BCS-BEC Crossover in a Two Dimensional Fermi Gas

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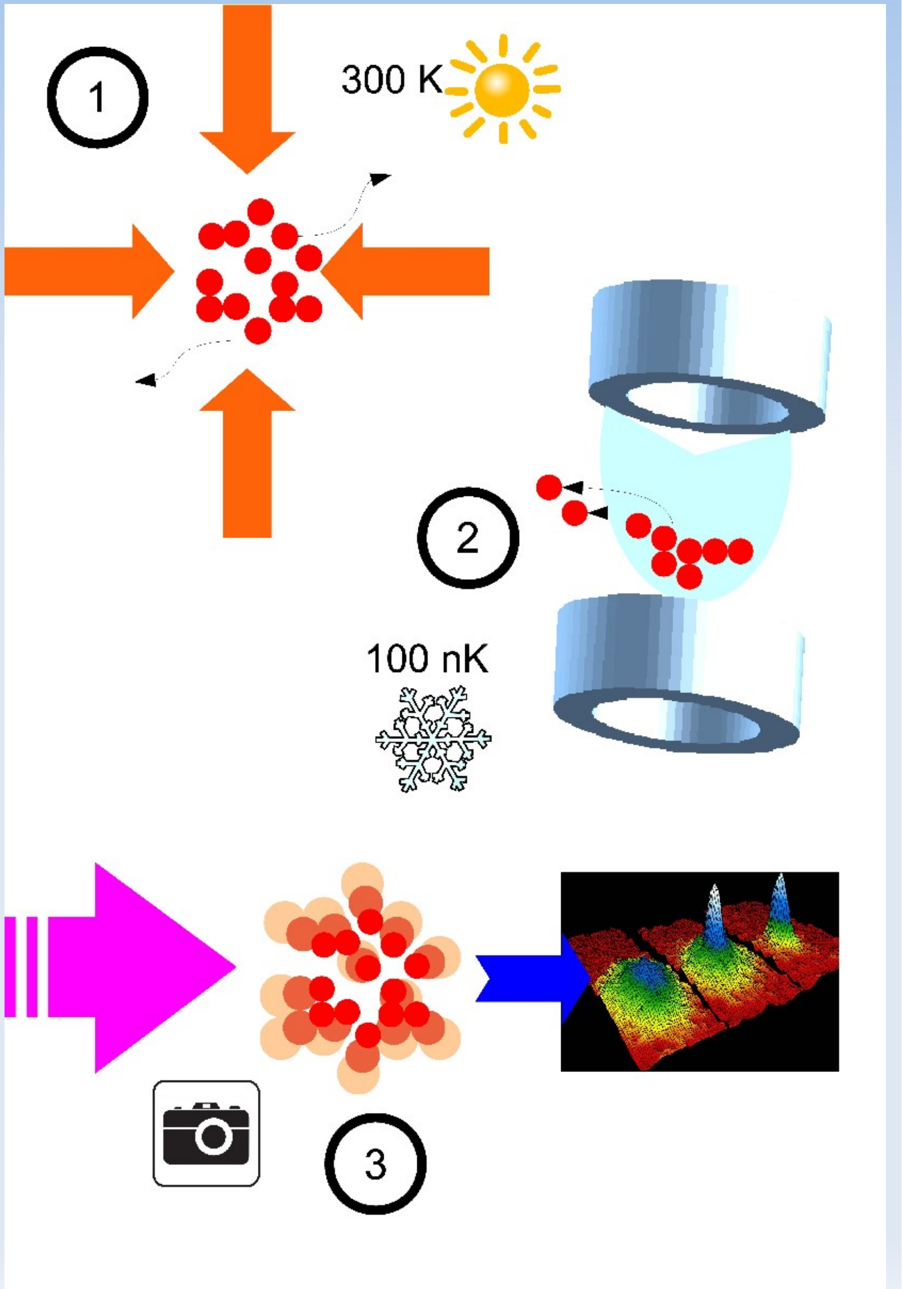
Outline

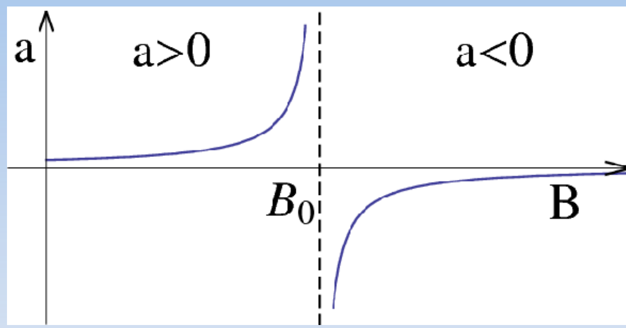
- Ultracold Fermi gases and BCS-BEC crossover
- Quantum Monte Carlo techniques
- BCS-BEC crossover in two dimensions

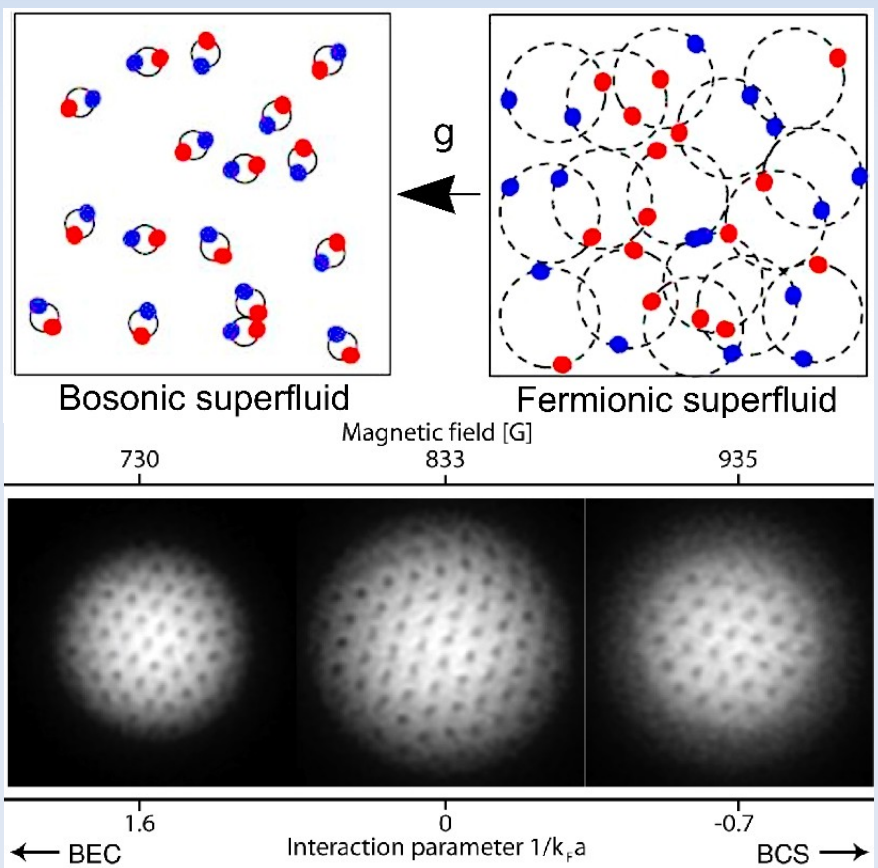
Ultracold Fermi gases

- Short range interaction R
- Diluteness $R \ll l$
- Low temperature $R \ll \lambda_T$
- 1) Laser cooling of alkali atoms (^{6}Li, ^{40}K ...)
- 2) Magneto-optical cooling and trapping (two hyperfine states)
- Quantum degeneracy $n \lambda_T^d > 1$
- 3) Imaging: density and spin density



BCS-BEC crossover





2 species of fermions with attractive interaction at T=0 (Feshbach resonance)

- Weak coupling: Cooper instability $$E/N \sim \alpha\, \varepsilon_F$$
- Strong coupling: Condensate of dimers $$E/N \sim \frac{\varepsilon_b}{2}$$
- Crossover:
  no phase transition and no small parameter for perturbation theory
  → QMC

Ketterle group, MIT (3D system)

Diffusion Monte Carlo (DMC)

Schroedinger equation in imaginary time

$$-\frac{\partial}{\partial \tau}\Psi = H\Psi \qquad \Psi(x,\tau) = e^{-H\tau}\Psi_T(x)$$

Eigenfunctions $\quad \psi_n(x,\tau) = e^{-E_n\tau}\psi_n(x)$

Energy

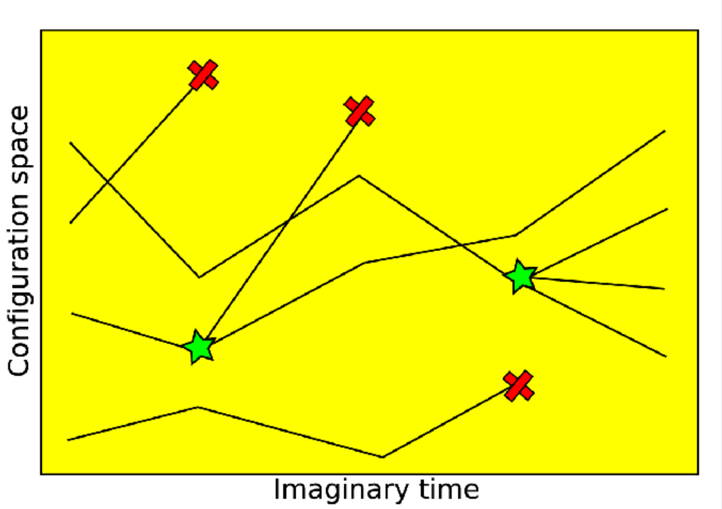
$$\langle H\rangle = \frac{\langle\psi_0|H|\Psi_T\rangle}{\langle\psi_0|\Psi_T\rangle} = \frac{\int \psi_0(x)\Psi_T(x)\frac{\langle x|H|\Psi_T\rangle}{\Psi_T(x)}dx}{\int \psi_0(x)\Psi_T(x)dx} = \int g(x)E_L(x)dx$$

Ψ_T Trial (guide) wavefunction

Basic Algorithm (small time-step expansion)

Random walk + Drift

Branching

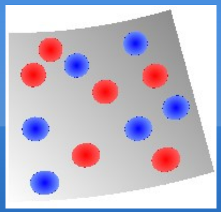


Bosons : *exact (ground state)*

Fermions : *Fixed Node approx.* (*variational principle*) $\Psi(x)\Psi_T(x) \geqslant 0$

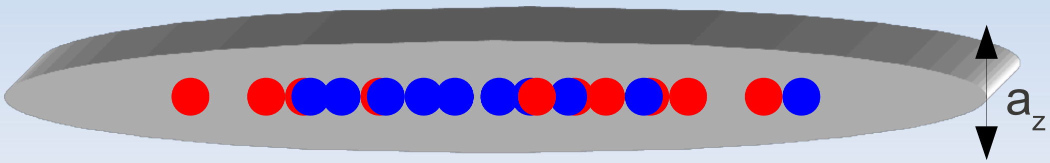
BCS-BEC crossover in 2D

G. Bertaina and S. Giorgini, Phys. Rev. Lett. 106, 110403 (2011).



Optically trapped gas: Quasi-2D $\varepsilon_F \ll \hbar\omega_z$

Experiments (2010): Turlapov, Koehl



Mapping: trapped 3D → Pure 2D

$$a_{2D} \propto a_z \exp\left(-\sqrt{\frac{\pi}{2}}\frac{a_z}{a_{3D}}\right)$$

Bound state $\varepsilon_b \propto -\frac{\hbar^2}{m a_{2D}^2}$

Model hamiltonian: Square well interaction (in universal regime $nR^d \ll 1$)

Trial Wavefunctions (used to fix the nodal surface in DMC):

- Weak coupling: $\Psi_{JS}(x) = J_{\uparrow\downarrow} D_\uparrow(N_\uparrow) D_\downarrow(N_\downarrow)$

  Jastrow factor: $J_{\uparrow\downarrow} = \prod_{ii'} f_{\uparrow\downarrow}(x_{ii'})$ f: two-body problem

- Strong coupling: $\Psi_{BCS} = A[\phi(x_{11'}) \dots \phi(x_{N_\uparrow N_\downarrow})]$

  ϕ: bound state

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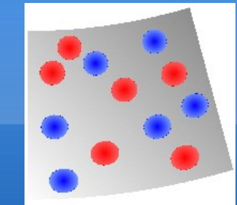
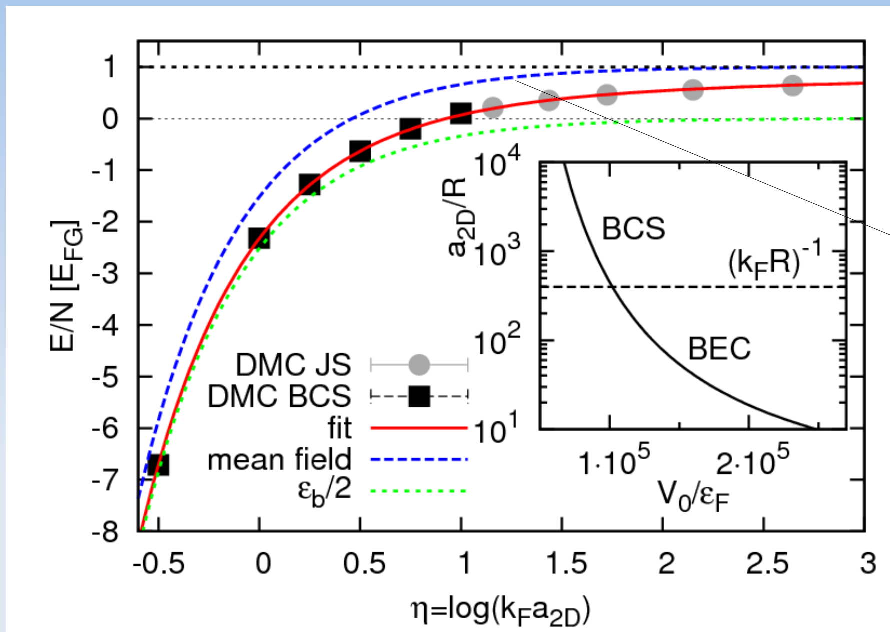




Energy per particle

Crossover parameter: $\eta = \log(k_F a_{2D})$

BCS selfconsistent theory (Randeria) misses interaction between bosons

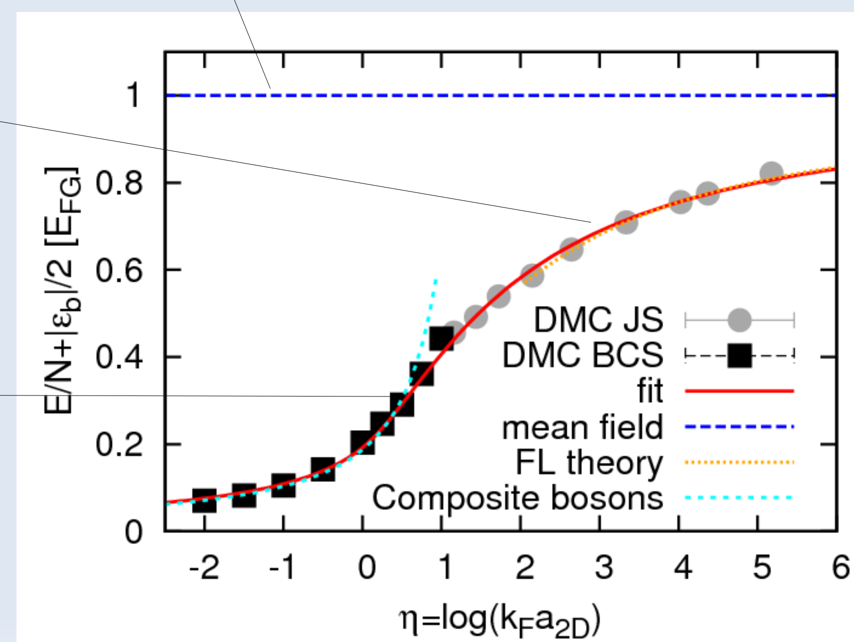
$$\frac{E}{N} = E_{FG} - \frac{|\varepsilon_b|}{2}$$

Fermi liquid energy functional (small gap)

$$\frac{E}{N} = E_{FG}\left(1 - \frac{1}{\eta} + \frac{A}{\eta^2}\right)$$

Composite bosons: $a_{dd} \sim 0.55(4)\, a_{2D}$ (Petrov)

$$\frac{E}{N} = -\frac{\varepsilon_b}{2} + \frac{\pi \hbar^2 n_d}{m_d} \frac{1}{\log \frac{1}{n_d a_{dd}^2}} \left[1 + 2^{nd}\, order\right]$$



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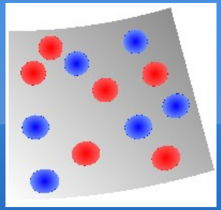
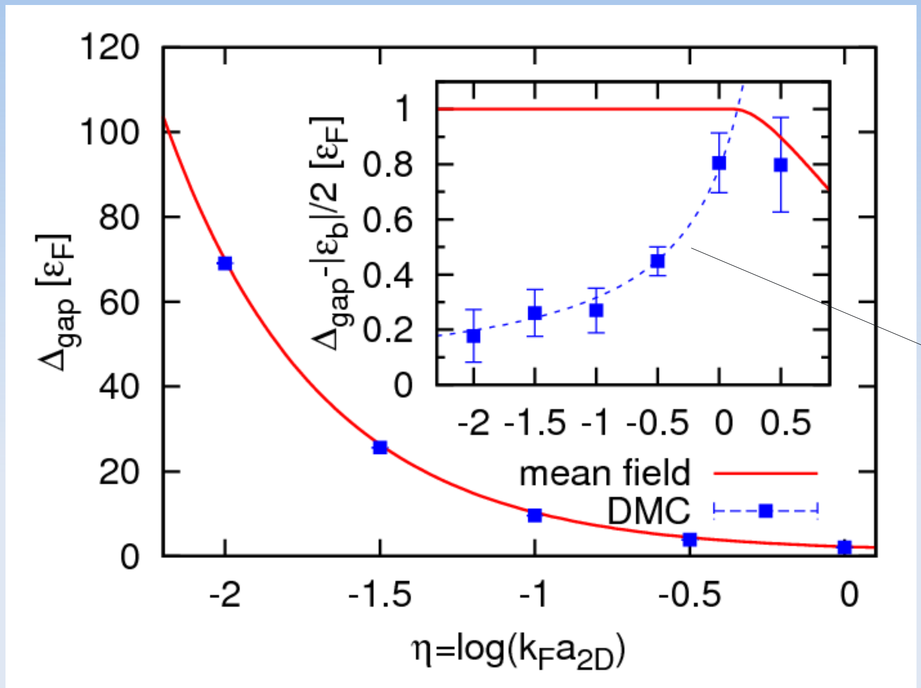




Gap in the spectrum

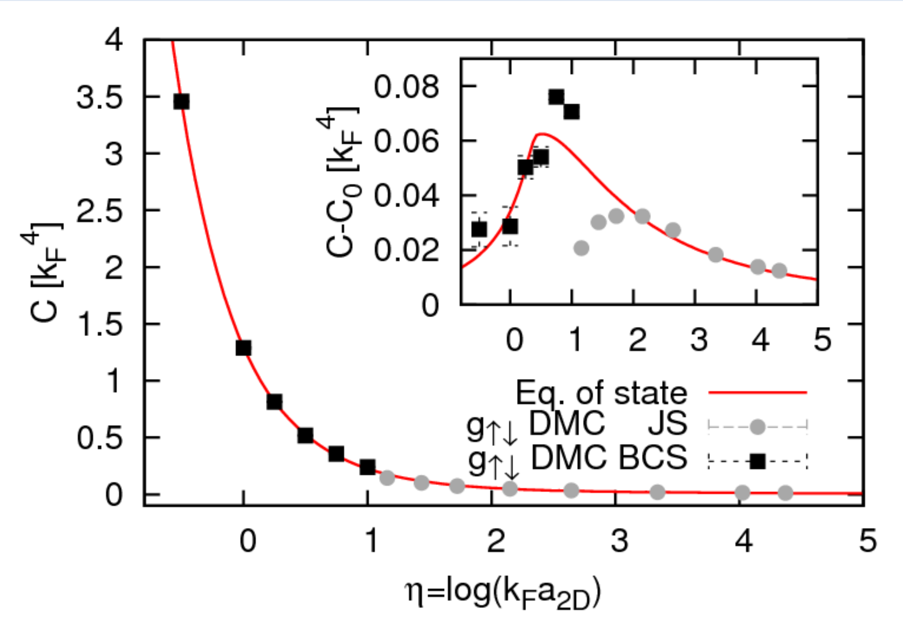
$$E(N_\uparrow+1, N_\downarrow) = E(N_\uparrow, N_\downarrow) + \mu_\uparrow + \Delta_{gap}$$

Interpretation:
One fermion in a BEC
$a_{ad} \sim 1.7(1)\, a_{2D}$

Short range interactions:
Contact parameter (Tan)

$$\frac{dE}{d\log(k_F a_{2D})} \propto C$$

$$g^{(2)}_{\uparrow\downarrow}(r) \underset{r\to 0}{\propto} C \log(r/a_{2D})^2$$



Conclusions

QMC shows absolute relevance of beyond mean-field corrections

The nodal surface of the used wavefunctions contains the relevant T=0 physics

Outlook

Polaron-molecule problem in 2D

Phase separation