\text{sign}(\lambda_2) \cdot \rho(r)$ onto low-values RDG isosurfaces

Formic acid cyclic dimer
RDG surfaces

The idea of Johnson et al.: plotting $\text{sign}(\lambda_2) \cdot \rho(r)$ onto low-values RDG isosurfaces

$\text{sign}(\lambda_2) \cdot \rho(r) > 0$
O⋯O, C⋯C nonbonded contacts

Formic acid cyclic dimer $\text{sign}(\lambda_2) \cdot \rho(r) < 0$
Strong H⋯O hydrogen bonds

The color bar is related to the strength of the interaction: red $\rightarrow$ strongly attractive; blue $\rightarrow$ strongly repulsive
On the other hand, flat and unstructured RDG surfaces, associated to $\text{sign}(\lambda_2) \cdot \rho(r)$ values close to 0, appear among facing aromatic rings.

*Is this a tool to ‘see’ dispersive interactions?*

**No!** Dispersive interactions are always present among interacting charge distributions; flat RDG surfaces mean that the ratio between the gradient vector module and the charge density is rather constant in space.

RDG surfaces: physical meaning

At the same time, they are colored green as the ED in region not dominated by directional interactions is low and the sign($\lambda_2$) factor is rather indeterminate.

Such surfaces mark zones of ED where no directional strong(er) interactions are present (steric clashes, hydrogen bonds...), being therefore dominated by dispersive/repulsive contributions.

Conclusions

The topological analysis of the \( \rho(r) \) scalar field according to the Quantum Theory of Atoms in Molecules allows one to unequivocally partition any molecule into disjoint atomic basins connected by atomic interaction lines. Though the Bader’s partitioning scheme is by itself as much arbitrary as any other one, it is rooted in quantum mechanics and allows to naturally recover the concept of atomicity from the charge density observable.

The Laplacian is related to the charge concentration and depletion regions of the charge density. Through the virial theorem in its local form, it provides information on the relative balance between potential and kinetic energy densities in a given region of space.
Conclusions

The Source Function provides insights on the properties of delocalization patterns and aromaticity.

The RDG-based NCI descriptor allows to distinguish regions dominated by different networks of non-covalent interactions.

All the topological descriptors are based on the charge density observable, amenable to experimental determination.

Single-crystal X-ray diffraction is much more than a tool to investigate the molecular structure. It can be used to experimentally grant access to a wealth of chemical information, including intermolecular interaction energies, nature and strength of weak intermolecular bonds, chemical correlations among distant atoms, nucleophilic or electrophilic hot-spots on the molecular surface, electrostatic moments, and so on.
Thank you for your kind attention
The quantum description

Any sensible description of the matter at the molecular and sub-molecular level must rely on quantum mechanics.

Electrons and nuclei should be described in terms of the probability amplitudes (wavefunctions, $\Psi$) of finding them in a certain region of the spacetime. The $\Psi$'s depend on the coordinates of electrons ($r$), nuclei ($R$), and possibly on time ($t$), if the system is not stationary.

$\Psi$ must satisfy the Schrödinger equation

$$\hat{H} \Psi(r, R, t) = i\hbar \frac{\partial \Psi(r, R, t)}{\partial t}$$
Quantum Theory of Atoms in Molecules

\[ \oint_{S(\Omega)} \nabla \rho(\mathbf{r}) \hat{n} dS = 0 \]

This choice avoids ambiguities in the definition of the electronic kinetic energy

\[
T(\Omega) = -\frac{\hbar^2}{4m} N \int_{\Omega} d\mathbf{r} \int \left( \Psi^* \nabla^2 \Psi + \Psi \nabla^2 \Psi^* \right) d\tau' = \\
= \frac{\hbar^2}{2m} N \int_{\Omega} d\mathbf{r} \int \left( \nabla \Psi \cdot \nabla \Psi^* \right) d\tau' - \frac{\hbar^2}{4m} \oint_{S(\Omega)} \hat{n} \cdot \nabla \rho(\mathbf{r}) dS
\]

Analysis of X-ray diffracted intensities

\[ I_{\text{meas}}(\mathbf{H}) \div |F_{\text{obs}}(\mathbf{H})|^2 \]

\[ F_{\text{obs}}(\mathbf{H}) = \int \rho(\mathbf{r}) \exp{(2\pi i \mathbf{H} \cdot \mathbf{r})} d\mathbf{r} \]

1. Conventional analysis of \( \rho(\mathbf{r}) \)

\[ \rho(\mathbf{r}) = \frac{1}{V} \sum_{hkl} F_{\text{obs}}(\mathbf{H}) \exp{(-2\pi i \mathbf{H} \cdot \mathbf{r})} \]

2. Multipole analysis of \( \rho(\mathbf{r}) \)


\[ \rho(\mathbf{r}; Q_e) = \sum_{p=1}^{N} \sum_{l=1}^{L} \left[ \sum_{m=0}^{l} C^e_{plm} \cdot B^e_{plm}(\mathbf{r} - \mathbf{R}_p) + \sum_{m=1}^{l} C^o_{plm} \cdot B^o_{plm}(\mathbf{r} - \mathbf{R}_p) \right] \]
## RDG surfaces vs. topological descriptors

<table>
<thead>
<tr>
<th></th>
<th>RDG-NCI</th>
<th>AIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atom-atom picture</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$\nabla \rho(r_{cp}) = 0$</td>
<td>Not strictly required</td>
<td>Bond critical point required</td>
</tr>
<tr>
<td>NCI classification</td>
<td>Attractive-repulsive</td>
<td>Open-closed shell; A cp always implies a stabilizing interaction</td>
</tr>
</tbody>
</table>
Critical points

Critical points are classified according to their rank (number of nonzero Hessian eigenvalues) and their sign (number of positive Hessian eigenvalues minus the number of Hessian negative eigenvalues).

- **Cage critical point** (minimum)
  - (3,+3) \( 0 < \lambda_1 < \lambda_2 < \lambda_3 \)

- **Ring critical point** (saddle)
  - (3,+1) \( \lambda_1 < 0 < \lambda_2 < \lambda_3 \)
Critical points

\[ \nabla^2 \rho(r_{cp}) = \frac{\partial^2 \rho(r_{cp})}{\partial x^2} + \frac{\partial^2 \rho(r_{cp})}{\partial y^2} + \frac{\partial^2 \rho(r_{cp})}{\partial z^2} \]

**ED Laplacian:** *the trace of the Hessian matrix.* Provided that the latter is diagonalized:

\[ \nabla^2 \rho(r_{cp}) = \lambda_1 + \lambda_2 + \lambda_3 \]

To understand the physical meaning of the ED Laplacian, integrate it in a neighborhood, small to desire, of a critical point:

\[ \int_V \nabla^2 \rho(r_{cp}) d\mathbf{r} = \int_V \nabla \nabla \rho(r_{cp}) d\mathbf{r} \]
The multipole formalism

Basis Functions

$B_{plm} (r - R_p)$

VALRAY code

XD code
The multipole formalism

Basis Functions

Fourier transform

Generalized scattering factors, $gsf$

$B_{plm}(r - R_p)$

VALRAY code

XD code
The multipole formalism

Basis Functions

Fourier transform

Generalized scattering factors, $gsf$

$B_{plm}(r - R_p)$

Positional parameters

Non lin. Comb.

Thermal parameters

Electron population parameters

VALRAY code

XD code

$C_{plm}$
The multipole formalism

Basis Functions

Fourier transform

Generalized scattering factors, \( gsf \)

Non lin. Comb.

VALRAY code

XD code

Iterative optimization

(least squares refinement)

\( B_{plm}(r - R_p) \)

Positional parameters

Thermal parameters

Electron population parameters

\( F_{hkl} \) calc

\( F_{hkl} \) obs

Least squares refinement
The multipole formalism

Basis Functions

Fourier transform

Generalized scattering factors, $g_{sf}$

$C_{plm} B_{plm} (r - R_p)$

Optimized electron population parameters

Positional parameters

Thermal parameters

Electron population parameters

Iterative optimization

(least squares refinement)

$F_{hkl}^{calc}$

$F_{hkl}^{obs}$

VALRAY code

XD code

Non lin. Comb.
Outline

1. The Quantum Theory of Atoms in Molecules

2. Charge density from single-crystal X-ray diffraction

3. Applications
Requirements for an accurate determination of $\rho(r)$

- High-quality crystals
- Excellent instrumentation
- Low Temperature
CHARGE DENSITY FROM THE EXPERIMENT

High-quality crystals

~ 0.5 mm, C$_6$H$_{12}$O$_5$

Austdiol

Ø ~ 0.35 mm, C$_{16}$H$_{10}$O$_2$

Bis-carbonyl[14]annulene
WARNING!

Daubreelite, $\text{FeCr}_2\text{S}_4$
CHARGE DENSITY FROM THE EXPERIMENT

Excellent instrumentation

Precise and accurate goniometers

Sensitive detectors
(state-of-art CCD, Silicon-pixel photoelectric detectors…)

Intense X-ray beams
(rotating anodes, liquid metal anodes, focusing optics…)
CHARGE DENSITY FROM THE EXPERIMENT

Low Temperature
CHARGE DENSITY FROM THE EXPERIMENT

Why low temperature?

Reduction of the Thermal Diffuse Scattering (TDS)

Increase of the number of measured intensities

Easier deconvolution of the thermal motion from the static electron density

\[ \bar{\rho}(r) = \sum_{m} \left\{ \frac{\exp(-E_m/kT)}{\sum_i \exp(-E_i/kT)} \int \chi_m^*(R) \rho(r;R) \chi_m(R) dR \right\} \]

Vibrational wavefunction