

Threshold Resummation in Semi-Inclusive B decays

Giancarlo Ferrera

Universitat de Barcelona, Barcelona, Spain & Universidad de Granada, Granada, Spain

We discuss threshold resummation in radiative and charmless semileptonic B decays. To deal with the large non perturbative effects, we introduce a model for NNLL resummed form factors based on the analytic QCD coupling. By means of this model we can reproduce with good accuracy the experimental data. Finally we briefly present an improved threshold resummed formula to deal with jets initiated by massive quarks as in the case of semileptonic charmed decays.

I. INTRODUCTION

The aim of the work presented in this talk is to analyze semi-inclusive B decays spectra measured at B -factories, which allows the extraction of the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements $|V_{ub}|$ and $|V_{cb}|$ [1], [2]. To this end is crucial to have a good control of the so called threshold region, defined as the region where the invariant mass of the inclusive hadronic state X is much smaller compared with its energy: $m_X \ll E_X$. This region is affected both by perturbative soft gluon radiation and by non-perturbative phenomena related to the “Fermi-motion” of the heavy-quark inside the meson [3]. We take in account such phenomena with a model based on soft gluon resummation to next-to-next-to-leading logarithmic accuracy (NNLL) and on analytic QCD coupling having no Landau pole. Our model, which does not contain non-perturbative free-parameters, gives a good description of experimental data of the B -factories and it allows an extraction of $\alpha_S(m_Z)$ which is in agreement with the current PDG average within at most two standard deviations.

II. THRESHOLD RESUMMATION

Let’s consider first radiative decays $B \rightarrow X_s \gamma$: factorization and resummation of threshold logarithms in such decays leads to an expression for the event fraction of the form

$$\frac{1}{\Gamma_r} \int_0^{t_s} \frac{d\Gamma_r}{dt} dt = C_r[\alpha_S(Q)] \Sigma[t_s; \alpha_S(Q)] + D_r[t_s; \alpha_S(Q)], \quad (1)$$

where Γ_r is the inclusive radiative width, $t_s \equiv m_{X_s}^2/m_b^2$, $C_r(\alpha_S)$ is a short-distance, process dependent hard factor, $\Sigma(t_s, \alpha_S)$ is the universal QCD form factor for heavy flavor decays resumming series of logarithmically enhanced terms to any order in α_S and $D_r(t_s, \alpha_S)$ is a short-distance, process dependent remainder function vanishing in the threshold region $t_s \rightarrow 0$.

An analogous formula can be written for the semileptonic decays $B \rightarrow X_u l \nu_l$. In this three-body decay case, the most general distribution is a triple differential dis-

tribution [4]

$$\frac{1}{\Gamma_s} \int_0^u \frac{d^3\Gamma_s}{dx dw du'} du' = C_s[x, w; \alpha_S(Q)] \Sigma[u; \alpha_S(Q)] + D_s[x, u, w; \alpha_S(Q)], \quad (2)$$

where $x = \frac{2E_l}{m_b}$, $w = \frac{2E_X}{m_b}$, $u = \frac{1 - \sqrt{1 - (2m_X/Q)^2}}{1 + \sqrt{1 - (2m_X/Q)^2}}$.

The hard scale of the B decays in the threshold region is fixed by the final hadronic energy E_X

$$Q = 2E_X = m_b \left(1 - \frac{q^2}{m_b^2} + \frac{m_X}{m_b^2} \right), \quad (3)$$

where q^μ is the 4-momentum of the probe (the real photon in the radiative decays and the lepton neutrino pair in the semileptonic ones). While in the radiative decays $q^2 = 0$ and the hard scale is always of the order of the beauty mass ($Q \simeq m_b$), in the semileptonic decays the lepton pair can have a large invariant mass $q^2 \sim m_b^2$, implying a substantial reduction of the hard scale, which is no more fixed but it depends on the kinematics. Since the hard scale Q appears in the argument of the infrared logarithms as well as in the argument of the running coupling, semileptonic spectra have in general a specific infrared structure, which is different from the invariant hadron mass distribution in the radiative decay. Semileptonic spectra are naturally classified depending on whether they involve or do not involve an integration over the hard scale. The main consequence is that additional long distance effects, related to small hadronic energies, occur in semileptonic decays, which cannot be extracted from the radiative decays [5].

The heavy flavor form factor has an exponential form in Mellin space [6], [7]:

$$\log \sigma_N(\alpha_S) = \int_0^1 \frac{dy}{y} [(1-y)^{N-1} - 1] \left\{ \int_{Q^2 y^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(Q^2 y)] + D[\alpha_S(Q^2 y^2)] \right\}, \quad (4)$$

where $\sigma_N(\alpha_S) = \int_0^1 (1-t)^{N-1} \sigma(t; \alpha_S) dt$, $\sigma(t, \alpha_S) = d\Sigma(t, \alpha_S)/dt$ and the functions $A(\alpha_S)$, $B(\alpha_S)$ and $D(\alpha_S)$ describe log-enhanced radiation and have a standard fixed order expansions in α_S .

As is clear from the k_\perp integral of Eq.(4), semi-inclusive processes are multi-scale processes, characterized by fluctuations with transverse momenta up to Q : a jet with a relative large invariant mass m_X — typically

$\Lambda_{QCD} \ll m_X \ll Q$ — can contain very soft partons, with transverse momenta of the order of the hadronic scale. That produces an ill-defined integration over the Landau pole and the form factor acquires an unphysical imaginary part for any N . A prescription for the low-energy behaviour of the running coupling is therefore needed to give a meaning to the formal expression in Eq.(4): our prescription is to use an effective QCD coupling which does not present the Landau pole.

III. EFFECTIVE COUPLING

The standard QCD coupling

$$\alpha_S^{lo}(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}, \quad (5)$$

where $Q^2 \equiv -q^2$ with q^μ the gluon momentum, has a physical cut for $Q^2 < 0$ related to the decay of a time-like gluon in secondary partons ($g^* \rightarrow gg, q\bar{q}$, etc.) and an unphysical simple pole (the Landau pole) for $Q^2 = \Lambda_{QCD}^2$, which signals a formal breakdown of the perturbative scheme.

The analytic QCD coupling is defined as having the same discontinuity of the standard coupling along the cut, while being analytic elsewhere in the complex plane [8],

$$\bar{\alpha}_S(Q^2) = \frac{1}{2\pi i} \int_0^\infty \frac{ds}{s+Q^2} \text{Disc}_s \alpha_S(-s). \quad (6)$$

At LO, it reads

$$\bar{\alpha}_S^{lo}(Q^2) = \frac{1}{\beta_0} \left[\frac{1}{\log Q^2/\Lambda^2} - \frac{\Lambda^2}{Q^2 - \Lambda^2} \right]. \quad (7)$$

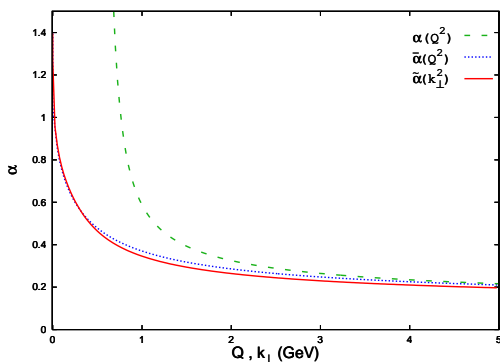


FIG. 1: QCD couplings at NNLO. Dashed line (green): standard coupling $\alpha_S(Q^2)$; dotted line (blue): analytic coupling $\bar{\alpha}_S(Q^2)$; continuous line (red): time-like coupling $\tilde{\alpha}_S(k_\perp^2)$.

The analyticization procedure has therefore the effect of subtracting the infrared pole in $Q^2 = \Lambda^2$, by means of a power-suppressed term, in a minimal way. The analytic coupling (Fig. 1) has a constant limit at zero momentum transfer: $\lim_{Q^2 \rightarrow 0} \bar{\alpha}_S(Q^2) = \frac{1}{\beta_0}$; furthermore, the subtraction term does not modify the high-energy behavior because it decays as an inverse power of the hard scale: $\bar{\alpha}_S(Q^2) \simeq \alpha_S(Q^2)$ for $Q^2 \gg \Lambda_{QCD}^2$.

Higher orders in the form factors have the main effect of replacing the tree-level coupling with a time-like one evaluated at the transverse momentum of the primary emitted gluon

$$\alpha_S \rightarrow \tilde{\alpha}_S(k_\perp^2) \equiv \frac{i}{2\pi} \int_0^{k_\perp^2} ds \text{Disc}_s \frac{\alpha_S(-s)}{s}. \quad (8)$$

By performing such integral exactly, one includes in the coupling absorptive effects related to the decay of time-like gluons.

The prescription at the root of our model is simply to replace the standard coupling on the r.h.s. of Eq. (8) with the analytic coupling [10] (see also [9]). Therefore we have a formula similar to Eq.(4) where the effective coupling have replaced the standard one. In order to include as many corrections as possible, we make the integration over y in Eq.(4) exactly, in numerical way; this is possible because the time-like coupling $\tilde{\alpha}_S(k_\perp^2)$ does not have the Landau singularity and is regular for any $k_\perp^2 \geq 0$.

The form factor in the physical space is obtained by inverse Mellin transform

$$\sigma(t; \alpha_S) = \int_{C-i\infty}^{C+i\infty} \frac{dN}{2\pi i} (1-t)^{-N} \sigma_N(\alpha_S), \quad (9)$$

where the constant C is chosen so that the integration contour in the N -plane lies to the right of all the singularities of $\sigma_N(\alpha_S)$. In order to correctly implement multi-parton kinematics, the inverse transform from N -space back to x -space is also made exactly, in numerical way. Let us note that no prescription — such as the minimal prescription in the standard formalism [11] — is needed in our model because $\sigma_N(\alpha_S)$ is analytic for any $\text{Re } N > 0$.

IV. PHENOMENOLOGY

In this section we compare the theoretical distributions obtained with our model with experimental data (see also [15], for other approaches see [12], [13], [14]). Since our model has no free parameters, it allows a straightforward extraction of the value of $\alpha_S(m_Z)$ from the experimental data.

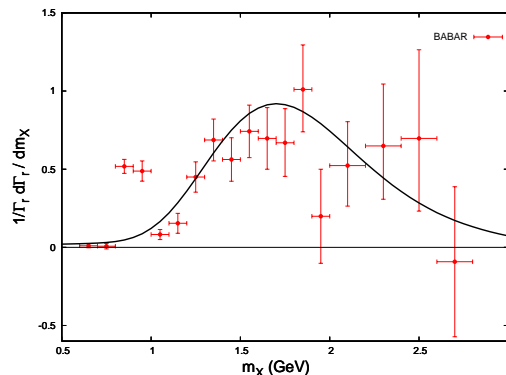


FIG. 2: $B \rightarrow X_s \gamma$: m_X distribution from BaBar [16]. The data show a rather pronounced K^* peak, which clearly cannot be accounted for in a perturbative QCD framework.

The electron spectrum and the m_X spectrum in the semileptonic decay are affected by a large background for $E_e < \frac{m_B}{2}(1 - m_D^2/m_B^2) \simeq 2.31\text{GeV}$ (i.e. for $\bar{x}_e > 0.125$) and for $m_X > m_D = 1.867\text{GeV}$ respectively, coming from the decays $B \rightarrow X_c l \nu_l$. This background is larger than the signal by two orders of magnitude because $|V_{ub}|^2/|V_{cb}|^2 \sim 10^{-2}$. Let us stress that the photon energy spectrum in radiative decay and the electron energy spectrum in semileptonic decay have to be convoluted with a normal distribution, in order to

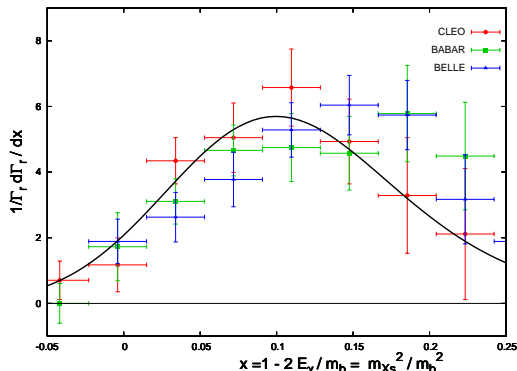


FIG. 3: $B \rightarrow X_s \gamma$: photon spectrum from CLEO (red), BaBar (green) and Belle (blue) [16, 17, 18], the Doppler effect is sufficient to completely eliminate the K^* peak.

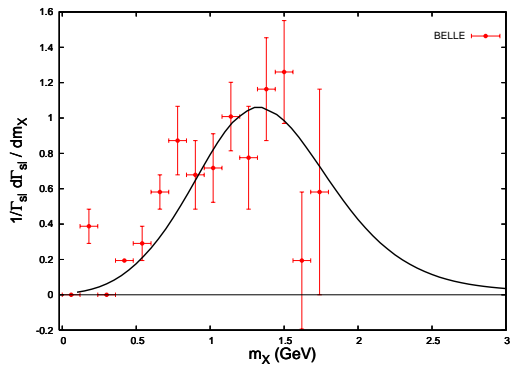


FIG. 4: $B \rightarrow X_u l \nu_l$: m_X distribution from Belle [17]. Data show the π and the ρ peak.

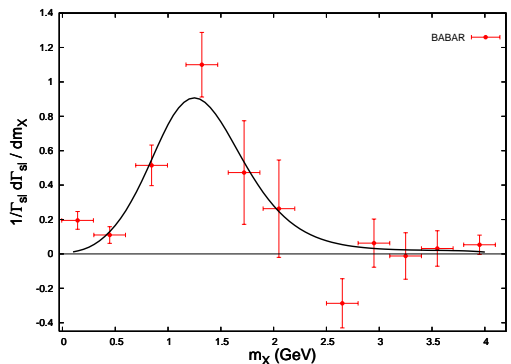


FIG. 5: $B \rightarrow X_u l \nu_l$: m_X distribution from BaBar [16]. Due to the larger binning only the π peak is visible.

model the Doppler effect, due to the motion of the B mesons in the $\Upsilon(4S)$ rest frame. The over-all agreement of the model with the data is good for what concerns all the distributions in the radiative decay and the m_X distributions in semileptonic decays in the region $m_X > 1$ GeV, below which single resonances are expected to have a substantial effect in the dynamics. The extracted values of $\alpha_S(m_Z)$ are in agreement with the world average at most within two standard deviations (see Tab. I).

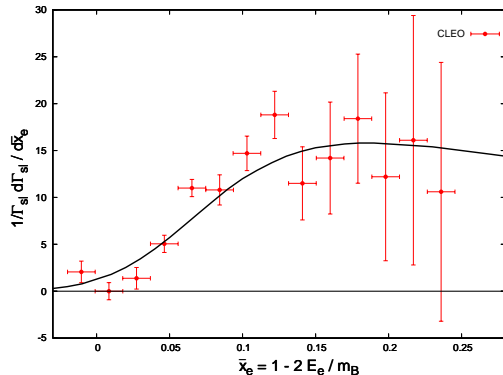


FIG. 6: $B \rightarrow X_u l \nu_l$: electron energy distribution from CLEO [18].

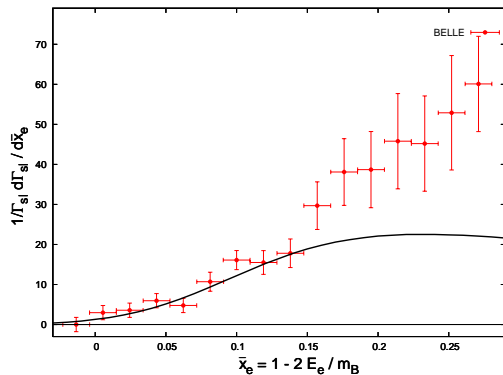


FIG. 7: $B \rightarrow X_u l \nu_l$: electron energy distribution from Belle.

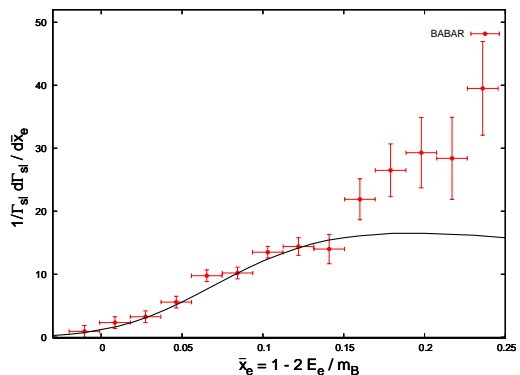


FIG. 8: $B \rightarrow X_u l \nu_l$: electron energy distribution from BaBar.

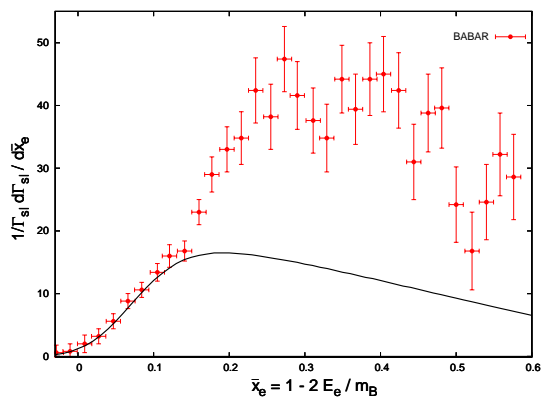


FIG. 9: $B \rightarrow X_u l \nu_l$: electron energy distribution from BaBar (preliminary, error bars indicate statistical errors only).

TABLE I: Extracted value of $\alpha_S(m_Z)$ compared with the PDG world average.

Spectrum	$\alpha_S(m_Z)$	Error
E_γ $B \rightarrow X_s \gamma$ CLEO	0.117	0.004
E_γ $B \rightarrow X_s \gamma$ BaBar	0.129	0.005
E_γ $B \rightarrow X_s \gamma$ Belle	0.130	0.005
m_X $B \rightarrow X_u l \nu_l$ BaBar	0.119	0.003
m_X $B \rightarrow X_u l \nu_l$ Belle	0.119	0.004
E_e $B \rightarrow X_u l \nu_l$ CLEO	0.117	0.005
E_e $B \rightarrow X_u l \nu_l$ BaBar	0.119	0.005
PDG	0.1176	0.0020

The theory-data agreement is less clear in the case of the electron spectra in semileptonic decay. The agreement is acceptable in the charm background free region, i.e. for $2.31 \text{ GeV} < E_e < 2.64 \text{ GeV}$. There is not a good agreement with the preliminary BaBar spectrum for small electron energies: our model predicts a broad maximum around $E_e = 2.1 \text{ GeV}$, while the data seem to peak at lower energies. We do not know whether this discrepancy is related to a deficiency of our model or to an under-estimated charm background.

V. SEMILEPTONIC CHARMED B DECAYS

To describe the semileptonic charmed B decays $B \rightarrow X_c l \nu_l$ we need a formalism to take in account the (non negligible) charm mass m_c . Standard threshold resummation has recently been generalized to the case of jets

initiated by massive quarks [19]. The inclusion of mass terms, in the N -moment space, results in a universal correction factor $\delta_N(Q^2; m^2)$

$$\log \delta_N(Q^2; m^2) = \int_0^1 \frac{dy}{y} [(1-y)^{r(N-1)} - 1] \quad (10)$$

$$\times \left\{ - \int \frac{m^2 y}{m^2 y^2} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] - B[\alpha_S(m^2 y)] + D[\alpha_S(m^2 y^2)] \right\},$$

so that the QCD form factor for massive quarks reads

$$\sigma_N(Q^2; m^2) = \sigma_N(Q^2) \delta_N(Q^2; m^2), \quad (11)$$

where $\sigma_N(Q^2) \equiv \sigma_N(Q^2; m^2=0)$ is the standard QCD form factor for a massless quark defined in Eq. 4 and $r \equiv m^2/Q^2 \ll 1$ is the corrections mass parameters.

Combining the above resummed formula with the full $O(\alpha_S)$ triple differential distribution recently computed [20], [21] and using the model coupling described in the previous section, we can analyze experimental data from semileptonic $b \rightarrow c$ transitions.

VI. CONCLUSIONS

We have presented a model for the QCD form factor describing radiative and semileptonic B decay spectra, based on soft-gluon resummation to NNLL accuracy and on a power expansion in an analytic time-like coupling. The latter is free from Landau singularities and resums absorptive effects in gluon cascades to all orders.

The agreement with invariant hadron mass distributions in radiative and semileptonic decays measured by CLEO, BaBar and Belle is pretty good. The extracted values of $\alpha_S(m_Z)$ are in agreement with the current PDG average within at most two standard deviations (see Tab. I).

The agreement with the electron spectra in semileptonic decays is instead, in general, not so good. At present, we do not know whether the discrepancy is due to a deficiency of our model or to an under-subtracted charm background. Let us stress however that non-perturbative effects are expected to be much smaller in the electron spectrum than in the case of the other analyzed spectra [5].

Using a new resummation formalism recently developed together with available fixed order calculation, we will carry out a phenomenological analysis of semileptonic charmed decays soon.

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