Math Empowerment: A Multidisciplinary Example to Engage Primary School Students in Learning Mathematics.

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Abstract

This paper describes an educational project conducted in a primary school in Italy (*Scuola Primaria Alessandro Manzoni* at Mulazzano, near to Milan). The school requested our collaboration to help improve upon the results achieved on the National Tests for Mathematics, in which students, aged 7, registered performances lower than the national average the past year.

From January to June, 2016, we supported teachers, providing them with information, tools and methods to increase their pupils' curiosity and passion for mathematics. Mixing our different experiences and competences (instructional design and *gamification*, information technologies and psychology) we have tried to provide a broader spectrum of parameters, tools and keys to understand how to achieve an inclusive approach that is 'personalised' to each student.

This collaboration with teachers and students allowed us to draw interesting observations about learning styles, pointing out the negative impact that standardized processes and instruments can have on the self-esteem and, consequently, on student performance.

The goal of this programme was to find the right learning levers to intrigue and excite students in mathematical concepts and their applications.

Our hypothesis is that, by considering the learning of mathematics as a continuous process, in which students develop freely through their own experiments, observations, involvement and curiosity, students can achieve improved results on the National Tests (INVALSI).

This paper includes results of a survey conducted by children -'About Me and Mathematics'.

Keywords: Self-Adaptation, Learning Strategy, Self-Esteem, Cognitive Science, Augmented Didactics, Gamification.

Glossary

INVALSI is an acronym of *Istituto Nazionale per la VALutazione del Sistema dell'Istruzione*. This is an Italian government institution that submits the annual standard tests (TEST INVALSI) to every Italian school in order to evaluate the National Education System. They are standard tests based on European guidelines and are focused on Italian (language and literature) and mathematics.

Currently the INVALSI tests are submitted at these following stages of education:

A. Second grade of primary school (7 years)

B. Fifth grade of primary school (10 years)

C. Third grade of secondary school (13 years)

D. Second grade of secondary school (15 years)

In this paper we consider the INVALSI TEST for the second grade of [rimary school.

Coding or computer programming is the process of designing, writing and testing the source code of computer programs. In the context of the project, the coding activity allows students to develop a problem solving attitude even when facing complex problems, teaching them to break down complex operations into simple ones (sub-problems).

Gamification is the process born from Games Theory and is used to describe the application of games design into 'no recreational' areas. The Oxford Dictionary defines it as: *'The application of typical elements of game playing (e.g. point scoring, competition with others, rules of play) to other areas of activity* […] *to encourage engagement with a product or service: gamification is exciting because it promises to make the hard stuff in life fun'.* We are applying gamification in education in order to encourage student engagement with mathematics.

Programme Description

Why

In the second grade of Italian primary school, pupils have to achieve the following mathematics competences :

<u>Numbers</u>

- ✓ Counting objects in ascending or descending order, and jumping two, three or more digits (e.g.: 4 8 12 16 ...)
- ✓ Reading and writing natural numbers up to 100 with awareness of quantity, and being able to compare and sort them.
- ✓ Mentally performing simple operations with natural numbers, and verbalizing the calculation procedures.

<u>Shape, Space and Measures</u>

- ✓ Locating positions in physical space both with respect to themselves and to other people or to objects using appropriate terms.
- ✓ Representing build and locate positions.
- ✓ Reading coordinates.
- ✓ Identifying paths on graph planes.
- ✓ Drawing, naming and describing some basic geometric shapes.
- ✓ Recognizing objects in the environment and along the most common plane, and recognizing solid geometric figures.
- ✓ Drawing objects and symmetrical figures.
- ✓ Making direct and indirect measurements of quantities, and expressing them with unconventional and conventional units of measure.

Data, Sets and Relations

- ✓ Classifying according to one or more properties, using appropriate representations, depending on the context and purpose.
- ✓ Arguing using the criteria that have been used to create classifications and assigned orders.

At this stage of education, the learning aims for pupils include:

- ✓ Acquiring a positive attitude towards mathematics.
- ✓ Guessing how the mathematical tools are used and useful in day-by-day practices.
- ✓ Being confident in mental arithmetic with natural numbers.
- ✓ Understanding texts that involve logical and mathematical aspects.
- ✓ Describing the procedure used to solve problems, and recognizing the different solution strategies.
- ✓ Reasoning with assumptions supporting their ideas, and confronting the points of view of other classmates.
- ✓ Developing cooperative and collaborative attitudes.

Who

The project involved three teachers, 23 boys and 23 girls of two second grade classes of primary school: in the first class (2A) there were 24 children and in the second class (2B) there were 22 children.

The teachers involved already recognized the value of innovative lessons based on interdisciplinary workshops and using mathematics as a ‘meta-language’. We also added new points of view - metrics and pedagogic approaches - to support them.

What

With teachers, we made the following activities:

- ✓ Analysis of educational strategies currently in place from cognitive and psycho-social points of view. .
- ✓ Application of standardized instruments to observe students’ behaviour in relation to psychological and cognitive parameters correlated to the mathematical skills.
- ✓ Application of gamification: students can identify and implement appropriate solution strategies learning tricks and tools to ‘find the interpretations’ of a mathematical problem.

How to

We have defined a series of cognitive-driven tests aimed at evaluating the logical-mathematical skills needed to face the INVALSI trials. However, with the same tests, we also aimed to assess the meta-cognitive skills required by the INVALSI problems in order to ‘situate’ the thought of the students to the specific task, which we named ‘face the INVALSI test’. We wanted to find out if the students were ready to approach the INVALSI test by answering the question ‘Have I got the knowledge and skills required for the INVALSI problems?’.

If it is possible to evaluate a student's cognitive 'situation' in math skills by the use of a standardized paper and pencil instrument, then we believe that the use of cognitive games, particularly implemented through technological media, may allow students to advance their cognitive position, developing skills that are transferrable to the standardized instrument in a fun and stimulating way (Kebritchi, Hirumi, Bai, 2010). This process takes time, because students must not only develop and consolidate these skills but also learn how to transfer them. In other words, students must determine how to use these skills in tasks that may appear totally different (e.g., from a computer game to a school trial).

In addition, the results of any mathematical test are mediated by emotional and motivational aspects, which often emerge as negative associations. Consequently, the INVALSI tests are regarded as serious and difficult to face; therefore, they are generally associated with negative attitudes (Pagani, 2016). As a result, even students with adequate skills who are predicted to achieve high scores could 'fail' simply due to a lack of motivation, a factor that can lower attention levels and inhibit a student's ability to find connections between the current test and the acquired skills (Pagani, 2016).

We speculate that it is important to support students in the development of 'situated thought', helping them to connect directly to the INVALSI test. At the same time, students must also learn to implement situated meta-cognitive strategies. For example, they should be able to reflect on the cognitive resources that a task requires and on the possible connections to other tasks previously faced. In this way, young students must learn to shift from one format to another, transferring a problem to a different format so to apply the same method.

In particular, we consider it to be essential that students learn to reflect on the representation of a problem. In this way, students develop a kind of operational thinking (or pre-logic) that can be associated with a number of key questions such as 'where have I seen this kind of problem? ',' what is similar in this problem compared to the previous one? 'and so on.

Through a series of cognitive-operational tasks mediated by the use of epistemic (technological learning) tools we aim to support the thought of children in their developmental stage. However, we will also contribute to the implementation of the most advanced forms of thought (adapted to prepare and enhance the physiological development), based on computational problem solving (Grover & Pea, 2013), the peer collaboration, and the use of different cognitive codes (verbal, visual, motor).

When

The activities described in this paper took place between the months of January and June, 2016, and the next phases will be implemented in the next school year.

Where

Mulazzano is a very small village near to Milan. As often happens in small places, we can feel a sense of community and affiliation that is very strong. Its primary school is modest and does not have lavish infrastructure but is soon to be equipped with the latest educational technologies.

Our research began by observing teachers at work with their pupils in the gym, in the garden and in the classroom. In these relaxed and joyful atmospheres, interesting dynamics have emerged.

The observations of pupils at work

The students of the two classes we observed were mixed and divided into three groups by their teachers who organized weekly mathematic games and experiments.

We took turns in every laboratory and each of us observed each group for 30 minutes. Overall, the observation of the work lasted about 1 hour and 30 minutes. We did not interact either with teachers or with children for the whole duration of the experiment. This was to reduce the novelty of having 'new people' and the alteration of their normal way of dealing with the various games.

The time sampling and methodology of observation does not allow for an accurate analysis of class dynamics, nor justifiable elements to draw scientific inferences about the relationship between the means used in the workshops and the learning objectives. Although we could not define precise models of interpretation, we have provided a number of insights and suggestions to enhance this approach already put in place by teachers.

Creative Laboratory: ***Build your scene hat for Turandot***

The aim of this workshop was the creation of 'scene elements' that children must wear during their participation at the Opera *Turandot* at *Teatro degli Archimboldi* in Milan the 15th of May, 2016. This premise is fundamental because such a pragmatic goal affects the organization of work and class dynamics. The costumes had to be completed by a due date, putting into the background other more general objectives.The activities were challenging and the teacher sometimes had to follow the students individually. The presence of a single teacher to support multiple students made the class dynamics complex, the evidence we gained was very interesting.

This lab allowed students to give a concrete shape to concepts such as 'point' 'line' 'measure' - basic elements of geometry that are sometimes presented in an abstract way. The pupils understood the concepts and theorems of geometry through simple operations, such as those implied by the creation of a hat.They demonstrated their ability to use available materials (ruler, tape measure, scissors, pencil and coloured paper) in a creative way, although somewhat departing from specific objectives, which we could trace in mathematical reasoning and the use of mathematical objects. In addition, students had fun in choosing the colour of their hat to measure the circumference of their heads and to give shape to their personal model.

When the teacher was busy supporting a single student, other students engaged themselves in various activities: some talked to each other while others measured body parts and compared results, and others impersonated the Princess Turandot or the teacher.

Scientific laboratory: numbers multiplication tables and code blocks


The use of computers, LIMs (*Lavagna Interattiva Multimediale* – Interactive Multimedia Blackboard) and the game *Angry Birds*, were used to capture student attention. Many of them already knew the game and the characters, so they were easily drawn into the activities and contributed comments and generalizations. Moreover, the teacher immediately identified a pupil who was competent in working on computers, and who acted as a helper for the other students (peer-to-peer). The teacher assumed the role of 'primus inter pares' instead of being the sole guide.

The aim of the *Angry Birds* game was to teach students 'algorithmic' thought, which is the ability to organize a series of instructions so as to achieve a simple goal. This activity should allow students to develop the basis of computational thinking, which is the ability to organize instructions to perform specific tasks.

Cognitive psychology emphasizes the importance of the interaction between different forms of thought in order to develop not only a sufficiently wide range of skills, but also integrated meta-cognitive skills, in order to organize a personal hierarchy of values, preferences and objectives (Steinbring, 2005). Thus, in general, the development of computational thinking must be understood as a powerful way to improve performance in logical-mathematical tasks and, more generally, in the formal problem solving.

The activity we observed in the classroom gave us the opportunity to note the student difficulty when 'decoding' their own thoughts. In particular, the generalization process does not appear to be linear because the transition from one kind of instruction to another appears to be difficult every time students attempt it. For example, the transition from 'take a step forward' to 'turn left' appears to be mediated by a complex understanding process. This fact implies at least two considerations:

- ✓ The 'decoding' activity required time and patience. The students not only needed to reach the specific outcome ('capture the pig and roast it'), but they also needed to consolidate this result over time through repeated trials and level advancements. Moreover, the kind of thinking that was put in place by the students (as evident from the comments and loud thought) seemed more heuristic-exploratory, driven by curiosity, rather than by the search for a logical basis. In some cases, this made it difficult for pupils in following the logical path suggested by the teacher, who sometimes introduced complications that were not useful to the achievement of objectives.

- ✓ The association with the INVALSI test appears more evident in this case. In fact, the development of formalized strategies will allow students to face the problem solving tasks present in the INVALSI test. However, once consolidated, the procedure (that is, established thought routines useful to address the problems, according to the instruction sequences) will be necessary to explain the association with the structure (that is, the actual shape) of the test. In fact, the main problem in this phase of the development of thinking is related to the transition from one form of representation (of the problem) to the other.

Students often developed meta-cognitive comments, captured by the teacher and incorporated to stimulate the development of higher forms of thought, not strictly linked to *hic et nunc* (here and now), but directing the students use of their 'cognitive tool'.

Students appeared attentive, interested, and motivated to be part of the lesson. The use of humor, alternating with formal calls from the teacher to pay attention, maintained a gaming approach even during the didactic activities. The students interacted with each other and independently took turns in using computers to ensure that all students interacted directly with the game, showing high sensitivity to the sharing of resources. When they found new solving mechanisms they spontaneously formulated meta-cognitive comments, also suggesting them to the teacher.

In each group, one student always tended to get distracted, even if not excessively, but there was no need for intervention (the children themselves involved each other). The achievement of a milestone in the game acted as a reward and reinforcement for students, who were free to express themselves. All the activity certainly facilitated the achievement of a series of educational objectives and a psychological relationship that went beyond the development of logical-mathematical thought.

Laboratory at the gym: Experiences on weights and measures

The practical activities used in the gym is a way to incorporate mathematical concepts (weights, measures, comparisons and that like) through direct experiences that allow students to embody abstract issues. For instance, we observed students estimating the weight of objects using different strategies and sensorial aspects. In this way, they had the opportunity to develop hypotheses by the use of objective cues, such as the object's size, and to test these hypotheses by the use of simple experiments, for example, holding two different objects, one in each hand, to evaluate which was the heaviest.

The proposed activities seemed particularly interesting to the involved students and, in the long run, they will probably be rewarding, since learning concepts through a direct embodied experience should facilitate the transferring of knowledge from school to everyday life (Tall, 2004). Instead, the move from the physical experience of concepts to the abstract realm of mathematics seemed to be more complicated, since students experienced difficulties in converting their experience into logical-mathematical terms. Furthermore, students needed to develop their cognitive system following their own particular and specific trajectory, so it was not possible to achieve the same result for all the involved students due to their different cognitive development rhythms, which implies physiological and spontaneous processes (Blakemore and Frith, 2005).

During the observed activities, the students were motivated and curious, and they demonstrated their understanding of the activities. However, these activities are distracting, time-consuming and trigger social dynamics that are not always easy to manage by teachers (Johnson, Johnson, Holubec, 1996). Furthermore, the involvement of children was not equally distributed. Although the equitable distribution of involvement cannot be achieved in a systematic way, it is still a factor to be considered when assessing results and their impact, both on performance and on the quality of social life at the school. It might be interesting to predict groups / teams with a reduced number of students (3-5) so that everyone can interact more frequently and participate assiduously in all the phases of the activities.

Students often provided spontaneous meta-cognitive comments within the activity. Probably, these comments could have been taken up, discussed and incorporated by the teacher to encourage the emergence of appropriate generalizations to other educational activities or disciplines. For example, when students were asked to form a 'half-moon shape', most had great difficulty in understanding the command. Only when a student offered the solution, saying 'we have to make the shape of the moon', did the groups form the right shape. This comment could be used to raise an interesting discussion on geometric shapes and on the relationship between shapes, space and thinking. Merging practical activities and metacognitive skill development could allow students to enhance their mathematical abilities, even in the case of low performance (Veenman and Spaans, 2005).

Implementing solutions using coding and robotics

The greatest difficulty for children (in particular, in addressing INVALSI test) consists of the 'narrative' problems (i.e. those which add narrative text to the requests for calculation) (Lee, 2006). We need to help children to overcome the anxiety caused by narrative problems (which cause difficulties even for adults), teaching them to extract meaningful data and to apply an 'algorithmic mentality', which is the ability to break down the main problem in a series of sequential (or not) steps to come to a solution. Considering the age of the children and the fact that problem-solving skills will mainly be developed, in general, between 11-12 years of age, this work will show long-term benefits. Nevertheless, it is crucial to start developing mathematical and scientific attitudes in students.

In the context of this project, the coding activity allows children to:

- Understand texts involving logical and mathematical aspects;
- Solve difficult mathematical problems;
- Describe the followed procedure and recognize different solution strategies;
- Think about their assumptions, supporting their ideas and comparing different points of view; and
- Develop cooperative and collaborative attitudes.

The coding indirectly constitutes a strategy for the INVALSI test, facilitating the development of their ability to decode the text of a problem and the development of a mathematical and scientific approach, making problems concrete and closer to daily solving mechanisms.

The survey

Mathematical skills assessment is usually based on a set of standardized scales with the main aim to check for any deficit (Bull and Scerif, 2001). Instead, in this study, we aimed at profiling students' abilities to find out their approach and attitudes to math but not to evaluate them in a strict sense. This characterization of the students has allowed the definition of individual competency profiles. We then compared the results of these profiles with the results obtained by students at the National Test INVALSI math (AA.VV., 2004).

The test we have used was based on the assumption that mathematical competence is a multidimensional construct. It is, therefore, important to explore all of these dimensions, albeit through short sub-scales (D'Amore, Fandiño Pinilla, Marazzani, 2004). In particular, we have evaluated the following dimensions:

- Knowledge of arithmetic facts
- Knowledge of basic mathematical language
- Knowledge of arithmetic combinations
- The fluidity of calculation
- Mathematical intuition, including:
  – Fluidity in estimating and judging quantity
  – Shifting from a problem representation to another
  – Judging the plausibility of results
  – Flexibility in mental calculation
  – Use of heuristic solutions
  – Self-efficacy, self-esteem and emotional aspects

It is important that the student is able to use mathematical language in a flexible and intuitive way, even when they do not yet formally approach the problem at school. To this aim, we introduced some advanced exercises (i.e. tasks that were deliberately more complex than those a 7-year-old child normally faces at school) (D'amore, 1993). The test also offered a recreational and aesthetic representation of numbers, in order to intuitively assess the approach of the child with math objects and to stimulate curiosity and associations between knowledge domains that are seemingly distant. Students may add comments and drawings without limitation on the test sheet. The test was undertaken with less formality than the INVALSI test. The teachers helped the students to understand that doing something in itself can be useful and fun.

The test was administered in section of about 15 pages each. To answer the questions, the students had a total of 15 minutes per section.

The instrument used consisted of 51 questions, divided as follows:

1. Evaluation student understanding and analysis of numerical sequences (4 items).
2. Evaluation of student ability to decode and transcode formats (e.g., the transition from Arabic to alphabetical or graphic format). This skill takes time to develop. For a student, it is cognitively complex to understand that two entirely different stimuli represent the same quantity. Also, from a neurological point of view, this competency requires the maturation and optimization of specific neuronal circuits (6 items).
3. Evaluation of students' number sensitivity. This is a complex dimension, difficult to formalize, that tries to measure the mathematical intuition of a student, a kind of implicit sensitivity to numbers, which children often show even before starting school, surprising parents and teachers (9 items).
4. Assessment of students' mathematical language (i.e. whether the child has acquired basic interpretation of symbols and their use (4 items).
5. Assessment of basic mathematical knowledge (e.g., operations and comparisons) (10 items).
6. Evaluation of student ability to manipulate the graphics formats and convert the Arabic number in analogical quantities using the graphic space offered. Transcoding number/graphical representation is complex to learn. Different formats are often used in the INVALSI test so as to assess the ability of students to understand the meaning of mathematical problems (6 items).
7. Assessment of advanced (anticipated) tasks that students are not expected to be ready to solve (3 items).
8. Assessment of selective attention mechanisms of students (4 items).
9. Evaluation the emotional attitude of students towards math and school (6 items).
10. Evaluation of the subjective attitude of students towards math and numbers (6 items).
11. Evaluation of the subjective attitude of students toward the INVALSI tests (2 items).

Each item was matched with a self-assessment evaluation about students' confidence in the given answer. In this way, we had the opportunity to assess whether students were giving the correct answer without a true understanding of the problem (children had to opt for 1 of 4 answers) (Eccles, 1999). Furthermore, by the use of self-rating items, we tried to focus responders' attention to the task, activating their awareness mechanisms.

In order to develop mathematical skills, students need to understand that mathematics is a series of dynamic processes and not a mechanical reproduction of 'algorithms' (Liverta Sempio, 1997). The tool used did not allow us to accurately measure all of the described dimensions. To this aim, it would be interesting to develop, in the future, a computerized version that can also take account of the response times and evaluate the implied cognitive dimensions, such as the development of automation, cognitive rigidity, working memory and sustained over selective attention.

Data Analysis

We collected 46 questionnaires from two classes (24 from Grade 2 A and 22 from Grade 2 B). The sample consisted of 23 males and 23 females.

First, we report the distributions of the scores (sum of the correct answers for each dimension) by gender and class section (A or B). As you can see from the Table 1 below, there are not any significant differences between the two sections; consequently, we can say that the students coming from A and B classes have the same mathematical competence. Even with respect to gender, there are no significant differences.

Table 1. Distributions of scoring across classes

		Mean	Std. Error	U	p
Number Sequences	A	3,455	,261	252,5	0,536
	B	3,500	,250		
Format Transcoding	A	8,364	,434	195	0,535
	B	7,917	,415		
Number Sense	A	7,909	,515	202,5	0,735
	B	7,750	,493		
Mathematical Language	A	3,273	,226	238	0,538
	B	2,833	,217		
Numerical Facts	A	7,455	,562	171,5	0,636
	B	7,250	,538		
Grafico	A	4,545	,276	145	0,078
	B	3,583	,265		
Advanced Tasks	A	1,818	,240	318,5	0,065
	B	2,167	,230		
Selective Attention	A	2,636	,164	239	0,936
	B	2,833	,157		

We found significant differences between males and females in their attitude towards school (see table 2) as more males reported negative emotions than females (Chi-Square = 5.825, p = .035). In Table 2, 0 represents no negative feeling reported towards school. The number of males reporting negative feelings were significantly higher than females.

Table 2. Attitudes towards school by gender

		Attitudes toward school		Total
		0	1	
Gender	f	21	2	23
	m	14	9	23
Total		35	11	46

In order to analyze the relationship between performance in the various domains and confidence in their answers (which is a good indication of self-efficacy and emerging meta-cognitive skills), we correlated the results of each dimension with confidence scores, using t the Spearman's rho index. We computed two confidence indices: the first is simply given by the sum of confidence scores at each single item (ranging from 1 'not confident at all' to 4 'very confident'); and the second is the sum of confidence scores for only the items for which a student gave the correct answer.

Table 3

	Sequence	Format	Sense	Language	Facts	Graphic	Advanced	Attention	Condifidence	confidence ToT
Sequence	1,000	,113	,510**	-,068	,243	,103	,230	-,016	,395	,236
		,493	,001	,675	,153	,532	,149	,923	,077	,330
Format	,113	1,000	,352*	,066	,411*	,614**	-,039	,062	,794**	,222
	,493		,035	,683	,012	,000	,808	,698	,000	,360
Sense	,510**	,352*	1,000	-,047	,762**	,197	-,179	-,069	,715**	,182
	,001	,035		,781	,000	,255	,289	,687	,000	,455
Language	-,068	,066	-,047	1,000	,171	,115	,273	,161	,276	,093
	,675	,683	,781		,305	,475	,073	,303	,226	,705
Facts	,243	,411*	,762**	,171	1,000	,225	-,312	-,003	,798**	,366
	,153	,012	,000	,305		,186	,057	,988	,000	,123
Graphic	,103	,614**	,197	,115	,225	1,000	,025	,276	,548*	,201
	,532	,000	,255	,475	,186		,875	,080	,010	,409
Advanced	,230	-,039	-,179	,273	-,312	,025	1,000	-,218	-,073	-,126
	,149	,808	,289	,073	,057	,875		,161	,753	,607
Attention	-,016	,062	-,069	,161	-,003	,276	-,218	1,000	,438*	,407
	,923	,698	,687	,303	,988	,080	,161		,047	,084
Condifidence	,395	,794**	,715**	,276	,798**	,548*	-,073	,438*	1,000	,545*
	,077	,000	,000	,226	,000	,010	,753	,047		,016
confidenceToT	,236	,222	,182	,093	,366	,201	-,126	,407	,545*	1,000
	,330	,360	,455	,705	,123	,409	,607	,084	,016	

The above table shows the correlation analysis. The total confidence index does not correlate to performance, but the correct answer index does. This means that pupils who have claimed to be sure of their answer most of the time actually gave correct answers, and when responders declared to be unconfident, often they committed some mistakes. This result, although very general, suggests that this score might be thought of as a good index of self-awareness, that is the ability that children have to judge their knowledge and skills. We may consider it as a rising index of meta-cognitive processes that will be fully developed in the following years. It is likely that these results are the consequence of the work done in the classroom and in labs that required students to reflect on what they did in relation to the development of logical-mathematical, spatial and linguistic skills.

It is also interesting to note the lack of correlation between the confidence indices and the advanced items score and knowledge of mathematical language. It is clear that these two areas have stumped responders, generating confusion and uncertainty, thus leading students to be unable to judge their performance.

The other correlations, for example between sense and knowledge of the number, though conceptually interesting, have no statistical value since the dimensions are not independent.

One of the most interesting subjective variables occurs in question 51, in which we asked the students to describe their state of mind while performing the INVALSI tests.

Table 4

		How do you feel when you do INVALSI tests					Total
		Missing	Bored	Happy	Concerned	Calm	
Section	2a	2	5	1	6	10	24
	2b	2	7	1	6	6	22
Total		4	12	2	12	16	46

As stated in the above table, half of the children (four have not responded) reported a positive mood (peaceful or happy), while the other half reported being concerned or bored. It is interesting to see if this feeling has any impact on performance or if it differentiates good results from bad ones.

Table 5

Dependent Variable		Mean	Std. Error
	Bored	3,571	,281
	Happy	4,000	,525
	Concerned	3,778	,248
	Calm	3,600	,235
	Bored	7,429	,560
	Happy	8,000	1,047
	Concerned	8,556	,494
	Calm	8,400	,468
	Bored	7,000	,555
	Happy	8,500	1,038
	Concerned	8,889	,489
	Calm	8,000	,464
	Bored	3,143	,311
	Happy	3,500	,582
	Concerned	3,000	,274
	Calm	2,800	,260
	Bored	6,143	,602
	Happy	8,500	1,125
	Concerned	8,333	,531
	Calm	7,500	,503
Graphic		3,333	,664
	Bored	3,714	,435
	Happy	4,500	,813
	Concerned	3,889	,383
	Calm	4,100	,364
	Bored	2,286	,298
	Happy	1,500	,558
	Concerned	2,000	,263
	Calm	1,800	,250
	Bored	2,714	,206
	Happy	3,000	,386
	Concerned	2,778	,182
	Calm	2,700	,173

The statistical analyses show that students who declared to be bored also have lower performances, especially in their sense of number and mathematical knowledge dimensions. There is, however, no relationship between the subjective feeling and confidence in responses.

It is noteworthy that students who expressed concern often have excellent performance. It is not so much the emotional value (positive vs. negative) in itself that differentiates respondents, but the specific emotion felt. Students who are ‘concerned’ are probably just afraid to make mistakes, being aware that the INVALSI tests are difficult. This concern may be the result of good meta-cognitive ability that allows them to understand how the INVALSI tasks can be misleading (though not yet knowing how not to fall into traps). On the other hand, the concern could arise from a state of anxiety that already characterizes the students of this age by virtue of their own or their parents’ expectations.

In regards to the results on ‘bored’ students, the low motivation and the low level of attention do not favour the learning of those automatic mechanisms that are fundamental to math competence. As a result, bored (low motivated) students will continue to make mistakes and this will have a negative effect on their self-efficacy and self-esteem.

Finally, we want to focus attention on the relationship between student attitudes towards school and performance. In this case, a value of 0 indicates a positive attitude, while a 1 indicates a negative attitude. In fact, respondents who reported a negative attitude showed to be less skilled in the use of different formats, having a lower sense of numbers, less mathematical knowledge, and more difficulties in using graphic representations. This group (children with a negative attitude towards school) consisted of just 11 children, among them 5 also declared to 'bored' by the INVALSI test. Consequently, it is clear that the two groups are in part overlapping, highlighting similar problems. This variable indicates more general difficulty, a cross-ambiguity that merits further investigation. It is likely that in some cases it is a self-esteem problem, perhaps linked to learning disabilities. It is difficult to say whether, in this small sample, some kind of emotional or psychological problems related to school experience are present.

In order to evaluate the relationship between the scores at the different dimensions evaluated and the performance on the INVALSI (note that students faced three different INVALSI tests across three consecutive weeks). The relationship between the results on the INVALSI tests (number of correct answers) and the scores on the study's instrument dimensions are reported in the following table.

Table 6

	Sequence	Format	Sense	Language	Facts	Graphic	Advanced	Attention	Confidence Corr.Ans.	Confidence	Invasi 1	Invasi 2	Invalsi 3
Sequence	1,000	,113	,510**	-,068	,243	,103	,230	-,016	,395	,236	,246	,025	-,144
		,493	,001	,675	,153	,532	,149	,923	,077	,330	,112	,871	,362
Format	,113	1,000	,352*	,066	,411*	,614**	-,039	,062	,794**	,222	,411**	,501**	,406**
	,493		,035	,683	,012	,000	,808	,698	,000	,360	,007	,001	,008
Sense	,510**	,352*	1,000	-,047	,762**	,197	-,179	-,069	,715**	,182	,169	,143	,098
	,001	,035		,781	,000	,255	,289	,687	,000	,455	,304	,385	,559
Language	-,068	,066	-,047	1,000	,171	,115	,273	,161	,276	,093	,042	,150	,236
	,675	,683	,781		,305	,475	,073	,303	,226	,705	,789	,331	,127
Facts	,243	,411*	,762**	,171	1,000	,225	-,312	-,003	,798**	,366	,116	,216	,185
	,153	,012	,000	,305		,186	,057	,988	,000	,123	,480	,186	,265
Graphic	,103	,614**	,197	,115	,225	1,000	,025	,276	,548*	,201	,276	,570**	,573**
	,532	,000	,255	,475	,186		,875	,080	,010	,409	,077	,000	,000
Advanced	,230	-,039	-,179	,273	-,312	,025	1,000	-,218	-,073	-,126	,211	-,055	-,039
	,149	,808	,289	,073	,057	,875		,161	,753	,607	,168	,723	,805
Attention	-,016	,062	-,069	,161	-,003	,276	-,218	1,000	,438*	,407	,008	,061	,000
	,923	,698	,687	,303	,988	,080	,161		,047	,084	,957	,694	1,000
Confidence Correct Ans.	,395	,794**	,715**	,276	,798**	,548*	-,073	,438*	1,000	,545*	,209	,459*	,429*
	,077	,000	,000	,226	,000	,010	,753	,047		,016	,362	,036	,041
Confidence	,236	,222	,182	,093	,366	,201	-,126	,407	,545*	1,000	,005	,238	,097
	,330	,360	,455	,705	,123	,409	,607	,084	,016		,983	,326	,693
Invasi 1	,246	,411**	,169	,042	,116	,276	,211	,008	,209	,005	1,000	,540**	,334*
	,112	,007	,304	,789	,480	,077	,168	,957	,362	,983		,000	,025
Invalsi 2	,025	,501**	,143	,150	,216	,570**	-,055	,061	,459*	,238	,540**	1,000	,910**
	,871	,001	,385	,331	,186	,000	,723	,694	,036	,326	,000		,000
Invalsi 3	-,144	,406**	,098	,236	,185	,573**	-,039	,000	,379	,097	,334*	,910**	1,000
	,362	,008	,559	,127	,265	,000	,805	1,000	,091	,693	,025	,000	

It is clear that the only two dimensions that appear to correlate with performances on the INVALSI tests are a graphic representation and format transcoding. Actually, at an early age, most mathematical difficulties arise from the inability to shift from one format to another one or to understand that two presentations of the same problem imply the same solution. The other dimensions are probably more important in tests designed for older students.

Also, in this analysis, the total confidence index does not correlate with the performance on the INVALSI tests, but the correct answer index does. This result suggests that the development of a valid self-confidence (when pupils are aware of their knowledge) is a good predictor of the success on the INVALSI tests.

Final considerations

The work done has allowed us to understand some of the underlying dynamics of the mathematical performance of the pupils involved. Collaborative, collegial and interdisciplinary work has allowed for mutual enhancement, facilitating the emergence of a learning path that might promote further achievements in the future by the involved students.

The current study presented two main results. First, meta-skills are probably more important than specific skills and knowledge with regard to the probability of obtaining good results on the INVALSI tests. This is particularly important since it suggests that teachers should spend more time allowing pupils to see the analogies between different formats of a same problem. In particular, it is important that pupils are able to acquire a mathematical language to describe problems so to be able to apply that language to different situations. It ''requires the development of particular ways of thinking, including analyzing relationships between quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, proving, and predicting'' (Cai and Knuth 2011). We argue that it is possible to introduce this method in early grades so to boost future achievements. Moreover, an early acquiring a proper mathematical language should help children to express their understanding and overcome some diffuse beliefs about mathematical ability, often considered too difficult to learn without an innate 'gift' (Dweck 2000; Lee 2006).

A very promising approach is to engage with early algebraic reasoning using a polyhedral method based on pragmatic settings, active learning, interactivity, interplay between supervised individuals and small group work, where the teacher may offer help, encouragement and timely feedback on pupils' work (Bjuland, 2012). A multi-approach method, based on facts, notions, activities, interaction and logical work (e.g. coding) may be strongly suggested when a mathematical boost is needed. All this work will allow pupils to grow cognitive structures rich in inter-domain connections. For instance, when a student experiences a mathematical fact in a gym exercise and, subsequently, find the same fact in a math book, she/he may draw a connection between the domain of 'doing' and the domain of 'thinking'. This is a basic step for the development of a more general cognitive tool that is a mathematical language.

A second important results relates to self-confidence. A pupil develops a self-confidence when she/he is able to predict when a correct answer is given. In this way, self-confidence does not indicate a more general self-esteem or an ingenuous feeling, but it may be considered a cognitive tool since it allows students to develop valid expectations about their behavior in relation to their abilities. Also, self-confidence was found to be a good predictor of the INVALSI test scores. More generally, pupils need to develop a positive attitude to mathematics so to enable them to be adaptive when facing and to continue learning. This attitude, also called mathematical resilience (Johnston-Wilder and Lee 2010), should be developed as early as possible so to boost new leaning as well as to sustain motivation. We argue that mathematical resilience is linked to two other fundamental concepts: self-efficacy, that is the feeling of being able to navigate successfully in a given field (Bandura, 1997; Pampaka et al. 2011); and an internal locus of control that pupils may acquire thanks to the ability of teachers to emphasis the importance of effort and of the pupils' work (Borich, 1996; Reynolds D & Muijs D, 1999).

Taken together, our results strongly suggest that using the INVALSI test format may be used to trigger the development of a mathematical language to be applied to a number of different cognitive domains. Instead of considering the INVALSI format as something extraneous to the normal teaching, it could be used as a way to help students finding analogies between domains and foster more general and abstract cognitive tools. Teaching mathematical facts and testing notions and knowledge do not allow pupils to see how math may be used in many other ways and contexts. Transforming the INVALSI test into an interactive multi-modal (numeric, graphics, motor) game is a first step toward the use of the INVALSI format as a teaching tool during the first two years of school. A second step requires students to decompose problems into more simple sub-problems and to apply already know algorithms so to approach complex problems using coding and computational thinking. We argue that this second step will be strongly eased by the development of self-confidence that allow pupils to know what they can already do correctly and what needs further effort.

It is also evident that some students lack motivation, while others feel insecure or show a low self-efficacy. On this front, we need to consider that the cognitive and emotional maturity of the pupils will gradually unveil the true nature of the difficulties found during this study.

The ultimate purpose of reinforcement does not reside in the development of specific skills, but in the consolidation of a cognitive ground, which allows the students to know their self, their limitations and their potential. A land where there are no school subjects or single concepts, but where the languages of knowledge meet and fertilize, promoting the harmonious development of the different attitudes of children.

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