European Journal of Physics

PAPER • OPEN ACCESS

A Cartesian diver to study oscillations and internal gravity waves in a stratified fluid

To cite this article: Marina Carpineti *et al* 2024 *Eur. J. Phys.* **45** 045803

View the article online for updates and enhancements.

You may also like

- Dose assessment following an overexposure of a worker at a Swiss nuclear power plant
Claude J Bailat, Jean-Pascal Laedermann, Sébastien Baechler et al.

- Hollow-microsphere composite offers depth-independent superior thermal insulation for diver suits
Jonathan Brown, John Oldenkamp, Raymond Gamache et al.

- Investigations into the diver method of particle size analysis
B H Kaye and G W James

This content was downloaded from IP address 151.40.24.254 on 21/03/2025 at 20:47

OPEN ACCESS
IOP Publishing

European Journal of Physics

Eur. J. Phys. **45** (2024) 045803 (18pp)

https://doi.org/10.1088/1361-6404/ad57a2

A Cartesian diver to study oscillations and internal gravity waves in a stratified fluid

Marina Carpineti[1], Irene Spongano[1], Fabrizio Croccolo[2] and Alberto Vailati[1]

[1] Dipartimento di Fisica Aldo Pontremoli, Università degli Studi di Milano, I-20133 Milano, Italy
[2] Universite de Pau et des Pays de l'Adour, E2S UPPA, CNRS, LFCR UMR5150, Anglet, France

E-mail: marina.carpineti@unimi.it

Received 28 February 2024, revised 29 May 2024
Accepted for publication 12 June 2024
Published 8 July 2024

Abstract

We propose a variation of the well-known Cartesian diver experiment where, instead of moving in a uniform fluid, the diver floats in a fluid stratified in density. In contrast to the original experiment, for a given external pressure the diver can stop in a stable equilibrium position within the fluid, at the depth where the surrounding density matches its own. By varying the applied pressure, the density of the diver changes and it moves until it reaches a new stable equilibrium condition at a different depth. When a sudden pressure pulse is applied, the diver, pushed off its equilibrium position, starts oscillating due to a restoring force that depends on the density gradient. The oscillations produce internal gravity waves that are typical of stratified fluids, when a portion of them is displaced and transmits its motion to the surrounding fluid. Although they are extremely difficult to observe, gravity waves are particularly interesting, as they typically occur in the atmosphere and in the stars. We propose a simple experiment and suggest a way to make the internal gravity waves visible. The experiment can be realized by students with easy-to-find household objects and used to improve their understanding of many concepts and laws of hydrodynamics, but also to introduce them to complex phenomena of general interest.

Keywords: pressure, fluids, hands-on experiment, Cartesian diver

Original content from this work may be used under the terms of the Creative Commons Attribution 4.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

© 2024 The Author(s). Published on behalf of the European Physical Society by IOP Publishing Ltd

1. Introduction

Students encounter many difficulties when studying simple concepts of hydrostatics. For example, they often mistakenly confuse the concept of pressure with that of force. One consequence is that it is difficult for them to understand Pascal's law, which states that a change in pressure that occurs at one point in an incompressible (or slightly compressible) fluid is transmitted to all points in the fluid [1–3]. Students struggle to connect the global behavior of fluids with a detailed local description. In addition, it is challenging for them to reconcile rules and formulas studied in books with practical examples occurring in fluids [1]. Stevin's law also creates some problems, as students often overlook the role of external pressure and focus their attention only on the variation of pressure with depth. Even concepts such as buoyancy and sinking are rarely included in an organized reasoning in which the role of pressure is clearly identified.

The Cartesian diver is a classical scientific experiment that, often used as a toy, can be proposed to teach students the principles of buoyancy and the properties of fluids in an engaging way [4, 5]. Its main feature is that the diver floating in water is partly filled with air, so that its density can be easily varied through the application of an external pressure on the container. The diver is open to the passage of liquid and can be prepared so that it contains a pocket of air large enough to make it float in water at atmospheric pressure. If the pressure applied to the container is increased, the volume of air inside the diver decreases, providing room for more liquid, so that the diver average density increases causing it to sink (figure 1). Studying the properties of this toy in different educational versions allows introducing some concepts and laws of hydrodynamics such as pressure in fluids, ideal gas law, Archimede's principle, and Stevin and Pascal laws [4–14]. Whatever the pressure applied, the Cartesian diver does not have a stable equilibrium point inside the liquid. On the contrary, it can either float or sink depending on the external pressure.

In this paper we propose an experiment where the Cartesian diver is combined with ideas from the 'magic cork' experiment [15], where a floating/sinking object is immersed in a density stratified liquid, realized through the dissolution of coarse kitchen salt in water.

In the present case, when a constant pressure is applied to the container, the diver sinks until it reaches a depth where its density is matched by the surrounding liquid. For times shorter than the one needed by the density profile to evolve significantly, this is a stable equilibrium position. The presence of a density gradient allows to change at will the equilibrium position of the diver by varying the average pressure on the fluid. Thanks to the same principle, if the system is suddenly perturbed through the application of a pressure pulse, the diver, pushed off equilibrium, moves back to its initial position as soon as the extra pressure is removed. In the case of a small vertical perturbation, the force acting on the diver is proportional to its displacement and causes it to oscillate around the equilibrium point at an angular frequency ω that can be shown to be simply related to the local density gradient $\frac{\partial \rho}{\partial z}$ of the stratified fluid.

This frequency is known as Brunt–Väisälä or buoyancy frequency [16], and is an important concept in atmospheric and oceanic science, where typically density stratification occurs. An interesting consequence of the oscillations is that they perturb the fluid surrounding the diver, and the disturbance propagates in the fluid as a transverse wave. These waves are known as internal gravity waves and are typical of density stratified fluids. They are very difficult to visualize as they only involve variations of the index of refraction.

We propose a series of experiments, designed with the aim of studying the Brunt–Väisälä frequency at different depths inside the fluid and the character of the internal gravity waves

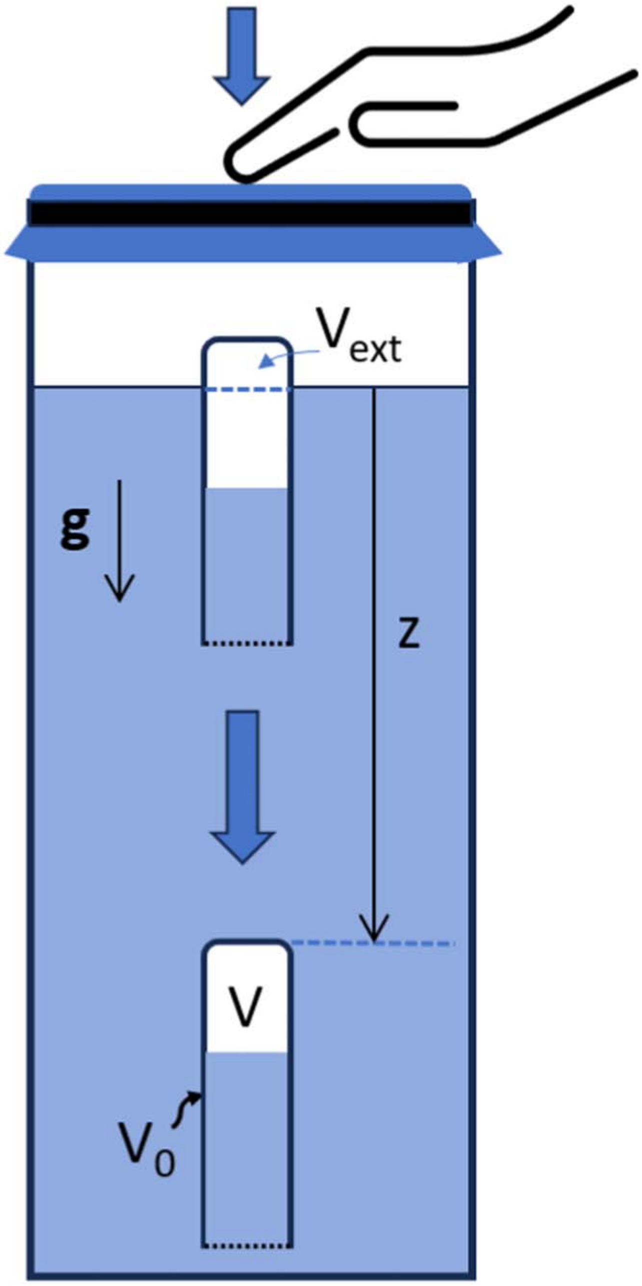

Figure 1. Simplified scheme of a traditional Cartesian diver experiment where the relevant physical parameters are sketched. V_0 is the total volume of the diver, V the volume of air in the diver, and V_{ext} the volume emerging from the surface of the water. The imposition of an external force on an elastic diaphragm placed at the top of the container determines the sinking of the diver.

that are generated by the oscillations of the diver. Students can perform the experiments easily at home or in a laboratory, using everyday objects, and dealing with both practical and conceptual problems that may arise. Indeed, in spite of their simplicity, these experiments shed light on several revealing theoretical concepts in the fields of hydrostatics, the physics of stratified fluids, and wave propagation.

Brunt–Väisälä oscillations and internal gravity waves are extremely interesting phenomena that can be observed, for example, in the atmosphere or in forming stars. Moreover, the stratification of density inside the oceans and the atmosphere strongly affects the motion and dispersion of pollutants and organisms [17]. The investigation of Brunt–Väisälä oscillations is also important for the understanding of the behavior of stratified fluids at the mesoscopic scale, where gravity strongly affects the relaxation of non-equilibrium concentration fluctuations inside the fluid [18], making experiments performed in microgravity highly desirable [19]. Therefore, the simple experiment proposed by us can also be used as an engaging cue to introduce and discuss phenomena of very general relevance with students.

The interest of this research topic is also witnessed by some recently published research papers, where the fundamental aspects of the dynamical behavior of a Cartesian diver in a stratified fluid are studied both experimentally and theoretically [20, 21]. The experimental work by Le Gal *et al* investigates the dynamics of a Cartesian diver forced to oscillate in a stably stratified fluid in a rectangular narrow container [20]. A sinusoidal modulation of the external pressure allows to keep the diver in a regime of forced oscillations around its equilibrium depth. The authors analyze in detail and model the resonant behaviour which occurs when the forcing frequency approaches the Brunt–Väisälä one, a condition that gives rise to a horizontal displacement of the diver. Inspired by the seminal work of Couder and coworkers on the wave-particle duality of droplets bouncing at the free surface of a vibrated fluid bath [22], the authors investigate the possibility of finding hydrodynamical quantum analogs determined by the association between the diver and the internal gravity waves generated by its oscillations. The very recent theoretical work by Voisin [21], provides an accurate modelling of the oscillations of buoyant bodies in stratified fluids, taking into account both the effect of friction and the one of the fluid inertia, which acts as a virtual added mass. Both papers are extremely interesting and report new research results. They can serve as valuable supporting materials when presenting our experiment to graduate and undergraduate students, enabling them to provide a comprehensive description of the phenomena. Conversely, we think that a simplified description where the damping effects are neglected can be more effective in conveying the fundamental concepts about pressure, when the experiment is proposed to high school students.

2. Cartesian diver, principles of operation

The physical principles behind the operation of a Cartesian diver have been dealt with in several publications (for an introduction see [6], while for a more detailed treatise see [7]). Here we recall some basic aspects of the theory that explain how a change of pressure can make the diver either float or sink. A change of pressure in a fluid can be determined either through Pascal's law by imposing an external force, or through Stevin's law by changing the depth of the diver inside the fluid in the presence of gravity. We will describe how the contributions to the pressure change arising from these two fundamental laws of hydrostatics affect the equilibrium condition of a Cartesian diver.

2.1. Conceptual scheme

We consider a Cartesian diver hosted in a sealed container partially filled with water, with a layer of air on top of it (figure 1). Under normal conditions, the air inside the container is at atmospheric pressure p_a, but the pressure can be changed to a different value $p_e = p_a + \Delta p_e$ by pushing on the external walls of the container, for example by imposing an external force to an elastic diaphragm that seals its top. The diver consists of a small open container of total volume V_0 opened at its bottom, which contains a pocket of air of volume V at its top, the remaining part of its internal volume being filled with water of density ρ.

2.2. Floating condition

Initially, the presence of the pocket of air allows the diver to float at the surface of the water, with a volume V_{ext} emerging from it. Neglecting the mass of air inside the diver and calling m_0 the mass of the diver when it is completely filled with water, the modulus of the weight force acting on the diver is $F_w = (m_0 - \rho V)g$, while the modulus of the buoyancy force is $F_b = \rho(V_0 - V_{\text{ext}})g$. Under mechanical equilibrium, the weight force is balanced by the buoyancy force and the floating condition can be written as:

$$m_0 - \rho V = \rho(V_0 - V_{\text{ext}}), \tag{1}$$

where the left term corresponds to the mass of the diver and the right term to the mass of the water displaced by it.

Note that both V_0 and m_0 are specific properties of a particular Cartesian diver that depend on the material and geometry adopted for its preparation.

2.3. Neutral buoyancy condition

The imposition of an external force determines an increase Δp_e of the pressure inside the liquid. According to Pascal's law, the pressure is transmitted uniformly at every point inside the fluid, including the walls of the container, and determines a decrease ΔV of the volume of the air pocket and in turn a decrease $\Delta V_{\text{ext}} = \Delta V$ of the volume of the diver emerging from the water surface. Under the condition

$$\Delta V = -V_{\text{ext}} \tag{2}$$

the diver becomes fully submerged immediately below the surface of the water and a neutral buoyancy equilibrium condition is reached where the mass of the diver matches that of the displaced water:

$$m_0 - \rho V = \rho V_0. \tag{3}$$

Under this condition, the diver is fully supported by the water around it.

2.4. Negative buoyancy condition

If the external pressure is increased further, the volume of the pocket of air becomes smaller and a negative buoyancy condition is reached where the mass of the diver becomes larger than the one of the water displaced by it:

$$m_0 - \rho V > \rho V_0. \tag{4}$$

Under this condition, the diver sinks until it reaches the bottom of the container.

2.5. Critical depth

When the diver has sunk to the bottom of the container and the excess external pressure is released, the volume of the pocket of air increases, but the diver might not rise back to the surface of the water. To understand this peculiar irreversible behavior that might arise in the motion of the diver we need to investigate in detail how the volume V of the pocket of air inside the diver is affected by the total pressure p acting onto it. These two quantities are related by the equation of state for ideal gases $pV = Nk_B T$, where N is the number of air molecules, k_B is the Boltzmann's constant, and T is the absolute temperature. If the pressure is increased slowly, we can assume that the process occurs under isothermal conditions, due to the fact that the air inside the diver is thermostated by the surrounding mass of water. Under these conditions, the equation of state implies that $pV = (p + \Delta p)(V + \Delta V)$, which can be used to express the volume variation ΔV as a function of the pressure variation Δp:

$$\Delta V = -\frac{V\Delta p}{p + \Delta p} \tag{5}$$

which holds for arbitrarily large Δp and ΔV. The pressure acting on the submerged diver is determined both by the external pressure p_e and by its depth z inside the fluid so that the total hydrostatic pressure is

$$p(z) = p_e + \Delta p_e + \rho g z. \tag{6}$$

When the external pressure is released ($\Delta p_e = 0$), the diver is still under the action of the pressure determined by its depth, which can be large enough to keep it fully submerged. Therefore, if the container is high enough, in its descent the diver will reach a critical depth z_c, beyond which it will not rise again, even when the pressure determined by the external force is removed. Combining equations (2), (5) and (6) the critical depth is given by

$$z_c = \frac{V_{\text{ext}} p_e}{\rho g (V - V_{\text{ext}})}. \tag{7}$$

When designing a Cartesian diver, it is important to estimate this critical value and to maintain it larger than the height of the container by keeping the ratio $\frac{V_{\text{ext}}}{V - V_{\text{ext}}}$ large enough (all the other quantities, p_e, ρ and g, are constant), so that it can be operated reversibly and either sink or float by imposing or removing an external pressure.

The only equilibrium conditions for the diver are either floating or neutral buoyancy. Floating represents a stable equilibrium condition and, in the presence of small perturbations of the depth of the diver, gives rise to vertical oscillations. Conversely, the neutral buoyancy condition can be realized only when the density of the diver matches the one of the surrounding water and represents an unstable equilibrium condition, because even by imposing with extreme precision the pressure necessary to ensure neutral buoyancy, any displacement of the diver from this position will alter the pressure exerted on the volume of air by the column of water above it, and cause an accelerated downward or upward motion that will move the diver away from its initial position.

3. Hydrostatic equilibrium in a stratified fluid and gravity waves

By using a fluid stratified in density, it is possible to realize a system where the diver can be in stable equilibrium at a given depth. An object immersed in a stratified fluid will move vertically until it reaches a layer of fluid with the same density, or sink to the bottom of the container if this layer is not met [23]. It is surprising and inspiring to read a perfect description of this phenomenon given in 1665 by Galileo Galilei in its Dialogue Concerning Two New Sciences [24, 25]: '*By means of another device I was able to deceive some friends to whom I had boasted that I could make up a ball of wax that would be in equilibrium in water. In the bottom of a vessel I placed some salt water and upon this some fresh water; then I showed them that the ball stopped in the middle of the water, and that, when pushed to the bottom or lifted to the top, would not remain in either of these places but would return to the middle. (Sagredo)*'. It is possible to use devices based on density gradients in very different fields, and this variety of applications can be discussed and deepened with students to contextualize the experiment [26]. In a previous work, we proposed an engaging educational experiment in which a cork weighted to sink in water is immersed in a container of water with a quantity of coarse salt at the bottom [15]. The gradual dissolution of the salt in time creates a stratified concentration c along the depth, and in turn, a density gradient given by:

$$\nabla\rho = \frac{\partial\rho}{\partial c}\nabla c. \tag{8}$$

After the salt starts to dissolve, the cork, initially at the bottom of the container, rises slowly within the fluid until it eventually floats at a depth where the density of the fluid matches its own.

Again, the phenomenon can be found in the Dialogue of Galileo [24] in the answer given by Salviati to Sagredo: '*This experiment is not without usefulness. For when physicians are testing the various qualities of waters, especially their specific gravities, they employ a ball of this kind so adjusted that, in certain water, it will neither rise nor fall. Then in testing another water, differing ever so slightly in specific gravity [peso], the ball will sink if this water be lighter and rise if it be heavier. And so exact is this experiment that the addition of two grains of salt to six pounds of water is sufficient to make the ball rise to the surface from the bottom to which it had fallen*'.

The link between what students are experiencing firsthand and the history of physics, witnessed directly in Galileo's words, can be very intriguing for students.

We can exploit the results of this first experiment and immerse a Cartesian diver in an analogously obtained stratified liquid, with the great advantage that the density of the diver $\rho_t(p)$ can be finely tuned by changing the external pressure p applied to the container [20, 21]. For any pressure, the diver will be in stable equilibrium at a depth z_0 of the layered fluid such that $\rho_t(p(z_0)) = \rho(z_0) = \rho_0$, where $\rho(z)$ is the density of the solution of salt and water.

In the following we will outline a simple frictionless linear model able to describe the oscillations of a density matched body immersed in a stably stratified fluid. For a recent exhaustive treatise including the effects of viscous dissipation see [21].

As a first order approximation, we assume that the density of the stratified fluid changes linearly around a given point. We will show later that this assumption is fully consistent with the experimental results that we obtain, which are described nicely by this linear model. Under these conditions, around a layer of density ρ_t, we can write the density of the salt-water solution as a function of z as:

$$\rho(z) = \rho_0 + \frac{\partial\rho}{\partial z}z, \tag{9}$$

where the depth z is taken equal to zero at the free surface of the liquid and directed vertically downwards, so that $\frac{\partial\rho}{\partial z}$ is positive.

From equation (9), we see that if a diver of density ρ_t is moved from its equilibrium position, it finds itself in a region of different density. If the density of the fluid is higher than that of the diver, the buoyancy force dominates over the weight force and the diver is pushed upwards. Conversely, if the density of the fluid is lower than ρ_t, the opposite occurs. The diver is therefore affected by a restoring force, and starts oscillating around its stable equilibrium position. The frequency of oscillation can be calculated by noticing that the total force acting on a body of volume V_0 immersed in a density stratified fluid is [15]:

$$F_T = -gV_0 \frac{\partial \rho}{\partial z} \Delta z, \tag{10}$$

where $\Delta z = z - z_0$ is the displacement from the equilibrium position. Therefore, by applying the second principle of dynamics $\rho_0 V_0 \ddot{\Delta z} = F_T$ we obtain the equation of motion of a harmonic oscillator:

$$\ddot{\Delta z} = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \Delta z. \tag{11}$$

The oscillation then occurs at the Brunt–Väisälä angular frequency ω, typically observed in stratified fluids [20, 21, 27]:

$$\omega = \sqrt{\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}}, \tag{12}$$

which is simply related to the local density gradient $\frac{\partial \rho}{\partial z}$. In deriving equation (12) we have assumed that the density ρ_t of the diver is not affected by the change of depth determined by its vertical oscillations and remains pinned at its equilibrium value ρ_0. This approximation holds when the amplitude of the oscillations is small. In the presence of large displacements, the variation with depth of the force exerted by the fluid on the diver determines a variation of the average density of the diver, which gives rise to a nonlinear equation of motion, leading to more complex dynamics, whose description is beyond the aim of this introductory work.

The oscillations can affect not only a body immersed in a stratified fluid, but also portions of the fluid itself. In this last case, when a portion with a given concentration of the layered fluid is pushed in a layer of different concentration, the excess concentration can be dissipated either by diffusion or by the buoyancy force, which brings back the portion of the fluid in a layer with the same density. When the portion of fluid affected by the displacement is large, the buoyancy force acts much faster than diffusion, and the parcel of fluid oscillates [28]. The local oscillations of a body immersed in a stratified fluid or of a parcel of the fluid determine a gradual transfer of mechanical energy to the surrounding fluid through propagating waves. In fact, part of the energy is dissipated by viscous drag, while the remainder is transferred through the stratified fluid in the form of an internal gravity wave up to a certain distance. These kinds of waves are similar to those that can be observed on the surface of a liquid, but they occur in bulk of the fluid, where the density gradient varies smoothly and no sharp variation is observed.

The theory predicts that gravity waves that propagate in a plane perpendicular to the oscillations should have the same frequency of the oscillating body [16, 27], namely the Brunt–Väisälä one. These waves are typically observed in nature on a large scale within the two main fluids on our planet: the ocean and the atmosphere [29]. In the ocean, wave behaviour occurs not only at the surface but also internally, when low-density water overlaps high-density one. The change in density is mainly due to variations in temperature and salinity: solar radiation heats the upper layers of water lowering the density of water near the surface, while the flow of rivers and ice dissolution make the bottom layers denser.

Internal gravity waves in the atmosphere are formed, for example, when a mass of air meets the wall of a mountain and cools down through an adiabatic transformation and increases its density while rising in altitude; this causes a slowdown in its ascent, which progressively becomes an accelerated downward movement due to the force of gravity. The fall of the dense mass of air causes it to heat up and decrease again its density, reverses the displacement, and makes it rise. Gravity waves are extremely difficult to visualize as they involve only index of refraction variations, but a condensation of vapor occurring in the atmosphere could make the waves visible in the form of clouds at the crests [29–31]. The link of a simple experiment as the one we are proposing with such general natural phenomena may be used to show the student how the investigation of a simple model laboratory system can be used to reach an understanding of the behavior of much more complex natural systems.

4. Cartesian diver in a stratified fluid

In a previous experiment, we studied the oscillations of a weighted cork of fixed density immersed in a stratified solution of sodium chloride and water. When the cork was in stable mechanical equilibrium in a layer of solution matching its density, we triggered its oscillation by pushing it off equilibrium with a thin stick [15]. The limit of this approach was that we could only investigate the liquid in a thin layer of fluid with the same average density of the cork. This prevented the investigation of the oscillation frequencies at different depths at the same time of dissolution of the salt, but only to follow the cork behaviour around a layer in the fluid that was slowly changing in time as a function of the salt dissolution process. Moreover, the pushing method was inevitably invasive due to the direct mechanical action on the fluid and necessarily perturbed the stratified fluid by altering its concentration and density profiles.

In the same experiment, we also proposed two different methods to measure the frequency of the internal gravity waves. One was to show that a second cork with approximately the same density as the original cork started oscillating with a delay and at the same frequency when the original cork was put in oscillation. The second method was to visualize the internal gravity wave, generated by the oscillation of the cork, through the deformation of the lines of a square sheet placed in the background of the container, due to the undulations in the refractive index produced by the passage of the wave [15]. Although both methods have demonstrated to be quite effective, they do not allow a direct visualization of the waves.

In the new experiment that we propose here, we suggest solutions and improvements to all the issues that emerged in the first experiment.

4.1. Materials and methods

In order for an object to behave like a Cartesian diver, it must have some particular characteristics: it needs to be elongated in shape, small compared to the container into which it fits, and it must be pervious at the bottom to allow water to enter. Another important feature is that the lower part of the diver must be heavier than the upper one to ensure that the centre of buoyancy is located above the centre of mass, so that the diver remains aligned in the configuration where an air bubble is stably trapped at its top. This configuration where the longitudinal axis is aligned along the vertical direction also limits the horizontal components of the oscillations.

Particularly suitable objects are small glass bottles such as the ones used as perfume samples. Other divers can be custom-made using for example marker caps sealed on top with waterproof tape, or rubber Pasteur pipette teats (figure 2(a)). The assembly of the container hosting the diver requires the availability of coarse kitchen salt, a glass container, an elastic silicone lid, and rubber bands (figure 2(b)).

Figure 2. (a) Examples of divers made from common objects such as a glass ampoule (left), a rubber dropper teat (center), and a small perfume sample vial; (b) materials needed to assemble the diver experiment: 1. Coarse kitchen salt, 2. Rubber bands, 3. Small syringe, 4. Divers, 5. Glass jar, 6. Silicone lid. (c) Assembled diver experiment. By putting a weight on top of the elastic membrane, the diver sinks until it reaches the layer of the water-salt solution matching its density.

Before the diver is placed in the container filled with water, coarse kitchen salt is deposited at the bottom, so that its slow dissolution produces a density stratification when water is poured inside the container; the use of fine kitchen salt would not allow an equally slow and gradual dissolution process. We used amounts of salt in the range 15–35 g l^{-1}. The shape and size of the container in which the diver is immersed require an accurate choice too, since the lateral walls determine the reflection of gravity waves, which influence the diver's oscillations after a certain time [15]. We chose a cylindrical glass container with a capacity of 1.5 litres and a diameter of 9 cm. After the diver is inserted into the container, a small syringe is used to achieve a fine adjustment of the quantity of water initially contained inside it. The container is then sealed with an elastic silicone lid of a diameter slightly smaller than its top aperture, and the sealing is reinforced with an elastic band around the opening (figure 2(c)).

Objects of different weights can be placed on the silicone membrane to change the pressure of the air above the fluid and to maintain it constant to set the desired equilibrium depth of the diver. To achieve a fine tuning of the external pressure, we used graduated containers filled with water with a capacity ranging from 100 ml to approximately 700 ml. To obtain heavier weights, we added metal objects such as wrenches or hammers up to 1.5 kg.

Our experimental setup is very simple and easy to realize with everyday objects. However, as reported in [20], more refined setups can be realized for research purposes. Not only Le Gal *et al* [20] use an extremely accurate pressure control, but they also create a desired density stratification and study it before and after the experiment. Moreover, as they are interested in the horizontal displacement of the diver, they use a quasi-two dimensional container to impose a precise direction to the horizontal motion. Conversely, in our case we preferred a circular symmetry, to reduce the spurious effects due to uneven reflections of gravity waves reported in [15].

To visualise the propagation of the gravity waves, we performed some experiments by adding to the solution small pieces of black acrylonitrile-butadiene-styrene (ABS), obtained by cutting a 1.75 mm diameter ABS wire into 1–2 mm segments. The choice of this material is based on the fact that ABS wire is widely available for 3D printing and its density is approximately 1030–1050 kg m^{-3}, slightly higher than that of water. Thus, in pure water, the tracers are deposited at the bottom of the container, while in the presence of a density gradient, they set in the fluid layer that has a density equal to their one. As the gravity wave

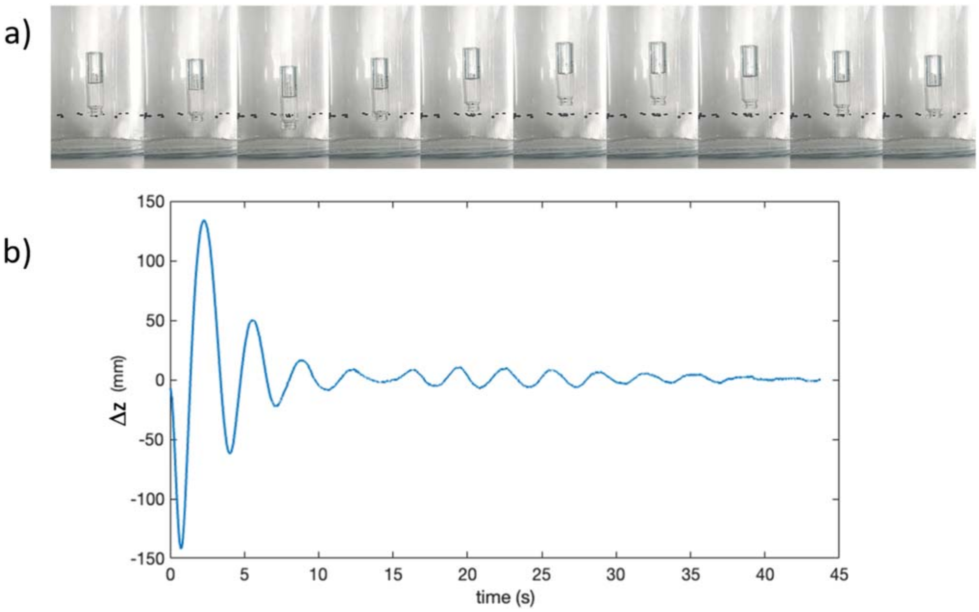

Figure 3. (a) Sequence of images during the diver oscillations. The motion is started by applying a pressure pulse on top of the container. (b) Position as a function of time of a reference point of the diver, put in oscillation by a pressure pulse on top of the container.

passes, these small fragments oscillate due to the perturbation of the layer where they float and give direct evidence of the internal wave.

To perform the measurements we used the camera of a smartphone mounted on an adjustable support to guarantee that it was perpendicular to the container and at the correct height to record the oscillations of the diver and/or the tracers without significant distortions.

5. Experimental results

A remarkable advantage of the experimental apparatus presented here is that the diver can be easily moved away from its equilibrium position without touching it, via the application of a pressure pulse on the membrane that seals the container. This action induces an abrupt variation of the diver density and, in turn, a temporary change of its equilibrium depth. The extra pressure lasts for a very short time and, as soon as it is removed, the diver density goes back to the original one that depends only on the static pressure on the container. As a consequence, the diver starts oscillating around its original equilibrium position at the Brunt–Väisälä frequency.

5.1. Diver oscillations

By taking a video of the diver, it is possible to study its oscillations and measure their frequency. To achieve this purpose, we used the software 'Tracker' [32], an open-source Java-based application, which allows to track the position of an object in a video at different time instants. In figure 3(a) we present a sequence of images taken at different times of a diver oscillating inside the container. The motion started without touching the diver, but by applying a pressure pulse onto the upper membrane. It can be noticed that the small bottle used as a diver, contains an air bubble that, at atmospheric pressure, allows the diver to float. The depth of the diver inside the liquid is guaranteed by a constant pressure applied to the container.

In figure 3(b) we show the position in time of the diver measured with Tracker, by using a reference point as a target. In order to have a quantitative estimate of the displacement of the diver as the one shown in the figure, it was necessary to perform a first spatial calibration of the distances by using a known length as a reference.

So far in the discussions of the problem we have considered a body with constant density, while for a Cartesian diver the density actually slightly varies when its position inside the fluid changes, due to the variation of the hydrostatic pressure with depth. For the whole discussion to be valid, we have assumed that the variation in density that occurs during the oscillations is negligible and that the only moment when the pressure increase is large enough to affect significantly the diver density is during the initial pressure pulse. As soon as the forcing pressure is released, the density goes back to the original value.

To assess whether this assumption is reasonable, we have estimated the amount of pressure applied during the pulse, by pressing a balance with the same intensity. We measured values from 3 and 4 Kg (corresponding to forces between 29, 4 and 39, 2 N). When applied on the area of the container opening equal to $63.6 \times 10^{-4}\ \mathrm{m}^2$, they correspond to an extra pressure on the fluid of the order of 4600–6200 Pa.

An order of magnitude estimate of the validity of the assumption of constant density of the diver during the oscillations can be obtained by considering that the typical oscillation amplitude after the pulse is around 4 mm and the corresponding pressure variation $\Delta p = \rho g \delta h \approx 40$ Pa determined by this change of depth is roughly two orders of magnitude smaller than the one applied during the pulse. Thus, the approximation of constant density during the oscillations can be considered reasonable.

In figure 3(b) we observe that the first two oscillations have an amplitude much larger than the following ones. Indeed, the first half cycle of oscillation corresponds to the displacement of the diver determined by the imposition and release of the pressure pulse that triggers the oscillations. When the impulsive force is interrupted, the diver still has a non-zero velocity and continues its downward motion before ascending back. Thus, we consider the first oscillations as anomalous ones and we do not use them for the data interpolation procedure. In the case discussed in [15], the amplitude of the oscillations was observed to slightly decrease and then increase again. This behaviour was due to beatings between the oscillations of the diver and the internal gravity wave reflected back by the container walls. In the present case, the phenomenon is less severe due to the smaller size of the diver and the careful choice of the container.

Thus, we decided to neglect beatings and to interpolate the position of the diver with the expression for a damped harmonic oscillator:

$$z(t) = A \sin[\omega(t - t_0) + \phi)]\mathrm{e}^{-\frac{t-t_0}{\tau}} + z_0, \tag{13}$$

where ω, A, t_0, ϕ and τ are free parameters. From the fitting of the oscillations it is then possible to measure the Brunt–Väisälä angular frequency ω.

As anticipated, a remarkable characteristic of this experiment is that we have the possibility to change the depth at which the diver is in equilibrium by varying the constant pressure on top of the container, and this allows to map the oscillation frequencies at different depths at any time during the salt dissolution process. In figure 4 we show the results obtained with a long measurement taken by adding $20\ \mathrm{g\ l^{-1}}$ of kitchen coarse salt to water and by collecting data for 13 d.

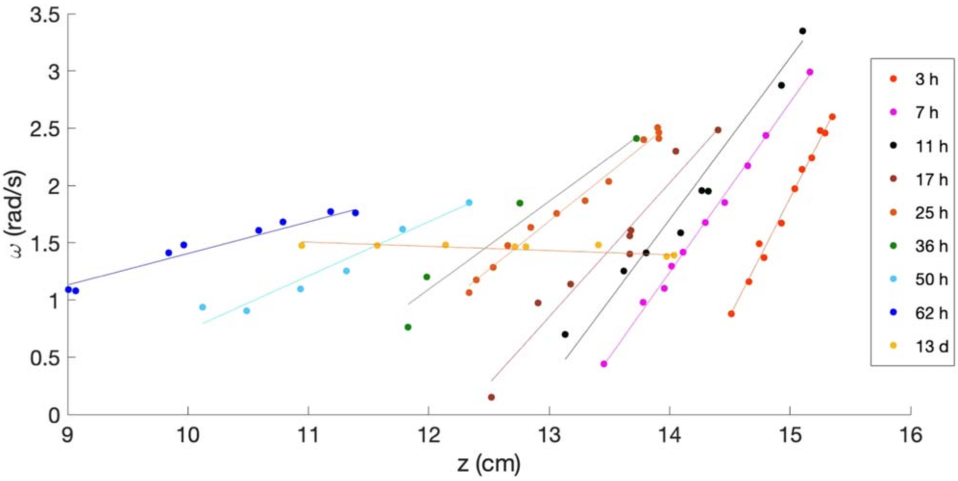

Figure 4. Brunt–Väisälä angular frequencies measured at different depths, for different times after the beginning of salt dissolution. The solid lines are a guide to the eye. The equilibrium position was changed by varying the pressure on top of the container. At the first stages of salt dissolution (right region of the graph), the diver can be easily pushed at high depths and the oscillation frequency is strongly dependent on the equilibrium depth. When the salt gradient expands, the diver remains in the upper region even when high pressures are applied, and ω is less dependent on z. After many days, the concentration gradient becomes almost constant, and the diver can be pushed again at high depths.

To describe the motion of the diver we have introduced as a relevant parameter the depth z_0 of its equilibrium position because this is the quantity that directly determines the hydrostatic pressure acting on it. In the case of a diver floating inside a stratified fluid, another relevant parameter is represented by the total depth of the liquid contained inside the container, 17 cm in our case, because the bottom of the container corresponds to the layer of fluid where the salt starts to dissolve. In fact, at the beginning of the salt dissolution process, the diver can be easily pushed at high depths and the oscillation frequencies are large and strongly dependent on the equilibrium depth z_0. This means that the concentration profile is not linear. Moreover, it provides an indication of the fact that the region interested by the density variation is initially located at the bottom of the container where $\frac{\partial \rho}{\partial z}$ is very large. As time passes by, the density profile becomes less and less clear-cut and the region in which $\frac{\partial \rho}{\partial z} \neq 0$ becomes increasingly larger. Under these conditions, it becomes difficult to push the diver at the bottom of the container, and the oscillation frequencies are less dependent on depth, because the value of $\frac{\partial \rho}{\partial z}$ in the various layers of the fluid is decreasing. After 13 d we measured approximately the same frequency at all the depths inspected, which means that the salt dissolution has created a density profile that varies linearly with depth.

5.2. Internal gravity waves

As anticipated, thanks to the plastic fragments dispersed in the stratified liquid, it is possible to visualize the passage of the internal gravity wave. In order to do so, it is necessary to set the equilibrium depth of the diver in the layer where the fragments are positioned. Then, when the pressure pulse is applied, the diver starts oscillating, followed by the fragments that start oscillating at the same frequency with a delay that depends on their distance from the diver. In

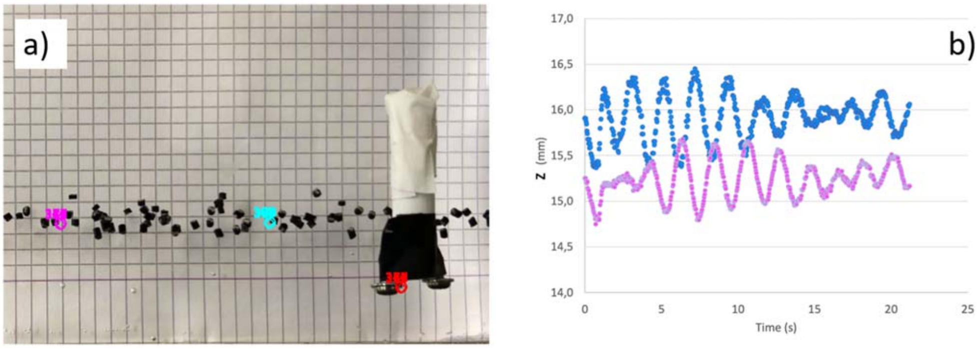

Figure 5. (a) By tuning the external pressure, the diver can be set in equilibrium at approximately the same depth as the trackers. As soon as it starts oscillating, its energy is partially transferred to the liquid as an internal gravity wave that makes the plastic fragments oscillate with a delay that depends on their distance from the diver. (b) The positions of the two plastic fragments evidenced in figure (a) have been tracked with Tracker. There is a phase delay in the oscillations due to the time needed for the internal gravity wave to propagate inside the sample from one fragment to the other. The frequency of oscillation is the same for both the tracers and coincides with that of the diver.

order to have a clear view of the liquid, free from distortions determined by the shape of the container, we used a rectangular plexiglass tank, 16.5 cm high, 25.7 cm long, and 4.1 cm thick, closed with water-resistant adhesive tape. To help the eye, we also used a chequered sheet placed in contact with the back window of the tank.

The velocity of the internal gravity wave can be measured by dividing the distance of a tracer from the diver by the time needed for a tracer to start oscillating after the diver has started to move. By taking different fragments we have estimated that for the particular case shown in figure 5 the wave velocity is approximately $1.5\ \mathrm{cm\ s^{-1}}$, while the Brunt–Väisälä angular frequency is $\omega \approx 2, 6\ \mathrm{rad\ s^{-1}}$, as the one measured for the diver.

6. Applications with students

The idea of this experiment was proposed to the undergraduate student Irene Spongano (co-author of this work) as a bachelor's degree thesis in Physics. She designed the experiment and performed the measurements in great autonomy, although under the guidance of her professors. We believe that the experiment resulting from this work is particularly suitable as an engaging activity for high school students and allows for an inquiry-based approach to introduce topics such as pressure in fluids, the ideal gas law, Archimedes principle, and Stevin and Pascal laws.

Research in Physics education has demonstrated that active learning methods foster independent thinking, critical questioning, and collaborative discussions among students [33–37]. Various approaches to active learning are possible, all of them being methods that engage students in the learning process. Among the possible methods, inquiry-based learning is particularly effective in improving students' understanding and critical thinking skills [36, 38, 39]. In inquiry-based lessons, the teacher should avoid assertive statements and instead cultivate an atmosphere conducive to exploration, where his or her role is that of a partner in inquiry. Within the framework of Inquiry-based Science Education (IBSE), students collaborate in groups, generate their own questions, make observations, design and execute experiments, gather and analyze data, develop hypotheses, and justify their conclusions based on the evidence collected [35, 38, 39].

In traditional teaching, teachers propose the problem, the procedure to reach the solution and, finally, the solution itself. In IBSE, some of these steps are left to students who have the chance to learn how to approach problems in a scientific way, grasp and organise their new knowledge by adapting and refining existing concepts, and incorporate new concepts based on their current understanding and beliefs. Inquiry-based teaching can be proposed with different levels that indicate the extent to which students are empowered and take ownership of their learning [40].

A possible and well-established method to propose to students a new problem with an inquiry-driven approach is through the 5E cycle [41], where the five Es are the learning phases and mean Engage; Explore; Explain; Extend and Evaluate, respectively.

We propose here a possible learning path that makes use of the modified Cartesian diver in the 5Es contexts. Clearly, this is just one of the possible ways to use this experiment for an active learning approach, and the proposal should be considered only as a working suggestion. In the first stage, the attention of the students is caught thanks to a surprising experiment. In this case, we can show them two Cartesian diver types of equipment that look apparently identical but behave in a completely different way. In one of them, when pressure is increased, the diver sinks to the bottom of the container, while in the other the diver stops at a certain depth. In the second phase, students should work in groups and try to find out some explanations. They should work with both the traditional and the modified Cartesian diver, exploring their behaviour. The first step is to understand how the traditional apparatus works; by putting their knowledge of fluids and pressure into practice, they should explore the characteristics of the other experiment. They should investigate what happens when the pressure on the container is varied and they should notice that the diver equilibrium position changes. There is the possibility to leave them completely unassisted or to guide their observations suggesting questions or conveying their attention to particular aspects. One of them could be the effect of a pressure pulse, another could be to look through the container. It is possible that some students are able to notice an inhomogeneity of the index of refraction in the modified device.

It is up to teachers to choose the way to proceed, although research on open inquiry or 'minimally guided instruction' has indicated that it may not always be the most effective approach [42]. Additionally, it can pose significant challenges, and may not align well with contemporary standards-based education systems. In contrast, a guided inquiry approach can offer a more practical strategy. It allows the teacher to maintain a certain level of responsibility, mitigate risks and ensure alignment with desired learning outcomes. Nevertheless, this method still grants students a significant degree of autonomy and the pleasure of discovery [43].

The next phase is perhaps the most delicate one, in which students are asked to propose an explanation of what they have observed and understood on the basis of their experiments. The role of the teacher is here particularly important as she/he is expected to suggest some possible further experiments to do, in order to test the effectiveness of the explanation proposed. A possible experiment to suggest is to immerse in water an object with a density slightly larger than water [15] and to add salt to the container. What do they expect to happen? What happens when the object is suspended at a certain depth and they push it off its equilibrium position? As the dissolution process is quite slow, we suggest using a large amount of fine salt ($>40\ \mathrm{g\ l^{-1}}$) for the first trials: this would shorten the time considerably.

In the final phase, students compare their findings and evaluate the effectiveness of the interpretation. Interesting discussions may arise and new ideas usually emerge. We plan to propose some high school teachers to test this experiment with their students.

However, from the experience gained with the bachelor's degree thesis, we are confident that this experiment can also be successfully proposed to undergraduate students, who might find it very engaging to attempt an approach in which they manage the different steps themselves and use the scientific method to arrive at a satisfactory explanation.

7. Conclusions

Research in physics education has amply documented that students often encounter difficulties in understanding the concept of pressure in fluids [1–3]. They frequently cannot see a connection between a local action, such as the application of a pressure, and the global effects occurring inside the fluid; a large percentage of both high school and university students do not think that buoyant forces are the resultant of pressure forces; they usually relate the pressure on a body to its depth inside the liquid, but they often do not take into account the external pressure [1].

The modified Cartesian diver experiment proposed in this paper not only involves all these aspects but has also the advantage of being easily connected to real-life contexts, a very important starting point in any active learning approach. The presence of a density gradient is typical of natural fluids well known by students such as seas, oceans, and the atmosphere. Moreover, severe density gradients are also present during star formation and have a significant role in the process. Gravity waves, which occur inside fluids where density gradients are present, are responsible for the formation of very peculiar clouds [31], and of internal ocean waves, documented by satellite images [29]. The fundamental investigation of their interaction with vortical modes in a fluid represents a field at the forefront of current research in fluid dynamics [44]. By proposing the experiment in the framework of an active learning approach, it will enable students to address some of their difficulties by having direct experience with the role of external pressure, on the global effects of local actions, and the fundamental relation between buoyancy and pressure. The density gradient inside the fluid makes it possible to obtain an stable equilibrium position for the diver and to change it at will by varying the external pressure. The different frequencies of oscillation measured at various heights and at different times of salt dissolution, allow to gain a guess about the density profile and its evolution in time.

Acknowledgments

We thank C Anzivino and N Manini for useful suggestions. Work supported by the European Space Agency (ESA) in the framework of the 'Giant fluctuations', NESTEX and 'Sedimenting Colloids' projects, and by the Italian Space Agency (ASI) through the projects 'Gravitationally TApping Colloids in Space (GTACS)—Sedimenting Colloids' (Number 2023-19-U.0) and 'Non-Equilibrium Phenomena in Soft Matter and Complex Fluids (NESTEX)' (Number 2023-20-U.0). FC acknowledges financial support from the CNES.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

Marina Carpineti https://orcid.org/0000-0002-8766-703X
Irene Spongano https://orcid.org/009-0005-6072-1664
Fabrizio Croccolo https://orcid.org/0000-0001-6832-400X
Alberto Vailati https://orcid.org/0000-0002-3119-6021

References

[1] Besson U and Viennot L 2004 *Int. J. Sci. Educ.* **26** 1083–110
[2] Glough E E and Driver R 1985 *Res. Sci. Technol. Educ.* **3** 133–44
[3] Kariotoglou P and Psillos D 1993 *Res. Sci. Technol. Educ.* **11** 95–108
[4] Amir N and Subramaniam R 2007 *Phys. Educ.* **42** 478
[5] Turner R C 1983 *Am. J. Phys.* **51** 475
[6] De Luca R and Ganci S 2011 *Phys. Educ.* **46** 528–32
[7] Güémez J, Fiolhais C and Fiolhais M 2002 *Am. J. Phys.* **70** 710
[8] Güémez J, Fiolhais C and Fiolhais M 2003 *Phys. Teach.* **41** 495
[9] Graham R M 1994 *Phys. Teach.* **32** 182
[10] Adler J D 1997 (University of British Columbia) https://open.libra ry.ubc.ca/collections/ubctheses/831/items/1.0054980
[11] Orwig L P 1980 *Am. J. Phys.* **48** 320
[12] Miller J S 1954 *Am. J. Phys.* **22** 235
[13] Ivanov D and Nikolov S 2020 *Phys. Educ.* **55** 025006
[14] Fakhruddin H 2003 *Phys. Teach.* **41** 53
[15] Carpineti M, Croccolo F and Vailati A 2021 *Eur. J. Phys* **42** 055011
[16] Tritton D J 1988 *Physical Fluid Dynamics* (Oxford Science Publications, Clarendon Press, Oxford)
[17] More R V and Ardekani A M 2023 *Ann. Rev. Fluid Mech.* **55** 157–92
[18] Castellini S, Carpineti M, Giraudet C, Croccolo F and Vailati A 2023 *J. Chem. Phys.* **158**
[19] Vailati A *et al* 2020 *Microgravity Sci. Technol.* **32** 873–87
[20] Le Gal P, Castillo Morales B, Hernandez-Zapata S and Ruiz Chavarria G 2021 *J. Fluid Mech.* **931** A14
[21] Voisin B 2024 *J. Fluid Mech.* **984**
[22] Couder Y, Protière S, Fort E and Boudaoud A 2005 *Nature* **437** 208–208
[23] Winant C D 1974 *Deep Sea Res. Oceanograph. Abstr.* **21** 445–53
[24] Galilei G 1950 *Dialogues Concerning Two New Sciences* (North-western University Press)
[25] Oster G and Yamamoto M 1963 *Chem. Rev.* **63** 257–68
[26] Oster G 1965 *Sci. Am.* **213** 70–9
[27] Nappo C J 2012 *An introduction to Atmospheric Gravity Waves* vol 102 (Elsevier)
[28] Croccolo F, García-Fernández L, Bataller H, Vailati A and Ortiz de Zárate J M 2019 *Phys. Rev.* E **99** 012602
[29] Atmospheric gravity waves imagery, https://svs.gsfc.nasa.gov/14445/
[30] Alexander S and Murphy D 2015 *J. Atmos. Sci.* **72** 1010–21
[31] Murphy D, Alexander S K A and PL 2015 *Aust. Antarct. Mag.* **28** https://www.antarctica.gov.au/magazine/issue-28-june-2015/science/refining-gravity-waves-in-climate-models/
[32] Tracker app: https://physlets.org/tracker/
[33] Driver R 1983 *The Pupil as Scientist?* (Open University Press)
[34] Dermott L C M, Shaffer P S and Constantinou C P 2000 *Phys. Educ.* **35** 411–6

[35] Rocard M, Csermely P, Jorde D, Lenzen D, Walberg-Henriksson H and Hemmo V 2007 *Science Education Now: A Renewed Pedagogy for the Future of Europe* (Office for Official Publications of the European Commission)
[36] Fazio C 2020 *Active Learning Methods and Strategies to Improve Student Conceptual Understanding: Some Considerations from Physics Education Research. In Guisasola Jenaro and Zuza Kristina (a cura di), Research and Innovation in Physics Education: Two Sides of the Same Coin* (Springer International Publishing) pp 15–35
[37] Fazio C, Carpineti M, Faletič S, Giliberti M, Jones G, Mcloughlin E, Planinšiç G and Battaglia O R 2021 Strategies for active learning to improve student learning and attitudes towards physics *Teaching-Learning Contemporary Physics. Challenges in Physics Education* ed B Jarosievitz and C Sükösd (Springer, Cham) pp 213–33
[38] Gormally C, Brickman P, Hallar B and Armstrong N 2009 *Int. J. Scholarship Teach. Learn.* **3** 16
[39] Duran M and Dökme M 2016 *EURASIA J. Math. Sci. Technol. Educ.* **12** 2887–908
[40] Tafoya E, Sunal D and Knecht P 1980 *Sch. Sci. Math.* **80** 43–8
[41] Bybee R W *et al* 1989 *Rowley, MA: The National Center for Improving Science Education* (The NETWORK)
[42] Tafoya E, Sunal D and Knecht P 2006 *Learn. Instr.* **16** 165–9
[43] Feyzioğlu E Y and Demirci N 2021 *J. Turkish Sci. Educ.* **18** 401–20
[44] Labarre V, Augier P, Krstulovic G and Nazarenko S 2024 *Phys. Rev. Fluids* **9**