Abstract: The application of AutoRegressive (AR) models to extract spectral parameters from residual ECG (rECG) signals is exploited. In particular, a new method based on AR spectra is employed to estimate the dominant atrial cycle length (DACL) on ECG obtained from patients undergoing episodes of atrial fibrillation (AF). The traditional FFT-based spectral approach will be compared with the new method. Potentialities and possible superior performances of AR spectra are documented and discussed.

INTRODUCTION:
In the last two decades, spectral analysis of the residual ECG signal (rECG, i.e. an ECG signal in which ventricular components were canceled through beat averaging techniques) has been employed to characterize atrial activities. The method is particularly relevant during atrial fibrillation (AF), after infusion of drugs or when various abnormal atrial rhythms are present. Typically, for a patient undergoing AF, a main spectral component $f_0$ is found in the range 3-12 Hz. The inverse of $f_0$ (the so called Dominant Atrial Cycle Length, DACL) has been related to atrial refractoriness and therefore to the probability of AF-maintenance through atrial electrical remodeling [1]. Recently, different authors have observed an increase DACL before spontaneous termination of AF [2].

The same technique was successfully employed for the analysis of long-term Holter recordings: in this framework however, movement artifacts and external noises may become critical factors for a reliable spectral estimation. A step forward would be the development of techniques able to provide robust, automatic estimation of rECG spectral parameters in long-term Holter recordings. Commonly, non-parametric, FFT-based methods, like the Welch's averaged, modified periodogram method, are employed for DACL computation [3-4]. Nevertheless, model-based spectral estimators have potential theoretical advantages when dealing with low-components, noisy signals. In addition, the use of a spectral-decomposition method [5] would provide a natural alternative for $f_0$ estimate. In this paper, rECG were constructed for each recording in the spontaneous termination AF database available from PhysioNet. Power spectra of rECG were obtained with estimators based on AR models and FFT. The different $f_0$ frequencies derived from the two approaches were then compared.

METHOD:
Atrial Fibrillation Database. We based our analysis on the 2004 PhysioNet/Computers in Cardiology Challenge Database. It includes a collection of 80 digitized two-channel ECG recordings. Each record is a one-minute segment of atrial fibrillation, sampled at 128 Hz. All the recordings therein were considered.

Signal pre-processing. Extraction of the residual ECG was obtained through beat-to-beat subtractions of an averaged QRST complex [2-4]. Despite the dataset already contained QRS annotations, ventricular beats were mainly misclassified. Thus, QRS detection was performed using a modified version of OSEA [6], a freely available ECG library. QRS onsets and widths were further refined by means of a second publicly available software, ECGPUWAVE [7]. Then, on a lead-by-lead basis, separate average templates were built for QRS and T waves. Subtraction of the templates was performed after a warping procedure and subsequent templates were connected via linear interpolation.

Spectral Estimation. The power spectral density (PSD) of rECG was firstly estimated using the Welch’s periodogram (512-points Kaiser’s
window with an overlap of 256 points). The technique ensured a spectral resolution of 0.25 Hz and a reasonably low variance of the PSD estimator (averaged over 29 windows). The non-parametric spectrum was finally inspected for a peak in the range 3-12 Hz. The frequency \( f_{NP} \) at which the peak maximum was occurring is the non-parametric estimate of \( f_0 \) (method M1).

**Autoregressive Spectral Estimation.** An autoregressive (AR) model

\[
y_n = -\sum_{i=1}^{M} a_i y_{n-i} + w_n,
\]

where \( w_n \) is a white Gaussian noise with zero mean and standard deviation \( \sigma_w \), was fitted for each rECG series (covariance method). The rECG series were preliminarily down-sampled at \( f_S = 32 \) Hz to simplify the fitting procedure. We found empirically that \( M = 8 \) was a reasonable value for the order of the AR model. Please refer to the concluding section for a discussion on this issue.

The PSD for an AR process is given by

\[
PSD(f) = \frac{\sigma_G^2 / f_s}{1 + \sum_{k=1}^{M} a_k e^{-j2\pi f / f_s}}.
\]

Using the Cauchy residue theorem, the total power of the rECG series, \( \sigma_{rECG}^2 \), can be decomposed [5] into M contributions, one for each pole of the AR model:

\[
\sigma_{rECG}^2 = \sigma_G^2 \sum_{k=1}^{M} \gamma_k,
\]

where \( \gamma_k \) is the residue relative to pole \( p_k \).

**RESULTS:**

The power spectral densities of two different rECGs are shown in Fig. 1.
In the paper three parametric methods to estimate $f_0$ were suggested. They proved to be in good agreement among them and with the classical FFT-based approach. In particular, M2 and M3 showed to be the most consistent among each other and they offered the smallest variation between estimates on different leads. Being M2 less computationally demanding, we selected it as our first choice method.

M4 performed poorly, compared to the other three methods. This is related to the order of the model we selected ($M=8$). When $M$ is small, the AR models extract the prevalent features of the rECG signals, among which the fibrillatory wave emerge clearly. In this situation, M2 and M3 excel. On the contrary, when $M$ is very high, the parametric spectrum tends to be identical to the non-parametric one. Thus M1 and M4 lead to similar results, while M2 and M3 are penalized by over-fitting effects. In this limit, the fibrillatory wave is buried among other details and the convenience of the AR approach over the classical one vanishes.

We numerically verified that values of $M$ between 7 and 10 offered the best compromise among these two limits.

References: