A Journey to the Center of the Earth: Cosmology and the Centrobaric Theory from Antiquity to the Renaissance

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Abstract: This chapter aims to throw light on the ways in which the concept of center of gravity interacted with some of the cosmological ideas conceived in antiquity and in particular with the idea of the figure of earth as presented in Aristotle's *De coelo*. Developing earlier research, this study provides a better understanding of the scientific discussion that took place during the crucial first stage in the development of modern science. The origins and earliest stages in the development of the concept of center of gravity in Ancient Greece was briefly studied by Duhem, whose cursory analysis of a few texts by Pappus and Archimedes was undertaken with the specific purpose of showing the supposed faults inherent in Greek statics. The chapter will begin with a discussion of these Greek sources and attempt to follow the intellectual recovery of this key concept in the Renaissance and the first decades of the seventeenth century, relying in particular on a thorough study of textbooks used for teaching astronomy in Jesuit schools, for example, *Commentaries on De sphera* of Sacrobosco.

Introduction

The theory of the center of gravity played an important role in mathematical and mechanical studies in antiquity. It was an essential part of Archimedes' geometrical works: its most thorough exposition is to be found in the treatise *Ἐπιπέδων ἰσορροπιῶν* (*On the Equilibrium of Planes*). The most important result of this work was the demonstration of the law of the equilibrium of the balance: this result was not an isolated case but was used to solve complex mathematical problems, such as, first of all, those concerning the area or volume of figures bounded by curved lines or surfaces, as can be seen in the book *On the Quadrature of the Parabola*, and further as a method for finding the ratios between the volumes of parts of regular solids inscribed and circumscribed within or without the solid for which the volume is sought. This second method using the centrobaric theory was utterly unknown before the discovery in 1906 of a letter written by Archimedes to Eratosthenes. This treatise, which is known as *On the Method* shows the great potential of such kinds of mechanical considerations for the solution of geometrical problems. But this potential does not seem to have been appreciated by later mathematicians. The concept of center of gravity, though peculiar to Archimedes' great mathematical talent, did not disappear from mathematical thought in later periods, but it was mostly and successfully employed in mechanics. During the Renaissance, the study of the so-called "simple machines" (balance, lever, pulley capstan or wheel-and-axle, wedge) became one of the main fields in which the centrobaric theory re-emerged.[1]

The thorough study made in the sixteenth century of the Greek and Roman writings on mechanics and technology played an essential role in the so-called Scientific Revolution and many recent scientific contributions by historians of science have correctly stressed the importance of the Archimedean tradition within this group of works. Compared to this great process of recovery of ancient texts, the slow and subdued penetration of the new learning into the works of the sixteenth century that were developed for teaching purposes has remained in the background. It has perhaps been often assumed that works of this kind were not open to new ideas, though in fact they also underwent important transformations. This for example is the case with the treatise *De sphera* by Johannes de Sacrobosco (John of Holywood). After having been the subject of several commentaries over four centuries, it experienced a substantial updating in the sixteenth century, which subsequently changed the way in which extensive sections of the text were understood. Of particular interest for this paper is the introduction in this treatise of the concept of 'center of gravity,' which, though it was already present in earlier times, was then completely revised and corrected.

In the following, I shall try to shed light on the ways in which the centrobaric theory interacted with some of the cosmological ideas conceived in antiquity and in particular with the idea of the figure of the earth presented in Aristotle's *De coelo*. This research, though not entirely new, is nevertheless necessary for a better understanding of the scientific discussion that took place during the crucial first stage of the development of modern science.

Prelude: The Different Treatments of the Centrobaric Theory by some Historians of Mechanics

Though there are many and various writings by seventeenth-century authors that deal with the center of gravity in Archimedes' works, up until the last decade of

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[1] To the study of the centrobaric theory in the field of mechanics was later joined a more strictly mathematical study concerning the determination of rigorous methods for finding the center of gravity of solids. The lack of ancient specific texts made this study more complex and difficult, entailing a discontinuity between the two stages of development.

the nineteenth century the complex development of the centrobaric theory has not been studied as an important topic by historians of mathematics or physics. Considering Pierre Duhem's numerous important studies on this subject, such a statement appears at first sight rather absurd: yet this was the situation that clearly emerged from reading some of the most important works in the field published before Duhem.

The state of affairs seems to have been a direct consequence of the new system of mechanical theory elaborate by Lagrange, who in the historical sections of the *Mecanique analytique* (1788) pointed out the main stages in the development of the mechanical principles and, in the case of statics, the three general principles at the foundation of the laws of equilibrium: the principle of the lever, the principle of the composition of forces and the principle of virtual velocities. From an historical point of view, the centrobaric theory had been closely connected with the study of the law of the lever, but this was not enough to assure the survival of the long tradition of the theory within the new development of mechanics outlined by Lagrange. The fragile connection linking the study of the centers of gravity with the research concerning the laws of equilibrium had in fact been broken forever. "The equilibrium results from the destruction of several forces that fight against each other and annihilate reciprocally the action they exert on each other."[2] In this brief definition, we find the main reason for the lack of interest in the centrobaric theory.

Studies on the center of gravity, which had been developed from antiquity to the earlier decades of the seventeenth century, were thus removed from the main branches of the history of mathematics. They did not disappear completely but left a trace of their history in the general principle of the lever. This faint trace, which the following historians of mechanics were inclined to ignore, came back into full view towards the end of the nineteenth century. This made it possible to gain a more precise picture of the development of this branch of mathematics and of its past achievements.

For the moment, the dominating view was that dictated by the *Mecanique analytique*, as can be seen from major nineteenth-century works on the history of mechanics such as Eugen Karl Dühring's *Kritische Geschichte der allgemeinen Principien der Mechanik* (1873), and Ernst Mach's *Die Mechanik in ihrer Entwicklung: Historisch-kritisch dargestellt* (Mach 1988) in which he presented a rather detailed analysis of what the Siracusan mathematician had written on equilibrium.

[2] "La Statique est la science de l'équilibre des forces […] L'équilibre résulte de la destruction de plusieurs forces qui se combattent et qui anéantissent réciproquement l'action qu'elles exercent les unes sur les autres; et le but de la Statique est de donner les lois suivant lesquelles cette destruction s'opère. Ces lois sont fondées sur des principes généraux qu'on peut réduire à trois ; celui du *levier,* celui de la *composition des forces,* et celui des *vitesses virtuelles*." Lagrange (1811, 1–2).

From the study of Archimedes' text, Mach went on to make important epistemological reflections, but without making any reference to the concept of center of gravity. By now, reflections on equilibrium seemed to be disengaged from any of the fundamental concepts that determined the discussions on this subject during most of the previous centuries. This, however, was not exactly the case. Mach had also found fault with the Archimedean demonstration from another point of view: according to him, on the basis of just those assumptions given at the beginning of the text, it would have been impossible to deduce the law of the lever (Mach 1988, 38–39). The answer to this criticism prompted the recovery of the Archimedean text relating to the centers of gravity.

What seemed to have completely disappeared from the history of mechanics reappeared thanks to the Italian scholar Giovanni Vailati, who in a paper presented at the International Congress of Historical Sciences (April 1–9, 1903) tried to show that Archimedes could have been able to deduce the law of the lever from

> certain reflexions concerning the center of gravity, to which he briefly refers several times in his demonstrations without insisting at length on them, as if it were a subject already discussed in some previous treatise which was lost (Vailati 1904, 245).

Mach briefly discussed Vailati's argument in the seventh edition of his *Die Mechanik* (1907), and this discussion, reproduced in all the subsequent editions of Mach's work, made it imperative to go back and study the ancient texts.

That same edition of 1907 contained a long *excursus* summarizing the result of Pierre Duhem's research made in the first years of the twentieth century. Mach's interest was mainly focused on the studies collected in the first volume of *Les origines de la statique* (Duhem 1905), where the French physician had brought back to light the medieval *scientia de ponderibus* ('science of weights') and presented its development from its beginning to its late followers (Mach 1988, 101–8). But for the history of the centrobaric theory, Duhem's studies collected in the second volume of its work (1906) are much more interesting: there he adopted for the first time a method of investigation which was going to be systematically practiced in the ten volumes of his *Le système du monde.*[3]

We have thus come to the great revival of the study of the theory of the centers of gravity promoted by the French historian's unremitting researches, but before discussing in detail his works we must also remember the important contributions to that study given by the Italian historian, Raffaello Caverni.

Though Caverni is less famous than Duhem, his work in the history of science is no less valuable, and he was perhaps more rigorous in his analysis of manuscript and printed sources. In his *Storia del metodo sperimentale in Italia*, volume IV (Caverni 1895), Caverni discussed at length the development of the studies on the center of gravity and on the equilibrium of the balance from antiquity to the

[3] Volumes I–V, 1913–1917; volumes VI–X were published posthumously by his daughter, 1953–1959.

seventeenth century. Caverni's research greatly contributed to the resumption of the Archimedean tradition in the sixteenth century, and it presented the development of ancient mechanics in an original though not entirely new way. Caverni assumed a clear continuity between Archimedes' work and the pseudo-Aristotelian *Mechanical Problems*. The principle of the balance as established in this work was thus connected with the origin of the concept of the center of gravity. On what was this belief based? The author of the *Mechanical Problems* had pointed out that each point on the radius of a rotating circle moves with different speed according to its distance from the center, which is at rest. He then distinguished two components of that motion: a rectilinear natural movement downwards and a lateral violent motion towards the stationary center of rotation, which increased the nearer the points were to the center, was the cause of the lesser speed of the points nearer to the center. This explanation showed that there was a relation between the weight that was moved, the moving power and the different speed of the points at which the power was applied. This result was based on the analysis of different motions. How could this way of proceeding be connected with the demonstration of the law of the lever later established by Archimedes? In Archimedes' surviving writings there is no reference whatsoever to possible movement or speed. Caverni nevertheless thought that what was established in the first book of Archimedes' *On the Equilibrium of Planes* had somehow originated from the *Mechanical Problems*.[4]

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[4] The explanation of the functioning of the lever in the *Mechanical Problems* is vague. This fact was pointed out more than once by Renaissance scientists following the Archimedean tradition. The vagueness was explained as a first imperfect result of the earliest investigations on the lever, which were later to be fully developed only in Archimedes' work and in the centrobaric theory. This point of view is clearly presented in Guidobaldo del Monte's preface to his edition of Archimedes' work *In duos Archimedis aequiponderantium libros paraphrasis*, Monte (1588, 4). The same point of view is present in Archimedes' biography by Bernardino Baldi: "Since Archimedes (as it is probable and as Guidobaldo himself guessed in the preface to Book One of *On the Equilibrium of Planes*) regarded this Aristotelian work as being based on solid principles, but not being very clear in explaining them, he wanted to make it more explicit and more easily understandable by adding mathematical demonstrations to physical principles. Aristotle solved the problem of why the longer the lever, the easier it moves the weight, by saying that this happens because of the greater length on the side of the moving power; this was true according his principle, in which he supposed that the things that are at greater distance from the center move more easily and with greater force; the cause of which he saw in the greater speed with which the bigger circle moves compared to the smaller circle. This cause is indeed true, but lacks precision; for given a weight, a lever and a power, I do not know how I should divide the lever in the point where it turns, so that the given power balances the given weight. Archimedes accepted Aristotle's principle, but went further; he was not satisfied with saying that the force would be greater on the longer side of the lever, but he determined how much longer it should be, that is, what proportion it should have with the shorter side, so that the given power would balance the given weight. [...] He established this with a brilliant demonstration in Book One of *On the Equilibrium of Planes*, which, as Guidobaldo pointed out, was the book of *Elements* of the whole field of mechanics. In the preface of his paraphrase of Archimedes' work, Guidobaldo showed that Ar-

Caverni's interpretation, based on an hypothetical reconstruction of Archimedes' lost work *περί ζυγῶν*, *On the Balance* was a bit strained and assumed that the treatment of questions relating to motion and the equilibrium of heavy bodies was uniform in the Ancient World. But this assumption is unfounded. However, it is on this assumption that Caverni described the origin of the centrobaric theory:

> We see that any body always falls by natural necessity when it lacks a support; and whether it falls freely or it is supported, it is always a very thin thread that marks the clear way or that obstructs the tendency to move. This observation, which is obvious and meaningless for common people, was the start of a scientific investigation for the philosophers who, considering how any weight could be prevented from falling by holding it by means of a very single thread, drew the conclusion that in the fall the weight's strength (*conato*) gathered in the vertical direction. This first important result was then further developed on the base of the experiences: seeing that the weight was keeping equally at rest from whatever point of its surface it was hanging, it was not difficult by means of geometry to draw the conclusion that all the strength of the falling body gathered not in a thin line, as it had seemed before, but at an invisible point which was determined by the intersection of two vertical lines that could be drawn across the hanging weight when it was taking now one now another position. (Caverni 1895, 101–102)

In this explanation Caverni was comparing different Aristotelian considerations drawn from *De coelo*: the definition of the center of gravity given by Pappus in Book Eight of *Mathematical Collections*, and Proposition One from the same Book. After the passage quoted above, Caverni made references to the balance and to Archimedes' treatment of this instrument. In his explanation Caverni seemed to regard all these passages as stages of a uniform development. But certainly things were a bit more complicated, as will be seen.

Pierre Duhem was without doubt aware of this greater complexity, but in his investigations he also assumed the existence of a close connection between the origin of the centrobaric theory and the philosophical considerations concerning the natural motion of heavy bodies. Therefore, I think it necessary to assess whether this assumption is correct in order to evaluate Duhem's general interpretation.

This chapter attempts to analyze this particular aspect of the history of the centrobaric theory by considering the use of the concept of center of gravity in the field of cosmography. The analysis is based on some works overlooked by previous historians, assuming that Pierre Duhem's studies on the philosophers belonging to the so-called Parisian school of the fourteenth century are already well known and do not need to be discussed.

I shall start by criticizing the interpretation given by the French physicist of some Aristotelian passages, and his attempt to see a first confused appearance in

chimedes had followed Aristotle entirely, as far as the principles were concerned, but he had added his own exquisite demonstrations." Baldi (1887, 54–55).

De coelo II, 14 of some ideas that were later combined to form the concept of center of gravity. First, I shall try to determine more exactly Aristotle's aim in that chapter of his work by specifying the function of the distinction between the center of the world and the center of the earth. This distinction was made in order to explain the process of formation of the earth, its shape, its position at the center of the world and its being there at rest. This distinction should not be related to the centrobaric theory but rather considered in a different way. At the center of my new interpretation is an explanation I will give of the *aporia* or difficulty contained in the chapter of *De coelo* mentioned earlier. My explanation will lead to a discussion of the role of the word ῥοπή in Aristotle. According to Aristotle, this word seems to abandon any reference to the movements of the balance and to the problem of the equilibrium to which it had been closely connected since Homer. Through a linguistic and conceptual analysis, I will try to explain the reason for Duhem's interpretation of *De coelo*'s passage, which will lead to an analysis of the origin of the word ἰσόρροπος before Archimedes.

In the second part of my essay I will show how the rediscovery of *De coelo* influenced the interpretation of Sacrobosco's *De sphera*, a textbook on cosmography which has been studied for centuries. I will also show how more and more space was dedicated to discussions on the natural motion of heavy bodies and how the model of the balance was increasingly used for the explanation of the earth being at rest at the center of the world. The change that took place after Simplicius's commentary on *De coelo* became known, which contained a direct reference to the centrobaric theory, will be discussed here only incidentally since it is a fact well known to historians of science such as Pierre Duhem, Giuseppe Boffito (1902) and Edward Grant (1984).

The last part of my essay will deal with the great changes that took place in the sixteenth century in those fields of science considered here. These changes were first introduced mainly as a consequence of geographic discoveries, which undermined previous ideas concerning the distinction between center of gravity and center of magnitude. To all this we should add the rediscovery of Pappus' *Mathematical Collections*, which in Book Eight presents a different treatment of the center of gravity. The author who best exemplifies the novelties is Francesco Maurolico, who in his *Cosmographia* introduced a radical change in the discussion of these questions in the Renaissance. Through Clavius' *Commentary on Sacrobosco's De sphera*, the new results obtained by Maurolico became part of the teaching of mathematics. Thus, towards the end of the sixteenth century, a new theoretical approach was introduced in the discussion of the role of the centrobaric theory within the science of 'cosmography.' This new role was connected to the development of the theory of simple machines, which was based more and more often on Archimedean principles. In all this movement of ideas, the Jesuits played an important role, as can be judged from the example of Giuseppe Biancani, who went far beyond his masters in his discussion of the question of the direction of movement of heavy and light bodies. He imagined that the different parts of a heavy body fall along parallel lines, a consequence of the fact that the center of gravity of the body moves along a line to the center of the earth. In this way one could discuss the question of the equilibrium of the balance in close connection with that of the movement of heavy bodies.

At the Beginning there was Aristotle: The Early Stages in the Development of Centrobaric Theory According to Pierre Duhem

The origin and the earliest stages in the development of the concept of center of gravity in ancient Greece were briefly studied by Duhem at the beginning of the second volume of his *Les origines de la statique*. The cursory way in which he discussed the contributions of the ancients is probably the consequence of the fact that his main interest was in the scientific results achieved by a series of Parisian philosophers of the fourteenth century, who were relatively unknown before Duhem's research, figures such as Jean Buridan, Nicole Oresme, and particularly Albert of Saxony, all of whom formed the so-called "Parisian school of science". Duhem's special focus on the importance of this school of thought had a negative influence on his reconstruction of the history of science. His lack of interest in a precise evaluation of the different stages in the development of the concept of center of gravity by ancient Greek mathematicians and philosophers resulted in a simplified view of that development and in an inaccurate discussion of that concept, which would only be rigorously defined by Torricelli. Duhem's view derived once again from Lagrange, though he pointed to some incorrect judgements made by the Italian mathematician, which could have been avoided if the latter had played greater attention and 'respect' to the medieval *scientia de ponderibus* (Duhem 1905, 1–6).

As a consequence of these preconceived ideas, Duhem was satisfied with a brief analysis of a few texts by Pappus and Archimedes, chosen for the precise purpose of showing the supposed faults presented in Greek statics. He then proceeded, with little respect for chronology, to a brief and insufficient discussion of Chapter 14 of Book Two of Aristotle's *De coelo*. In this text concerning the determination of the spherical shape of the earth, Duhem saw a first and still confused idea of a doctrine that would have a long and successful history. He described this doctrine as follows:

> In each heavy body there is a point where its weight or gravity is concentrated: this is the *center of gravity*. In each heavy body the gravity is its desire to unite this center of gravity with the center of the world. If its center of gravity coincides with the center of the world,

> the heavy body is at rest. If the center of gravity is outside the center of the world, the first point aims at joining the second and, if it is not stopped from doing that, moves towards it in a straight line. The earth is a heavy body like all other heavy bodies; therefore it joins its center of gravity to the center of the world; and it is for this reason that the earth is at rest at the center of the world (Duhem 1905, 9–10).

But is this a correct way of putting it? Is it right to explain Aristotle's thought in the light of a concept that was introduced only later and, according to Duhem himself, was unknown to the Greek philosopher? Duhem's explanation should have been based on a deeper and more elaborate analysis of the Aristotelian text. It is therefore necessary to re-examine Chapter 14 of Book Two of *De coelo*.

How the Spherical Shape of the Earth Was Formed

In the chapter ending Book Two, Aristotle concludes the discussion he began in Chapter 13 of various questions concerning the earth: its situation and shape, and whether it is at rest or in motion. He had started by reviewing previous theories, in particular, the doctrine developed by the Pythagoreans who held that the earth revolves like a planet round a center, which is occupied by fire: by proposing this picture of the world they were denying the idea that the earth was at rest at the center of world. But Aristotle also discussed at length many other theories that assumed this idea and explained why the earth was at rest at the center of the world.

This problem was closely linked with that of the shape of the earth, and was connected with Aristotle's doctrine of the natural motion of the elements: at this point he was breaking with the previous tradition of thought. What up to then had been explained by a violent action (such as, for instance, the motion of the heavens around the earth suggested by Empedocles) for Aristotle was a consequence of the nature of the elements: these moved in a straight line towards their natural places, the heavy bodies towards the center of the world, the light bodies towards the concave surface of the sphere of the moon; and both kinds of bodies stopped moving when they reached their natural places. Based of these principles it would be meaningless to ask why the earth was at rest in the middle of the world; likewise it would be absurd to ask the same question concerning fire being in its natural place (apart from the circular motion imparted to fire by the motion of the sphere of the moon.)

The earth was not at rest because, as Anaximander had argued, being at the center and in exactly the same relation to the extreme parts of the world around it, it had no reason to move in one direction rather than in another. This explanation would make the earth's immobility depend only on its position. If this explanation were true, then any other element, for instance fire, if placed at the center, would remain there like the earth. But the earth not only rests at the center but also moves towards it. This indicates that the doctrine of natural places, and not the argument from the position at equal distance from the extreme parts of the world, gives the true explanation. If we judge Anaximander's argument from the point of view of the doctrine presented in Aristotle's *Posterior Analytics*, it would be considered as a reasoning based on extremely general principles, rather than on the principles of the science of nature, where bodies are always considered as being either heavy or light.

Put in these terms, the question concerning the position of the earth at rest at the center of the world was closely related to the study of the motion of heavy bodies, which became an essential part of the argument together with the observation of the rising and setting of the constellations. One could positively state for certain that heavy bodies move naturally towards the center of the earth and that they do not fall in parallel lines, but so as to make similar angles with its surface. This statement was considered essential by Aristotle who repeated it twice in chapter 14. This indicated that the earth, towards which their fall was directed, was spherical and placed at the center of the world. The motion of heavy bodies was regarded as perpendicular not to the center of the earth, which was only accidentally the natural place towards which the heavy bodies tended, but rather to the center of the world, which was the true limit of all the motions of these bodies.

It was on the base of these premises, ignored by Duhem in his brief exposition, that Aristotle in chapter 14 of *De coelo* discussed in detail the question of the shape of the earth. The distinction between the two centers previously mentioned was here introduced in a dubitative manner in order to strengthen his criticism of his predecessors made in chapter 13, and at the same time to develop more clearly his arguments concerning the natural motions of elementary bodies. Beginning his historical reconstruction of the Aristotelian doctrine on the basis of this passage from the text of *De coelo*, the French scholar was moving from a completely different presupposition, which induced him to interpret the whole chapter in light of the medieval discussion on the different centers of the sphere of the earth and of that of water, and as a consequence, to distinguish between the center of magnitude and the center of gravity. Based on this interpretation, there was the idea that the concepts developed by Aristotle and other ancient philosophers could be applied to those medieval problems, but this assumption needed a rigorous analysis of the texts within an exact historical perspective. But what emerges from reading the texts tends rather to deny Duhem's assumption.

If Aristotle had kept to the principle of the eternity of the world, his analysis of the motion of natural bodies previously mentioned would have been sufficient to establish the spherical shape of the earth. But his argument had been developed in opposition to the cosmogonic theories of the Presocratic philosophers, who were incapable, according to him, of explaining in a non-contradictory manner the generation of the cosmos and its present shape. The validity of his doctrine of natural motions had therefore to be proved by a demonstration based on its application to a process of formation of the sublunar world taking place over time. How can the formation of a spherical agglomeration by the natural motion of earth towards the center of the world be explained?

In the case of a motion towards the center of similar parts coming from all directions and from equal distances, the problem was easy to solve because it seemed evident that by adding similar parts coming from all directions the result would have been a spherical body. As in the previous case, all the parts pushed by their weight would have continued to move towards the center of the world until they reached it. But what would have happened if a larger part pushed a smaller one? This question cannot be answered on the basis of the principles of modern physics: we do not have here two rigid bodies that collide in space but a completely different situation. It is not very clear what Aristotle means when he says that "when a smaller part is pressed on by a larger, it cannot surge round it [like a wave] (οὐχ οἷόντε κυμαίνειν), but is rather squeezed together with the other and combines with it (συγχωρεῖν) until they reach the center" (Aristotle and Guthrie 1939, 246–247/297a9–12).

It would seem that in this case the smaller part could still offer a certain resistance, and this would entail a reciprocal "action" between the two parts. The final result of this process of bidirectional buckling would be the formation of one single body, that is, the inclusion of the smaller part inside the larger. But it is not clear whether at the end of this process it would still be possible to single out in some way the two parts that have combined to form a new body.

It would seem that this explanation could be referring to the way in which the artisans made things from clay, but a "physical" explanation could not refer to an external cause such as the artisan, but must be based on principles relating to natural bodies, that is, the "weight" and the tendency to move downwards (i.e. towards the center of the world).

The Aporia in De coelo II,14

How can the process previously described be understood? What happens when the smaller part, which is already placed at the center of the world, comes into contact with the larger one? Being in its natural place, the smaller part, according to Aristotle, would have been able to offer a certain resistance to the larger one that was pushing it, and both buckled together would have formed a single spherical body, remaining then at rest at the center of the world. Should we perhaps think of the smaller part as a smaller immobile sphere stuck at the center? Or could we believe that the smaller part has been moved slightly away from the center of the world by the larger part that pushes it? These questions show that here we have a problem that must have already been considered in Aristotle's time, seeing that he discussed the following *aporia,* or perplexing difficulty, which is similar to our previous considerations.

> If, the Earth being at the centre and spherical in shape, a weight many time its own were added to one hemisphere, the centre of the Universe would no longer coincide with that of the Earth. Either, therefore, it would not remain at the centre, or, if it did, it might even as it is be at rest although not occupying the centre, i.e. though in a situation where it is natural for it to be in motion (Aristotle and Guthrie 1939, 248–249/297a31–297b1).

How must the expression "a weight added to one hemisphere" be understood? If we consider Aristotle's solution of the difficulty, this thing does not seem very important:

> It is not hard to understand [the difficulty], if we makes a little further effort and define the manner in which we suppose any magnitude, possessed of weight, to travel towards the centre. Not, clearly, to the extent of only touching the centre with its edge: the larger portion must prevail until it possesses the centre with its own centre, for its impulse extends to that point. It makes no difference whether we posit this of any chance portion or clod, or of the Earth as a whole, for the fact as explained does not depend on smallness or greatness, but applies to everything which has an impulse towards the centre. Therefore whether the Earth moved as a whole or in parts, it must have continued in motion until it occupied the centre evenly all round, the smaller portion being equalized by the greater under the forward pressure of their common impulse (Aristotle and Guthrie 1939, 248–251/297b2–14).

This is the passage where Duhem, following Simplicius, saw the first still confused appearance of what will be subsequently called the center of gravity. This interpretation will be discussed later; up until now we have considered the first part of Aristotle's statement concerning the motions of bodies towards the center. Do we face reasoning that implies the displacement of a rigid body caused by another rigid body placed on it? No, we do not. What could have caused this sudden change? Was it perhaps a different way of dealing with this kind of problem by the presumed author of the *aporia*? But it is probably Aristotle himself who is the source of this *aporia*.

To understand this part of Aristotle's reasoning we must return to what was said earlier. There we have a weight much larger than the earth. The earth takes the place of the smaller sphere immobile at the center of the world. Though offering some resistance, after a reciprocal deformation it will eventually merge with the larger body. When could the arrangement of the two parts—to form a single body—be considered to accomplish? And what form will this body have? Aristotle does not say it explicitly, but on the basis of the text of the *aporia* we may suppose that the final result is a spherical body since there is no mention of a possible different figure. But Aristotle adds that this *aporia* still holds valid since it is supposed that the formation of the new body ends when the edge of the larger part touches the center of the world. Why did he conceive such an idea? It may be that this is a naïve concept of the center as a support, almost a *floor*, which stops the weight from going further down. But in this case the new shape of the earth would be similar to a *pear*. But it seems correct to assume that Aristotle was thinking of something more subtle.

Let us therefore imagine that the *aporia* describes the meeting of two spheres of different magnitudes. We may then think of a deformation of the smaller one, that is, the earth, similar to squeezing its two hemispheres along the diameter parallel to the line passing through the tangent point. This would be a symmetric process, which would take place at both sides divided equally by the diameter, and which would come to an end when the external surface of the new body reaches the center of the world. If it were possible to single out the two different deformed bodies at the end of this process, they would appear as two bodies that are tangent to one another at the point previously occupied by the center of the smaller sphere, that is, the center of the world.

According to this reasoning, the end of the process of formation of the new body would be the final moment. But this would agree only partially with what Aristotle has said since in this case a fundamental aspect of the problem would be ignored, that the natural tendency of heavy bodies to move towards the center of the world, a tendency that does not cease to act until the bodies come to equal distances from the center. If, on the contrary, we consider the newly formed body only in relation to the center, without referring to its natural tendency, we get a situation exactly equal to that described in the *aporia*. We can then refer to the earth in its final placement, which is enclosed in the new body and therefore no longer in a central position, but actually displaced. Or we can consider the earth in relation to its original center, from which it has never moved away, despite being squeezed. But now this point is no longer its center since it is placed on the surface, and is to an even lesser extent the point towards which every part having weight tends to move.

Once we have established that the idea on which the *aporia* is based is wrong, how can we solve the problem? Going back to the process mentioned above, we can state that the body does not stop when its edge touches the center of the world; the larger body continues to act on the smaller and this would continue until the center of the new body coincides with the center of the world.

For Aristotle, it was of paramount importance to conceive a cosmogonic process, that is, a process for the generation of the world that was based on a rigorous doctrine of the natural motion of bodies. The main points of this doctrine have already been mentioned: heavy and light bodies have a natural tendency to move towards their natural places along a straight line, the heavy bodies towards the center of the world, the light ones in the opposite direction from the center towards the concave surface of the sphere of the moon. When they reach those places, the bodies no longer have that tendency and remain at rest. The reasoning by which Aristotle had tried to solve the problems of the placement and of the shape of the earth has already shown how important it was that all its parts were at equal distance from the center since only in this way was the earth at rest. But how must we understand this equal distance? Can we simply refer it to a 'mechanical' context, as Duhem did? Or is it necessary, also in this case, to be cautious? In other words, must we consider the parts of the earth placed at equal distances, from the point of view of the equilibrium of the balance? It is now necessary to analyze briefly the concepts of βάρος (weight) and ῥοπή (tendency of the heavy bodies to move downwards, that is, towards the center of the world) as they are employed in *De coelo*, Book Two, Chapter XIV.

The Different Ways in which the Terms βάρος and ῥοπή are Used in De coelo II,14

We must first note that both terms are peculiar to Chapter XIV since they occur there more often than in any other Aristotelian text, and they are particularly frequent in the passages quoted above: they are employed (1) to determine the true natural place of the motion of heavy bodies and to show that the center of the earth only accidentally coincides with it; (2) to deny that heavy bodies fall along parallel lines and to prove that they move along lines that converge to the center of the spherical earth placed at the center of the world; (3) to show that parts of different dimensions meet to form a single body; (4) to solve the *aporia* of the weight placed on one hemisphere and to state the truth of what was said in point (3) for bodies of any dimension. For our problem, the last two points are particularly interesting.

The question raised in point (3) is mentioned twice by Aristotle: the first time at the beginning of the discussion concerning the shape of the earth; the second time after discussing the formation of a spherical body by the motion towards the center of equal parts coming from all directions and from equal distances, and just before enunciating and discussing the *aporia*. This repetition is not casual, given the general character of the explanation suggested, and it does not seem that there is a substantial difference between the two passages, though in the first case Aristotle emphasizes the way in which the two parts come together to form a single body, whereas in the second case he stresses the necessity for this process to take place. Aristotle employed the two terms, βάρος and ῥοπή, in a rather indiscriminate way. In the initial passage, the contact and the following reciprocal 'action' between the parts is possible because "every one of its parts has weight until it reaches the center (ἕκαστον γὰρ τῶν μορίων βάρος ἔχει μέχρι πρὸς μέσον)" (Aristotle and Guthrie 1939, 246–247/297a8–9); whereas in the later passage "a greater mass must always drive on a smaller mass in front of it, if the inclination of both is to go as far as the center (τὸ γὰρ πλεῖον ἀεὶ τὸ πρὸ αὑτοῦ ἔλαττον προωθεῖν ἀναγκαῖον μέχρι τοῦ μέσου τὴν ῥοπὴν ἐχόντ ων ἀφοῖν)" (ibid. 248–249/297a27–29). But perhaps in this second case the term ῥοπή was used to avoid linguistic confusion, since the parts were differentiated here by their weight rather than their dimension, as had been done before.

In the following passage, where the presuppositions of the *aporia* are criticized, the term employed by Aristotle is always ῥοπή, and this time without any possibility of misunderstanding. The supposition that the formation of a new body was completed when the edge of the larger part touched the center of the world was wrong; as a matter of fact, it was necessary for the center of the newly formed body to coincide with the center of the world since the larger part continues to act on the smaller part and both had an inclination to go as far as the center (μέχρι τούτου [μέσου] γὰρ ἔχει τὴν ῥοπὴν) (ibid. 297b6–7.)

For Aristotle, the determining factor in this process was not so much the size of the two parts, nor their weight, but rather what was common to all parts, that is, the impulse to go towards the center (ἀλλὰ κατὰ παντὸς τοῦ ῥοπὴν ἔχοντος ἐπὶ τὸ μέσον) (ibid. 297b–9). And whether the earth moved as a whole or in part "it must have continued in motion until it occupied the center evenly all round
(ἀνανκαῖος μέχρι τούτου φέρεσθαι ἕως ἂν πανταχόθεν ὁμοίως λάβῃ τὸ μέσ ον)" (ibid. 297b11–12). It was therefore "under the forward pressure of their common impulse" that the smaller portions were *equalized* by the greater (ἀνισαζομένων τῶν ἐλαττόνων ὑπὸ τῶν μειζόνων τῇ προώσει τῆς ῥοπῆς) (ibid. 297b12–14).

The solution of the *aporia* and the whole process of formation of the earth were based on the concept of ῥοπή. But how can this Aristotelian concept be related to the idea of equilibrium? Can we with the help of Duhem come to the conclusion that Aristotle's reasoning contains a first vague idea of what was going to be called the center of gravity? Some further considerations induce us to utterly exclude such a possibility.

The Concept of ῥοπή before Aristotle

The term ῥοπή acquires a particular meaning in Aristotle's writings, which amounts to a deep change of this concept as compared to its traditional meaning. In texts written before the fourth century B.C., the concept of ῥοπή was most often related to the idea of a scale, particularly to the idea of the inclination of the scale and precisely to the idea of the heavier scale pan going down towards the ground. The scale was considered from the point of view of common experience, without any reference to its structure or to the motion of heavy bodies placed on it. What was considered important was the moving away of the beam from the position parallel to the ground, which disrupts the equilibrium. This image, when being metaphorically applied to human destiny, evoked decisive moments in a man's life. The use of the term ῥοπή in this context goes back to the earliest times of the Greek world, and it occurs already in Homer where it characterizes one of the meanings of the verb ῥέπειν.

In Book Eight of the "Iliad," Zeus placed the fates of the Achaeans and of the Trojans on a scale and, lifting it, he weights them. The scale pan holding the fate of the Achaeans then "inclined downwards" (ῥῆπε) and rested on the ground, whereas the Trojans' scale pan "rose to the sky," and so the battle, which had lasted the whole morning with alternate fortune, was settled in the Trojans' favor:

> The Sire of Gods his golden scales suspends,
>
> With equal hand: in these explore'd the fate
>
> Of Greece and Troy, and pois'ed the mighty weight,
>
> Press'd with its load, the Grecian balance lies
>
> Low sunk on earth, the Trojan striks the skies. (Homerus and Pope 1760, book 8/verses 88–92).

Similarly to Homer, at a date closer to Aristotle, Euripides employed the term ῥοπή in his "Helen" (1090) with the same meaning but without making a direct reference to the scale. At the time, the phrase must have been so familiar to the spectators of the tragedy that it could be used in the right context without further illustration. It is the moment in the drama when Helen and Menelaus, after persuading Theonoes not to reveal to his brother Theoclimenus that the news of Menelaus' death was not true, start to plan their escape from Egypt. They have to get away from Theoclimenus, who wants to marry Helen, but at the same time they need his help. Therefore they try to cheat him, and it is Helen who has to face this risky endeavor. Being aware of her situation, she then exclaims: "I see two outcomes (ῥοπάς). Either I must die, if my tricks are discovered, or I return to my fatherland and save your life" (Euripides and Kovacs 2002, 132–133). Once again, the scale could sink on either one side or the other, but this time the outcome would have decided not the destiny of two groups of warriors fighting against each other, but that of a single person. The term ῥοπή meant life on the one side, death on the other.

We could give further examples, and even more if we consider the various meanings of the verb ῥέπειν, but this would only confirm the variety of contexts in which this term has been used and which determine its meaning of 'inclination,' also in many modern languages.

Let us go back to our problem. A comparison between the concept of ῥοπή as employed by Aristotle and the concept of it as used by Homer and Euripides shows clearly that in the first case the "tendency to go towards the center of the world" results in reaching a state of rest, whereas in the second case the action described by the term is that of leaving the state of rest. For Aristotle, the action described by the term ῥοπή is the necessary condition for reaching a state of equilibrium, whereas for the other authors it is the cause of disruption of equilibrium.

It is true that the two concepts appear to be complementary; it could be said that they agree on an essential condition of the state of equilibrium, that is, that what is at rest must be equidistant from something that is without motion, that is, the center of the world for Aristotle or the ground for those refer to the scale. But is this condition sufficient to establish a satisfying doctrine of the equilibrium? Certainly not. Both conceptions in fact underwent further developments. In the end, it was the development of the studies concerning the scale that produced the greatest results, whereas the direction taken by Aristotle turned out to be rather sterile. Aristotle in fact gave up any reference to the scale and continued to analyze the concepts of rest and equilibrium simply in terms of equidistance. On the other side, however, the studies of these concepts were developed mainly in terms of equal ratios of weight and distances. What we state here is confirmed by the transformation that the concept of ῥοπή underwent in other passages in Aristotle's works.[5]

Already in *De coelo*, Aristotle did not hesitate to make use of that term also with reference to upward motions. From his point of view, the term ῥοπή could indeed be conceived as a general tendency of bodies to move towards their natural places and therefore it would have been more correct to definite the meaning of this term without specifying any preferred direction; but, as we have seen, the common usage of that word was still prevalent in his writings. One of the clearest examples of the new broader meaning of the term can be found in Book Three, Chapter II of *De coelo*, where after showing that "every body has a natural motion performed neither under compulsion nor contrary to nature," Aristotle states briefly that "some bodies must owe their impulse to weight or lightness (ὅτι δ'ἔνια [τῶν σωμάτων]

[5] A first stage in the transformation of the term ῥοπή can be seen in a passage from *De justo* ascribed to Plato, where the verb ῥέπειν usually refers to the motion of a heavy body downwards, seems to be used also with reference to the motion of a light body upwards. But the thing seems rather dubious. This brief text attempts to define what is *just* through a Socratic discussion, where the solution of the problem is found through a series of short questions and answers concerning less problematic fields of knowledge than the one discussed here. Among these is the doctrine dealing with the concepts of *heavy* and *light*. The question then is: How can we judge whether a body is heavy or light? By its weight. How can the weight of a body be assessed? By the art of weighing. Now the art of weighing makes use of the scale, and therefore "what on the scale inclines downwards is heavy, what *inclines* upwards is light" (τὸ μὲν κάτω ῥέπον ἐν τοῖς ζυγοῖς, βαρύ· τὸ μὲν ἄνω, κοῦφον). In the second part of this sentence, the verb is missing, but it seems correct to assume that the term ῥέπον should be repeated. But this not a question of a natural tendency of the light body to move upwards but rather of the simple observation of what happens on the scale: the equilibrium has been disrupted, one body goes downwards, the other goes upwards. We should not think of a scale for weighing the lightness, but of a determination of *heavy* and *light* as relative terms.

ἔχειν ἀναγκαῖον ῥοπὲν βάρους καὶ κουφότητος)" (Aristotle and Guthrie 1939, 276–277/301a22–23). To show this, he argued that "if that which moves has no natural impulse, it cannot move either towards or away from the center (εἰ δὲ μὴ ἕξει φύσει ῥοπὲν τὸ κινούμενον, ἀδύνατον κινεῖσθαι ἢ πρὸς τὸ μέσο ν ἢ ἀπὸ τοῦ μέσου)" (ibid. 276–277/301a24–26).

Here the meaning of the term ῥοπή has lost any connection with the inclination of the scale. Its meaning here is so general that it could be used not only to prove the spherical shape of the earth but also to explain the formation of the sphere of fire. But the process by which light bodies are placed in the space between the earth and the concave surface of the sphere of the moon is analyzed by Aristotle in a completely different way. Surely fire also moves towards its natural place along a straight line and meets the edge of that place at equal angles. But that is all. Moreover, this kind of reasoning would not even have been necessary for Aristotle: none of his predecessors had even conjectured that the world, or at least the sublunar region, could have been formed through a process of *emanation* from a single point, as understood for instance by Robert Grosseteste (c. 1168–1253), who in his treatise *On light* perceived light, as the first form to be created in the prime matter, as being propagated from an original point into a sphere, thus giving rise to spatial dimensions and to everything else, according to optical laws.

Aristotle followed a different theory, which introduced a sharp distinction between absolutely heavy and light elements, such as earth and fire, and elements that were heavy and light only in relation to other elements, such as water and air. On the basis of this theory, he solved the problem of placing the elements in the sublunar region. This solution was of great consequence for later developments of the scientific thought.

Critical Discussion of Duhem's Interpretation. The Use of the Term ἰσόρροπος before Archimedes

Let us go back to Duhem and his interpretation of the *De coelo*. Without making any reference to its context, he simply translated the passage concerning the question of locating the true natural place to which heavy bodies move. In this passage, Aristotle had tried to explain that the center of the earth was only accidentally the point towards which they moved. After quoting this passage, Duhem proceeded immediately to the other passage that presents the *aporia* concerning a weight placed on one of the hemispheres of the earth, and focused his attention on the passage stating the need to place the center of the heavy body at the center of the world. Following Simplicius, and without considering the whole argument developed by Aristotle, Duhem concluded:

> Aristotle's doctrine is still vague on this point; the Philosophers does not define this center, τὸ μέσον, which in any heavy body tends to reach the center of the universe; he does not make it identical with the center of gravity, which he did not know (Duhem 1905, 11).

The French scholar, by interpreting Aristotle's text from the point of view of the historical development of the concept of center of gravity, regards it as an unsuccessful attempt to describe the conditions of equilibrium of a heavy body placed at the center of the world. All his analysis is based on the assumption that the Greek philosopher regarded the earth as a rigid body, and that those passages from his work were the earliest documents of a tradition of thought which lasted until the first half of the seventeenth century. The first assumption is a bit overstretched, but the second is the result of adopting without criticism the interpretation of the Aristotelian text by ancient and medieval authors. Our previous considerations lead us to quite different conclusions.

On the basis of what he states in the *De coelo*, it is probable that Aristotle regarded the earth as a very malleable body. To show the validity of the principles of his physical theory, he confronted earlier thinkers and developed a cosmogony based on a rigorous doctrine of natural motions, according to which one could explain the central position, the state at rest and the spherical shape of the earth. In the concept of ῥοπή, which in his work underwent a substantial change of meaning, Aristotle found a physical principle that explained the present shape of the earth, its place at the center of the world and it being at rest. As soon as a condition of rest/equilibrium is established, this active principle no longer holds: the condition is defined by the equidistance of each earthly part from the center of the world. From this point of view, each part having ῥοπή will converge towards the center and acquire a spherical shape. Any other solid geometrical figure, since the bodies are not rigid, will not remain unchanged at the center of the world but will change shape by effect of its ῥοπή and will always result in a spherical body having a center coinciding with the center of the world. In other words, the earth would behave like *a fluid body*. The interpretation of Aristotle's *aporia* based on a concept of equilibrium modeled on the idea of a scale is misleading. Such a concept emerged much later in developing the idea of equilibrium conceived in terms of equal ratios between weights and distances. It is imperative to put aside this idea, though it might not be so easy since it has influenced all subsequent interpretations and commentaries of *De coelo*.

A clear example of such a misleading interpretation is offered by Duhem's translation of the final passage of the *aporia*. Aristotle says that "whether the Earth moved as a whole or in parts, it must have continued in motion until it occupied the center evenly all round, the smaller portions being *equalized* by the greater under the forward pressure of their common impulse (Ὥστε εἴτε ὅλη ποθὲν ἐφέρετο εἴτε κατὰ μέρος, ἀναγκαῖον μέχρι τούτου φέρεσθαι ἕως ἂν πανταχόθεν ὁμοίως λάβη τὸ μέσον, ἀνισαζομένων τῶν ἐλαττόνων ὑπὸ τῶν μειζόνων τῇ προώσει τῆς ῥοπῆς)" (Aristotle and Guthrie 1939, 250–2551/297b10–14).

The analysis of the concept of ῥοπή makes this process of "equalizing" very clear, but for the one who could not free himself from the Archimedean idea of equilibrium, this was very difficult to see. For Duhem, the arrangement of the parts around the center would not result from the pressure of the bigger parts on the smaller, but from the fact that "the tendency of the different parts to move cause them to counterbalance each other." For the French historian, the meaning of the verb ἀνισάζειν became unconsciously identified with that of the verb ἰσορροπεῖν, and Aristotle's vague idea was replaced with the more precise Archimedes' concept. The Archimedean concepts of equilibrium resulting from an equal ῥοπή of heavy bodies placed at certain distances from the center of a scale was inserted into the Aristotelian text, where it assumed a new aspect. But Aristotle could not have conceived in this way the arrangement of the parts of earth at equal distances from the center, because with such an arrangement their tendency to go towards the center of the world would have completely ceased.

Was there any reason to justify such a change of meaning? Was there any passage in Aristotle's works that would support such an interpretation? In order to answer these questions, we must analyze the term ἰσόρροπος and discuss the ways in which it was used by ancient authors. This discussion will complete and confirm what we have said concerning the term ῥοπή.

The concept of ἰσόρροπος is closely related to the idea of a scale: it refers to the state of equilibrium and the absence of inclination. The prefix ἰσο– signifies the idea of equality between inclinations and their reciprocal elimination. The impossibility of one weight prevailing over the other results in the immobility of the scale and the equidistance of all its parts from the ground. In Homer's and Euripides' passages quoted above, the term ἰσόρροπος does not appear, but it would have stood for the first moment of the operation of weighing: if on a scale in the state of equilibrium two weights are placed and they are not equal, the scale would incline on one side. To this situation it could be added, as in the case of Homer, that the greater weight would come down to the ground.

If the idea of equilibrium could be tacitly assumed in the situation just described, a very different situation would occur if the initial state were one of non-equilibrium. It is in such a situation that the term ἰσόρροπος occurs for the first time in an ancient author, that is, in Aeschylus' *Persians* (346). In the battle between the Greeks and Persians, the destiny of the former seemed to be already decided: the number of Xerxes' ships was by far greater than that of the Greek's, but then some divinity destroyed the Persian fleet "weighting the scales so that even fortune did not fall out (τάλαντα βρίσας οὐκ ἰσορρόπῳ τύχῃ)" (Aeschylus and Sommerstein 2008, 137–138). Here the inclination is not going to take place in the immediate future, but is already present. The scale is not in equilibrium and the inclination must be reversed.

Aeschylus' image is very evocative, but I have recalled here it because of the earliest occurrence of the term and of its connection with the scale. Plato's use of the term ἰσόρροπος is much more significant for us because in this case it refers to the kind of questions which were later discussed by Aristotle, and the position taken by his master is very important in this context. The most interesting passage in which the term occurs is in the *Phaedo*, where Socrates presents his idea of the shape, place and dimension of the earth. He begins to describe the position of the earth in the cosmos in a way similar to that later followed by Aristotle.

> If the Earth is round in the middle of the heavens, it needs neither the air nor any other similar force to keep it from falling, but its own equipoise (τῆς γῆς αὐτῆς τὴν ἰσορροπίαν) and the homogeneous nature of the heavens on all sides suffice to hold it in place; for a body which is in equipoise (ἰσόρροπον), and is placed in the centre of something which is homogeneous cannot change its inclination (κλιθῆναι) in any direction, but will remain always in the same position without inclination (ὁμοίως δ'ἔχον ἀκλινὲς μενεῖ) (Plato and Flowler (1914, 374–375/109A).

Plato's last sentence is very similar to one in *De coelo* describing Anaximander's position as reported by Aristotle: "That which is situated at a position at the center and is equably related to the extremes (ὁμοίως πρὸς τὰ ἔσχατα ἔχον) has no impulse to move in one direction or either upwards or downwards" (Aristotle and Guthrie 1939, 234–235/295b12–14).

The fact that Plato insists here on using such terms as ἰσορροπίαν and ἰσόρροπον shows that for him the state of equilibrium of the earth was an essential supposition assumed as a basis of his argument. Unfortunately, he does not say anything about the way in which such equilibrium came about. He states it as a fact, which shows that the equidistance of the earth from all sides of the heavens is not sufficient to assure its remaining at the center of the world. The earth, as a scale, must be in such condition that all of its parts have equal inclinations, which eliminate each other. The beam of the scale remained equidistant from the ground as long as it remained in equilibrium, that is, as long as one weight could not prevail over the other.

If this was the true meaning of the term, it is clear why Aristotle avoided using it. He had based the process of formation of the earth on the concept of ῥοπή, the tendency of heavy bodies to move towards the center of the world, a tendency destined to disappear as soon as they reached their natural place, where they were arranged to form a spherical figure and remained at rest.

Our suggestion that Duhem's translation of Aristotle's passage is misleading can be further confirmed by the fact that the term ἰσόρροπος occurs rather infrequently in Aristotle's work and is used with the common meaning. The way in which this term is employed in *Nicomechean Ethics* (Book Nine, Chapter I) is surely less relevant to our enquiry: there concerning "those who have imparted instruction in philosophy" Aristotle says that "the value of their service is not measurable in money, and no honour paid then could be an equivalent (τιμή τ'ἰσόρροπος οὐκ ἂν γένοιτο)" (Aristotle and Rackham 1926, 520–521/1164b2–4). The term here has the metaphoric meaning of "well-balanced" or "well-matched" or "having the same value": no honor could have such a value as to adequately counterbalance the value of their service. In other words, the imaginary balance with which we could weight the value of wisdom cannot be put in a state of equilibrium by placing on one of the dishes either money or honor.

From the point of view of the history of mechanics, it is more interesting to consider the use of the term ἰσόρροπος in a passage from Aristotle's *Parts of Animals* where the structure of birds is analyzed in terms of equal distribution of their weight similar to the placing of equal weights on the scales of a balance:

> Quadrupeds have forelegs to support their forward weight; birds, however [...] nor have forelegs, because they have wings instead. By way of compensation, Nature has made the ischium long, reaching to the middle of the body,
> (μακρὸν ἡ φύσις τὸ ἰσχίον ποιήσασα εἰς μέσον προσήρεισεν), and has fixed it fast, while beneath it she has placed the legs, so that the weight may be equally distributed on either side and the bird enabled to walk and to stand still
> (ὅπως ἰσορρόπου ὄντος τοῦ βάρους ἔνθεν καὶ ἔνθεν πορεύεσθαι δύνηται καὶ μένη)
> (Aristotle and Rackham 1926, 520–521/1164b2–4).

The concept of equilibrium derived from the idea of balance was essential for understanding how birds could walk and stand still. This sort of explanation would later lead to important developments in the study of animal anatomy. The body of a bird, supported by legs placed in a central position, did not keep the same 'inclination,' and could therefore not be immediately related to the idea of the beam of a balance equidistant from the ground. The body of a bird, usually in a slanting position, would continuously bend down, first on one side and then on the other, but in spite of that the animal would remain in equilibrium. Such a situation would have offered the opportunity for a deeper analysis of this phenomenon, but the complexity involved prevented the development of further research in antiquity. The advancement in the study of the anatomical structure of animals became possible only in the sixteenth and seventeenth centuries thanks to the revival and new development of the doctrine of the center of gravity.

Aristotle's passage quoted above shows a widening and transformation of the meaning of ἰσόρροπος, which had already been used in relation to human beings. It had been employed in the field of medicine in some of the works forming the *corpus hippocraticum*, but there the term was still connected to the idea of equidistance or setting in a line. For instance, in the case of a fracture of the thigh bone, it was advised to pull the limb strongly so as to avoid ending up with one leg shorter than the other. This kind of operation, if done unskillfully, would have made the patient wish that both his legs had been broken, for then at least he would be in equilibrium (ἰσόρροπος γοῦν ἂν εἴη αὐτος ἑωυτῷ), that is, he would not be lame. In this case, real bodies, and not simple distances, are equal, and, being used to support something placed above, they could cause an unwanted inclination. Such a lack of equilibrium would surely be of no help for normal walking.

The Multiplication of the Centers. The Study of Aristotle's and Sacrobosco's Texts from the Middle Ages to the Beginning of the Sixteenth Century

Johannes de Sacrobosco's *De sphera* enjoyed great renown during the Middle Ages and from the middle of the thirteenth century onwards it was taught at all the schools of Europe: during more than three centuries it served as the main textbook on cosmology for the students in medieval and Renaissance universities. The work presented in a simplified form some fundamental astronomical concepts and required only knowledge of elementary mathematics. Written probably around 1220, it was the subject of numerous commentaries that continued to be produced up until the first decades of the seventeenth century. *De sphera* was repeatedly enlarged with additions and developments based on Aristotle's *De coelo* and other mathematical and astronomical works by Greek and Arab authors, and on the new geographical discoveries that produced a better knowledge of the sky of the southern hemisphere. The observation of the new stars from 1572 onwards helped to raise doubts about the incorruptibility of the celestial spheres and, as a consequence, to overcome the traditional contraposition between the heavenly and sublunar world.

Within this complex process of assimilating new ideas and revising old ones, reviving the concept of center of gravity played a significant role: in particular, it was employed mainly to explain how parts of the terrestrial sphere emerged from water. This problem had been discussed in the previous centuries within the field of the exegesis of the Holy Scriptures, particularly "Genesis" and the "Psalms." But with the recovered knowledge of Aristotle's natural philosophy, this problem was seen in a new light. The whole question was the result of a sort of misunderstanding of Aristotle's texts, which led to suppose that in the sublunar world each element was entirely surrounded by the element next to it, and that on the basis of examples quoted by Aristotle of transformation of a thick substance into a thin one, the thinner element was taking up a space ten times larger than that taken up by the thicker element. This supposition, in connection with the doctrine of natural place, suggested the idea that the sphere of water was ten times larger than the earth.

Moving away from this supposition, two fundamental questions were raised that led to a revival of the doctrine of the center of gravity: what had caused the water surrounding the earth to shift away? What allowed the parts of the earth that had emerged to remain so without being again submerged by water?

The various answers given to these questions were discussed by several scholars, among them Pierre Duhem, Giuseppe Boffito and Edward Grant. It seems useless to discuss these questions again after all the work invested by these scholars. But both Boffito and Grant have scarcely considered the centrobaric doctrine, while Duhem, though focusing his attention on it, treated it within an idea of the historical development of mechanics which I intend to correct here. Thus it will be appropriate to discuss once again the authors who had tried to solve these questions with the help of the concept of center of gravity.

The Form, the Position and the Immobility of the Earth in Sacrobosco's De sphera

The question of the arrangement of the elements within the sublunar world is discussed by Sacrobosco at the beginning of Chapter One of *De sphera*, after giving two definitions of a sphere according to Euclid and Theodosius and analyzing the real sphere on the base of the concept of 'substance' and 'accident.' The division of the sphere in two separate regions, the heavenly or *ethereal* sphere and the sphere of the elements, is followed by a description of the elements which are concentrically arranged around the earth placed at the center of the world so that each of them completely surrounds the element below, with the exception of "that dry part of the earth which withstands the flow of water to keep safe the lives of living things."[6] Sacrobosco's brief sentence needs some explanation but he says nothing else on this point in his work.

This sentence is surely not related to a general doctrine of the motions of heavy bodies, which is not even mentioned in *De sphera* where the process of formation of the earth is not discussed. Furthermore, Sacrobosco shows no interest in the cosmogonic problem and is therefore satisfied with a very simple argument based on motion to prove the spherical shape of the earth. This argument is similar to that developed in *De coelo*: if equal parts of earth move from all directions and from equal distances towards the center of the world, they will form a spherical body around it. Except that here the tendency of the parts of earth to move towards their natural place is not considered only in relation to the center of the world, as

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[6] The original reads: "trium quorum [elementa] quodlibet terram orbiculariter undique circumdat, nisi quantum siccitas terre humori aque obsistit ad vitam animantium tuendam." Thorndike (1949, 78–79).

was the case for the Aristotelian ῥοπή in Chapter XIV of Book Two of *De coelo*, but also with regard to their moving away from the rotating motion of the celestial spheres.[7] It is not clear what the author of *De sphera* had in mind, but this is all that can be found in this work concerning the process of formation of the earth.[8]

In Sacrobosco's work the earth is presented as a body already formed, and its spherical shape is proved entirely on the basis of the observation of the motion of the celestial bodies. In truth, there is still a brief mention of the natural motion of the earth, at the end of the section discussing the position of the earth at the center of the world, and it is made in order to prove its immobility. But this does not suffice to modify our judgment. Let us see, therefore, how the questions concerning the shape of the earth and of water are discussed in the first chapter of *De sphera*: these questions, it must be remembered, are not here related to what had been said concerning the size of the elements and are therefore treated separately.

That the earth is spherical from east to west derives from the fact that the stars do not rise and set at the same time, but at different times depending on the location. The stars rise first for those who live in the east and later for those who live in the west. This is evidently shown also by observations of lunar eclipses: the same eclipse observed by us around the first hour of the night (after sunset) is visible by those who live in the east around the third hour of the night. This shows the sun rises earlier in the east than in the west.

The mere observation of the rising and setting of stars is enough to prove the spherical shape of the earth from north to south: an observer moving from north to south at a certain point sees those stars rising that were previously always visible; the opposite happens to an observer moving in the opposite direction. All of these phenomena can be explained only if the earth has a spherical shape.

Naturally, the same arguments that were used to prove the spherical shape of the earth could have been used to find out if water is also spherical. But the difficulty in measuring distances and the position of ships during navigation prevented the application of this same method. Although it is almost impossible to sail for long distances in the same pre-established direction, this kind of experience could have been used to solve the question. It would have been enough to take into ac-

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7 "Omnia etiam preter terram mobilia existunt, que ut centrum mundi ponderositate sui magnum extremorum motum undique equaliter fugiens rotunda spere medium possidet." Thorndike (1949, 79).

8 Sacrobosco might have deliberately avoided a discussion of this type of problem in his treatise. It would seem that he abstained from referring to Aristotle's *De coelo*. At the time that Sacrobosco wrote his *De sphera*, Aristotle's work was already available both through Gerardo of Cremona's translation, and through the Pseudo-Avicenna's *Liber de celi et mundi*, which was much more common. This work was a paraphrase of some sections of *De coelo* and at the time of Sacrobosco, generally regarded as an original work by Aristotle, Gutman (1997, 121–8). The tenth chapter of the *Liber de celi et mundi* which is titled *Quod figura terre sperica est*, contains an interesting discussion of the problem I have been considering here. I will discuss the content of this chapter on another occasion. Gutman (2003, 181–183).

count an experience familiar to sailors: a sailor standing at the top of the mast can see the coast before it is visible to those standing on deck.

This experience was actually referred to by those who tried to prove the spherical shape of water. After placing a signal on the coast, the ship sailed away from the land until an observer on deck could no longer see the signal. After stopping the ship, the observer climbed to the top of the mast so that he could see the signal again. Since the distance between the observer and the signal was shorter when he was on deck, the only explanation that could be given for his not seeing it on deck was the interposition of the water, which could be eliminated by climbing to a higher position.

This observation could only prove that the surface of the water was curved, not that it was actually spherical. The author of *De sphera* must have been aware of this fact, as is apparent from the words he uses to introduce the argument: “Quod autem aqua habeat tumorem et accedat ad rotunditatem sic patet,” that is, “that water has a swollen shape and is almost round is thus evident” (Thorndike 1949, 83). To this *proof* he added an argument that referred to the natural tendency of water to assume a spherical shape. This tendency, which can be observed in drops of water and in dewdrops, could also be ascribed to seawater because of its homogeneity and of the assumed identity of nature between the parts and the whole.[9]

Not based on a general physical principle, these arguments, being disconnected and contradictory, needed further development. It is possible that Sacrobosco intended to leave out of his work the doctrine of natural motions because it was a subject belonging to natural philosophy. With this decision, he would have confirmed the strict distinction between disciplines that were observed by medieval universities. But if this is true, his attitude was outdated. With the revival of the study of Aristotle’s works on natural philosophy, and particularly of *De coelo*, the discussion of subjects thus far treated separately, increasingly involved a unitary approach. The subjects treated in *De sphera* and in *De coelo* were so close that it became impossible to discuss them separately.

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[9] It is interesting to observe that even in this case Sacrobosco never refers to *De coelo* II, 4 which would have given him a good argument to prove that water takes on a spherical shape. In the passage concerned, Aristotle presents a geometrical demonstration of the type of *reductio ad absurdum*, in which the conclusion is not in contradiction with the assumed geometrical principle, but rather it conflicts with the evidence of sense experience: water runs by nature from higher places to lower places. It would have been impossible for Sacrobosco to refer to the proposition at the beginning of Archimedes’ *On Floating Bodies*, which was later translated by William of Moerbeke. Only during the Renaissance were references to Aristotle and Archimedes made by those authors who wrote commentaries on the *Sphere*, thus showing the need for more rigorous demonstrations on this important point.

The Rediscovery of De coelo and Cosmography

The first document testifying to this new approach is the *Commentary on De sphera of Sacrobosco* (*Super auctorem Sperae cum questionibus expositio*)" ascribed to Michael Scot (†c. 1235), where this author's continuous referral to Aristotle's works turns his *expositio* into a proper and detailed *quaestio*.[10] Thus, for instance, the discussion concerning the elements, which had been only briefly mentioned by Sacrobosco, was developed by Scot into a thorough examination of the various positions taken by Aristotle on that subject in *Metaphysics*, *Physics*, *De coelo*, *De generatione et corruptione* and in *Meteorologica*. But when Michael Scot discussed the passage in *De sphera* concerning the arrangement of earth and water, he abandoned any reference to the Aristotelian text and relied instead on direct observation and the "Holy Scriptures." He strongly rejected what was stated at the beginning of the *De sphera* concerning the arrangement of the element in the sublunary world, and maintained that it is rather the earth that is placed above the water, in the same way as the islands are placed in the middle of the sea. This truth was confirmed also by Psalm 103 "Qui fundasti terram super aquas," that is, "You who firmly based the earth on the water" and by Psalm 29 "Quia ipse super maria fundavit eam," that is, "Because he himself firmly based it on the sea."

How can this change of approach be explained? Were there no passages in the Aristotelian works that could help to clarify this point? Perhaps some assertion that could confirm the literal meaning of the commented text? Before this, Michael Scot had in fact just written: "From one fistful of earth ten fistfuls of water are generated by rarefaction and by thinning. And inversely from ten fistfuls of fire one fistful of air is generated by condensation and by thickening."[11] How can this statement, based on a partial interpretation of some Aristotelian passages (mainly from *De generatione et corruptione*, Book Two, Chapter VI), be made compatible with the solution of the problem taken from the Holy Scriptures and based on observation? Unfortunately, it is not possible to give a clear answer and find out whether the commentator had intentionally avoided facing an apparent contradiction or whether he was able or not to understand the Aristotelian text cor-

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[10] This must be obviously related to Michele Scoto's activity as a translator of Aristotelian works (with Averroës' *Commentaries*): he had translated into Latin an Arabic translation of *De coelo* before 1230. Another, perhaps not complete, translation was made by Robert Grosseteste soon after 1230. After these works became available, it was unlikely that those who wrote on cosmography could ignore Aristotle's work.

[11] The original reads: "ut ex pugillo terre per rarefactionem et subtiliationem fiunt pugille aque decem etc. Et econverso ex decem pugillis ignis per condensationem et inspissationem fit unus pugillus aeris et deinceps." Thorndike (1949, 263–264).

rectly.[12] Pierre Duhem accurately explained this difficult point of Aristotle's philosophy in his *Système du Monde*, where the reader could find further explanation (Duhem 1958, 91–97).

Concerning the question of the spherical shape of the earth, Michael Scot's commentary is more dependent on Aristotelian texts, although the proof of the spherical shape derived from the doctrine of the motion of heavy bodies is only briefly mentioned in his *Expositio*. This proof is presented within a discussion concerning the local motion of the elements seen as a process that leads to the complete actualization of the form.

The passages from *De coelo*, which are more frequently quoted in the discussion of the shape and position of the earth, are often given different interpretations that are not always true to the original. This is the case for the brief mention of the Aristotelian text contained in Bartholomaeus Anglicus' *De proprietatibus rerum* (Chapter I): the medieval author seems to use the term *equilibratus* with the same meaning as that of ἰσόρροπος, so that the passage concerning the equal arrangement of parts of earth around the center of the world caused by ῥοπή is interpreted as stating a situation of equilibrium resulting from equal and opposite inclinations.

Such interpretation is clearly based on the idea of balance: the earth seems to be in a state of equilibrium because its heavy parts have a tendency to go towards the center of the world as a consequence of their weight; and the earth, because of the tendency and inclination of its parts, is *hanging* at the center in a state of equilibrium and remains at rest in the same place.[13]

[12] Further on in the text just quoted (Thorndike 1949, 296), Michael Scot discussed the same question from the point of view of concept of place by investigating the reason for the placement of the earth within the sphere of the immediately higher element. The internal surface of the sphere of water, as it was not in contact with all parts of the sphere of the earth, could not be considered as the place of the earth, whereas in some passages in Aristotle's works it appeared that the internal part of the sphere of any element was the place of the element immediately lower. The solution given by Michael Scot was similar to that briefly mentioned by Sacrobosco: according to the form proper to each element, the earth should have been contained by water, but the world was not perfect and the great majority of the animals and of the plants could not live in water, therefore part of the earth had been cleared of water. No reasons are given to explain how this situation came about.

[13] Bartholomaeus Anglicus (1505, sign. [s 3v]) "Terra, ut dicet Philosophus, est propriis equilibrata ponderibus. Quelibet enim suarum partium suo pondere nititur ad mundi medium, quo nisu et inclinatione singularium partium, tota circa centrum equilibrata suspenditur, et equaliter immobilis retinetur." The first part of this passage seems to be in relation with Publius Ovidius Naso (*Metamorphoseon*, I, 1, vv. 12–13) "nec circumfuso pendebat in aere tellus ponderibus librata suis…"

The Acquaintance with Simplicius's Commentary on De coelo: Through its Medieval Latin Translation, the Centrobaric Theory Becomes a Part of Cosmography

Around the mid-thirteenth century, the Aristotelian texts had already been given different interpretations, and their use to explain Sacrobosco's *De sphera* became a generalized practice. Only during the second half of the century were all those questions discussed more deeply and on the base of new texts. This development was made possible by the Latin translation of Simplicius' *Commentary on De coelo,* made by William of Moerbeke in 1267. Through this work, the Western philosophers became acquainted with the doctrine of the center of gravity: with this new conceptual instrument they could give new answers to the traditional cosmological questions.

In truth, knowledge of the part of Simplicius' work that concerned these questions did not spread very rapidly. Thomas Aquinas, who was one of the first to know about Moerbeke's translation, did not use it in his *Lectio* XXVII on Book Two of *De coelo*, where he gave an explanation of the reciprocal action between parts of earth that is different from Aristotle's text and replaced the term *inclinatio* with *pondus*, in a way similar to that which we have already seen:

> (versio vetus) Plus enim semper quod ipso minus propellere est necessarium, usque ad medium inclinationem habentibus ambobus, et graviori propellente usque ad hoc minus grave.
>
> (comm. Thomas) Nam si versus unam partem terrae sit maior quantitas, ad hoc quod ipsa magis appropinquet medio, depellit minorem partem per violentiam a medio *quousque aequale pondus ex omni parte inveniatur*.[14]

At the beginning of the fourteenth century, the philosophers of the Parisian school approached questions of the position and shape of the earth from a new point of view. By making an exhaustive analysis of the Aristotelian concept of 'place' based on parallel passages from Book 4 of *Physica* and from *De coelo*, these authors put the study of the formation of the earth in second place and gave prominence to the problem of the arrangement of the "elements" and of the receding of water from land populated by men and animals. They were able to satisfactorily tackle these problems by introducing a distinction between center of magnitude and center of gravity of a body. This distinction, which had already been made by Simplicius in his commentary on *De coelo*, was employed by the Parisian philosophers in their discussion of the *aporia,* mentioned earlier.

In explaining the shifting of the center of gravity of the spherical body placed at the center of the world as a result of adding a great weight to one hemisphere, they were using an argument belonging to the Archimedean tradition, but said

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[14] *De caelo et Mundo*, Book. 2, lectio, 27, De Aquino (1866, 142–143). Author's italics.

nothing about the place of the two centers, one of gravity and the other of magnitude, within one single nor homogeneous spherical body. This problem was related to another question in Simplicius' commentary, namely the question of the perfectly spherical shape of the earth, which he discussed by referring to Alexander of Aphrodisia's statement on the existence of non-homogeneous parts in heavy bodies. This observation necessarily involved a distinction in a body between center of gravity and center of magnitude: one center was important for determining the motion of a heavy body downwards, whereas the other was irrelevant from this point of view and had a purely geometrical meaning. In consequence, it was possible to place a heavy body differently with respect to the center of the world according to whether it was homogenous or not.

The author, who in the Middle Ages developed this argument in the most rigorous way, was Albert of Saxony. In his *Questiones subtilissime in libros de celo et mundo* (1520), Albert of Saxony discussed the problem of the place of the earth in the world by making use of that distinction. There were thus two different ways of considering the position of the earth at the center of the world: either in relation to its center of magnitude, or in relation to its center of gravity. The fact that parts of land were not covered by water excluded the first case and involved the necessity of placing the two centers differently. The earth was therefore to be placed in the world with respect to its center of gravity because (to quote Albert's paraphrase of the Aristotelian passage) "the heavier part pushes" the less heavy one "until the center of gravity of the whole earth is at the center of the world," for "then the two parts will have the same gravity, though one is bigger and the other smaller with regard to their magnitude": the inverse proportion between the two parts is the same as that of two different weights placed on a balance in equilibrium.[15]

The transformation of the Aristotelian concept of ῥοπή could not be expressed more clearly. Here, it is no longer the question of several parts of earth of different size, which by interacting with one another end up making a spherical body, but rather of weights of different magnitude which, as in Proposition 3 of Book One of *On the Equilibrium of Planes*, are in equilibrium when their common center of gravity is placed nearer to the heavier weight.[16]

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[15] "Quod terra est in medio mundi quo ad centrum sue gravitatis. Probatur: nam omnes partes terre tendunt ad medium per suam gravitatem, sicut dicit Aristoteles in littera; et verum est. Modo pars que esset gravior depelleret aliam tam diu quod medium gravitatis totalis terre esset in medio mundi; e tunc starent due partes eque graves; licet una maior et alia minor quantum ad magnitudinem contra se invicem; sicut duo pondera in equilibra." Albert of Saxony (1520, 40r, lib. 2, quaestio 25).

[16] We could even think that Albert refers to the law of lever as presented by Archimedes in Proposition 6 of Book one, but from the Latin passage quoted in the note above it would seem that Albert only points out that in this case the heavier weight is nearer the center of the world, and the lighter weight is further away from it.

But in addition to this, we also have to take into consideration other factors that alter the weight of the different parts of the earth. The cause of the difference in weight between the two hemispheres is the sun, the heat of which makes the land not covered by water less dense, and therefore lighter. It is a cause which endures over time: the changing action, together with other changes taking place on the surface of the earth, determine a continuous shifting of its center, which therefore can move in a straight line.[17]

The New Worlds and the Centrobaric Theory: Franciscus Maurolycus, Christoph Clavius, Giuseppe Biancani

Around 1540 the geographical discoveries made during oceanic voyages by Spanish and Portuguese sailors had changed the picture of the earth and raised doubts about many ideas on which medieval cosmology was based. The impossibility of reconciling old ideas with the knowledge of new facts made it necessary to introduce substantial corrections in the astronomy textbooks used in the schools. The passages in these textbooks, which needed corrections, were those concerning the inhabited lands rising from water, as a consequence of the discovery of new countries in the West Indies. Thus there was no longer any reason for referring to the concept of center of gravity in the commentaries on *De sphera,* though it was still considered necessary to go back to the Aristotelian text.

In the discussion concerning the position, the immobility and the shape of the earth, the situation remained unchanged and the doctrine of the motion of heavy bodies still played an important role in this context. But during the sixteenth century things began to change. In addition to the "old world" represented by Aristotle's works, a "new world" made its first appearance in the form of texts that were almost unknown up to this point: the *Mechanical Problems,* at that time ascribed to Aristotle; Archimedes' *On Floating Bodies* and *On the Equilibrium of Planes;* the works by Hero of Alexandria; and those by Pappus. These new texts offered a more complex idea of the motion of bodies, based on the observation of motion in certain machines. For the first time, scholars could familiarize themselves with works containing rigorous discussion of problems concerning the moving and lifting of heavy loads, and could learn how simple machines functioned. The concept of center of gravity was not used systematically in all these works but it did play a central role in Archimedes' texts. Through Book Eight of Pappus' *Mathematical*

[17] By referring to what Aristotle says in chapter XIV of Book Two of *Meteorologica*, Albert probably thought that the continuous changing location of lands, rivers and seas involved a continuous changing arrangement of the weighing parts of the earth.

Collections scholars became aware of the only definition of center of gravity to be handed down from antiquity.[18]

The Rediscovery of Pappus' "Mathematical Collections": The Center of Gravity of Bodies: From Cosmography to Mechanics

The fact that all these new sources of information became available produced important changes in the use of the concept of center of gravity within the discussion on cosmology. This new development started with Franciscus Maurolycus, one of the major mathematicians of the sixteenth century, who in his *Cosmographia* (Maurolico 1543) introduced radical changes both in the content and form of exposition. Written in the form of three dialogues, this work included in a systematic way new ideas derived from the two "worlds" mentioned above and focused attention on many questions that were later discussed in similar works. Maurolycus' new discussion on the shape and place of the earth and of water was included in its entirety, without any mention of the source, in Clavius' *Commentary on De sphera*, a work that was used as an astronomy textbook by generations of scholars of the Jesuit Order.

On the model of Sacrobosco's *De sphera*, Maurolycus discussed the question of the spherical shape of the earth and of water just after presenting the proofs for the spherical shape of heavens.[19] The whole argument is developed by following, from the beginning, the line of reasoning traced by the Sacrobosco but it is made more precise and articulate in a similar way to that adopted by many commentators of Sacrobosco's work. In the case of the first proof for the spherical shape of

[18] Here, I deliberately disregard the tradition of the medieval Latin *Scientia de ponderibus*, which greatly influenced discussions of the problem of the equilibrium of balances during the Renaissance. This is not because I assume there is no relation between the question I discuss and those treated in the works ascribed to Jordanus Nemorarius. On the contrary, the premises of this work treat explicitly the relation between *gravitas* and *rectitudo* of the path of descent of heavy bodies (with reference to the line that ends in the center), Moody and Clagett (1960, 128–129). I chose not to deal with this tradition because the problems of the centrobaric theory are not mentioned. For an extensive treatment of the dispute between followers and opponents of the *Scientia de ponderibus* in the sixteenth century, see Renn and Damerow (2012). In the Pseudo-Aristotelian *Mechanicals Problems* there is also no reference to the concept of center of gravity, but in this case there is a tendency to adopt Archimedes' method, on the assumption of a continuity between Archimedes' work and the mechanical tradition of Pseudo-Aristotle.

[19] Maurolycus' peculiar way of phrasing the question must be noted. For him, the problem was the need to explain "ut hanc terrae marisque congeriem conglobatam esse." Maurolico (1543, 7r/v). This seemed to go back to the tradition of discussions on the causes of why the earth was not completely covered by water. Sacrobosco instead discussed separately, one after the other, the question of the spherical shape of the earth ("Quod terra etiam sit rotunda sic patet," Thorndike (1949, 81) and of water ("Quod autem aqua habeat tumorem et accedat ad rotunditatem sic patet," Thorndike (1949, 83).

the earth, Maurolycus specified that the lunar eclipse must be observed from two towns on the same latitude. This condition was essential to guarantee the truth of the reasoning, which ended with an observation made in different places on the same parallel: only then would it be possible to establish that the differences in time were proportional to the differences in places, and to prove rigorously the spherical shape of the earth (Maurolico 1543, 7r/v).

Concerning the shape of water, Maurolycus sensibly departed from the medieval author. At the beginning, he seemed to move in the same direction as Sacrobosco and to rely again on the common experience of sailors who, when approaching the coast, first see the tops of mountains, bell towers and high buildings. But immediately afterwards, Maurolycus considered the whole terrestrial globe, formed by earth and water, and discussed the proof of its spherical shape by observing the shape of the earth's shadow on the surface of the moon during the lunar eclipse. This proof was based on the fact that water has a natural tendency to move towards lower places and, owing to its instability, stops moving only when its surface has the same height everywhere: this situation occurs when the surface of water is equidistant from the center of the world, which is the lowest point that a heavy body can reach (Maurolico 1543, 7v–8r).

Any other argument thus fell short of proving anything, including the presumed tendency of water to take on a spherical shape: the phenomenon observed in dewdrops and in drops of water could not take place in the whole element of water; this element took on a spherical shape in order to keep the equilibrium of its parts, which struggled equally to move towards the center (servet aequilibrium aequaliter ad centrum connitens), whereas dewdrops and drops of water had a spherical shape as a result of the action of their opposite, that is, the dry element (Maurolico 1543, 8r). The cause of the spherical shape of water could therefore be found in its nature as a heavy body.

This explanation was no different from that given by Aristotle for the spherical shape of earth, which, though it was not made to flow and have its parts uniformly arranged, could not have naturally taken any other shape but the spherical one (Maurolico 1543, 8r). It would never have a perfectly spherical shape like that of water but its lack of perfection was not so important. Mountains and valleys could be regarded as having no tangible dimensions compared to the great size of the earth, even though for us they appeared to be very large (Maurolico 1543, 8v).

The question of the shape of the earth and of water was thus inevitably related to the doctrine of the motion of heavy bodies and to the concept of natural place. Everything encouraged the scholars to believe that the center of the terrestrial globe, formed by earth and water, coincided with the center of the world. But what had been said so far was not sufficient to prove the truth of this conception. Other proofs were needed.

The proofs adduced by Maurolycus were various and also in this case went further than what had been said by Sacrobosco. Their starting point was the same, that is, the absurdities that would follow if the earth were placed outside the center of the world: the stars would appear to come close to and move away from the earth; it would have been impossible to see half of the celestial sphere and therefore to observe the rising and setting of six signs of the zodiac. All of these absurd consequences were contradicted by observation. To these arguments, which were the only ones presented by Sacrobosco, Maurolycus added other arguments based on astronomy and, more interestingly, on some thoughts concerning the weight and place of the elements in the sublunar region.

Both the arrangement of the elements on the basis of their being heavy or light and their motions *from* or *towards* the center of the world confirmed what had been said by Aristotle: heavy bodies move towards their natural place along lines meeting at one single point and tend to keep their tendency to move until they reach a place where all their parts are at equal distances from the center, thus forming a spherical figure (Maurolico 1543, 8r). The heavy bodies move along lines perpendicular to the surface of the earth and of water, as can be seen from observations made with a plumb-line; and this ensures that both elements, that is, water and earth, are placed around the same center (Maurolico 1543, 8r). That the lines along which heavy bodies fall are convergent is not immediately visible: to an observer they seem to fall along parallel lines, but this impression is due to the short distance they are observed to fall. As in the case of the walls of two buildings or of a well, the apparent parallel direction is the result of their small dimensions compared to the size of the earth: if we imagine the size of the two buildings to be increased enormously, they would appear to be more and more divergent, whereas the walls of the well would in the end converge at the center of the earth (Maurolico 1543, 15v–16r).

These are traditional examples, which had already been mentioned by Albert of Saxony in the corollaries to the *Quaestio* XXVII on Book Two of *De coelo*. But now they are related to the question of the *antipodes* which, after the geographical discoveries, made it impossible to believe in the existence of an enormous sphere of water not concentric with the sphere of the earth. To be able to stand upright, the inhabitants of any zone of the earth must put their feet and head on the straight line passing through their body and reaching the center of the world. In this way two people standing in diametrically opposed places on the surface of the earth are placed in similar positions in relation to the lowest point of the world; they are in the same situation as two hanging weights which, if allowed to drop, would converge towards that point.[20]

[20] "Nimirum, quod utrique nos erectos arbitramur, verum est; quod vero nos illos, illique nos in caput versos putamus, falsum. Siquidem utrique recti stamus; ipsumque terrae centrum locus est

After briefly mentioning earlier discussions of the reasons why the inhabited lands were not covered by water (with reference to Nicholas de Lyra, Paul of Burgos and Matthias Döring), Maurolycus focused his attention on some apparently paradoxical consequences resulting from the application to the whole terrestrial globe of concepts that are usually defined with regards to smaller portions of its surface (Maurolico 1543, 17r/v). How can the construction of a leveled floor be defined? This is done by using a "level," that is, an instrument indicating a line parallel to the plane of the horizon and determining the horizontal position of a surface to which it is applied. This is unproblematic as long as we are dealing with small portions of a spherical surface: the curvature of the surface in this case is so slight that it can hardly be perceived. But if we imagine the floor as an enormously extended plane that is tangent to the surface of the earth, that floor cannot be defined as being "leveled," since a body moving on it would not remain at the same distance from the center, but this distance would diminish or increase according to whether the body moves towards or away from the point of tangency.[21] In this case, a man walking on a horizontally leveled plane would be moving upwards or downwards with regard to the center of the earth.

Another paradoxical consequence would result from imagining a very large vase full of water placed on the surface of the earth or near to its center: the curvature of the surface of the water would be different according to the position of the vase and, in consequence, the vase would contain a greater quantity of water in the area close to the center. This difference would not be noticeable on the earth, even if we were able to place the vase on top of the highest mountains or at the bottom of the deepest pits, because the distances would be too short compared to the radius of the terrestrial globe. Nevertheless, these paradoxical consequences were directly deduced from the general principles given by Aristotle (Maurolico 1543, 18r/v).

The natural tendency of heavy bodies to move towards the center of the world along straight lines and their arrangement at equal distances from the center required that the nature of the resulting spherical agglomerate be homogeneous, since only in this case would the center of magnitude and the center of gravity coincide. But did the terraqueous globe formed of land and water have such a homogeneous nature? Were the two elements arranged around a single center? The observations collected during the oceanic voyages of Colombo and Vespucci were able to give positive answers to these questions and exclude the possibility of the terrestrial globe not being homogeneous, as the medieval authors had thought to explain the existence of lands not covered by water. The earth and water were not

infimus utrisque communis; ad quem sane duo pondera utrinque suspensa pendent, et dimissa concurrerent." Maurolico (1543, 16v).

[21] This argument was also derived from Albert of Saxony (1520, 41v.), *Quaestiones subtilissime*, in the 4th corollary to Question XXVII (already mentioned) of Book Two of *De coelo*.

arranged around the center of the world as a spherical body made of stone or wood would have been, that is, with the center of gravity placed nearer to the heavier part: they did not have different centers (Maurolico 1543, 18v).

Does this mean that the center of gravity was no longer an important concept in the field of cosmology? Not really. Though the arrangement of land and water on the earth no longer needed to be established, the center of gravity now became an essential concept for defining in a rigorous way the motion of any heavy body. The center of gravity of a body hanging in any position was always placed on the perpendicular line extending to the center of the world and, if dropped and not stopped by any obstacle, at the end of its movement it would encounter this center.[22] This way of defining the center of gravity with regard to its possible future motion was different from Pappus' definition, where mainly the static condition of a hanging heavy body was taken into consideration. The new "definition" includes some important aspects of the method for finding the center of gravity, following Pappus in the First Proposition of Book Eight of his *Mathematical Collections*. The text of this proposition was reproduced almost *ad litteram* by Maurolycus:

> Let the body be suspended in any way so that it would be freely hanging; now from the point from which it is suspended let a straight line be drawn perpendicular to the horizontal plane, as Euclid shows in Proposition XI of Book Eleven of the "Elements." Let the same body now again be suspended in a similar way from another point, from which a new line may be drawn again perpendicular to the horizon. Both perpendicular lines should surely extend through the center of gravity, since this center is always found on the same perpendicular line, however the body may be suspended. Therefore, the point at which the perpendicular lines intersect each other will without doubt be the center of gravity that was sought.[23]

Sacrobosco's *De sphera* Revised and Corrected: Cristophorus Clavius' Commentary

As mentioned earlier, this section of Maurolycus' work was almost entirely incorporated by Clavius into his *Commentary on Sacrobosco's De sphera* (Clavius 1570), within a long discussion on "whether the Earth and water formed a single

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[22] "Punctum videlicet, quod utcunque ac quotiescunque suspenso corpore, semper versus universale centrum pendet ad perpendiculum; quodque, dimisso corpore, modo absint obstacula, ipsi universali centro connitur." Maurolico (1543, 18v).

[23] The original reads: "Suspendatur proposita res utcunque, ut libere pendeat; mox ab ipso suspensionis signo ad horizontis planum perpendicularis recta ducatur, quemadmodum Euclidis in 11 undecimi docet. Rursum ab alio signo similiter res ipsa appendatur; et a signo rursum perpendicularis agatur ad horizontem. Oportebit nanque utranque perpendicularium per centrum incedere gravitatis, quandoquidem tale centrum in ipsa semper ad horizontem perpendiculari, utcunque res pendeat, invenitur. Punctum igitur, in quo se vicissim perpendiculares intersecane, erit proculdubio quaesitum gravitatis centrum." Maurolico (1543, 19r/v).

globe, that is whether the convex surfaces of these elements had the same center."[24] These questions were not directly related to Sacrobosco's text and, being in a way autonomous, were discussed in a section added to the commentary. The discussion started with a close criticism of those who maintained that the spheres of the earth and water had different centers. The first argument had a long tradition: the water placed outside the center of the world would require supernatural mediation to keep in that position; but such a miracle was really not necessary and therefore should be excluded. Immediately after this explanation, Clavius referred to the geographical explorations that had revealed the existence of dry lands at the antipodes. This discovery, though it showed the falsity of the hypothesis mentioned above, could not prove anything concerning the actual position of the two centers. So what were the proofs supporting the coincidence of the centers of the two spheres? And how could their placement at the center of the world be demonstrated? This could be done first of all by observing the motion of heavy bodies, then by using astronomical arguments such as the observation of lunar eclipses, and lastly by resorting to the testimonies of sailors who had crossed the ocean and observed variations in the rising and setting of stars, similar to those observed on land. Here, we are only interested in the first argument.

Although he used most of Maurolycus' arguments based on the center of gravity of bodies, Clavius changed the order he followed and divided the proof into two parts, based on the observation on natural motion. He did so in a way that at first sight does not seem suitable to develop rigorous reasoning. Maurolycus had shown: (1) that both the earth and water had a spherical shape; (2) that their centers coincided on the basis of experience with the plumb line. Only after proving this did he criticize those who thought the center of magnitude was different from the center of gravity, and his criticism was mainly based on the new geographical discoveries, without making any reference to the motion of heavy bodies. This motion was instead taken into account in defining the center of gravity, and this argument was connected on some points with the experience of the plumb line.

In order to prove by geometrical demonstration that the centers of both earth and water could not be different, Clavius repeatedly resorted to the motion of heavy bodies. He demonstrated first of all that if all bodies have a tendency to move towards the center of the world, then they must form a single spherical body and, on the contrary, that if they did not result in the formation of a single spherical body, they could not have a tendency to move towards the same center (Knobloch 1999, 59). He then went on to prove that the center of gravity and the center

[24] Clavius (1611). This edition was included in volume 3 of Clavius' *Opera omnia*, anastatic reprint with an introduction by Eberhard Knobloch. The original reads: "An ex terra et aqua unus fiat globus, hoc est, an horum elementorum convexae superficies idem habeant centrum." Knobloch (1999, 57).

of magnitude of the body formed by both the earth and water coincided by observing that the angles formed by the plumb line on the surface of the earth and water were equal (Knobloch 1999, 59–60). In the first part of his demonstration, Clavius had taken from Maurolycus both the definition of center of gravity and the method for finding it. But these two texts seem misplaced because they are included in a geometrical demonstration that avoids any reference to the angles formed by the straight lines directed towards the center of the world.[25]

The general impression that can be gathered from this section of Clavius' *Commentary* is that different materials have been assembled without any attempt at connecting and further developing them. Such assemblage of texts could be very useful for teaching, but did not help to develop this key concept in mechanics which was destined to undergo deep transformations. This impression is confirmed by the last part of Clavius' long digression on the spherical shape of the globe formed by both the earth and water: the apparent paradoxical consequences of this doctrine are once more discussed at length, such as perpendicular buildings would not be parallel, that the motion on a plane tangent to the surface of the earth would not be horizontal but upwards or downwards, and that a vase full of water would contain a greater or smaller quantity of liquid depending on whether it were nearer to air or farther away from the center of the earth.[26]

Giuseppe Biancani's Work: Cosmography and Mechanics at the Beginning of Modern Science

Though based on more recent astronomical works, Clavius' *Commentary on De sphera* still relied on observations made by the naked eye. In the 1611 edition of his work, the German mathematician mentioned the new astronomical discoveries accomplished with the telescope but died soon afterwards without being able to include them in the discussion contained in his work. Other members of the Jesuit Order immediately realized that these novelties ought to be taken into account in any discussion concerning astronomical matters, and that they would have a great effect on the Ptolemaic model of the world. The dispute between the followers of different cosmological systems was still at the beginning and the need to reassess the nature of the motion of heavy bodies within different astronomical hypotheses

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[25] Knobloch (1999, 57). In the 1581 edition published in Rome, the definition of the center of gravity taken from Maurolycus is followed by the one taken from Pappus' *Mathematical Collections*, Book Eight. Since any reference to the motion of the body let loose from the point at which it was hanging is missing, Pappus' definition would have been more suitable for Clavius' argument; but its inclusion in this argument is in no way justified. Any comparison between the two texts is missing, and the new definition seems to be an unnecessary addition.

[26] Knobloch (1999, 65–66). See Maurolico (1543, 18r/v).

was not yet felt. This new situation can already be seen in Giuseppe Biancani's *Sphera mundi*; its full title mentioned the "new findings" (*novis adinventis*) of Brahe, Kepler, Galilei, and "other astronomers." However, when discussing the position and the mobility or immobility of the earth, Biancani repeated traditional arguments (Blancanus 1620).

In Part Three, Treatise II of *On the elements*, the question of the direction of the motion of heavy and light bodies was discussed with the same arguments used by previous authors, though in a more rigorous and coherent way. Biancani, more outspoken and critical than Clavius, was utterly contemptuous of those who believed that such motion took place along parallel lines: he regarded this idea as childish and shared by ignorant people who believed that the world was like an "oven" that is, a hemisphere with an endless bottom. In such a world, heavy bodies would fall along parallel lines and men on the surface of the earth would be in an upright position only in relation to those parallel lines. For Biancani, this is contrary to experience, which shows that heavy bodies move towards the center of the world and that the upright position of men is in relation to lines passing through the same center. For him, this truth seemed to be confirmed by a simple observation and could therefore be verified by anyone standing on any point of the surface of the earth, even by those inhabiting the antipodes.

All this is based on the presupposition that the earth has a spherical shape, a thing that Biancani had not yet proved, though he thought he could reject the common experience as utterly erroneous. The traditional view of the world, which defines the structure of the elementary region of the world on the basis of the two natural motions "away from" and "towards" the center of the world, had such an influence on direct observation that it seemed unnecessary to resort to a geometrical demonstration in order to establish the true direction of the motion of heavy and light bodies. Within a naïve and primitive picture of the world, the possibility of regarding the motion of such bodies as taking place along parallel lines was excluded (Blancanus 1620, 69–70).

To correctly understand the motion of heavy bodies towards the center of the world, it was necessary to know where the center of gravity of a body was placed: only after establishing the place of that center would it be possible to draw the straight line that the body would actually follow in its movement. For Biancani, Pappus' definition of the center of gravity was the ideal picture of the body before it began to fall, and the fall was imagined as the simple motion of a point along a straight line towards another point. Once the line of direction of the fall of a heavy body had been established in this way, the movements of the other points of the body needed to be analyzed. This aspect of the question had never been considered before and it directly concerned the general concept of equilibrium.

Pappus had established that a body hanging from its center of gravity, no matter how it was positioned, would never move from its initial position because on both its sides there would be two equally balanced parts (ἰσόρροπα δύο μέρη), that is, two parts having equal and contrasting tendencies to move towards the center of the world. A body would thus be conceived as being formed, in a way, of a group of innumerable balances, all positioned with different inclinations with regards to the perpendicular line that stretched to the center of the world, but all in a state of equilibrium. Such a body, when falling, would not rotate in any way so that each one of its points would fall along a line parallel to the perpendicular (Blancanus 1620, 70). This description would make it possible to discuss in a unified way the conditions of equilibrium of all bodies, both those placed at fixed distances from the center, and those in motion towards the center or away from it: this approach was very useful for a deeper study of the problem of the balance, which after the publication in 1577 of Guidobaldo del Monte's *Mechanicorum liber* had reached a degree of complexity never achieved before.[27]

A study of this problem from a strictly mechanical point of view had actually been made by Biancani in his *Aristotelis loca mathematica*, when he discussed the second question of the *Mechanical Problems* ascribed to Aristotle. This question asks why "if the cord [more correct "support"] supporting a balance is fixed from above, when after the beam has inclined the weight is removed, [does] the balance return[s] to its original position"? (Aristotle and Hett (1936, 347–349/850a2–6). Biancani first recalled the discussion of previous authors, and then tried to answer the Aristotelian question on the basis of the concept of center of gravity.[28]

[27] Monte (1577, 5v–21v; 1581, 5r–25r). In these pages Guidobaldo criticized the solution of the problem of the equilibrium of the balance given by the medieval *Scientia de ponderibus*.

[28] Blancanus (1615, 155–157). In this work the Jesuit scholar had also discussed the passage on the *aporia* in Chapter XIV of Book Two of *De coelo*. On the basis of the concept of center of gravity, the motion of a heavy body towards the center of the world and its resting at this center at the end of the motion were immediately understandable: as soon as the center of gravity of the body and the center of the world coincided, the body would stop moving. But Aristotle could not have conceived of this center in this way since the concept of center of gravity was first used by Archimedes. Therefore Biancani thought that Aristotle meant the center of magnitude and hence was wrong. "Iuxta mathematicos duplex esse medium, sive centrum cuiusvis magnitudinis: aliud enim est centrum molis, aliud est centrum gravitatis. [...] Quando igitur Aristoteles ait, grave descensurum, donec ipsius medium, sive centrum, mundi centrum attingat, bene dicit, si de medio gravitatis intelligat, male autem si de medio molis, quia gravia omnia ratione centri gravitatis ponderant, neque manent, nisi ipsum maneat; quare nisi ipsum attingant centrum mundi, semper gravitabunt, et movebuntur. Verum enim vero ex antiquorum monumentis manifestum est, Archimedem, qui multo post Aristotelem floruit, primum omnium de centro gravitatis esse philosophatum, qua ratione dicendum esset, Aristotelem de centro, molis loquutum esse, et perinde non usquequaque vere." Blancanus (1615, 81). The explanation of the phenomenon described in the *aporia* was simplified and transformed by Biancani: the greater part that pushes the smaller one is regarded as being inside the solid body. "Sensus Aristotelis est, debere nos existimare, quod si quaepiam gravis magnitudo descendat ad centrum mundi, eam non permansuram, statim ac ipsius extremum centrum mundi attigent; sed eo usque descensuram, quosque ipsius medium, mundi medium, sive centrum assequutum sit; maior enim ipsius pars, in qua scilicet medium est, minorem partem propellit, donec utrinque a centro mundi aeque emineat. Omne enim grave hucusque habet propensionem, sive hucusque gravitat, v.g. si lapis illuc de-

Biancani did not repeat this discussion in his *Sphera*, but preferred to include some arguments taken from the *Commentaries and Disputations on St. Thomas Summa* by Grabriel Vásquez, one of the leading Jesuit theologians of the time (Vásquez 1606, 464–465). These arguments were based on a strict application of the mechanical model derived from the balance, which presupposed a continuous slight shifting of the center of gravity of the earth. Any weight, however small, even the weight of a bird flying from one place to another, would alter the equilibrium of the weight existing in the terrestrial body and, in consequence, would cause a shifting and a new placement of it around the center of the world.[29] Although this shifting could not be perceived by human beings, this would not be considered a sufficient reason to deny its validity. This idea, which was a direct consequence both of the doctrine of equilibrium expounded in books on mechanics and of the customary practice of weighing by means of a balance, was the final stage of a long development of thought on the concept of center of gravity, which had begun with Maurolycus' work. At this point, Aristotle's and Sacrobosco's texts were only vague starting points and the medieval discussions had now lost any relevance. The terraqueous globe was now no longer considered from the point of view of the process by which it had been formed, but as a body already formed, having a spherical shape, and placed at the center of the world. The motions of heavy bodies proved the necessity of these assumptions and the laws of the equilibrium of weights confirmed them. But these same laws entailed the impossibility of an absolute immobility of the earth, since weights on it moved continuously; and that caused a continuous shifting of its center of gravity. A theoretical model stated the existence of a fact that could not be perceived by the senses; similar phenomena would occur more frequently in modern science.

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scenderet, non quiesceret statim ac prima ipsius pars ad mundi centrum pertingere, sed reliquae ipsius partes adhuc gravitarent, sicque ulterius primam partem impellerent, donec lapidis medium mundi medio congrueret." Blancanus (1615, 80–81).

29 "Terrae moles ita circa mundi centrum constituta est, ut in aequilibrio sita sit, idest, partes eius circa mundi centrum aeque ponderent, ac propterea immota consistat; quae vero in aequilibrio manent, quovis minimo ex una parte addito, vel ablato pondere, ab aequilibrii situ dimoventur, ut experientia quotidiana in lancibus, ac stateris ostendit, et rationes Mechanicorum evincunt. Cum igitur perpetuo circa Terram, res variae modo illi addantur, modo demantur (ut eum lapis in altum proiicitur, vel cum aves ab ea avolant, et ad eandem advolant, aut cum aliquid super eam saltat) necessarium esse videtur ipsa in perpetua quadam trepidatione insensibili tamen, titubare, ac vacillare." Blancanus (1620, 76).

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