ESSAYS ON OIL PRICE SHOCKS AND THE MACROECONOMICS OF OIL EXPORTING COUNTRIES

CANDIDATO: ALI SARZAEEM

SUPERVISORI: PROF. DR. EMANUELE BACCHIOCCHI

COORDINATORE: PROF. DR. MICHELE SANTONI

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ACKNOWLEDGMENTS

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Contents

Introduction

1. Oil Price Shocks and Business Cycles of Oil Exporting Countries
   a. Introduction
   b. The Econometric Framework
   c. Data
   d. Empirical Results
   e. Conclusion
   f. References
   g. Appendix

2. Oil Price Shocks and Iranian Economy: A Structural Cointegrated VAR Approach
   a. Introduction
   b. Oil Price and the Economy: Review of Literature
   c. A Structural Cointegrated VAR analysis
   d. Empirical Results
   e. Conclusions
   f. References
   g. Appendix

3. Quality of Institution and the Composition of Government Expenditure
   a. Introduction
   b. Review of Literature
   c. Model
   d. Conclusion
   e. References
   f. Appendix
Introduction

The central issue that this dissertation tries to address is the relationship between oil price shocks and the macroeconomics of oil exporting countries. For all those who had a chance to live in an oil exporting countries, it is evident that oil revenue plays a major role to induce ups and downs in economy and also in politics. I tried to address this phenomenon in the first chapter in which the relationship between oil price shocks and business cycles is investigated. In the first chapter it is shown that in some oil exporting countries like Kuwait, Venezuela and Libya, the impact of oil price shock is conditional on the cycle of the economy. An oil price shock has different effect when the economy is in boom than when it is in bust. Iran is not included in the sample of countries of this chapter since exogenous radical political events like revolution, persistent war, sanction, short term civil war and so on, makes the performance of the economy too volatile to be able to fit its data with regime switching models. This approach works well when changes are smooth. Therefore, I analyzed the case of Iran in a separate chapter through the VAR and SVECM framework. This paper examines the well-known “Dutch disease” hypothesis for the case of Iran. The result of second chapter partially contradict the prediction of this hypothesis since it is shown that positive oil price shock has permanent positive effect on GDP which is consistent with the finding of Esfahani, Mohades and Pesaran (2009). Success of populist candidate in presidential election in Iran and Venezuela motivated the writing of the first paper. This paper explores the relationship between quality of institution and the composition of government budget. It shows that when the quality of institution is low, majority of constituency prefer direct transfer rather than public spending on necessary and productive public goods, although investment in public goods is a prerequisite for development. But when the quality of institution is high, voters choose public investment. That is the cause of the difference in the public choice of two countries: Norway and Venezuela. This is in line with the finding of the literature on political economy of oil. Therefore, all three chapters although are different in method and content but concern a central issue which is the role of oil in the economy of oil exporting countries.
Abstract

In this paper I empirically investigate the relationship between oil price changes and output in a group of oil exporting countries. The dynamics of business cycles in Libya, Saudi Arabia, Nigeria, Kuwait, Venezuela and Qatar are modelled by alternative regime switching models. I show that the extension of univariate Markov Switching model in order to include oil revenue improves dating business cycles in these economies. For all countries, the optimal specification suggested by the data is to consider three cycles or regimes, namely, high growth, mild growth, and recession. These three regimes can be associated to high positive oil shock, mild positive oil shock and negative oil price shock. An interesting finding of the paper is that there is a variety of relationships between oil price shocks and business cycles. Thus, in order to see the effects of an oil price shock one should take into consideration the economic regime when the oil price shock hits the economy. Therefore, it is not possible to talk about a general relationship between oil price shocks and macroeconomic variables for all the main oil exporting countries.

*I have to thank professor Emanuele Bacchiocchi and professor Matteo Pelagatti for their comments. All remaining faults belong to me.
1 Introduction

It is widely believed that oil price shocks have major effects on economic activity and macroeconomic variables in both oil importing and oil exporting countries. It is assumed that an increase in oil price precedes economic boom in oil exporting countries since it enables government to increase its expenditure and increases aggregate demand accordingly. Conversely, negative oil price shock predicts arrival of periods of recession due to reduction in aggregate demand. An opposite direction of changes is expected for oil importing countries, leading to several researches about the nature of relationship between oil price shock and macroeconomics of developed countries (see Hamilton (1983,1996a, 1996b and 2003),Burbidge and Harrison (1984), Hooker (1996a, 1996b), Mork (1989) and Cunado and Perez de Gracia (2003) among all).

Although there is a vast literature on this issue for oil importing countries, there are very few studies about oil exporting countries (Eltony and Al-Awadi, 2001, on Kuwait, Olomola and Adejumo, 2006, on Nigeria, Berument and Ceylan, 2005, on sample of countries in MENA, and Esfahani, Mohaddes and Pesarani, 2009, on Iran). The traditional methodology used in these studies is based on vector autoregressions (VAR). In particular, many of them concentrate on the issue of asymmetric response of gdp to oil shocks (for instance Farzanegan and Markwardt (2009), Mehrara and Oskoui (2007), and Mendoza and Vera (2010)). Recent research on this topic for developed countries has moved to model business cycles and to investigate the role of different shocks including oil price shock in transition between cycles. However, research for oil exporting countries is still behind this frontier.

The aim of this paper is to study the nature of business cycle in oil exporting countries and the role of oil price shocks in dating business cycles. For this purpose, Markov-Switching (MS) models are used as econometric framework. This approach is a common method of studying business cycles in different countries and regions. Hamilton (1989) and Chauvet and Hamilton (2004) applied this method for dating business cycles in US while Raymond and Rich (1997), Clements and Krolzig (2002) and finally Holmes and Wang (2003) used the same approach for studying the impact of oil price shock on the business cycle in UK and US. Salkoglu et al (2003) applied MS framework to study business cycle in Turkey. Ginters (2010) employed the same approach to study aggregate variation in Latvia. Stanca (1999) applied MS method to the case of Italy while Cologni and Manera (2009) studied business cycles of a sample of developed countries through MS models. Thus, to the best of my knowledge, there is no paper that applies this method to oil exporting countries. This paper fills such a gap.

2 The Econometric Framework

The traditional approach for studying the relationship between oil price shocks and macroeconomic variables is VAR, see among many others Hamilton (1983),
Burbidge and Harrison (1984), Bernanke, Gertler and Watson (1997) and Cuñado and Perez de Gracia (2003). Negative oil price shocks during 80’s confirm the failure of the most predictions in the past and led Lee, Ni and Ratti (1995) to distinguish between periods in which oil price was stable and periods in which oil price was volatile. This paper shows the necessity of addressing different regimes in the economy and investigating the effect of oil shocks in each regime. Therefore, it seems that Markov-Regime Switching is appropriate for that purpose.

A seminal paper by Hamilton (1989) introduced Markov-Switching models as a framework for analysing business cycles. In fact, MS-VAR model is a modern parametric alternative to the more traditional non-parametric approach to business cycle measurement in the Burns-Mitchell tradition. This method assumes that the parameters of a time series model of some macroeconomic variables depend upon an unobservable regime variable $S_t \in \{1, ..., M\}$ which represents the state of the business cycle. It is assumed that regime $S_t$ is generated by a Markov chain.

In a general form, one can specify the Markov-Switching models in the following statement:

$$y_t - \mu(s_t) = v(s_t) + A_1(s_t)[y_{t-1} - \mu(s_t)] + ... + A_p(s_t)[y_{t-p} - \mu(s_{t-p})] + \varepsilon_t$$

where $y_t = (y_1, y_2, y_3, ..., y_t)$ is a n dimensional time series vector, $v$ is the vector of intercept, $A_1,...,A_p$ are the matrices of auto-regressive parameters and $\varepsilon_t$ is a white noise vector process such that $\varepsilon_t \mid s_t \sim NID(0, \Sigma(s_t))$.

Krolzig (1997) introduced a notation which is widely used as a common specification of Switching models. Based on variation of mean and intercept, Krolzig call them Markov-Switching in mean (MSM) and Markov-Switching in intercept (MSI) as shown below:

$$y_t - \mu_{s_t} = A_1(s_t)[y_{t-1} - \mu(s_t)] + ... + A_p(s_t)[y_{t-p} - \mu(s_{t-p})] + \varepsilon_t$$

$$y_t - \mu_t = v(s_t) + A_1[y_{t-1} - \mu] + ... + A_p[y_{t-p} - \mu] + \varepsilon_t$$

If the variance of error terms are not constant and also depend on a latent variable $s_t$, these models are called MSMH and MSIH respectively. If auto-regressive parameters depend on latent variable $s_t$, then it is called MSIAH. The following Table 1 summarizes the different typologies of Markov-Switching models:

It is worth mentioning that Hamilton (1989) used MSM(2)-AR(4) as follows:

$$y_t - \mu(s_t) = A_1[y_{t-1} - \mu(s_t)] + ... + A_p[y_{t-p} - \mu(s_{t-p})] + \varepsilon_t$$

where $\varepsilon_t \mid s_t \sim NID(0, \Sigma(s_t))$, with two different regimes, $s_t = 1, 2$. One basic assumption is that unobserved state follows a first order Markov process, meaning that each regime depends on the previous regime only, but not further in the past.
Table 1: Typology of Markov Switching Models

<table>
<thead>
<tr>
<th>Notation</th>
<th>mean</th>
<th>intercept</th>
<th>Variance matrix of autoregressive parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSM-AR</td>
<td>varying</td>
<td>-</td>
<td>invariant</td>
</tr>
<tr>
<td>MSMH-AR</td>
<td>varying</td>
<td>-</td>
<td>varying</td>
</tr>
<tr>
<td>MSI-AR</td>
<td>-</td>
<td>varying</td>
<td>invariant</td>
</tr>
<tr>
<td>MSIIH-AR</td>
<td>-</td>
<td>varying</td>
<td>varying</td>
</tr>
<tr>
<td>MSIAH-AR</td>
<td>-</td>
<td>varying</td>
<td>varying</td>
</tr>
</tbody>
</table>

\[ P\{s_t = j \mid s_{t-1} = i, s_{t-2} = k, \ldots \} = P\{s_t = j \mid s_{t-1} = i \} = P_{ij} \]

\( P_{ij} \) shows the probability that state \( j \) follows state \( i \). Given the fact that there are \( N \) states, one can define a transition matrix as follows:

\[
P = \begin{bmatrix}
P_{11} & P_{21} & \cdots & P_{N1} \\
P_{12} & P_{22} & \cdots & P_{N2} \\
\vdots & \vdots & \ddots & \vdots \\
P_{1N} & P_{2N} & \cdots & P_{NN}
\end{bmatrix}
\]

It is clear that the sum of probabilities of transition to different states should be one (\( \sum_{j=1}^{N} P_{ij} = 1, \ i = 1 \ldots N \) and \( 0 \leq P_{ij} \leq 1 \)).

In this paper, I will follow the specification used by Hamilton (1989) by assuming deviation of output growth from its mean follow a \( p \)-th order autoregressive process as shown below:

\[
\Delta gdp_t - \mu(s_t) = v(s_t) + A_1(s_t) [\Delta gdp_{t-1} - \mu(s_t)] + \ldots + A_p(s_t) [\Delta gdp_{t-p} - \mu(s_{t-p})] + \varepsilon_t
\]

If we assume a three state model (for example recession, low growth and high growth), then the above specification will be as follows:

\[
\Delta gdp_t = w_1 + A_{11}.\Delta gdp_{t-1} + \ldots + A_{p1}.\Delta gdp_{t-p} + \varepsilon_t
\]

\[
\Delta gdp_t = w_2 + A_{12}.\Delta gdp_{t-1} + \ldots + A_{p2}.\Delta gdp_{t-p} + \varepsilon_t
\]

\[
\Delta gdp_t = w_3 + A_{13}.\Delta gdp_{t-1} + \ldots + A_{p3}.\Delta gdp_{t-p} + \varepsilon_t
\]

Following Cologni and Manera (2009), the test for the existence of non-linearity (i.e. regime shift) is done based on a statistic with asymptotic \( \chi^2(q) \) distribution where \( q \) represents the number of restrictions. Cologni and Manera (2009) used Akaike Information Criterion (AIC) as suggested by Psaradakis and Spagnolo (2003) in order to determine the optimal number of regimes. As is common, the optimal number of lags is tested based on a Likelihood Ratio Test (LR). Besides taking into account these criterion, my key criteria for selection of model are goodness of fit, value of log-likelihood function and reasonable probability of regime switching. Moreover, as reported below, normality, Arch and Portmanteau tests were calculated for all estimated model.
3 Data

In this paper I focus on six major oil exporting countries: Libya, Saudi Arabia, Kuwait, Nigeria, Venezuela and Qatar. All data are annual. The GDP data of all these countries have been extracted from IFS database. I intentionally use oil revenue from export as a proxy for oil price. The data is collected from IFS dataset or from the OPEC website.

Regarding the sample sizes and specific country information, data of Libya are from 1960 to 2009 and data for oil revenue was obtained from OPEC website. Data of Kuwait are from 1962 to 2008 and are obtained from IFS database. Data of Saudi Arabia are from 1968 to 2008 and are from IFS database. Data of Qatar come from IFS dataset and range from 1980 to 2008. Data of Nigeria, instead, cover the period 1973-2003 and are obtained from IFS database.

Figures 1-6 show the trend of oil revenue changes and GDP growth. The first general comment refers to the evidence of co-movements between the two variables, clearly highlighted in all graphs. As it is shown in Figure 1, Saudi Arabia has benefited from oil price increase during 70’s especially at the time of second oil price shock (due to the Iranian revolution) and its economic growth was negligible or even negative during 80’s when negative oil price shocks hit oil market. Moreover, it is worth mentioning that Saudi Arabia also benefited from the recent positive oil price shocks (after 2004).

Libya is an interesting case in the sense that co-movements of oil revenue and economic growth is clearer than any other countries which confirm high dependency of this economy to revenue from oil export. In Figure 2, recession of Libyan economy during 80’s is concurrent with recession in global oil market and low price for petroleum while economy was in the boom during 70’s and the first years of the third millennium.

Co-movement of oil price and economic growth in Venezuela is also confirmed in the Figure 3 but this correlation is weaker than any other country in the sample. Mild growth during 70’s and mild recession is the main feature of this economy.

The pattern of macroeconomic trend in a small oil exporting country like Kuwait is not different from its neighborhood (like Saudi Arabia) because these countries implemented similar economic policies in the past. The only exceptional policy measure Kuwait took was to create a fund for saving oil revenue for next generations. Figure 4 indicates that Kuwait experienced boom at the time of positive oil shock and bust at the time of negative oil price shock.

Economic performance of Qatar is more successful since this economy has experienced less volatility and managed the risk of oil price shocks as shown in Figure 5. This became possible due to long-term oil and gas contracts between Qatar and major oil companies.

Nigerian economy follows the same pattern of performance but Figure 6 shows much less variation in aggregate output. Too much volatility in oil revenue of this country is the result of political and armed conflicts in the region over

\[^{1}\text{I have used OxMetrics6 for all econometric analysis.}\]
the oil fields. These conflicts sometimes prevent operation of oil extraction, transportation and exportation. To summarize, all countries in the sample of our study show the same pattern of co-movements of oil revenue and aggregate output.

4 Empirical Results

In this section, I perform an econometric analysis to better understand the stylized facts observed in the previous section. As already mentioned, the econometric analysis is based on the specification and estimation of Markov Switching bivariate models for each of the mentioned countries. For each country, the dependent variable is represented by the first differences of log GDP (i.e. the growth rate of the economy) which is supposed to be a function of some of its lagged values, as well as of contemporaneous and lagged values of the growth rate of oil revenue. This simple econometric specification allows us to understand the dynamic effects that oil shocks exert on the pattern of the economy of this group of oil exporting countries.

In what follows, I discuss, in details, the specification of the model and the main findings for each of these countries. Tables for estimated results and graphs for the regimes are reported in Appendix 2 and Appendix 3, respectively.

For the case of Libya, the specification analysis suggests a model with three regime switching in mean and one auto-regressive component. The results are reported in Table 2 and Figure 7. The first regime (regime 0) captures periods of recessions which accounts for 17.3% of all samples and lasts on average one year. Concerning the other two regimes, regime 1, which accounts for 58.7% of all periods and lasts on average 4.5 years depicts low growth periods, and regime 2 which accounts for 24% of all cases and lasts 1.1 years shows high growth periods. Transition probabilities show that recessions are temporary and the economy will move rather fast from recession into high growth phase. If the economy is trapped into low growth rate, with a probability of 75% it will remain there and will transit into either recession or high growth rate with a probability of 14% and 10%, respectively. Periods of high growth rate will not last for a long time and with high probability (50%) will transit into low growth periods and with a probability of 36% will transit into recession and with probability of 11% will remain in that regime. Concerning the estimated coefficients, Table 2 shows that oil shocks have a greater effect during recessions (regime 0) and high growth periods (regime 2). It is worth noting that the sign of the coefficients of most variables are different in regime 0 and regime 2 which implies the existence of different relationships in each regime.

Figure 8 shows that a model with three regime switching in intercept and one lag fits data of Saudi Arabia well. The first regime (regime 0) indicates periods of mild growth which accounts for two thirds of the whole sample with an average duration of 5.2 years. Regime 1, indicating low or negative growth rate, represents 15% of all periods with an average duration of 1 year. Regime 2, instead, indicates high growth periods and lasts on average 1.75 years. The
probabilities of transition from mild growth to either recession or high growth is rather low (less than 10%) whereas the probability of transition from recession (or low growth) to mild growth is very high (around 80%) and to high growth regime is around 20%. Based on these findings, when economy is in high growth periods, it will remain there with a probability of 47% or will transit into low growth or recession with a probability of 52%. The results of this regression show different and significant values for intercept in different regimes. Based on Table 3, I find that the impact of oil shocks on the economy does not depend on the cycle of the economy.

For Venezuela, econometric results reported in Table 4 and Figure 9 suggest a three regime switching in mean with three lags. Regime 0 indicates periods of low or negative growth rate which comprises 25% of the whole sample period with a duration of 1.2 years. Regime 1 indicates periods of mild growth rate which amounts to 43% of the whole period with a duration of 2.1 years. Regime 2 shows periods of high growth rate which constitutes 31% of the whole sample periods with average duration of 2.14 years. Probability of remaining in recession or low growth period is low (18%) but the probabilities of transition to mild growth and high growth are 53% and 27%, respectively. The probability of remaining in mild growth regime is quite high (55%) with equal probability (22%) of transition to either high or low growth phase. The coefficients of oil price variables indicate different effects of oil price on output in each regime which confirms the claim that the impact of oil price depends on the regime of the economy. Thus, it is not possible to have a general judgement about the impact of oil shocks on Venezuela. Instead one should see in which context (economic cycle) an oil shock hit the economy.

For Kuwait, a three regime switching in mean (with heteroskedasticity) with two lags fits the data well as pointed out in Table 5 and Figure 10. These results, however, highlight that the three regimes do not have clear economic interpretation. Regime 0 represents 20% of the whole period with average duration of 1 year while regime 1 forms 50% of the whole period with average duration of 1.5 years and regime 2 makes up 30% with average duration of 1 year. If the economy is in regime 0, it will move out of it for sure and probably to enter into regime 1 (83%) and regime 2 (17%). If it is in regime 1, it will not move into regime 0 but will move either into regime 2 with probability of 54% or remain there with probability of 45%. If it is in regime 2, it won’t remain there and will move to regime 1 with probability of 54% or will transit into regime 1 with probability of 45%. An interesting result in Table 4 is the difference between coefficients of the same variables in different regimes which indicates that in each regime there is a specific relationship between macro variables.

For Nigeria, Table 5 and Figure 12 display a three regimes switching in intercept. Regime 0 shows periods of recessions while regime 1 shows periods of high growth rate and regime 2 indicates periods with mild growth. 27% of the whole period is in regime 0 with an average duration of 2 years and 27% of the whole period is in regime 1 with an average duration of 2 years while regime 2 indicates whole period from 1991 up to 2003 (13 years). If Nigerian economy is in regime 0 then it will remain there with probability of 53% and
will transit to regime 1 with probability of 46%. Conversely if it is in regime 1, then it will move to regime 0 with probability of 39% and will remain in regime 1 with probability of 48% and will remain in regime 2 with probability of 11%. If Nigerian economy is in regime 1, then it will remain there forever. An interesting result is the significant difference between coefficients of intercept in different regimes. In this economy, oil revenue plays a negligible role and its impact decreases significantly after one period.

For the case of Qatar, the specification analysis suggests an optimal model with three regimes switching in intercept (with heteroscedasticity) as described in Table 7 and Figure 11. Regime 0 indicates periods of recession or low growth while regime 1 indicates periods of mild growth and regime 2 indicates periods of high growth. Regime 0 comprises 53% of the whole observations and regime 1 makes up 28.5% and regime 2 forms 17.8% of the whole period. If this economy is trapped in regime 0, it will remain there with 46% and will move into regime 1 with 42% and into regime 2 with 11% of probability. If it is in regime 1, it will not remain there and will move into regime 0 probably (84%) and will transit into regime 2 with a probability of 15%. If it is in regime 2, it will remain in that period with probability of 43% or alternatively will move into regime 1 with probability of 56%. The intercept in these regimes are significantly different while the impact of oil shock on the economy does not depends on whether the economy is in boom or in recession.

5 Concluding Remarks

This paper reports empirical results that may be of interest in dating business cycles in oil exporting countries. For this purpose, different specifications of Markov Switching models are proposed and estimated for each country included in the analysis.

The common finding is that, given the volatility of oil market, it is possible to discern three cycles or regimes in these economies, namely, high growth, mild growth, and recession. These three regimes can be associated to high positive oil shock (like the one in the 70’s), mild positive oil shock and negative oil price shock. As a by-product of the performed econometric analysis, it is possible to estimate duration of each cycle and the probability of transition from one cycle to another in each economy.

An interesting finding of the paper is that there is a variety of relationships between oil price shocks and business cycles. In Saudi Arabia and Qatar, the impact of oil price shocks does not depend on the business cycle of the economy while in Kuwait, Venezuela and Libya, the effects of oil price shocks are different in each regime. Thus, one should take into consideration the business cycles (economic regimes) when an oil price shock hits the economy. In Nigeria oil price shocks have negligible influence on the economy. Therefore, it is not possible to talk about a general relationship between oil price shocks and macroeconomic variables for all the main oil exporting countries. Instead, one should consider the dependence of these relationship on business cycle in each country.
Bibliography


Appendix 1: Co-Movement of Oil Revenue Changes and GDP Growth

Figure 1: Saudi Arabia: Trend of Oil Revenue Changes and GDP Growth

Figure 2: Libya: Trend of Oil Revenue Changes and GDP Growth
Figure 3: Venezuela: Trend of Oil Revenue Changes and GDP Growth

Figure 4: Kuwait: Trend of Oil Revenue Changes and GDP Growth
Figure 5: Qatar: Trend of Oil Revenue Changes and GDP Growth

Figure 6: Nigeria: Trend of Oil Revenue Changes and GDP Growth
Appendix 2: Tables of estimated results

Table 2: Libya - Estimation results of MS regression

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (0)</td>
<td>-0.077***</td>
</tr>
<tr>
<td>Constant (1)</td>
<td>-0.0015</td>
</tr>
<tr>
<td>Constant (2)</td>
<td>0.073***</td>
</tr>
<tr>
<td>Doil(0)</td>
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</tr>
<tr>
<td>Doil(1)</td>
<td>0.09***</td>
</tr>
<tr>
<td>Doil (2)</td>
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<td>Doil-1(2)</td>
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<td>Dlgdp-1(0)</td>
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<tr>
<td>Dlgdp-1(1)</td>
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<td>Dlgdp-1(2)</td>
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<td>Sigma</td>
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<td>Log-likelihood</td>
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<td>AIC</td>
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<tr>
<td>LR linearity Test</td>
<td>35.17**</td>
</tr>
<tr>
<td>Normality Test= Chi2(2)</td>
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</tr>
<tr>
<td>Arch 1-1 test= F(1,27)</td>
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<tr>
<td>Portmonteau(6)= Chi2 (6)</td>
<td>7.7396</td>
</tr>
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Transition Probabilities

<table>
<thead>
<tr>
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<th>regime 0</th>
<th>regime 1</th>
<th>regime 2</th>
</tr>
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<tbody>
<tr>
<td>regime 0</td>
<td>0</td>
<td>0.1434</td>
<td>0.3665</td>
</tr>
<tr>
<td>regime 1</td>
<td>0</td>
<td>0.7505</td>
<td>0.5191</td>
</tr>
<tr>
<td>regime 2</td>
<td>1</td>
<td>0.1</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Significant at *** 1 percent - ** 5 percent - * 10 percent
Table 3: Saudi Arabia - Estimation results of MS regression

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficients</th>
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<tbody>
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<td>Constant (1)</td>
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<td>Constant (2)</td>
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<td>Dlgdp-1</td>
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<td>Sigma</td>
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<td>AIC</td>
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<td>LR linearity Test</td>
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<td>2.73</td>
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Significant at *** 1 percent - ** 5 percent - * 10 percent
Table 4: Venezuela - Estimation results of MS regression

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Significant at *** 1 percent - ** 5 percent - * 10 percent
Table 5: Kuwait - Estimation results of MS regression

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Significant at *** 1 percent - ** 5 percent - * 10 percent
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Significant at *** 1 percent - ** 5 percent - * 10 percent
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Significant at *** 1 percent - ** 5 percent - * 10 percent
Appendix 3: Figures of Fitted Values

Figure 7: Libya: fitted values and smoothed probabilities
Figure 8: Saudi Arabia: fitted values and smoothed probabilities

Figure 9: Venezuela: fitted values and smoothed probabilities
Figure 10: Kuwait: fitted values and smoothed probabilities

Figure 11: Qatar: fitted values and smoothed probabilities
Figure 12: Nigeria: fitted values and smoothed probabilities
Chapter Two: Oil Price Shocks and Iranian Economy: A Structural Cointegrated VAR Approach*

Abstract

Oil price movements are known to be a major cause of business cycle in oil importing and also oil exporter countries. Iran, as a major holder of oil and gas reserves in the world and an important producer of energy, is not an exception. Its dependency on oil revenue makes it vulnerable to oil price changes. This paper investigates dynamic relations between oil price shocks and major macroeconomic variables in Iran through a structural vector error correction model (SVECM). Based on one long-run cointegrating relations, impulse response functions provide evidence that oil price shocks have a permanent positive effect on gdp despite the fact that they decrease exchange rate. This is in line with well-known Dutch disease hypothesis.

*I have done this research under the supervision of Prof. Emanuele Bacchiocchi. I am indebted to him for his intellectual and insightful suggestions.
1 Introduction

Nowadays, energy plays a central role in the functioning of modern economies. During 70’s and following Arab OPEC embargo associated with the Yom Kippur war and revolution of Iran, oil price shocks accompanied with a worldwide recession in most major developed economies and a rise in inflation and unemployment. Accordingly, a considerable amount of research has concentrated on identifying the channels through which oil price can affect macroeconomic variables. Economists proposed several explanations for the relationships between oil price and aggregate economic performance.

The first explanation was supply shock. As energy is considered as an important input for many industries and production technologies, a rise in oil price leads to lower output and labor productivity. Others (see, for example, Pindyck and Rotemberg (1983)) have emphasized on the adjustment cost that is imposed on firm to change their technology (from energy intensive technology to energy saving one) or to move from an energy intensive sector to energy efficient sector. Since adoption to new technologies and adjustment of firms to a new situation takes time, there will be, as a result, an increase in unemployment and underutilization of resources. The more recent research attributes the magnitude and timing of recession during 70’s to monetary policies at that time. Referring to the effects of price control in the early 1970 and collapse of pegged currency system, they rejected the idea that oil shock was the only or the main cause of recessions. Earlier, James Tobin (1980), comparing the ratio of energy consumption to GDP, refused to attribute recession to oil shocks.

In contrast, in oil exporting counties, oil shocks were believed to be a kind of bless that enables them to generate wealth and to escape from poverty trap. But, surprisingly, countries with oil or other natural resources not only failed to grow faster than those without these endowments, but also on average have had lower growth over the last four decades. To illustrate it more vividly, Sachs and Warner (1997) sketched the following figure in which 95 developing countries are located according to their growth rate and the share of natural resource-based export to GDP. They found a negative relationship even after controlling for a number of variables introduced in previous growth studies.
This phenomenon is usually referred to as paradox of plenty or “resource curse”. There are huge amount of theoretical and empirical works to examine it (see, for example van der Ploeg (2009), Frankle (2010)). The first explanation of resource curse was “Dutch Disease” hypothesis. According to this hypothesis, extra wealth generated by the sale of natural resources causes an appreciation of the real exchange rate. At the same time, an increase in natural resource revenue boosts national income and domestic demand for both tradable and non tradable goods. Given that the world prices of tradable goods are constant, real exchange rate appreciation leads to an increased import and contraction of manufacturing. Simultaneously, an increase in the demand for non tradable goods results to an expansion of non tradable sector. The literature assumes that "learning by doing" in manufacturing sector increases growth in the long run. Hence, the contraction of manufacturing and expansion of non tradable sector caused by resource windfall results in slow down of "learning by doing" which in turn leads to permanently lower growth rate (van Wijinbergen (1984), Krugman (1987)).

In contrast to “dutch disease” hypothesis, historical evidence shows that a resource boom can indeed lead to growth expansion. For example, an increase in the price of primary commodity raises the rental rate on land used in the production of primary goods and boosts the demand for labor in those areas and shifts factors away from subsistence farming to mining. Salient examples like Norway, Canada and United States (in 19th century as a resource rich economy) cast doubt on the validity of “dutch disease” hypothesis in all cases. Recent study by Brunnschweiler and Bulte (2008) found little support for resource curse argument and
presented evidence in favor of the argument that resource abundance positively affects growth.

Inspired by the institutional economic school of thought, some scholars referred to institutional setting of countries as a determinant of whether oil revenue could be a blessing or a curse. Mehlum, Moene and Torvik (2006) show that the key determinant is whether the institutions are production friendly or rent grabbing friendly which means that whether established institutions motivate people to invest their resources in productive activities like manufacturing or in rent seeking behavior like lobbying to get import licenses and so on. In this line of research, the more recent literature focuses on political economy aspects and argues that more resource revenue creates budget deficits due to voracity effects (i.e., it provides incentives for more claims of the government budget simultaneously by different social groups, see Tornell and Lane (1999)) or motivate civil conflicts (Collier and Hoefler (2004)) or increase political corruption and reduce the quality of candidates (Brollo, Nannicini, Perotti and Tabellini (2010)).

To summarize, evidence from empirical works are not clear cut whether oil revenue is a blessing or a curse. Dutch disease hypothesis assumes that oil revenue is temporarily. But when an economy like Iran is blessed with huge oil reserves that generate oil revenue for a long time, it is not clear whether that hypothesis applies.

Iran is a major oil exporting country. After Saudi Arabia, Iran has the biggest proved reserves of oil across the world which is estimated to be around 137.6 billion barrel (BP (2010)). Production of oil in Iran is reported to be around 4216 thousand barrel per day (BP (2010)). In the past decades, revenues from oil export constitute a major part of government budget in Iran. As tax rate is really low in Iran comparing with developed countries, government depends increasingly on oil revenue. Naturally, government investment plays a major role in Iran economy. To have a clear understanding of the role of oil revenue in the economy of Iran, fig 2 shows the ratio of the value added of oil and gas sector to national income. It is evident that this ratio has increased from 14 percent to around 30 percent.

Figure 3 shows the share of energy export to total government budget across time in Iran. As it is clear, Iranian government is highly dependent on oil revenue and in some period, oil revenue constitutes up to 80% of government budget.

2 Oil Price and Economy: Literature Review

Up to now, most of the research addressing the relationship between oil price shock and macroeconomic variables have focused on developed
Figure 1: The ratio of the value added of oil and gas sector to national income

Figure 2: The share of oil and gas revenue in government budget
countries. Regardless of those researches (see for example, Tatom (1988) among all which followed production function approach, as discussed briefly in previous section), most of the research is influenced by the seminal work of Hamilton (1983). Based on a linear model, he concluded that oil price changes were an important factor in almost all US recession after World War II. Moreover, he found that oil price movements Granger cause variation in unemployment and output in US. Later work by Hamilton (1996a, 1996b and 2003) was concerned about asymmetric effects of oil shocks which provoked critics (see, for example Hooker (1996a, 1996b and 1996c) and Mork (1989)) and further research (see for example Hooker (1999, 2000) and Mork (1994a, 1994b), Mory(1993)).

During 80's, application of Vector Auto Regressive (VAR) models for discovering the relationship between oil price and macro variables started with the contribution of Burbidge and Harrison (1984). They showed that oil price shock has a significant negative impact on industrial production. Hamilton (1996), by introducing net oil price increase (NOPI) as a new variable, used VAR model to show the robustness of the relationship between oil prices and real GNP. Brown and Yucel (1999) also constructed a VAR model of US economy and found that the economy responds to an oil price with a decrease in real GDP, an increase in interest rate, and in price levels.

Research by Eltony and Al-Awadi (2001) on Kuwait, Olomola and Adejumo (2006) on Nigeria and Berument and Ceylan (2005) on selected countries in Middle East and North Africa are among the exceptions that considered oil exporting countries.

The closest works to this paper are Farzanegan and Markwardt (2009), Esfahani, Mohaddes and Pesaran (2009) and Sarzaeem (2008). The paper by Farzanegan and his coauthor investigates the effect of oil price shocks on the economy of Iran through a VAR model but the main difference between that paper and our approach is on the choice and the type of variables. They used quarterly data for a short period of time (1988 till 2004) while I am using a more lengthy annual and an updated dataset. They have followed the literature of asymmetric response of macroeconomic variables to oil price shocks and distinguished positive oil shocks and negative oil shocks while recent work by Kilian (2009) shows that this method of separating positive and negative oil price shocks leads to "inconsistent parameter estimates, and that the implied impulse responses have been routinely computed incorrectly". In the present paper, this distinction is not considered because the main focus is not on the asymmetric behavior of oil shocks. Moreover, having in mind the “Dutch Disease” phenomenon, they used industrial production as proxy for output while in this paper national income is used. Thirdly,
they used effective exchange rate as a measure of exchange rate while in this paper nominal exchange rate is used. Fourthly, they have not tried to identify long run cointegration relations which I examined.

Another seminal work belongs to Esfahani, Mohaddes and Pesaran (2009). In search for finding a long run effect of oil shocks, they developed a growth model and derived conditions under which oil revenue are likely to have a lasting impact. In order to test their model, they applied an error correction model in which world output, world inflation and real exchange rate are used. They found that when the effect of oil export is taken into account, a convergence in output growth between Iran and the rest of the world appears. What distinguishes this research from the others is their emphasize on trade while I emphasize on the interrelation between domestic variables.

Finally, Sarzaeem (2008) performed a similar econometric analysis but based on unofficial quarterly dataset while this paper uses official annual data with an updated dataset. The data used in Sarzaeem (2008) starts from 1970 and lasts up to 2000 and does not include recent oil price hike which is widely believed to have a clear effect on Iranian economy.

3 A Structural Cointegrated VAR Analysis

3.1 The Econometric Framework

Vector Autoregressive (VAR) models, since its introduction, have been widely used in empirical research. The VAR system is based on regularities in the data and being a reduced form, (it is not based on specific economic theory), interpretation of its result is not possible. According to definition, all variables are regressed on its own lagged values and the lagged values of all other variables in the system as follows:

\[ Y_t = (Y_{1t}, Y_{2t}, Y_{3t}) \]

\[ Y_t = A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + u_t \]

where \( A_i \) is \( K \times K \) coefficient matrix and \( u_t = (u_{1t}, u_{2t}, u_{3t}) \) is an unobservable error term. It is commonly assumed that \( u_t \) is a white noise stochastic vector with \( u_t \sim (0, \Sigma_u) \). This is a reduced form formulation which arose several critics due to lack of economic meaning of the result. Conventional shocks with economic meanings are oil shock, money supply shock and exchange rate shock. These shocks are unobservable directly and there is a need for assumptions in order to identify them. So, structural restrictions are required to identify the relevant
innovations and impulse responses. The resulting model is called Structural VAR (SVAR) (see, e.g., Sims (1980, 1986, 1992), Bernanke (1986) and Shapiro and Watson (1988)). In this method, identification focuses on the errors of the system rather than on the autoregressive coefficients. The structural shocks should be uncorrelated and hence orthogonal. Such an orthogonality enables us to track the dynamic effects of an isolated shock. Given the fact that there are some relations between shocks, decomposition is necessary. There are mainly three ways to use non-sample information in specifying innovations which are known as A-model, B-model and AB model. If both sides of the VAR equation are multiplied by $A$ and rename $Au_t$ as $B\varepsilon_t$ then structural disturbances will be $\varepsilon_t \sim (0, I_k)$ and the reduced form residuals are related by $A\varepsilon_t = Bu_t$. Most applications use $A = I_k$ (B model) or $B = I_k$ (A model). When some or all variables are stationary, then the VAR system has a vector error correction representation named VECM(p):

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma_1 Y_{t-2} + \cdots + \Gamma_{p-1} Y_{t-p+1} + u_t$$

where $\Pi = -(I - A_1 - A_2 - \cdots - A_p)$ and $\Gamma_i = -(A_{i+1} + \cdots + A_p)$ for $i = 1, \ldots, p - 1$. Since $\text{rank}(\Pi) = r < k$, $\Pi$ could be decomposed as $\Pi = \alpha\beta^\prime$. $\beta$ is the cointegrating matrix and $\alpha$ is the loading matrix. Likewise VAR, structural definition of VECM models is possible which are known as SVEC. In order to analyze the impact of unanticipated shocks on macroeconomic variables in SVAR or SVEC models, impulse response function (IRF) and forecast error variance decomposition are commonly used. Impulse response functions and variance decompositions are obtained from the moving average representation of the SVECM model.

### 3.1.1 The Macroeconomic Model

In this section, we consider restrictions on long-run and short-run relationships inspired by economic theories. Long run restrictions are linear restrictions on the cointegrated vectors whereas short term restrictions are imposed on the residual covariance matrix according to economic theories. One possible long-run relation is between output, oil price and exchange rate. In the VAR model, endogenous variables are $y = (\text{oil}, \text{exchangerate}, \text{gdp})$. We expect oil price to have a positive effect on gdp. So, I use the following formulation as a restriction on the long run relationships:

$$gdp = \beta_0 + \beta_1 \text{oilprice} + \beta_2 \text{exchangerate}$$

Concerning the short-run restrictions, oil shocks ($\varepsilon_0$) are independent from other variables since oil price are determined in the global oil market.
and this market is so big that macroeconomic variables of Iran can not have an influence on it. Hence we have:

\[ u_1 = \alpha_{11} \varepsilon_0 \]

I assume that oil price shocks contemporaneously affects exchange rate which is consistent with the literature of “Dutch Disease”

\[ u_2 = \alpha_{21} \varepsilon_0 + \alpha_{22} \varepsilon_e \]

With regard to output, I assume that oil price shocks are a key determinant factor of GDP in resource economies. The main channel that connects domestic economy to world economy is exchange rate and should be included especially when import and export are not included in the model:

\[ u_3 = \alpha_{31} \varepsilon_0 + \alpha_{32} \varepsilon_e + \alpha_{33} \varepsilon_y \]

The structuralization of the model, thus, refers to the wellknown Å model in Lütkepohl (2005), in which the short-run restrictions are summarized in the following identification matrix:

\[
\begin{pmatrix}
    u_1 \\
    u_2 \\
    u_3
\end{pmatrix} =
\begin{pmatrix}
    \alpha_{11} & 0 & 0 \\
    \alpha_{21} & \alpha_{21} & 0 \\
    \alpha_{31} & \alpha_{32} & \alpha_{33}
\end{pmatrix}
\begin{pmatrix}
    \varepsilon_0 \\
    \varepsilon_e \\
    \varepsilon_y
\end{pmatrix}
\]

In the above equation, \( \varepsilon_0, \varepsilon_e \) and \( \varepsilon_y \) are structural disturbances, i.e. oil shock, exchange rate shock and output shock. The order of shocks shows that both oil shock and exchange rate shock influence GDP while exchange rate is under the influence of oil price shock. Oil price shock is not affected by either of them.

4 Empirical Results

4.1 Data and Its Statistical Properties

The dataset consists of annual observation for the period of 1975 to 2009 (\( T = 35 \) observations). This interval includes the post revolution period. The variables used in this paper are output, exchange rate and oil revenue. It is a common practice to include government expenditure in this type of modelling but I intentionally excluded it since 1) the revenue of oil export goes directly to the budget of the government, and 2) non oil revenue consists of tax revenue but it is widely believed that economic activity in non oil sector and accordingly tax revenue is also highly correlated to the oil revenue. Oil price instead of oil revenue is used since oil revenue is determined by oil price and exchange rate policy. Since
annual data for employment is not available, employment is not considered. In addition, since interest rate is administratively controlled in Iran, I have decided to not include it in the model. All variables are in logarithm. All data are taken from the website of the Central Bank of Iran.

All data are in real term. Data of GDP was already in constant price of 1997/1998. Real oil price is the average of Iranian light and heavy crude oil revenue deflated by US consumer price index. Data of the price of Iranian light and heavy crude oil is available from OPEC website. As official exchange rate is administratively set and is nearly fixed for a long period, the exchange rate at the black market is used which reflects the market price much better\(^1\).

### 4.2 Econometric results

The results of ADF unit root tests reported in Table 1 confirm the presence of non-stationarity in the time-series included in the VAR. This evidence suggests to check for possible co-integrating relationships among the series. Tables 2-4 provide useful information for the verification of the VAR model, and suggest that the choice of 1 lag is sufficient for obtaining non-correlated and normally distributed residuals.

Then, the number of cointegrating relations should be determined. "Maximum eigenvalue" and "trace test" as developed by Johanson are two criteria for the determination of the number of co-integration relations in a VAR framework. As reported in Table 5, both tests unanimously suggest that there is one cointegrating relation among variables.

We assume the cointegration relation to be the relationship between gdp, oil price and exchange rate. We impose the following restrictions: all coefficients are normalized to the coefficient of output \((\text{lgdp})\). Estimation results for the cointegration relationship are shown in Table 6. According to this estimation, one standard deviation in oil price leads to 0.17 percentage change in GDP. More interesting is the effect of exchange rate which is comparable with oil revenue. This means that exchange rate policy can be very effective in influencing output.

Given the long-run relationship, we can examine the short-run linkages among the variables under study. In this regard, impulse response function should be employed as it traces the effects of different structural shocks to other variables over time. Figures 4-6 show the response of the gdp and exchange rate to (one standard deviation) oil price and exchange rate shocks.

The figures above clearly show that oil price shock enhances output and this increase reaches a maximum within five periods but this effect

\(^1\)Econometric software Eviews6 is used for all estimation and test in this paper.
Figure 3: Response of GDP to impulse of Oil Price

Figure 4: Response of exchange rate to impulse of oil price
is temporary and the increase in output diminishes through time but it does not revert to its original level. Thus, a positive permanent effect remain forever due to the non-stationarity of the series, and to the cointegrating relationship highlighted before. Moreover, it is shown that oil shocks lead to appreciation of exchange rate which is consistent with the “Dutch Disease” hypothesis. Equally interesting is the effect of an exchange rate shock. A standard deviation shock in exchange rate leads to an increase in output. This result has a useful policy implication: depreciation of exchange rate can increase output which is double as much the effect of an increase in oil price shock. One should take into account that an increase in exchange rate is a policy tool in the hand of policy makers but oil price increase is beyond their control.
5 Conclusions

In this paper, I investigated a small-scale structural vector error correction model (SVECM) for the Iranian economy which includes macroeconomic variables such as oil price, exchange rate and output. The analysis shows that oil price shock has a permanent positive effect on gdp in addition to a hike in output in the short run. This result is in line with the findings of Esfahani, Mohaddes and Pesaran and contradicts the prediction of the “Dutch Disease” hypothesis. So, (positive) oil price shock is a kind of bless rather than curse as is expected in the literature. The “Dutch Disease” hypothesis correctly predicts that oil shocks lead to appreciation of exchange rate which is confirmed by my empirical results. My finding shows that exchange rate depreciation is more effective in increasing gdp than an oil price shock. So, exchange rate policy can be considered as powerful policy tool for enhancing growth in Iran.
References


49, pp. 1057–1072.


Appendix: Statistical Results

Table 1: Unit Root Test

<table>
<thead>
<tr>
<th>variable</th>
<th>t-statistics</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.81</td>
<td>0.99</td>
</tr>
<tr>
<td>exchange rate</td>
<td>-1.85</td>
<td>0.34</td>
</tr>
<tr>
<td>Oil Price</td>
<td>-1.5</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Test Critical Values: 1% level: -3.63; 5% level: -2.94; 10% level: -2.61

Table 2: Lag Order Selection Criteria

<table>
<thead>
<tr>
<th>Lag</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.34</td>
<td>4.47</td>
<td>4.38</td>
</tr>
<tr>
<td>1</td>
<td>-3.8</td>
<td>-3.35*</td>
<td>-3.7</td>
</tr>
<tr>
<td>2</td>
<td>-4.17</td>
<td>-3.23</td>
<td>-3.84*</td>
</tr>
<tr>
<td>3</td>
<td>-4.04</td>
<td>-2.71</td>
<td>-3.58</td>
</tr>
<tr>
<td>4</td>
<td>-4.09</td>
<td>-2.36</td>
<td>-3.49</td>
</tr>
<tr>
<td>5</td>
<td>4.2*</td>
<td>-2.06</td>
<td>-3.46</td>
</tr>
</tbody>
</table>
Table 3: Residual Serial Correlation LM Tests

<table>
<thead>
<tr>
<th>Lags</th>
<th>LM statistics</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.98</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>4.53</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 4: VAR residual Portmonteau Test for Autocorrelations

<table>
<thead>
<tr>
<th>Lags</th>
<th>Q-statistic</th>
<th>Adj. Q-statistics</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.7</td>
<td>16.17</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>20.17**</td>
<td>20.91**</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>31.04**</td>
<td>32.79**</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>41.22**</td>
<td>44.28**</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>46.05</td>
<td>49.92*</td>
<td>36</td>
</tr>
</tbody>
</table>

Significant at *** 1%, ** 5%, and * 10%

Table 5: Unrestricted Cointegration Rank Test

<table>
<thead>
<tr>
<th>Hypot. # of CE(S)</th>
<th>Eigenvalue</th>
<th>Trace Stat.</th>
<th>0.05 Crit. Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None*</td>
<td>0.46</td>
<td>35.06**</td>
<td>29.79</td>
<td>0.011</td>
</tr>
<tr>
<td>At most 1*</td>
<td>0.29</td>
<td>12.97*</td>
<td>15.49</td>
<td>0.115</td>
</tr>
<tr>
<td>At most 2</td>
<td>0.0255</td>
<td>0.905</td>
<td>3.84</td>
<td>0.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hypot. # of CE(S)</th>
<th>Eigenvalue</th>
<th>Lambda Max.</th>
<th>0.05 Crit. Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None*</td>
<td>0.46</td>
<td>22.08</td>
<td>21.13</td>
<td>0.037</td>
</tr>
<tr>
<td>At most 1*</td>
<td>0.29</td>
<td>12.07</td>
<td>14.26</td>
<td>0.108</td>
</tr>
<tr>
<td>At most 2*</td>
<td>0.0255</td>
<td>0.905</td>
<td>3.84</td>
<td>0.340</td>
</tr>
</tbody>
</table>
Table 6: Vector Error Correction Estimates with one Cointegration Relation

<table>
<thead>
<tr>
<th>Cointegration Restrictions: B(1,1) = -1, A(2,1) = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR test for binding restrictions (rank=1)</td>
</tr>
<tr>
<td>Chi-square (1) = 0.419</td>
</tr>
</tbody>
</table>

Cointegrating Equation  | Cointegration Equation 1 |
------------------------|--------------------------|
GDP(-1)                 | 1.0                      |
                        | (0.000)                  |
Exchange Rate (-1)      | 0.19**                   |
                        | (10.5)                   |
Oil Price(-1)           | 0.17*                    |
                        | (2.52)                   |
Constant                | 11.3                     |

Error Correction:       | Cointegration Equation 1 |
------------------------|--------------------------|
D(GDP)                  | 0.18                     |
                        | (0.06)                   |
                        | [2.86]                   |
D(Exchange Rate)        | 0.000                    |
                        | (0.000)                  |
                        | [NA]                     |
D(Oil Price)            | -0.8                     |
                        | (0.28)                   |
                        | [-2.87]                  |
Chapter Three: Quality of Institution and Composition of Government Expenditure

June 2011

Abstract

This paper investigates the effect of redistributive politics on the composition of government expenditure. By applying a dynamic and complete information game, it is shown that quality of institution is a key determinant of how much of government revenue is spent on public goods. The main finding of this paper is that in democracies with high quality of institution, less transfer is demanded through election but when the quality of institutions is low, voters prefer more direct transfer despite the high positive externality of public goods on production (and income). This model can explain why in some occasions, majority of the population vote for a populist candidate who promises extreme redistributive policies at the expense of other policies which are harmful for economic growth in the long-run.

*I have done this research under the supervision of professor Micael Castanheira. I am indebted to him for his intellectual and insightful suggestion. Professor Giovanni Facchini kindly reviewed the paper and gave me detail comments for which I am thankful.
1 Introduction

Different historical cases have shown that populism is harmful for economic growth at least in the long run. Populists generally choose policies which seem attractive in the first look and beneficial in the short time but are shown to be harmful in the long-run (Dornbush and Edwards 1991a, 1991b). Then, a question arises and that’s why rational voters may vote for a populist candidate? A simple answer could be that voters are myopic or do not have enough knowledge about the effects of different policies and hence can not discern good policies and candidates from the others. But, economists are reluctant to accept exception to their universal assumption about rationality, perfect information and farsightedness. This paper tries to rationalize voting for populism through a game of redistribution. The model developed in this paper shows the conflict between rich and poor over tax rate and government budget. Each part reveals his/her preference through voting for different candidates who promise to implement policies according to voter’s preferences.

The main contribution of this paper is that rivalry between poor and rich goes beyond determining tax rate and affects the composition of government expenditure. There is a large literature about redistribution and dozens of models try to analyze the consequence of redistributive concerns on voting and selecting among different economic policies in general and tax rate in particular. The tax rate is the most common tool for redistribution in modern economies (let alone other forms such as expropriation and so on). The existing models shows how inequality and/or inequality aversion leads to a tax rate different from the one that is optimal based on pure efficiency point of view. In other words, the main concern of current research is the size of redistribution and because government is the only legitimate entity that uses his legitimate power for taxation and allocation of his resources, the size of the government budget is at the
heart of political economy research. The point of departure of this paper from the existing literature is its move beyond the size of government expenditure and to investigate how redistributive concern can affect the composition of these expenditures. In this paper, I will show that voters’ perception of the quality of institutions, hand in hand with a concern for redistribution, can affect the composition of government expenditure. This theoretical paper shows that the better are institutions, the less transfer are demanded by voters and the more public goods are provided by government. In my view, this explains the nature of populism. Despite the fact that there is no consensus over the definition of populism, extreme redistribution which is proven to be harmful for growth is promised by populist and such promise is welcome by majority of voters in some occasions (like Iran, Venezuela, Peru in contemporary time).

2 Literature review

The models by Roemer (1975) and Meltzer and Richard (1981) are widely recognized as the main workhorse models for preferences for redistribution. Although both models are static and very simple, they show how the tax rate determination is at the core conflict between affluent individuals and needy persons. The type of regime determines who has the upper hand in the tax rate determination and the direction of redistribution. In non democratic regimes which were the most prevalent type of political structure in the past, poor are taxed to finance the government who were either an instrument of the rich (as marxists claim) or have common interests with them. Democracies happened to reverse this flow of income redistribution by giving the right to vote to all. As the majority of voters are not rich, they can exploit their voting power to extract as much as possible from the richer groups of society. It was Acemoglu and Robinson (2006) who for the first time interpret democratization as a process of con-
ict over redistribution. In the past, democratization was known as a process
in which liberty was the main issue, but Acemoglu and Robinson (2000 and
2001) provide evidence that the extension of the franchise and of voting rights
was demanded for redistributive concerns. Their theoretical model is based on
Meltzer and Richard (1981) and by the use of a dynamic game between elites
and citizens, they specify conditions under which democratization, revolution,
repression will happen.

There is another strand of literature on inequality and growth that has orig-
ninated from the work of Persson and Tabelini (1994) and Alesina and Rodrick
(1994). They show that severe inequality leads to more redistribution and higher
taxation of capital which in turn can lead to lower growth rate. They provide
weak empirical support for their claim but more recent empirical work on this
matter did not support their result at all (see for example, Perotti (1996) and
Benabou (1996)). The main contribution and the point of departure of this
paper from the existing literature on inequality and growth is on the extent to
which redistributive concerns affect fiscal policy. While existing research shows
how inequality can influence the tax rate and accordingly the size of the gov-
ernment revenues, this paper claim that this can affect also the composition of
government expenditure. So the extent of inequality not only determines the
size of the pie of the government budget, but also changes the composition of
the government spending in terms of transfers and provision of public goods.
Moreover, the paper by Persson and Tabellini (1994) shows that inequality leads
to less capital formation which in turn leads to lower growth rate (for a simple il-
lustration of this model, see Person and Tabellini (2000)). But this paper shows
another channel through which redistributive concerns harm growth, namely a
reduction in the provision of public goods.

This paper is not the first contribution taking into account the effects of
political economy consideration on the composition of government spending.
Recently Moslehi and Creedy (2009, 2010) have published several papers to analyze the political economy determinants of the composition of government expenditure. In the empirical part of their papers, they restricted themselves to the best practice countries in terms of quality of institution and draw their inference accordingly but in this paper, imperfectness of institution is at the heart of an analysis and it is the pivotal factor that determines the ratio of transfer and public goods to the whole government budget.

Another line of research that is close in many of concepts and problem framing is the recent work on populism. Although it is hard to define what exactly populism means and how to differentiate a populist leader from a non-populist one, it is almost true that redistribution is an ever present issue in all promises that populist give to voters and general public. The emphasise of populists on direct transfers comes at the expense of higher tax rates and lower provision of public goods. The early paper by Dornbush and Edwards (1991a) and the subsequent book edited by them (1991b) are the main non technical contribution on populism, in which a description of different populist experiences in Latin America provides common character of populist leaders. Recently, Acemoglu, Egorov and Sonin (2010) have built a theoretical model in which re-election concerns can motivate rivals to choose policies to the left of their bliss point when there is a possibility that politician grab part of government revenue as rent and can receive bribe from the rich segment of society. They describe this kind of behavior as populism.
3 Model

3.1 Basic Setup

In order to capture the redistributive nature of politics, I assume that society consists of only two groups: rich and poor. I assume that poor have nothing else but their labour force to supply and accordingly they receive a wage when they are employed. We assume that the unemployment rate is zero and all poor are employed. In contrast, rich individuals are endowed with capital and based on their endowment they will earn a return on capital. Labor and capital join together to produce output. Production function is assumed to be Cobb-Douglas with constant returns to scale. This hypothesis implies imperfect substitution between capital and labour and perfect competition in input and product market. I normalize the size of the population to 1 so that the poor and rich constitute $n$ and $1 - n$ respectively and the poor have the majority.

One crucial assumption in this paper is that the government provision of the public goods in form of infrastructure (if provided by government) has a positive externality on production. This is captured by $A(G)$ and the production function takes the following form:

$$Y = F(G, K, n) = A(G)K^n n^{1-\alpha}$$

I should mention that this is not an arbitrary assumption. The role of infrastructure in growth is widely acknowledged in the growth literature and this form of modeling is suggested by Barro and Sala-i Martin (1995, Section 4.4). They consider public goods (infrastructures) as an element similar to human capital. As human capital can increase the productivity of the labor force, infrastructure enhances the productivity of capital. Since we intend a special form of public good which is infrastructure, then we do not consider public goods as a consumption goods to be included in utility function.
A can be interpreted as Total Factor Productivity or Technology. $K$ stands for total capital as $L$ represents total employment. The assumption of constant returns to scale enables us to determine wage and return to capital directly from the production function. In a competitive market, labour force and capital owners receive their marginal product as wage and return to capital. Hence,

$$w = \frac{dF}{dn} = (1 - \alpha) A (K/n)^\alpha$$

$$r = \frac{dF}{dK} = \alpha A (K/n)^{\alpha - 1}$$

Thus, the share of output accrues to the labor force and capital owners are $1 - \alpha$ and $\alpha$ respectively.

$$\frac{w}{n} = (1 - \alpha) Y$$
$$\frac{rK}{W} = \frac{dF}{dK} = \alpha Y$$

Note that $\alpha$ is a measure of inequality. If one divides return of capital by the return to labor, will see that the result depends on $\alpha$. The higher is $\alpha$, the greater is inequality.

$$\frac{rK}{w} = \frac{\alpha Y}{(1 - \alpha) Y} = \frac{\alpha}{(1 - \alpha)}$$

To emphasize the redistribution, I assume that only rich pay the taxes and all poor are exempt from tax payment. In the basic model by Roemer (1975) and Meltzer and Richard (1981) and in the subsequent models by Acemoglu and Robinson (2006), both poor and rich groups pay the tax but as both receive a lump sum transfer, the poor are better off and the rich are worse off since poor pay less than what they receive as transfer and rich receive less. Naturally, poor prefer a higher tax rate and rich prefer a lower one. In this model, the conflict between rich and poor is similar but not exactly the same. In this setup, only the rich pay the tax and in a democratic regime, only poor receive the transfer.
which means that I ruled out the possibility of redistribution from poor to rich in both democratic and non democratic political system. This simplification is not realistic but makes our analysis extremely simple. As a result, part of the return to capital is paid for tax and the rest is accrued to rich for their consumption.

\[ rK = \alpha Y (1 - t) + \alpha Y^t \]

Another feature of this model is that the political system has a direct redistributive consequence. In a democracy, the poor have the majority and can force their redistributive will upon the rich who are a minority. This redistributive will is reflected in the form of both tax rate and direct transfer determination. But when the political system is not a democracy, the poor are deprived of this right. In non democratic regime, minority elite who are rich or those whose interest are in line with rich, set the tax rate and for sure refuse redistribution. This type of modelling and analysis is inspired to a large extent by Acemoglu and Robinson (2006) who describe the democratization process as a redistributive conflict between a minority of rich and majority of poor. In other words, in the redistribution conflict, the poor have an upper hand in a democracy but it is the rich who have an upper hand in non democracies. In the context of current model, this superiority is also translated into first move of the game by poor.

Now, a famous question in political economy arises: "If in a democracy the poor have an upper hand in the redistribution conflict, why do they not expropriate all the wealth of the rich?". One answer points to the economic impossibility of imposing high tax rate due to extreme disincentive effect. This stylized fact is represented in the laffer curve and in the existence of an optimal taxation. The rationale behind the laffer curve is the cost of taxation which increases at a higher pace when the tax rate increases, and this results in a point
where the cost of taxation exceeds its benefit. But it is not the only answer.
A political economy answer is that the poor cannot impose an excessively high
tax rate, since rich individuals are wealthy enough to form an interest group
and lobby to receive a tax exemption or convince the government not to accept
a high tax rate (Rodriquez, 2004). This political economy cocept is considered
in my model in another way. Although the poor set a tax rate and direct
transfer in democracy but the rich can start rent seeking and gain something
that they lose through taxation. Rent seeking is a costly activity or a business
that only rich can enter in it. By paying the cost of rent seeking, they acquire
some amount of rent. Thus the net benefit of rent seeking is determined by the
difference between the amount of rent acquired and the cost of rent seeking. It
is clear that the quality of institution is a major determinant of the cost of rent
seeking. In countries where the quality of institutions is high, rent seeking is so
costly that it is not worth pursuing, but in countries where institutions are not
of high quality, the cost of rent seeking is low, and rent seeking is a lucrative
business. In this model, the quality of institution is shown by $h$. A higher $h$
shows a better institution or investment friendly institutions but low $h$ indicates
low quality and enviroment which is not welcome investment and competition
but encourage corruption and rent seeking. The rationale to separate rent from
public goods is that they are in fact two different issues but they appear under
one title in the government budget "Public good". From Tullock (1967) we
know that often rent seeking activities are carried out under the guise of public
good provision. Tullock says that whenever a project like building a road is
started, one should think of who will built this road to understand why is this
road built. I do not want to be that much pessimistic about the nature of public
goods but let us assume that part of what is provided as "public Good" in fact is
not a real public good and will not produce a positive externality on production.
There are a lot of examples of abandoned airports and many useless roads that
have been financed in developing countries. For this reason, I explicitly made a
distinction between a real public good and a real rent.

Based on what has been described, we can claim that the income of poor
consists of wage \( (w) \) and direct transfer from government \((T)\) and income of rich consist of return to capital (after taxation) \( rK(1-t)\) plus net rent acquired through rent seeking \((NR)\). As we assume the price of the only good produced in this economy to be 1, utility of agents derived from the consumption of the only good available in economy is linearized to 1. Thus, utility maximization translates to incomes maximization:

\[
y^p = w + T
\]

\[
y^r = rK(1-t) + NR
\]

With these explanations, it becomes clear that government spending consists
of three parts: direct transfer to poor, rent grabbed by the rich and finally public good expenditure. This formulation of government spending is similar to Persson and Tabellini (2000) with one exception. In their model, rent was grabbed by politician but here, politicians are absent from the analysis and the main conflict is between poor and rich.

\[
B = T + R + G
\]

In the above formula, \( B \) represents government budget, \( T, R \) and \( G \) represent direct transfer, rent and public goods. In the remainder of the paper, the ratio of these three elements to the total budget will be used and will be denoted by \( \varphi, \rho, 1 - \varphi - \rho \). Thus we have the following

\[
\varphi = \frac{T}{B}
\]

\[
\rho = \frac{R}{B}
\]
\[ 1 - \rho - \varphi = \frac{G}{B} \]

On the revenue side, government receives tax. Thus, the size of the budget government is determined through following formula in which \( C(t) \) represents cost of taxation and by definition should be a convex function of tax rate. In order to deter government from grabbing all revenue of people as tax, I imposed cost of taxation.

\[ B = rK(t - C(t)) \]

Another assumption is that poor and rich had solved collective action problems among themselves and can act in a unified and united manner. So, we will consider the poor and the rich as if there are only two actors in this political game. So the income of the poor as whole is as follows:

\[ y^p = w.n + T = (1 - \alpha) Y + \varphi B \]

Similarly, the income of the rich is as follows:

\[ y^r = rK(1 - t) + NR = [\alpha (1 - t) Y + R - H (\rho)] = [\alpha (1 - t) Y + \rho B - H (\rho)] \]

In the above equation, \( H (\rho) \) represents cost of rent seeking.

### 3.2 One Period Model (Exogenous Government Revenue)

Before describing the model, let us start with a simple model to grasp the intuition behind the complete model. Having understood the intuition in a simple but unrealistic models facilitates understanding a two period model. In this hypothetical model, we assume the budget of the government as given and we do not model the revenue sources. Thus in this section I concentrate on redistribution of the cake (budget) rather than on the size. So, the size of the budget of government is fixed \( (B) \).
3.2.1 Dictatorship

Now, we can define the timing of the game in a dictatorship as a benchmark to democracy. Non-democracy by definition is a political system in which a minority elite can impose its will on the whole population unilaterally. The timing of events is as follows:

1. The rich decides how much to allocate to the poor as direct transfer.
2. The rich decides whether to do rent seeking and if yes, how much.
3. The government provides public good with the remaining money in his budget.
4. Production takes place.
5. All agents receive their pay off.

The rich maximize their income.

\[
\max_{y^r} = rK(1-t)+NR = A(G)K^\alpha n^{1-\alpha} + NR = A \left[ (1 - \varphi - \rho)B \right] K^\alpha n^{1-\alpha} + \rho B - H(\rho)
\]

In non democracy, rich are dominant over poor and do not have any incentive to give any transfer to them through government budget. Thus,

\[
\varphi = 0
\]

From this first order condition, we have

\[
A' \left[ (1 - \rho)B \right] K^\alpha n^{1-\alpha} = B - H'(\rho)
\]

This says that the marginal benefit of doing less rent seeking should equal marginal cost. In other words, if the positive externality of having public goods is high enough, then the rich prefer to do less rent seeking and let the money remain in the government budget in order to be spent on public good (i.e. infrastructure).
3.2.2 Democracy

In a democracy the poor make the decisions and they move first. This fact reflected into different timing of the game as follows:

1. Poor decide how much to receive as a direct transfer to themselves.
2. Then rich decide how much to grab as a rent.
3. what remain in the budget of government is allocated to the provision of public good.
4. production takes place.
5. Each player receives his own pay off.

This is a dynamic complete information game and should be solved by backward induction which means that first rich determine their level of rent seeking given a specific amount of direct transfer. Then poor, decides how much to receive as a transfer. Thus we should start from optimal decision making by the rich.

The rich maximize their income.

\[
\max_{\rho} y^r = \left[ \alpha (1 - t) Y + \rho \bar{B} - H (\rho) \right] \\
= \left[ \alpha (1 - t) A (G) K^{\alpha} n^{1-\alpha} + \rho \bar{B} - H (\rho) \right] \\
= \left[ \alpha (1 - t) A \left[ (1 - \rho (\varphi) - \varphi) \bar{B} \right] \cdot K^{\alpha} n^{1-\alpha} + \rho \bar{B} - H (\rho) \right]
\]

First order condition is determined through differentiating wrt \( \rho \):

\[
-\alpha (1 - t) \dot{A}_G' \cdot B K^{\alpha} n^{1-\alpha} + \dot{B} - H' (\rho) = 0 \text{(reaction function of rich)}
\]

Again, this means that the rich compare cost and benefit of rent seeking in order to determine the level of their investment in rent seeking. They grab as much as they can up to a point where more rent seeking is not beneficial. At this point, they prefer more public goods is provided since they will also benefit from more public good as it increases their return to capital. This equation
represents reaction function of the rich. In order to have a specific solution, we need to specify a cost function for rent seeking and a functional form for the positive externality of the public good. I assume that $H(\rho) = \frac{hB\rho^2}{2}$ is a cost function for rent seeking. It is convex and increases at a higher pace when the rich want to grab all the budget as a rent. Thus, this functional form deters rich from claiming all the government budget. In this formula, $h$ is a variable that stands for quality of institution. Moreover, I assume a logarithmic form for the externality of infrastructure (i.e. public goods) on production ($A(G) = \log(G)$). The reason for choosing this specific functional form is the possibility of reaching a clear analytical solution. Otherwise no analytical solution will be found and all equations should be solved numerically.

Since $H(\rho) = \frac{hB\rho^2}{2}$ and $A(G) = \log(G)$ then $H(\rho) = hB\rho$ and $A'(\rho, \varphi) = \frac{1}{\sigma} = \frac{1}{(1-\rho-\varphi)B}$. By plugging these two in the reaction function of rich, we will have:

$$(1 - \rho - \varphi)(1 - h\rho) = \frac{\alpha(1-t)K^\alpha n^{1-\alpha}}{B}$$

This is an equation of second order and one can compute $\rho$ in terms of $\varphi$ and $h$. Like all equation of order two, there are two solutions but one of them is not acceptable since $\rho$ can not be negative. As a result, we focus on

$$\rho = \frac{((1-\varphi)h+1) + \sqrt{((\varphi-1)h+1)^2 + 4h}\frac{\alpha(1-t)K^\alpha n^{1-\alpha}}{B}}{2h}$$

Moreover, it can be shown that $-1 < \frac{\partial \rho}{\partial \varphi} < 0$ (see appendix 1 for proof) which shows interaction between two players (rich and poor). As it was expected, $\frac{\partial \rho}{\partial n} < 0$ (see appendix 2 for the proof) which means that as the quality of the institutions increases, the level of rent seeking decreases. Now, it is the turn of the poor to decide about $\varphi$. The poor maximize their income by setting $\varphi$, but they take into account the effect of their decision on the decision of rich.

$$\max_{\varphi} y^p = w + T = (1 - \alpha)Y + \varphi B = (1 - \alpha) A [(1 - \rho(\varphi) - \varphi) ] K^\alpha n^{-\alpha} + \varphi B$$
Thus, first order condition with respect to \( \varphi \) yields:

\[
(1 - \alpha) \cdot A_G'(\rho, \varphi) \cdot (-\rho' - 1) \cdot \bar{B} \cdot K^{\alpha n^{1-\alpha}} + \bar{B} = 0
\]

\[
\frac{\rho'(\varphi) + 1}{1 - \rho - \varphi} = \frac{\bar{B}}{(1 - \alpha) \cdot K^{\alpha n^{1-\alpha}}} \quad \text{(reaction function of poor)}
\]

This equation is the reaction function of poor. By inserting \( \varphi \) and \( \frac{d\rho}{d\varphi} \) from the reaction function of rich, we will have then

\[
\varphi = 1 + \left(1 \pm \sqrt{\frac{h}{(1 - \alpha) \cdot K^{\alpha n^{1-\alpha}}}} \right)^2 - 4h \cdot \frac{\alpha (1-t) K^{\alpha n^{1-\alpha}}}{\bar{B}}
\]

This shows the optimal amount of transfer in terms of the parameters of the model. An interesting comparative static result concerns the behavior of \( \varphi \) with respect to changes in \( h \), i.e. the parameter that captures the quality of the institution. As it is shown in appendix 3,

\[
\frac{d\varphi}{dh} < 0
\]

this suggest that as the quality of the institution improve, the poor prefer less redistribution since they believe that the rich will grab less and more public goods will be provided but when quality of institution is low, poor people prefer high redistribution since they are not sure what they do not acquire will be spent on public goods. A direct consequence of this result is that the variation of public good provision is in line with variation of quality of institution\(^1\).

\[
\frac{\partial G}{\partial h} > 0
\]

Corollary: When there is an election, voters prefer less transfer as the quality of institutions increases and more government budget is allocated to provision of public goods.

\(^1\)we know that the public good is determined according to \( G = (1 - \rho - \varphi)B \) hence \( \frac{\partial G}{\partial h} = -\frac{d\rho}{dh} - \frac{d\varphi}{dh} = -(\frac{d\rho}{dh} + \frac{d\varphi}{dh}) > 0 \), thus \( \frac{\partial G}{\partial h} > 0 \)
I should notice that the way both actors (i.e. rich and poor) play the redistributive game is similar to Stackelberg game in which two firms sequentially decides how much to produce.

3.3 Two Period Model (Endogenous Government Revenue)

In the previous section the amount of government revenue is considered as exogenous and this assumption made the model imperfect. In this section, I relax the assumption of exogenous government revenues and I endogenize it by giving a voting right over the tax rate to voters. Thus, poor people who are in majority should determine both tax rate and the amount of direct transfer. In one period, people decides about the tax rate and then taxes are collected and in the next period government spend collected revenues. Thus, the complete model should have two period but as will be shown soon the result does not change. In this setting, the timing of events is as follows:

1. Poor vote over tax rate and direct transfer
2. Rich decides how much to grab as rent
3. Production takes place (Without any public goods)
4. Government tax the rich
5. Direct transfer and rent grabbing happens.
7. The same timing of event repeats for second period.

The difference between this case with the previous one is on the number of periods. Thus, we should take into account the income in both periods.

\[ Y_p = w_1 + T_1 + w_2 + T_2 \]

\[ Y_r = r_1K + NR_1 + r_2K + NR_2 \]
Moreover, it should be taken into consideration that public goods is provided in the second period and its positive externality appears in the production function in the second period. Thus the production in the first period happens in the absence of public good.

\[ Y_1 = K^\alpha n^{1-\alpha} \]

\[ Y_2 = A(G)K^\alpha n^{1-\alpha} = A((1 - \rho - \varphi)B_1)K^\alpha n^{1-\alpha} \]

Therefore, payoff are defined accordingly as follows:

\[ w_1 = (1 - \alpha)K^\alpha n^{-\alpha} = (1 - \alpha)Y_1 \]

\[ w_2 = (1 - \alpha)A(G_1)K^\alpha n^{-\alpha} = (1 - \alpha)\log(G_1)K^\alpha n^{-\alpha} = (1 - \alpha)Y_2 \]

\[ r_1K = \alpha K^\alpha n^{1-\alpha} = \alpha Y_1 \]

\[ r_2K = \alpha A(G_1)K^\alpha n^{1-\alpha} = \alpha Y_2 \]

The budget of government will be financed through taxation as follows:

\[ B_1 = (t_1 - C(t_1))r_1K = (t_1 - C(t_1))\alpha K^\alpha n^{1-\alpha} = (t_1 - C(t_1))\alpha Y_1 \]

\[ G = (1 - \rho - \varphi)B_1 = (1 - \rho - \varphi)(t_1 - C(t_1))\alpha K^\alpha n^{1-\alpha} \]

\[ B_2 = (t_2 - C(t_2))r_2K = (t_2 - C(t_2))\alpha \log(G)K^\alpha n^{1-\alpha} = (t_2 - C(t_2))\alpha Y_2 \]
The game is a dynamic complete game and as before the method of solving is backward induction. Hence we should start our analysis from the second period. As the world will finish at the end of the second period, there is no incentive for both sides to leave anything to be provided as public goods. Because poor are the first mover, they ask for all the government budget as direct transfer since whatever remain will be grabbed as rent totally. Thus $\varphi_2 = 1, \rho_2 = 0$ and $G_2 = 0$. The value of $t_2$ could be determined by maximizing the income of poor which consist of wage and transfer in two periods.

\[
Y_p = w_1 + T_1 + w_2 + T_2 = (1 - \alpha)K^\alpha n^{-\alpha} + \varphi B_1 + (1 - \alpha)\log(G)K^\alpha n^{-\alpha} + B_2 = (1 - \alpha)Y_1 + \varphi(t_1 - C(t_1))\alpha Y_1 + (1 - \alpha)Y_2 + (t_2 - C(t_2))\alpha Y_2
\]

\[
Y_p = (1 - \alpha)K^\alpha n^{-\alpha} + \varphi(t_1 - C(t_1))\alpha K^\alpha n^{1-\alpha} + (1 - \alpha)\log [(1 - \rho - \varphi)(t_1 - C(t_1))\alpha K^\alpha n^{1-\alpha}] K^\alpha n^{-\alpha} + (t_2 - C(t_2))\alpha \log [(1 - \rho - \varphi)(t_1 - C(t_1))\alpha K^\alpha n^{1-\alpha}] K^\alpha n^{1-\alpha}
\]

This can be done by differentiating with respect to $t_2$ as follows

\[
\frac{dY^p}{dt_2} = 0 \Rightarrow \left(1 - C'(t_2)\right) = 0 \Rightarrow t_2 = C'^{-1}(1)
\]

with a similar reasoning with can find $t_1$

\[
\frac{dY^p}{dt_1} = \varphi \left(1 - C'\right)\alpha K^\alpha n^{1-\alpha} + (1 - \alpha) \left[(1 - \rho - \varphi)\left(1 - C'\right)\alpha K^\alpha n^{1-\alpha} \right] \frac{1}{(t_1 - C(t_1))} K^\alpha n^{-\alpha} + \ldots + (t_2 - C(t_2))\alpha \left[(1 - \rho - \varphi)\left(1 - C'\right) \frac{1}{(t_1 - C(t_1))} K^\alpha n^{1-\alpha} \right] K^\alpha n^{1-\alpha} = 0
\]

\[
\Rightarrow \left(1 - C'(t_1)\right) = 0 \Rightarrow t_1 = C'^{-1}(1)
\]
The equilibrium \( t_1 \) and \( t_2 \) shows the maximum point on the laffer curve. That means that poor decides to increase the size of government revenue (cake) irrespective of the fraction they will receive in the redistributive conflict with rich. Now, the value of \( \varphi_1 \) and \( \rho_1 \) should be determined through the optimal decision making of political agents. First the rich maximize their two periods income.

\[
\max_{\rho} Y^r = r_1 K + NR_1 + r_2 K + NR_2 = \alpha K^{\alpha} n^{1-\alpha} + (\rho B_1 - \frac{h \rho^2}{2} B_1) + \alpha \log(G) K^{\alpha} n^{1-\alpha} + 0
\]

\[
= \alpha K^{\alpha} n^{1-\alpha} + (\rho - \frac{h \rho^2}{2}) B_1 + \log [(1 - \rho - \varphi) B_1] \alpha K^{\alpha} n^{(1-\alpha)}
\]

First order condition by differentiating with respect to \( \rho \) results

\[
\frac{dY^r}{d\rho} = (1 - h \rho) B_1 + \frac{-1}{(1 - \rho - \varphi)} B_1 \alpha K^{\alpha} n^{(1-\alpha)} = 0
\]

and from that we find reaction function of rich which is exactly similar to the one period model

\[
(1 - h \rho)(1 - \rho - \varphi) = \alpha K^{\alpha} n^{(1-\alpha)} \text{reaction function of rich}
\]

Thus we will have the same result as before:

\[
\rho = \frac{((1 - \varphi) h + 1) + \sqrt{((\varphi - 1) h + 1)^2 + 4h \frac{\alpha (1-\varphi) K^{\alpha} n^{1-\alpha}}{B}}}{2h}
\]

\[-1 < \frac{\partial \rho}{\partial \varphi} < 0 \]

\[-1 < \frac{\partial \rho}{\partial h} < 0 \]

Then poor should maximize their income given the value of \( \rho \).

\[
Y^p = (1 - \alpha) K^{\alpha} n^{-\alpha} + \varphi(t_1 - C(t_1)) \alpha K^{\alpha} n^{1-\alpha} + (1 - \alpha) \log [(1 - \rho - \varphi)(t_1 - C(t_1)) \alpha K^{\alpha} n^{1-\alpha}] K^{\alpha} n^{-\alpha} + (t_2 - C(t_2)) \alpha \log [(1 - \rho - \varphi)(t_1 - C(t_1)) \alpha K^{\alpha} n^{1-\alpha}] K^{\alpha} n^{1-\alpha}
\]
By differentiating with respect to $\varphi$ we will have the following equation

$$
\frac{dY^p}{d\varphi} = (t_1 - C(t_1))\alpha Y_1 + [(1 - \alpha) + (t_2 - C(t_2))\alpha] \frac{dY^c}{d\varphi} \quad \ldots
$$

$$
= (t_1 - C(t_1))\alpha Y_1 + [(1 - \alpha) + (t_2 - C(t_2))\alpha] \frac{-(1 - \rho')}{(1 - \rho - \varphi)} \quad \ldots
$$

$$
\ldots \quad (t_1 - C(t_1))\alpha K^\alpha n^{1-\alpha} Y_1 = 0
$$

$$
\Rightarrow \frac{1 + \rho'}{(1 - \rho - \varphi)} = \frac{1}{[(1 - \alpha) + (t - C(t))\alpha] K^\alpha n^{1-\alpha}}
$$

The resulting equation is a reaction function of poor that is again similar to the previous equation and we can have the same conclusions:

$$
\frac{\partial \rho}{\partial h} < 0, \quad \frac{d\varphi}{dh} < 0, \quad \frac{\partial G}{\partial h} > 0
$$

Again, it shows that when the quality of institution is high, the rich will invest lower amount in rent seeking behavior and the poor who are majority of voters prefer less direct transfer of money from government and the government invest more in public goods.

It is worth noting that if I follow the common way of modelling by accepting taxation from poor, the result of redistributive conflict over government expenditure will remain as before but the value of optimal tax will change and the model would become unnecessary more complicated.
4 Conclusion

This paper analyzes populism in the context of a redistributive conflict between poor and rich and describes the consequence of such conflict on the composition of government. It shows that the quality of the institutions is a crucial factor in determining how much transfer is allocated to poor and how much public goods is provided by government. This paper shows that inequality not only affects the amount of tax rate but also the composition of government expenditure. A natural prediction of the model is as follows: when the quality of the institutions is high, greater part of government expenditure should be allocated to the provision of public goods but when the quality of institution is low, then transfer will constitute a major part of government budget. This claim should be evaluated empirically in the future researches.
References


5 Appendix 1

\[ 1 - \varphi + (\varphi h - h - 1)\rho + h\rho^2 = \frac{\alpha L^{1-\alpha} K^{\alpha} (1 - t)}{B} = \zeta > 0 \Rightarrow h\rho^2 + ((\varphi - 1)h - 1)\rho + 1 - \varphi - \zeta = 0 \]

\[ [2h\rho + ((\varphi - 1)h - 1)] \partial \rho + [\rho h - 1] \partial \varphi = 0 \Rightarrow \frac{\partial \rho}{\partial \varphi} = -\frac{1 - \rho h}{1 - \rho h + h(1 - \varphi - \rho)} < 0 \quad (1) \]

\[ \rho = \frac{((1 - \varphi)h + 1) + \sqrt{((\varphi - 1)h + 1)^2 + 4h\zeta}}{2h} \Rightarrow \frac{\partial \rho}{\partial \varphi} = \frac{1}{2} \left[ -1 + \frac{((1 - \varphi)h + 1)}{\sqrt{((\varphi - 1)h + 1)^2 + 4h\zeta}} \right] < 0 \quad (2) \]
\section{Appendix 2}

\[ \rho^1, \rho^2 = \frac{(1 - \varphi)h + 1 \pm \sqrt{(\varphi - 1)h - 1} + 4h \zeta + 4h(\varphi - 1)}{2h} = \frac{(1 - \varphi)h + 1 \pm \sqrt{(\varphi - 1)h + 1} + 4h \zeta}{2h} \]  

(3)

for positive sign \[ \rho^1 = \frac{(1 - \varphi)h + 1 + \sqrt{(\varphi - 1)h + 1} + 4h \zeta}{2h} \]  

(4)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ (1 - \varphi)h + \frac{2((\varphi - 1)h + 1)(\varphi - 1)h + 4h \zeta}{2\sqrt{(\varphi - 1)h + 1} + 4h \zeta} - ((1 - \varphi)h + 1) - \sqrt{(\varphi - 1)h + 1} + 4h \zeta \right] = \]  

(5)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ \frac{(1 - (1 - \varphi)h)(\varphi - 1)h + 2h \zeta}{\sqrt{(\varphi - 1)h + 1} + 4h \zeta} - \sqrt{(\varphi - 1)h + 1} + 4h \zeta - 1 \right] = \]  

(6)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ \frac{(1 - (1 - \varphi)h)(\varphi - 1)h + 2h \zeta}{\sqrt{(\varphi - 1)h + 1} + 4h \zeta} - 1 \right] = \]  

(7)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ \frac{(-1 - (1 - \varphi)h) - 2h \zeta}{\sqrt{(\varphi - 1)h + 1} + 4h \zeta} - 1 \right] = \]  

(8)

\[ = \frac{1}{2h^2 \sqrt{(\varphi - 1)h + 1} + 4h \zeta} \left[ -(1 - (1 - \varphi)h) - 2h \zeta - \sqrt{(\varphi - 1)h + 1} + 4h \zeta \right] \]  

(9)

\[ = \frac{1}{2h^2 \sqrt{(\varphi - 1)h + 1} + 4h \zeta} \left[ -(1 + (1 - \varphi)h) - 2h \zeta - \sqrt{(\varphi - 1)h + 1} + 4h \zeta \right] \]  

(10)

\[ = \frac{1}{2h^2 \sqrt{(\varphi - 1)h + 1} + 4h \zeta} \left[ -(1 + (1 - \varphi)h) - 2h \zeta - \sqrt{(-\varphi - 1)h - 1} + 4h \zeta \right] \]  

(11)

\[ = \frac{1}{2h^2 \sqrt{(\varphi - 1)h + 1} + 4h \zeta} \left[ -(1 + (1 - \varphi)h) - 2h \zeta - \sqrt{(-\varphi - 1)h - 1} + 4h \zeta \right] < 0 \]  

(12)

since \[ -1 + (1 - \varphi)h < \sqrt{-1 + (1 - \varphi)h} + 2h \zeta \]  

(13)

for negative sign \[ \rho^2 = \frac{(1 - \varphi)h + 1 - \sqrt{(\varphi - 1)h + 1} + 4h \zeta}{2h} \]  

(14)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ (1 - \varphi)h - \frac{2((\varphi - 1)h + 1)(\varphi - 1)h + 4h \zeta}{2\sqrt{(\varphi - 1)h + 1} + 4h \zeta} - ((1 - \varphi)h + 1) + \sqrt{(\varphi - 1)h + 1} + 4h \zeta \right] = \]  

(15)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ \frac{(1 - (1 - \varphi)h)(\varphi - 1)h + 2h \zeta}{\sqrt{(\varphi - 1)h + 1} + 4h \zeta} + \sqrt{(\varphi - 1)h + 1} + 4h \zeta - 1 \right] = \]  

(16)

\[ \frac{\partial \rho}{\partial h} = \frac{1}{2h^2} \left[ \frac{-(1 - (1 - \varphi)h)(\varphi - 1)h - 2h \zeta + ((\varphi - 1)h + 1) + 4h \zeta}{\sqrt{(\varphi - 1)h + 1} + 4h \zeta} - 1 \right] \]  

(17)

\[ \frac{\partial \rho}{\partial h} = 1 \left[ (1 - (1 - \varphi)h) + 2h \zeta \right] \]  

(18)
7 Appendix 3

\[
\frac{\rho'(\varphi) + 1}{1 - (\rho + \varphi')} = \frac{\bar{B}}{(1 - \alpha) \cdot K^\alpha n^{1-\alpha}} = L
\]

\[
\Rightarrow \frac{1 - \frac{1 - \rho h}{1 - (\rho + \varphi')}}{1 - (\rho + \varphi')} = \frac{\frac{h(1 - \varphi - \rho)}{1 - \rho h + h(1 - \varphi - \rho)}}{1 - (\rho + \varphi')} = \frac{h}{1 - \rho h + h(1 - \varphi - \rho)} = L
\]

\[
\Rightarrow \frac{h}{1 + h - h\varphi - ((1 - \varphi)h + 1) \pm \sqrt{(\varphi - 1)h + 1 + 4h\zeta}} = L
\]

\[
\Rightarrow 1 + h - h\varphi - \left[((1 - \varphi)h + 1) \pm \sqrt{(\varphi - 1)h + 1 + 4h\zeta}\right] - \frac{h}{L} = 0
\]

\[
\Rightarrow \mp \sqrt{(\varphi - 1)h + 1 + 4h\zeta} - \frac{h}{L} = 0
\]

\[
((\varphi - 1)h + 1) + 4h\zeta = \left(\frac{h}{L}\right)^2
\]

\[
\Rightarrow (\varphi - 1)h + 1 = \pm \sqrt{\left(\frac{h}{L}\right)^2 - 4h\zeta}
\]

\[
\Rightarrow \varphi = 1 + \frac{-1 \pm \sqrt{\left(\frac{h}{L}\right)^2 - 4h\zeta}}{h}
\]

Only positive sign is acceptable since \(\varphi\) can not be negative. Thus,

\[
\frac{d\varphi}{dh} = \frac{-1}{h^2} \left[1 + \frac{\left(\frac{h}{L}\right)^2 - 2h\zeta}{\sqrt{\left(\frac{h}{L}\right)^2 - 4h\zeta}} - \sqrt{\left(\frac{h}{L}\right)^2 - 4h\zeta}\right] = \frac{-1}{h^2} \left[1 + \frac{2h\zeta}{\sqrt{\left(\frac{h}{L}\right)^2 - 4h\zeta}}\right] < 0
\]