ITERATIVE DECONVOLUTIONS TO COMPENSATE WAVELET STRETCHING ON 4TH ORDER TRAVELTIME KINEMATIC

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Introduction. Normal move-out (NMO) correction applied to common-midpoint (CMP) gathers is needed for stacking, AVO analysis and for other seismic processing steps. The result of this operation are CMP gathers where the offset and velocity effects have been removed and thus all the traces, recorded at variable source to receiver offset, simulate a zero-offset kinematic. The NMO corrected CMP gathers are then employed for AVO analysis and for building stacked images. However, it is well known that the traditional sample by sample NMO correction introduces the stretching of the reflected wavelets (Buchholtz, 1972). These distortions are caused by the non-parallelism of the local travelttime of each reflected event, or, in the frequency domain, as the consequence of the nonphysical energy changes introduced by the non-stationary time shifts applied. Additionally, there are other drawbacks of the standard NMO correction, such as the partial duplication of the recorded events and the time inversion of the samples of a reflection (Masoomzadeh et al., 2010). Typically, in order to avoid such negative effects, which may compromise the quality of the stacked images, a mute function is applied to the distorted part of the corrected CMP gathers. Yet, the application of the mute function limits the stacking process to only the near vertical reflections. For example, in case of long-offset acquisitions the wavelets used in the stacking process are constrained to a limited portion of the recorded data and therefore the muting reduces the exploitable information provided by the wide-angle wavelets. As a matter of fact, there are many instances where long-offset acquisitions are crucial, such as sub-basalt exploration or seismic undershooting. In these cases the far offset reflections on the CMP gathers are necessary for imaging purposes (Colombo, 2005).

Since in exploration seismology it is now common to acquire and process data with more than 10 km of offset, a nostretch NMO correction procedure is of interest. Many authors have proposed alternative methods (e.g. Perroud and Tygel 2004; Masoomzadeh, 2010). However, these techniques are affected by limitations, in particular they do not account for interfering reflections and offset variations of the waveforms, which are commonly present in wide-angle CMP gathers. Moreover, they may introduce horizontal coherent noise that after stacking can mimic true reflected events. In this paper we propose an extension of the algorithm of the normal moveout through iterative partial corrections and deconvolutions which deals with long-offset data and offset varying waveforms (Mazzotti et al., 2005). We test our method on a synthetic seismic gather which presents long-offset traces, interfering events and amplitude and phase variations with offset of the reflected waveforms. In addition to these features, we add random noise to the gather and we simulate under-shooting acquisitions using offsets greater than 1000 m (Fig. 1). We also apply our nostretch algorithm to a subset of an offshore marine line (Rocchi et al., 2007) simulating again an under-shooting pattern by muting the short offset traces. We start illustrating the normal moveout through partial corrections (NMOPC) and then we show the synthetic and real applications.

Method. The Normal Moveout through partial corrections can be divided into three phases: wavelet estimation, partial NMO correction and shaping deconvolution. We based our algorithm on the 4th order traveltime approximation proposed by Taner and Koehler (1969). For the wavelet estimation, we employ temporal-offset windows which slide both in time and offset. The short-offset window is projected along the 4th order curve corresponding to the central time of the window. These windows enable to select portions of the wavefield where the reflected wavelets can be considered as stationary. The time samples and the traces within each window constitute a matrix which contains reflected and interfering events plus random noise. The singular value decomposition (SVD) is then applied to each
of these matrices and from the 1\textsuperscript{st} eigenvalue the reflected waveform can be separated from interfering events and random noise. Finally, an average across the columns is computed to obtain the estimated wavelet. The short-offset window is then shifted by a predetermined time amount, which ranges between 50\% and 80\% of the window’s time length, and the whole estimation procedure is repeated. Thus, we end up with an estimation of the offset variant wavelets for each 4\textsuperscript{th} order traveltime curve derived from the velocity analysis spectrum. These time and offset variant wavelets will constitute our desired output in the successive deconvolution steps that will be carried out in an iterative fashion after the application of each partial NMO correction.

It is worth noting that in order to avoid edge effects on the representative wavelets, all the adjacent estimation windows are both time and offset overlapped, typically this overlap is between 20\% and 50\% of the considered dimension. The short-offset window time length is approximately equal to the wavelet duration, while the offset width depends on whether the waveform variations along the offset are strong or mild.

The second phase of our method is the NMO partial correction. The purpose of this operation is to limit the waveform distortions, which would be greater if the corrections were performed in a single step. We regulate the amount of NMO correction by means of the NMO percentage parameter that is defined as follow:

\[ \alpha = \frac{t_1 - t_{1\text{\,new}}}{t_1 - t_0}, \]

where \( t_1 \) is the traveltime at the maximum offset for the uncorrected trajectory, \( t_{1\text{\,new}} \) is the traveltime at the same offset for the partially corrected trajectory and \( t_0 \) is the final correction time.

Once \( \alpha \) has been set, it is possible to move the original traveltime curve \( t_{\text{old}} \) to the partially NMO corrected trajectory \( t_{\text{new}} \) using the following equations:

\[ \beta = \frac{t_{1\text{\,new}}^2 - t_0^2}{c_{2\text{\,old}}x^2 + c_{3\text{\,old}}x^4}, \]

\[ t_{\text{new}} = \sqrt{t_0^2 + \beta c_{2\text{\,old}}x^2 + \beta c_{3\text{\,old}}x^4}, \]

where \( c_{2\text{\,old}} \) and \( c_{3\text{\,old}} \) are the coefficients of the fourth order equation, \( X \) is the maximum offset and \( x \) is the offset. The result of the partial correction is a CMP where the reflections have been only partly moved to their final horizontal alignment so that they have been subjected to only a minimal stretch.

The limited distortions introduced by the partial correction can be removed by shaping the stretched wavelets to their original (unstretched) status by the application of Wiener shaping filters. In fact, knowing the relations between the old and the new traveltime curves (equation 3) and using the estimated wavelets, it is possible to build the partially corrected unstretched CMP gather which constitutes the desired output. To enhance the coherent reflections and to attenuate the influence of the noise we may weigh the samples of the desired output CMP with the semblance values computed from the velocity analysis.

To create the unstretched CMP gather, that is our desired output, the estimated wavelets are inserted into an empty CMP panel centered along the corresponding partially corrected 4\textsuperscript{th} order curves. We employ the same time and offset overlaps we used in the wavelet estimation. Then we apply a trace by trace shaping deconvolution to the partially corrected and stretched CMP gather having as the desired output the partially corrected unstretched CMP. The procedure of partial NMO correction and shaping deconvolution is then reiterated until a complete NMO correction is achieved.
Fig. 1 – Synthetic gather with random noise obtained using a ray-tracing routine (Cerveny, 1985) on the six layers earth model of Tab. 1. To demonstrate the efficacy of the NMOPC the traces up to 1000 m offset have been removed.

**Synthetic example.** To test our nostretch routine, we apply the NMOPC to a synthetic gather where the near-offset traces are completely missing, in fact, the minimum offset in the CMP of Fig. 1 is 1000 m. In this example the receiver interval is 50 m and the maximum offset is 5000 m. This synthetic gather is built using the earth layered model of Tab. 1. In this model at far offset the traveltimes deviate from the hyperbolic trajectories particularly for the second reflection. Note that the ray-tracing algorithm we used (Cerveny, 1985) properly computes the amplitude and phase variations with offset of the reflected waveforms. Six reflections with offset variations are present, and the two deeper events are nearly overlapped due to the small thickness of the sixth layer (50 m only). Moreover, the noise level is such that the fourth reflection is almost buried in the noise.

Tab. 1 – Earth layered model used for the computation of the synthetic CMP gather of Fig. 1.

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>Velocity (m/s)</th>
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Fig. 2a shows the NMO corrected synthetic data without any mute applied, obtained employing the conventional NMO technique. Several well known artifacts can be observed in this image. First, the
two shallower reflections are affected by significant over-stretching and distortions in the interfering part. Second, the traveltime curves which do not correspond to a reflected event but are nevertheless used by the NMO procedure, cause wavelet repetitions, as indicated by the arrows in the corrected panel. Finally, the other reflections are affected by waveform stretching in particular at far offset. The superimposed red line indicates the mute function with a 50% of stretching mute. This mute function rejects completely the first and the second corrected events and the long-offset part of the gather as well. Conversely, the synthetic gather corrected using the NMOPC does not present the distortions of the traditionally corrected CMP (Fig. 2b). The long-offset waveforms are not stretched, the intersecting events are completely separated, there are not any spurious wavelets and also the corrected events have retained much of their amplitude and phase characteristics. Furthermore, since the NMOPC is based on a SVD estimation process and on a series of shaping deconvolutions that use semblance weighted wavelets, the random noise is highly damped. Note that the fourth reflection, which is completely obscured in Fig. 2a, can be detected in this corrected CMP. The minor artifacts that do appear would be effectively attenuated by the stacking process.

Real data application. We carry out a real data application on a marine seismic line which pertains to a subset of the 3D data set described in Rocchi et al. (2007) and acquired on the Senegal passive margin. The acquisition parameters for this line are: source interval 100 m, receivers interval 12.5 m, minimum offset 380 m, maximum offset 8100 m and fold coverage of 125. The result of the stacking procedure employing the whole offset range is shown on Fig. 3a. To test the NMOPC, we simulate an undershooting configuration dropping the data acquired with offset shorter than 2300 m. To this data set we apply the conventional NMO technique and the NMOPC to proceed with the stacking process. The stacked section obtained with the traditional NMO correction and mute (30% of stretching threshold) is displayed on Fig. 3b. Many reflected events are muted in this sections, especially in the right hand part, where a low velocity anticline is present. Therefore, the shallow structure is completely lost. On the contrary, the NMOPC stacked section (Fig. 3c) presents all the reflected events, including the shallow ones and shows a signal-to-noise ratio comparable with the full offset section of Fig. 3a.
Fig. 3 – a) Stacked section of a subset of the data described in Rocchi et al. (2007) employing all the recorded traces. b) Stacked section obtained using the undershooting test data after correction with the traditional NMO algorithm. c) Stacked section obtained using the NMOPC routine applied on the undershooting test data.

**Conclusion.** The conventional NMO correction cannot deal with long-offset recordings and may cause serious problem when it is applied to interfering events and data with amplitude and phase variations with offset. Not only is the NMOPC effective in avoiding stretching distortions caused by the NMO corrections, but it is also able to maintain much of the amplitude and phase features of the reflected wavelets. This can turn out useful in many situations such as when employing undershooting techniques or when useful reflections occur at large source-receiver offsets. Moreover, the NMOPC corrected CMPs can be employed for AVO and PVO analysis. The efficacy of our method is demonstrated by either the synthetic example and the real data application. The NMOPC is rather flexible and can adapt to different data. In fact the estimation window dimension and overlaps can be varied according to specific needs and targets. The same applies to the number of iterations controlled by the NMO percentage. The main purpose of the NMOPC is not to replace the traditional NMO correction, but to provide information in such cases when the over-stretch distortions cannot be avoided unless by the application of a muting function.

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References