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Supervisore: Professor Claudio GRILLO

Co-supervisore: Professoressa Amata MERCURIO

Coordinatrice: Professoressa Roberta VECCHI

Tesi di Dottorato di:

Giovanni Granata

Matr. R12991

Anno Accademico 2022/23

Commission of the final examination:

External Referee:

Dottorssa Annalisa Pillepich

External Member:

Professor Henk Hoekstra

Internal Member:

Professor Marco Bersanelli

Final examination:

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Contents

List of Figures	ix
List of Tables	xvii
Introduction	xviii
Motivation	xix
Overview of the objectives of this Thesis	xx
List of Publications	xxiv
Part I : Clusters of galaxies	3
1 Basic phenomenology and dynamics of elliptical galaxies	5
1.1 Introduction to the formation and evolution of galaxies in the Λ CDM framework	5
1.2 The morphological classification of galaxies	8
1.3 Dynamics of elliptical galaxies	11
1.4 The Fundamental Plane of elliptical galaxies and its projections	17
2 Basic phenomenology and cosmological simulations of galaxy clusters	19
2.1 Basic phenomenology of galaxy clusters	19
2.2 Cosmological hydrodynamical simulations of galaxy clusters	25
Part II : Overview of strong lensing and galaxy kinematics	33
3 An introduction to strong gravitational lensing	35
3.1 Gravitational lensing in the framework of General Relativity	35
3.2 Lensing properties of the singular isothermal sphere and its generalisations	40
3.3 Parametric strong lensing modelling with <code>LensTool</code>	43
4 Photometric and spectroscopic observations of strong lensing clusters	47
4.1 Hubble Space Telescope imaging	47
4.2 Spectroscopy with the Very Large Telescope	48

5	Measured kinematics of the cluster member galaxies	53
5.1	The spectral energy distribution of early-type galaxies	53
5.2	Inferring the parameters of the LOSVD of a galaxy from its spectrum	54
5.3	Velocity dispersion measurement of cluster galaxies from VLT/MUSE spectroscopy	58
 Part III : Improved strong lensing modelling of galaxy clusters using the Fundamental Plane		65
6	Improving the strong lensing model of Abell S1063	67
6.1	Abell S1063: Data and analysis	68
6.2	Building the strong lensing model	71
6.3	Comparison and discussion	76
6.4	Mass profile decomposition	80
7	Comparison with cosmological simulations	85
7.1	An excess of galaxy-scale strong lenses in massive clusters	85
7.2	Comparison with halo occupation distribution studies	89
7.3	Comparison with cosmological hydrodynamical simulations	91
7.4	Conclusions of Part III	98
 Part IV : Insights on the mass structure of cluster galaxies with galaxy-scale strong lensing		103
8	Exploring the low-mass regime of galaxy-scale strong lensing in clusters	105
8.1	Reference strong lensing models	106
8.2	The galaxy-galaxy strong lensing systems	107
8.3	Strong lensing modelling	113
9	Insights into the mass structure of cluster galaxies	121
9.1	Investigating the compactness of cluster galaxies	121
9.2	Conclusions of Part IV	130
 Part V : Towards the calibration of the velocity dispersion function of quiescent cluster galaxies		135
10	First steps towards the velocity dispersion function of quiescent galaxies in lensing clusters	137
10.1	Introductory remarks on the velocity function of quiescent galaxies	137
10.2	Building the velocity dispersion function for the quiescent members of the HFF clusters	139
 Summary, conclusions and outlook		153

Appendices	157
A Velocity dispersion measurements from simulated MUSE spectra of cluster members	161
A.1 Simulated MUSE spectra of cluster members	161
Bibliography	165
Acknowledgments	173

List of Figures

- 1.1 Schematic representation of the Hubble classification diagram. From esahub-
ble.org 9
- 1.2 Colour-magnitude distribution of early- and late-type galaxies. Colour-
magnitude relation for approximately 70,000 galaxies from the SDSS sur-
vey. The number distributions of the early- and late-type galaxies have
been marked in red and blue, respectively. From Baldry et al. (2004b). 9
- 1.3 Spectra of early- and late-type galaxies. Combined rest-frame spectra of
several galaxies of the cluster MACS J1206.20847 ordered in six different
spectral types. From Girardi et al. (2015). 10
- 2.1 DM and ICM mass distribution of six massive galaxy clusters superim-
posed to the cluster galaxies. The DM distribution is modelled with grav-
itational lensing, while the ICM is probed with the Chandra X-ray emis-
sion, and the cluster galaxies have been observed with the *Hubble* Space
Telescope. A clear difference can be observed in the X-ray emission be-
tween relaxed clusters, with a regular ICM morphology, and more irregu-
lar clusters. Credits: NASA, ESA, STSci, CXC. 21
- 2.2 Total mass distribution profile of a galaxy cluster and its probes. Cumula-
tive projected total mass profiles of MACS J1206.2-0847 determined with
galaxy dynamics, gravitational lensing, and ICM hydrostatic equilibrium.
From Rosati et al. (2014). 22
- 2.3 Number fraction of the different morphological galaxy types (as defined
in the text) as a function of the environment. In the upper panel, the frac-
tion is presented for all galaxies as a function of the galaxy density. In the
lower panel, it is obtained for cluster galaxies only, as a function of the
projected distance from the cluster centre, normalised for the cluster virial
radius. From Goto et al. (2003). 24
- 2.4 Overview of the different boxes of the Magenticum Pathfinder simula-
tions, from the Gpc to the kpc scales. The panels show the distribution of
gas particles, colour-coded according to the gas temperature from lower
(brown) to higher (blue/cyan) values. Galaxies and stars forming in the
simulation are shown in white. Credit: Dolag et al. (2015), Teklu et al. (2015) 28

2.5	Circular velocity and cumulative mass profiles per particle type in simulated clusters. We show DM (left column), and stars (right column) for cluster sub-haloes with total masses in the range $1 - 6 \times 10^{11} M_{\odot}$, for varying resolution choices and feedback prescriptions, indicated with the same naming scheme adopted in the text. For each setup, we plot the minimum resolved scale for the stellar mass distribution ($2.8\epsilon_*$, where ϵ_* is the stellar softening length) as vertical dashed lines. The profiles have been obtain as an average of six Dianoga clusters with a virial mass in the range $3 - 11 \times 10^{14} M_{\odot}$. Figure from Ragagnin et al. (2022).	30
3.1	Schematic representation of a gravitational lens (adapted from Bartelmann & Schneider 2001, using the same notation as in this Thesis).	38
3.2	Visual summary of the scope and utilisation of <code>LensTool</code> .	46
4.1	Transmission curves of the HST ACS and WFC3 filters of the bands used for the CLASH survey (from Postman et al. 2012).	48
4.2	The 20 X-ray selected galaxy clusters of the CLASH survey: Chandra X-ray images (from Postman et al. 2012).	49
4.3	The galaxy clusters of the HFF survey. RGB image and parallel field of the six HFF clusters (credits: NASA, ESA, STScI, and the HFF team).	49
4.4	Three-dimensional distribution of the galaxies in a cluster field. Spatial and CLASH-VLT redshift distribution of the galaxies in the field of the cluster MACSJ0416.1–2403. Red dots indicate the spectroscopically confirmed cluster members. Blue dots indicate foreground and background objects. From Rosati et al. (2014).	51
4.5	HFF RGB images of the core of a HFF cluster (MACSJ0416.1–2403). Left panel: the cluster members included in the most recent SL model are marked with small circles. Spectroscopically confirmed members are marked in red and green. The MUSE pointings are overlaid (from Bergamini et al. 2021). Right panels: the multiple images are included in the most recent SL model are marked with small circles (from Bergamini et al. 2023b).	51
4.6	Resolution of MUSE as a function of wavelength.	52
5.1	Spectrum of a red star (SDSS J105206.40+005433.7). From the SDSS data release 14	54
5.2	Spectra of SDSS galaxies with prominent emission (marked in blue) and absorption (in red) lines. Upper panel: the spiral galaxy SDSS J123421.84+081216.6. Lower panel: the elliptical galaxy SDSS J124249.82+024115.9. From the SDSS data release 14	55
5.3	pPXF fit of the spectrum of a cluster member (member 83042 of MACS J0416.1–2403). The observed spectrum (in black) is plotted together with the best-fit spectrum (in red). The fit residuals are shown at the bottom of the figure (in green). The masked regions are marked in grey, the residuals are drawn in blue. We report the recovered σ_{LOS} (in km s^{-1}) and the spectral S/N .	60
6.1	Colour composite image of the core of AS1063 created from HFF photometry. Blue, green and red channels are obtained by combining the filters F435W, F606W + F814W, and F105W + F125W + F140W + F160W, respectively. (From Granata et al. 2022).	69

- 6.2 Distribution around the FP of the 30 members selected to determine its parameters, with the 1σ uncertainties obtained through the error propagation. For the physical quantities, we use the units listed in the main text. (From Granata et al. 2022). 74
- 6.3 Values of the rms difference between the observed and model-predicted positions of the 55 multiple images used in our model as a function of the ratio between the truncation and effective radii of the cluster members. (From Granata et al. 2022). 75
- 6.4 Total mass of the cluster members as a function of their velocity dispersion. In blue: values predicted by our model. In red: values predicted by the model presented in B19. (From Granata et al. 2022). 77
- 6.5 Stellar fraction of the cluster members as a function of their central velocity dispersion for the 63 galaxies with $\sigma_{\text{FP}} > 80 \text{ km s}^{-1}$: comparison between the values obtained with our model (in blue) and those from the model in B19 (in red). (From Granata et al. 2022). 78
- 6.6 Logarithmic values of the total mass as a function of the logarithmic values of the stellar mass, both projected within the effective radius, of all members with $\sigma_{0,\text{FP}} > 80 \text{ km s}^{-1}$. The plot also shows the relation found by Grillo (2010). (From Granata et al. 2022). 79
- 6.7 Posterior probability distributions of the core radius r_c and fiducial velocity dispersion σ_{LT} values of the main DM halo and degeneracy between them for our model (in blue) and that by B19 (in red). The 16th, 50th and 84th percentiles of the marginalised probability distributions are marked with dashed vertical lines for both models. The values of these percentiles for our model are reported above the histograms. (From Granata et al. 2022). 79
- 6.8 Reconstructed surface brightness distribution of all cluster members used in the SL model in HST F814W band. The field of view is 700 kpc wide and centred on the BCG to cover the entire region where the cumulative mass profile is obtained. (From Granata et al. 2022). 81
- 6.9 Upper panel: cumulative two-dimensional mass profiles for all baryonic and dark components of AS1063. The total mass profile and those of the diffuse DM haloes and of DM sub-haloes obtained from the model in B19, which differ from ours, are represented with dashed lines. Lower panel: cumulative stellar-, gas- and baryonic-to-total mass fractions from our model. The value of the cosmological baryon fraction from the Planck Collaboration et al. (2020) is also shown. (From Granata et al. 2022). 82
- 7.1 Comparison between SL models and hydrodynamical simulations. Upper panel: GGSL probability as a function of the source redshift for the Dianoga simulations (in red), for the models of B19 (in blue), for the CLASH models presented in Caminha et al. (2019) (in purple), and for the SL models of all remaining HFF clusters (in cyan). Lower panel: maximum circular velocity of cluster sub-haloes as a function of their total mass for the SL models presented in B19 (black line) and for the Dianoga simulations (dots colour coded according to the projected distance of the sub-halo from the cluster centre normalised to the virial radius) (From Meneghetti et al. 2020). 87

- 7.2 Sub-halo compactness predicted by the Hydrangea simulations. Upper panel: maximum circular velocity of cluster sub-haloes in the Hydrangea hydrodynamical and DM-only simulations as a function of their total mass. In the case of the hydrodynamical simulation, the points are colour-coded for their baryon fraction. Lower panel: maximum circular velocity of cluster sub-haloes in the Hydrangea (pink) and IllustrisTNG300 (green) hydrodynamical simulations as a function of their total mass. (From Bahé 2021). 88
- 7.3 Comparison between the values of the stellar over total mass fraction of the cluster members obtained with our lensing model (in blue) and with the stellar-to-halo mass relation from Girelli et al. (2020) (in orange). Only the 63 cluster members with $\sigma_{\text{FP}} > 80 \text{ km s}^{-1}$ are shown. Upper panel: stellar over total mass fraction of members as a function of their stellar mass. Lower panel: stellar-to-halo mass relation. (Adapted from Granata et al. 2022). 90
- 7.4 Comparison between the values of the stellar over total mass fraction of sub-haloes with velocity dispersion greater than 80 km s^{-1} as a function of their total mass. In blue: values reconstructed from our model. In orange: values predicted by the Dianoga hydrodynamical simulations. (Adapted from Granata et al. 2022). 92
- 7.5 Maximum circular velocity of the cluster members as a function of their total mass. In red: values predicted by the model presented in B19. Our values are colour-coded according to the predicted stellar over total mass fraction. Only members with $M > 10^{10} M_{\odot}$ are shown. (Adapted from Granata et al. 2022). 93
- 7.6 Cumulative sub-halo mass function for the members within a projected radius of $0.15r_{\text{vir}}$ predicted by our model (in blue) and by that presented in B19 (in red). (Adapted from Granata et al. 2022). 94
- 7.7 Average cumulative number of satellite sub-structures with total mass larger than M_{SH} and lower than $10^{13} M_{\odot}$ (left column) and for stellar masses larger than $M_{\star}$, computed within an aperture of 30 physical kpc. We show data for the four $1 \times \text{R15}$ (cyan), $1 \times \text{RF18}$ (pink), $10 \times \text{B20}$ (black), and $1 \times \text{B20}$ (magenta) simulations in common between the four suites. For each simulation we average data over three orthogonal projections. Upper panels consider sub-haloes within the virial radius and lower panels consider sub-haloes with projected cluster-centric distance $R < 0.15R_{\text{vir}}$. Observations are from the central region ($R < 0.15R_{\text{vir}}$) of cluster AS1063 presented in B19 and in this Thesis. The simulated clusters have an average redshift of $z \approx 0.4$. (Adapted from Ragagnin et al. 2022). 96
- 7.8 Median velocity dispersion $\sigma = v_{\text{max}}/\sqrt{2}$ (top panel), median half-total-mass projected radius $R_{50\%}$ (second panel), median half-stellar-mass projected radius $R_{\star,50\%}$ (third panel), and median total stellar mass $M_{\star}$ (fourth panel), all as a function of the total sub-halo mass M_{SH} . Simulation suites are colour-coded as in Fig. 7.7. We also show median values of observations from cluster AS1063 presented in this Thesis (orange solid lines with diamonds) and in B19 (blue solid lines with crosses). (Adapted from Ragagnin et al. 2022). 97

- 8.1 *Hubble* Frontier Fields RGB (F105W+F125W+F140W+F160W, F606W+F814W, F435W) image of MACS J0416.1–2403 ID14. The two cluster members are identified as in B23 (IDs 8971 and 8980). The two components of the source are identified as 1 and 2. The four multiple images are indicated with the letters a, b, c, and d. (From Granata et al. 2023) 108
- 8.2 *Hubble* Frontier Fields RGB (F105W+F125W+F140W+F160W, F606W+F814W, F435W) image of MACS J0416.1–2403 ID16. The two cluster members are identified as in B23 (IDs 8785 and 9129). (From Granata et al. 2023) 110
- 8.3 *Hubble* Frontier Fields imaging of MACS J0416.1–2403 ID16 in two different bands. Upper panel: the two cluster members (ID 8785 and 9129) in the ACS F814W band. Lower panel: the multiple images in the ACS F435W band. The two components of the source are identified as 1 and 2. The four multiple images are indicated with the letters b, c, d, and e. The critical lines predicted by the model by B23 for a source at redshift $z = 2.095$ (the same as ID16) are marked in red. (From Granata et al. 2023) 111
- 8.4 CLASH RGB image (based on all 12 CLASH filters) of MACS J1206.2–0847 ID14. The three cluster members are identified as in B19 (IDs 3910, 3920, and 2541). The two components of the source are identified as 1 and 2. The three multiple images are indicated with the letters a, b, and c. (From Granata et al. 2023) 112
- 8.5 Fitting of the LOSVD of the MUSE spectrum of member 8971 with pPXF . The observed spectrum is shown in black, the red curve is the best-fit model, while the green points show the difference between the data and the model. The blue shaded regions along the wavelength axis were excluded in the fitting procedure, due to the presence of sky subtractions around emission lines or laser lines in the spectrum. (From Granata et al. 2023) 116
- 9.1 Projected cumulative total mass profiles for the lens galaxies studied in this Thesis compared with the predictions of B23 and B19. From the top to the bottom panel: member 8971, member 8785, and member 3910. The vertical bars indicate the inferred values of the truncation radius. Shaded regions indicate the 16th and 84th percentiles for the mass profile and the truncation radius. The projected distances of the observed multiple images from the lens centre are marked with vertical black lines. (Adapted from Granata et al. 2023) 122
- 9.2 Truncation radius of members 8971, 8785, and 3910 superimposed to the RGB cutout of the respective galaxy-scale SL system. We show the median value as a solid line and the 1σ uncertainty range with dashed lines. (From Granata et al. 2023) 123
- 9.3 Relation between the values of the velocity dispersion and of the truncation radius for the three cluster members included in this study (in blue) and for a satellite of SL2S J08544–0121, as measured by Suyu & Halkola (2010) (in red). The inferred uncertainty on the velocity dispersion of member 8785 is too small to be visible in this plot. (Adapted from Granata et al. 2023) 124

- 9.4 Comparison between the M -to- σ relation estimated by our analysis and those adopted for the cluster galaxies in the SL models of MACS J0416 (B23), AS1063 (B19,G22), MACS J1206 (B19), and A2744 (Bergamini et al. 2023a). Top panel: comparison with the SL models describing the cluster galaxies according to Eqs. (3.57) and (3.58). Bottom panel: comparison with the SL model of AS1063 by G22, based on the FP. (Adapted from Granata et al. 2023) 125
- 9.5 Comparison between the magnification values close to the three galaxy-scale lenses predicted in this Thesis and in B23 and B19. The colour map is based on the value of $\xi = \log \left(\left| \frac{\mu - \mu_{CS}}{\sigma_\mu} \right| \right)$ close to members 8971, 8785, and 3910, respectively. The red crosses indicate the observed positions of the multiple images included in this Thesis. (From Granata et al. 2023) 127
- 9.6 Comparison between the $v_{\max}$ -to- M relation obtained from our analysis and those predicted by the cosmological hydrodynamical simulations described in Ragagnin et al. (2022). The naming convention for the different suites is presented in Chapter 2. (Adapted from Granata et al. 2023) 128
- 9.7 Stellar mass of lens galaxies: comparison between this Thesis and the 85 SLACS lens galaxies presented in Auger et al. (2009). Upper panel: Stellar mass as a function of the velocity dispersion. Lower panel: Two-dimensional stellar-to-total mass fraction measured within the effective radius; the 68% confidence interval found by Grillo (2010) for a selected sample of 2×10^5 SDSS early-type galaxies is also shown. (Adapted from Granata et al. 2023) 129
- 10.1 Velocity dispersion function across cosmic time. Evolution with redshift of the σ function of 5500 quiescent galaxies from the Newfirm Medium Band Survey COSMOS and UKIDSS UDS fields (from Bezanson et al. 2012). 138
- 10.2 Velocity dispersion function in the field and in clusters. Comparison between the velocity function of quiescent galaxies in the field and in the Coma and Abell 209 clusters (from Sohn et al. 2017a). 139
- 10.3 Increase of the S/N . Percentage variation of the spectral S/N between the spectra extracted within a fixed aperture with a $R = 0.8''$ radius and with the weighted spectral extraction procedure described in this Chapter. We show here the variation for the cluster members of A2744 (top panel) and of MACS J0416 (bottom panel) for which we are able to measure the velocity dispersion. 141
- 10.4 CM relations for the four clusters included in our sample. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. Red and blue points are used to distinguish red cluster members. Diamond markers identify members with $n > 2.5$. Solid and dashed lines indicate the best-fit relation and a 1σ scatter (2σ for A2744). The best-fit values of the parameters a and b are reported above the plots. 143
- 10.5 Cluster members on which we base the calibration of the FP plotted around the CM relations for the four clusters included in our sample. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. Solid and dashed lines indicate the best-fit relation and a 1σ scatter (2σ for A2744). 145

- 10.6 Comparison between the values of the velocity dispersion σ measured using weighted MUSE spectra and $\sigma_{ap}(R_e) = \sigma_e$. We show here the comparison for the cluster members of A2744 (top panel) and of MACS J0416 (bottom panel) for which we are able to measure the velocity dispersion. 146
- 10.7 FP relations for the four clusters included in our sample in the projection $(\log R_e, \alpha \log \sigma_0 + \beta \text{SB}_e + \gamma)$. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. The solid lines indicate the best-fit FP relation, the black dot the observed members used for the plane calibration and the observational uncertainty. We report the values of the α and β parameters. 148
- 10.8 FP relations for the four clusters included in our sample in the projection $(\log \sigma_0, (\log R_e - \beta \text{SB}_e - \gamma) / \alpha)$. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. The solid lines indicate the best-fit FP relation, the black dot the observed members used for the plane calibration. We report the values of the α and β parameters. 149
- 10.9 Distribution of the members with $\sigma_0 < 100 \text{ km s}^{-1}$ around the FP. The relations for the clusters A2744 (top panel), and MACS J0416 (bottom panel) have been calibrated including only the selected members with $\sigma_0 > 100 \text{ km s}^{-1}$. The solid lines indicate the best-fit FP relation, the black dot the members used for the plane calibration, the red dot those that were excluded as their $\sigma_0 < 100 \text{ km s}^{-1}$. We report the values of the α and β parameters. 150
- 10.10 Distribution of all the red members of the clusters A2744 (top panel), and MACS J0416 (bottom panel) around the FP relation. The solid lines indicate the best-fit FP relation, the black dot the members used for the plane calibration, the red dot those that were excluded as their $n < 2.5$. We report the values of the α and β parameters. 151
- 10.11 Velocity dispersion function of the four clusters included in our sample. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. The red lines indicate the σ_0 function for all the red cluster members included in our morphological analysis. The orange lines indicate the σ_0 function for all the red cluster members for which we have a reliable velocity dispersion measurement. 152
- A.1 Difference between the measured and true values of σ , as a function of the true value, for the 10000 simulated MUSE spectra of cluster members described in Table A.1. We also report the polynomial fit that we use in this Thesis to correct the measured σ values for this effect. We mark the threshold of $\sigma = 30 \text{ km s}^{-1}$ with a vertical line. 163
- A.2 Relative difference between the measured and true values of σ , as a function of the spectral S/N , for the 10000 simulated MUSE spectra of cluster members. We mark the threshold of $S/N = 10$ with a vertical line. 164

List of Tables

6.1	Best-fit values and 1σ errors of the parameters of the FP.	73
6.2	Median values and 1σ errors of the parameters of our SL model, compared with those obtained in B19. The positions of the centres of the haloes are referred to the centre of the BCG.	75
6.3	rms and maximum likelihood $\mathcal{L}$ values for our model and for the one presented in B19.	76
6.4	Projected mass within a two dimensional radius $R = 350$ kpc of the baryonic components.	81
6.5	Comparison between the projected mass within a two-dimensional radius $R = 350$ kpc for the DM components in our model and in that by B19.	82
8.1	Reference lensing models of MACS J0416 (B23) and MACS J1206 (B19): relevant parameters. N_m and N_m^s indicate the number of cluster members included in the model and the number of those with measured velocity dispersion, respectively. N_i and N_s indicate the number of multiple images included in the model and the number of sources, respectively.	107
8.2	Multiple image positions for the GGSL system MACS J0416 ID14. In cols. 2 and 3, we report the relative positions of the multiple images with respect to the centre of cluster member 8971, for which we provide the values of R.A. and Dec of the centre. The images are identified as in Fig. 8.1. We also report the position of the centre of cluster member 8980.	109
8.3	Multiple image positions determined from HFF photometry for the GGSL system MACS J0416 ID16. In cols. 2 and 3, we report the relative positions of the multiple images with respect to the centre of cluster member 8785, for which we provide the values of R.A. and Dec of the centre. The images are identified as in Fig. 8.2. We also report the position of the centre of cluster member 9129.	110
8.4	Multiple image positions determined from CLASH photometry for the GGSL system MACS J1206 ID14. In cols. 2 and 3, we report the relative positions of the multiple images with respect to the centre of cluster member 3910, for which we provide the values of R.A. and Dec of the centre. The images are identified as in Fig. 8.4. We also report the position of the centre of the cluster member 2541.	112
8.5	Best-fit values and 1σ errors of the parameters describing the cluster member 8971 in the SL model of MACS J0416 by B23.	114

8.6	Best-fit values and 1σ errors of the parameters of our galaxy-scale SL models of MACS J0416 ID14. In the first column 8971 is described as a S1St, in the second one as a S1E.	115
8.7	Best-fit values and 1σ errors of the parameters of our galaxy-scale SL models of MACS J0416 ID16. In the first column 8785 is described as a S1St, in the second one as a S1E.	117
8.8	Best-fit values and 1σ errors of the parameters of our galaxy-scale SL models of MACS J1206 ID14. In the first column 3910 is described as a S1St, in the second one as a S1E.	118
A.1	Number of simulated spectra N of cluster members in six bins of velocity dispersion, and values intervals for σ , and S/N of the simulated spectra.	162

Motivation

Galaxy clusters are a crucial bridge between Astrophysics and Cosmology: they are privileged tracers of the formation of massive structures by hierarchical mergers and of the evolution of galaxies in dense environments. During their formation, smaller haloes are subsequently accreted into larger ones and they are subject to several physical processes: tidal stripping and heating can remove a significant fraction of their mass, and baryonic processes such as cooling or energy feedback can re-shape their internal structure and influence the star formation of the galaxy they host. As such, the dark matter (DM) mass distribution of galaxy clusters bears the traces of the processes that drive their formation and evolution, including, crucially, the micro-physics of DM and of its interplay with baryons. Cosmological hydrodynamical simulations trace the evolution of clusters, allowing us to test our hypotheses on the processes that shape them.

With the support of several dedicated photometric and spectroscopic surveys, strong gravitational lensing has become an extremely accurate probe of the total mass distribution in the cores (out to a few hundreds of kiloparsecs from the centre) of massive galaxy clusters (Caminha et al. 2016; Limousin et al. 2016; Bergamini et al. 2023b). When combined with baryonic mass probes, strong lensing allows us to disentangle the mass distribution of cluster- and galaxy-scale DM haloes, which can be compared to the predictions of high-resolution cosmological simulations. This makes galaxy clusters excellent astrophysical laboratories to test the Λ CDM paradigm and our hypotheses on the nature of DM (Grillo et al. 2015; Natarajan et al. 2017). Several recent studies have highlighted a discrepancy between the observed sub-halo mass distribution of cluster galaxies and the predictions of cosmological hydrodynamical simulations. Meneghetti et al. (2020) reported a significant discrepancy between the probability of observing galaxy-galaxy strong lensing events in massive galaxy clusters, as predicted by a sample of state-of-the-art strong lensing models of observed clusters, and by galaxy clusters of similar mass extracted from the most recent cosmological hydrodynamical simulations. This discrepancy can be interpreted as a consequence of observed member galaxies being more compact than their simulated counterparts. Identifying the origins of this discrepancy can shed light on galaxy formation and evolution in dense environments and point towards open issues with the Λ CDM paradigm.

This question calls for a careful analysis of the accuracy of strong lensing models in constraining the mass distribution properties of cluster galaxies, and for improvements to the techniques adopted to model them. The reconstruction of the mass structure of the member galaxies by strong lensing models of galaxy clusters is particularly sensitive

to parametric degeneracies that can bias the recovered compactness (Bergamini et al. 2019). The wealth of deep MUSE observations of the cores of lensing clusters is the key to overcoming this limitation, allowing us to probe observationally the stellar kinematics of the cluster galaxies, which provides independent information on their mass structure.

Overview of the objectives of this Thesis

Main objectives

In light of the motivations detailed above, the present Thesis is devoted to the study of the mass structure of the member galaxies of massive galaxy clusters. We proceed by enhancing the current strong lensing modelling techniques with information on the kinematics of the cluster galaxies obtained from MUSE integral-field spectroscopy. The main results of the Thesis are summarized below.

Improved strong lensing modelling of galaxy clusters using the Fundamental Plane:

We address an important limitation of strong lensing models of galaxy clusters: the mass distribution of the cluster sub-haloes is typically defined by two parameters – their velocity dispersion and their truncation radius – whose values are determined through power-law scaling relations with respect to the total observed luminosity of the galaxy they host. These relations are too simple to describe the mass distribution of the cluster members and are prone to the degeneracies noted above, which impact on the recovered galaxy compactness. We aim at improving the description of the mass structure of the cluster members by replacing the power-law relation adopted to connect their velocity dispersion to their total luminosity with a newly-calibrated Fundamental Plane relation (Dressler et al. 1987; Djorgovski & Davis 1987; Bender et al. 1992). To this purpose, we take advantage of the deep Hubble Frontier Fields photometry, and data from the Multi Unit Spectroscopic Explorer on the Very Large Telescope, with the objective of developing an improved strong lensing model of the cluster, that will allow us to decompose the cluster cumulative projected total mass profile into its stellar, hot gas and dark matter mass components.

Investigating the discrepancy in sub-halo compactness between observed and simulated galaxy clusters:

To test whether the new modelling techniques has a significant impact on the reported discrepancy between observed and simulated compactness, we will compare our predictions on the mass structure of the members with a suite of high-resolution hydrodynamical simulations of galaxy clusters, built from the Dianoga simulation set (Planelles et al. 2014; Rasia et al. 2015). We will probe the impact of the simulation set-up on the sub-halo compactness, testing a higher mass particle resolution and a different implementation of the sub-grid active-galactic-nuclei- and supernovae-driven baryonic feedback mechanisms – resulting in a more or less efficient suppression of gas over-cooling and star formation.

Insights into the mass structure of cluster galaxies with galaxy-scale strong lensing:

We focus on galaxy-galaxy strong lensing events within clusters as a following step in the study of their sub-halo mass distribution: these are strong lensing events in which a cluster member galaxy acts as a primary lens (multiple images of the same source are observed close to it). We will build strong lensing models for three lens galaxies within the massive clusters MACS J0416.1–2403 and MACS J1206.2–0847, reliably measuring their truncation radii without any scaling relation, thus avoiding any bias they might introduce.

Towards the calibration of the velocity dispersion function of the quiescent members of the Hubble Frontier Fields clusters: The stellar velocity dispersion is a good tracer of the total mass distribution of early-type galaxies, particularly of their central gravitational potential well, and thus of their compactness, and the recent, deep integral-field spectroscopic observations available for several strong lensing clusters open new possibilities to probe their sub-halo velocity function. We will perform the preliminary procedure required to calibrate the velocity dispersion function of the early-type members of the five Hubble Frontier Fields (Lotz et al. 2017) clusters for which deep MUSE data is available.

Organizational note

The present Thesis consists of five Parts, for a total of ten Chapters. Part I is composed of Chapters 1 and 2 and presents a phenomenological description of elliptical galaxies and of galaxy clusters, the main object of our studies. Part II spans from Chapter 3 to Chapter 5 and introduces the methodological tools, specifically strong gravitational lensing and stellar kinematics of elliptical galaxies, and the observations on which our research is based. Part III, composed of Chapters 6 and 7 presents our work to build a strong lensing model of the massive cluster Abell S1063 based on the Fundamental Plane. Part IV, or Chapters 8 and 9 describes the construction of strong lensing models for three galaxy-scale lenses within the massive clusters MACS J0416.1–2403 and MACS J1206.2–0847. Finally, Part V, which actually corresponds solely to Chapter 10, is dedicated to the presentation of our work in progress to calibrate the velocity dispersion function of the quiescent members of a sample of massive strong lensing clusters. Parts III and IV have appeared as refereed publications in scientific journals (Granata et al. 2022, 2023). Finally, Appendix A presents a set of simulated MUSE spectra of cluster galaxies that we have built in order to test the accuracy of our pipeline to measure the velocity dispersion of the cluster members. Some variations have been made in the presentation of previously published results, to maintain consistency of style and content structure through the manuscript.

In this Thesis, we adopt a flat Λ CDM cosmology with $\Omega_m = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. All magnitudes are expressed in the AB system.

List of Publications

As of 12 March 2024

Refereed publications

- D’Addona, M., Mercurio, A., Rosati, P., Grillo, C., Caminha, G. B., Acebron, A., Angora, G., Bergamini, P., Bozza, V., **Granata, G.** et al. 2024: “The powerful lens galaxy cluster PLCK G287.0+32.9 ($\theta_E \sim 43''$)”. A&A, forthcoming.
- Bergamini, P., Schuldt, S., Acebron, A., Grillo, C., Meštrić, U., **Granata, G.**, Caminha, G. B., Meneghetti, M., Mercurio, A., Rosati, P. et al. 2024: “Augmenting the power of time-delay cosmography in lens galaxy clusters by probing their member galaxies II. Cosmic chronometers”. A&A, 682, L2.
- Acebron, A., Schuldt, S., Grillo, C., Bergamini, P., **Granata, G.**, Meštrić, U., Caminha, G. B., Meneghetti, M., Mercurio, A., Rosati, P. et al. 2023: “Augmenting the power of time-delay cosmography in lens galaxy clusters by probing their member galaxies I. Type Ia supernovae”. A&A, 680, L9.
- **Granata, G.**, Bergamini, P., Grillo, C., Meneghetti, M., Mercurio, A., Meštrić, U., Ragagnin, A., Rosati, P., Caminha, G. B., Tortorelli, L., and Vanzella, E., 2023: “Exploring the low-mass regime of galaxy-scale strong lensing: Insights into the mass structure of cluster galaxies”. A&A, 679, A124.
- Meneghetti, M., Cui, W., Rasia, E., Yepes, G., Acebron, A., Angora, G., Bergamini, P., Borgani, S., Calura, F., Despali, G., et al. (including **Granata, G.**), 2023: “A persistent excess of galaxy-galaxy strong lensing observed in galaxy clusters”. A&A, 678, L2.
- Bergamini, P., Grillo, C., Rosati, P., Vanzella, E., Mestric, U., Mercurio, A., Acebron, A., Caminha, G. B., **Granata, G.**, Meneghetti, M. et al., 2022: “A state-of-the art strong lensing model of MACS J0416.1-2403 with the largest sample of spectroscopic multiple images”. A&A, 674, A79.
- Tortorelli, L., Mercurio, A., **Granata, G.**, Rosati, P., Grillo, C, Nonino, M., Acebron, A., Angora, G., Bergamini, P., Caminha, G.B. et al., 2023: “The Kormendy relation of early-type galaxies as a function of wavelength in Abell S1063, MACS J0416.1-2403, and MACS J1149.5+2223”. A&A, 671, L9.

- Meneghetti, M., Ragagnin, A., Borgani, S., Calura, F., Despali, G., Giocoli, C., Granato, G. L., Grillo, C., Moscardini, L., Rasia, E. et al. (including **Granata, G.**), 2022: “*The probability of galaxy-galaxy strong lensing in hydrodynamical simulations of galaxy clusters*”. A&A, 668, A188.
- Ragagnin, A., Meneghetti, M., Bassini, L., Ragone-Figueroa, C., Granato, G. L., Despali, G., Giocoli, C., **Granata, G.**, Moscardini, L., Bergamini, P. et al., 2022: “*Galaxies in the central regions of simulated galaxy clusters*”. A&A, 665, A16.
- **Granata, G.**, Mercurio, A., Grillo, C., Tortorelli, L., Bergamini, P., Meneghetti, M., Rosati, P., Caminha, G. B., and Nonino, M., 2022: “*Improved strong lensing modelling of galaxy clusters using the Fundamental Plane: Detailed mapping of the baryonic and dark matter mass distribution of Abell S1063*”. A&A, 659, A24.

Publications in conference proceedings

- Bergamini, P., Grillo, C., Rosati, P., Acebron, A., Vanzella, E., Mercurio, A., Meneghetti, M., Angora, G., **Granata, G.**, Meštrić, U. et al. 2023: “*High-precision strong lensing models of galaxy clusters in the JWST era*”. IAU Proceeding Series, accepted for publication.
- **Granata, G.**, Mercurio, A., Grillo, C., Tortorelli, L., Bergamini, P., Meneghetti, M., Ragagnin, A., Rosati, P., Caminha, G. B., and Nonino, M., 2023: “*Investigating the discrepancy in sub-halo compactness between observed and simulated galaxy clusters with improved strong lensing modelling*”. Memorie della SAIt, accepted for publication.
- **Granata, G.**, 2022: “*Improved strong lensing modelling of galaxy clusters using the Fundamental Plane: detailed mapping of the baryonic and dark matter mass distribution of Abell S1063*”. The Hypatia Colloquium 2022 book of proceedings.

Part I

Clusters of galaxies

In Part I, we introduce the basic observational and phenomenological framework to the study of strong lensing galaxy clusters and their members, the main target of the studies described in this Thesis.

A large part of these has been especially focused towards the description of the physical structure of the cluster galaxies: in Chapter 1, we provide a general, basic, treatment of the main morphological and dynamical properties of galaxies. Special attention is given to elliptical galaxies, which are the most frequent and massive amongst the cluster members.

In Chapter 2, we turn our attention to galaxy clusters, with a general description of their structure and mass composition. Several of the objectives of this Thesis are prompted by some discrepancies recently emerged between the physical properties of observed and simulated cluster galaxies. We thus also provide a general introduction to cosmological hydrodynamical simulations, to which we will refer when comparing their results to the observations.

Basic phenomenology and dynamics of elliptical galaxies

In this Chapter, we present the traditional Hubble morphological classification of galaxies, as a brief prelude and to introduce and describe some of the terms and concepts that will be used in the next Chapters. In the work presented throughout this Thesis, we study the physical properties and mass distribution of elliptical cluster member galaxies by enhancing strong gravitational lensing with the study of their stellar kinematics. As such, in this Chapter, we focus on the physical properties of elliptical galaxies, giving special attention to their dynamics and stellar kinematics. As a preliminary step, in the next Section, we provide a general introduction to the formation and evolution of galaxies in the currently accepted cosmological framework, which will allow us to link their properties to their history and thus to the processes that shape them.

1.1 Introduction to the formation and evolution of galaxies in the Λ CDM framework

The current observational picture on the large-scale inhomogeneities and on the expansion rate of the Universe supports a concordance cosmological model, dominated by a cosmological constant Λ , responsible for the accelerated expansion of the Universe, and with collisionless, cold (i.e. non-relativistic at the time of the equivalence between matter and radiation) DM particles (CDM Peebles 1982; Bond et al. 1982; Blumenthal et al. 1984). The anisotropy of the cosmic microwave background (CMB) shows the traces of an inhomogeneous Universe already during the recombination era ($z \sim 1000$), and the current three-dimensional distribution of galaxies, clusters of galaxies, superclusters, and voids is the result of the growth of the density fluctuations responsible for these initial inhomogeneities across cosmic time.

The amplitude of the CMB temperature anisotropy ($\Delta T/T \sim 10^{-5}$) is an indication of the correspondingly small amplitude of the density inhomogeneities at recombination. To describe quantitatively the relative amplitude of the density fluctuations in a position $\mathbf{r}$ and at a time t , the relative density contrast

$$\delta(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}, \quad (1.1)$$

is typically adopted, where $\rho(\mathbf{r}, t)$ is the matter density and $\bar{\rho}(t)$ its average across the Universe. In the lower-redshift Universe, we observe an average relative density contrast greater than 200 within a radius of 1.5 Mpc from the centre of massive galaxy clusters (these objects will be described in greater detail in Chapter 2), clearly indicating the growth of the inhomogeneities of the Universe. An over-dense region produces a stronger gravitational field, and will expand more slowly than the average, leading to a

slower-than-average decrease of the matter density. This entails an increase in the relative density contrast, which leads to an even stronger gravitational field, supporting the continuation of the growth of the density perturbation. A symmetric argument can explain the growth of the under-dense regions.

The statistical properties of the field of density fluctuations is encompassed by its Fourier transform, or power spectrum $P(k, t)$, where k is a co-moving wave number. As for the initial power spectrum $P_0(k)$, Harrison (1970), Zeldovich (1972), and Peebles & Yu (1970) first proposed a scale-invariant (slope $n_s = 1$) power-law form $P_0(k) \propto k^{n_s}$. The current picture presented in the Planck 2018 results (Planck Collaboration et al. 2020) instead finds, in the Λ CDM framework, a slope slightly smaller than 1 of $n_s = 0.965 \pm 0.004$. If DM is cold, as in the Λ CDM paradigm, no free-streaming is possible for its particles, allowing the density fluctuations to grow on all scales. Such power spectrum leads to a hierarchical, or bottom-up, growth of the density perturbations, with the smaller DM haloes forming first, and the most massive systems resulting from subsequent mergers between smaller ones.

In the initial regime $|\delta| \ll 1$, the linear treatment is a good approximation for the description of the growth of the density perturbations, but for $|\delta| \sim 1$, this description becomes inadequate. In the case of a perturbation power spectrum with $n_s \sim 1$, perturbations with a small length scale (or, correspondingly, above a certain wave number k^*), will be the first to evolve according to a non-linear collapse. For instance, in the case of an Einstein-de Sitter cosmology (with matter and cosmological constant density parameters $\Omega_m = 1$ and $\Omega_\Lambda = 0$, respectively), any spherical region with a relative density contrast $\delta_0 > \delta_C \sim 1.69(1+z)$ will be collapsed at redshift z^1 . As the density perturbations grow, the co-moving wavelength scale R that defines a co-moving volume within which the condition $\delta_R > \delta_C$ is true progressively increases. We can associate a mass M to a density perturbation with a wavelength R (and a density contrast $\delta_M = \delta_R$) as the mass within the co-moving volume $\frac{4}{3}\pi R^3$, to the density perturbation: this implies that haloes with increasing mass will experience non-linear collapse. This is responsible for the hierarchical formation of the DM haloes that host galaxies and clusters of galaxies, and all the smaller sub-haloes that surround them. Smaller-scale inhomogeneities in the collapsing structures give rise to fluctuations in the gravitational potential that scatter the infalling particles. More details on this process, referred to as violent relaxation (Lynden-Bell 1967), will be provided in the continuation of this Chapter and in Chapter 2.

The process we have just outlined is responsible for the formation of gravitationally-bound, virialised DM haloes on a large mass spectrum. In the transition between the $k \ll k^*$ and $k \gg k^*$ regimes, the perturbations grow quasi-linearly, giving rise to large-scale pancakes and filaments. The resulting DM mass distribution of the Universe is a complex “cosmic web”, where virialised haloes are found within a network of large-scale filaments within low-density regions, or voids. Modelling of non-linear spherical collapse allows for an estimate of the number density of DM haloes as a function of their mass: the Press-Schechter model (Press & Schechter 1974) estimates the mass fraction contained within collapsed haloes with a mass greater than a threshold M , at a given redshift z , based on the probability of having $\delta_M > \delta_C(1+z)$ for a given density field. In the case, described so far, of a Gaussian random field of density perturbations in a Λ CDM Universe with a power-law initial power spectrum, the Press-Schechter model

¹The value of δ_C can be estimated in a more general cosmological framework.

predicts a (co-moving) number density of relaxed DM haloes of mass M and redshift z

$$n(M, z) = \frac{\rho_{cr}\Omega_m}{\sqrt{\pi}} \frac{\gamma}{M^2} \left(\frac{M}{M^*(z)} \right)^{\gamma/2} \times \exp \left(- \left(\frac{M}{M^*(z)} \right)^{\gamma} \right), \quad (1.2)$$

where γ depends on the value of n_s . The Press-Schechter function is a power law in M for masses much smaller than a redshift-dependent threshold $M^*(z)$, and has an exponential cut-off for masses much larger than the threshold.

The linear and non-linear evolution of the cosmic density field is extremely complex and involves significantly different scales, and the problem is typically tackled with cosmological N -body simulations. In this Thesis, cosmological simulations will be used as a benchmark to compare our observational findings on galaxy clusters with theoretical predictions and expectations: as such, we dedicate a more thorough introduction to them in Chapter 2.

As shown by cosmological hydrodynamical simulations, baryonic particles, under the form of gas, initially follow the behaviour of DM. As the gas becomes denser, physical phenomena such as heating, friction, and cooling start to exert significant influence, leaving DM unaffected. The cooling of gas within haloes is the trigger of star formation, particularly in denser regions where the cooling rate is higher. Gas tends to accumulate due to dissipational processes, and the increased density makes star formation more efficient leading to the formation of disc-like galaxies (Rees & Ostriker 1977; White & Rees 1978; Blumenthal et al. 1984). These structures may then evolve into spiral galaxies: while some phenomenological information on their morphology will be given in the next Section, the description of their formation and evolution is outside of the scope of this Thesis.

Our picture of the formation of elliptical galaxies is extremely complex and still incomplete. As detailed in the continuation of this Chapter, elliptical galaxies are dynamically hot systems, which means that dissipational cool-down of gas is insufficient to justify their kinematics. As we will see, collisionless violent relaxation is a sensible framework to justify their current observed structure. Major mergers between disc galaxies of a similar mass can significantly perturb the kinematic state of galaxies and lead to the high values of stellar velocity dispersion observed for elliptical galaxies. They have been invoked as the main scenario for the formation of elliptical galaxies (Toomre 1977; Barnes 1989; Kormendy & Sanders 1992), accounting for several inconsistencies affecting a monolithic collapse scenario, where the stellar formation is expected to entirely coincide with the collapse of elliptical galaxies to their equilibrium state (Partridge & Peebles 1967; Larson 1975). However, mergers struggle to reproduce the observed properties of bright ellipticals, for which several studies suggest that dry (dissipationless) mergers of gas-poor elliptical progenitors is required (Khochfar & Burkert 2005; Nipoti et al. 2003). The relative lack of evolution of the luminosity (e.g. Kauffmann et al. 1993) and velocity dispersion functions for $z < 1$, as we will see in Chapter 10, suggests that formation of ellipticals takes place primarily at early cosmic times.

In conclusion, the observed properties of galaxies bear the traces of the formation and growth of cosmic structures, and of their evolution across cosmic times. In this Thesis, the study of the morphology and kinematics of galaxies will be adopted as a tool to gain insights on the processes that shape them, and on the underlying cosmological framework. In the next Section, we will lay out some fundamental facts on their observed morphology and structure, which will be relevant for the work we will present.

1.2 The morphological classification of galaxies

According to a traditional, photometry-based view, the classification of galaxies is based on their morphology as observed in the visible wavelengths and was proposed by Edwin Hubble (Hubble 1926, 1936). The classification is visually represented with a diagram, usually drawn as a tuning fork, showing elliptical, or early-type, galaxies on the left, lenticular galaxies as a transition group in the centre, and spiral, or late-type, galaxies on the right, separated into normal and barred spirals. The early- and late-type terminology is dated, as spiral galaxies are not the evolution of ellipticals with time as initially speculated, but is indeed still often adopted. All galaxies that do not belong to these categories are called irregular galaxies. While the classification was proposed based on the optical morphology of the galaxies, several other galactic properties concerning, for instance, their stellar population and gas content, have been found to correlate well with their Hubble category. The classification is typically codified as follows (Schneider 2006a):

- elliptical galaxies have elliptical isophotes and are categorised with the code E followed by a number $n = 10(1 - q)$, where q indicates the axis ratio of the isophotes. Much of this is caused by projection effects, but, as we will see, further categories have been introduced to classify ellipticals;
- spiral galaxies have a disc-like structure, with a central bulge and spirals. They are categorised with the codes S and SB depending on the presence of a central bar (normal or barred spirals), followed by the letter a, b, or c, depending on the brightness ratio between bulge and spiral (descending from a to c) and the definition and regularity of the arms (ascending from a to c);
- lenticular galaxies, typically coded as S0 or SB0 depending on the presence of a bar, are an intermediate category between the first two: disc galaxies, which do not show a spiral structure;
- irregular galaxies (Irr) is a category indicating galaxies with little or no regular structure.

Galaxies can also be classified according to the colour-magnitude relation and at their spectral features. On the colour-magnitude diagram, where the magnitudes of the galaxies in the r band are plotted against their colour, defined as $u - r$, galaxies show two prominent peaks: a red sequence of early-type at bright magnitudes and red colours, and one at fainter magnitudes an blue colours made up by late-type galaxies (e.g. Baldry et al. 2004b; Tortorelli et al. 2018, and Fig. 1.2). As noted by Baldry et al. (2004a), the two groups seem to be separated by the typical stellar mass scale of $2 - 3 \times 10^{10} M_{\odot}$. The spectrum of red galaxies shows little dust and very few signs of star formation. Blue galaxies are gas- and dust-rich and show emission lines, which can trace the stellar formations. An intermediate class comprises early-type, non-star-forming galaxies with strong Balmer absorption lines, indicating a younger or starburst stellar population (Girardi et al. 2015; Mercurio et al. 2004). Finally, Cappellari et al. (2011b) suggested that the morphological properties of the galaxies correlate with their kinematics, proposing an alternative classification into fast rotators (spirals and lenticular galaxies) and slow rotators (ellipticals).

1.2.1 Light profile of elliptical galaxies

Elliptical galaxies span a very large range in magnitude (B band absolute magnitude M_B from -8 to -25) and total mass ($10^7 - 10^{14} M_{\odot}$). The giant, intermediate, and compact

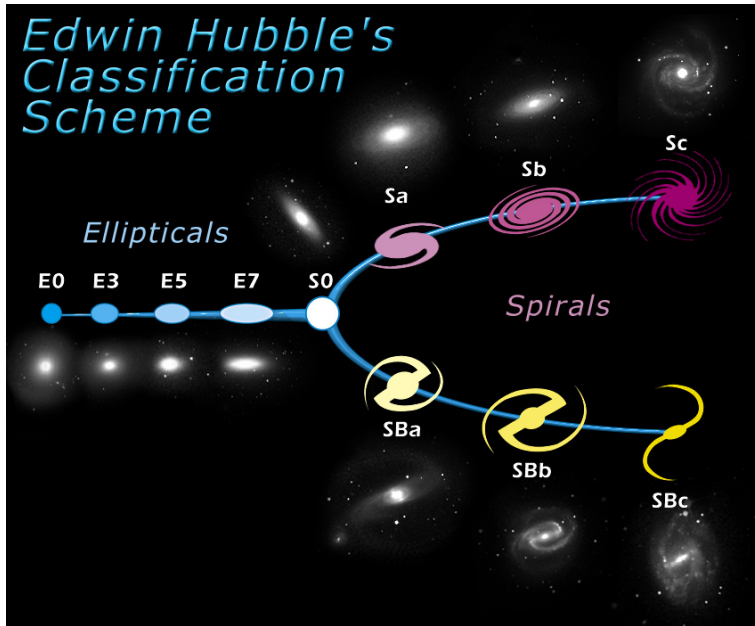


Figure 1.1: Schematic representation of the Hubble classification diagram. From esahubble.org

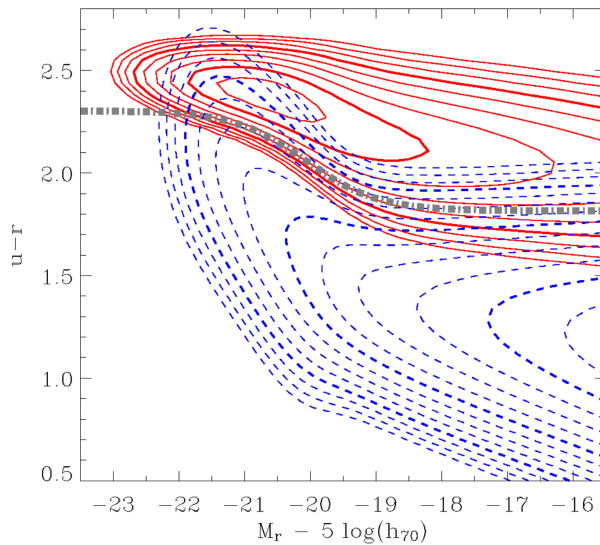


Figure 1.2: Colour-magnitude distribution of early- and late-type galaxies. Colour-magnitude relation for approximately 70,000 galaxies from the SDSS survey. The number distributions of the early- and late-type galaxies have been marked in red and blue, respectively. From Baldry et al. (2004b).

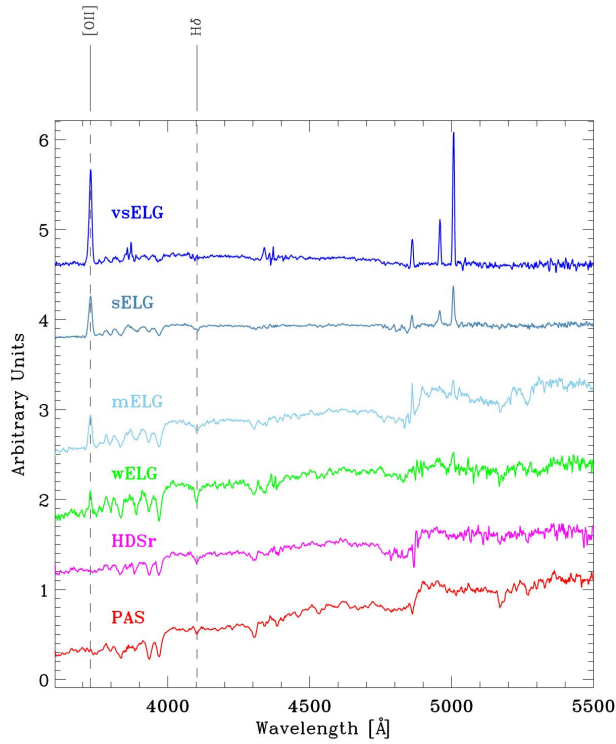


Figure 1.3: Spectra of early- and late-type galaxies. Combined rest-frame spectra of several galaxies of the cluster MACS J1206.20847 ordered in six different spectral types. From Girardi et al. (2015).

elliptical galaxies are usually grouped as “normal ellipticals” (Schneider 2006a). They are distinct from the “dwarf ellipticals”, small galaxies with a very low surface brightness and a high metallicity, and from the “dwarf spheroidals”, the faintest of the group, with an even lower surface brightness and a high DM fraction. On the opposite end of the spectrum, the cD galaxies are extremely bright, large (more than 100 kpc in radius), and massive ellipticals, often with active galactic nuclei, generally found close to the centre of galaxy clusters. This category includes the brightest cluster galaxies (BCGs).

Taking advantage of the axial symmetry of elliptical galaxies, we can define their surface brightness profile as a function of R , the length of the isophotal semi-major axis. In this Thesis, we will mostly focus on normal ellipticals and cDs. The surface brightness profile $I(R)$ of the former follows the De Vaucouleurs Law (de Vaucouleurs 1948), a particular case of the Sérsic Profile (Sérsic 1963)

$$I(R) = I_0 \exp \left(-b(n) \left(\frac{R}{R_e} \right)^{\frac{1}{n}} \right) = I_e \exp \left(-b(n) \left(\frac{R}{R_e} \right)^{\frac{1}{n}} - 1 \right), \quad (1.3)$$

where n is a concentration parameter, and $b(n)$ is a coefficient (Ciotti & Bertin 1999). I_0 is the central surface brightness, R_e is the effective radius, $I_e = I(R_e) = I_0 \exp(-b(n))$. The De Vaucouleurs Law is obtained from the Sérsic Profile of Eq. (1.3) in the case of $n = 4$, with $b(4) \approx 7.67$. Unlike normal ellipticals, cDs show a significant luminosity excess with respect to the De Vaucouleurs Law in its outskirts (Schombert 1986).

The total luminosity of a source described by a Sérsic surface brightness profile can be obtained from Eq. (1.3) as $L = \int_0^\infty I(R) 2\pi R dR$. If we consider an isotropic flux F emitted by a galaxy of size-radius R_g at angular-diameter and luminosity distances $D_A(z_g)$ and $D_L(z_g)$, then $F = \frac{L}{4\pi D_L^2}$. If we call $\Omega = \frac{\pi R_g^2}{D_A^2}$ the solid angle occupied by the in the sky, then the mean surface brightness will be

$$\langle I \rangle = \frac{F}{\Omega} = \frac{L}{\pi R_e^2} (1+z)^{-4}, \quad (1.4)$$

where we exploited $\frac{D_A^2}{D_L^2} = (1+z)^{-4}$. As R_e is the half-light radius, we have

$$\langle I \rangle_e = \frac{F}{\Omega} = \frac{L}{2\pi R_e^2} (1+z)^{-4}, \quad (1.5)$$

where we indicate as $\langle I \rangle_e$ the mean surface brightness within R_e ². The Sérsic Profile for I can also be expressed in the units of mag arcsec², defined as $\mu = -2.5 \log(I)$,

$$\mu(R) = \mu_0 + 1.086 b(n) \left(\frac{R}{R_e} \right)^{\frac{1}{n}} = \mu_e + 1.086 b(n) \left(\left(\frac{R}{R_e} \right)^{\frac{1}{n}} - 1 \right), \quad (1.6)$$

where $\mu_0 = -2.5 \log(I_0)$ and $\mu_e = -2.5 \log(I_e)$.

1.3 Dynamics of elliptical galaxies

In this Section, we briefly describe the stellar dynamics of elliptical galaxies, which are intimately linked to their mass distribution and can be probed, as we will see in Chapter

²Note that $\langle I \rangle_e \neq I_e$. For instance, if $n = 4$, then $\langle I \rangle_e \approx 3.61 I_e$.

5, from observing their spectra. First and foremost, it is important to note that ellipticals are “hot” stellar systems: the kinematics of the stars are dominated by chaotic motions, and their velocity dispersion is much larger than their rotation velocity $\sigma > v_{\text{rot}}$. The flattening of ellipticals is thus not caused by their rotation.

1.3.1 Elliptical galaxies as violently relaxed collisionless systems

We now ask ourselves whether elliptical galaxies are relaxed (i.e. thermalised) systems, where the two-bodies interactions between stars have driven their velocity distribution towards an isotropic Maxwell-Boltzmann function. If we define the relaxation time from two-bodies collisions as the time required for a star to deflect its trajectory by 90° due to pair interactions, we find that (Chandrasekhar 1942; Schneider 2006a)

$$t_{\text{rel}} \approx \frac{v^3}{8\pi n G m^2 \ln N}, \quad (1.7)$$

where v is the typical velocity of stars, n is their number density, m is their typical mass and N is their number. Equation (1.7) can be re-written as

$$t_{\text{rel}} \approx \frac{R}{v} \frac{N}{\ln N} = t_{\text{cross}} \frac{N}{\ln N}, \quad (1.8)$$

where R is the size of the stellar system. Here, we have introduced $t_{\text{cross}} = \frac{R}{v}$, the typical timescale for a star to cross the stellar system. For typical values of $R = 30 \text{ kpc}$ and $v = 200 \text{ km s}^{-1}$, then $t_{\text{cross}} \approx 10^8 \text{ yr}$. Taking $N = 10^{12}$, then $\ln N \approx 30$, which leads to a t_{rel} which is significantly larger than the age of the Universe (Hubble time): this indicates that two-body interactions between stars are extremely rare and do not account for relaxation. Stellar systems are thus collisionless systems and must be accordingly treated. The ellipticity results from the anisotropy of the distribution of the stars in the velocity space (i.e. different values of σ in different directions).

Despite their very low degree of collisionality, galaxies are still subject to relaxation processes known as collisionless violent relaxation (Lynden-Bell 1967), as shown by cosmological simulations. During the gravitational collapse that leads to the formation of galaxies, the high granularity in the mass distribution and the instabilities that emerge during the collapse determine quick fluctuations of the gravitational potential that deflect the trajectories of the galaxies. This type of processes takes place on a dynamical time-scale (i.e. on the order of t_{cross}). The resulting re-distribution of energy leads to a quasi-relaxed state that approximately satisfies the virial equation (which will be introduced more in detail) and simulations suggest that the predicted light profile is well described by the De Vaucouleurs Law (van Albada 1982). As a consequence, elliptical galaxies are approximately described by an isotropic Maxwell-Boltzmann velocity distribution function near the core, while the stars in the envelope have mostly radial orbits (van Albada 1982).

1.3.2 The Jeans Equation

In this Sub-section, we introduce the dynamical equations that can be used to describe the motions of the stars in a galaxy, seen as a collisionless system under the effect of its own gravity (for a more complete treatment, see Binney & Tremaine 1987; Bertin 2014; Mo et al. 2010).

A collisionless system of many particles whose dynamics are regulated by the potential $\Phi(\mathbf{x}, t)$, is described by a probability density distribution function $f(\mathbf{x}, \mathbf{v}, t)$ in the phase space $(\mathbf{x}, \mathbf{v})$. Its evolution is described by the collisionless Boltzmann Equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^3 v_i \frac{\partial f}{\partial x_i} - \sum_{i=1}^3 \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0, \quad (1.9)$$

where $i = 1, 2, 3$ indicates the three spatial directions. Integrating Eq. (1.9) over $\mathbf{v}$, we have

$$\frac{\partial}{\partial t} \int f d^3 \mathbf{v} + \sum_{i=1}^3 \frac{\partial}{\partial x_i} \int f v_i d^3 \mathbf{v} - \sum_{i=1}^3 \frac{\partial \Phi}{\partial x_i} \int \frac{\partial f}{\partial v_i} d^3 \mathbf{v} = 0. \quad (1.10)$$

Assuming f vanishes for large velocities, the divergence Theorem implies that the third term is zero. By definition, the particle spatial density is

$$\nu(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{v}, t) d^3 \mathbf{v}, \quad (1.11)$$

while the mean velocity in the direction of the i -th coordinate is

$$\langle v_i \rangle = \frac{1}{\nu} \int v_i f(\mathbf{x}, \mathbf{v}, t) d^3 \mathbf{v}. \quad (1.12)$$

The substitution of these quantities into Eq. (1.10) allows us to re-write it as

$$\frac{\partial \nu}{\partial t} + \sum_{i=1}^3 \frac{\partial}{\partial x_i} (\nu \langle v_i \rangle) = 0, \quad (1.13)$$

which is a continuity equation for ν . Similarly, multiplying Eq. (1.10) for v_j and taking the integral in $d^3 \mathbf{v}$, we obtain

$$\frac{\partial}{\partial t} \nu \langle v_j \rangle + \sum_{i=1}^3 \frac{\partial}{\partial x_i} (\nu \langle v_i v_j \rangle) + \nu \frac{\partial \Phi}{\partial x_j} = 0, \text{ for } j = 1, 2, 3, \quad (1.14)$$

where $\langle v_i v_j \rangle$ can be re-written as $\langle v_i \rangle \langle v_j \rangle + \sigma_{ij}^2$. The Jeans Equation is now obtained by subtracting from Eq. (1.14) the product between Eq. (1.13) and $\langle v_j \rangle$

$$\nu \frac{\partial \langle v_j \rangle}{\partial t} + \nu \sum_{i=1}^3 \langle v_i \rangle \frac{\partial \langle v_j \rangle}{\partial x_i} = - \frac{\partial (\nu \sigma_{ij}^2)}{\partial x_i} - \nu \frac{\partial \Phi}{\partial x_j}, \text{ for } j = 1, 2, 3. \quad (1.15)$$

It is useful to re-write the Jeans equation in spherical coordinates (r, θ, ϕ) . The radial component of the Equation becomes

$$\begin{aligned} \nu \frac{\partial \langle v_r \rangle}{\partial t} + \nu \left(\langle v_r \rangle \frac{\partial \langle v_r \rangle}{\partial r} + \frac{\langle v_\theta \rangle}{r} \frac{\partial \langle v_r \rangle}{\partial \theta} + \frac{\langle v_\phi \rangle}{r \sin \theta} \frac{\partial \langle v_r \rangle}{\partial \phi} \right) = & - \frac{\partial (\nu \sigma_{rr}^2)}{\partial r} - \frac{1}{r} \frac{\partial (\nu \sigma_{r\theta}^2)}{\partial \theta} - \\ & \frac{1}{r \sin \theta} \frac{\partial (\nu \sigma_{r\phi}^2)}{\partial \phi} - \frac{\nu}{r} (2\sigma_{rr}^2 - \sigma_{\theta\theta}^2 - \sigma_{\phi\phi}^2 - \langle v_\theta \rangle^2 - \langle v_\phi \rangle^2 + \sigma_{r\theta}^2 \cot \theta) - \nu \frac{\partial \Phi}{\partial r}, \end{aligned} \quad (1.16)$$

where $v_r = \frac{\partial r}{\partial t}$, $v_\theta = r \frac{\partial \theta}{\partial t}$, and $v_\phi = r \sin \theta \frac{\partial \phi}{\partial t}$.

1.3.3 The Jeans Equation for a spherically symmetric system in hydrostatic equilibrium

Equations (1.15,1.16) have the form of the Euler Equation for a density ν and a velocity v , where the isotropic pressure term is replaced by the stress tensor $\nu\sigma^2$. Equations (1.13) and (1.15,1.16) make up a system of four equations for the six components of σ^2 and the three of x . This means that the equations do not form a closed system and thus cannot be solved except for the cases in which the symmetry of the system allows for a simplification. For instance, if the system is in equilibrium $\langle v_r \rangle = \frac{\partial \langle v_r \rangle}{\partial r} = 0$. If the system is also spherically symmetric, then $\langle v_\theta \rangle = \langle v_\phi \rangle = \sigma_{r\theta}^2 = \sigma_{r\phi}^2 = \sigma_{\phi\theta}^2 = 0$, $\langle v_r^2 \rangle = \sigma_{rr}^2$, and $\langle v_\theta^2 \rangle = \langle v_\phi^2 \rangle = \sigma_{\theta\theta}^2 = \sigma_{\phi\phi}^2$. In this situation, Eq. (1.16) becomes

$$\frac{d(\nu \langle v_r^2 \rangle)}{dr} + 2\frac{\nu}{r} (\langle v_r^2 \rangle - \langle v_\theta^2 \rangle) = -\nu \frac{d\Phi}{dr}. \quad (1.17)$$

The necessity of adopting symmetry or simplification arguments to solve the Jeans equation is the main limitation to its description of real stellar systems. Elliptical galaxies are ellipsoidal, but spherical modelling is widely used (Binney & Mamon 1982; Agnello et al. 2014) to avoid over-complicating the modelling in light of the large uncertainty on the observed kinematics and of the shape of the galaxy mass distribution. In this approximation, we can easily link the Jeans Equation to observable quantities. It is convenient to define the anisotropy parameter $\beta(r) = 1 - \frac{\langle v_\theta^2 \rangle}{\langle v_r^2 \rangle}$, to re-write Eq. (1.17) as

$$\frac{d(\nu \langle v_r^2 \rangle)}{dr} + 2\frac{\nu}{r} \beta(r) \langle v_r^2 \rangle = -\nu \frac{d\Phi}{dr}. \quad (1.18)$$

While $\nu(r)$ is the stellar mass density, $\phi(r)$ is the total gravitational potential, so $\frac{d\Phi}{dr} = \frac{Gm(r)}{r}$. If we define $J_\beta(r, r') = \int_r^{r'} \frac{2\beta(s)}{s} ds$ and integrate Eq. (1.18) with respect to r , we obtain the radial velocity dispersion

$$\langle v_r^2 \rangle(r) = \sigma_{rr}^2 = \frac{G}{\nu(r)} \int_r^\infty \frac{m(r')\nu(r')}{r'^2} J_\beta(r, r') dr'. \quad (1.19)$$

From Eq. (1.18), we also derive the circular velocity (Binney & Mamon 1982)

$$v_c(r) = \frac{Gm(r)}{r} = -\langle v_r^2 \rangle \left(\frac{d \ln \nu}{d \ln r} + \frac{d \ln \langle v_r^2 \rangle}{d \ln r} + 2\beta \right). \quad (1.20)$$

With spectroscopic observations, we probe the line-of-sight (LOS) velocity distribution along the optical axis z . If we take a star at a distance r from the galaxy centre, we can link $\langle v_{\text{LOS}}^2 \rangle$, the expected value of the squared velocity along the LOS, to $\langle v_r^2 \rangle(r)$ and $\langle v_\theta^2 \rangle(r)$, and to the angle α between the LOS and the radius on which the star lies, namely $\langle v_{\text{LOS}}^2 \rangle = \langle (v_r \cos \alpha + v_\theta \sin \alpha)^2 \rangle = \langle v_r^2 \rangle \cos^2 \alpha + \langle v_\theta^2 \rangle \sin^2 \alpha$. Weighing this last relation with the projected light profile $I(R) = 2 \int_R^\infty \frac{\nu(r)r}{\sqrt{r^2 - R^2}} dr^3$, we obtain the LOS velocity dispersion profile

$$\begin{aligned} \sigma_{\text{LOS}}^2(R) &= \frac{1}{I(R)} \int_{-\infty}^\infty (\langle v_r^2 \rangle \cos^2 \alpha + \langle v_\theta^2 \rangle \sin^2 \alpha) dz = \\ &= \frac{2}{I(R)} \int_R^\infty \left(1 - \beta(r) \frac{R^2}{r^2} \right) \nu(r) \langle v_r^2 \rangle(r) \frac{r}{\sqrt{r^2 - R^2}} dr. \end{aligned} \quad (1.21)$$

³In this passage and others hereafter, we project 3D functions to the galactic plane (x, y) using the Abel's transform $F(R) = \int_{-\infty}^\infty f(r) dz = 2 \int_0^\infty f(r) dz = 2 \int_R^\infty \frac{f(r)r}{\sqrt{r^2 - R^2}} dr$.

We can replace Eq. (1.19) in Eq. (1.21) to obtain

$$\sigma_{LOS}^2(R) = \frac{2G}{I(R)} \int_R^\infty \frac{\nu(r)m(r)}{r^2} \left(\sqrt{r^2 - R^2} + k_\beta(r, R) \right) dr, \quad (1.22)$$

where $k_\beta(r, R) = \int_R^r \frac{(2r'^2 - 3R^2)\beta(r')J_\beta(R, r')}{r'\sqrt{r'^2 - R^2}} dr'$. As will be detailed in Chapter 5, the LOS velocity dispersion is measured by studying the spectral energy distribution of galaxies; spectroscopic observations are usually performed by collecting the spectrum through either a slit or a circular fibre⁴. In this case, we measure the LOS velocity dispersion, as defined in Eq. (1.22), weighted by the stellar light distribution within an aperture

$$\sigma_{ap}^2(R) = \frac{2\pi}{L(R)} \int_0^R R' I(R') \sigma_{LOS}^2(R') dR' = \quad (1.23)$$

$$= \frac{4\pi G}{L(R)} \int_0^R R' \int_{R'}^\infty \frac{\nu(r)m(r)}{r^2} \left(\sqrt{r^2 - R'^2} + k_\beta(r, R') \right) dr dR', \quad (1.24)$$

where $L(R) = \int_0^\infty 2\pi R' I(R') dR'$ is the total galaxy luminosity.

The choice of the anisotropy profile $\beta(r)$ is crucial for dynamical studies. As clear from the previous equations, the observed velocity dispersion profile is not enough to determine the mass profile without hypotheses on the anisotropy of the stellar orbits. This is also known as the “mass-orbit” or “mass-anisotropy” degeneracy. The degeneracy is typically broken either with the assumption of constant anisotropy (or even isotropic orbits), or with other physically motivated anisotropy profiles. As anticipated, violent collisionless relaxation is expected to lead to isotropic orbits ($\beta = 0$) in the galactic core and radially-biased orbits ($\beta \approx 1$) in the outskirts. This is well fitted with the Osipkov-Merritt anisotropy profile (Osipkov 1979; Merritt 1985).

1.3.4 Connecting the kinematics of galaxies to their mass distribution

The Virial Theorem

An important consequence of the previously described collisionless violent relaxation is that the system is quasi-virialised, which means that the Virial Theorem approximately holds. We will omit here a full proof of the Virial Theorem (for a more detailed treatment, see e.g. Mo et al. 2010), which can be obtained from the second moment with respect to $x_i x_j$ of Eq. (1.14), and then expressed in a scalar form by taking the trace of the resulting tensors

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2K + U + \Sigma, \quad (1.25)$$

where I is the moment of inertia of the system, K and U are its total kinetic and potential energy, respectively, and Σ the work from the outside of the system. For a static and isolated system, the first and the last terms are zero, thus

$$2K + U = 0, \quad (1.26)$$

$$E = K + U = -K = \frac{U}{2}, \quad (1.27)$$

⁴Alternatively, integral-field spectroscopy is also possible, but in the case of cluster galaxies, the low signal-to-noise of the observations typically still requires the selection of all pixels within a circular aperture to measure the velocity dispersion.

where E is the total energy of the system. In the case of an elliptical galaxy for which we use a spherical, isotropic model, then $\langle v^2 \rangle = \sigma^2$. It follows that

$$K = \frac{1}{2} \sum_{i=1}^N m_i \sigma_i^2 = \frac{1}{2} M_{\text{tot}} \langle \sigma^2 \rangle = \frac{3}{2} M_{\text{tot}} \langle \sigma_{\text{LOS}}^2 \rangle, \quad (1.28)$$

where N is the number of particles, M_{tot} is the total galaxy mass, and we have taken advantage of the isotropy of the system to substitute $\sigma^2 = 3\sigma_{\text{LOS}}^2$. In Eq. (1.28), the values of σ^2 and $3\sigma_{\text{LOS}}^2$ are mass-weighted averages. However, elliptical galaxies are not found to be thermalised due to the lack of collisionality, which allows us to approximate $\langle \sigma_{\text{LOS}}^2 \rangle = \sigma_{\text{LOS}}^2$. As for the total potential energy

$$U = - \sum_{i>j} \frac{Gm_i m_j}{r_{ij}} = - \frac{GM_{\text{tot}}}{r_g}, \quad (1.29)$$

where $r_g = 2m^2 \left(\sum_{i \neq j} \frac{m_i m_j}{r_{ij}} \right)^{-1}$, is a typical scale of the stellar system, and r_{ij} is the distance between the i -th and j -th star. From the virial theorem, we obtain

$$M_{\text{tot}} = \frac{3\sigma_{\text{LOS}}^2 r_g}{G}. \quad (1.30)$$

This relation links the total mass of the galaxy to the LOS velocity dispersion of its stars.

In Chapter 5, we will present a method to link the properties of the spectral features of an elliptical galaxy to its LOSVD to obtain a dynamical estimate of the total galaxy mass through Eq. (1.30). This approach will require us to assume that elliptical galaxies are spherically symmetric and, as we will see, that the stellar velocity dispersion measured from the spectral properties of the observed galaxies is an effective probe of σ_{LOS} . The impact of these approximations will be discussed while presenting our studies and results in Chapters 6-10. In this Thesis, we will compare and combine mass measurements from strong lensing and stellar kinematics, which will help us break the degeneracies that affect them singularly.

The total mass profile of elliptical galaxies

As previously detailed, violent collisionless relaxation leads to a velocity distribution for the elliptical galaxies, which can be well approximated by a Maxwell-Boltzmann function. We pass to the continuous limit defining the mass density profile $\rho(r) = \langle m \rangle n(r)$, where $\langle m \rangle$ is an average star mass and n the stellar number distribution. A system of stars with such a distribution will be in dynamical equilibrium if the gravitational attraction on a mass element $\rho(r)dV$ is balanced by the gradient of the dynamical pressure $p(r) = \rho(r) \frac{\langle v^2 \rangle}{3} = \rho(r) \sigma^2$, where $\sigma^2 = \sigma_{\text{LOS}}^2$, and the last relation is motivated by the isotropic velocity distribution. The dynamical equilibrium equation in the radial direction thus becomes

$$\frac{d}{dr} \left(\sigma^2 \frac{d\rho(r)}{dr} \right) + 4\pi G r^2 \rho(r) = 0, \quad (1.31)$$

also known as the Lane-Emden equation. An analytical solution of the Lane-Emden equation is the singular isothermal sphere (SIS) mass density profile

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \quad (1.32)$$

where σ is the only free parameter.

As elliptical galaxies lack an obvious extended kinematic tracer, analogous to the atomic hydrogen emission of spiral galaxies, obtaining convincing dynamical evidence of the presence of DM was a less straightforward process (Bertin & Stiavelli 1993; Gerhard et al. 2001; Bacon et al. 2001; Cappellari et al. 2006, 2011a). The formation of DM haloes can be thought of as the result of similar collisionless processes. As such, we expect that the total mass distribution of elliptical galaxies will be described by Eq. (1.32). The combination of strong lensing and stellar kinematics has confirmed this picture (Koopmans & Treu 2002; Treu & Koopmans 2004; Bolton et al. 2006; Treu et al. 2006; Grillo et al. 2008, 2009; Grillo 2012; Shajib et al. 2021). This power-law profile is observed even though neither the stellar nor the DM components follow an isothermal profile if taken singularly (bulge-halo conspiracy), as we will see in the continuation of this Thesis.

1.4 The Fundamental Plane of elliptical galaxies and its projections

1.4.1 The Fundamental Plane

Scaling relations between the main physical quantities and observables describing galaxies are a precious tool to guide galaxy evolution models and, as we will see, instrumental in reducing the number of parameters in strong lensing models of galaxy clusters. Observations have shown that early-type galaxies are described by a tight scaling relations linking the kinematic properties, which descend from their total mass distribution to the luminosity, which depends on the distribution of the stars. This empiric law is known as the Fundamental Plane (FP) and links the central LOS velocity dispersion σ_0 (measured within an aperture of $R_e/8$) to the effective radius R_e and the average surface brightness within the effective radius $\langle I \rangle_e$ (Dressler et al. 1987; Djorgovski & Davis 1987; Bender et al. 1992)

$$R_e \propto \sigma_0^\alpha \langle I \rangle_e^\delta. \quad (1.33)$$

The FP is typically presented in its logarithmic form, where $\langle \mu \rangle_e$ is often indicated as SB_e

$$\log R_e = \alpha \log \sigma_0 + \beta SB_e + \gamma. \quad (1.34)$$

We know that $\alpha \approx 1.4$, depending on the band, and $\beta \approx 0.34$, implying $\delta \approx -0.85$ (Jorgensen et al. 1996). The FP can be justified by considering that $L = 2\pi R_e^2 \langle I \rangle_e \propto R_e^2 \langle I \rangle_e$. Furthermore, Eq. (1.30) implies that $M_{\text{tot}} \propto \sigma^2 R$. As such, $\frac{M_{\text{tot}}}{L} \propto \frac{\sigma^2}{R_e \langle I \rangle_e}$. The FP then implies $\frac{M_{\text{tot}}}{L} \propto \frac{\sigma^{0.6}}{\langle I \rangle_e^{0.15}}$, which can be re-written as

$$\frac{M_{\text{tot}}}{L} = \kappa L^{0.25}. \quad (1.35)$$

This discrepancy from a constant mass-to-light ratio for elliptical galaxies is known as the tilt of the FP, and can be interpreted in two ways. If there is a homology in the equilibrium state of elliptical galaxies (κ is approximately the same for all of them), then their mass to light ratio depends on luminosity and is higher for more luminous galaxies. If, instead, we assume that all ellipticals have the same stellar population (M_{tot}/L is approximately the same for all of them), then the virialisation state depends on the luminosity of the galaxy. Although some studies find a prominent role for non-homology in causing the tilt of the FP (Trujillo et al. 2004) the current consensus is that this is a secondary factor. The leading cause of the tilt is a combination of systematic variations in the stellar population and in the DM concentration, with high-luminosity galaxies being

inhabited by stars with a higher $\frac{M}{L}$ and having a higher DM fraction (Cappellari et al. 2006; Grillo & Gobat 2010). More details on the calibration and on applications of the FP to strong lensing models will be given in Part III.

The Faber-Jackson Law

Before the introduction of the FP, the Faber-Jackson Law (Faber & Jackson 1976) was calibrated as a power-law relation between the total luminosity and the central velocity dispersion of elliptical galaxies

$$L \propto \sigma_0^\zeta, \quad (1.36)$$

with $\zeta \approx 4$, which implies a FP with $\alpha = 2$ and $\beta = -0.5$. The discrepancy with the actual values is indicative of the fact that the Faber-Jackson Law is not an edge-on projection of the FP, and has large, correlated residuals.

The Kormendy Relation

Another projection of the FP is the Kormendy relation (Kormendy 1977), between R_e and the average surface brightness within $\langle I \rangle_e$. It is often presented in logarithmic form

$$\langle \mu_e \rangle = \nu \log R_e + \xi, \quad (1.37)$$

with $\nu \approx 3.25$ in the optical bands, indicating that larger and brighter ellipticals have a lower surface brightness. This reflects the difference in the origin of bright and faint ellipticals and bulges. In Tortorelli et al. (2023), building upon the results of (Tortorelli et al. 2018), we study the Kormendy relations of the members of three massive galaxy clusters, Abell S1063, MACS J0416.1–2403, and MACS J1149.5+2223, as a function of wavelength, using very deep Hubble Space Telescope photometry. The simultaneous measurement of the magnitude and effective radius values of a high number of galaxies required the development of a new Python package called MORPHOFIT (Tortorelli & Mercurio 2023).

Basic phenomenology and cosmological simulations of galaxy clusters

In this Chapter, we give a brief phenomenological introduction to the main components and to the properties of galaxy clusters, which are the main target of the studies presented here. This Thesis especially focuses on investigating the physical structure of the cluster member galaxies: we will detail the properties of this population of galaxies, which result from a different evolution history compared to that of field galaxies, due to the effects of the high environmental density.

Galaxy clusters have emerged as crucial astrophysical laboratories, allowing us to test the physical implications of our cosmological hypotheses. Several works, including some to which this Thesis has contributed, have adopted cosmological simulations as a tool to perform comparative tests between our theoretical expectations, descending from our cosmological paradigms, and observations. We provide a brief introduction to the objectives and implementation of cosmological hydrodynamical simulations of massive clusters, to which we will reference while comparing our results with simulations.

2.1 Basic phenomenology of galaxy clusters

Redshift surveys clearly demonstrate that galaxies are not uniformly distributed in the Universe, and tend to create over-dense regions of galaxies referred to as groups or clusters of galaxies. Groups and clusters are found at the crossings of the DM-dominated large scale structures. Galaxy clusters are typically distinct from groups by being made up of more than 50 galaxies within a radius of 1.5 Mpc, and are the most massive gravitationally bound structures in the Universe.

2.1.1 Morphology and mass components of galaxy clusters

Galaxy clusters have a total mass in the range $10^{14} - 10^{15} M_{\odot}$, inferred, for instance, from the kinematics of the galaxies they host. Since the first observations of the kinematics of the member galaxies within clusters, it was clear that stars in member galaxies can only provide a relatively small fraction of their total mass (Smith 1936; Zwicky 1937). Around 95% of their baryonic mass actually comes from a very hot plasma, the intra-cluster medium (ICM), whose temperature is generally at least 10^7 K. The ICM mass is itself only about 10 – 15% of the total mass: the remaining 85 – 90% of the mass budget is under the form of DM. Dark matter in clusters is found both in diffuse cluster-scale haloes and in smaller haloes, or sub-haloes, including those that host the galaxy cluster members. More details on the mass function and on the mass density distribution of DM haloes in clusters will be provided in the continuation of this Chapter.

While optical and near-infrared observations of clusters are dominated by the stars in their member galaxies, due to its high temperature the ICM is very bright in the X-ray band. Their X-ray luminosity, L_X , is usually between 10^{43} and 10^{45} erg s $^{-1}$ and originates from an extended region (often more than 1 Mpc in radius). The spectral energy distribution (SED) is typical of an optically thin, free-free emission from a thermal population of electrons undergoing Coulomb scattering by ions (thermal bremsstrahlung). The emissivity of thermal bremsstrahlung is, apart for a quantum mechanical correction factor of order unity,

$$\epsilon_{\nu}^{ff} = \frac{32\pi Z^2 e^6 n_e n_i}{3m_e c^3} \sqrt{\frac{2\pi}{3kTm_e}} e^{-\frac{h\nu}{kT}}, \quad (2.1)$$

where e is the elementary charge, n_e and n_i the number density of electrons and ions, Z the charge of ions, m_e the electron mass and T the temperature of the emitting gas (in general T can vary within the cluster and across clusters of different total mass). The plasma is approximately uncharged, so $n_e \approx n_i$ and we will indicate them as n . The SED thus shows a plateau for low frequencies, where the emissivity scales as $n^2 T^{-\frac{1}{2}}$, and an exponential cut for ν_c such that $h\nu_c = kT$. The observed X-ray emissivity provides us with a chance to map the hot-gas mass distribution (e.g. Bonamigo et al. 2017, 2018). When combined with measurements of the stellar mass of the cluster members, which can be performed from optical and near-infrared photometry and spectroscopy, this provides us with a determination of the whole baryonic mass budget of clusters. This makes finding probes of the total cluster mass crucial, as subtracting the baryonic mass to it gives us a rare chance of accurately mapping the DM mass distribution.

The emission from the ICM is also very useful to study the morphology of clusters, as its distribution is much simpler than the stellar one, although some clusters have a perturbed ICM, typically due to mergers (see Fig. 2.1). Regular clusters usually have a quasi-spherical medium, with a single luminosity peak, while irregular ones often have multiple peaks and are therefore less easy to describe with a simple model. The sound-crossing time of the ICM, which can be used as an approximation of the typical time-scale of the deviations from pressure equilibrium, is significantly shorter than the Hubble time. As such, the ICM can be considered in hydrostatic equilibrium, which leads to the relation

$$\nabla P = -\rho_g \nabla \phi, \quad (2.2)$$

where P is the gas pressure, ρ_g its density and ϕ the total gravitational potential. This equation is particularly useful as it establishes a relation between some observable gas properties and the total gravitational potential, providing us with a probe of the total mass of the cluster. Imposing spherical symmetry, we obtain

$$\frac{1}{\rho_g} \frac{dP}{dr} = -\frac{GM(r)}{r^2}. \quad (2.3)$$

Knowing that $P = \rho_g \frac{kT}{\mu m_p}$, where $\mu = 0.5 - 0.6$ is the mean molecular weight and m_p is the mass of a proton, the Eq. (2.3) becomes

$$M(r) = -\frac{kTr}{G\mu m_p} \left(\frac{d \ln \rho_g(r)}{d \ln r} + \frac{d \ln T(r)}{d \ln r} \right). \quad (2.4)$$

The quantities $\rho_g(r)$ and $T(r)$ can be obtained by combining the spatially resolved emissivity and integrated emissivity. We did not use hydrostatic total mass estimates for the results of this Thesis: as such, we will not discuss further details of this probe (see, e.g. Ettori et al. 2019; Ettori & Eckert 2022).

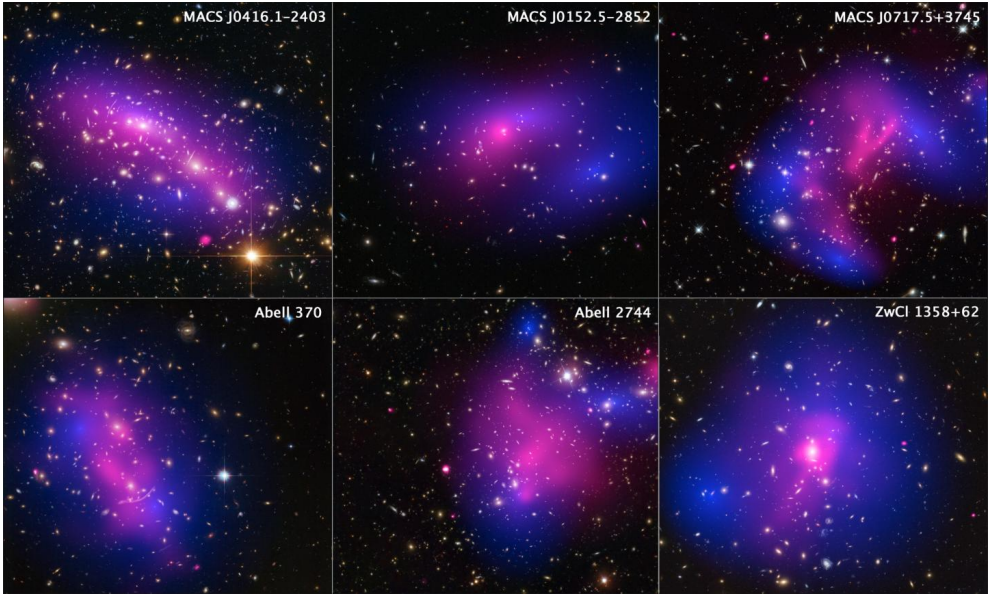


Figure 2.1: DM and ICM mass distribution of six massive galaxy clusters superimposed to the cluster galaxies. The DM distribution is modelled with gravitational lensing, while the ICM is probed with the Chandra X-ray emission, and the cluster galaxies have been observed with the *Hubble* Space Telescope. A clear difference can be observed in the X-ray emission between relaxed clusters, with a regular ICM morphology, and more irregular clusters. Credits: NASA, ESA, STScI, CXC.

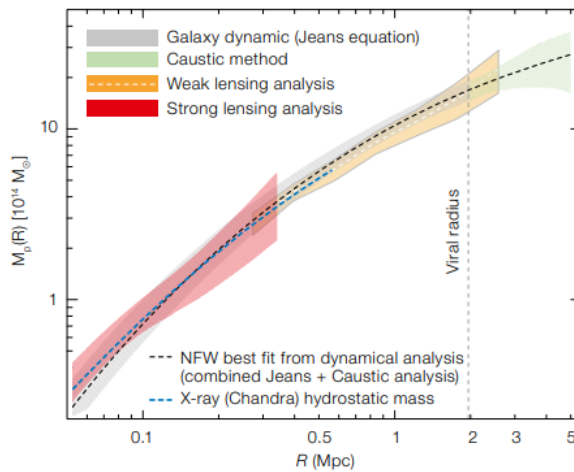


Figure 2.2: Total mass distribution profile of a galaxy cluster and its probes. Cumulative projected total mass profiles of MACS J1206.2-0847 determined with galaxy dynamics, gravitational lensing, and ICM hydrostatic equilibrium. From Rosati et al. (2014).

In analogy with the stellar dynamics in galaxies, the dynamics of the cluster members within the cluster are a second total mass probe. The formalism and techniques have a strong analogy with those presented when introducing the dynamical modelling of galaxies from their stellar kinematics, in Chapter 1. Given that we will not make use of the galaxy kinematics for the results of this Thesis, we will not present any technical details regarding this mass tracer (for a more complete dissertation, see e.g. Mamon et al. 2013; Biviano et al. 2013, 2023; Sartoris et al. 2020). Similarly to what we found in the previous Chapter for elliptical galaxies, galaxy clusters are not relaxed, as their relaxation time is larger than the Hubble time. However, they are in a state of quasi-relaxation, which allows us to consider them as approximately virialised. Again, this state is determined by collisionless incomplete violent relaxation, a mechanism that we have already discussed in Chapter 1.

A third probe of the total mass distribution of galaxy clusters is gravitational lensing, in the strong and weak regimes. Given the importance of this probe for the results of this work, we will devote Chapter 3 to its detailed description. These different techniques complement each other independently, and trace the total mass distribution in different radial ranges. For instance, Rosati et al. (2014) compared, in Fig. 2.2, the projected total mass density profile of MACS J1206.2-0847 obtained from dynamical analyses (galaxy dynamics and caustic), strong lensing, weak lensing, and X-ray data, reporting remarkable agreement between the probes in the overlapping regions.

2.1.2 The population of galaxies in dense environments

Observations have shown that the population of galaxies in dense environments (groups and clusters) differs from that of field galaxies, with a smooth transition from the least dense to the densest regions. As found by Dressler (1980), while 70% of field galaxies are spirals, the majority of cluster galaxies are elliptical or irregular, whose fraction increases from the outskirts towards the denser central regions. This morphology-density relation was confirmed for the Sloan Digital Sky Survey by Goto et al. (2003), by dividing the

galaxies in all environments into S0, late disc (mostly Sc), early disc (mostly Sa), and early type galaxies. Fig. 2.3 shows a notable drop of the frequency of late discs with increasing density, compensated by a clear increase in the fraction of ellipticals. The same trend is observed within the same cluster with decreasing (projected) distance from the cluster centre, normalised by the cluster virial radius, as shown in the lower panel of Fig. 2.3.

This morphological gradient observed for galaxies is strongly related to an analogous variation in colour and star-formation rate for high-density environments. Baldry et al. (2004a), working on the same sample of galaxies adopted to build Fig. 1.2, find that the fraction of red galaxies is also significantly increasing, together with the fraction of ellipticals, in denser environments. The same trend is reported by Tanaka et al. (2004) for the fraction of passive galaxies in high-density regions (the fraction of star forming galaxies is, conversely, found to be decreasing with density). Intriguingly, they find that faint galaxies are responsible for most of the gradient, suggesting that star-formation quenching is driven by density for low-mass galaxies.

These observational trends are the effect of the physical mechanisms affecting galaxies in dense environment and producing a transformation of their properties, both influencing their structure and their star-formation rate. Cluster galaxies evolve via interaction with the ICM, influenced by the properties of the hot gas (e.g. Treu et al. 2003), tidal interactions between galaxies and with the cluster potential, which depends on the local galaxy density, and dynamical friction.

The most important galaxy–ICM interactions are ram-pressure stripping and suffocation, caused by the ram pressure on the intra-galactic gas of member galaxies. In the former instance, ram pressure strips galaxies of most of their gas content, quenching their star formation (Gunn & Gott 1972). Ram-pressure stripping has been invoked to explain the deficit of gas-rich, star-forming spiral galaxies, and the consequent over-abundance of S0 disc galaxies in comparison to the field. When ram-pressure stripping of the gas reservoir of a galaxy is limited to the more loosely bound outskirts, this prevents its replenishing and thus causes star formation to only continue until the remainder of the gas is exhausted. This process is known as strangulation (Larson et al. 1980), and can be used to explain the lower star-formation rates of late-type discs in clusters (Balogh et al. 2000; Diaferio et al. 2001).

Due to their high velocity dispersion, cluster galaxies experience high-speed encounters with other members, which increase their internal energy and makes them less bound. This process is known as galaxy harassment. These tidal interactions may lead to gas inflow and strong star formation activity (Moore et al. 1996), and disturb the spiral structure of disk galaxies, causing a strong heating effect on their dynamically cold disks, and determining an evolution into a spheroidal shape (Moore et al. 1996, 1998, 1999). Galaxy harassment is especially impactful on the open and thin Sc late-type galaxies, which have a longer dynamical time than Sa/Sb spirals, and are therefore less able to adiabatically respond to the perturbation given by the galaxy encounters. Furthermore, these processes are especially important for galaxies that have never been through the cluster centre, and can also occasionally lead to a merger between galaxies of similar mass. On the other hand, a steep cluster potential characterises the high-density regions: here we can expect that tidal stripping determined by the intense gravitational potential of the cluster is dominant.

Finally, the distribution of galaxies within the cluster is driven by dynamical friction: the most massive cluster members tend to attract gas and less massive galaxies gravitationally, while moving in the cluster. The smaller members thus accumulate on the path of the more massive galaxies, especially behind them, slowing them down considerably and causing them to accumulate in the central regions of the cluster. Satellite galaxies in

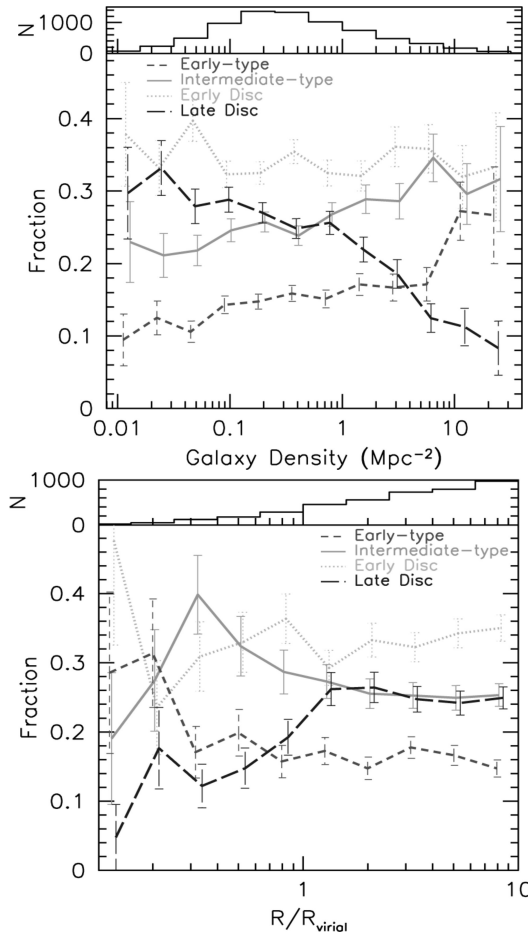


Figure 2.3: Number fraction of the different morphological galaxy types (as defined in the text) as a function of the environment. In the upper panel, the fraction is presented for all galaxies as a function of the galaxy density. In the lower panel, it is obtained for cluster galaxies only, as a function of the projected distance from the cluster centre, normalised for the cluster virial radius. From Goto et al. (2003).

this region are also shaped by mergers with the central galaxy. This process is known as galactic cannibalism and has two important effects: it is at the basis of the cD morphology of the BCGs that dominate many clusters in their central regions (see Chapter 1), and it causes a depletion of massive satellite galaxies.

2.2 Cosmological hydrodynamical simulations of galaxy clusters

Galaxy clusters are a crucial crossing point where Astrophysics meets Cosmology. On one hand, their overall dynamics are dominated by gravity, and their sub-halo mass hierarchy and baryonic fractions are crucial signatures of the underlying cosmology. On the other, their structure bears the traces of the Astrophysical processes that take place at a galactic scale, and that strongly depend on the evolution of all three cluster mass components (DM, ICM and stars). As such, studying the physical properties of galaxy clusters provides us with a wealth of information on our cosmological framework. Given the non-linear nature of the phenomena that shape the DM and baryonic mass distribution of galaxy clusters, numerical simulations are the most effective way to link their properties to the underlying cosmology, and to test how our cosmological hypotheses and paradigms interplay with astrophysical processes.

2.2.1 Introduction to cosmological simulations of galaxy clusters in a Λ CDM framework: gravity

As briefly discussed in the beginning of Chapter 1, in the framework of the Λ CDM paradigm, DM haloes form hierarchically, with the most massive systems resulting from subsequent mergers between smaller ones. As such, high-mass DM haloes, such as those that host galaxy clusters, contain a full hierarchy of lower-mass substructures, also known as sub-haloes. In Eq. (1.2), we have presented a model of the resulting DM halo mass spectrum. Depending on their mass and environment, galaxy cluster sub-haloes may have been the site of stellar formation and thus may now host a cluster galaxy. As anticipated, the non-linear gravitational collapse of DM haloes during the formation of clusters can be effectively described by cosmological N -body simulations, which have allowed for much progress and success in the description of the large scale structure of the Universe. As detailed, galaxy clusters are dominated by large, diffuse haloes, whose total mass can exceed $10^{15} M_{\odot}$, surrounded by a hierarchy of lower-mass sub-haloes whose mass distribution decreases radially. Collisionless simulations are able to reproduce the sub-halo distribution accurately (Nagai & Kravtsov 2005), find that the Press-Schechter model under- (over-) estimates the DM sub-halo mass function for higher (lower) mass values by a factor of up to 20% (Jenkins et al. 2001; Warren et al. 2006). In terms of the halo mass distribution, N -body simulations predict a universal 3D mass density functions for (sub-)haloes of all scales (Wang et al. 2020), typically fit by a Navarro-Frenk-White (Navarro et al. 1996, 1997) profile,

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}, \quad (2.5)$$

which scales as r^{-1} for $r \ll r_s$ and as r^{-3} for $r \gg r_s$, where r_s is a scale radius.

The coarse-grained nature of N -body simulations has been mitigated during the years by significant efforts to increase their resolution and size, which brought, for instance, already in 2005, to the Millennium simulation (Springel et al. 2005), following $N \approx 10^{10}$ particles from $z = 127$ within a cubic region with a size of 500 Mpc. N -body

simulations are also adopted to model the disruption processes that affect sub-haloes during their formation, mostly due to tidal stripping and heating, although these predictions are valid within the mass and spatial ranges allowed by the adopted numerical resolution.

Several methods are then employed to identify haloes within simulations, often based on the Friend of Friend (FoF) algorithm (Davis et al. 1985), which evaluates the distance between particles to link them together. As anticipated, galaxy-cluster-scale haloes are surrounded by sub-haloes which host their satellite galaxies. These sub-structures are identified by algorithms like SUBFIND (Springel et al. 2001b), which look for density peaks within the main haloes.

2.2.2 Introduction to cosmological simulations of galaxy clusters in a Λ CDM framework: baryonic physics and galaxy formation

Cluster sub-haloes contain a significant fraction of baryons, under the form of gas, stars, and supermassive black hole (SMBH). Understanding how galaxies populate DM sub-haloes requires a detailed modelling of how the complex stellar formation processes are linked to the growth of DM structures, which cannot be encompassed by collisionless simulations. This can be accounted through semi-analytical prescriptions to populate DM haloes (White & Frenk 1991), or through a fully hydrodynamical simulation that includes gas and stars. The main advantage of the latter method is that it is self consistent, does not require the strong a priori hypotheses of for semi-analytical models, and can reproduce the back-reaction of baryonic feedback onto the DM clustering. However, fully hydrodynamical simulations require a more complex underlying physical model, and therefore larger computing times with respect to collisionless simulations, in order to describe the baryonic physics phenomena taking place on small scales. As such, cosmological hydrodynamical simulations typically focus on a smaller region, often a single galaxy cluster, zoomed-in from a N -body collisionless simulation, where galaxy formation and evolution is simulated. These limitations have partly been overcome in the last decade, when a new generation of hydrodynamical simulations has modelled galaxy populations within larger volumes. Some notable examples of such achievements can be found, for instance, within the Illustris simulation¹ (Vogelsberger et al. 2014) and its successor IllustrisTNG² (Pillepich et al. 2018b; Weinberger et al. 2017), the Virgo Consortium’s Evolution and Assembly of GaLaxies and their Environments simulation³ (EAGLE Schaye et al. 2015), and the Magneticum simulation⁴ (Dolag et al. 2016). Finally, we note that the hydrodynamic equations describing the physical processes driving galaxy formation can be implemented on a grid (Eulerian formalism) or with a smoothed-particle hydrodynamics approach (SPH, or Lagrangian formalism), which have the advantage of automatically over-sampling denser regions, without the need for a grid refinement. For instance, the EAGLE and Magneticum simulations use the SPH code Gadget 3 (Springel 2005), while Illustris uses Arepo, an intermediate approach based on a moving mesh which follows the gas flow.

As detailed, the baryonic mass budget of galaxy clusters is dominated by gas, initially close to the primordial mixture of hydrogen and helium. During the early stages of cluster formation, the gas is heated to temperatures higher than 10^7 K and reaches hydrostatic equilibrium. In the following collapse, the gas can cool and part of it can feed

¹<https://www.illustris-project.org/>

²<https://www.tng-project.org/>

³<https://eagle.strw.leidenuniv.nl/>

⁴<http://www.magneticum.org/>

SMBH accretion and, crucially, star formation. The cooling efficiency is influenced by the gas metallicity, enriched by the stellar evolution processes, and by a time-dependent UV background. Hydrodynamical simulations that only include gas cooling lead to an excess of star formation and therefore of the amount of stars compared to observations. The missing element is energy or momentum injection from supernovae (SN) and active galactic nuclei (AGN), which transfers energy to the ICM quenching star formation. Specifically, kinetic feedback driven by a SN is often implemented in the form of galactic winds that can remove gas from the relatively shallow galaxy-scale potential well of the low- and intermediate-mass sub-haloes, regulating their stellar fraction. On the other hand, the AGN feedback from SMBHs can supply sufficient energy to quench star formation in the more massive cluster members. While the most recent high-resolution simulations include gas particles with a mass close to $10^6 M_\odot$, star formation processes such as cloud collapse happen on even smaller scales ($10^3 - 10^4 M_\odot$), and thus its description requires a sub-grid treatment with prescriptions to model the sub-resolution star formation. Finally, to avoid a divergence of the gravitational potential descending from the coarse-grained nature of the simulations, a softening length ϵ is implemented in the evaluation of the distance between particles required for the force estimation. The softening length is also useful to estimate the scale below which mass density profiles are not resolved, which corresponds to roughly 2.8ϵ (Hernquist 1987), at least for gravity-only calculations.

In recent years, several efforts have been performed to extend hydrodynamical simulations to larger or smaller sizes (see Fig. 2.4), to probe different phases and regimes of the formation of structures. Comparison between the measured and simulated properties of the dark and baryonic components of galaxies and clusters can guide further improvements to the simulations themselves, and provide the chance to test our hypotheses on the cosmological paradigms, and thus any discrepancy between them may signal a systematic issue within the simulation, or perhaps a potential problem with the Λ CDM model itself. Finally, cosmological DM-only or hydrodynamical simulations can be used to characterise the observational signature of alternative DM models, such as warm DM (WDM, see Lovell et al. 2014), and self-interacting DM (SIDM, see Vogelsberger et al. 2012), which can then be compared against observations. The hypothesis of SIDM is especially impactful on the properties of the central regions of haloes, significantly changing the DM mass density profile slope (Despali et al. 2019), possibly creating a central, extended core.

2.2.3 The Dianoga hydrodynamical simulations

Within the Magneticum cosmological hydrodynamical simulations, based on the SPH code Gadget 3, the Dianoga suite (Planelles et al. 2014; Rasia et al. 2015; Bassini et al. 2020) has been particularly useful to test the effect of different feedback prescriptions in hydrodynamical simulations, and to compare the structure of observed and simulated cluster members. As such, it has been adopted as a reference for several works developed in connection with this Thesis. Dianoga⁵ is a set of 24 simulated galaxy clusters with a virial mass in the range $5 - 20 \times 10^{14} M_\odot$, extracted from a low-resolution DM simulation of 1024^3 particles, which covers a box size of 1 Gpc in a standard WMAP-7 Λ CDM cosmological model, with a present matter density parameter $\Omega_{m,0} = 0.24$ and baryon density parameter $\Omega_{b,0} = 0.04$, and a Hubble constant value of $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$. From this simulation 24 Lagrangian regions around the most massive clusters were extracted and initial conditions were re-generated at a higher resolution for these zoomed-

⁵<https://wwwmpa.mpa-garching.mpg.de/kdolag/Simulations/>

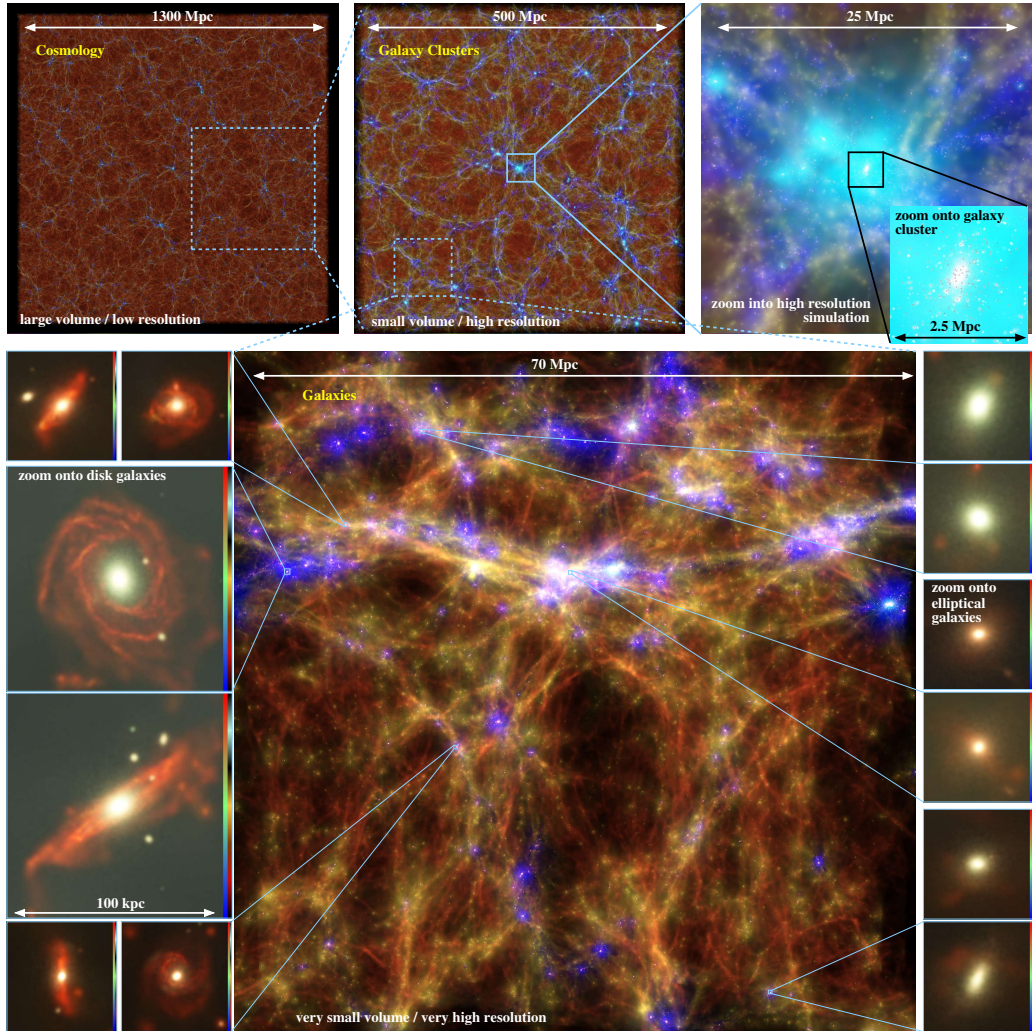


Figure 2.4: Overview of the different boxes of the Magenticum Pathfinder simulations, from the Gpc to the kpc scales. The panels show the distribution of gas particles, colour-coded according to the gas temperature from lower (brown) to higher (blue/cyan) values. Galaxies and stars forming in the simulation are shown in white. Credit: Dolag et al. (2015), Teklu et al. (2015)

in regions. Finally, fully hydrodynamical re-simulations of the 24 clusters were performed using the TreePM–SPH GADGET–3 code (Springel 2005; Beck et al. 2016) with a set of different resolutions, softening lengths, star formation prescriptions, and feedback schemes. The Dianoga simulations were adopted as a reference by Meneghetti et al. (2020) and the subsequent studies, to which this Thesis contributes, given their availability to us and the fact that the simulated clusters span a similar total cluster mass range as the observed SL clusters studied in this Thesis. We will therefore describe more in detail some of the baryonic feedback implementations that we compared with our observational results.

The reference simulations described in Rasia et al. (2015) (hereafter $1\times$ R15 model: we follow the naming conventions introduced by Ragagnin et al. 2022; Meneghetti et al. 2022) have a mass particle for DM and gas of $8.47 \times 10^8 M_\odot$ and $1.53 \times 10^8 M_\odot$, respectively. For the gravitational force, a softening length of 3.75 ckpc (comoving kiloparsecs) is used for DM and gas particles, whereas 2 ckpc is used for SMBH and star particles. The radiative cooling is implemented following Wiersma et al. (2009), while star formation, stellar feedback, SMBH accretion, and AGN feedback follow the models by Springel & Hernquist (2003) and Steinborn et al. (2015). Both mechanical and radiative SMBH feedback is included. The model is parameterised by an outflow efficiency that regulates gas heating power used for mechanical feedback, while the radiative efficiency regulates the luminosity of the radiative component (see Steinborn et al. 2015). The effects of a UV time-dependent uniform background are modeled as in Haardt & Madau (2001).

Alternative implementations of the baryonic feedback within the Dianoga simulations have been presented in Ragone-Figueroa et al. (2013) and Ragone-Figueroa et al. (2018) (hereafter $1\times$ RF18 scheme), and in Bassini et al. (2020) with different mass particle resolutions (hereafter $1\times$ and $10\times$ B20). The $1\times$ RF18 simulation implemented a mechanism of cold-cloud evaporation, where the removal of gas particle to avoid star formation is performed if gas in the cold phase is heated by the AGN energy to a temperature that exceeds the average gas temperature. These simulations have the same resolution and a larger softening compared to $1\times$ R15: 5.62 ckpc is used for DM and gas particles, while 3 ckpc is used for SMBH and star particles. The $1\times$ and $10\times$ B20 simulations differ in resolution by a factor ten, where $1\times$ B20 has the same mass particle resolution as $1\times$ R15 and $1\times$ RF18. The feedback scheme similar to $1\times$ RF18, but does not implement cold cloud evaporation. This simulation was used to test the effects on the properties of cluster galaxies of a higher mass resolution and of a higher feedback efficiency, akin to the one implemented by the Hydrangea simulations⁶ (Bahé et al. 2017), part of the Cluster-EAGLE (C-EAGLE) project. The consequence of the more efficient feedback is an over-production of very massive galaxies. As evidenced by Fig. 2.5, hydrodynamical simulations allow for detailed reconstruction of the mass distribution and dynamics of the cluster members.

⁶<https://home.strw.leidenuniv.nl/bahe/hydrangea.html>

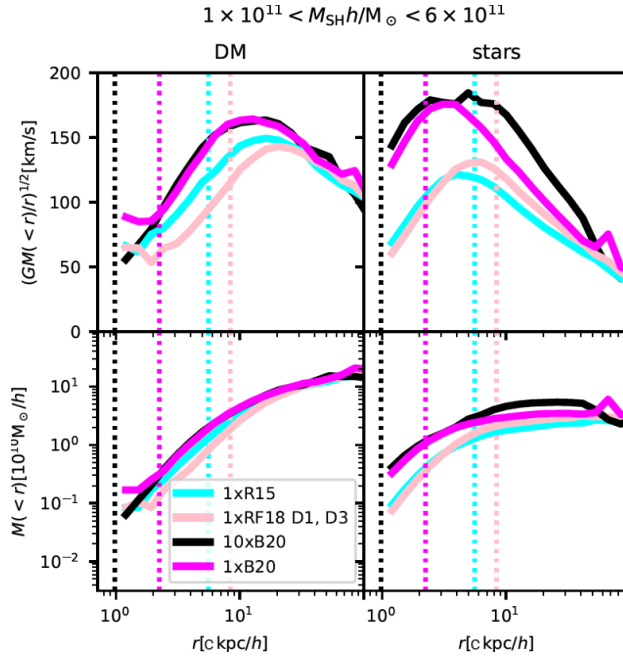


Figure 2.5: Circular velocity and cumulative mass profiles per particle type in simulated clusters. We show DM (left column), and stars (right column) for cluster sub-haloes with total masses in the range $1 - 6 \times 10^{11} M_{\odot}$, for varying resolution choices and feedback prescriptions, indicated with the same naming scheme adopted in the text. For each setup, we plot the minimum resolved scale for the stellar mass distribution ($2.8\epsilon_*$, where ϵ_* is the stellar softening length) as vertical dashed lines. The profiles have been obtained as an average of six Dianoga clusters with a virial mass in the range $3 - 11 \times 10^{14} M_{\odot}$. Figure from Ragagnin et al. (2022).

Part II

Overview of strong lensing and galaxy kinematics in massive clusters

In Part II, we introduce the main observations and methods on which the results of this Thesis are based.

We will study the mass distribution of massive galaxy clusters and of their members using strong gravitational lensing. In Chapter 3, we provide a short theoretical introduction and then we describe the statistical and computational tools employed to build the strong lensing models.

Given the accuracy of the information provided by strong lensing on the inner mass structure of galaxy clusters, several imaging and spectroscopy surveys have focused on lensing clusters. In Chapter 4, we describe the observational campaigns which collected the data we have used for our studies.

Finally, to study the baryonic and DM mass distribution of the member galaxies, we complement the information provided by strong lensing with the study of the stellar line-of-sight velocity dispersion of the members. In Chapter 5, we link the kinematic properties of the cluster galaxies to their spectra and describe the procedure we adopt to measure their velocity dispersion values.

An introduction to strong gravitational lensing

One of the key predictions of the equivalence principle is that light rays are deflected by the gravitational field of massive bodies. This effect can be precisely described in the framework of General Relativity (Einstein 1916, 1936; Zwicky 1937): as gravity warps space-time, light-like geodesics follow the space-time curvature and bend towards massive objects. This effect is known as gravitational lensing, and affects the light coming from distant astronomical sources and reaching an observer on Earth. As a consequence of the deflection of their light, many astrophysical objects appear in a different position compared to where they would be observed in the absence of lensing. Differential deflection of light rays leads to a change in the observed shape and size of these objects: as photons are neither created nor destroyed during the process, this results in the conservation of the surface brightness of a lensed source and thus in its (de-)magnification. This, crucially, allows us to observe faint objects which, if not lensed, would fall below the luminosity detection threshold of our telescopes. Finally, multiple paths are occasionally possible for the light of a distant object to reach us, leading to the observation of multiple images of the same source. This phenomenon is also known as strong gravitational lensing (SL). The paths have different lengths and travel through a gravitational potential of differing intensity, which results in a different amount of time needed for the light rays to reach us. In the case of a time-variable strongly-lensed source this is observed as a time delay between the multiple images.

In this chapter, we provide a concise formal introduction to the phenomenon (for a more complete treatment, see Bartelmann & Schneider 2001; Schneider 2006b; Meneghetti 2022), and we describe some useful mass distribution models typically adopted to describe astrophysical objects. Finally, we detail how SL modelling of the mass distribution of galaxies and galaxy clusters is implemented in this work.

3.1 Gravitational lensing in the framework of General Relativity

3.1.1 Light deflection

Let us consider a light ray propagating in a region with a gravitational potential Φ . We will work in the approximation $\Phi \ll c^2$, which always holds true in the systems studied in this Thesis. The space-time metric can thus be written as a perturbation of the Minkowski metric

$$g_{\mu\nu} = \begin{pmatrix} 1 + \frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & -(1 - \frac{2\Phi}{c^2}) & 0 & 0 \\ 0 & 0 & -(1 - \frac{2\Phi}{c^2}) & 0 \\ 0 & 0 & 0 & -(1 - \frac{2\Phi}{c^2}) \end{pmatrix}, \quad (3.1)$$

so the line element becomes (repeated indexes are summed)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\Phi}{c^2}\right) (d\mathbf{x})^2. \quad (3.2)$$

The velocity of a light ray ($ds = 0$) is therefore

$$c' = \frac{|d\mathbf{x}|}{dt} = c \sqrt{\frac{1 + \frac{2\Phi}{c^2}}{1 - \frac{2\Phi}{c^2}}} =: c/n, \quad (3.3)$$

where n is analogous to the refraction index of a medium, although completely achromatic. At first order in $\frac{\Phi}{c^2}$, n can be re-written as

$$n \approx 1 - \frac{2\Phi}{c^2}. \quad (3.4)$$

In General Relativity, an equivalent form of the Fermat Principle holds true. A light ray will travel from point A to point B along the trajectory that requires the shortest time, or alternatively, that determines a stationary point for the functional

$$\int_A^B n(\mathbf{x}(l)) dl = \int_{\lambda_A}^{\lambda_B} n[\mathbf{x}(\lambda)] \left\| \frac{d\mathbf{x}}{d\lambda} \right\| d\lambda. \quad (3.5)$$

Defining $\dot{\mathbf{x}} := \frac{d\mathbf{x}}{d\lambda}$, one can thus solve the Euler-Lagrange equations

$$\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{\mathbf{x}}} - \frac{\partial L}{\partial \mathbf{x}} = 0, \quad (3.6)$$

for the Lagrangian

$$L(\dot{\mathbf{x}}, \mathbf{x}, \lambda) = n[\mathbf{x}(\lambda)] \left\| \frac{d\mathbf{x}}{d\lambda} \right\|. \quad (3.7)$$

If $\|\dot{\mathbf{x}}\| = 1$, defining $\mathbf{e} := \dot{\mathbf{x}}$, the Euler-Lagrange equations become

$$n\dot{\mathbf{e}} = \nabla n - \mathbf{e}(\nabla n \cdot \mathbf{e}) \Rightarrow \dot{\mathbf{e}} = \frac{1}{n} \nabla_\perp n \approx -\frac{2}{c^2} \nabla_\perp \Phi. \quad (3.8)$$

The deflection angle of a light ray $\hat{\alpha}$ can therefore be obtained as

$$\hat{\alpha} = \frac{2}{c^2} \int_{\lambda_A}^{\lambda_B} \nabla_\perp \Phi d\lambda. \quad (3.9)$$

If the lens is thin, that is to say its extension along the LOS is much smaller than the distances between source, lens and observer, then the Born approximation can be used. The total deflection is computed on an unperturbed path, parallel to the optical axis (the line connecting the observer to the centre of the lens) and separated from it by a certain impact parameter b . The deflection angle becomes

$$\hat{\alpha}(b) = \frac{2}{c^2} \int_{-\infty}^{\infty} \nabla_\perp \Phi dz. \quad (3.10)$$

For a point mass M , $\Phi = -\frac{GM}{r}$ and the deflection angle, in polar coordinates on the lens plane, is

$$\hat{\alpha}(\mathbf{b}) = \frac{4GM}{c^2 b} \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix}. \quad (3.11)$$

$\hat{\alpha}$ depends linearly on M , therefore we can apply the superposition principle to a system of point masses M_i , with $1 \leq i \leq N$. If ξ is the two-dimensional coordinate vector on the lens plane and ξ_i the position on the same plane of the i -th point mass, the total deflection angle for a light ray crossing the lens plane in ξ will be

$$\hat{\alpha}(\xi) = \frac{4G}{c^2} \sum_{i=1}^N M_i \frac{\xi - \xi_i}{\|\xi - \xi_i\|^2}. \quad (3.12)$$

This expression can be generalised to a continuous three-dimensional mass distribution, described by a density $\rho(\xi, z)$, so the sum in (3.12) will become the integral

$$\hat{\alpha}(\xi) = \frac{4G}{c^2} \int_{-\infty}^{\infty} \int_{\mathbb{R}^2} \rho(\xi', z') \frac{\xi - \xi'}{\|\xi - \xi'\|^2} d\xi' dz'. \quad (3.13)$$

Introducing the surface mass density, obtained from the total mass density by projecting it on the lens plane $\Sigma(\xi) := \int_{-\infty}^{\infty} \rho(\xi, z') dz'$, (3.13) becomes

$$\hat{\alpha}(\xi) = \frac{4G}{c^2} \int_{\mathbb{R}^2} \Sigma(\xi') \frac{\xi - \xi'}{\|\xi - \xi'\|^2} d\xi', \quad (3.14)$$

again valid for a generic mass distribution.

We define a map that traces back the path of a light ray from the lens plane to its source. This relation is usually referred to as the ray-tracing equation.

It is now convenient to define D_l , D_s and D_{ls} as the angular-diameter distances respectively between observer and lens, observer and source, and between lens and source. If θ e β are the angular positions of image and source, as in Figure (3.1), then, for a small deflection angle, we obtain

$$\beta D_s = \theta D_s - \hat{\alpha} D_{ls}. \quad (3.15)$$

The expression can be made adimensional by introducing the reduced deflection angle.

$$\alpha(\theta) = \frac{D_{ls}}{D_s} \hat{\alpha} \left(\theta = \frac{\xi}{D_l} \right) = \frac{D_l D_{ls}}{D_s} \frac{4G}{c^2} \int_{\mathbb{R}^2} \Sigma(\theta') \frac{\theta - \theta'}{\|\theta - \theta'\|^2} d\theta', \quad (3.16)$$

so the equation (3.15) becomes

$$\beta = \theta - \alpha(\theta). \quad (3.17)$$

The equation (3.16) to compute the reduced deflection angle can be simplified by introducing the critical surface density, Σ_{cr} , a typical mass density scale for the system

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_{ls}} \Rightarrow \alpha(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^2} \frac{\Sigma(\theta')}{\Sigma_{cr}} \frac{\theta - \theta'}{\|\theta - \theta'\|^2} d\theta'. \quad (3.18)$$

We then introduce $\kappa(\theta)$, or convergence, an adimensional surface mass density, as

$$\kappa(\theta) = \frac{\Sigma(\theta)}{\Sigma_{cr}} \Rightarrow \alpha(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^2} \kappa(\theta') \frac{\theta - \theta'}{\|\theta - \theta'\|^2} d\theta'. \quad (3.19)$$

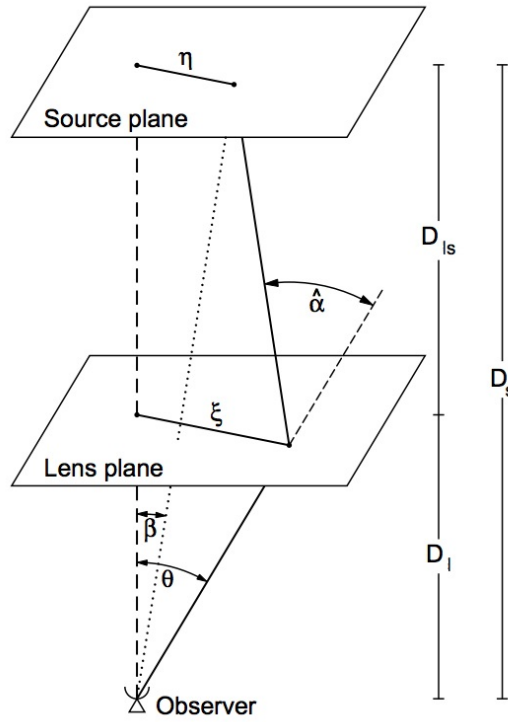


Figure 3.1: Schematic representation of a gravitational lens (adapted from Bartelmann & Schneider 2001, using the same notation as in this Thesis).

Another fundamental quantity describing gravitational lensing is the lensing potential $\psi(\boldsymbol{\theta})$, especially useful when predicting the number of images than a lens can form (Bartelmann and Schneider 1999). It is defined as

$$\psi(\boldsymbol{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} \kappa(\boldsymbol{\theta}') \ln \|\boldsymbol{\theta} - \boldsymbol{\theta}'\| d\boldsymbol{\theta}', \quad (3.20)$$

and, as suggested by (3.19), we can use it to express α as

$$\alpha(\boldsymbol{\theta}) = \nabla \psi(\boldsymbol{\theta}). \quad (3.21)$$

From (3.20) and (3.21) we can build a relation between ψ and κ , i.e. a two-dimensional Poisson equation linking the potential and the surface mass density

$$\nabla^2 \psi(\boldsymbol{\theta}) = 2\kappa(\boldsymbol{\theta}). \quad (3.22)$$

If it is reasonable to assume that the lensing effect is approximately uniform in a small region of the lens plane, we can linearise the ray-tracing equation, writing it as $\mathbf{y} = \mathbf{y}_0 + A\mathbf{x}$. $\mathbf{y}_0$ is a constant deflection, whose effect is not measurable. A is instead the Jacobian matrix of the ray-tracing function

$$A = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} \right) = (\delta_{ij} - \psi_{ij}), \quad (3.23)$$

where δ_{ij} is the Kroenecker delta and $\psi_{ij} := \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$. In order to distinguish two different contributions to the deflection, the Jacobian can be separated into a trace-less part and a term proportional to the identity

$$A = \frac{\text{tr} A}{2} \mathbb{1} + \left(A - \frac{\text{tr} A}{2} \mathbb{1} \right) = \quad (3.24)$$

$$\begin{pmatrix} 1 - \frac{1}{2}(\psi_{11} + \psi_{22}) & 0 \\ 0 & 1 - \frac{1}{2}(\psi_{11} + \psi_{22}) \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}(\psi_{11} - \psi_{22}) & -\psi_{12} \\ -\psi_{12} & \frac{1}{2}(\psi_{11} - \psi_{22}) \end{pmatrix}. \quad (3.25)$$

Starting from (3.22) and imposing $\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22})$ and $\gamma_2 = \psi_{12} = \psi_{21}$, we get

$$A = (1 - \kappa) \mathbb{1} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} = (1 - \kappa) \mathbb{1} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix}, \quad (3.26)$$

where $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$ and ϕ is such that $\gamma_1 + i\gamma_2 = \gamma e^{2i\phi}$.

From (3.26) we notice that, at first order, the deflection can be seen as the superposition of two effects. In the first term, proportional to the identity, we find again the convergence κ , determining a diagonal matrix that magnifies or de-magnifies the image preserving its shape. The second term, instead, is a shear term (γ_1, γ_2) , that deforms and stretches the image along a fixed direction. A is a real symmetric matrix, so it has an orthogonal basis of eigenvectors. Its eigenvalues are

$$\text{tr} A = 2(1 - \kappa), \det A = (1 - \kappa)^2 - \gamma^2 \Rightarrow \lambda_{\pm} = (1 - \kappa) \pm \gamma. \quad (3.27)$$

If $\det A \neq 0$, $|\det A|$ locally gives the ratio between surface elements on the source and image plane. A magnification tensor can therefore be defined locally as $M = A^{-1}$. M is linked to the total magnification μ , the radial and the tangential magnifications μ_r and μ_s , by the relation

$$\mu = |\det M| = \frac{1}{|\det A|} = \frac{1}{|(1 - \kappa)^2 - \gamma^2|}, \quad \mu_r = \frac{1}{1 - \kappa + \gamma}, \quad \mu_t = \frac{1}{1 - \kappa - \gamma}. \quad (3.28)$$

If $\det A = 0$, then at least one of the eigenvalues of A is zero, so either the radial or tangential magnification goes to infinity: this generates the giant arcs, typical of cluster scale SL. The points in which A is singular, or alternatively the critical points of the function $\theta - \alpha(\theta)$, are also called critical lines. Their counter-images on the source plane are referred to as caustic lines. The critical lines also mark the points on the lens plane where the number of equations of the ray-tracing map changes. This results in the necessary and sufficient condition for SL

$$\exists \bar{\theta} : \det A(\bar{\theta}) < 0 \Leftrightarrow \text{the lens can produce multiple images.} \quad (3.29)$$

Alternatively, a sufficient condition is for the convergence κ to be greater than the unity somewhere on the lens plane

$$\exists \bar{\theta} : \kappa(\bar{\theta}) > 1 \Rightarrow \exists \bar{\beta} : \text{a source in } \bar{\beta} \text{ is multiply imaged.} \quad (3.30)$$

3.2 Lensing properties of the singular isothermal sphere and its generalisations

A gravitational lens is axisymmetric when its surface mass density only depends on the distance from the centre of the lens, or $\Sigma(\xi) = \Sigma(|\xi|) = \Sigma(\xi)$. For the Gauss Theorem, in a thin lens approximation, if a ray crosses the lens plane with an impact parameter ξ , the deflection is the same as the one determined by a point mass with $M = M(< \xi)$, the total mass enclosed within a cylinder of radius ξ around the optical axis, placed in the centre of the lens. The reduced deflection angle is

$$\alpha(\theta) = \frac{M(\theta)}{\pi D_L^2 \Sigma_{cr} \theta} = \frac{2 \int_0^\theta \theta' \kappa(\theta') d\theta'}{\theta} = \theta \bar{\kappa}(\theta), \quad (3.31)$$

where $\bar{\kappa}(\theta)$ is the average value of κ within a distance θ from the lens centre. We can use this function to re-write the ray-tracing equation as

$$\beta = \theta(1 - \bar{\kappa}(\theta)). \quad (3.32)$$

We note that if $\bar{\kappa}(\theta) = 1$ then $\beta = 0$, so a source placed behind the centre of the lens will produced a ring-like image exactly where the average enclosed surface mass density is equal to the critical density. This phenomenon is often observed around galaxy-scale lenses and is typically called an Einstein ring.

All results obtained for axisymmetric lenses can be generalised to the case of an oblate ellipsoid (Golse & Kneib 2002). A possible implementation of this will be presented in the next Section.

3.2.1 The singular isothermal sphere

The SIS profile was introduced in Chapter 1 to describe the total mass distribution of elliptical galaxies. We have introduced the expression of its total mass density in Eq.

(1.32). The cumulative enclosed mass as a function of the distance from the centre of the sphere is

$$M(r) = \frac{2\sigma^2}{G}r. \quad (3.33)$$

Integrating Eq. (1.32) along the line of sight, we derive the surface mass density and the enclosed 2D mass for a SIS

$$\Sigma(R) = \frac{\sigma^2}{2GR}, \quad (3.34)$$

$$M(R) = \frac{\pi\sigma^2}{G}R. \quad (3.35)$$

In order to write the ray-tracing equation for this particular case, the Einstein radius is introduced

$$\theta_E = 4\pi \left(\frac{\sigma}{c} \right)^2 \frac{D_{ls}}{D_s}. \quad (3.36)$$

The Einstein radius can be estimated from the parameters of the system as

$$\theta_E = 1.15'' \left(\frac{\sigma}{200 \text{ km s}^{-1}} \right)^2 \left(\frac{D_{ls}}{D_s} \right). \quad (3.37)$$

The ray-tracing equation is

$$\beta = \theta - \theta_E \text{sgn}(\theta). \quad (3.38)$$

If a source is located at $\beta \geq \theta_E$ we obtain a single image in $\theta_+ = \beta + \theta_E$, while if $\beta < \theta_E$ we obtain two images found at

$$\theta_{\pm} = \beta \pm \theta_E, \quad (3.39)$$

From (3.38), we can compute the magnification of the two images.

$$\mu_{\pm} = 1 \pm \frac{\theta_E}{\beta}. \quad (3.40)$$

3.2.2 The dual pseudo-isothermal mass distribution

The SIS mass density profile diverges for small values of r , and its total integrated mass is unbounded for large radii. To avoid these two non-physical behaviours, the dual pseudo-isothermal ellipsoidal (dPIE) mass distribution profile was introduced in Elíasdóttir et al. (2007) and Limousin et al. (2005). Its spherical case is a generalisation of the SIS with the introduction of a central core determining a finite central density. The density profile is also truncated for large radii, where $\rho(r) \propto r^{-4}$. The profile is parametrised as

$$\rho(r) = \frac{\rho_0}{(1 + r^2/r_c^2)(1 + r^2/r_t^2)}, \quad (3.41)$$

where

$$\rho_0 = \frac{\sigma^2}{2\pi G} \frac{r_c + r_t}{r_c^2 r_t}. \quad (3.42)$$

We notice how r_t parametrises the radial scale of the transition from an isothermal profile to the $\rho \propto r^{-4}$ regime. From eq. (3.41), we can derive the total enclosed 3D mass, the

surface mass density, and the enclosed mass within a 2D radius for a spherical dPIE (see Elíasdóttir et al. 2007)

$$\Sigma(R) = \frac{\sigma^2}{2G} \frac{r_t}{r_t - r_c} \left(\frac{1}{\sqrt{R^2 + r_c^2}} - \frac{1}{\sqrt{R^2 + r_t^2}} \right), \quad (3.43)$$

$$M(r) = \frac{2\sigma^2}{G} \frac{r_t}{r_t - r_c} \left(r_t \arctan \left(\frac{r}{r_t} \right) - r_c \arctan \left(\frac{r}{r_c} \right) \right), \quad (3.44)$$

$$M(R) = \frac{\pi\sigma^2}{G} \frac{r_t}{r_t - r_c} \left(\sqrt{R^2 + r_c^2} - r_c - \sqrt{R^2 + r_t^2} + r_t \right). \quad (3.45)$$

The total mass of a spherical dPIE can be obtained from Eq. (3.44) or (3.45) in the limit of $r \rightarrow \infty$

$$M_{\text{tot}} = \frac{\pi\sigma^2}{G} r_t. \quad (3.46)$$

As we will see, galaxy cluster members are typically modelled as spherical dPIEs with a vanishing core (r_c much smaller than r_t). In this context, r_t is approximately a half-mass 3D radius, as a sphere of radius $r = r_t$ encloses $m(r_t) \approx 0.5M_{\text{tot}}$. Looking at the 2D cumulative mass profile, instead, $R = r_t$ and $R = 5r_t$ enclose, respectively, around 60% and 90% of M_{tot} (see Bergamini et al. 2019; Granata et al. 2022). The generalisation of the surface mass density presented in Eq. 3.43 to an elliptical symmetry is performed substituting the projected distance from the centre R with $\hat{R}$ such that (Elíasdóttir et al. 2007)

$$\hat{R}^2 = \frac{x^2}{\left(\frac{2a}{a+b}\right)^2} + \frac{y^2}{\left(\frac{2b}{a+b}\right)^2}, \quad (3.47)$$

where a and b are the major and minor projected semi-axes of the ellipsoid, and x and y are the coordinates along them¹.

3.2.3 Kinematic profiles of a dual pseudo-isothermal mass distribution

We now derive the kinematic properties of a galaxy whose total mass profile $\rho(r)$ is described by a dPIE, while the luminous mass profile is parameterised with a function $\nu(r)$. From Eq. (1.20), we can derive the circular velocity of a dPIE

$$v_c^2(r) = \frac{GM(r)}{r} = 2\sigma^2 \frac{r_t}{r_t - r_c} \frac{1}{r} \left(r_t \arctan \left(\frac{r}{r_t} \right) - r_c \arctan \left(\frac{r}{r_c} \right) \right). \quad (3.48)$$

The maximum value of the circular velocity, indicated as $v_{c,\text{max}}$ is a crucial parameter to compare SL models and simulations. In the case of a SIS, $v_c = \sqrt{2}\sigma$, but more in general, for the typical values of r_c and r_t found for cluster members, $v_{c,\text{max}}$ is approximated by $\sqrt{2}\sigma$ with a lower bias of less than 3% (Bergamini et al. 2019). If we assume a constant stellar-to-total-mass ratio, then $\nu(r)$ is a dPIE with the same parameters as $\rho(r)$. In the isotropic case ($\beta(r) = 0$), from Eq. (1.22), we can compute the projected line-of-sight

¹Equation (3.47) implies that, compared to the spherical case, the area enclosed by a given iso-surface-density contour changes by a factor $(4ab)(a+b)^{-2}$.

(LOS) velocity dispersion profile

$$\sigma_{\text{LOS}}^2(R) = \sigma^2 \frac{4}{\pi} \frac{r_c + r_t}{r_c^2 r_t} \left(\frac{1}{\sqrt{R^2 + r_c^2}} - \frac{1}{\sqrt{R^2 + r_t^2}} \right)^{-1} \times \int_R^\infty \frac{r_t \arctan\left(\frac{r}{r_t}\right) - r_c \arctan\left(\frac{r}{r_c}\right) \sqrt{r^2 - R^2}}{r^2 \left(1 + \frac{r^2}{r_c^2}\right) \left(1 + \frac{r^2}{r_t^2}\right)} dr. \quad (3.49)$$

The hypothesis of isotropic stellar orbits is a simplification which we will apply to the kinematic studies presented in this work, as an approximation valid in the central regions of early-type galaxies. The anisotropy profile of early-type galaxies can be probed with resolved, high-signal-to-noise spectroscopic observations, but the compact structure of cluster galaxies makes this difficult to achieve. The light-weighted aperture LOS velocity dispersion can then be obtained (see 1.23 and Bergamini et al. 2019) as

$$\sigma_{\text{ap}}^2(R) = \sigma^2 c_P^2(R), \quad (3.50)$$

where

$$c_P^2(R) = \frac{4}{\pi} \frac{r_c + r_t}{r_c^2 r_t} \left(\sqrt{R^2 + r_c^2} - r_c - \sqrt{R^2 + r_t^2} + r_t \right) \times \int_0^R R' \int_{R'}^\infty \frac{r_t \arctan\left(\frac{r}{r_t}\right) - r_c \arctan\left(\frac{r}{r_c}\right) \sqrt{r^2 - R'^2}}{\left(1 + \frac{r^2}{r_c^2}\right) \left(1 + \frac{r^2}{r_t^2}\right)} \frac{1}{r^2} dR' dr. \quad (3.51)$$

Our definition differs from that in Bergamini et al. (2019) by a factor of $\sqrt{3/2}$. For the typical values of r_t and r_c of cluster members in SL models, c_p varies between 0.9 and 1. Bergamini et al. (2019) also tested the case in which the projected light distribution is described by a Sérsic profile (Sérsic 1963), finding differences on the predicted value of σ of no more than 5%. We can thus connect the observed values of $\sigma_{\text{ap}}^2(R)$ with the parameters of the dPIE mass haloes introduced in SL models, to complement and enhance the information provided us by lensing with an independent kinematic probe.

3.3 Parametric strong lensing modelling with **LensTool**

In this Section, we give an overview of the parametric methods used for the SL modelling presented in this Thesis. Parametric methods reconstruct the mass distribution of a strong gravitational lens by superposition of clumps of matter: in the case of galaxy clusters these are introduced to represent the diffuse DM and ICM mass components and, on a smaller scale, the total mass distribution of the member galaxies. Each of the clumps is defined by a set of parameters describing their mass density profile, to be optimised in an effort to reproduce the observed lensed images. Alternative non-parametric (free-form) methods are also adopted: in this case the cluster is subdivided into a pixelised mesh, and the mass of each pixel is optimised (e.g. Diego et al. 2023; Cha & Jee 2023). Parametric models are currently the most widely used method for SL reconstruction of massive galaxy clusters (e.g. Grillo et al. 2015; Jauzac et al. 2015; Caminha et al. 2016; Sharon et al. 2020; Acebron et al. 2022a). On one hand, they require a priori hypotheses on the lens mass distribution, such as the number and position of the mass haloes and their parametrisation. The use of smooth mass clumps, with a simple density

profile may be a simplification with respect to the clumpy DM mass distribution predicted for galaxy clusters by cosmological simulations. On the other hand, parametric methods allow for a natural inclusion of astrophysical priors on the parameters that describe the physical properties of the cluster and its member galaxies (Jullo et al. 2007), and avoid the over-fitting problems that might affect free-form lensing modelling in light of the very high number of free parameters they require (Meneghetti et al. 2017).

3.3.1 Bayesian parametric inference with *LenSTool*

The SL modelling work presented in this Thesis is performed using the parametric code *LenSTool*² (Kneib et al. 1996; Jullo et al. 2007; Jullo & Kneib 2009), which implements a Bayesian optimisation procedure to sample the posterior distribution of the parameter space. We remind here the Bayes' Theorem (Bayes 1764)

Theorem 1 (Bayes'). *Let θ be a set of parameters describing a model M of a system and let d be the available experimental data. Then the posterior probability distribution for the parameters set θ given the data d is*

$$P(\theta|d) = \frac{\mathcal{L}(d|\theta)P(\theta)}{P(d)}, \quad (3.52)$$

where $\mathcal{L}(d|\theta)$ is the likelihood of the data given the parameters, $P(\theta)$ is the prior, i.e. the a priori knowledge of the parameters, and $P(d)$ is a normalisation that does not depend on θ , called evidence.

To guarantee the correct normalisation,

$$P(d) = \int \mathcal{L}(d|\theta)P(\theta)d\theta, \quad (3.53)$$

where the integral is performed on the parameters space.

The likelihood of a model is based on its ability to reproduce the observed positions of the multiple images. If we have N multiply-imaged sources, the i -th source is imaged n_i times and the position of the j -th image of the i -th source is known with an uncertainty of $\Delta x_{i,j}$, the likelihood of a model defined by the parameter set θ is (Jullo et al. 2007)

$$\mathcal{L} = \prod_{i=1}^N \prod_{j=1}^{n_i} \frac{1}{\Delta x_{i,j} \sqrt{2\pi}} e^{-\frac{\chi_i^2}{2}}, \quad (3.54)$$

with

$$\chi_i^2 = \sum_{j=1}^{n_i} \frac{\|\mathbf{x}_{i,j}^{\text{obs}} - \mathbf{x}_{i,j}(\theta)\|^2}{\Delta x_{i,j}^2}. \quad (3.55)$$

Here $\mathbf{x}_{i,j}^{\text{obs}}$ and $\mathbf{x}_{i,j}(\theta)$ are, respectively, the observed and the model-predicted positions of the j -th image of the i -th source.

LenSTool uses the Markov chain Monte Carlo (MCMC) sampler *BayeSys*³ (Skilling 2004). Ten chains run simultaneously and the new steps are extracted, at each step, with a Metropolis Hastings algorithm (Hastings 1970). The convergence of the chains to the posterior probability distribution is guaranteed by a thermodynamic integration technique (Ò Ruanaidh & Fitzgerald 1996) called selective annealing. During an initial

²Version 6.8.1, <https://projets.lam.fr/projects/lenstool/wiki>

³<https://bayesian-ai.eecs.qmul.ac.uk/bayesys/>

burn-in phase, the likelihood is introduced in the Bayes' Theorem as $\mathcal{L}^\lambda$, and λ increases progressively from 0 to 1. This improves the capability for the MCMC of exploring the parameter space, and significantly reduces the possibility for them to converge in a secondary peak of the likelihood.

3.3.2 Parametrising the mass distribution of galaxy clusters

As already pointed out, SL is a linear phenomenon, i.e. the lensing deflection of a system of point masses (or of a continuous mass distribution) can be obtained as the sum (the integral) of the contributions of each point (infinitesimal mass element). As such, the superposition principle can be applied, and we can dissect the total mass of a galaxy cluster into its mass components, at the different scales. The total mass of clusters is typically decomposed into cluster- and galaxy-scale haloes. The former represent both the diffuse DM clumps hosting clusters and the ICM. The latter represent the total mass of the cluster galaxies. In addition, perturbation terms, such as foreground or background objects, and shear terms, are sometimes introduced, to take into account the lensing effect of mass structures nearby (or on the LOS), and occasionally to compensate for the . The total gravitational potential of a galaxy cluster is thus (see Natarajan & Kneib 1997; Grillo et al. 2015)

$$\Phi_{\text{tot}} = \sum_{i=1}^{N_{\text{cs}}} \Phi_{\text{cs},i}(\theta_{\text{cs},i}) + \sum_{j=1}^{N_{\text{gal}}} \Phi_{\text{gal},j}(\theta_{\text{gal},j}) + \sum_{k=1}^{N_{\text{p}}} \Phi_{\text{p},k}(\theta_{\text{p},k}), \quad (3.56)$$

where $\Phi_{\text{cs},i}$ is the gravitational potential of the i -th (out of N_{cs}) cluster-scale halo, $\Phi_{\text{gal},j}$ is the gravitational potential of the j -th (out of N_{gal}) galaxy, and $\Phi_{\text{p},k}$ is the gravitational potential of the k -th (out of N_{p}) perturbation. $\theta_{\text{cs},i}$, $\theta_{\text{gal},j}$, and $\theta_{\text{p},k}$ are the free parameters of the i -th cluster-scale halo, the j -th galaxy, and the k -th perturbation, respectively.

In this Thesis, similarly to most recent state-of-the-art parametric SL models of massive clusters, we use dPIE haloes (see Eqs. (3.41,3.42,3.47)) to parametrise the total mass distribution of both cluster- and galaxy-scale components (see Caminha et al. 2016, 2017a,b; Bonamigo et al. 2018; Bergamini et al. 2019; Acebron et al. 2022a,b; Bergamini et al. 2023b,a). As we detailed in Chapter 2, cosmological simulations suggest that DM haloes of any mass have a similar mass density profile, with a central cusp, usually described using the Navarro-Frenk-White (Navarro et al. 1996, 1997) or Einasto (Einasto 1965) profiles (e.g. Wang et al. 2020). Adopting a NFW profile for the diffuse DM components would, however, increase the computational time required for the model optimisation beyond practical limits. Furthermore, as shown by Bonamigo et al. (2018), a superposition of a few dPIEs is, in projection, very well fit by a 2D NFW profile. The haloes representing the mass distribution of the ICM can be fixed from X-ray observations, following Eq. (2.1) (Bonamigo et al. 2017, 2018). The ICM dominates the baryonic mass budget of the diffuse component of the cluster: when its mass distribution is fixed, we can disentangle its contribution from the total cluster mass distribution. As a consequence, the remaining cluster-scale clumps that we optimise (introduced with the term $\sum_{i=1}^{N_{\text{cs}}} \Phi_{\text{cs},i}(\theta_{\text{cs},i})$ to the gravitational potential in Eq. (3.56)) are representative of the diffuse DM haloes of the lens cluster.

The total mass distribution of cluster galaxies is instead modelled with spherical dPIEs with a vanishing core radius, centred in the centre of light of the member galaxy, leading to two free parameters for each halo. Up to a few hundreds of cluster members are included in the most recent SL models, which would imply a number of free parameters greater than the number of observational constraints. In order to reduce the

Inputs	Functional and parametric assumptions	Inferred parameters
• Lensed multiple image families • Positions of the multiple images	• Parametrisation of the total mass distribution with an array of cluster- and galaxy scale haloes • Functional form for the mass distribution of the haloes • Scaling laws for the parameters describing the cluster members	• Mass distribution of the cluster-scale haloes • Parameters of the scaling laws modelling the properties of the cluster members

Figure 3.2: Visual summary of the scope and utilisation of *LenSTool*.

number of free parameters during the model optimisation, power-law scaling relations are calibrated between the velocity dispersion and the truncation radius of the cluster members and their total luminosity. These relations are usually expressed as (Jullo et al. 2007; Brainerd et al. 1996)

$$\sigma_i = \sigma^{\text{ref}} \left(\frac{L_i}{L_0} \right)^\alpha, \quad (3.57)$$

$$r_{t,i} = r_t^{\text{ref}} \left(\frac{L_i}{L_0} \right)^\beta. \quad (3.58)$$

The scope and functioning of *LenSTool* is summarised in Fig. 3.2. Further discussion on the possible systematics introduced by this approach can be found in Parts III and IV.

Photometric and spectroscopic observations of strong lensing clusters

Due to their high mass and deep gravitational potential well, galaxy clusters often act as lenses causing SL of several tens of background sources. As anticipated, SL allows for a detailed reconstruction of the total mass profile of galaxy clusters and, when combined with other probes, for its decomposition into all baryonic and DM components. As such, massive clusters have been extensively observed by deep surveys. In this chapter, we briefly describe and summarise the imaging (at the Hubble Space Telescope, hereafter HST) and spectroscopy (at the Very Large Telescope, hereafter VLT) campaigns whose data outputs are the basis of the work presented in this Thesis.

4.1 Hubble Space Telescope imaging

Building the SL model of a massive galaxy cluster requires deep photometry and very high-precision astrometry. The positions of the multiple images are the primary observational constraint for the mass distribution profile optimisation, and they need to be known with an uncertainty of a few hundredths of arcseconds. The 2D structure of the cluster should be reconstructed accurately, distinguishing the cluster members from foreground and background objects, whenever spectroscopic redshifts are unavailable. Finally, as we will see, the structural parameters of the cluster members are a crucial prior on their mass distribution parameters.

We base the studies presented in this Thesis on imaging from the HST¹. The HST was launched in 1990, orbits at an altitude of 600 km and is equipped with two cameras: the Advanced Camera for Surveys (ACS) and the Wide Field Camera 3 (WFC3). Both cameras have a set of broad- and narrow-band-pass UV- to near-IR-wavelength filters. The two cameras have, respectively, a field of view (FoV) of $202'' \times 202''$ and $123'' \times 137''$. In particular we analyse the deep and accurate images provided by two surveys: the Cluster Lensing And Supernova survey with Hubble and the Hubble frontier Fields Programme which will be described in the following Sub-sections.

4.1.1 The Cluster Lensing And Supernova survey with Hubble

The Cluster Lensing And Supernova Survey with Hubble² (CLASH, Postman et al. 2012) is a 524-orbit Multi-Cycle Treasury Program at the HST, targeting 25 massive galaxy clusters to reconstruct their total mass distribution with SL. Galaxy clusters with over-dense cores, or irregular morphology, or dynamically young, i.e. showing signatures of recent

¹<https://www.stsci.edu/hst>

²<https://www.stsci.edu/postman/CLASH/>

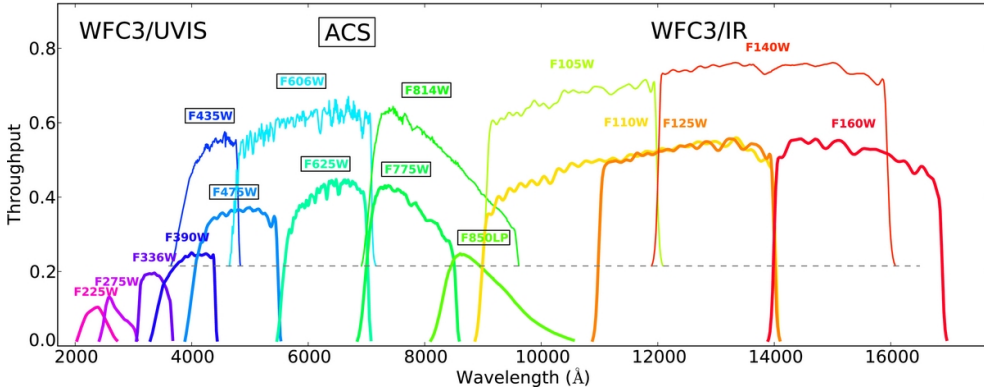


Figure 4.1: Transmission curves of the HST ACS and WFC3 filters of the bands used for the CLASH survey (from Postman et al. 2012).

or ongoing merger of substructures, have a significantly higher probability of causing SL events. A SL-selected sample of objects would therefore be biased in morphology and relaxation state with respect to the general population of clusters. To obtain a more representative sample of galaxy clusters, 20 out of the 25 targets (shown in Fig. 4.2) were selected based on their bright ($kT > 5$ keV) and smooth X-ray emission, indicative of a high mass and regular morphology, and only five of them were known strong lenses ($\theta_{\text{Einstein}} > 35''$ at $z = 2$), with a high number of multiple images. The 25 clusters were observed in the wavelength range 2000 – 17000 Å, using the 16 filters of the HST WFC3 and ACS cameras (see Fig. 4.1). CLASH photometry allowed for the observation of high-redshift lensed galaxies, the study of DM and baryons in cluster, and the measurement of cosmological parameters (see e.g. Coe et al. 2013; Umetsu et al. 2016; Riess et al. 2018).

4.1.2 The Hubble Frontier Fields programme

The Hubble Frontier Fields programme³ (HFF, Lotz et al. 2017) provided us with very deep imaging for a smaller sample of six SL clusters, shown in Fig. 4.3, three of which were included in the CLASH survey. They were selected for their SL features and the absence of bright stars in the FoV, and observed for 840 orbits (140 per cluster) with the F435W, F606W, and F814W ACS filters, and the F105W, F125W, F140W, and F160 WFC3 filters. The observations push to a 5σ point-source depth of 29 ABmag, approximately 1.5 mag deeper than CLASH. This makes the survey especially well suited to build accurate SL models to study high-redshift lensed sources (e.g. Bergamini et al. 2023a,b).

4.2 Spectroscopy with the Very Large Telescope

Spectroscopy plays a crucial role in SL modelling of galaxy clusters. As shown in Eq. (3.31), the observed deflection angle simultaneously depends on the lens mass and the distances between the observer, lens, and source, i.e. on their redshifts. Secure redshift measurements for the multiply imaged source are thus necessary to derive an accurate total mass profile from lensing observations. Breaking the degeneracy between lens mass and source redshift is also required to turn the relation between z and the cosmological

³<https://frontierfields.org/>

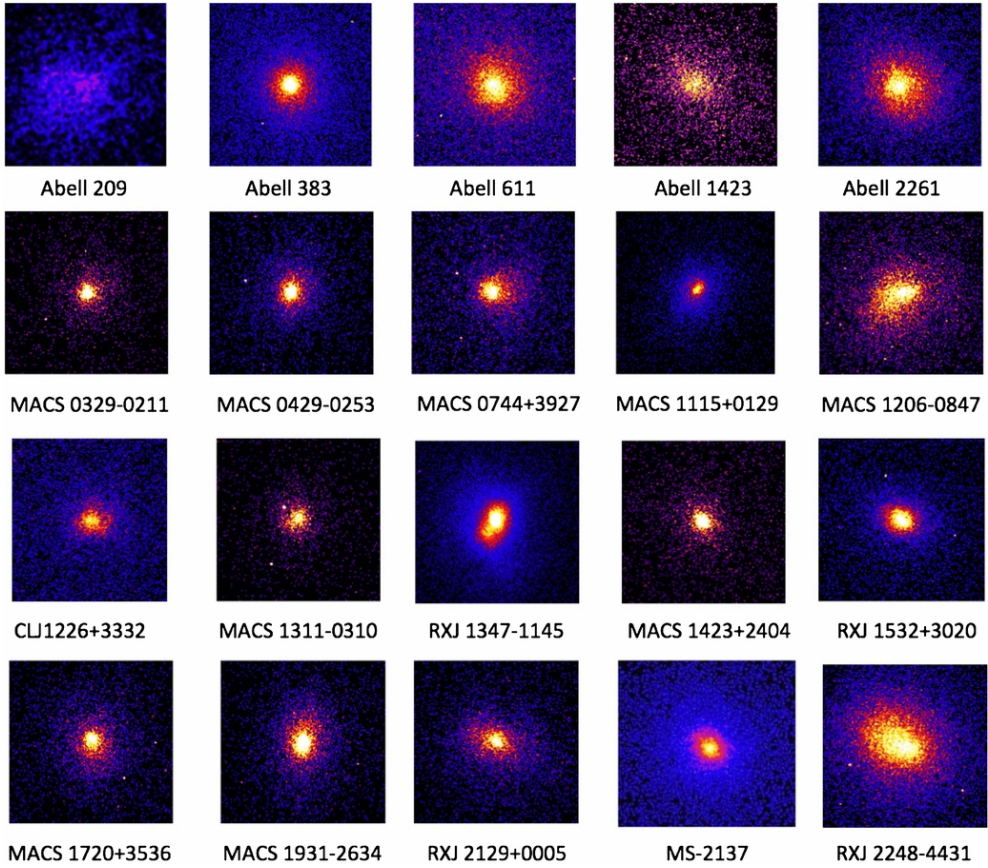


Figure 4.2: The 20 X-ray selected galaxy clusters of the CLASH survey: Chandra X-ray images (from Postman et al. 2012).

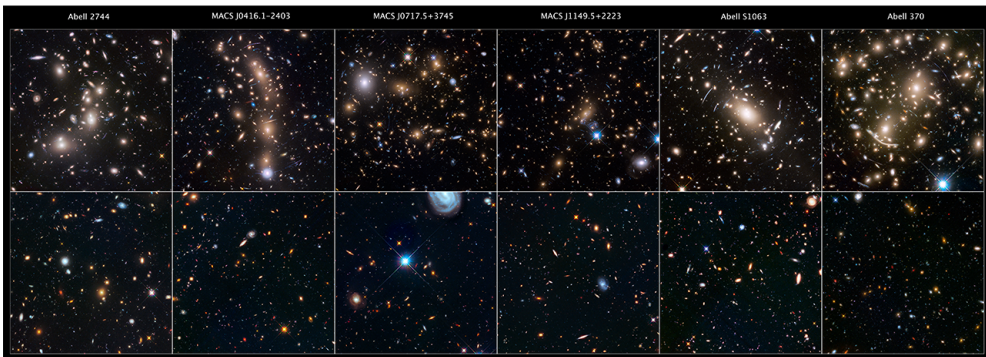


Figure 4.3: The galaxy clusters of the HFF survey. RGB image and parallel field of the six HFF clusters (credits: NASA, ESA, STScI, and the HFF team).

parameters into constraints on the values of the latter. This is especially true when the redshift values span a wide range, making the multiple images configurations more sensitive to the underlying cosmology (Caminha et al. 2022).

In this Thesis, we use spectroscopic data collected at the VLT⁴, at the European Southern Observatory of the Cerro Paranal in Chile. The telescope consists of an array of four Unit Telescopes (UT), with mirrors of 8.2 m in diameter, and four auxiliary movable telescopes (AT), with a diameter of 1.8 m. The UT telescopes allow for adaptive optics (AO) to correct for the effects of the atmosphere seeing and to improve the point spread function (PSF) of the observations, i.e. the function whose convolution accounts for the spreading of point sources in the observed images. It is important to distinguish between multi-object spectrographs and integral-field units: in the first case, a collection of targets on the FoV is defined before observations, and slits or fibres are placed in correspondence with their positions. In the second, the spectrum is spatially resolved across the instrument FoV. Integral-field spectroscopy allows for a blind search of multiple images, unlike multi-unit spectroscopy. This allows for a significant increase of the number of multiple images detected, as some emission-line multiply-imaged objects, such as Lyman- α high-redshift sources, may not have a strong enough optical counterpart to be detected. This comes with the disadvantage of a considerably smaller FoV, which limits the observations to a relatively small region of the cluster, typically the core.

4.2.1 The CLASH-VLT programme

Strong lensing modelling of the CLASH clusters required spectroscopic data to complement the HST imaging. This was guaranteed, for 12 out of the 25 clusters, by the CLASH-VLT⁵ programme (Rosati et al. 2014). The spectroscopic data were obtained by the Visible wide field Imager and Multi-Object Spectrograph (VIMOS), with a spectral of 3600 – 10000 Å, and a resolution in the range $R \approx 200 - 600$. The wide FoV of VIMOS (made of 4 quadrants of $7' \times 8'$ each) led to the spectroscopic confirmation of at least 500 members per cluster (see Fig. 4.4), allowing for an extensive study of its dynamics (e.g Mercurio et al. 2021; Sartoris et al. 2020; Biviano et al. 2023). More than 200 lensed images per cluster were identified (e.g. Rosati et al. 2014; Grillo et al. 2015), providing detailed constraints on its mass distribution.

4.2.2 The Multi Unit Spectroscopic Explorer at the VLT

The Multi Unit Spectroscopic Explorer⁶ at the VLT (MUSE, Bacon et al. 2010) is an integral-field spectrograph at the VLT. The instrument, installed in 2014, has a spectral range between 4650 Å and 9300 Å, a spectral resolution between $R \approx 1750$ at 4650 Å and $R \approx 3750$ at 9300 Å, and a spectral sampling of 1.25 Å/pixel. When working in wide-field mode (WFM), it has a FoV of $1' \times 1'$, and a pixel scale of $0.2'' \times 0.2''$. This field size makes it very useful for SL modelling, as most of the core of galaxy clusters at $z > 0.3$ can be covered by one or a few pointings, as shown in Fig. 4.5. This allows us to spectroscopically confirm faint cluster members, to blindly search multiply imaged sources, especially high-redshift emission-line sources (Bergamini et al. 2023b), and to study the kinematics of the cluster member galaxies (Bergamini et al. 2019; Granata et al. 2022). MUSE spectra will be the observational basis of several results presented in this Thesis, especially as far as the physical properties of the cluster members are concerned.

⁴<https://www.eso.org/public/teles-instr/paranal-observatory/vlt/>

⁵<https://sites.google.com/site/vltclashpublic/home>

⁶<https://www.eso.org/sci/facilities/develop/instruments/muse.html>

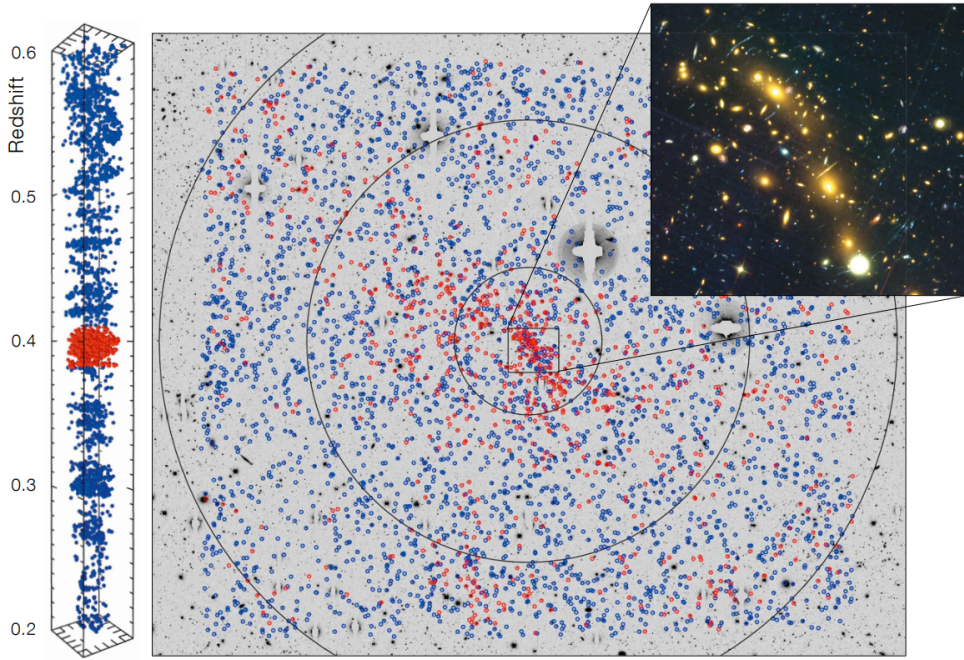


Figure 4.4: Three-dimensional distribution of the galaxies in a cluster field. Spatial and CLASH-VLT redshift distribution of the galaxies in the field of the cluster MACSJ0416.1–2403. Red dots indicate the spectroscopically confirmed cluster members. Blue dots indicate foreground and background objects. From Rosati et al. (2014).

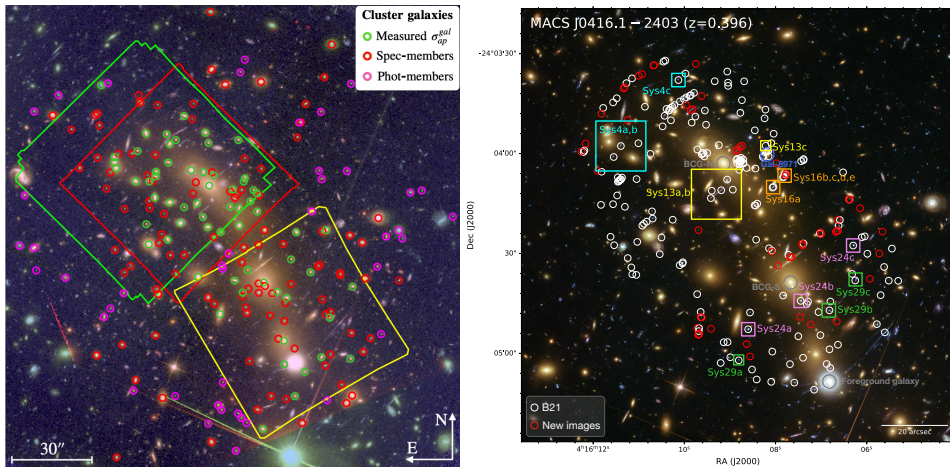


Figure 4.5: HFF RGB images of the core of a HFF cluster (MACSJ0416.1–2403). Left panel: the cluster members included in the most recent SL model are marked with small circles. Spectroscopically confirmed members are marked in red and green. The MUSE pointings are overlaid (from Bergamini et al. 2021). Right panels: the multiple images are included in the most recent SL model are marked with small circles (from Bergamini et al. 2023b).

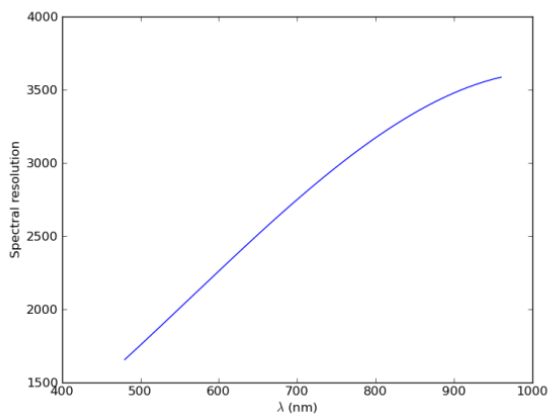


Figure 4.6: Resolution of MUSE as a function of wavelength.

From <https://www.eso.org/sci/facilities/paranal/instruments/muse/inst.html>

Measured kinematics of the cluster member galaxies

One of the main objectives of this Thesis is the combination and the comparison of the information provided by SL and galaxy kinematics on the structure and physical properties of cluster members. In Chapter 3, we set out the theoretical and phenomenological basis of SL, detailing how its observation and study provide us with detailed information on the projected mass distribution profile of lens galaxies. On the other hand, stellar kinematics is a completely independent probe of the three-dimensional mass distribution, subject to different and unrelated systematics. As such, the combination of SL and dynamics allows us to break the degeneracies affecting the determination of the total mass distribution of galaxies. Insights on the stellar kinematics of galaxies can be obtained from the study of their spectral energy distribution (SED). In this chapter, we provide a general observational framework on galaxy dynamics, and on its probes from observed galactic SED.

5.1 The spectral energy distribution of early-type galaxies

The internal kinematics of galaxies can be inferred from the study of the broadening of the absorption lines of their spectrum. Galaxy spectra are the result of the luminosity-weighted output from billions of stars (an example of a stellar spectrum is shown in Fig. 5.1) and the surrounding interstellar medium. Thus spectra encode the physical and chemical properties of stellar populations and the gas content within each galaxy. The content of stars in each galaxy is the result of the combination between a stellar initial mass function (IMF) and a star formation history.

The stellar IMF is the mass function of the stars of a galaxy $\phi(M)$ at the time of their formation. The IMF of the milky-way (MW) is well described by a Chabrier (Chabrier 2003) or Kroupa (Kroupa 2001) IMF, with $\phi \propto M^{-2.35}$ for $M \gtrsim 0.5 M_{\odot}$ and a decrease for lower masses. Spectral studies (e.g. Vazdekis et al. 2016), as well as lensing and dynamical models (e.g. Grillo & Gobat 2010; Sonnenfeld et al. 2012, 2019; Cappellari et al. 2012, 2013a,b; Cappellari 2016), find evidence in support of massive early-type galaxies having a steeper-than-MW IMF at low masses (see Smith 2020), also referred to as a bottom-heavy IMF, typically fit with a power-law Salpeter IMF (Salpeter 1955). The IMF evolves according to a star formation rate, defined as the fraction of gas mass converted into stars per unit time. As the amount of gas available decreases with time, this is usually described with an exponentially decreasing function with respect to the time ($\psi(t)$). Another factor determining the spectrum of a galaxy is its metallicity at a given time $Z(t)$, as a higher metallicity leads to redder stars with a larger mass-to-light ratio. Due to the metals created by stellar evolution, $Z(t)$ is an increasing function. From a stellar IMF and $Z(t)$ we can derive $F_{\lambda,Z}(t)$, the SED of a galaxy at time t . $F_{\lambda,Z}(t)$ can be

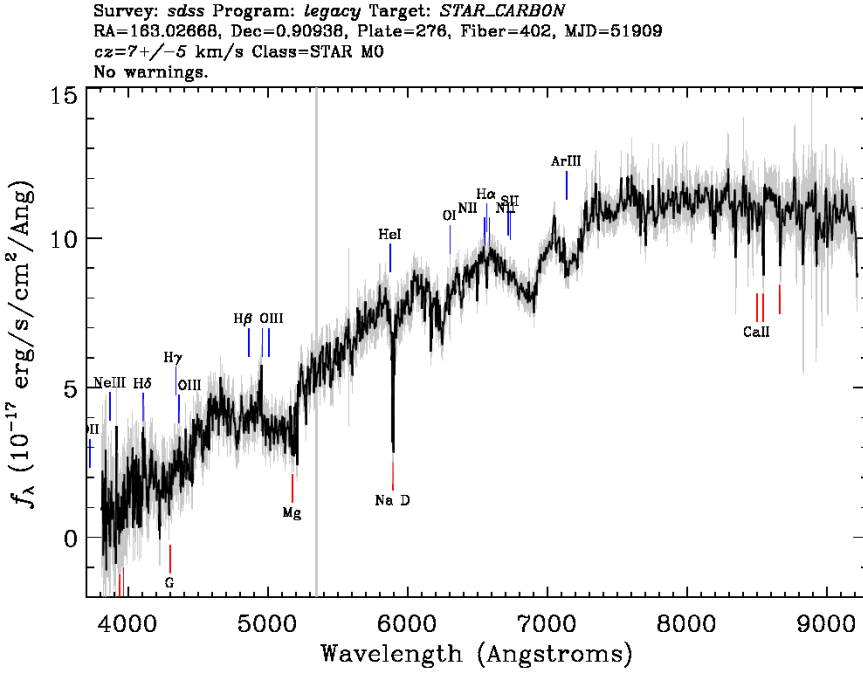


Figure 5.1: Spectrum of a red star (SDSS J105206.40+005433.7). From the SDSS data release 14

convolved with the star-formation function to derive the observed spectrum of a galaxy at time t , $F_\lambda(t)$

$$F_\lambda(t) = \int_0^t \psi(t - t') F_{\lambda,z}(t') dt'. \quad (5.1)$$

As anticipated in Chapter 2, the population of galaxies in the cluster centers is morphologically dominated by ellipticals. Elliptical galaxies are generally characterized by old stellar populations, dominated by red, lower-mass stars and show very little star formation. As a consequence, in contrast to spiral galaxies, they do not show prominent emission lines, as clear from Fig. 5.2. Some prominent features are a 4000 Å break, caused by absorption from the metals in the atmospheres of the old stars, and the deep CaII H and K absorption lines at rest-frame wavelengths of, respectively, 3934 Å and 3969 Å. At higher wavelengths, as clear from Fig. 5.2, we notice the G-band Fraunhofer line and the Ca II triplet.

5.2 Inferring the parameters of the LOSVD of a galaxy from its spectrum

In Chapter 1, we have introduced the main concepts and equations regulating the dynamics of elliptical galaxies, which we have linked, in Chapter 3, to the mass density profiles commonly adopted for SL models of galaxy cluster members. In particular, we

¹<https://skyserver.sdss.org/dr14/en/home.aspx>

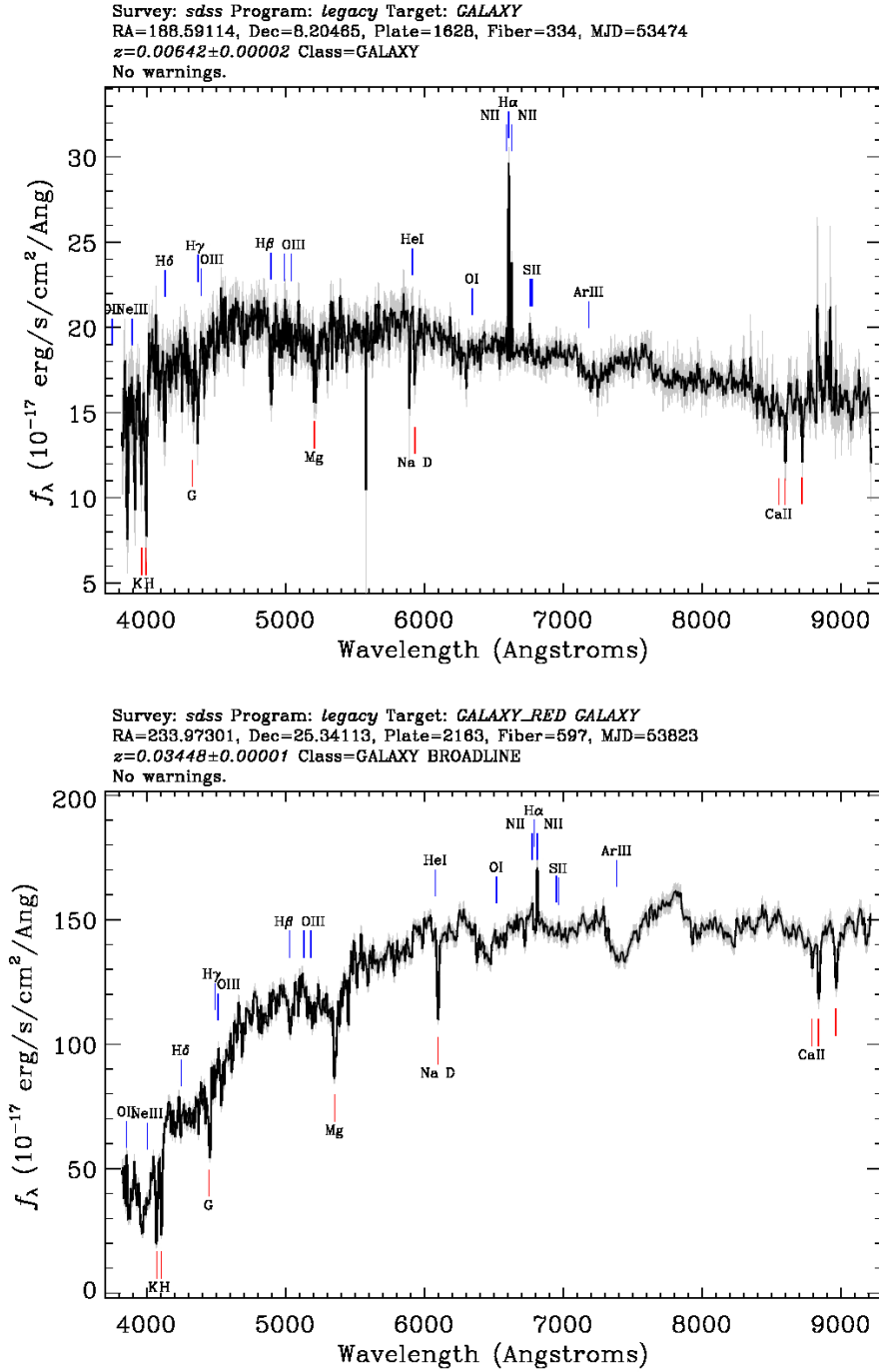


Figure 5.2: Spectra of SDSS galaxies with prominent emission (marked in blue) and absorption (in red) lines. Upper panel: the spiral galaxy SDSS J123421.84+081216.6. Lower panel: the elliptical galaxy SDSS J124249.82+024115.9. From the SDSS data release 14¹.

have seen how the distribution function links a galaxy's gravitational potential to its stellar kinematics. The line-of-sight velocity distribution (LOSVD) of the stars is reflected into Doppler shifts for their spectra: the spectrum of a star that is moving with respect to the galaxy centre-of-mass with a LOS-velocity component v_{LOS} will be subject to a Doppler shift $\lambda \rightarrow \lambda' = \lambda + \Delta\lambda$, where

$$\Delta\lambda = \left(\frac{v_{\text{LOS}}}{c}\right) \lambda. \quad (5.2)$$

Equation (5.2) is valid for the comparatively small v_{LOS} values ($v_{\text{LOS}}/c \lesssim 0.01$) found when observing the motion of member galaxies with respect to the centre of mass of galaxy clusters, while it is ill-defined for higher values. The shift is naturally reflected by the observed wavelength of the known spectral features. The observed spectrum of a galaxy is thus not only determined by its stellar population and evolution, as presented in Eq. (5.1), but also by the LOSVD of its stars. The LOSVD of the stars in a galaxy is typically described by a function $\mathcal{L}(v_{\text{LOS}})$, such that the fraction of stars with LOS velocity between v_{LOS} and $v_{\text{LOS}} + dv_{\text{LOS}}$ is $\mathcal{L}(v_{\text{LOS}})dv_{\text{LOS}}$. From $\mathcal{L}(v_{\text{LOS}})$, we can also compute the moments of the LOSVD of a galaxy: the first is its peculiar LOS velocity, the second is the LOS velocity dispersion

$$\langle v_{\text{LOS}} \rangle = \int_{-\infty}^{+\infty} \mathcal{L}(v_{\text{LOS}}) v_{\text{LOS}} dv_{\text{LOS}}, \quad (5.3)$$

$$\sigma_{\text{LOS}}^2 = \int_{-\infty}^{+\infty} \mathcal{L}(v_{\text{LOS}}) (v_{\text{LOS}} - \langle v_{\text{LOS}} \rangle)^2 dv_{\text{LOS}}. \quad (5.4)$$

The second moment of the LOSVD does not influence the central wavelength of the spectral features, but leads to their broadening. At first approximation, an ideal spectral feature of infinitesimal width will result in a line-spread-function (LSF) approximately equal to $\left(\frac{\sigma_{\text{LOS}}}{c}\right) \lambda$. This line broadening emerges from the comparison between the width of the lines in the spectrum of a galaxy (Fig. 5.2) and a single star (Fig. 5.1). In practice, the observed σ_{LOS} is always weighted by the light distribution (as in Eq. 3.50), and we will measure it within circular apertures centred in the centre of light of the target galaxies.

Equation (5.2) shows that the Doppler shift of a line at a rest-frame wavelength λ_0 for a given v_{LOS} is proportional to λ_0 : it is then convenient to define the variable $x = \ln \lambda$. In this case, a star moving with a LOS velocity v_{LOS} will be subject to a constant shift

$$x \rightarrow x' = \ln \lambda' = \ln \lambda + \frac{v_{\text{LOS}}}{c} = x + \frac{v_{\text{LOS}}}{c}. \quad (5.5)$$

This formalism is particularly well suited to the inference of the parameters of the LOSVD of a galaxy from its observed spectrum $G(x)$. If we build a model galaxy spectrum $G_M(x)$ as the convolution of a template spectrum $T(x)$, typically the combination of a library of spectra of single stars of different types, then

$$G_M(x) = (T * \mathcal{L})(x) = \int_{-\infty}^{+\infty} \mathcal{L}(v_{\text{LOS}}) T\left(x - \frac{v_{\text{LOS}}}{c}\right) dv_{\text{LOS}}. \quad (5.6)$$

This means that, in principle, we can infer the best-fit parameters of $\mathcal{L}(v_{\text{LOS}})$ by comparing $G(x)$ and $G_M(x)$. The problem is made especially critical by the fact that it requires simultaneously fitting for the template combination that best reproduces the

spectral properties of the galaxy and for the parameters of the LOSVD. In the next Sub-section, we detail the solution adopted in this Thesis, proposed by Cappellari & Emsellem (2004), and refined in Cappellari (2017, 2023), implemented in the Penalized Pixel Fitting (pPXF²) code.

5.2.1 Parametric line-of-sight velocity distribution modelling with pPXF

In this Sub-section, we describe the pPXF method (Cappellari & Emsellem 2004; Cappellari 2017, 2023) adopted for the LOSVD analyses on which the results of this Thesis are based. Equation (5.6) will be the basis of the procedure: a spectral model of the target galaxy is obtained from the combination of a set of templates convolved for a LOSVD, and compared to the observed spectrum.

The observed spectra (and the templates, if necessary) are brought to rest-frame as a first step. A complication emerges when the template spectra are not observed with the same spectrograph as the fitted galaxy. High-resolution templates are typically used to measure lower values of σ . However, the spectral resolution impacts on the intrinsic width of the spectral features, implying that the observed line broadening may not entirely be the result of the convolution with the LOSVD. The templates thus need to be convolved with a kernel $K(x, \lambda)$, in general dependent on λ , so that $\hat{T}(x) = T(x) * K(x, \lambda)$ has the same resolution as the observed spectra. This passage is handled by pPXF using a Gaussian kernel with variance $var = \sigma_{\text{diff}}^2(\lambda) = \sigma_{\text{obs}}^2(\lambda) - \sigma_{\text{temp}}^2(\lambda)$, where σ_{obs} and σ_{temp} are, respectively, the Gaussian dispersion of the LSF of the observed spectrum and of the templates. The observations and the templates are then re-binned to the same logarithmic scale, introduced as x in the previous Section. The pixel scale Δx naturally defines a velocity scale

$$V := c\Delta x = c\Delta \ln \lambda. \quad (5.7)$$

The value of V is crucial to convert pixel differences between the model and the observed spectrum into an estimate of the LOSVD moments.

Equation (5.6) is implemented by pPXF as

$$G_M(x) = \sum_{n=1}^N w_n \left(\left(\hat{T}_n(x) * \mathcal{L}_n(cx) \right) \sum_{k=1}^K a_k P_k(x) \right) + \sum_{l=1}^L b_l P_l(x) + \sum_{j=1}^J c_j S_j(x). \quad (5.8)$$

Here $\hat{T}_n(x)$ is part of a set of N stellar templates, each one convolved with a LOSVD $\mathcal{L}_n(cx)$ and multiplied by a weight w_n that sets its contribution to the galactic spectrum. $P_l(x)$ are additive polynomials, from the truncated Fourier series, to compensate for template mismatch and adjust the intensity of the spectral lines. $P_k(x)$ are multiplicative polynomials, from the Legendre series, to compensate for inaccurate spectral calibrations or dust reddening. Finally, $S_j(x)$ are the sky contributions to the spectrum.

A Gauss-Hermite parametrisation is chosen for the LOSVD: if we define the parameter $y = \frac{v_{\text{LOS}} - \langle v_{\text{LOS}} \rangle}{\sigma_{\text{LOS}}}$, then

$$\mathcal{L}(v_{\text{LOS}}) = \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}\sigma} \left(1 + \sum_{m=3}^M h_m H_m(y) \right), \quad (5.9)$$

where $H_m(y)$ are Hermite polynomials and h_m are the moments of the LOSVD with $m \geq 3$. The set of moments of the LOSVD $(v_{\text{LOS}}, \sigma_{\text{LOS}}, h_m)$, and the weights (w_n, a_k, b_l, c_j) are

²Version 8.0.2,
https://pypi.org/project/ppxf/

https://www-astro.physics.ox.ac.uk/cappellari/software/#ppxf,

thus optimised to minimise the difference between $G(x)$ and $G_M(x)$. The optimisation is handled by a penalised χ^2

$$\chi_p^2 = \chi^2(1 + \xi^2 \mathcal{D}^2). \quad (5.10)$$

χ^2 is the standard measure of the agreement between model and observation

$$\chi^2 = \sum_{n=1}^{N_{\text{pix}}} \frac{(G_M(x_n) - G(x_n))^2}{G(x_n)^2}, \quad (5.11)$$

where the residuals are computed for all pixels (for $n = 1, 2, \dots, N_{\text{pix}}$). The term $\mathcal{D}^2$, weighted by ξ^2 , is a measure of the difference between the best-fit LOSVDs obtained, respectively, when all the LOSVD moments are optimised, and when only the first two are (Gaussian approximation of the LOSVD). The penalisation term biases the fit towards a Gaussian LOSVD when the recovered σ is comparable to (or smaller than) the spectral resolution and when the signal-to-noise of the spectrum is low. As detailed and motivated in the next Sub-section, in this Thesis we only focus on recovering the first two moments of the LOSVD, which implies that we use a Gaussian LOSVD and $\mathcal{D}^2$ is thus zero. Therefore, we will set the bias parameter ξ to zero during the spectral fits described in this Thesis.

5.3 Velocity dispersion measurement of cluster galaxies from VLT/MUSE spectroscopy

In this Section, we describe the pipeline built for the kinematic studies performed in this Thesis, and based on the code `pPXF`, which we presented previously. The main objective of our pipeline is to allow for a reliable measurement of the σ_{LOS} of the largest possible sample of member galaxies in SL clusters for which deep MUSE pointings are available. This is crucial in order to calibrate accurate scaling laws to describe the members in SL models of massive clusters (Bergamini et al. 2019, 2021; Granata et al. 2022; Bergamini et al. 2023b; Beauchesne et al. 2024; Bergamini et al. 2023a). The main application of this pipeline is an ongoing work presented in Chapter 10, which aims at a study of the velocity dispersion function of the quiescent cluster members of the HFF clusters for which a deep MUSE pointing is available (Abell 2744, MACS J0416.1–2403, MACS J1149.5+2223, Abell S1063, and Abell 370), which will allow us to probe the evolution of cluster member galaxies and compare the velocity function of observed and simulated quiescent members.

5.3.1 pPXF fit of σ_{LOS}

In this Sub-section, we describe the `pPXF` configuration for the fit of the LOS velocity dispersion of the member galaxies of massive lensing clusters. As anticipated, we only focus on measuring the value of σ_{LOS} , the second moment of the LOSVD. In these conditions, no penalty term is added to the χ^2 from Eq. (5.10); thus, the bias value does not need to be set. Bergamini et al. (2019), in a similar work, fit the first four moments of the LOSVD, finding a degeneracy between σ_{LOS} and h_4 for low values of σ_{LOS} , which they empirically corrected for. The correction is typically a few km s^{-1} , which is comparable with the uncertainty on the velocity dispersion. Like Bergamini et al. (2019) and D’Ago et al. (2023), we decide to use only additive polynomials, as we do not wish to make stellar population studies and our fits do not suggest the need for a further correction for calibration or reddening problems. We choose a degree of the additive degrees of

12, as suggested by D’Ago et al. (2023), which allows us to precisely recover the σ_{LOS} of simulated spectra (see Appendix A).

We choose to fit the LOSVD on the spectral range $[3700 - 5400] \text{ \AA}$, to include the main spectral absorption features of the spectra of elliptical galaxies while excluding the regions of low S/N due to the MUSE sensitivity curve and those affected by sky-lines residuals. The average spectral resolution of MUSE, introduced in Fig. 4.6, is approximately 2600. This corresponds to a redshift-independent instrumental dispersion of approximately 50 km s^{-1} , and to a $\Delta\lambda$ resolution of approximately $2.6 \text{ \AA}$, which needs to be de-redshifted to the rest-frame. As stellar templates, we use the spectra from the second data release (DR2) of the X-shooter Spectral Library (XSL), presented in Gonneau et al. (2020). We limit ourselves to the UVB branch $[3000 - 5560] \text{ \AA}$, which covers the spectral range we are focusing on at the redshift of the clusters considered in this Thesis. These spectra have a nominal resolution of $R = 9700$, corresponding to a dispersion of approximately only 13 km s^{-1} . As we notice a few anomalies in the wavelength axes of a few of these templates, we re-bin them on the declared wavelength scale of $0.15 \text{ \AA}$. We only include the templates with a signal-to-noise per unit wavelength larger than $100 \text{ \AA}^{-1}$, for a total of 463 templates. Galaxy emission lines, sky lines, telluric lines, and the adaptive-optics laser emission have been masked, in the same way as Bergamini et al. (2019). An example of a σ_{LOS} fit is shown in Fig. 5.3, for a member galaxy of the massive cluster MACSJ0416.1–2403.

In Appendix A we describe a set of simulations of cluster member spectra, which were used to test the accuracy of the velocity dispersion fitting procedure based on `pPXF` for the typical values of σ_{LOS} and spectral signal-to-noise ratio (S/N) found for MUSE pointings of the cores of massive clusters. We report here just the results and the implications for our procedure: we find that for spectra with $S/N \geq 10$ the value of σ_{LOS} is recovered without any bias besides those associated with under-sampling (see Cappellari & Emsellem 2004), which slightly biases (by up to a few km s^{-1} , see Fig. A.1) the recovered σ_{LOS} values below $\approx 100 \text{ km s}^{-1}$, which correspond to approximately 2 pixels. We report here the required correction to the measured velocity dispersion values for $\sigma_{\text{LOS}} < 250 \text{ km s}^{-1}$, as derived in Eqs. (A.1,A.2)

$$\sigma(\text{km s}^{-1}) = \sigma_m(\text{km s}^{-1}) - \sigma_{\text{corr}}(\text{km s}^{-1}), \quad (5.12)$$

where σ_m is the measured value of σ_{LOS} measured in km s^{-1} and σ_{corr} is a correction bound to σ_m by

$$\sigma_{\text{corr}}(\text{km s}^{-1}) = (3.32 - 1.23 \times 10^{-2} \sigma_m + 4.82 \times 10^{-6} \sigma_m^2) (\text{km s}^{-1}). \quad (5.13)$$

The LOSVD fit performed on observed cluster members shows that for $S/N < 10$ the fit often completely fails, or struggles to recover the observed shape of the absorption features. As such, we only include spectra with $S/N \geq 10$ for our sample of measured velocity dispersions. In any case, we visually check all spectral fits for anomalous residuals around the spectral features and all of the MUSE cube cutouts for light contamination. Even for $S/N > 10$, the noise determines random offsets between the recovered and true values of the velocity dispersion, which make the determination of the best-fit template combination required to fit the observed spectrum more difficult. In Appendix A we also estimate this contribution to the error budget, which we will sum in quadrature to the statistical uncertainty determined by `pPXF` during the LOSVD fit in order to take this effect into account. The contribution of the noise to the relative statistical uncertainty on the velocity dispersion is linked to the S/N value by the following relation derived in

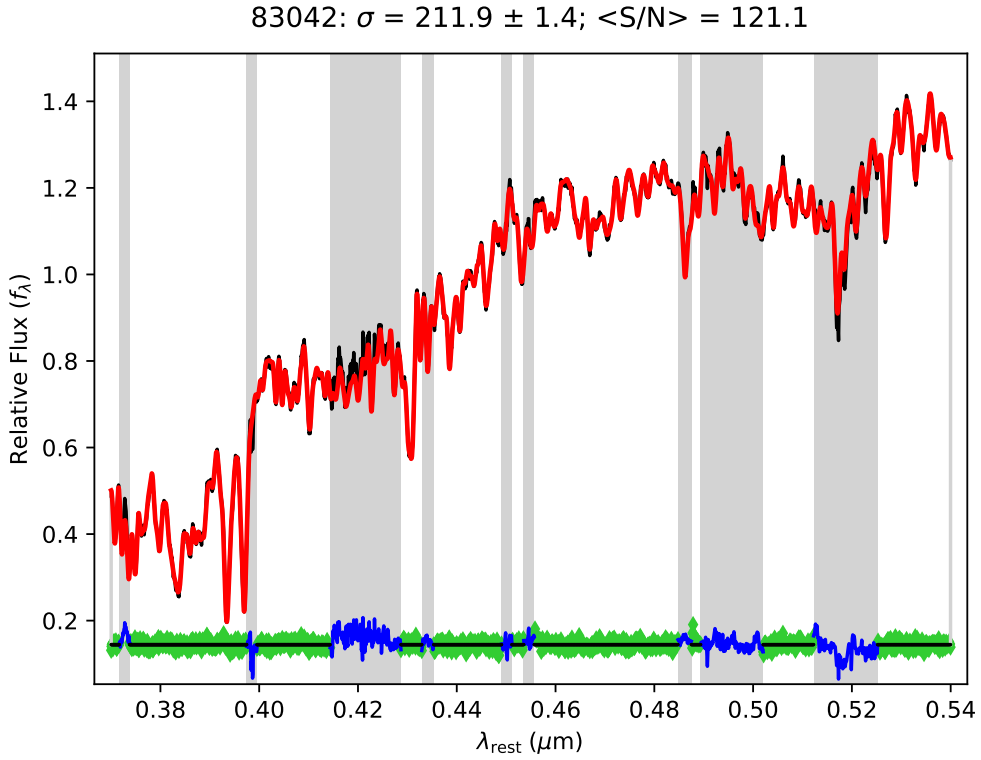


Figure 5.3: pPXF fit of the spectrum of a cluster member (member 83042 of MACS J0416.1–2403). The observed spectrum (in black) is plotted together with the best-fit spectrum (in red). The fit residuals are shown at the bottom of the figure (in green). The masked regions are marked in grey, the residuals are drawn in blue. We report the recovered σ_{LOS} (in km s^{-1}) and the spectral S/N .

Eq. (A.3)

$$\delta\sigma_{\text{noise}}/\sigma = 7.62 \times 10^{-2} - 3.28 \times 10^{-3}(S/N) + 5.30 \times 10^{-5}(S/N)^2 - 2.72 \times 10^{-7}(S/N)^3, \quad (5.14)$$

where σ is the measured value of the LOS velocity dispersion provided by `pPXF`. As such, the contribution of the noise to the absolute uncertainty will be $\delta\sigma_{\text{noise}} = (\delta\sigma_{\text{noise}}/\sigma) \sigma$, and the final estimate of the absolute uncertainty on the velocity dispersion measurement will be $\delta\sigma = \sqrt{\delta\sigma_{\text{noise}}^2 + \delta\sigma_{\text{pPXF}}^2}$, where $\delta\sigma_{\text{pPXF}}$ is the velocity dispersion uncertainty estimated by `pPXF` during the fit of the observed spectrum to the best-fit template combination.

Part III

Improved strong lensing modelling of galaxy clusters using the Fundamental Plane

In this Part, we present a new strong lensing modelling of the galaxy cluster Abell S1063, in which we obtained an improved description of the mass distribution of the cluster galaxies by using the Fundamental Plane relation to link the velocity dispersion of the members to their structural parameters. This technique, combined with accurate measurements of the stellar and gas mass distribution of the cluster has allowed us to obtain a more detailed mapping of its dark and baryonic mass components. The enhanced constraints on the mass structure of the cluster galaxies have allowed us to perform a meaningful comparison with the most recent cosmological hydrodynamical simulations.

The Part is organised as follows. In Chapter 6, we give details on our photometric and spectroscopic data and on how they were analysed and reduced for our objectives, and we describe our strong lensing model built with the FP. In Chapter 7, we present the analysis and discussion of the results of the optimisation, and the comparison with the previous model by Bergamini et al. (2019). We show the cumulative, projected mass profiles for all the baryonic and DM components of our model, and we compare our results with the predictions of stellar-to-halo mass relations derived with halo occupation distribution studies based on N -body simulations. Furthermore, in Chapter 7, we compare the stellar-to-total mass fraction and the compactness of the sub-haloes in our model with those suggested by recent high-resolution hydrodynamical simulations. Finally, we summarise our most important conclusions.

The main scientific results presented in this Part are reported in Granata et al. (2022), whilst Chapter 7 includes part of the analysis performed in Ragagnin et al. (2022) taking advantage of the results of Granata et al. (2022).

Improving the strong lensing model of Abell S1063

In the past few years, gravitational lensing has become one of the most effective techniques to study the total mass distribution of galaxy clusters (e.g. Natarajan & Kneib 1997), as it does not discriminate between baryons and DM. Strong lensing is especially effective in the central regions, or cores, of massive clusters (i.e. within approximately 1 arcmin or approximately 350 kpc, at $z = 0.5$), where several multiple images of background sources are usually found. In these regions, two important discrepancies between the predictions of cosmological simulations and lensing models have emerged. First, several SL models suggest that the main DM haloes of clusters might have central flat cores (Sand et al. 2004), extended up to around 100 kpc (Grillo et al. 2015; Annunziatella et al. 2017; Bergamini et al. 2019, hereafter B19). This result seems to be confirmed on a homogenous and unbiased sample of massive lens galaxy clusters (Postman et al. 2012). A second discrepancy appears when comparing the amount and distribution of DM substructures (or sub-haloes). As found in Grillo et al. (2015), Munari et al. (2016) and Bonamigo et al. (2018), cosmological simulations under-estimate the number of massive substructures near the cores of massive clusters. Furthermore, the number of predicted galaxy-scale SL events seems to imply that the mass distributions of these substructures are more concentrated than suggested by hydrodynamical simulations (Meneghetti et al. 2020). More details on this last issue will be provided in Chapter 7.

Data from the numerous spectro-photometric surveys that focused on lensing galaxy clusters, introduced in detail in Chapter 4, have allowed for the construction of detailed SL models, whose accuracy can be assessed considering the distance between the observed positions of multiple images and those predicted by a model. The root mean square (rms) of this discrepancy can be as low as $0.3''$ (e.g. Caminha et al. 2019, B19), however still higher than the observational uncertainty on the position of the images, which can be less than a tenth of arcsecond. This might suggest that the total mass modelling of galaxy clusters needs to be further improved (see Meneghetti et al. 2017; Remolina González et al. 2018; Raney et al. 2020).

Strong lensing observations permit a very effective determination of the total mass enclosed within the multiple images of a single background source, but the same amount of mass can be obtained with relatively different mass distribution models (e.g. Limousin et al. 2016; Ghosh et al. 2021). This implies that the parameters that define a model often have a certain degree of degeneracy between them, which corresponds to a possible transfer of mass between the various components, introduced in Eq. (3.56).

Parametric degeneracy has a strong impact on the recovery of the mass distribution of the cluster galaxies. Due to the high total mass of lensing galaxy clusters, multiple images of background sources appear separated by up to a few tens of arcseconds. As such, their positions strongly depend on the total mass of the cluster galaxies, but they

are not very sensitive to the details of its distribution. As a consequence, the scaling relations typically introduced to constrain their velocity dispersion and truncation radius values, introduced in Eqs. (3.57) and (3.58), suffer from a strong degeneracy that affects the compactness of cluster galaxies: the same total mass can be obtained with a higher (lower) value of σ and a lower (higher) value of r_t . Information on the kinematics of galaxies can significantly reduce the degeneracy between the parameters describing the sub-halo component of a SL model, offering an independent prior on the value of σ . In recent years, velocity dispersion measurements have also become available for several members in some galaxy clusters. Monna et al. (2015) used spectrography from the Hectospec fiber spectrograph at the MMT and early CLASH data for 21 members of the cluster Abell 383, showing a significant reduction of the degeneracy between the SL model parameters. B19 refined and significantly extended this method to three CLASH-VLT clusters: MACS J1206.2–0847, MACS J0416.1–2403, and Abell S1063. The integral field data provided by MUSE permitted to obtain velocity dispersion measurements for 40–60 galaxies for each cluster. These data have been exploited to estimate the values of the slope and normalisation of the Faber-Jackson relation (as introduced in Eqs. (1.36) and (3.57)) for those members. This has, in turn, been used as a prior to determine the exponent of the power-law relation which binds the total mass of sub-haloes to their luminosity. The new data and procedure have allowed for a more accurate mass assignment to the diffuse and sub-halo components, reducing the degeneracy between their parameters. This is crucial both to infer the DM mass density profile in the central regions of a cluster and to determine precisely its sub-halo mass function.

This technique has enabled a first step towards a more accurate mass modelling of the cluster members, but a single power-law relation between their total mass and luminosity may be inaccurate on the very large total mass range of member galaxies ($10^9 - 10^{13} M_\odot$). In order to improve on the adopted scaling relations for members, in this Thesis we will consider the Fundamental Plane relation, introduced in Eq. (1.33), which has a lower scatter than the Faber-Jackson law. We will focus on one of the three clusters considered by B19, namely Abell S1063 (AS1063). We choose it due to its rather regular morphology and because we have at our disposal line-of-sight velocity dispersion measurements for a sizable subset of cluster members, for which we also have an estimate of their effective radius and magnitude. We can therefore use these galaxies to obtain the best-fit values of the parameters that define the FP.

6.1 Abell S1063: Data and analysis

Abell S1063, first identified in Abell et al. (1989), is a massive galaxy cluster at redshift $z = 0.348$. A virial mass value of $(2.17 \pm 0.06) \times 10^{15} M_\odot$ was estimated by Sartoris et al. (2020). Despite its regular shape, it appears to have recently undergone an off-axis merger, as inferred by Gómez et al. (2012) and Rahaman et al. (2021) from X-ray data, and by Mercurio et al. (2021) from the dynamics of the members. Its high mass and redshift make it a very effective gravitational lens, which allows us to use the observed position of 55 spectroscopically confirmed multiple images from 20 background sources, distributed over a wide range of redshifts ($z = 0.73 - 6.11$, see Caminha et al. 2016), to build a SL model. This high number of spectroscopically confirmed sources is crucial for an accurate SL model. Indeed, the well-known degeneracy between the total mass of a lens and the redshift of a multiply-imaged source can be broken when several multiply-imaged sources in a wide redshift range are available and their redshifts are measured.

The cluster is also a very bright X-ray source ($L_X \approx 2.5 \times 10^{45} \text{ erg s}^{-1}$ in the 0.1 – 2.4 keV band), due to the emission of the hot ($T \approx 12 \text{ keV}$) ICM (Rahaman et al. 2021): it

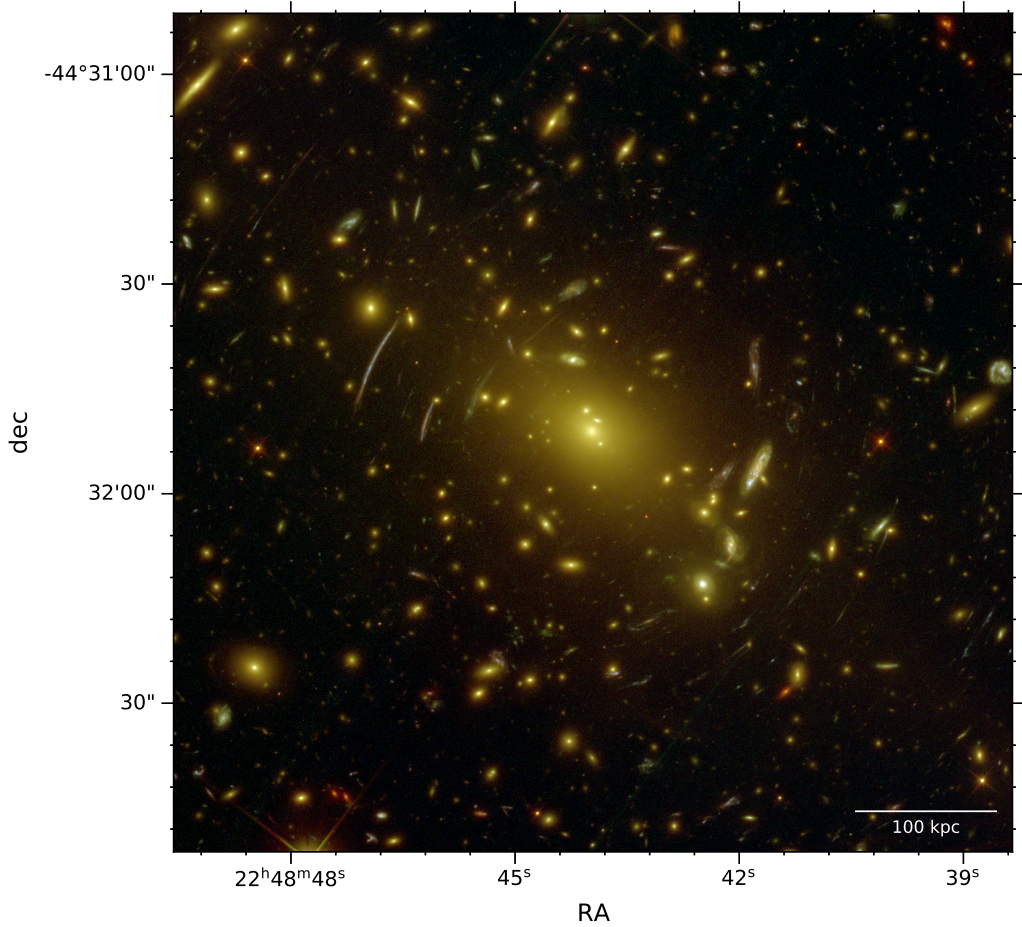


Figure 6.1: Colour composite image of the core of AS1063 created from HFF photometry. Blue, green and red channels are obtained by combining the filters F435W, F606W + F814W, and F105W + F125W + F140W + F160W, respectively. (From Granata et al. 2022).

was identified as RXJ 2248.7–4431 in the ROSAT All-Sky Survey (De Grandi et al. 1999; Guzzo et al. 1999).

6.1.1 Photometric data

Following Tortorelli et al. (2018), we will use the HFF photometric data in the F814W band, which corresponds to a rest-frame R-band for the cluster redshift, as it allows for a good signal-to-noise ratio down to magnitudes of around 28 and it has a lower full width at half maximum (FWHM) of the PSF when compared to the near-IR bands.

In order to obtain the parameters of the FP and consequently derive the values of the velocity dispersions of the cluster members, we need robust measurements of their magnitudes and effective radii. We adopt the prescriptions developed in Tortorelli et al. (2018, 2023), while analysing the 222 galaxies that we include in our model. The galaxy surface brightness profile fitting consists of iterative steps, where we consecutively in-

crease the size of the fitted images. We start by measuring the photometric properties with `SEXTRACTOR`¹ (Bertin & Arnouts 1996). These properties are used as initial guesses for the more accurate galaxy light profile analysis with `GALFIT`² (Peng et al. 2010). We cut stamps around each galaxy, fitting its light profile and those of the closest galaxies. Then, the best-fitting parameter values are used as starting points for the following iterations, where we simultaneously fit all galaxies falling in the nine regions we separate our image into. Finally, the parameters obtained in the previous step are used as a starting point for the final simultaneous light profile fitting of all galaxies in the whole image.

The surface brightness profiles of the galaxies are fitted with `GALFIT` using single Sérsic light profiles with seven free parameters (the two centre coordinates, major-to-minor axis ratio, position angle, major-axis effective radius, Sérsic index, and magnitude). For every galaxy, `GALFIT` is provided with different combinations of PSFs, background value estimates, and noise images, as described in Tortorelli et al. (2018). This allows us to average out the contribution of systematic effects in the surface brightness fit.

The SED obtained from the multi-band HST photometry also permits us to infer the stellar mass of the member galaxies, using composite stellar population models. Further details are provided in Mercurio et al. (2021), where the authors use the HST photometry plus the MUSE spectra to fit the SEDs of the members with a delayed exponential star formation history, solar metallicity, and a Salpeter (Salpeter 1955) stellar initial mass function (IMF). The presence of dust is taken into account following Calzetti et al. (2000).

However, MUSE data are only available for a fraction of our sample. For those galaxies we compare their stellar mass values obtained including information from spectra with those derived without. We notice only a small systematic offset between the two estimates, thus we choose to consider, for all the members, the stellar masses obtained by using only the photometric data, so as to have a uniform sample. From this comparison, we also fix a conservative error of 0.22 on the decimal logarithmic value of the stellar mass.

6.1.2 Spectroscopic data

Abell S1063 was targeted by the spectroscopic programmes presented in Chapter 4. Within the CLASH-VLT survey, a total of 16 masks were used on a field of approximately 25 arcmin across, allowing for the measurement of 3607 reliable redshifts, 1109 of which identify cluster members (Mercurio et al. 2021). Abell S1063 was also observed with two different MUSE pointings, centred in the NE and SW regions of the cluster (Karman et al. 2015; Caminha et al. 2016; Karman et al. 2017), with an exposure time of 3.1 and 4.8 hours and a seeing of 1.1'' and 0.9'', respectively. MUSE provided a total of 175 additional reliable redshifts, 104 of which refer to cluster members. Following Caminha et al. (2016), we identify as spectroscopic cluster members those galaxies whose redshift falls in the range 0.336 – 0.362, corresponding to a rest-frame velocity range of $\pm 3000 \text{ km s}^{-1}$. Moreover, multi-band CLASH data permit to identify photometric members with the method presented in Grillo et al. (2015).

The MUSE data are deep enough for the measurement of the line-of-sight stellar velocity dispersion of a considerable fraction of the cluster members (80 out of 222). As in B19, we use the `pPXF` software (penalized pixel-fitting) by Cappellari & Emsellem (2004) for the measurement, but instead of using a fixed circular aperture of 0.8'' for all members, we choose a wider one, of 2'', and weigh each pixel with its corresponding

¹Version 2.5.0, <https://www.astromatic.net/software/sextractor/>

²Version 3.0.5, <https://users.obs.carnegiescience.edu/peng/work/galfit/galfit.html>

best-fit surface brightness profile obtained with `GALFIT` for HFF F814W band images. This procedure is similar to that presented in Chapter 5, and has been found to provide velocity dispersion values equivalent to the central ones (σ_0). The aperture was reduced to $0.5''$ for a member whose velocity dispersion measurement was strongly influenced by the effects of the diffuse light of the BCG. In this case, we used `pPXF` a set of stellar templates from the MILES library (Vazdekis et al. 2010).

6.2 Building the strong lensing model

We build our SL model using that by B19 as a basis and taking advantage of the information provided by the FP relation for a more accurate determination of the total mass of the cluster members. We will then compare our model with that by B19 to understand how the new modelling choices on the mass of the sub-haloes influence the reconstruction of the diffuse mass component of the cluster.

As in B19, we start from the same 55 spectroscopically confirmed multiple images produced by 20 sources presented in Caminha et al. (2016). Likewise, we model the various lens mass components with dual pseudo-isothermal elliptical (dPIE) mass density profiles (Limousin et al. 2005; Elíasdóttir et al. 2007), which are the ellipsoidal generalisation of a truncated isothermal sphere with a central core. As in B19, we use the publicly available SL code `LensTool` (presented in Chapter 3) in order to find the best-fit values of the parameters of our model. The dPIE profile, introduced in Eqs. (3.41) and (3.43), has seven free parameters: two for the centre position, the ellipticity e and its position angle θ_e , the central velocity dispersion σ , and the core and truncation radii r_c and r_t , respectively. The two diffuse, cluster-scale, DM haloes are represented with an elliptical dPIE, close to the BCG position, and a spherical dPIE with vanishing core radius (hereafter SIsT), whose position is free to vary on an area of $150'' \times 120''$ centred in the NE region of the cluster, respectively. The second halo is necessary to reproduce some arcs in the NE region and its presence is compatible with the post-merger scenario of the cluster. As in B19, the hot gas mass component is completely fixed by the Chandra X-ray data, reduced in Bonamigo et al. (2018). This component is modelled with three dPIE mass profiles, whose parameters are not optimised. The position of each multiple image and source is identified by two coordinates. As the positions of the sources are not known, 55 multiple images and 20 sources correspond to, respectively, 110 observables and 40 free parameters. The two cluster-scale DM clumps have nine further parameters: the values of the coordinates of the centre and of the velocity dispersions are free for both haloes, while those of the core radius, ellipticity and orientation angle are only optimised for the main halo. There is therefore a total of 49 free parameters, leading to 61 degrees of freedom.

The main difference with the model obtained in B19 is in the way we assign mass to the cluster members. In both cases, they are modelled as SIsT, so that their total mass is entirely fixed by their values of velocity dispersion and truncation radius. In B19, a fixed Faber-Jackson law with no scatter, is imposed to link the velocity dispersion of the members with their total luminosity, in a wide mass range. In our model, instead, we use the FP relation to completely fix the value of the velocity dispersion of the members from their measured effective radius and total magnitude values. This procedure has several advantages. First, the FP has a tighter scatter on the logarithmic value of the line-of-sight velocity dispersion (Jorgensen et al. 1996). The best-fit parameters of the relation are determined with an optimisation performed on a selected sample of members, to avoid the effects of systematics affecting the observations. Finally, the value of the velocity dispersion of a member depends on two observed quantities, R_e and SB_e , instead of

just the total luminosity. This method is therefore less sensitive to possible measurement errors or biases. Late-type galaxies are included with early-type galaxies and modelled in the same way. We are aware that this is an approximation, but, as clearly visible in Fig. 6.1, late-type galaxies are a very small fraction of the members located in the cluster central regions.

As for the truncation radii of the members, we choose them as proportional to their observed half-light radii. This *ansatz* is suggested by the abundance matching results of Kravtsov (2013), who found that the virial and half-light radii of galaxies are approximately proportional, as previously suggested by some theoretical studies (e.g. Mo et al. 1998). The ratio between the two mostly depends on the angular momentum acquired from tidal torque by the halo during its formation. The relation was observed for galaxies within a very wide range of stellar masses and morphologies, including elliptical galaxies in the Virgo cluster. In our case, rather than the virial radii of members, we are interested in estimating their truncation radii, which are significantly smaller. Galaxies in the cluster core might have a generally self-similar total mass profile, leading to an approximately constant ratio between truncation and virial radii, as a consequence of similar evolution processes. This leads to an approximately proportional relation between effective and truncation radius, which is the *ansatz* we use in this Thesis. Compared to previously adopted power-law relations between the truncation radius and the total luminosity of a member, our assumption relates in a more direct way two spatial scales of the members. We calibrate the proportionality relation by testing several values for the truncation-to-effective radius ratio, optimising all the free parameters describing the other mass components of the cluster each time. Finally, we choose the ratio value that allows for the lowest rms value of the discrepancy between the observed and model-predicted positions of the multiple images considered in the optimisation of the model.

6.2.1 The Fundamental Plane of Abell S1063

In this Part, the FP is defined in the form presented in Eq. (1.34), which we remind here for clarity,

$$\log R_e = \alpha \log \sigma_0 + \beta \text{SB}_e + \gamma. \quad (6.1)$$

We consider R_e as measured in kpc, and σ_0 in km s^{-1} , whereas SB_e is defined as $\left(\frac{m}{1 \text{ mag}} + 2.5 \log \left(\frac{2\pi R_e^2}{1 \text{ kpc}^2} \right) \right) [\text{mag arcsec}^{-2}]$, where m is the total magnitude measured using GALFIT.

As anticipated, MUSE observations and their subsequent analysis provide us with line-of-sight central stellar velocity dispersions, σ_0 , for 80 out of the 222 member galaxies included in the model. The FP relation has different parameters for early- and late-type galaxies (Zhu et al. 2024). As detailed in Chapter 2, the population of cluster members is dominated by early-type galaxies, which are also the most massive, providing the most important contribution to the SL model. As such, we build the FP for the early-type members, and ensure that the values of its parameters are not influenced by late-type galaxies. There are different ways of selecting early-type galaxies, based on their morphologies, colours, and stellar populations. In Tortorelli et al. (2018), it has been shown that early-type galaxies selected via a Sérsic index value $n > 2.5$, elliptical morphology and a passive stellar population constitute a coeval population. Therefore, we select early-type galaxies as those galaxies with Sérsic index $n > 2.5$. Furthermore, we abide by the criteria introduced in B19, based on the robustness of the velocity dispersion measurements by pPXF on simulated spectra, thus including only those with a spectrum signal-to-noise ratio greater than 10 and $\sigma_0 > 80 \text{ km s}^{-1}$. We also exclude two further

Table 6.1: Best-fit values and 1σ errors of the parameters of the FP.

Parameter	Value
α	0.99 ± 0.17
β	0.323 ± 0.029
γ	-10.3 ± 1.0

galaxies: one of them is in the diffuse light of the BCG, while the second is very close to another member which influences the velocity dispersion measurement. This leaves us with a total of 30 spectroscopically confirmed cluster members, out of the 80 for which we had a measurement of the stellar velocity dispersion, that we use as a basis to estimate the parameters of the FP relation.

To obtain the FP parameter values, we perform an optimisation using the code `ltsfit`³, developed by Cappellari et al. (2013b) for robust fits of lines or planes. A linear regression algorithm is used on the data to derive a first value for the parameters. A σ dispersion around the plane is defined as the range in which 68% of the values fall. The points that lie at more than 3σ from the plane determined by the first regression are clipped and the code repeats the procedure iteratively with the remaining points, until no new member is discarded and the final parameter values are obtained. As for the errors on the observables, we use the propagated experimental uncertainty. Tortorelli et al. (2018) find, for the same cluster, an average uncertainty of 0.09 kpc on R_e and of $0.19 \text{ mag arcsec}^{-2}$ on SB_e for galaxies with $m_{F814W} \leq 22.5$, which are significantly higher than what we obtain through the error propagation for several members. We choose 0.09 kpc and $0.19 \text{ mag arcsec}^{-2}$ as lower limits for those uncertainties. We list the best-fit values of α , β , and γ obtained with `ltsfit` in Table 6.1.

The values of α and β , which do not depend on redshift, are compatible with those reported in the literature (e.g. Barr et al. 2006).

As clear from Fig. 6.2, there are just very few outliers around the FP: their photometric and spectroscopic analyses have been checked and confirmed. They do not however influence the values of the FP parameters, as they are clipped during the optimisation procedure. To evaluate the accuracy of the calibrated FP at predicting cluster member central velocity dispersions, we can compare the measured values of σ_0 with those obtained through the relation for the 30 members used to derive the parameters. We call these values $\sigma_{0,\text{FP}}$. The median value of the ratio $\frac{\sigma_{0,\text{FP}}}{\sigma_0}$ is 0.98, with a standard deviation of 0.20, corresponding to a $\chi^2_{\text{red}} = \sum_{i=1}^{30} \frac{(\sigma_{0,\text{FP},i} - \sigma_{0,i})^2}{30\sigma_{0,i}^2} = 0.05$, denoting a low scatter around the plane, as shown by Fig. 6.2. Furthermore, we do not notice any systematic offsets between the observed values of σ_0 and those predicted by the FP.

6.2.2 The best-fit model

From the best-fit FP parameters, we can compute the value of the central stellar velocity dispersion for all members. In our case, due to the vanishing core radius of the members, the velocity dispersion parameter σ of the SSt profile, defined in Eq. (3.42), is well approximated by the measured stellar central velocity dispersion σ_0 (see Appendix C of B19). This allows us to use the σ_0 measurements obtained for all members through

³Version 5.0.19,
<https://pypi.org/project/ltsfit/>

<https://www-astro.physics.ox.ac.uk/cappellari/software/#lts>,

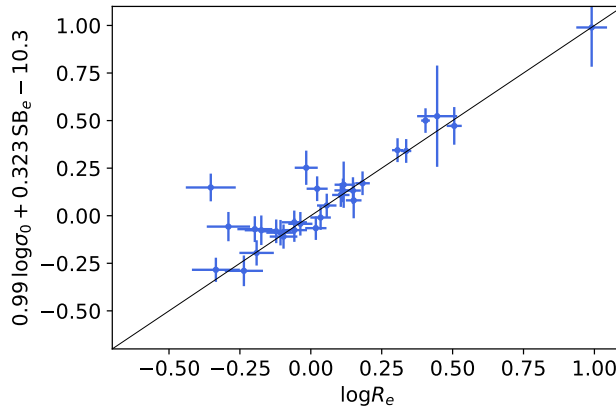


Figure 6.2: Distribution around the FP of the 30 members selected to determine its parameters, with the 1σ uncertainties obtained through the error propagation. For the physical quantities, we use the units listed in the main text. (From Granata et al. 2022).

the FP as a good approximation of their values of the parameter σ in the SL model. As anticipated, we take the value of the truncation radius of the members as a multiple of their measured effective radius, testing several proportionality constant values and re-optimising the values of all the parameters describing the diffuse DM components each time.

To determine the values of the parameters of the best-fit model, we use the χ^2 minimisation implemented in `LensTool`. The χ^2 function is defined as in Eq. (3.55), and the optimisation procedure is described in Chapter 3. As in Bonamigo et al. (2018) and B19, we assume an error of $0.5''$ on the HST multiple images and of $1''$ for the multiple images only found in the MUSE data.

We find that the lowest values of the χ^2 are obtained when the proportionality constant varies between 2.2 and 2.5, as clear from Fig. 6.3. In particular, we obtain the absolute lowest value of χ^2 for $r_t = 2.3R_e$. In this case, we find that the minimum χ^2 has a value of 84.92, with 61 degrees of freedom, which corresponds to a rms value of $0.60''$.

In Table 6.2, the median values and 1σ uncertainty of the free parameters describing the cluster-scale DM haloes for this model and for that by B19 are listed. Note that in this Chapter we often report the parametrisation adopted by `LensTool` for the velocity dispersion of dPIE haloes, which we indicate as σ_{LT} , connected to the dPIE parameter σ by $\sigma_{LT} = \sqrt{2/3}\sigma$. In the table, the index 1 refers to the main, central halo, the index 2 to the secondary, spherical one. The positions of the clumps are indicated with respect to the centre of the BCG. The orientation angle is measured counter-clockwise from the positive x axis. For both models, we determine the statistical uncertainties on the parameters using the Monte Carlo Markov chains as implemented by `LensTool`, based on the `BayeSys` algorithm (Jullo et al. 2007). To obtain a reliable estimate of the uncertainties on the parameter values from the Monte Carlo sampling, the observational uncertainty on the positions of the multiple images is increased to get a χ^2 value close to the number of degrees of freedom.

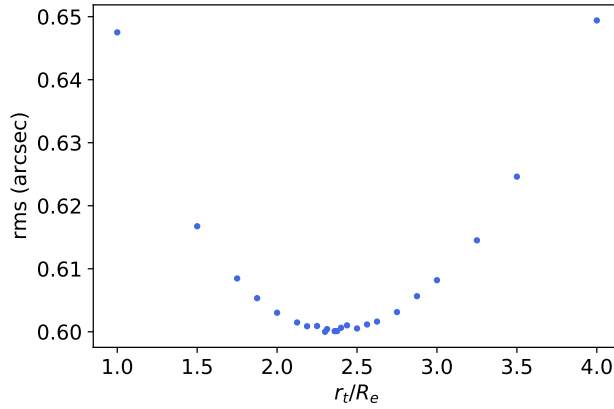


Figure 6.3: Values of the rms difference between the observed and model-predicted positions of the 55 multiple images used in our model as a function of the ratio between the truncation and effective radii of the cluster members. (From Granata et al. 2022).

Table 6.2: Median values and 1σ errors of the parameters of our SL model, compared with those obtained in B19. The positions of the centres of the haloes are referred to the centre of the BCG.

Parameter	Physical meaning	This Thesis	B19
x_1 (")	x position of halo 1	$1.21^{+0.24}_{-0.24}$	$1.40^{+0.23}_{-0.23}$
y_1 (")	y position of halo 1	$-0.69^{+0.19}_{-0.19}$	$-0.74^{+0.16}_{-0.17}$
e	Ellipticity of halo 1	$-0.63^{+0.01}_{-0.01}$	$-0.63^{+0.01}_{-0.01}$
θ_e (°)	Orientation angle of halo 1	$-38.79^{+0.25}_{-0.26}$	$-38.75^{+0.22}_{-0.23}$
r_c (")	Core radius of halo 1	$17.37^{+0.35}_{-0.37}$	$18.06^{+0.53}_{-0.52}$
$\sigma_{\text{LT},1}$ (km s $^{-1}$)	Velocity dispersion of halo 1	$1159.1^{+6.8}_{-7.4}$	$1162.4^{+6.4}_{-6.7}$
x_2 (")	x position of halo 2	$-50.4^{+4.5}_{-5.6}$	$-50.2^{+3.7}_{-4.4}$
y_2 (")	y position of halo 2	$27.5^{+3.2}_{-2.7}$	$26.8^{+2.4}_{-2.2}$
$\sigma_{\text{LT},2}$ (km s $^{-1}$)	Velocity dispersion of halo 2	214^{+29}_{-26}	221^{+24}_{-22}

Table 6.3: rms and maximum likelihood $\mathcal{L}$ values for our model and for the one presented in B19.

Parameter	This Thesis	B19
Observables	110	110
Free parameters	49	51
$\ln \mathcal{L}$	-65.94	-58.43
rms	0.60''	0.55''

6.3 Comparison and discussion

In this Section, we compare our model with that presented in B19, which is our starting point and reference. Using the same code, `LenSTool`, B19 obtain a minimum value of χ^2 of 69.90, which corresponds to a value of the rms on the positions of the 55 multiple images of 0.55'', slightly lower than our value of 0.60''. The maximum values of the likelihood for the two models, provided by `LenSTool`, are reported in Table 6.3. This small increase in the rms value with respect to B19 was expected, as in our model the mass component representing the member galaxies has less freedom. In B19, the power-law scaling relations defined in Eqs. (3.57) and (3.58) are

$$\begin{aligned}\sigma_{\text{LT},i} &= \sigma_{\text{LT}}^{\text{ref}} \left(\frac{L_i}{L_0} \right)^{0.27}, \\ r_{t,i} &= r_t^{\text{ref}} \left(\frac{L_i}{L_0} \right)^{0.66},\end{aligned}\tag{6.2}$$

where L_0 is the luminosity of the BCG, and the two exponents have been derived using kinematic observational priors and the hypothesis of a total-mass-to-light ratio for the members scaling as $M/L \propto L^{0.2}$. The reference values in the relations ($\sigma_{\text{LT}}^{\text{ref}}$ and r_t^{ref}) are free to vary, increasing the number of free parameters from 49 to 51. This additional freedom corresponds to some extent to a possible mass exchange between the member galaxies and the diffuse cluster-scale DM haloes. In our model, the total mass component of the cluster galaxies is completely fixed by the observations through the FP, and only the two main DM haloes have their mass free to vary.

We first focus on how mass is assigned to the member galaxies. For both models, cluster members are modelled as dPIEs with vanishing core radius and zero ellipticity, so their total mass can be obtained from Eq. (3.46). In B19, r_t and σ_0 are derived, for each member, with the two power law scaling relations reported in Eq. (6.2), which imply that $M \propto \sigma^{4.44}$. Such a precise and simple relation between M and σ , with no scatter, might be an oversimplification, given the large range in total mass value and the great variety in galactic morphology. The use of two measured observables and a more accurate relation (the FP) should allow for a more complex and realistic dependence of M from the other observables. This is shown in Fig. 6.4: the $M(\sigma)$ relation has a visible scatter, which the model by B19 could not include. The relation also shows a shallower slope (a bi-logarithmic fit on the members with $\sigma_{\text{FP}} > 80 \text{ km s}^{-1}$ gives a slope value of around 2.4). The two models are similar in the high-mass regime and agree on the prediction of the total mass of the BCG. Despite being fixed by the FP with the same procedure as for all other cluster members, our model predicts a BCG total mass value of $3.16 \times 10^{12} M_\odot$, very close to that found in B19 ($3.35 \times 10^{12} M_\odot$), where it is *de facto* a free parameter. Furthermore, the new procedure also leads to a different degeneracy between

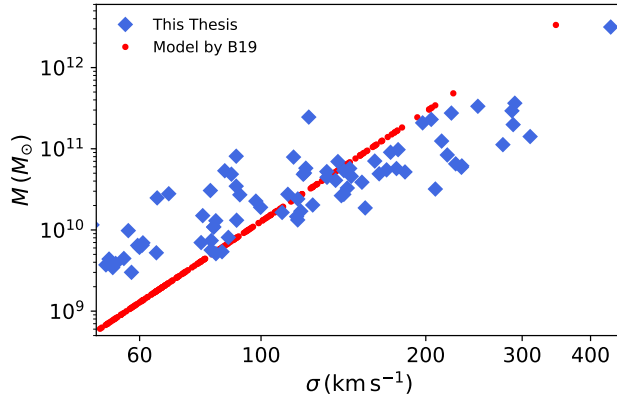


Figure 6.4: Total mass of the cluster members as a function of their velocity dispersion. In blue: values predicted by our model. In red: values predicted by the model presented in B19. (From Granata et al. 2022).

the parameters that describe the total mass of the sub-haloes. While the introduction of a kinematic prior in B19 already allows for a significant reduction of the degeneracy between the values of r_t and σ for the various cluster members, the two reference values that appear in Eq. (6.2) are still clearly degenerate (as shown in Fig. 4 from B19). In this Thesis, instead, the values of σ are fixed *a priori* from observations, while the values of r_t are obtained at a later stage, testing several values for the proportionality constant with respect to the observed values of the effective radius.

We measure the stellar mass values of all 222 member galaxies (Mercurio et al. 2021) from HST multi-band photometric data, using a Salpeter IMF. By combining this with the results of Eq. (3.46), we can estimate the stellar mass fraction of the cluster members, computed as the ratio between their stellar mass and total mass values. As lensing models are more sensitive to the effects of high-mass members, we will focus only on the 63 galaxies with $\sigma_{FP} > 80 \text{ km s}^{-1}$. For this subset of members, the median value of the stellar fraction is 0.47, with a standard deviation of 0.34. From the model presented in B19, using the same stellar mass values, we obtain a value of 0.25. Therefore our model predicts, on average, slightly higher values for the stellar fraction, as illustrated in Fig. 6.5.

We now compare our stellar fraction values with those of other massive early-type galaxies. Grillo (2010) studied the projected DM fraction within the effective radius for a sample of nearly 1.7×10^5 early-type galaxies from the Sloan Digital Sky Survey (SDSS) Data Release Seven (DR7), selected to reproduce well the physical properties of the lenses in the Sloan Lens ACS Survey (SLACS, Koopmans et al. 2009). He considered the values of the galaxy stellar mass as reported in the MPA/JHU catalogue, and obtained the values of the galaxy projected total mass within the effective radius, by assuming a one-component isothermal model, parametrised by the value of the measured central stellar velocity dispersion. He found that the best-fit linear relation between the logarithm of the projected total and stellar mass values within the effective radius, $M(< R_e)$ and $M_*(< R_e)$, respectively, is

$$\log(M(< R_e)) = -0.58 + 1.09 \log(M_*(< R_e)). \quad (6.3)$$

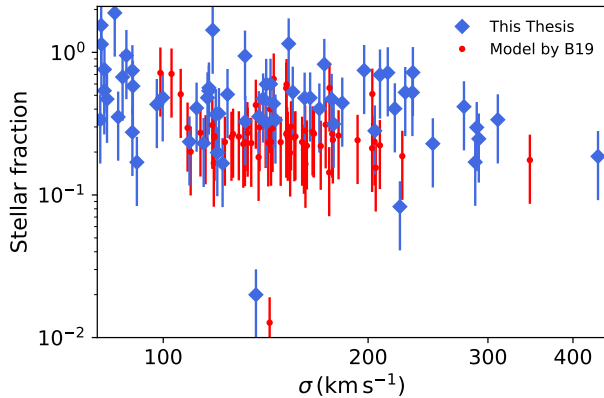


Figure 6.5: Stellar fraction of the cluster members as a function of their central velocity dispersion for the 63 galaxies with $\sigma_{\text{FP}} > 80 \text{ km s}^{-1}$: comparison between the values obtained with our model (in blue) and those from the model in B19 (in red). (From Granata et al. 2022).

As far as the cluster members of AS1063 are concerned, assuming that light traces stellar mass, $M_*(< R_e)$ is half of their total stellar mass, as R_e is defined as the two-dimensional radius within which half of the total light is emitted. The stellar mass values of our cluster members have been obtained assuming a Salpeter stellar IMF, while those presented by Grillo (2010) are derived adopting a Chabrier stellar IMF (Chabrier 2003). To compare the two samples, we use a constant conversion factor (0.585, from Speagle et al. 2014) between the stellar mass values obtained with a Salpeter and a Chabrier stellar IMF. As for the projected total mass, the mass of a dPIE profile with a vanishing core radius enclosed within a two-dimensional radius R is defined in Eq. (3.45). For $r_t = 2.3R_e$, Eqs. (3.45) and (3.46) give $M(R_e) \approx 0.344M$, where M is the total mass. As Fig. 6.6 shows, we find that our members follow well the relation found by Grillo (2010). This is significant, as we are considering in both cases massive elliptical galaxies, but the values of their stellar fractions are obtained with different methods. The observed agreement suggests that the inner (i.e. within R_e) mass structure of the massive cluster members of AS1063 and of massive early-type galaxies living in different environments might be very similar. Note that the relation shown in Fig. 6.6 has been adapted from Eq. (3.45) for the case of stellar mass values determined using a Salpeter IMF.

Looking instead at the diffuse cluster-scale components of the two models, from the Monte Carlo sampling of the marginalised posterior probability we see a drop of more than 30% of the uncertainty on the value of the core radius of the main DM halo (see Fig. 6.7). The scatter between the core radius and the velocity dispersion is also considerably reduced. This indicates that a more accurate estimate of the total mass of the member galaxies has resulted in distributions of the parameters with a lower statistical uncertainty.

Monte Carlo Markov chains also show that the core of the main diffuse DM halo has a radius of $86 \pm 2 \text{ kpc}$. Our result is in line with those from SL models of similar galaxy clusters (e.g. B19, Grillo et al. 2015; Annunziatella et al. 2017; Lagattuta et al. 2017; Caminha et al. 2019; Diego et al. 2020), which found that cluster haloes are very well described by cored isothermal mass density profiles. Fig. 6.7 also shows a clear degeneracy, with a positive correlation, between the velocity dispersion and the core

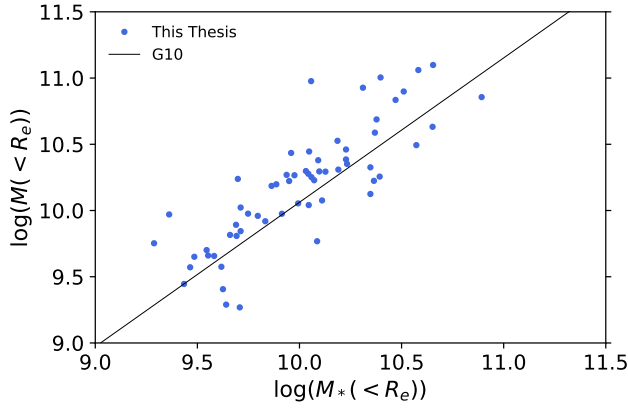


Figure 6.6: Logarithmic values of the total mass as a function of the logarithmic values of the stellar mass, both projected within the effective radius, of all members with $\sigma_{0,\text{FP}} > 80 \text{ km s}^{-1}$. The plot also shows the relation found by Grillo (2010). (From Granata et al. 2022).

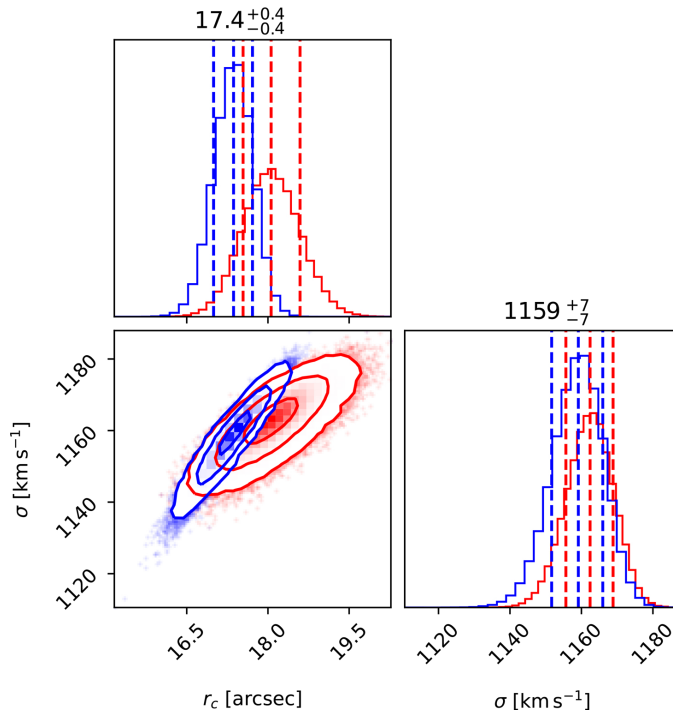


Figure 6.7: Posterior probability distributions of the core radius r_c and fiducial velocity dispersion σ_{LT} values of the main DM halo and degeneracy between them for our model (in blue) and that by B19 (in red). The 16th, 50th and 84th percentiles of the marginalised probability distributions are marked with dashed vertical lines for both models. The values of these percentiles for our model are reported above the histograms. (From Granata et al. 2022).

radius of the main DM halo. This is expected, as different combinations of these two parameters can determine the same value of the projected mass enclosed within a certain radius from the cluster centre. In particular, a higher value of central density scale, here included in the definition of σ_{LT} , can be compensated by a larger core in the density profile, and vice versa, to obtain the same mass within the multiple images of the same family.

Finally, from Table 6.2, we notice that the values of our and B19 best-fit parameters of the diffuse DM component are very similar. The only significant discrepancy is the value of the core radius of the main DM halo of the cluster. This can be explained by considering that, far from the cluster centre, the total mass distribution of the cluster is dominated by the cluster-scale DM halo component. The positions of the multiple images observed in these regions provide strong constraints on the values of all the parameters of this halo. The core radius of the cluster DM halo is more sensitive to the total mass profile of the cluster close to its centre, which is more directly related to the mass distribution of the cluster members located in this region. A change in the mass contribution of the cluster members is therefore reflected in a small change of the DM halo core radius.

6.4 Mass profile decomposition

In this Section, we will obtain and present the cumulative projected mass profiles of all the cluster mass components: the stellar mass and the DM of cluster members, the ICM, and the diffuse, cluster-scale DM. We will only consider the mass distribution out to a projected radius R of 350 kpc from the centre of the BCG, to avoid including regions not covered by our photometric data, where the cluster member catalogue is incomplete.

6.4.1 The baryonic mass components

To measure the stellar mass profile of the cluster, we start from the surface brightness best-fit model of each of the cluster members in the HST F814W band, shown in Fig. 6.8, which we already used to derive their structural parameters. Fig. 6.8 shows that our procedure does not allow to completely account for the intra-cluster light (ICL) distribution, which also contributes to the cluster stellar mass budget. However, the ICL in AS1063, especially far from its BCG, is less important than in other clusters of comparable mass. Similarly to Annunziatella et al. (2017), we estimate an average value for the stellar mass-to-light ratio (M_*/L) of all cluster members within a projected radius of 350 kpc from their total luminosity and total stellar mass. The median value of the stellar mass-to-light ratio for these members is $1.0 M_\odot/L_\odot$, estimated in the HST F814W band. The choice of considering a constant ratio is an approximation, as the value may change in the different regions of each galaxy, as well as among galaxies. However, several galaxies are included in each radial bin above 50 kpc, and this should reduce the impact of this approximation. We then use this ratio to convert the cumulative luminosity profile into a cumulative stellar mass profile. As above, we consider an error of 0.22 on $\log(M_*)$ of each galaxy, and we apply the same uncertainty on the total value in each radial bin of the projected profile.

We take the cumulative projected ICM mass profile from Bonamigo et al. (2018): following their result, we impose a relative error of 1% on each bin. The total projected mass of the three haloes used to model the hot gas, within a bi-dimensional radius of 350 kpc, is reported in Table 6.4.

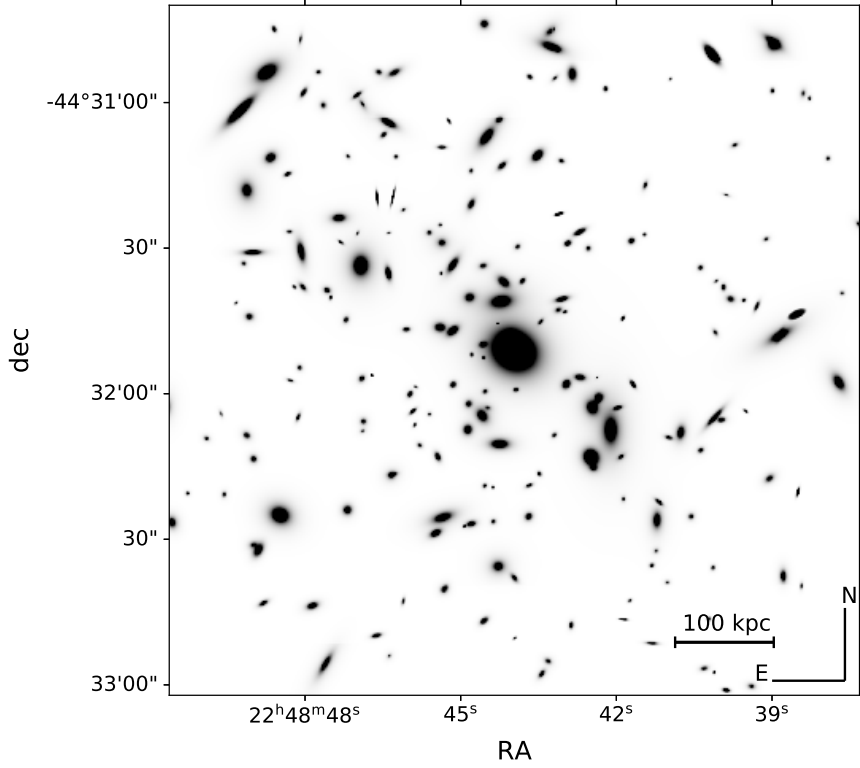


Figure 6.8: Reconstructed surface brightness distribution of all cluster members used in the SL model in HST F814W band. The field of view is 700 kpc wide and centred on the BCG to cover the entire region where the cumulative mass profile is obtained. (From Granata et al. 2022).

Table 6.4: Projected mass within a two dimensional radius $R = 350$ kpc of the baryonic components.

Component	$M(10^{12} M_{\odot})$
Stars	2.4 ± 0.6
ICM	63.3 ± 0.6

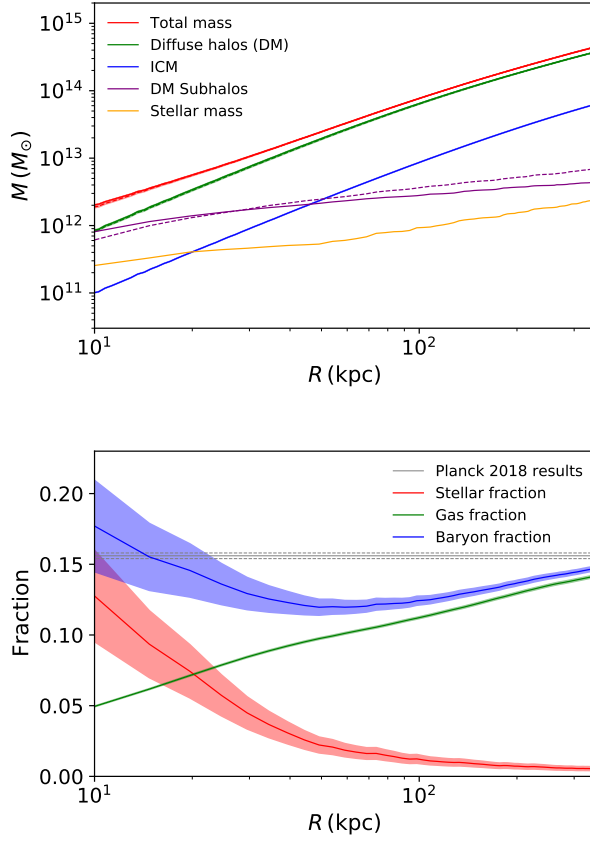


Figure 6.9: Upper panel: cumulative two-dimensional mass profiles for all baryonic and dark components of AS1063. The total mass profile and those of the diffuse DM haloes and of DM sub-haloes obtained from the model in B19, which differ from ours, are represented with dashed lines. Lower panel: cumulative stellar-, gas- and baryonic-to-total mass fractions from our model. The value of the cosmological baryon fraction from the Planck Collaboration et al. (2020) is also shown. (From Granata et al. 2022).

Table 6.5: Comparison between the projected mass within a two-dimensional radius $R = 350$ kpc for the DM components in our model and in that by B19.

Component	Our model $M(10^{12} M_{\odot})$	Model from B19 $M(10^{12} M_{\odot})$
Diffuse DM	377 ± 2	377 ± 2
DM sub-haloes	4.4 ± 0.6	6.9 ± 0.9
Total	447 ± 2	449 ± 2

6.4.2 The dark matter mass distribution

We previously described our modelling of the DM mass distribution: an ellipsoidal, pseudo-isothermal, non-truncated clump, and a less massive, spherical one represent the cluster-scale DM haloes. On the other hand, 222 spherical, truncated, isothermal haloes are introduced to describe the total mass of cluster members. To disentangle the mass distribution of DM sub-haloes, we therefore need to subtract the stellar mass profile to it.

The parameter space sampling obtained with the Monte Carlo method can be used to estimate the error on the DM cumulative mass profiles. We extract 100 random sets of parameter values from the chains and derive the cumulative mass profiles of each corresponding model. In Fig. 6.9 and in Table 6.5, the error bars show the 16th and 84th percentiles of the mass profiles associated to all the models extracted from the chains. We perform this analysis for our model and repeat it for the model by B19. In the first case, the total mass of the cluster members is fixed, so the uncertainty on the DM sub-halo mass profile is only associated to the error on the stellar mass. In the second case, instead, the mass of the cluster members is also optimised: Monte Carlo Markov chains provide us with an uncertainty on their mass, which is combined with that on M_* to obtain the error on the DM sub-halo cumulative mass profile.

We notice that the total mass of the cluster and the mass of the diffuse DM component are very well constrained by SL: the two models find very similar values and their statistical uncertainties are around 1%. As a matter of fact, the two profiles are almost completely superimposed in Fig. 6.9, and the error bars are very small. The mass of the DM sub-haloes, instead, slightly differs between the two models, especially in the outer regions, where our model predicts a lower value. A reason for this discrepancy might be in the different scaling laws that determine r_t in the two models. Comparing our results with the total mass profiles obtained, for the same cluster, with a full dynamical reconstruction and with the X-ray hydrostatic method (Sartoris et al. 2020), and with combined weak and SL (Umetsu et al. 2016), we find very good agreement in the radial range considered. These two profiles are shown in Fig. 7 by Sartoris et al. (2020).

The profiles measured from our model also allow us to obtain the two-dimensional, cumulative, stellar-, gas-, and baryonic-to-total mass fractions as a function of R , out to 350 kpc. In Fig. 6.9, we report the results and compare the baryonic fraction profile with the cosmological baryon fraction value reported in the Planck 2018 results: 0.156 ± 0.002 , obtained from the ratio between $\Omega_b h^2$ and $\Omega_m h^2$, with information from the CMB (Cosmic Microwave Background) power spectrum and lensing reconstruction (see Planck Collaboration et al. 2020). The error bars of the profiles result from the propagation of the uncertainty on the cumulative values of the mass components. The fraction obtained at $R = 350$ kpc is 0.147 ± 0.002 . This value is similar to the cosmological value and consistent with what Bonamigo et al. (2018) reported for the same cluster at a comparable radius.

Comparison with cosmological simulations

7.1 An excess of galaxy-scale strong lenses in massive clusters

Within massive SL galaxy clusters, the member galaxies can act as smaller, secondary gravitational lenses, perturbing the potential well of the galaxy cluster. Depending on the position of these objects within the cluster and on their mass and compactness, they can create secondary critical lines, which allow for the formation of further multiple images of background sources, very close to the cluster members that created the secondary critical line. In other words, the cluster member acts as a primary galaxy-galaxy (GG) lens.

This image configuration is a powerful probe of the physical properties of cluster member galaxies. As anticipated in Chapter 6, cluster-scale lenses produce multiple images at a typical distance of a few tens of arcseconds from one another: as such, their position is not very sensitive to the details of the galaxy-scale mass distribution, resulting in a degeneracy between the velocity dispersion and the truncation radius of the member galaxies that cannot be broken by SL alone. When a single member galaxy (or a few) determines several multiple lensed images of the same background source, their observed positions strongly depend on the lens galaxy mass structure, including its compactness. GGSL events provide us with the chance of breaking the degeneracy on the physical parameters of cluster galaxies using exclusively lensing information, as we will show in Chapter 8. Furthermore, the gravitational potential of the cluster increases the size of the critical lines of the cluster members, allowing us to observe GGSL events determined by low- and intermediate-mass galaxies. If these galaxies were in the field, their Einstein radius would be too small to be resolved by current observations. As such, GGSL events in clusters give us a tool to study a new mass regime for SL.

The probability for a galaxy cluster to produce GGSL events for sources at redshift $z = z_S$ can be estimated when a model of the mass distribution of the galaxy cluster is known, and obtained as (Meneghetti et al. 2020)

$$P_{\text{GGSL}}(z_S) = \frac{\sigma_{\text{GGSL}}(z_S)}{A(z_S)}, \quad (7.1)$$

where $A(z_S)$ is the area of the cluster considered when building the SL model mapped on the source plane at $z = z_S$ through the lens equation, and $\sigma_{\text{GGSL}}(z_S)$ is the sum of the areas of the secondary caustics on the source plane produced by cluster members, and is thus equivalent to a GGSL cross section. Starting from the hypothesis of a known number density of sources as a function of redshift and surface brightness $n(S, z_S)$, the total predicted number of GGSL events can be obtained as (Meneghetti et al. 2020)

$$N_{\text{GGSL}} = \int_{S_l}^{\infty} \int_{z_L}^{\infty} n(S, z_S) \sigma_{\text{GGSL}}(z_S) dz_S dS, \quad (7.2)$$

where S_l is the observable surface brightness limit, and z_L is the lens redshift.

Meneghetti et al. (2020) compared the expected value of N_{GGSL} predicted for a sample of state-of-the-art SL models of 17 massive clusters from the CLASH and HFF sample and for 25 simulated clusters with a virial mass $< 5 \times 10^{14} M_\odot$, in the redshift range $[0.25 - 0.55]$ extracted from the Dianoga simulation suite (Planelles et al. 2014; Rasia et al. 2015), presented in Chapter 2. Three spatial projection were considered for each simulated cluster. The prediction of N_{GGSL} was performed adopting the $n(S, z_S)$ function observed by the Hubble-Ultra-Deep-Field (HUDF) survey in the band F850LP (Beckwith et al. 2006), with photometric redshift from Rafelski et al. (2015). As clear from Fig. 7.1, Meneghetti et al. (2020) find an order-of-magnitude discrepancy between the values of N_{GGSL} predicted by observations and simulations, where N_{GGSL} is under-estimated by the simulations. Meneghetti et al. (2020) tested the impact on the results of the assumptions of SL models on the mass density profile of the haloes and of contributions along the line-of-sight, finding that these cannot account for the observed mismatch.

The discrepancy can be interpreted as arising from a difference in the compactness of the cluster sub-haloes. A possible proxy for this is their maximum circular velocity, defined as $v_{\text{max}} = \max(v_c(r))$, where v_c has been defined in Eq. (1.20) (for an isothermal model, $v_{\text{max}} = \sqrt{2}\sigma$). At a fixed total mass, a higher (lower) value of v_{max} implies a more (less) compact mass distribution. Meneghetti et al. (2020) compared the values of v_{max} found by the Dianoga simulations with those recovered by B19 for the SL models of the three clusters for which kinematic priors on Eq. (3.57) are available. As shown by Fig. 7.1, the Dianoga simulations predict sub-haloes with lower values of v_{max} at a fixed total mass compared to the SL models presented in B19, indicating that their mass distribution is significantly less compact than observed. Some other recent results, using the higher-resolution simulations from the Hydrangea/C-EAGLE suite (see Chapter 2 and Bahé et al. 2017; Barnes et al. 2017), do not claim the same large discrepancy in terms of galaxy scale lensing events and of sub-halo compactness (Robertson 2021; Bahé 2021). Studying simulated Hydrangea/C-EAGLE clusters, Bahé (2021) reported a bimodal $v_{\text{max}}(M)$ function, with a lower sequence of sub-haloes with a baryon fraction smaller than 0.1 dominating for $M < 10^{11} M_\odot$, and a second sequence for higher sub-halo mass values, clearly visible in the upper panel of Fig. 7.2. This last branch yields results consistent with those of the lensing models presented in Meneghetti et al. (2020). On the contrary, repeating their analysis with the highest resolution IllustrisTNG300 simulation (see Chapter 2 and Marinacci et al. 2018; Naiman et al. 2018; Nelson et al. 2018; Pillepich et al. 2018a; Springel et al. 2018), finding these to predict, on the contrary, lower $v_{\text{max}}(M)$ values at a fixed total mass than the Hydrangea simulation, and a compactness much closer to that reported by the Dianoga simulations studied in Meneghetti et al. (2020) (see the lower panel of Fig. 7.2).

As previously detailed, in the SL models by B19 σ and r_t are connected to the total luminosity of the members with fixed-exponent power-law scaling relation, presented in Eqs. (3.57) and (3.58), which lead to a power-law relation between their total mass and their velocity dispersion, and consequently between their total mass and their maximum circular velocity. The model presented in this Thesis allows for a more complex and realistic parametrisation of the sub-halo compactness: v_{max} is determined from σ_0 through the FP for all members (see Eq. (6.1)), and SL observables are used to determine the ratio r_t/R_e (see Fig. 6.3). In the remainder of this Chapter, we test whether a more realistic parametrisation of the physical properties of the members can mitigate the observed discrepancy between SL models and hydrodynamical simulations.

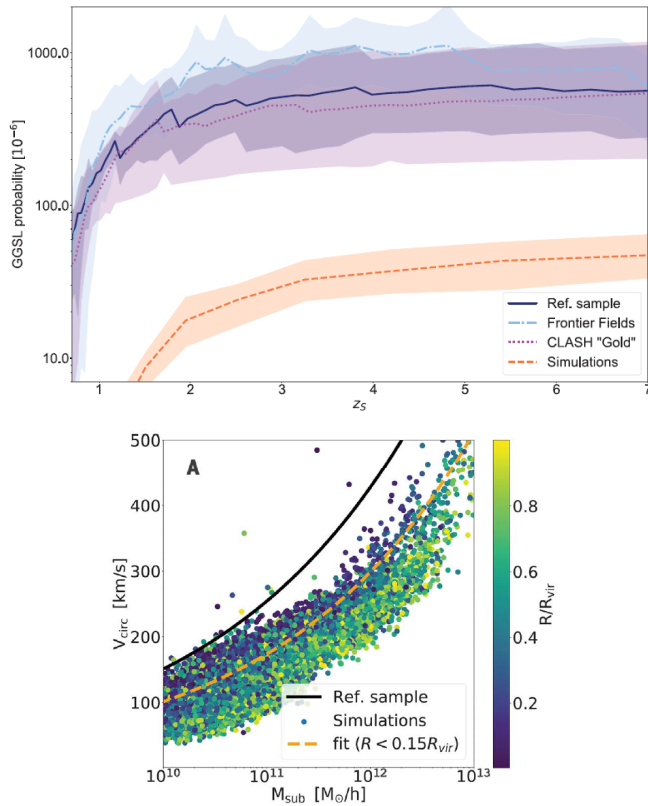


Figure 7.1: Comparison between SL models and hydrodynamical simulations. Upper panel: GGSL probability as a function of the source redshift for the Dianoga simulations (in red), for the models of B19 (in blue), for the CLASH models presented in Caminha et al. (2019) (in purple), and for the SL models of all remaining HFF clusters (in cyan). Lower panel: maximum circular velocity of cluster sub-haloes as a function of their total mass for the SL models presented in B19 (black line) and for the Dianoga simulations (dots colour coded according to the projected distance of the sub-halo from the cluster centre normalised to the virial radius) (From Meneghetti et al. 2020).

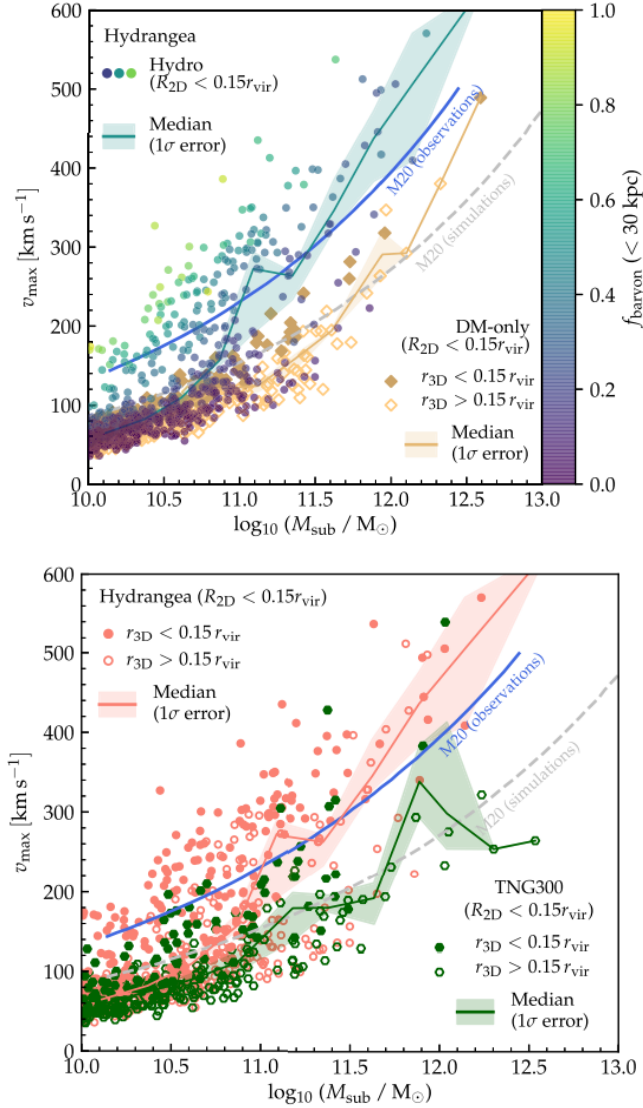


Figure 7.2: Sub-halo compactness predicted by the Hydrangea simulations. Upper panel: maximum circular velocity of cluster sub-haloes in the Hydrangea hydrodynamical and DM-only simulations as a function of their total mass. In the case of the hydrodynamical simulation, the points are colour-coded for their baryon fraction. Lower panel: maximum circular velocity of cluster sub-haloes in the Hydrangea (pink) and IllustrisTNG300 (green) hydrodynamical simulations as a function of their total mass. (From Bahé 2021).

7.2 Comparison with halo occupation distribution studies

In this Section, we compare the values of the total mass predicted for cluster members by our model with those obtained from the measured stellar mass values using halo occupation distribution (HOD) studies, based on cosmological N -body (DM only) simulations. In Chapter 6, we have found that both our model and that by B19 lead to median values of the stellar-to-total mass ratio of massive member galaxies of 0.25 at least. As we will see, this result differs significantly from the predictions of statistical (sub)-halo abundance matching studies. We compare our results with the recent stellar-to-halo mass relation (SHMR) presented in Girelli et al. (2020). They consider the halo mass function derived from the cosmological N -body collisionless Λ CDM DUSTGRAIN-*pathfinder* simulation (Giocoli et al. 2018), which is complete for halo masses $M_h > 10^{12.5} M_\odot$, and extend it to lower masses. The simulation is built from a sample that also includes some cluster members, especially in the high-mass range. Then, taking the observed galaxy stellar mass of the COSMOS field (Scoville et al. 2007), they perform an abundance matching via a deterministic relation between halo and stellar mass. The SHMR is found by Girelli et al. (2020) to be in good agreement both with the observed clustering of massive galaxies and with the SHMR of clusters selected from spectroscopic data from the SDSS. The inclusion of some scatter on the value of the stellar mass leads to consistent results for the relation, so we will not consider it in the following analysis.

The obtained SHMR is valid from stellar mass values between approximately $10^8 M_\odot$ and $10^{12} M_\odot$, from $z = 0$ to $z = 4$. The relation, using the parametrisation proposed by Moster et al. (2010), can be written as

$$\frac{M_*(z)}{M_h} = 2A(z) \left[\left(\frac{M_h}{M_A(z)} \right)^{-\beta(z)} + \left(\frac{M_h}{M_A(z)} \right)^{\gamma(z)} \right]^{-1}, \quad (7.3)$$

where $A(z)$ is a normalisation factor, $M_A(z)$ a characteristic halo mass, and $\beta(z)$ and $\gamma(z)$ are the slopes of the relation in the low- and high-mass ranges, respectively. We adopt the values of the parameters found by Girelli et al. (2020) at the cluster redshift. We can then proceed by using Eq. (7.3) to compute the total halo mass of the members and, in turn, the values of their truncation radii from Eq. (3.46). We will finally compare these values of the truncation radii with those we obtained in our model, from the effective radii of the members. We do not consider any uncertainty on the parameter and stellar mass values, as we will show that the differences are large and cannot be reconciled by the statistical errors. We notice that the the SHMR presented by Girelli et al. (2020) was derived adopting a Chabrier stellar IMF. To make use of the relation to estimate the halo mass of the cluster members from their stellar mass, we adopt the same conversion factor of 0.585 between the Salpeter and Chabrier IMFs that we previously used.

Using the SHMR, the projected mass of the DM sub-haloes within a bidimensional radius of 350 kpc grows to $6.1^{+0.1}_{-0.1} \times 10^{13} M_\odot$, larger by a factor of almost 14 compared to our results presented in Table 6.5. The ratio $r = \frac{M_{\text{SHMR}}}{M_{\text{model}}}$, where M_{SHMR} is the total mass obtained for a member from SHMR, and M_{model} is the corresponding total mass as in the SL model, has a median value of 8.6 for the members with $\sigma_{\text{FP}} > 80 \text{ km s}^{-1}$. For these selected high-mass members, as shown in Fig. 7.3, the median value of the stellar fraction decreases from 0.47 to 0.055. The value of r is also that of the ratio between the value of the truncation radius of a sub-halo obtained from simulations and that adopted in our model.

We perform a new run of the model, fixing the total mass of each cluster member to the value obtained from the SHMR. Optimising the free parameters of the two diffuse

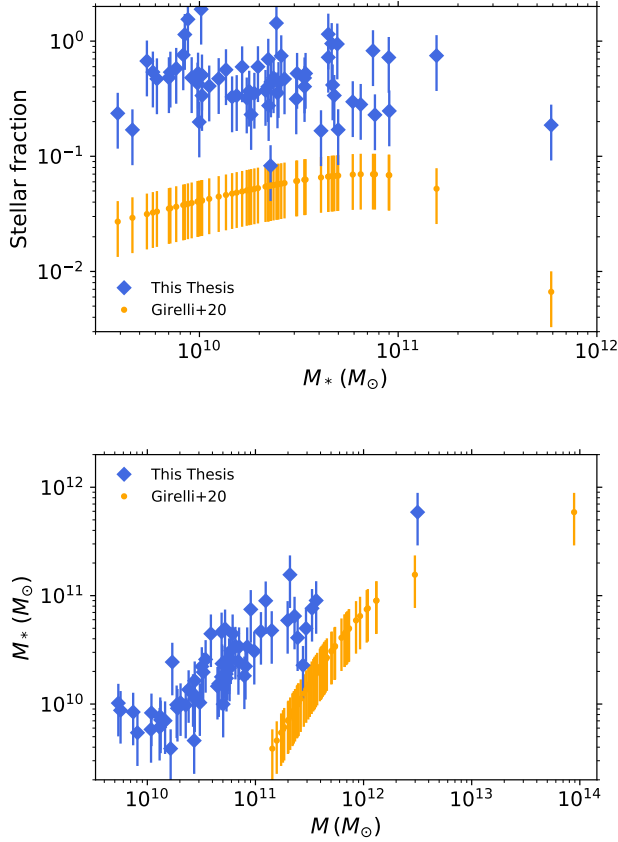


Figure 7.3: Comparison between the values of the stellar over total mass fraction of the cluster members obtained with our lensing model (in blue) and with the stellar-to-halo mass relation from Girelli et al. (2020) (in orange). Only the 63 cluster members with $\sigma_{\text{FP}} > 80 \text{ km s}^{-1}$ are shown. Upper panel: stellar over total mass fraction of members as a function of their stellar mass. Lower panel: stellar-to-halo mass relation. (Adapted from Granata et al. 2022).

DM haloes, the value of the rms between the observed and predicted positions of the 55 multiple images grows by more than 25% (to $0.76''$), despite having the same number of degrees of freedom.

These results seem to indicate a significant discrepancy between SL models and the predictions of a SHMR derived from N -body cosmological simulations. The total halo mass values predicted by the SHMR are significantly higher than what is predicted by both our SL model and that by B19, and their inclusion in a SL model leads to a significant decrease in its accuracy. An important reason for this discrepancy may be the fact that the Λ CDM DUSTGRAIN-*pathfinder* simulation only includes collisionless particles. Thus, it does not consider the effects of the interplay between DM and baryons, which are very important during galaxy formation. Several complex phenomena taking place on small spatial scales, such as star formation, radiative gas cooling, and active galactic nuclei (AGN) feedback can have a non-negligible effect on the physical properties of galactic haloes (see Chapter 2 and Kravtsov & Borgani 2012; Planelles et al. 2014; Barnes

et al. 2017). Moreover, the SHMR presented by Girelli et al. (2020) describes both cluster and field galaxies with the same law. During the formation of a cluster, galaxies fall into the cluster potential well. At first, they undergo a phase of star formation triggered by shocks in their gas component. At a later stage, both the baryonic and DM components of the members are stripped by ram pressure and gravitational tidal interactions (see, for instance, McCarthy et al. 2008; van den Bosch et al. 2008). This phase could determine a SHMR different from that of field galaxies: Smith et al. (2016), for instance, found that the DM stripping happens earlier than star stripping, leading to an increase of the ratio between the effective and virial radii of the sub-haloes. This could justify a higher value of the stellar mass fraction of cluster members when compared to field galaxies. While the most massive galaxies considered by Girelli et al. (2020) to calibrate the SHMR are mostly hosted in clusters and groups, the low-mass galaxies considered are mainly field galaxies, which have evolved in an environment different from that of the cluster members we focus on here. To address both the role of baryons and DM interactions and the influence of the dense environment of clusters in the evolution of galaxies, Sirks et al. (2022) used the Hydrangea/C-EAGLE suite (Bahé et al. 2017; Barnes et al. 2017) cosmological hydrodynamical simulations to build two distinct SHMRs, introducing a distinction between cluster and field galaxies. As clear from Fig. 5 in Sirks et al. (2022), the SHMR of cluster members yields stellar-to-total mass fraction values that are almost an order of magnitude higher than those of field galaxies, at a fixed stellar mass. As we will see in the next Section, such an effect will alleviate the reported discrepancies between the stellar fraction values predicted by SHMRs and reconstructed from our SL model.

7.3 Comparison with cosmological hydrodynamical simulations

In this Section, we contrast the features of the sub-haloes in our new SL model with the predictions of the most recent cosmological hydrodynamical simulations, presented in Chapter 2. In the first Sub-section, we consider the stellar over total mass fraction of sub-haloes. In the second Sub-section, we study the relation between their maximum circular velocity and total mass. In the third, we compare the compactness of the members as predicted by our new SL model and by a suite of cosmological hydrodynamical simulations, to test whether the discrepancy between them can be mitigated by a higher resolution or a different implementation of the baryonic feedback in the simulations.

7.3.1 Stellar fraction of cluster members

As reported in Chapter 6, our model predicts a median stellar-to-total mass fraction value of 0.47, with a standard deviation of 0.34, for the 63 cluster members with $\sigma_{0,FP} > 80 \text{ km s}^{-1}$. We have seen that a SHMR obtained with a HOD procedure on the results of N -body cosmological simulations underestimates this value with respect to our model by almost an order of magnitude. We will now repeat the comparison with high-resolution cosmological hydrodynamical simulations from the Dianoga suite (Planelles et al. 2014; Rasia et al. 2015), carried out with the GADGET-3 code (Springel et al. 2005). To perform a comparison between the features of sub-haloes as predicted by the simulations and by our model, we consider 18 simulated clusters with $M > 10^{15} M_{\odot}$ at redshift $z = 0.38$ and perform three spatial projections of their mass distributions. We then extract their sub-haloes from the SUBFIND algorithm (Springel et al. 2001a; Dolag et al.

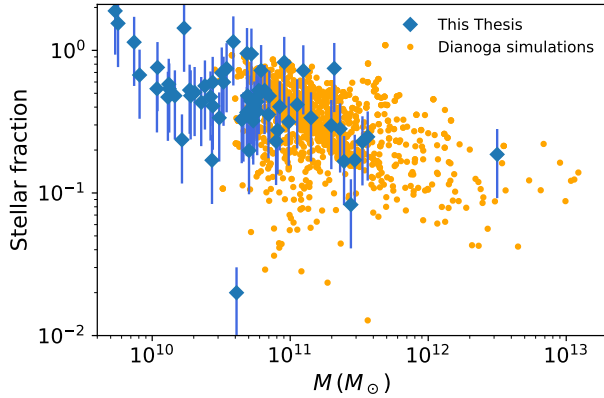


Figure 7.4: Comparison between the values of the stellar over total mass fraction of sub-haloes with velocity dispersion greater than 80 km s^{-1} as a function of their total mass. In blue: values reconstructed from our model. In orange: values predicted by the Dianoga hydrodynamical simulations. (Adapted from Granata et al. 2022).

2009) catalogue¹. We impose a minimum value of the velocity dispersion of 80 km s^{-1} , as in the previous analyses carried out in this Thesis, and a maximum projected distance from the cluster centre of $R = 0.15r_{\text{vir}}$, where r_{vir} is the cluster virial radius. We thus obtain a catalogue of 1756 sub-haloes, and we compute the stellar mass fraction for each of them. Again, as the Dianoga simulations use a Chabrier IMF to obtain the stellar mass of sub-haloes, we use the previously presented conversion factor to compare them with our measured stellar mass values, for which Mercurio et al. (2021) used a Salpeter IMF. We obtain a median stellar mass fraction value of 0.37, with a standard deviation of 0.18, in good agreement with the value suggested by the lensing model. Fig. 7.4 also shows that the Dianoga simulated sample has a similar relation between the stellar mass fraction and the total subhalo mass with respect to our model. We note that the sub-haloes extracted from the simulated catalogue have a slightly higher (around $3 \times 10^{10} M_{\odot}$) lower limit for the total mass compared to those extracted from the SL model adopting the same criterion. This seems to suggest that at a fixed velocity dispersion simulated sub-haloes have, on average, a higher total mass. In the next Sub-section, we will address this issue in light of the recent studies that have focused on it.

7.3.2 Compactness of the cluster sub-haloes

As detailed in a previous Sub-section, significant differences between the sub-halo compactness obtained from lensing models and cosmological hydrodynamical simulations have recently been reported by Meneghetti et al. (2020). They considered 11 SL models of massive galaxy clusters, including the model of AS1063 by B19, and a set of 25 clusters of similar mass and redshift from the Dianoga cosmological simulations presented in Chapter 2, which incorporate gas cooling, star formation, and energy feedback from supernovae and accreting SMBHs. They found that the probability of producing galaxy-scale strong lensing events is for observed clusters around an order of magnitude

¹The simulation snapshots and simulated subhalo catalogues used are available at <https://www.ict.inaf.it/index.php/DOI/122-ds-2020-1>.

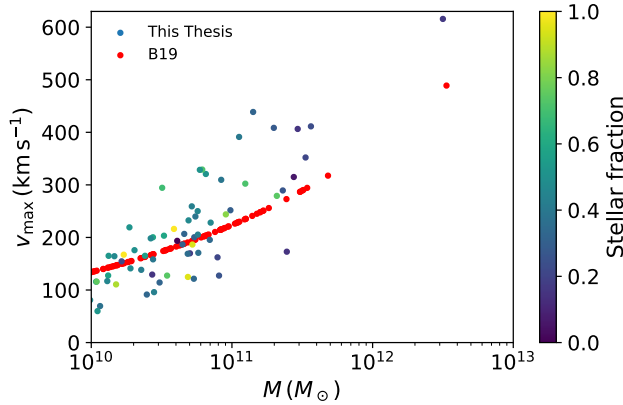


Figure 7.5: Maximum circular velocity of the cluster members as a function of their total mass. In red: values predicted by the model presented in B19. Our values are colour-coded according to the predicted stellar over total mass fraction. Only members with $M > 10^{10} M_{\odot}$ are shown. (Adapted from Granata et al. 2022).

larger than it is for simulated clusters. To interpret this discrepancy, they considered the relation between the mass of the sub-haloes in the model and their maximum circular velocity. As anticipated, Meneghetti et al. (2020) found that, at fixed total sub-halo mass, the value of the maximum circular velocity of cluster sub-haloes is larger than what cosmological hydrodynamical simulations predict. This seems to indicate that simulated cluster members are, on average, less compact than observed.

All SL models included in Meneghetti et al. (2020) used two optimised power-law scaling relations to model the truncation radius and the velocity dispersion of the cluster members from their luminosity (B19, Caminha et al. 2019). As all cluster models represent sub-haloes with isothermal profiles, this also leads to a fixed power-law relation between the maximum circular velocity (which is proportional to the velocity dispersion) of each sub-halo and its mass. On the other hand, our new method to assign mass to sub-haloes again avoids a fixed power law between the two quantities, allowing for the inclusion of some scatter. In Fig. 7.5, we show the $v_{\max}(M)$ relation obtained from our model for all the members with $M > 10^{10} M_{\odot}$ compared with that from B19: the two models agree in the entire mass range considered.

Comparing our results with those presented in Bahé (2021), shown in Fig. 7.2, we notice that our relation matches fairly well with the higher-mass branch of the Hydrangea hydrodynamic simulations, both in terms of circular velocity and of the stellar fraction of the cluster members at a fixed total mass. However, we note that the high-mass branch that agrees with our model in terms of maximum circular velocity is not found, for instance, in the Illustris-TNG300s simulation, which have a comparable resolution, as shown in the lower panel of Fig. 7.2. Furthermore, Ragone-Figueroa et al. (2018) also found that the Hydrangea simulations produce galaxies with a very high total mass which may not agree with observations. On the other hand, as already visible from Fig. 7.5, we do not find the large number of sub-haloes with $M < 10^{11} M_{\odot}$, $v_{\max} < 100 \text{ km s}^{-1}$, and a low stellar fraction predicted by the lower sequence by Bahé (2021). As a consequence, our model predicts, on average, higher maximum circular velocities with respect to simulations for sub-haloes with mass between 10^{10} and $10^{11} M_{\odot}$. In this to-

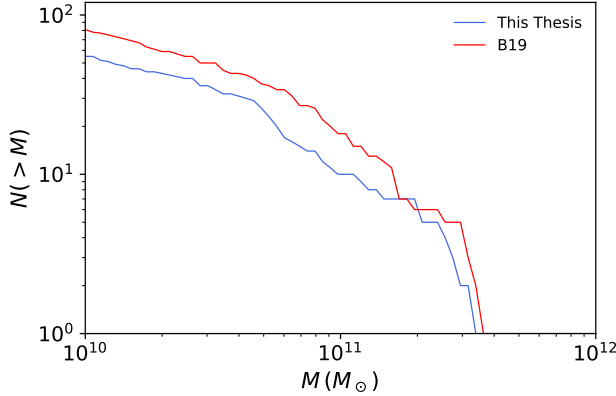


Figure 7.6: Cumulative sub-halo mass function for the members within a projected radius of $0.15r_{\text{vir}}$ predicted by our model (in blue) and by that presented in B19 (in red). (Adapted from Granata et al. 2022).

tal mass range, we find just a few members with maximum circular velocity lower than 100 km s^{-1} , while in Bahé (2021) most of the galaxies lie below this threshold. This seems to confirm the discrepancy reported by Meneghetti et al. (2020) in terms of galaxy-scale SL events, as most of these events are caused by haloes in this mass range, at least for AS1063 and the other clusters with a similar mass. We finally note that in Bahé (2021) the values of the baryon fractions are computed within a radius of 30 kpc, while in this Thesis they are the ratio between the total stellar mass and the total mass of each sub-halo. To compare the values obtained in the two cases, one must keep in mind that our model predicts that, for the high-mass cluster members, a median value of approximately 94% of the total mass is enclosed within a sphere with a radius of 30 kpc.

The comparative lack of low-mass members in our model is reflected by the cumulative sub-halo mass function within a projected radius of $0.15r_{\text{vir}}$, shown in Fig. 7.6: our model agrees with the results presented in B19, which in turn found a lower cumulative mass function compared to that of the Dianoga simulations, particularly considering sub-haloes with $M < 10^{11} M_{\odot}$ and $v_{\text{max}} < 100 \text{ km s}^{-1}$, as reported by Bahé (2021) in Fig. 1. In the same mass range, our cumulative sub-halo mass function has a slope similar to that by B19, significantly shallower than that from the cosmological simulations presented in Bahé (2021). In the radial range considered, our sample of photometric cluster members is highly complete down to $m_{\text{F160W}} = 24$.

7.3.3 Impact of the simulation set-up on the mass structure of the cluster sub-haloes

In Chapter 2, we have introduced a series of alternative set-ups for the Dianoga simulation suite, which differ in the mass particle resolution, in the softening length, and in the baryonic feedback scheme and its implementation. In particular, we have described four set-ups indicated as $1 \times \text{R15}$, $1 \times \text{RF18}$, $1 \times \text{B20}$, and $10 \times \text{B20}$. In this Sub-section, we report on a comparison between the mass function and the physical properties of the observed and simulated cluster sub-haloes, which was presented in Ragagnin et al. (2022).

Sub-halo mass function

In Fig. 7.7, we show the average cumulative satellite sub-halo mass number count (left column for total mass and right column for stellar mass), both within the projected virial radius R_{vir} (upper panels) and within a projected distance $R < 0.15R_{\text{vir}}$ (bottom panels) for the four simulation suites. The lower panels also compare the theoretical estimates against the observed mass distributions for AS1063 presented in Chapter 6 and in B19. The lower-resolution ($1\times$) simulation suites deviate from the power-law behaviour at masses $\lesssim 2 \times 10^{10} M_{\odot}$, indicating that resolution limits become significant at these mass scales. As a consequence, we exclude the sub-haloes below this mass limit from our analyses.

If we focus on the total-mass count within $0.15R_{\text{vir}}$ (lower panels), we find that different set-ups produce significantly different numbers of sub-haloes, and set-ups with larger softening produce a too small number of SHs. This is probably due to the fact that large-softening simulations produce more fragile SH cores, which are less resistant to tidal forces. We studied the stellar-mass function of the sub-haloes by showing the cumulative number count of sub-haloes with a given stellar mass. We present the cumulative number within R_{vir} in the top right panel of Fig. 7.7, where we see that $1\times R15$ and $1\times RF8$ produce sub-haloes with very similar stellar masses, while $1\times B20$ has many more bright galaxies (as expected by its lower feedback efficiency). $1\times B20$ and $10\times B20$ have nearly the same stellar mass counts. The stellar mass count inside $0.15R_{\text{vir}}$ (Fig. 7.7 bottom right panel) shows that a better resolution increases the number of high stellar mass galaxies.

Both the feedback scheme and the resolution of hydrodynamical simulations strongly affect the sub-haloes population in the cores of galaxy clusters, especially when it comes to high-mass sub-haloes: there are more massive sub-haloes in the core of a galaxy cluster when the resolution is increased or the feedback efficiency is reduced. Looking at the comparison between observations and simulations, all simulation set-ups seem to over-predict the number of high (total- and stellar-)mass sub-haloes: however, the most striking difference is observed with lower-feedback-efficiency simulations, which predict a significantly different stellar mass function compared to our observations.

Sub-halo compactness

In the first panel of Fig. 7.8, Ragagnin et al. (2022) show the median sub-halo v_{max} as a function of their total mass for all simulation suites, for B19 and for the SL model presented in Chapter 6. We note that the sub-haloes in the $1\times B20$ and $10\times B20$ simulations generally have higher maximum circular velocities than in the $1\times R15$ and $1\times RF18$ simulations. The difference grows with the mass of the sub-haloes. Consequently, the $v_{\text{max}} - M_{\text{SH}}$ relations in the B20 simulations are significantly steeper than the others. Meneghetti et al. (2020) already showed that the sub-haloes in the $1\times R15$ simulations have a lower maximum circular velocity than in B19 at all masses. Fig. 7.8 shows that the same result holds for the $1\times RF18$ simulations. On the other hand, the gap between observations and simulations is reduced when the feedback is less efficient, in the two B20 simulations. This behaviour is akin to that reported by Bahé (2021), which may indicate that the sub-grid model implemented in Hydrangea is less efficient at a fixed halo mass compared to the reference Dianoga simulations. At lower masses, the median sub-halo v_{max} in all our simulations reaches a value of $\sim 100 \text{ km s}^{-1}$, almost independently of the resolution and feedback model. This value is very similar to the median value reported by Bahé (2021) in the same SH mass-range, and is significantly below the observational findings of this Thesis and B19.

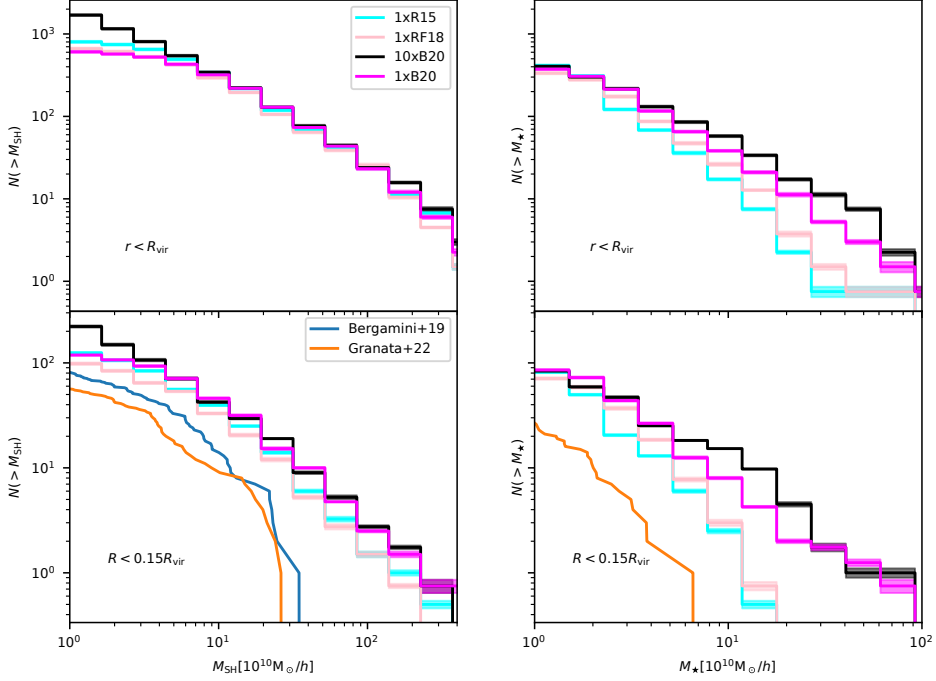


Figure 7.7: Average cumulative number of satellite sub-structures with total mass larger than M_{SH} and lower than $10^{13} M_{\odot}$ (left column) and for stellar masses larger than M_{*} , computed within an aperture of 30 physical kpc. We show data for the four $1 \times \text{R15}$ (cyan), $1 \times \text{RF18}$ (pink), $10 \times \text{B20}$ (black), and $1 \times \text{B20}$ (magenta) simulations in common between the four suites. For each simulation we average data over three orthogonal projections. Upper panels consider sub-haloes within the virial radius and lower panels consider sub-haloes with projected cluster-centric distance $R < 0.15 R_{\text{vir}}$. Observations are from the central region ($R < 0.15 R_{\text{vir}}$) of cluster AS1063 presented in B19 and in this Thesis. The simulated clusters have an average redshift of $z \approx 0.4$. (Adapted from Ragagnin et al. 2022).

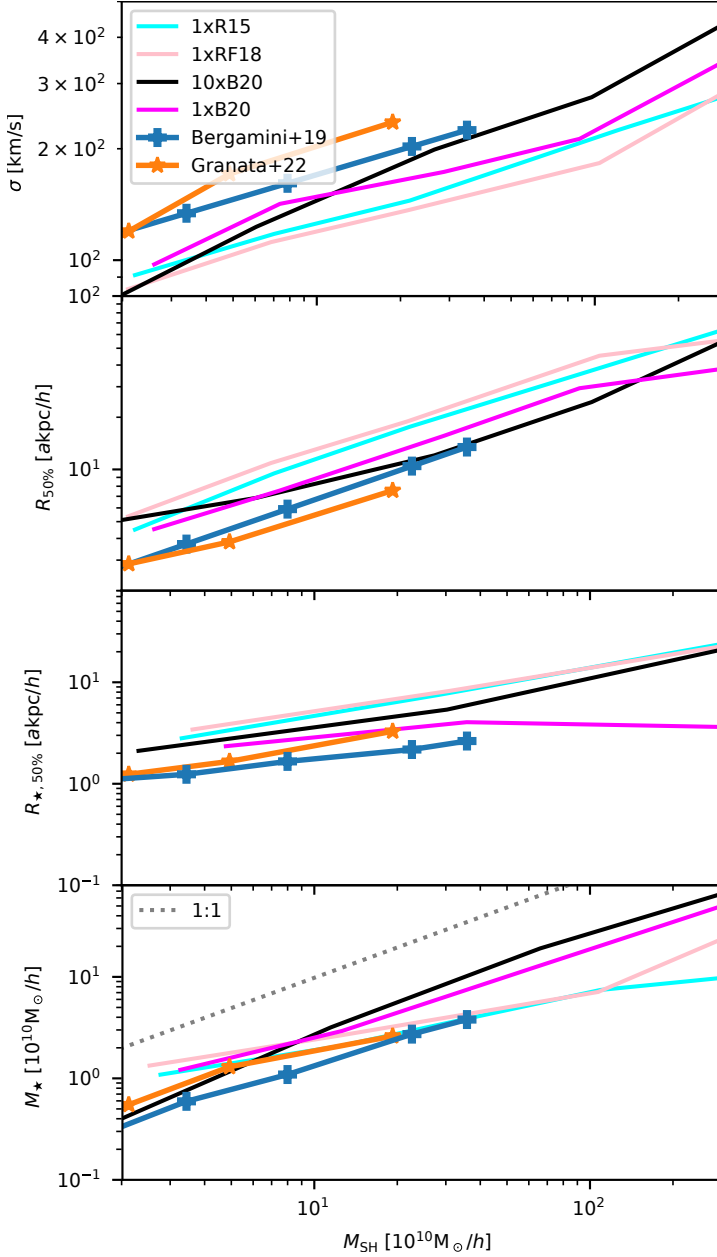


Figure 7.8: Median velocity dispersion $\sigma = v_{\max}/\sqrt{2}$ (top panel), median half-total-mass projected radius $R_{50\%}$ (second panel), median half-stellar-mass projected radius $R_{*,50\%}$ (third panel), and median total stellar mass M_* (fourth panel), all as a function of the total sub-halo mass M_{SH} . Simulation suites are colour-coded as in Fig. 7.7. We also show median values of observations from cluster AS1063 presented in this Thesis (orange solid lines with diamonds) and in B19 (blue solid lines with crosses). (Adapted from Ragagnin et al. 2022).

We see the same behaviour in the half-stellar-mass radius presented in the third panel of Fig. 7.8, where we show the half-mass radii of the stellar distribution of the sub-haloes versus their total mass. The panel shows that the simulation suites considered do under-estimate stellar component compactness. In the fourth panel of Fig. 7.8, we compare the stellar-to-total mass relation of simulated sub-haloes with observations, where we see that the stellar masses from simulated low-mass sub-haloes ($M_{\text{SH}} < 10^{11} M_{\odot}$) do overlap the values from observations. We therefore conclude that simulated low-mass sub-haloes have a realistic stellar fraction. Concerning high-mass members ($M > 4 \times 10^{11}$), instead, we know that simulations produce too many sub-haloes, as we pointed out previously. We note that the B20 simulation set-ups, which had the largest stellar-mass-function discrepancy with observations, also predict higher-than-observed stellar mass fractions, indicating that the lower feedback efficiency may bias the results.

7.4 Conclusions of Part III

In this Part, we have presented an improved SL model of the HFF galaxy cluster AS1063, thanks to new measurements of the structural parameters, namely magnitude, effective radius, and Sérsic index, of the cluster members. We have also exploited the values of the central stellar velocity dispersion of several luminous members, obtained from MUSE integral-field spectroscopy, to calibrate the FP relation for the cluster.

We use the FP to estimate the value of the central velocity dispersion of each cluster member from its observed magnitude and effective radius and we choose to consider a proportionality relation between the values of the effective R_e and truncation r_t radii. This allows us to assign a value of total mass to every cluster member, which does not depend only on the total luminosity of a galaxy. The main results and conclusions of our analysis are here summarised:

1. We find that the lowest value of the rms difference between the observed and model-predicted positions of the 55 considered multiple images (from 20 background sources) is obtained when $r_t = 2.3R_e$ for all cluster members.
2. Our method allows us to determine a relation between the total mass and velocity dispersion of the cluster members which is more realistic than the simple power law used in B19. Specifically, the new relation has some visible scatter and is less steep than that recently found in B19.
3. A more accurate estimate of the total mass of the cluster members allows for a reduction of the scatter on the determination of some parameters of the diffuse component. In particular, the statistical uncertainty on the value of the core radius of the main DM halo decreases by more than 30% compared to previous studies.
4. We confirm that modelling the main DM haloes of massive galaxy clusters with isothermal mass density profiles requires the presence of significantly large core radii. The value of the core radius of the main DM halo of AS1063 is 86 ± 2 kpc.
5. From our model and from the sampling of the posterior probability distribution of the parameters, we obtain the cumulative projected mass profiles of the various cluster components. Thanks to Chandra X-ray observations and to the new measurements of the stellar mass values of the cluster members, we can disentangle the hot gas and stellar mass profiles. At a distance from the cluster centre of 350 kpc, the cumulative, projected stellar, hot-gas, and baryonic fractions are, respectively,

- 0.6%, 14.1%, and 14.7%, the latter close to the cosmological value found by Planck Collaboration et al. (2020).
6. For the high-mass members included in our model, the median value of the stellar over total mass fraction is 0.47, with a standard deviation of 0.34. Considering, instead, the ratio between the stellar and total mass projected within the effective radius, we find good agreement with the relation obtained by Grillo (2010) on a wide sample of SDSS early-type galaxies.
 7. We use the SHMR from Girelli et al. (2020), based on the Λ CDM DUSTGRAIN-*pathfinder* N -body simulation, to obtain the halo mass values of the cluster members from their measured stellar mass values. We find that the SHMR predicts a median value of the stellar fraction of the high-mass cluster members smaller by almost an order of magnitude with respect to our model.
 8. We compare the high-mass cluster members with a catalogue of simulated sub-haloes. We extract the sub-haloes from the projected cores of 18 clusters from recent cosmological hydrodynamical simulations (Dianoga suite, Planelles et al. 2014; Rasia et al. 2015), imposing the same selection criterion on their velocity dispersion. We report good agreement between the values of the stellar over total mass fraction of the two samples.
 9. We study the relation between maximum circular velocity and total mass of the sub-haloes, which is an indication of their compactness. Comparing our results with those obtained by Bahé (2021) from the simulated clusters of the Hydrangea/C-EAGLE suite, we obtain consistent results for sub-halos with total mass greater than $10^{11} M_{\odot}$. On the contrary, we do not observe the large number of sub-haloes with mass below $10^{11} M_{\odot}$ and low baryon fraction predicted by Bahé (2021). In this sub-halo mass range, our relation predicts, on average, higher $v_{\max}$ at a fixed total mass, in agreement with B19 and Meneghetti et al. (2020). This corresponds to haloes that are more compact than those in the hydrodynamical simulations. As a consequence, in the same total mass range our cumulative sub-halo mass function has a shallower slope compared to what reported by Bahé (2021), very similar to that by B19 and Meneghetti et al. (2020).
 10. Varying resolution levels, softening lengths, and feedback schemes do not significantly impact the $v_{\max} - M_{\text{SH}}$ relation in the sub-halo total-mass range of interest for galaxy-galaxy SL events (i.e. sub-haloes with $M_{\text{SH}} < 10^{11} M_{\odot}$), and all the Dianoga re-simulations that we considered are unable to reproduce the sub-halo compactness predicted by this Thesis and B19.
 11. Some set-ups ($1 \times \text{B20}$, $10 \times \text{B20}$, and Hydrangea simulations) produce sub-haloes with a very high stellar mass which match our observed compactness. However, these objects are very rarely observed in the sample of clusters studied by Meneghetti et al. (2020).

Part IV

**Insights into the mass structure of
cluster galaxies with galaxy-scale
strong lensing**

In this Part, we present galaxy-scale SL models for three GGSL systems within the massive galaxy clusters MACS J0416.1–2403 and MACS J1206.2–0847, which provided us with a measurement of the truncation radius of the three lens member galaxies. We find that a parameterisation of the physical properties of cluster galaxies in SL models based on power-law scaling relations with respect to the observed total luminosity cannot accurately describe the compactness of the members over their full total mass range. Instead, our results agree with recent cluster member modelling based on the Fundamental Plane relation, presented in Chapters 6 and 7. Finally, we report good agreement between our predicted values of the stellar-to-total mass fraction within the effective radius and those of early-type galaxies from the Sloan Lens ACS Survey. Our work significantly extends the regimes of the current samples of lens galaxies towards the mass range that will be probed by the *Euclid*, *Rubin*, and *James Webb* Telescopes.

The Part is organised as follows. In Chapter 8, we give details on the reference SL models for MACS J0416.1–2403 and MACS J1206.2–0847, published in Bergamini et al. (2023b) and B19, respectively. We present our observations and the layout of the three GGSL systems, and we build a SL model for the lens galaxies. In Chapter 9, we discuss their inferred properties, focusing on their compactness.

The main scientific results presented in this Part are reported in Granata et al. (2023).

Exploring the low-mass regime of galaxy-scale strong lensing in clusters

In this Chapter, we aim to directly measure the compactness of a few selected galaxy-scale lenses in massive clusters. This provides us with a probe of the mass structure of a few cluster galaxies that does not depend on any scaling relation, thus testing the accuracy of the hypotheses and parametrisations adopted to describe the members in SL models of galaxy clusters. We considered the reference sample of lens clusters included in Meneghetti et al. (2020): AS1063, at $z = 0.348$, MACS J0416.1–2403 (MACS J0416), at $z = 0.396$, and MACS J1206.2–0847 (MACS J1206), at $z = 0.439$. We examined all main GGSL events in the three clusters, looking for systems with a clear morphology, in which several multiple images are observed very close to a cluster member, providing us with stringent constraint on its total mass profile, and therefore on its compactness. We selected two GGSL systems in MACS J0416, identified as ID14 and ID16 in Bergamini et al. (2023b) (hereafter B23), and one in MACS J1206, identified as ID14 in B19. Grillo et al. (2014) performed the first analysis of this system..

Gravitational lensing, in combination with stellar dynamics, has allowed for a considerable improvement in our understanding of the internal structure of galaxies, such as their stellar-to-total mass fraction and the mass density distribution of their DM haloes (see Shajib et al. 2022). In particular, observational campaigns such as the Lenses Structure and Dynamics (LSD; Treu & Koopmans 2004), the Sloan Lens ACS Survey (SLACS; Bolton et al. 2006; Treu et al. 2006; Auger et al. 2010), the Strong Lensing Legacy Survey (SL2S; Gavazzi et al. 2012), and the Dark Energy Survey (DES; Abbott et al. 2019) have mostly focused on isolated, massive early-type galaxies, due to observational limitations. Their total mass distribution is very well fit by an isothermal profile out to a large distance from their centre (Treu et al. 2006). In the next few years, new, wide, and deep surveys such as *Euclid* and the *Rubin* Observatory Legacy Survey of Space and Time (LSST) will boost the number of known galaxy-scale lenses by a few orders of magnitude, and significantly extend the lower mass threshold of observed lenses (Collett 2015).

The intense gravitational field of the cluster leads to secondary critical lines forming around several faint galaxies. In this Thesis, we build SL models for galaxies whose mass is too low for current surveys of lens galaxies to detect them in the field, probing a total lens mass range which will only be fully explored by the upcoming surveys. Furthermore, SL models of clusters favour truncated mass profiles for the cluster members, with a half-mass radius a few times larger than the half-light radius (Granata et al. 2022, hereafter G22) (see Chapters 6 and 7). In this Thesis, we evaluate the robustness of the assumption of truncated mass profiles for the cluster members and test possible alternative parametrisations.

In this Chapter, as in the previous, we use a flat Λ CDM cosmology with $\Omega_m = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, in which $1''$ corresponds to a scale of 5.34 kpc at $z = 0.396$,

the redshift of MACS J0416, and of 5.68 kpc at $z = 0.439$, the redshift of MACS J1206. All magnitudes are expressed in the AB system.

8.1 Reference strong lensing models

As anticipated, we considered the most recent versions of the SL models of MACS J0416 and MACS J1206 adopted by Meneghetti et al. (2020). They were presented in B23 and B19, respectively. Both models were built using the publicly available code `LenSTool`, introduced in Chapter 3. All mass components were described with a dPIE mass density profile (3.41), which is the ellipsoidal generalisation of a truncated isothermal sphere with a central core, and the ellipticity is introduced through Eq. (3.47).

The diffuse DM haloes and the ICM were modelled with cluster-scale dPIEs. The total mass distributions of each cluster member was described with a spherical coreless dPIE. An external shear term was taken into account while modelling MACS J1206. Following Bonamigo et al. (2018), the ICM mass distribution was fixed from X-ray observations, while all remaining components were optimised.

Therefore, a large sample of multiple images is crucial to increase the number of observational constraints to determine the best-fit set of parameters. The SL model of MACS J0416 presented in B23 is based on the largest sample of spectroscopically confirmed multiple images ever built for such scope: 237 multiple images from 88 background sources in the redshift range $z = 0.94 - 6.63$. The best-fit model has $\Delta_{\text{rms}} = 0.43''$ and includes 213 cluster members. The SL model of MACS J1206 from B19, on the other hand, includes 82 multiple images from 27 background sources in the redshift range $z = 1.01 - 6.01$. The best-fit model has $\Delta_{\text{rms}} = 0.46''$ and includes 258 cluster members. The values of Δ_{rms} found by B23 and B19 are significantly higher than the astrometric uncertainty on the positions of multiple images (typically smaller than $0.01''$). However, they are consistent with the state of the art of parametric SL models of massive clusters. Despite the availability of exquisite samples of spectroscopic multiple images, models are still affected by systematics that have an impact on their predicted image positions, such as the choice of the parametrisation of the cluster total mass distribution and the lensing effects of the cluster environment (Meneghetti et al. 2017; Acebron et al. 2017; Grillo et al. 2016; Johnson & Sharon 2016). For instance, as found by Chirivì et al. (2018), the line-of-sight mass distribution of MACS J0416, which is not included in B23, has a significant impact on the reconstruction of the multiple images, although its inclusion is not sufficient to reconcile observations and model predictions. Galaxy-scale SL models are not as affected by intrinsic systematics: we need to fully propagate the uncertainty impacting the cluster-scale mass distribution on the determination of the galaxy-scale parameters.

8.1.1 Modelling the mass distribution of the cluster members

As described in the previous Sub-sections, the cluster members of MACS J0416 and MACS J1206 were modelled with spherical dPIEs with a vanishing core radius. Their total mass only depends on two parameters: their velocity dispersion parameter σ , and their truncation radius r_t . In order to reduce the number of free parameters during the model optimisation, power-law scaling relations were calibrated between the two free parameters and the total luminosity of the cluster members (presented in Eqs. (3.57) and (3.58)).

Respectively 64 and 58 measured velocity dispersions have been included in the SL models of MACS J0416 and MACS J1206 to calibrate the Faber-Jackson relation. This pro-

Table 8.1: Reference lensing models of MACS J0416 (B23) and MACS J1206 (B19): relevant parameters. N_m and N_m^s indicate the number of cluster members included in the model and the number of those with measured velocity dispersion, respectively. N_i and N_s indicate the number of multiple images included in the model and the number of sources, respectively.

Cluster	N_m (N_m^s)	N_i (N_s)	Δ_{rms}
MACS J0416	213 (64)	237 (88)	0.43''
MACS J1206	258 (58)	82 (27)	0.46''

cedure partly breaks the degeneracy between the two parameters describing the cluster members. Using the SL forward modelling code `GravityFM`, B23 reconstructed the original surface brightness profile of some sources lensed by MACS J0416 and then traced back their images to the lens plane. They showed that, despite the parametric degeneracies involved, SL models can faithfully reproduce the observed morphology of the GGSL images, and that the inclusion of a larger sample of observed multiple images as a constraint for the optimisation of the model enhances the accuracy of the reconstruction. The degeneracy between the values of the velocity dispersion and of the truncation radius of the cluster members has also been addressed by G22 by replacing the Faber-Jackson law from Eq. (3.57) with a newly-calibrated FP: a more complex scaling law of which the Faber-Jackson is a projection (see Chapters 6 and 7). This allowed to estimate the central velocity dispersion for all the cluster members included in the model from their observed magnitude and half-light radius.

The $\sigma - r_t$ degeneracy has a very significant impact on the estimate of the compactness of the cluster members. The same total mass value for the cluster members can be obtained with different combinations of the two parameters resulting in a more (or less) compact mass distribution. On the other hand, in a GGSL system, the positions of the multiple images are stringent constraints on the critical lines around the lens galaxies, which strongly depend on their compactness, thus breaking the degeneracy.

8.2 The galaxy-galaxy strong lensing systems

In this Section, we lay out i) the structure of the three GGSL systems (MACS J0416 ID14, MACS J0416 ID16, MACS J1206 ID14) on which we focus this Thesis, ii) the data available for the cluster members, and iii) the procedure adopted to identificate the multiple images we later included in our lensing analyses.

8.2.1 MACS J0416.1–2403 ID14

The source ID14, studied in detail by Vanzella et al. (2017), is composed of a pair of faint, young, and compact stellar systems. It has been included in B23 with a spectroscopic redshift of $z = 3.221$ (Balestra et al. 2016). The gravitational potential of MACS J0416 splits it into three images separated by up to 50''. One of the three images, shown in Fig. 8.1, falls close to a pair of elliptical cluster galaxies (members 8971 and 8980 in the reference SL model) and is further split into four images. Following Vanzella et al. (2017), we refer to the two sources as 1 and 2, and to the four images as a, b, c, and d. Knots 1 and 2 have similar magnitudes in images a and b. Image 1c is the brightest observed, while 2c is much fainter, and its position is hard to measure precisely. A detailed photometric study of the lensing system has been performed to build the most recent lensing models:

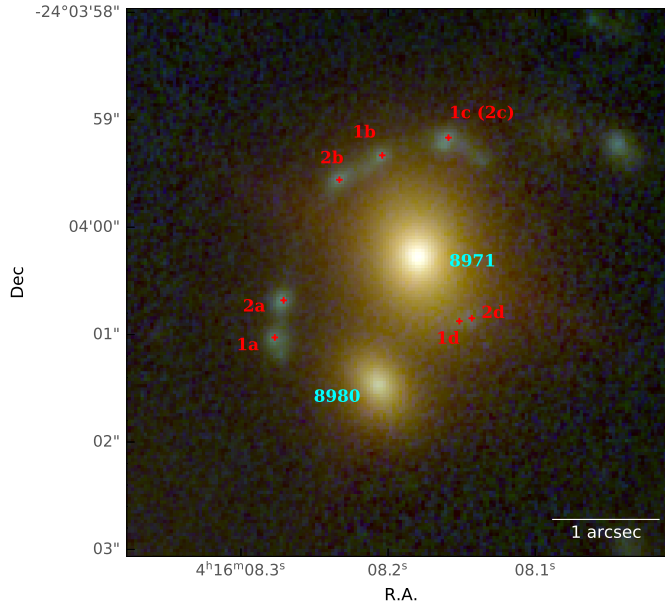


Figure 8.1: *Hubble* Frontier Fields RGB (F105W+F125W+F140W+F160W, F606W+F814W, F435W) image of MACS J0416.1–2403 ID14. The two cluster members are identified as in B23 (IDs 8971 and 8980). The two components of the source are identified as 1 and 2. The four multiple images are indicated with the letters a, b, c, and d. (From Granata et al. 2023)

we adopted the positions reported for the images a, b, and c. As such, we also chose to consider 2c in the same position as 1c for SL modelling, with a larger uncertainty on its value. In the case of image d, unlike B23, where only image 1d was included in the model, we identified the two components of the source and measured their positions separately. The adopted multiple-image positions are reported in Table 8.2.

8.2.2 MACS J0416.1–2403 ID16

The double source identified as ID16 in Bergamini et al. (2021), at a spectroscopic redshift of $z = 2.095$ (Balestra et al. 2016), is split into three multiple images by the gravitational potential of the cluster main halo. One of the three images falls very close to a pair of elliptical cluster members (8785 and 9129 in the reference SL model). The light distribution in the bluest of the HFF bands (ACS F435W) suggests that two further multiple images might fall very close to the centre of cluster member 8785, one almost completely superimposed on it. In order to correctly measure the position of this image and include it in our lensing model, we separated its light from that of the cluster galaxy. We did this in the HFF ACS F606W band, where the cluster member and multiple images are visible. To perform this task, first, we masked the light from the lensed image. The masked pixels are determined in the HST F435W band, where ID16 is less affected by the light of the cluster member. This way, we could run `GALFIT` (Peng et al. 2010) on the masked F606W image to model the light of the cluster member and measure its structural parameters. In the `GALFIT` run, the position of the cluster member and its Sérsic index $n = 0.5$ (Gaussian light distribution) were kept fixed. Parameters such as the magnitude and the half-light radius of the cluster member were manually tuned and fixed to minimise the

Table 8.2: Multiple image positions for the GGSL system MACS J0416 ID14. In cols. 2 and 3, we report the relative positions of the multiple images with respect to the centre of cluster member 8971, for which we provide the values of R.A. and Dec of the centre. The images are identified as in Fig. 8.1. We also report the position of the centre of cluster member 8980.

Reference	64.034084	−24.066743
Position	x (")	y (")
1a	−1.34	−0.77
2a	−1.26	−0.42
1b	−0.34	0.93
2b	−0.74	0.70
1c (2c)	0.27	1.09
1d	0.49	−0.59
2d	0.38	−0.57
8980	−0.36	−1.20

residual image obtained after its subtraction. No significant over- or under-subtraction regions were observed in the residual image, and the residual values are less than 20% of the original image value in every pixel. Therefore, we used the residual F606W image to measure the positions of all the multiple images, determining their x and y coordinates with Gaussian fits of the light profile. We refer to the two sources as 1 and 2 and to the three images produced by cluster member 8785 as c, d, and e. We also included image b in the model, which is only at a distance of around $1''$ from the cluster galaxy, and for which we have also measured the positions of the two components. The two sources are not clearly resolved in images d and e: we only included the brightest source, identified as 2, in our model. In Fig. 8.3, we confirm the new identification of two additional multiple images using the reference cluster-scale SL model. Even when only images a, b, and c are included in the optimisation process, the critical lines close to cluster member 8785 create images d and e. As shown in B23, SL forward modelling of the source ID16 performed with GravityFM allows us to reproduce the observed multiple image configuration and surface brightness distribution accurately. As such, our new multiple image catalogue, presented in Table 8.3, has been adopted by B23.

8.2.3 MACS J1206.2–0847 ID14

The double source ID14 was first included by Grillo et al. (2014) in a similar galaxy-scale lensing study before a cluster-scale lensing model for MACS J1206 was available. It is treated by B19 as a single source, with a spectroscopic redshift of $z = 3.753$ (Biviano et al. 2013; Caminha et al. 2017b). Five multiple images are observed, three of them very close to the centre of a cluster member (ID 3910), on which we focus our attention. Their positions are also influenced by the deflection caused by the second brightest cluster member (ID 2541). We refer to the two sources as 1 and 2 and to the three images as a, b, and c, as shown in Fig. 8.4. We measured their x and y positions, reported in Table 8.4, using CLASH photometry in the ACS F435W band.

Table 8.3: Multiple image positions determined from HFF photometry for the GGSL system MACS J0416 ID16. In cols. 2 and 3, we report the relative positions of the multiple images with respect to the centre of cluster member 8785, for which we provide the values of R.A. and Dec of the centre. The images are identified as in Fig. 8.2. We also report the position of the centre of cluster member 9129.

Reference	64.032442	−24.068485
Position	x (")	y (")
1b	−0.51	−0.44
2b	−0.70	−0.64
1c	0.23	0.42
2c	0.09	0.25
2d	−0.03	0.18
2e	0.05	−0.004
9129	−1.02	0.82

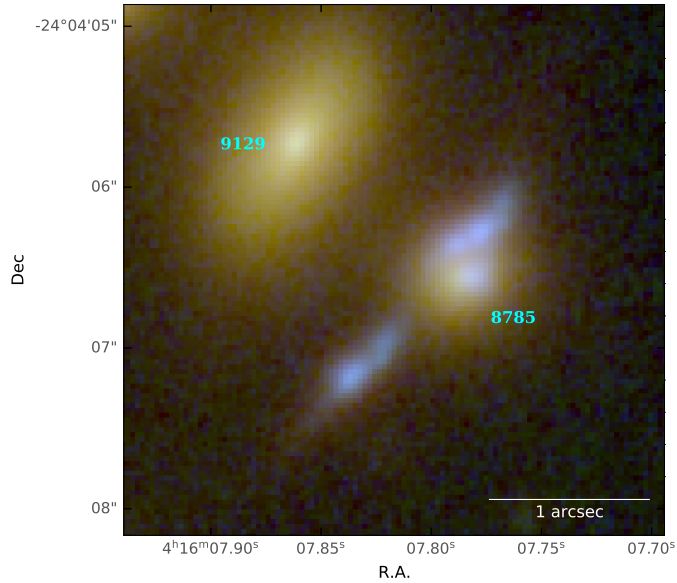


Figure 8.2: *Hubble* Frontier Fields RGB (F105W+F125W+F140W+F160W, F606W+F814W, F435W) image of MACS J0416.1−2403 ID16. The two cluster members are identified as in B23 (IDs 8785 and 9129). (From Granata et al. 2023)

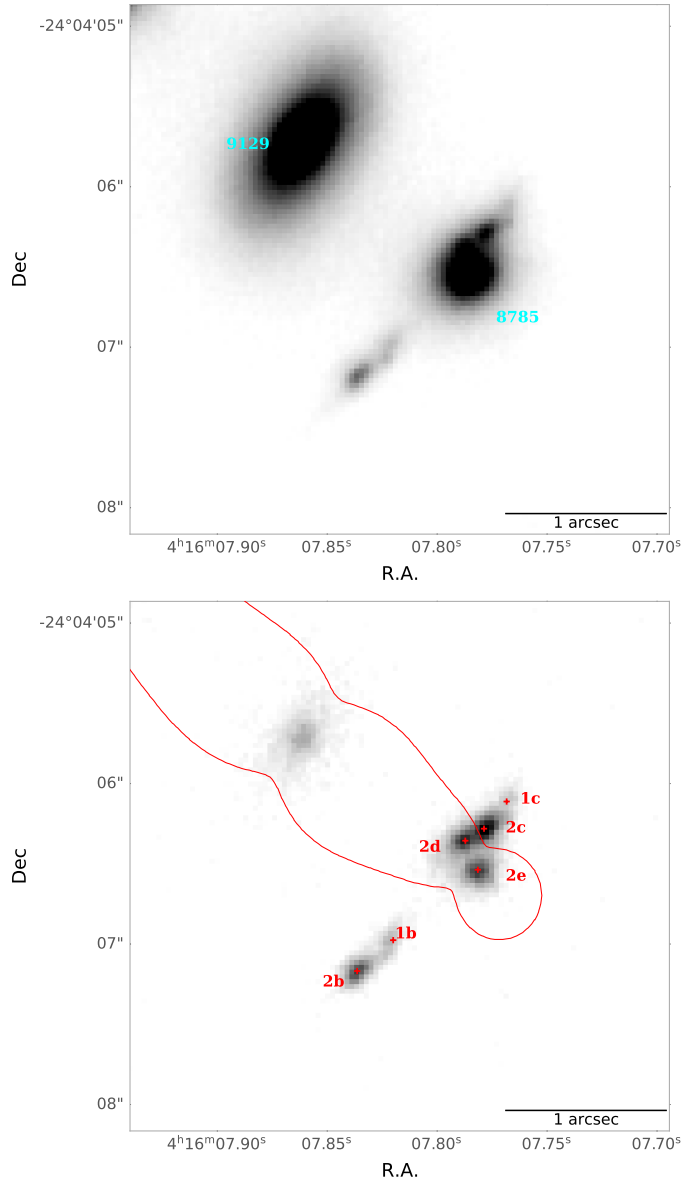


Figure 8.3: *Hubble* Frontier Fields imaging of MACS J0416.1–2403 ID16 in two different bands. Upper panel: the two cluster members (ID 8785 and 9129) in the ACS F814W band. Lower panel: the multiple images in the ACS F435W band. The two components of the source are identified as 1 and 2. The four multiple images are indicated with the letters b, c, d, and e. The critical lines predicted by the model by B23 for a source at redshift $z = 2.095$ (the same as ID16) are marked in red. (From Granata et al. 2023)

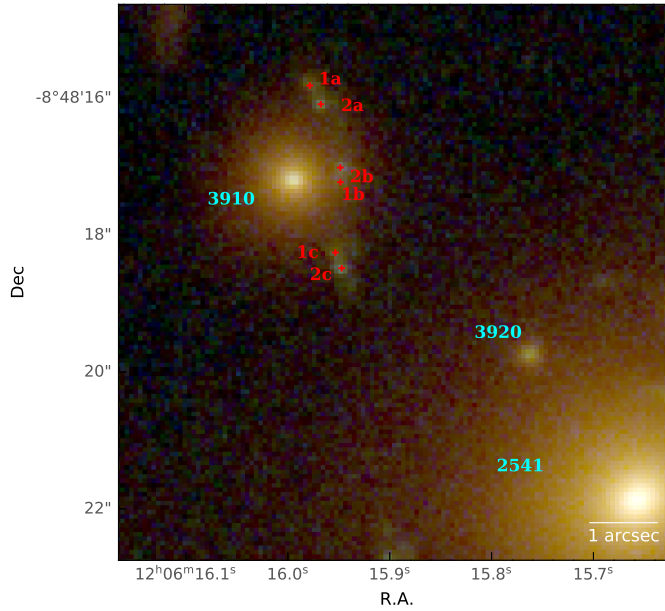


Figure 8.4: CLASH RGB image (based on all 12 CLASH filters) of MACS J1206.2–0847 ID14. The three cluster members are identified as in B19 (IDs 3910, 3920, and 2541). The two components of the source are identified as 1 and 2. The three multiple images are indicated with the letters a, b, and c. (From Granata et al. 2023)

Table 8.4: Multiple image positions determined from CLASH photometry for the GGSL system MACS J1206 ID14. In cols. 2 and 3, we report the relative positions of the multiple images with respect to the centre of cluster member 3910, for which we provide the values of R.A. and Dec of the centre. The images are identified as in Fig. 8.4. We also report the position of the centre of the cluster member 2541.

Reference	181.566661	−8.804784
Position	x (")	y (")
1a	0.26	1.36
2a	0.40	1.10
1b	0.68	−0.04
2b	0.68	0.18
1c	0.61	−1.06
2c	0.70	−1.29
2541	5.02	−4.67

8.3 Strong lensing modelling

Our main aim is to use galaxy-scale SL modelling to constrain the compactness of the cluster members directly and to compare our results with those obtained with cluster-scale modelling; this will allow us to study their effectiveness at recovering the mass distribution of the cluster sub-structures. In previous galaxy-scale SL studies in clusters (e.g. Grillo et al. 2014; Parry et al. 2016), a simplified description of the cluster-scale mass distribution was included in the SL models to account for its effects on the image deflection. An incorrect determination of the cluster-scale mass distribution can significantly hinder the description of galaxy-scale lenses, especially as far as their azimuthal structure is concerned. In our case, instead, the cluster-scale mass distribution is constrained with accuracy, owing to SL models based on up to ≈ 200 multiple images of background sources.

In SL models of massive galaxy clusters, the diffuse and galaxy-scale mass components are optimised with the same constraints. The cluster-scale DM haloes dominate the total mass budget and determine the position of its primary critical lines. Therefore, the parameters that describe their mass distribution have the largest impact on the positions of the multiple images (Meneghetti et al. 2017; Limousin et al. 2022). In addition, the scaling laws connecting the parameters adopted to model the cluster galaxies are influenced mainly by the properties of the most massive members, and power-laws can be too simple to describe the wide mass and morphology range of the cluster members (see G22, Chapters 6 and 7 and Beauchesne et al. 2024). The total mass distribution of the low- and intermediate-mass cluster galaxies is therefore not entirely constrained by cluster-scale SL models, as shown by the difference between the results obtained by describing them with different scaling laws (see Chapters 6 and 7). When modelling a galaxy-scale lens, we thus only optimised the parameters defining the cluster galaxies whose mass distribution significantly impacts the positions of the multiple images, keeping fixed the mass distribution of the rest of the galaxy cluster. To build the χ^2 function driving the parametric optimisation, we only included the multiple images close to the main galaxy-scale lens (within $2''$ from its centre). More distant multiple images could provide additional constraints on the position of the source. Still, their position is primarily determined by the mass distribution of other cluster mass components, whose description is subject to a higher uncertainty than the determination of the mass profile of the galaxy-scale lens. As such, their inclusion may propagate systematics affecting the total cluster mass onto the determination of the galaxy-scale mass distribution.

To account for this uncertainty on the cluster mass distribution, we did not limit ourselves to considering the best-fit mass models, which may be affected by a systematic bias in the studied region. Instead, we extracted 100 random sets of parameter values from the Markov Chain Monte Carlo (MCMC) sampling of the posterior probability distribution of the reference cluster-scale SL model. We fixed the mass distribution of the cluster-scale components and all the other members to one of the 100 realisations, and then optimised the galaxy-scale lenses. We repeated the optimisation for each of the 100 mass models of the cluster. This bootstrapping procedure also allowed us to estimate the uncertainty in the determination of the parameters of the galaxy-scale lenses and the degeneracies between them. Similarly to the reference models, we modelled the main lens galaxy of each galaxy-scale SL system as a spherical truncated dPIE. We also tested an elliptical non-truncated dPIE mass distribution to understand whether the preference for truncated models could arise from the insufficient azimuthal complexity of the spherical total mass model adopted. We performed all SL optimisations using `LensTool`.

Table 8.5: Best-fit values and 1σ errors of the parameters describing the cluster member 8971 in the SL model of MACS J0416 by B23.

Parameter	Value
σ (km s $^{-1}$)	134^{+7}_{-6}
r_t (")	$18.6^{+8.5}_{-8.3}$
e	$0.52^{+0.06}_{-0.11}$
θ_e ($^\circ$)	-40^{+20}_{-15}

8.3.1 MACS J0416.1–2403 member galaxy 8971

The galaxy-scale SL system MACS J0416 ID14 was described in the previous Section. Throughout the SL modelling procedure, we adopted the multiple image catalogue presented in Table 8.2, comprising eight multiple images from two background sources. All of the multiple images of the source ID14 are observed close to the cluster member 8971 (hereafter member 8971), at an average angular distance of $1.07''$: we thus focused on constraining its truncation radius. Cluster member 8980 significantly influences the multiple-image configuration, so we also optimised its mass distribution.

In B23, member 8971 was not included in the scaling relations adopted to describe the remaining cluster members, and it was modelled separately as a truncated dPIE profile. The values of the parameters of its total mass distribution are reported in Table 8.5, with an uncertainty provided by the MCMC sampling of their marginalised posterior probability distribution. The ellipticity of the halo converges to the upper limit of its prior, indicating that it is poorly constrained and perhaps compensates for some unaccounted shear.

As anticipated, we first described member 8971 as a spherical truncated dPIE with vanishing core (hereafter SIS_t), whose centre is fixed at the light centre. The two free parameters are thus the velocity dispersion σ and the truncation radius r_t . The alternative model, an elliptical non-truncated dPIE with a vanishing core (hereafter SIE), has three free parameters in σ , the ellipticity e , and the orientation angle θ_e . As we wish to focus on studying the radial structure of member 8971, we considered simpler models for the mass distribution of cluster member 8980. We tested spherical truncated and non-truncated isothermal models, and we noticed some degeneracy between the value of r_t for member 8980 and the parameters describing member 8971. To avoid this, we describe member 8980 with a SIS mass profile.

We optimised the two galaxy-scale SL models that we just outlined (with member 8971 parametrised as a SIS_t and a SIE, respectively) for each of the 100 cluster-scale total mass distribution realisations obtained as described earlier. With this bootstrapping procedure, we obtained a set of 100 best-fit values for the parameters of the galaxy-scale SL system: we used the median, the 16th, and the 84th percentiles of the resulting distribution of best-fit values to estimate of their value and its uncertainty. Studying the marginalised distribution of the 100 best-fit parameters also gives us insights into the degeneracy between them. Unlike in B23, where `LensTool` struggled to recover r_t and the ellipticity of member 8971 at the same time, all the parameters of the SIS_t and SIE models are well constrained, with low uncertainty. Their values are reported in Table

Table 8.6: Best-fit values and 1σ errors of the parameters of our galaxy-scale SL models of MACS J0416 ID14. In the first column 8971 is described as a SIsT, in the second one as a SIE.

Parameter	SIsT	SIE
Degrees of freedom	8	7
σ_{8971} (km s $^{-1}$)	$164.9^{+6.8}_{-7.5}$	$132.0^{+1.5}_{-1.2}$
$r_{t,8971}$ (")	$1.14^{+0.43}_{-0.20}$	—
e	—	$0.20^{+0.03}_{-0.03}$
θ_e (°)	—	$57.5^{+3.8}_{-3.5}$
σ_{8980} (km s $^{-1}$)	$67.2^{+3.5}_{-5.8}$	$45.6^{+3.1}_{-4.4}$
$\langle\Delta_{\text{rms}}\rangle$ (")	0.08	0.04

8.6. In the case of the SIsT mass profile, the total mass can be obtained as

$$M = \frac{\pi\sigma^2 r_t}{G}. \quad (8.1)$$

We can thus estimate a total mass value of $M = 1.2^{+0.3}_{-0.1} \times 10^{11} M_\odot$ for 8971.

For each best-fit model, we estimate the value of Δ_{rms} , and we refer to the average value of Δ_{rms} obtained for the 100 realisations of the model as $\langle\Delta_{\text{rms}}\rangle$: we find $\langle\Delta_{\text{rms}}\rangle = 0.08''$ for the SIsT models and $0.04''$ for the SIE model. We note that the SIE model allows for lower values of Δ_{rms} , which might be due to having one additional parameter compared to the SIsT model. This may imply that a spherical mass distribution is too simple to describe the total mass distribution in the region, perhaps also due to unaccounted shear from the cluster-scale mass distribution. This is also suggested by the high value of ellipticity reported by B23.

Given the significant difference in the best-fit value of σ between the models, we compare them with the measured stellar line-of-sight velocity dispersion (LOSVD) for member 8971. The NE region of MACS J0416 was included in the MUSE Deep Lens Field Vanzella et al. (2021), a very deep (17.1h integration time) MUSE observation with a seeing of approximately $0.6''$. The value of σ recovered by SL models is a central three-dimensional mass density scale, while the value of LOSVD is a projected stellar velocity dispersion, which depends on the light distribution and on the point spread function of the observations, so the two values do not need to be identical. The information provided by SL and stellar kinematics can be used to jointly constrain the mass model parameters by building self-consistent SL and dynamical models. This has been performed for several galaxy-scale lenses in the field, successfully enhancing the information provided by SL (Oldham & Auger 2018; Schuldt et al. 2019; Tan et al. 2023), but in our case it would be significantly complicated by the need to include the environment effects determined by the cluster-scale mass distribution.

We extracted the spectrum of member 8971 within a circular aperture centred in the centre of light of the galaxy and with a radius equal to the seeing of $0.6''$, to probe the central regions of the cluster galaxy. We measured the LOSVD using `pPXF` (penalised pixel-fitting) by Cappellari & Emsellem (2004) and Cappellari (2017, 2023), comparing the observed spectra with a set of 463 UVB stellar templates from the X-shooter Spectral Library (XSL) DR2 (Gonneau et al. 2020) with a signal-to-noise ratio (S/N) greater than $100 \text{ \AA}^{-1}$, combined and convolved with a LOSVD, as described in Chapter 5. The fit,

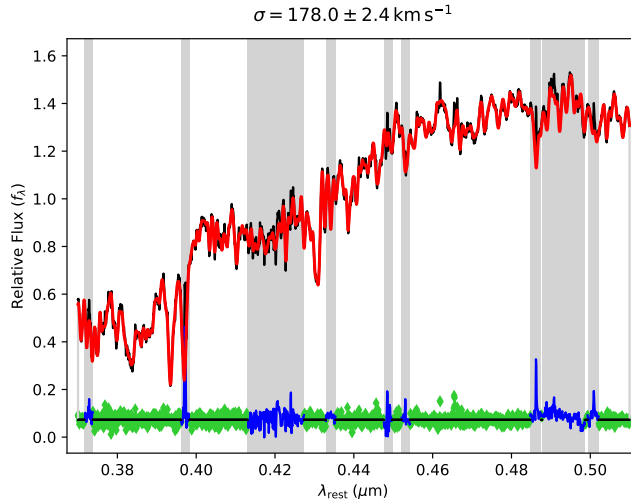


Figure 8.5: Fitting of the LOSVD of the MUSE spectrum of member 8971 with pPXF. The observed spectrum is shown in black, the red curve is the best-fit model, while the green points show the difference between the data and the model. The blue shaded regions along the wavelength axis were excluded in the fitting procedure, due to the presence of sky subtractions around emission lines or laser lines in the spectrum. (From Granata et al. 2023)

whose results are shown in Fig. 8.5, minimises a χ^2 function between the observed spectrum and a model and provides a measured LOSVD value of $178.0 \pm 2.4 \text{ km s}^{-1}$, for an average S/N value of 64.6 over the spectrum. The very low error on the measured value only includes the statistical uncertainty of the final LOSVD fit and does not include possible systematics introduced by the choice of the stellar templates adopted to fit the spectrum. The velocity dispersion value found by our SISt galaxy-scale SL model is consistent within approximately 2σ with the measured LOSVD. The comparison with the value found by our SIE model is less straightforward due to the change in the surface area within a given iso-density contour at a fixed σ determined by the introduction of the ellipticity through transformation defined in Eq. (3.47). However, we note that our ellipticity value of $e = 0.18$ implies a change in iso-density areas of only 1% with respect to the spherical case, which allows us to compare the two values. While the SL and the kinematic estimates of σ do not probe exactly the same physical quantity, we would expect them to have similar values, and only our SISt model seems to allow for that. This comparison is not performed in the following Sections for members 8785 and 3910, whose spectra have low S/N values, probably due to a very faint lens magnitude and a lower MUSE exposure time, respectively. The predicted compactness of the cluster galaxy 8971 and the evidence for a truncated total mass density profile are discussed in Chapter 9.

8.3.2 MACS J0416.1–2403 member galaxy 8785

The galaxy-scale SL system MACS J0416 ID16 was described in the previous Section. Throughout the SL modelling procedure, we adopted our new multiple image catalogue, presented in Table 8.3 and comprising six multiple images from two background sources. All six of the multiple images of the source ID16 are observed close to the clus-

Table 8.7: Best-fit values and 1σ errors of the parameters of our galaxy-scale SL models of MACS J0416 ID16. In the first column 8785 is described as a SISt, in the second one as a SIE.

Parameter	SISt	SIE
Degrees of freedom	5	4
$\sigma_{8785} \text{ (km s}^{-1}\text{)}$	$57.6^{+0.3}_{-0.3}$	$55.5^{+0.6}_{-1.2}$
$r_{t,8785} \text{ ("}\text{'})$	$0.74^{+0.12}_{-0.08}$	—
e	—	$0.18^{+0.08}_{-0.06}$
$\theta_e \text{ (}^\circ\text{'})$	—	110^{+10}_{-8}
$\sigma_{9129} \text{ (km s}^{-1}\text{'})$	$98.8^{+2.4}_{-2.2}$	$88.3^{+1.6}_{-2.1}$
$\langle \Delta_{\text{rms}} \rangle \text{ ("}\text{'})$	0.01	0.01

ter member 8785 (hereafter member 8785), at an average angular distance of only $0.43''$ from its centre. Thus, we focused on constraining the truncation radius of member 8785 and optimised the mass distribution of member 9129, which significantly influences the configuration of multiple images, as well.

In B23, member 8785 was modelled using the scaling relations (Eqs. 3.57 and 3.58) leading to a velocity dispersion value $\sigma = 83.3^{+2.7}_{-6.7} \text{ km s}^{-1}$ and a truncation radius $r_t = 0.8^{+0.2}_{-0.1}''$, resulting in a total mass $M = 2.1^{+0.3}_{-0.2} \times 10^{10} M_\odot$. We first described member 8785 as a SISt, and we studied an alternative SIE model. Again, we tested several models for member 9129, and we chose a SIS mass profile to avoid degeneracies between parameters.

We ran the SISt and SIE models for each of the 100 cluster-scale total mass distributions extracted from the MCMCs. In Table 8.7, we present the results of our bootstrapping procedure: the median values of the parameters of the best-fit galaxy-scale lensing models, and the 1σ uncertainties derived from their 16th and 84th percentiles.

The offsets between the observed and model-predicted positions of the multiple images are extremely small, with a value of $\langle \Delta_{\text{rms}} \rangle$, the average of Δ_{rms} for the 100 model realisations, of $0.01''$ for both the SISt and the SIE models. This value is lower than the observational uncertainty on the positions of the multiple images. The distribution of the best-fit values of the free parameters, for the 100 cluster-scale mass distributions considered, suggests that they are well constrained, with low uncertainty and without clear degeneracies between them. Comparing our results for the SISt model with those from B23, we find lower values for σ and r_t , corresponding to a lower total mass value of $M = 1.0^{+0.2}_{-0.1} \times 10^{10} M_\odot$. This discrepancy may be due to a bias introduced by the scaling relations adopted in B23: cluster-scale SL models that describe the sub-halo component with the FP find that power-law scaling relations can over-predict the total mass (see Chapters 6 and 7) and the velocity dispersion (Beauchesne et al. 2024) of low- and intermediate-mass cluster galaxies. In Chapter 9, we discuss the inferred compactness of member 8785.

8.3.3 MACS J1206.2–0847 member galaxy 3910

The galaxy-scale SL system MACS J1206 ID14 was described in the previous Section. Throughout the SL modelling procedure we adopted our new multiple image catalogue, presented in Table 8.4 and comprising six multiple images from two background sources. All six of the multiple images of the source ID14 are observed close to the cluster member

Table 8.8: Best-fit values and 1σ errors of the parameters of our galaxy-scale SL models of MACS J1206 ID14. In the first column 3910 is described as a SIS_t, in the second one as a SIE.

Parameter	SIS _t	SIE
Degrees of freedom	3	4
σ_{3910} (km s ⁻¹)	$129.2^{+5.1}_{-3.6}$	$113.8^{+2.5}_{-3.0}$
$r_{t,3910}$ (")	$0.90^{+0.20}_{-0.22}$	—
e	—	$0.77^{+0.06}_{-0.09}$
θ_e (°)	—	$96.53^{+0.29}_{-0.21}$
σ_{2540} (km s ⁻¹)	347^{+18}_{-16}	399^{+1}_{-5}
$r_{t,2540}$ (")	$1.75^{+0.20}_{-0.18}$	$0.49^{+0.03}_{-0.02}$
$\langle\Delta_{\text{rms}}\rangle$ (")	0.01	0.03

3910 (hereafter member 3910), at an average angular distance of $1.11''$ from its centre: we thus focused on constraining its truncation radius. Cluster member 2541, at a distance of $6.86''$ from the centre of 3910, is the second brightest cluster galaxy, with a predicted total mass of $9.55 \times 10^{11} M_\odot$ in B19. Closer to 3910 is the low-mass ($M = 1.97 \times 10^9 M_\odot$) cluster member 3920. Due to its high total mass, we also optimised the mass distribution of cluster member 2541. In contrast, we kept the parameters describing 3920 fixed to the values predicted by cluster-scale modelling.

B19 described member 3910 with the two scaling relations (Eqs. 3.57 and 3.58), leading to best-fit values of the velocity dispersion and of the truncation radius of $\sigma = 136.6^{+7.7}_{-6.6}$ km s⁻¹ and $r_t = 0.53^{+0.09}_{-0.07}''$, for a total mass $M = 4.2^{+0.4}_{-0.4} \times 10^{10} M_\odot$. Again, we first described 3910 as a SIS_t, and we tested an alternative SIE model. Regarding cluster member 2541, we note that in this case, we can constrain its truncation radius without unforeseen parametric degeneracies, and thus, we adopted a SIS_t model.

We ran the SIS_t and SIE models for each of the 100 cluster-scale total mass distributions extracted from the MCMCs. In Table 8.8, we present the median values of the parameters of the best-fit galaxy-scale lensing models, as derived from the bootstrapping procedure, with 1σ uncertainty derived from their 16th and 84th percentiles.

As in the case of MACS J0416 ID16, we find very small offsets between the observed and predicted multiple images, with an average Δ_{rms} value of $\langle\Delta_{\text{rms}}\rangle = 0.01''$ for the SIS_t model and $0.03''$ for the SIE model. Despite having fewer free parameters, the SIS_t model allows for a more accurate reconstruction of the lensing observables, and all parameters are estimated with low statistical uncertainty, as clear from Table 8.8. On the other hand, the SIE model cannot constrain well the value of the velocity dispersion of member 2540, which tends to the upper limit of our prior, and the predicted ellipticity of member 3910 is very high¹, suggesting that it might be needed to compensate for the lack of truncation in the total mass profile. Comparing our SIS_t model with B19, we find lower values for σ and significantly higher values for r_t , for a total mass value of $M = 6.3^{+1.0}_{-1.1} \times 10^{10} M_\odot$, higher than the best-fit value from B19. Again, the Faber-Jackson law adopted by B19 may be biasing the value of σ and, therefore, of the truncation radius. Grillo et al. (2014) use a one-parameter SIS mass profile to describe the cluster member, finding $\sigma = 97 \pm 3$ km s⁻¹. The lower value of σ is expected, given the lack of a truncation of the mass profile. In the next Chapter, we perform a more meaningful comparison, looking at the

¹As such, the value of σ_{3910} of the SIS_t model cannot be compared directly to that of the SIE model.

mass value within the effective radius.

Insights into the mass structure of cluster galaxies

9.1 Investigating the compactness of cluster galaxies

9.1.1 Truncation radius of the cluster members

As detailed in the previous Section, our galaxy-scale SL modelling procedure was aimed at estimating the truncation radius of three lens galaxies in massive clusters. The recovered values of r_t for members 8971, 8785, and 3910 are $6.1^{+2.3}_{-1.1}$ kpc, $4.0^{+0.6}_{-0.4}$ kpc, and $5.2^{+1.3}_{-1.1}$ kpc, respectively. To better illustrate the radial scales at play, we built a cumulative total mass profile of the three members by comparing the 100 best-fit mass profiles obtained with the bootstrapping procedure and by taking the 50th percentile at each projected radius. We also estimated the statistical uncertainty on the total mass profile from the 16th and 84th percentiles. We performed the same procedure on the SL models of the three cluster members by B23 and B19, using their MCMC sampling of the parametric posterior probability distribution to quantify the uncertainty on their total mass distributions. These profiles are shown in Fig. 9.1. We note that this procedure only accounts for the statistical uncertainty on the mass distribution parameters, and not for the possible systematics.

As shown in Figs. 9.1 and 9.2, the measured values of the truncation radius are, for all three members, within the range of the observed positions of the multiple images. In this radial range, SL allows for the highest accuracy in the reconstruction of the total mass profile of a lens, and is therefore more sensitive to its slope. In all three cases, we have tested an alternative non-truncated SIE mass parametrisation to inquire whether the truncated profile could actually be compensating for an insufficient description of the lens azimuthal structure. Despite having 4 free parameters, as opposed to the 3 of the SIS model, only in one of the three cases (member 8971), the SIE model leads to a slightly lower value of Δ_{rms} . In one case (member 3910), the SIE model predicts a very high value of the lens ellipticity, which is not supported by the light distribution. In conclusion, in all three systems we find that a spherical truncated total mass distribution for the lens galaxy is able to reproduce the SL observations with a small Δ_{rms} value and provide the value of r_t with a low statistical error. In two out of the three cases, a non-truncated model with a higher number of free parameters does not improve the description of the system, in one case leading to unrealistic parameter values.

Suyu & Halkola (2010) performed a similar truncation radius measurement for a satellite galaxy ($z = 0.351$) of the massive elliptical lens SL2S J08544–0121, which influences the shape of an Einstein ring determined by the main lens. They found a value of r_t close to its theoretically predicted tidal radius. Figure 9.3 shows that the value of r_t estimated in Suyu & Halkola (2010) seems to agree with the σ -to- r_t relation found in this Thesis. Monna et al. (2015) repeated the procedure for the CLASH-VLT galaxy

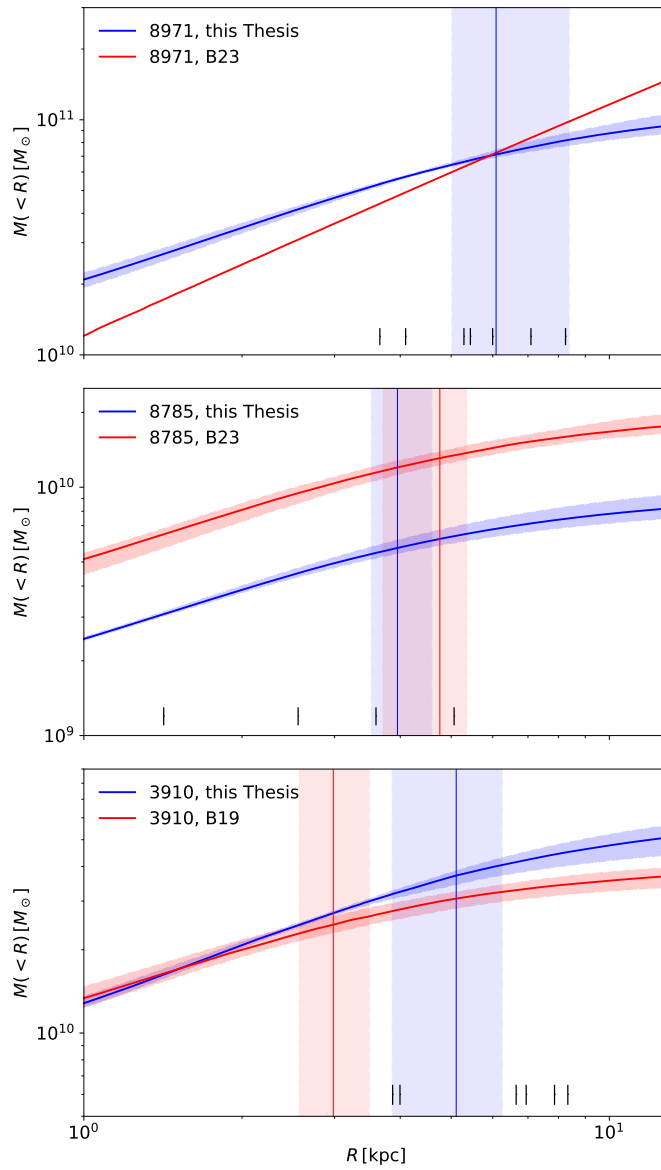


Figure 9.1: Projected cumulative total mass profiles for the lens galaxies studied in this Thesis compared with the predictions of B23 and B19. From the top to the bottom panel: member 8971, member 8785, and member 3910. The vertical bars indicate the inferred values of the truncation radius. Shaded regions indicate the 16th and 84th percentiles for the mass profile and the truncation radius. The projected distances of the observed multiple images from the lens centre are marked with vertical black lines. (Adapted from Granata et al. 2023)

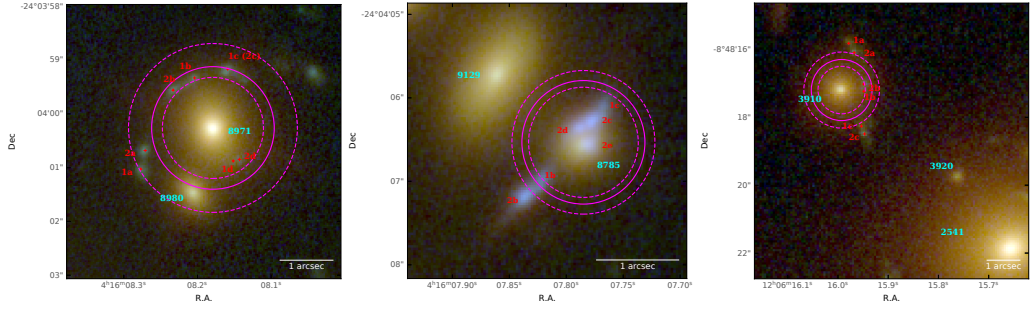


Figure 9.2: Truncation radius of members 8971, 8785, and 3910 superimposed to the RGB cutout of the respective galaxy-scale SL system. We show the median value as a solid line and the 1σ uncertainty range with dashed lines. (From Granata et al. 2023)

cluster Abell 383. They combined SL with observational priors on the observed velocity dispersion for two high mass (total mass greater than $10^{12} M_{\odot}$) members close to two lensed arcs of the same sources. The recovered value of r_t is greater than 50 kpc for both galaxies, significantly higher than predicted by the scaling law calibrated for the other cluster members of Abell 383. The authors suggest that very bright members may have not undergone strong stripping as a result of being a central galaxies prior to accretion on the cluster. Both Suyu & Halkola (2010) and Monna et al. (2015) based their SL modelling procedure on surface brightness reconstruction of extended arcs, finding that this significantly improves the accuracy of the constraints on the value of the truncation radius. In this Thesis, we only used the centroid position of the multiple images, rather than their full surface brightness. However, unlike in Suyu & Halkola (2010) and Monna et al. (2015), the cluster-scale mass distribution determines several different multiple images of the background source close to the galaxy-scale lens, rather than a single extended lensed arc. In our case, several multiple images are observed at different projected distances from the lens centre, providing us with detailed information about the galaxy total mass profile and its slope.

Finally, we tested whether the inferred values of r_t may be biased by the parametrisation chosen by B23 and B19 for the total mass profile of the haloes included in the model. To do so, we built an alternative cluster-scale model for MACS J1206 in which all of the cluster members are described with singular isothermal sphere (SIS) mass profiles (i.e. with infinite r_t) and repeated the statistical analysis described above. We find that this does not influence significantly the estimated parameter values for the lens galaxy 3910: their values are compatible within 3σ with those found in Chapter 8, suggesting that our modelling procedure is robust with respect to the parametrisation choices adopted for the remaining cluster mass components.

9.1.2 Compactness of the cluster galaxies

Galaxy-scale SL events in massive clusters allowed us to infer the physical properties of some selected cluster galaxies without relying on the scaling laws typically adopted to describe them. Cluster-scale SL modelling is mostly sensitive to the total mass of the cluster galaxies, rather than to the details of their mass density profiles. As such, σ and r_t suffer from a strong degeneracy and cannot be separately constrained in absence of an observational prior: this significantly limits the insights on the compactness of the cluster galaxies. To break the degeneracy between σ and r_t , B19 and B23 obtained a

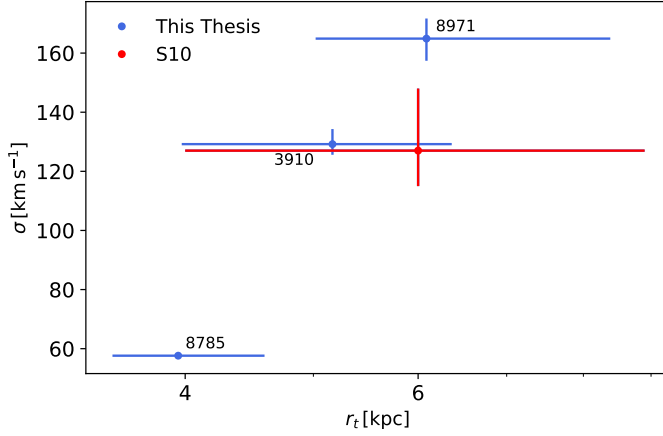


Figure 9.3: Relation between the values of the velocity dispersion and of the truncation radius for the three cluster members included in this study (in blue) and for a satellite of SL2S J08544–0121, as measured by Suyu & Halkola (2010) (in red). The inferred uncertainty on the velocity dispersion of member 8785 is too small to be visible in this plot. (Adapted from Granata et al. 2023)

kinematic prior on the value of the slope of the Faber-Jackson law (marked as α in Eq. 3.57). Assuming a total mass-to-light ratio $M/L \propto L^\gamma$ leads to $\beta = \gamma - 2\alpha + 1$ (β is defined in Eq. 3.58). From Eq. (8.1), one can then derive a total mass-to- σ scaling law

$$M \propto \sigma^{\frac{1+\gamma}{\alpha}}. \quad (9.1)$$

B19 and B23 assumed $\gamma = 0.2$, as suggested by the FP. The reference values of the scaling laws (σ_t^{ref} and r_t^{ref} in Eqs. 3.57 and 3.58) are mostly determined by the high-mass cluster members, which contribute significantly to the positions of the observed multiple images. On the other hand, once γ is fixed, the value of α fixes the slopes of the two scaling laws, determining the description of the total mass distribution of the low- and intermediate-mass cluster members.

Considering the reference sample of clusters included in Meneghetti et al. (2020), B19 find $\alpha = 0.28^{+0.02}_{-0.02}$ for MACS J1206 and $\alpha = 0.27^{+0.04}_{-0.04}$ for AS1063, while Bergamini et al. (2021) and B23 find $\alpha = 0.30^{+0.03}_{-0.03}$ for MACS J0416, corresponding to $M \propto \sigma^{4.3}$, $M \propto \sigma^{4.4}$, and $M \propto \sigma^{4.0}$, respectively. The same procedure was adopted by Bergamini et al. (2023a) for the HFF cluster Abell 2744 (A2744), finding a value of $\alpha = 0.40^{+0.03}_{-0.03}$ (implying $M \propto \sigma^{3.0}$). In the top panel of Fig. 9.4, we compare the M -to- σ scaling law for these four cluster scale SL models with the values of total mass and velocity dispersion for the galaxy-scale SL systems. In cluster-scale models, only the values σ_t^{ref} and r_t^{ref} are optimised: we estimated the uncertainty on the determination of the M -to- σ relation based on the MCMC sampling of the posterior probability distribution for these two parameters. We notice that the scaling relations used in cluster-scale models have significantly different normalisation and slope values. In particular, the SL model of MACS J0416 consistently predicts higher mass values at a fixed σ compared to those of MACS J1206 and AS1063. On the other hand, the model of A2744 has a significantly lower slope, predicting higher total mass values at low σ . Looking at the three cluster members studied in this Thesis, only the M -to- σ relation found for A2744 seems to represent well their compactness.

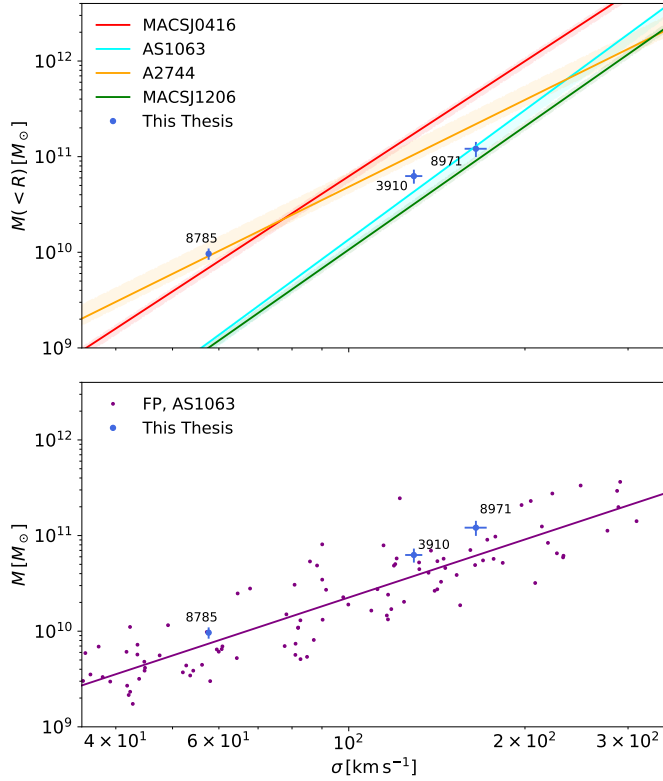


Figure 9.4: Comparison between the M -to- σ relation estimated by our analysis and those adopted for the cluster galaxies in the SL models of MACS J0416 (B23), AS1063 (B19,G22), MACS J1206 (B19), and A2744 (Bergamini et al. 2023a). Top panel: comparison with the SL models describing the cluster galaxies according to Eqs. (3.57) and (3.58). Bottom panel: comparison with the SL model of AS1063 by G22, based on the FP. (Adapted from Granata et al. 2023)

In the SL model of AS1063 by G22 (see Chapters 6 and 7), the values of σ were fixed from the observed luminosity and half-light radius R_e of the cluster galaxies, while a proportionality law was calibrated between the observed R_e and r_t . As such, the M -to- σ relation is not a power-law and is able to include a realistic scatter. As shown by the bottom panel of Fig. 9.4, the relation significantly differs from those obtained with a power-law approach: a bi-logarithmic fit of the relation predicts a slope of 2.0. The panel also shows that the three galaxy-scale lenses that we have modelled in this Thesis lie within the scatter of the scaling relation derived by G22 using the FP. While a sample of three objects is very small, it is interesting to note that the M -to- σ relation for the galaxy scale lenses, derived exclusively with SL, is very close to the predictions of G22, where lensing observables are only used to estimate the ratio between R_e and r_t . A more complex description of the cluster galaxies based on the FP leads to inferred properties which are compatible with our analysis. Beauchesne et al. (2024) recently modelled AS1063 adopting an intermediate approach between B19 and G22, where galaxies are described with the Faber Jackson law or the FP depending on the observations available for them. The value of α was optimised together with the parameters of the FP to avoid inconsistent slopes. Choosing $\gamma = 0$, they find $\alpha = 0.34$, obtaining to $M \propto \sigma^3$. On the other hand, fixed power-law scaling relations do not seem to be able to correctly describe the spatial structure of the cluster galaxies on the whole mass range included in SL models. As shown by Fig. 9.1, we predict significantly different properties for members 8785 and 3910 compared to B23 and B19: the former has a similar value of r_t but a higher σ , while the latter has a significantly larger r_t .

These differences may have a non-negligible impact on the magnification factor predicted close to the galaxy-scale lenses, which connects the observed and the unlensed magnitude for the multiply imaged sources. We measured the magnification predicted by the three galaxy-scale models. For each of them, we used the 100 models from our bootstrapping procedure to estimate the uncertainty on the magnitude value. In Fig. 9.5, we map the value of $\xi = \log \left(\left| \frac{\mu - \mu_{CS}}{\sigma_\mu} \right| \right)$, where μ is the magnification predicted by our models, σ_μ is the uncertainty on its value, and μ_{CS} is the magnification from the cluster-scale models (by B23 and B19). The upper panel of Fig. 9.5 shows that this Thesis and B23 predict significantly different magnifications close to member 8971. However, these differences are probably due to the different mass parametrisation chosen by the two models and are less pronounced close to the positions of the multiple images, where the total mass distribution is better constrained. As expected, close to members 8785 and 3910, the value of ξ is closer to zero as a consequence of choosing the same mass parametrisation in the two models. In all three cases, with few exceptions, the difference between the predicted values of the magnification is relatively small close to the multiple images, where the total mass distribution of the lens is better reconstructed. In conclusion, the systematics related to the mass modelling of the cluster members affect the predicted magnification map of lens clusters close to member galaxies. These predictions are more robust in proximity of the observed positions of the multiple images, but a more realistic description of the total mass properties of lens galaxies can benefit the accuracy of the studies of high-redshift lensed sources.

9.1.3 Comparison with cosmological simulations

In this Sub-section, we compare the compactness of the cluster members as obtained from our galaxy-scale SL models with the theoretical predictions of cosmological simulations. This study offers an excellent opportunity to contrast the properties of observed

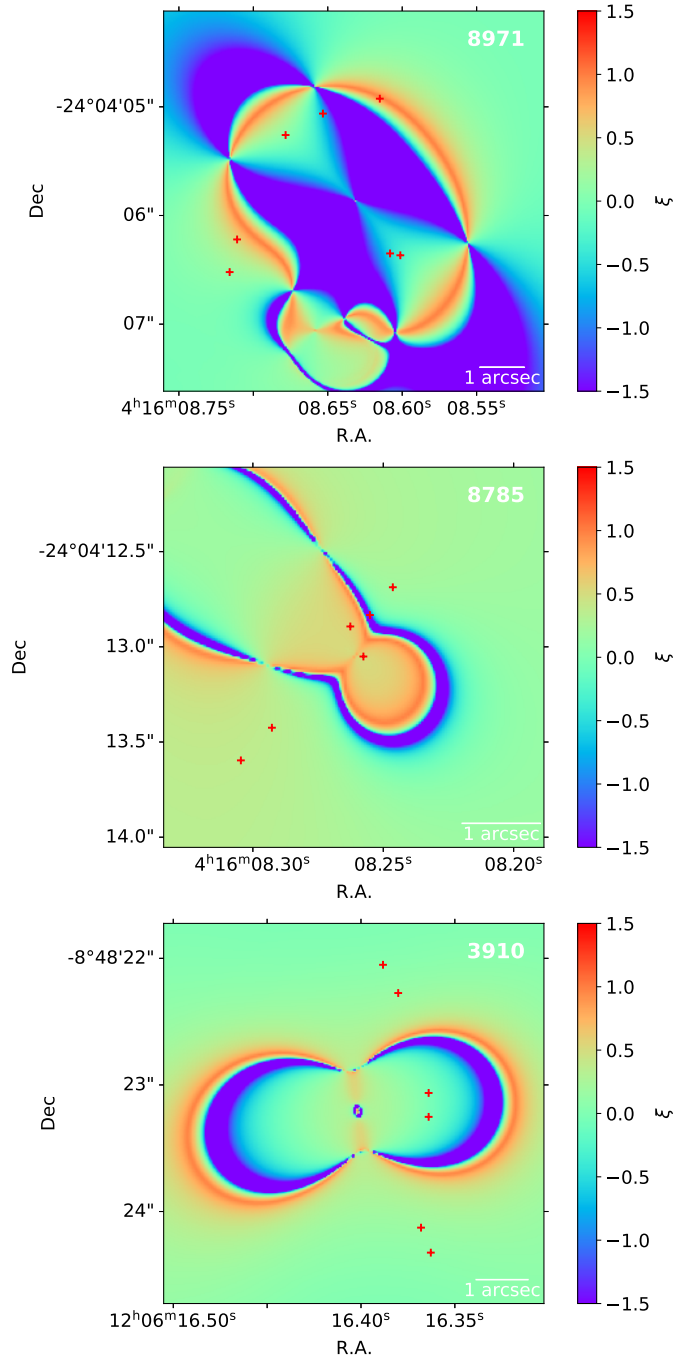


Figure 9.5: Comparison between the magnification values close to the three galaxy-scale lenses predicted in this Thesis and in B23 and B19. The colour map is based on the value of $\xi = \log \left(\left| \frac{\mu - \mu_{CS}}{\sigma_\mu} \right| \right)$ close to members 8971, 8785, and 3910, respectively. The red crosses indicate the observed positions of the multiple images included in this Thesis. (From Granata et al. 2023)

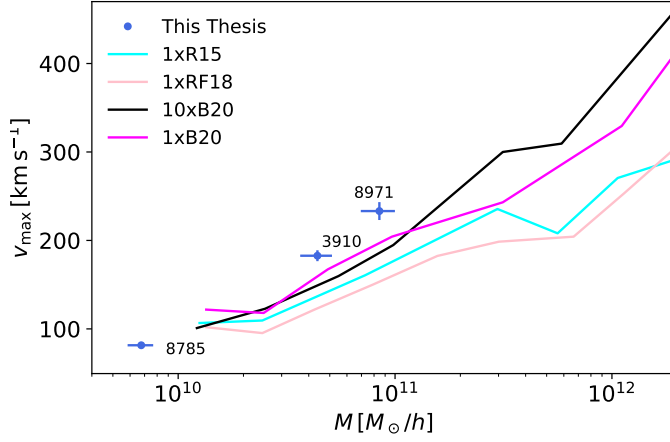


Figure 9.6: Comparison between the $v_{\max}$ -to- M relation obtained from our analysis and those predicted by the cosmological hydrodynamical simulations described in Ragagnin et al. (2022). The naming convention for the different suites is presented in Chapter 2. (Adapted from Granata et al. 2023)

and simulated galaxy-scale lenses in massive clusters (although we note that member 8785 is not the main lens responsible for the secondary critical line that produces system ID16). As in Meneghetti et al. (2020), we adopt the maximum circular velocity of the cluster members, defined as

$$v_{\max} = \max \left(\sqrt{\frac{GM(< r)}{r}} \right), \quad (9.2)$$

where $v_{\max} = \sqrt{2}\sigma$ for a singular non-truncated isothermal model, as a proxy for the compactness of cluster galaxies (see Meneghetti et al. 2020, 2022, 2023; Ragagnin et al. 2022): more compact objects have higher values of $v_{\max}$ at a fixed total mass. In Fig. 9.6, we compare the $v_{\max}$ -to- M relation for this Thesis with those found in Ragagnin et al. (2022) for a set of zoom-in re-simulations of the Dianoga suite of simulated galaxy clusters, presented in Chapter 2. These setups differ from one another in terms of their softening and feedback schemes.

Figure 9.6 shows that members 8971 and 3910 both have a maximum circular velocity value higher than those for simulated sub-haloes with the same total mass, irrespective of the feedback scheme or the resolution considered in Ragagnin et al. (2022), indicating a level of compactness higher than that predicted by simulations. This seems to suggest that the discrepancy between SL models and simulations cannot be entirely ascribed to systematics affecting the former, as a result of the adoption of power-law scaling relations to describe the cluster members, as noted already by G22. Member 8785 falls in a total mass range ($M < 10^{10} M_{\odot}$) which was excluded from the analyses of Meneghetti et al. (2020), because the current cosmological simulations do not have enough mass resolution.

9.1.4 Stellar mass of the cluster members

In this Sub-section, we study the stellar-to-total mass fraction of the cluster members. Measuring its value within the effective radius is an important probe of the interplay

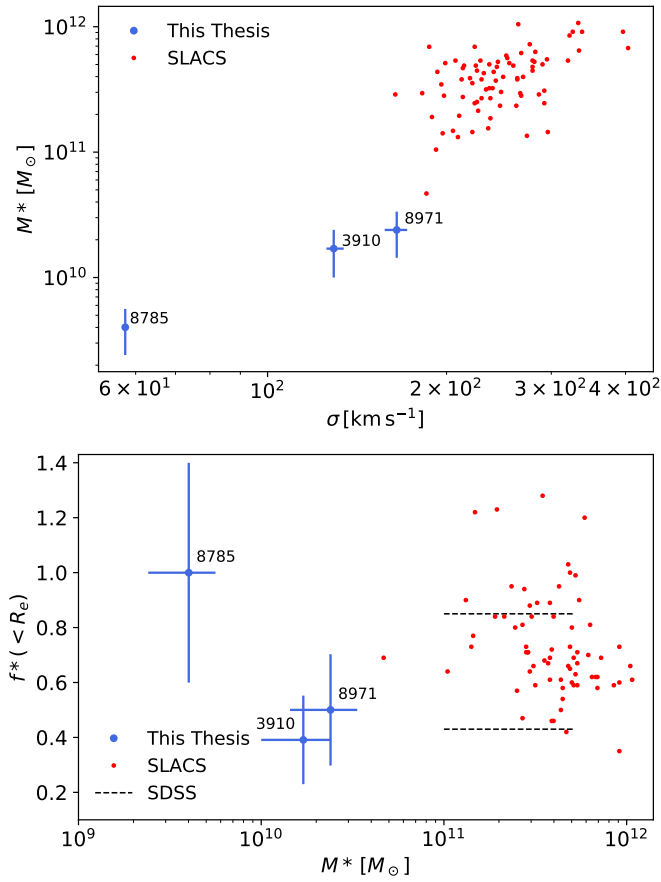


Figure 9.7: Stellar mass of lens galaxies: comparison between this Thesis and the 85 SLACS lens galaxies presented in Auger et al. (2009). Upper panel: Stellar mass as a function of the velocity dispersion. Lower panel: Two-dimensional stellar-to-total mass fraction measured within the effective radius; the 68% confidence interval found by Grillo (2010) for a selected sample of 2×10^5 SDSS early-type galaxies is also shown. (Adapted from Granata et al. 2023)

between the effects of the baryonic feedback processes and the gravitational potential of DM during galaxy formation (see Shajib et al. 2022; Smith 2020), and of stellar populations in early-type galaxies. These processes are particularly significant for lower-mass galaxies, which should have a stellar-to-total mass fraction higher than that of the very massive early-type galaxies that dominate the samples of lens galaxies in the field. This Thesis significantly extends the typical mass range of current studies, similarly to what will be performed with the upcoming samples of galaxy-scale lenses unveiled by *Euclid* and LSST (see Collett 2015).

The values of the total stellar mass of the three lens galaxies have been measured in recent works. For members 8971 and 8785, we followed the best-fit relation between the stellar mass and the magnitude in the HST F160W band found by Grillo et al. (2015), $\log(M^*(M_\odot)) = 18.541 - 0.416 \times F160W$, where M^* is the stellar mass of the cluster member. To consider the scatter about this mean relation, we chose a 40% uncertainty on the derived stellar mass values of $(2.4 \pm 1.0) \times 10^{10} M_\odot$, and $(4.0 \pm 1.6) \times 10^9 M_\odot$,

respectively. For member 3910, we adopted the value measured by Grillo et al. (2014) of $(1.7 \pm 1.0) \times 10^{10} M_{\odot}$. In both cases, the stellar mass measurements were based on the HST spectral energy distribution (SED) of the lens galaxies, using composite stellar population models, and a Salpeter (Salpeter 1955) stellar initial mass function (IMF). In Fig. 9.7, we show the relation between the stellar mass and the velocity dispersion for the three cluster members studied in this Thesis and for the 85 SLACS lenses presented in Auger et al. (2009), which are representative of the currently known population of lens galaxies. We note that the SLACS lenses were modelled with non-truncated total mass profiles, which could affect the recovered values of the velocity dispersion. The figure showcases the significant extension of the range of stellar mass and velocity dispersion values probed in this Thesis compared to current samples of lens galaxies.

Using GALFIT on the HFF (for MACS J0416) or CLASH (for MACS J1206) F814W band images, we measured the effective radii, R_e , of the three galaxies, finding $1.41 \pm 0.02 \text{ kpc}^1$, $0.77 \pm 0.03 \text{ kpc}$, and $2.13 \pm 0.09 \text{ kpc}$, respectively. They correspond to a ratio between the truncation and the half-light radius $\frac{r_t}{R_e}$ of 4.3 ± 1.6 , 5.13 ± 0.72 , and 2.77 ± 0.47 , all higher than the average value of 2.3 found by G22 for the cluster members of AS1063 (see Chapters 6 and 7), although they fall within the scatter around their mass-to- σ relation. From the total mass profile derived for each of the 100 best-fit models of the three galaxies, we obtained the total projected (two-dimensional) mass enclosed within the effective radius $M(< R_e)$ and its uncertainty. The stellar-to-total mass fraction within the effective radius for the three members is therefore

$$f^*(< R_e) = \frac{M^*/2}{M(< R_e)}. \quad (9.3)$$

We find 0.51 ± 0.21 , 1.0 ± 0.4 , and 0.39 ± 0.16 , for members 8971, 8785, and 3910, respectively.

We compare our values of the stellar-to-total mass fraction as a function of the stellar mass with the analogous relation found for the 85 SLACS lens galaxies presented in Auger et al. (2009) (similar results had previously been obtained by Grillo et al. 2008). We take their stellar mass values measured with a Salpeter stellar IMF and the stellar-to-total mass fraction within the effective radius measured in the rest-frame V -band (comparable with the F814W band for the two clusters considered in this Thesis). As clear from Fig. 9.7, we probe a lower stellar mass range, but find compatible values between the two samples. Our values agree with the results of G22 for the cluster members of AS1063. We also included in Fig. 9.7 the values found, starting from the same hypotheses, by Grillo (2010) for a selected sample of 2×10^5 SDSS early-type galaxies, which we find to be compatible both with this Thesis and with that by Auger et al. (2009). These results suggest that the tidal truncation to which the three lens galaxies are subject to, by virtue of the dense cluster environment in which they reside, only marginally affects their structure within the effective radius. This is in agreement with the conclusions of Grillo & Gobat (2010), Grillo et al. (2014), Parry et al. (2016), and G22, who compared the stellar fraction within the effective radius of cluster and field early-type galaxies.

9.2 Conclusions of Part IV

In this Part, we have presented the measurement with SL of the truncation radius of three cluster galaxies. We considered the reference sample of galaxy clusters included in the analysis by Meneghetti et al. (2020) and selected galaxy-scale SL systems with a

¹Compatible with the value measured by Tortorelli et al. (2023).

clear morphology and several multiple images close to one or a few member galaxies. We chose to focus on members 8971 and 8785 of MACS J0416, and member 3910 of MACS J1206. We built galaxy-scale SL models for the three cluster members and for the neighbouring galaxies which influence the lensing system the most.

We accounted for the lensing effects of the remaining mass components of the cluster according to the predictions of the most recent and accurate SL models of MACS J0416 and MACS J1206, presented in B23 and B19, respectively. To properly consider the uncertainty on the total mass distribution of the two clusters, we sampled the posterior probability distribution of the parameters of the two models, and extracted 100 points, corresponding to 100 realisations of the cluster-scale mass distribution. For each of them, we optimised the models of the mass distribution of the three galaxy-scale lenses. This bootstrapping procedure allowed us to obtain a realistic estimate of the uncertainty on the lens parameters and of the degeneracy between them. We described the three members on which we focus our analysis with spherical truncated profiles and test alternative ellipsoidal non-truncated models. The main conclusions of our analyses are summarised as follows:

1. We measured a truncation radius value of $6.1^{+2.3}_{-1.1}$ kpc, $4.0^{+0.6}_{-0.4}$ kpc, and $5.2^{+1.3}_{-1.1}$ kpc for members 8971, 8785, and 3910, respectively. These values correspond to a total mass of $M = 1.2^{+0.3}_{-0.1} \times 10^{11} M_{\odot}$, $M = 1.0^{+0.2}_{-0.1} \times 10^{10} M_{\odot}$, and $M = 6.3^{+1.0}_{-1.1} \times 10^{10} M_{\odot}$, respectively, and to velocity dispersion values of $164.9^{+6.8}_{-7.5} \text{ km s}^{-1}$, $57.6^{+0.3}_{-0.3} \text{ km s}^{-1}$, and $129.2^{+5.1}_{-3.6} \text{ km s}^{-1}$, respectively.
2. The values of r_t are well constrained, with a low statistical uncertainty. We compare our results with those of Suyu & Halkola (2010) for the satellite galaxy of the lensing system SL2S J08544–0121, finding very similar r_t values for galaxies in the same total mass range.
3. In the case of member 8971, the SIE model leads to a lower value of Δ_{rms} , but the comparison between the SL-derived σ and the measured LOSVD value of $178.0 \pm 2.4 \text{ km s}^{-1}$ strongly favours the SIsT model.
4. In the other two instances, SIE models do not lead to an improved accuracy of the description of the SL observations, in spite of their more complex azimuthal structure and a higher number of free parameters. In the case of member 3910, the parameters of the non-truncated models are not well constrained and show clear degeneracies.
5. Our inferred values of σ and r_t for the three cluster galaxies differ significantly from the results of B23 and B19, especially in the case of members 8785 and 3910, where they were derived with power-law scaling relations with respect to the galaxy total luminosity.
6. We compare our results with the total-mass-to- σ relations for MACS J0416, MACS J1206, AS1063, and A2744 from B23, B19, and Bergamini et al. (2023a), obtained with the power-law approach. We find that the scaling relations cannot consistently describe all three members studied in this Thesis. Our results instead agree with the mass-to- σ relation derived by G22 for AS1063, based on the FP relation and showing a larger scatter.
7. We juxtapose the estimated compactness of the three lens galaxies with the predictions of the hydrodynamical cosmological simulation suites presented in Ragagnin

et al. (2022), which differ in feedback and softening scheme, and mass resolution. Members 8971 and 3910 fall in the total mass range included in the analyses performed on the simulations. Their measured compactness is higher than what is found for simulated sub-haloes of the same total mass, independently of the simulation set-up considered, confirming the discrepancy between observations and simulations first reported in Meneghetti et al. (2020).

8. We measured the stellar mass and the effective radius, and the stellar-to-total mass fraction within the effective radius for the three cluster galaxies. For the latter parameter, we find 0.51 ± 0.21 , 1.0 ± 0.4 , and 0.39 ± 0.16 for members 8971, 8785, and 3910, respectively. Our values span the same range as those observed for the members of AS1063 by G22, for the 85 SLACS lens galaxies presented in Auger et al. (2009), and for a selected sample of early-type SDSS galaxies by Grillo (2010), suggesting that the tidal truncation of cluster galaxies does not significantly affect their structure within the effective radius.

As clear from Fig. 9.7, this Thesis significantly extends the mass range probed by current SL studies of early-type galaxies, towards the regimes that will be systematically explored by the upcoming lens surveys with the *Euclid* and the *Rubin* telescopes. On-going integral-field observations with the *James Webb* Space Telescope Near Infrared Spectrograph will provide us with spatially resolved kinematic constraints of galaxy-scale lenses in clusters, allowing for a more detailed reconstruction of the mass structure of member galaxies.

Part V

**Towards the calibration of the
velocity dispersion function of
quiescent cluster galaxies**

In this Part, I report some preliminary results on my work currently in progress, which aims to exploit the combination of galaxy- and cluster-scale SL with stellar kinematics on samples of cluster members larger than that those previously analysed.

The Part consists of Chapter 10, where we present the preliminary procedures required to calibrate the velocity-dispersion-function of the member galaxies of a sample of HFF galaxy clusters. We describe the pipeline adopted to extract the cluster member spectra from VLT/MUSE data and to select the reliable velocity dispersion measurements, and we propose a method to complete the function with the FP relation.

First steps towards the velocity dispersion function of quiescent galaxies in lensing clusters

Throughout the previous Chapters, we have pointed out on several occasions how the stellar LOSVD is a direct tracer of the total mass distribution of early-type galaxies, which suffers from fewer model dependencies and photometric biases when compared to other probes such as their luminosity or their stellar mass. In particular, the central velocity dispersion is an effective probe of the central gravitational potential well of early-type galaxies (hereafter ETGs), providing us with a connection between galaxy formation models and the observed galaxy population. Statistical study of the velocity dispersion function of cluster members offers an important test bed of the evolutionary processes affecting galaxies in dense environments, by comparison with the velocity function of field galaxies. Furthermore, pushing the velocity function down to low values of σ for a uniform sample of galaxies at a fixed redshift and environment reduce observational biases that may affect less deep surveys.

Furthermore, combining the mass and velocity dispersion functions of SL clusters offers a chance to further compare the compactness of observed and simulated cluster galaxies. As detailed in Chapter 7, SL modelling alone is sensitive to the total mass of the cluster members, but struggles to recover their compactness due to parametric degeneracy. As we have shown, providing SL models with more accurate priors on the velocity dispersion of the members, and its relation with their remaining observed properties, provides us with insights on their mass structure and compactness, which is a further probe of galaxy formation and of the cosmological paradigms. As we previously pointed out, the mass-to-velocity function of simulated cluster sub-haloes differs from our findings in SL models, but only in three of the clusters studied in Meneghetti et al. (2020) priors on the kinematics of members were used. Combining the observed velocity function of the HFF galaxy clusters with their SL models will allow us to further probe the compactness discrepancy more thoroughly and for a larger sample of clusters.

10.1 Introductory remarks on the velocity function of quiescent galaxies

Accurate measurements of the velocity dispersion function of ETGs were made possible by the large spectroscopic surveys of the last two decades, such as SDSS and the Baryon Oscillation Spectroscopic Survey (BOSS) (see Sheth et al. 2003; Choi et al. 2007; Chae 2010; Bezanson et al. 2011; Montero-Dorta et al. 2017; Sohn et al. 2017a), or fields such as NMBS COSMOS and UKIDSS UDS (Bezanson et al. 2012). These studies have also allowed the effects of galaxy morphology, environment, and redshift on the galaxy velocity dispersion function to be investigated (see Fig 10.1). Following Sheth et al. (2003),

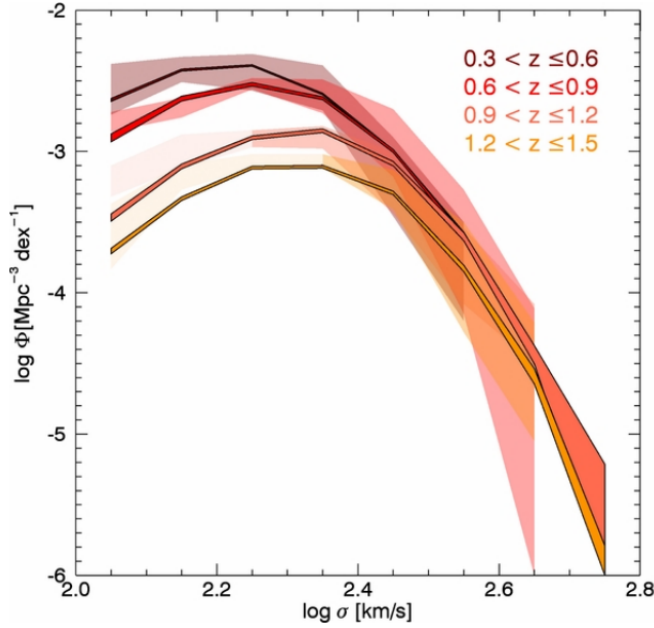


Figure 10.1: Velocity dispersion function across cosmic time. Evolution with redshift of the σ function of 5500 quiescent galaxies from the Newfirm Medium Band Survey COSMOS and UKIDSS UDS fields (from Bezanson et al. 2012).

the velocity dispersion function of quiescent galaxies is typically fit with a modified Schechter function,

$$\phi(\sigma)d\sigma = \phi^* \left(\frac{\sigma}{\sigma^*}\right)^\alpha \frac{\exp\left(-\left(\frac{\sigma}{\sigma^*}\right)^\beta\right)}{\Gamma(\alpha/\beta)} \beta \frac{d\sigma}{\sigma}, \quad (10.1)$$

where ϕ^* is the number density of galaxies, σ^* is a scale velocity dispersion, and α and β dictate the shape of the function for high- and low- σ values, respectively. Equation (10.1) descends from the luminosity function being well fit by a Schechter function Schechter (1976) and by the luminosity and the velocity dispersion of early-type galaxies being linked by a power law (Faber-Jackson relation, see Chapter 1). The parameters of the σ function of quiescent field galaxies in the local Universe show a significant variability between different works, probably due to the inconsistent definitions of early-type galaxies included in the sample and to its incompleteness. All of them have a positive value of α , which describes a drop of the σ function for $\log(\sigma(\text{km s}^{-1})) < 2.35$ (see Fig. 10.1). Bezanson et al. (2012) find that the high- σ end of the velocity dispersion function of quiescent galaxies does not significantly evolve with redshift, indicating that these object form at high redshift, while the low- and intermediate- σ regimes see a growth of the σ function, as a consequence of star formation shutting off in those objects (see Fig. 10.1).

From SDSS and MMT Hectospec spectroscopic data, Sohn et al. (2017a) measure the σ function for the quiescent galaxies of the nearby clusters Coma and Abell 209. As evidenced by Fig. 10.2, the velocity function of quiescent galaxies in the two clusters differs from that of field galaxies. Firstly, it appears flat for $\log(\sigma(\text{km s}^{-1})) < 2.35$. However,

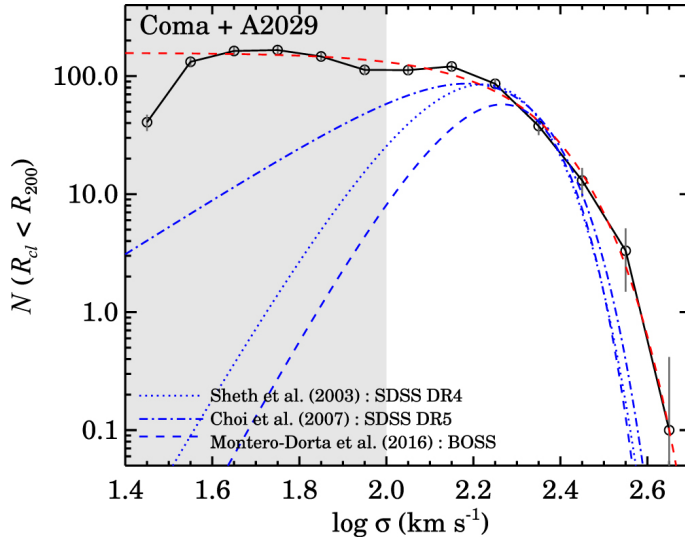


Figure 10.2: Velocity dispersion function in the field and in clusters. Comparison between the velocity function of quiescent galaxies in the field and in the Coma and Abell 209 clusters (from Sohn et al. 2017a).

this discrepancy can be reconciled completing the sample used to calibrate the σ function in the field (Sohn et al. 2017b). A second difference is an excess of the σ function of cluster galaxies for $\log(\sigma(\text{km s}^{-1})) > 2.4$. This can be seen as a consequence of the presence of BCGs, extremely massive galaxies with a high velocity dispersion found within the core of the cluster. The same surveys were then used to replicate the σ function calibration for nine SL clusters at $0.18 < z < 0.29$ in Sohn et al. (2020). They confirm the results of Sohn et al. (2017a), finding an analogous excess of cluster galaxies with $\log(\sigma(\text{km s}^{-1})) > 2.4$ with respect to the population of quiescent field galaxies and results compatible with Coma and Abell 209 down to the completeness limit of $\log(\sigma(\text{km s}^{-1})) \approx 2.05$, and find that more massive clusters generally have a smaller fraction of quiescent members with large σ compared to less massive clusters. These missing galaxies have presumably undergone dry mergers with the BCG.

10.2 Building the velocity dispersion function for the quiescent members of the HFF clusters

In this Section, we present the preliminary steps performed to calibrate the velocity dispersion function for the quiescent members of the five HFF clusters with deep MUSE data. During the development of the pipeline, we focused on the four clusters in the sample for which we already have measured the multi-band structural parameters of the members, i.e. A2744 ($z = 0.307$), AS1063 ($z = 0.348$), MACS J0416 ($z = 0.396$), and MACS J1149.5+2223 (hereafter MACS J1149, at $z = 0.544$). For now, we exclude Abell 370, which we will include in the final work. Several MUSE pointings of the cores of the four clusters have been reduced and combined to obtain the observations on which the velocity dispersion measurements have been performed. The core of A2744 was observed with five MUSE pointings with exposure times in the range 2 – 5 h (Mahler et al. 2018; Richard et al. 2021; Bergamini et al. 2023a); the NE and SW regions of the core of

MACS J0416 were observed by MUSE for 17.1 h (Vanzella et al. 2021) and 11 h (Caminha et al. 2017a), respectively (see also Chapter 8); the core of MACS J1149.5+2223, hereafter MACS J1149 was observed with a 4.8 h MUSE pointing (Grillo et al. 2016); finally, we used the two MUSE pointings of AS1063 introduced in Chapter 6, which have exposure times of 3.1 h and 4.8 h (Karman et al. 2015, 2017). We follow the criteria presented in Bergamini et al. (2023a,b), Caminha et al. (2016), and Grillo et al. (2016) to select 258, 158, 128, and 94 spectroscopic cluster members for A2744, AS1063, MACS J0416, and MACS J1149, respectively.

10.2.1 Measuring the velocity dispersion of the cluster members

In Chapter 5, we described the pipeline, based on `pPXF`, that we used to fit the LOSVD of the MUSE spectra of cluster galaxies. We now detail the procedure to extract these spectra from the MUSE reduced data cubes. At the redshift of the HFF clusters ($z \approx 0.3 - 0.55$), the extent of the FoV of a MUSE pointing ($1'$) corresponds to a physical size of approximately 270 – 380 kpc. Thus, a single or a few pointings are sufficient to cover most of the cluster core, measuring the spectrum of up to a few hundreds of objects (see Mercurio et al. 2021; Caminha et al. 2017a).

Integral-field spectroscopy gives us the advantage of not being forced to extract spectra from a fixed aperture and provides us with resolved spectral information with a spaxel of $0.2''$, albeit in practice the resolution is limited by the PSF, which is typically in the range $0.6'' - 1.0''$. If we wish to measure σ_{LOS} for a sample of galaxies, we need to make sure that the spectrum is extracted within a comparable aperture for all of them, and the natural scale to compare galaxies is their half-light radius R_e . For instance, as we saw in Chapter 2, one of the variables of the FP is σ_0 , i.e. the value of σ_{LOS} measured within a circular aperture of $R_e/8$. However, cluster members have different sizes, and R_e , for most member galaxies, is smaller than the PSF of the observations. We tackle this by extracting the spectra from larger apertures (with a radius of $1.5''$) and by using the light of the cluster members to weigh the spectrum of each spaxel (Mercurio et al. 2021; Granata et al. 2022). To perform this procedure, we use the light profile of the member galaxies in the band F814W, which mostly overlaps with the wavelength range used in the spectral fitting. The light profile can either be obtained from a fit of the structural parameters, as done in Chapter 6 (Granata et al. 2022), when they are available, or by simply taking the surface brightness in the F814W band and degrading it to the PSF of the MUSE observations. In this Chapter, we adopted the latter procedure as it is easier to generalise.

As we show in Fig. 10.3, the light-weighting procedure leads to a 10 – 20% increase in the S/N values compared to those found without weighing the MUSE cubes and extracting the spectra within apertures with a radius of $0.8''$, as done by Bergamini et al. (2019). As found by Bergamini et al. (2019) and confirmed by our tests presented in Appendix A, LOSVD fits are generally unreliable when the spectral S/N is smaller than 10: this procedure thus allows us to increase the sample of cluster galaxies with a sufficient S/N for our studies.

We measured the velocity dispersion for all the spectra with a $S/N > 10$. The large aperture from which the weighted spectra are extracted ($1.5''$), however, exposes us to risks of contamination. We visually inspected all instances of observed objects within a distance of $3''$ from the centre-of-light of the cluster members and excluded those showing the effects of light contamination. We also removed all the `pPXF` fits with anomalous residuals from our samples. This left us with 77, 57, 51, and 34 cluster members in A2744, AS1063, MACS J0416, and MACS J1149, respectively, for which we obtained a reliable

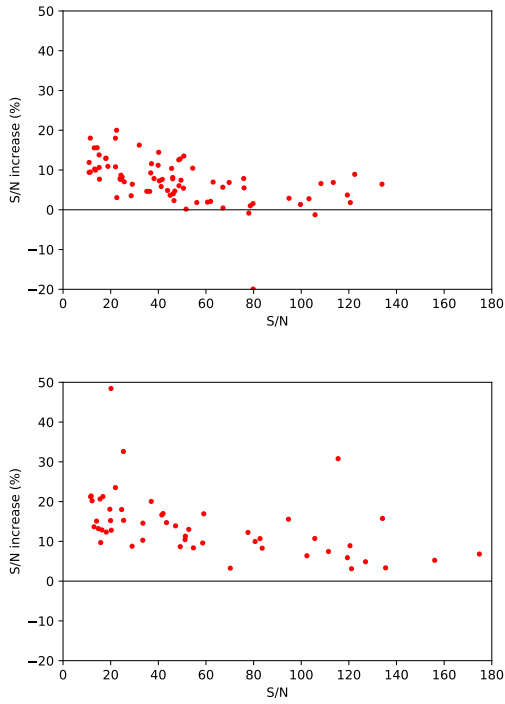


Figure 10.3: Increase of the S/N . Percentage variation of the spectral S/N between the spectra extracted within a fixed aperture with a $R = 0.8''$ radius and with the weighted spectral extraction procedure described in this Chapter. We show here the variation for the cluster members of A2744 (top panel) and of MACS J0416 (bottom panel) for which we are able to measure the velocity dispersion.

measurement of the value of σ . We corrected the velocity dispersion value and its error for the effects of spectral sampling and noise using the relations presented in Eqs. (5.13) and (5.14), and derived in Appendix A.

Once this sample of 219 members is completed with the addition of the members of Abell 370, it will push the limits of what we can achieve with the current IFU spectroscopic observation of HFF cluster cores. However, we still need to account for the regions of the cluster cores not covered by MUSE pointings and for the incompleteness of the measurements at lower σ values.

10.2.2 Completing the measured velocity dispersion function with the Fundamental Plane

Early-type galaxies lie on the FP, introduced in Chapter 1 (see Eq. 1.33), which links their central velocity dispersion $\sigma_0 = \sigma_{ap}(R_e/8)$ to their structural parameters R_e and SB_e . Thanks to the deep HFF photometry, we have measured the structural parameters in all available bands for the member galaxies of the four clusters that fall within the footprints of the MUSE pointings using the code `MORPHOFIT`¹ (Tortorelli & Mercurio 2023). The structural parameter catalogues of MACS J0416, MACS J1149, and AS1063 were presented in Tortorelli et al. (2023). For each cluster in our sample, we selected a pure sample of early-type galaxies for which we have a reliable σ to calibrate the FP. We then adopted the calibrated relation to estimate the central velocity dispersion of all quiescent member galaxies within the MUSE pointings from their measured structural parameters.

As the FP relation applies to early-type galaxies (see Chapter 1), we need to identify them within our sample. We aim at defining a procedure that can be applied to galaxies for which only morphological and colour information are available. Taking advantage the availability of multi-colour photometry, a first criterion to identify the population of early-type cluster members is by their red colour: using the fitting code `Ltsfit`, we calibrated a colour-magnitude (CM) relation for all the cluster members, defined as

$$m_{606} - m_{814} = a + b m_{814}, \quad (10.2)$$

where m_{606} and m_{814} are the `Galfit` total magnitudes in the HFF F606W and F814W bands, respectively. Following Tortorelli et al. (2018), we defined red galaxies as redder than the best-fit CM relation minus 1σ . We made an exception for A2744, where we included galaxies redder than the best-fit CM relation minus 2σ , as the scatter provided by the fitting procedure is significantly smaller than all other clusters and appears potentially underestimated. The best-fit CM relations and the red cluster members are shown in Fig. 10.4. The values of the slope b of the CM relations of all clusters are compatible.

As demonstrated by Tortorelli et al. (2018), who calibrated the Kormendy Relation (KR) of MACS J1149 and AS1063, different selection criteria adopted to identify the sample of elliptical cluster galaxies can influence the values of the KR parameters. The same may be true for the FP, of which the KR is a projection. Tortorelli et al. (2018) suggest that identifying ellipticals by imposing a Sérsic index $n > 2.5$ is a more restrictive criterion than selecting them based on their colour. While the first criterion occasionally leads to spectroscopically passive galaxies being excluded, the second may lead to the inclusion of a few late-type galaxies. Our sample confirms, as evidenced in Fig. 1.2, that several red galaxies have a Sérsic index $n < 2.5$. As in Chapter 6 and in Granata et al. (2022), we chose to calibrate the FP using the purest possible sample of early-type

¹<https://github.com/torluca/morphofit>

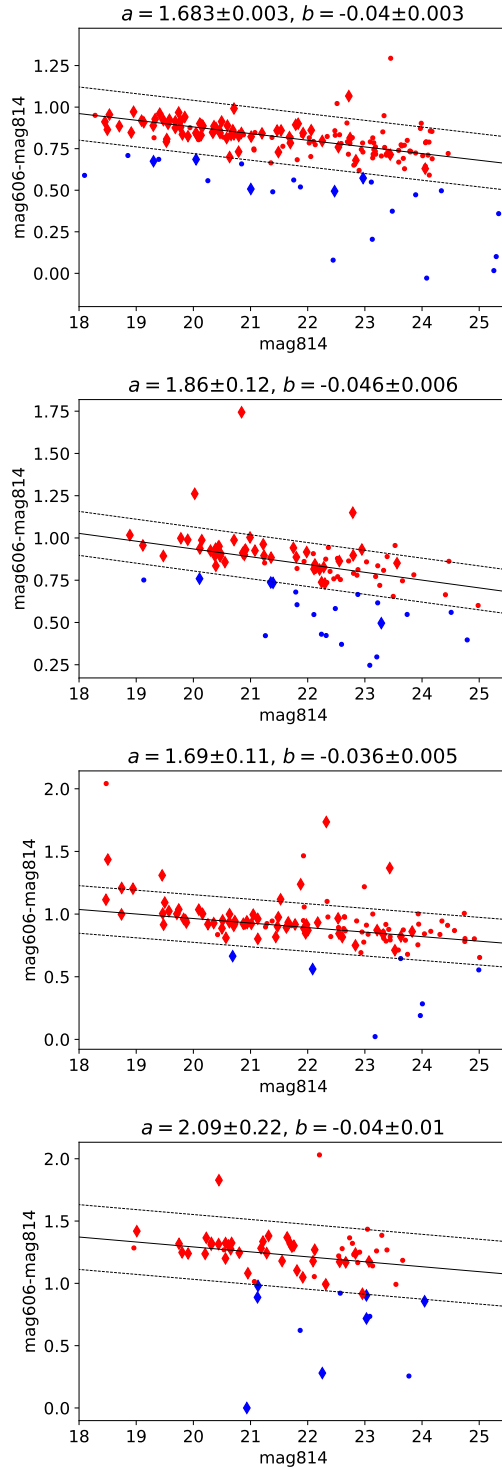


Figure 10.4: CM relations for the four clusters included in our sample. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. Red and blue points are used to distinguish red cluster members. Diamond markers identify members with $n > 2.5$. Solid and dashed lines indicate the best-fit relation and a 1σ scatter (2σ for A2744). The best-fit values of the parameters a and b are reported above the plots.

galaxies, and thus only considered the red galaxies with $n > 2.5$. On the other hand, for now, we applied our procedure to all red galaxies to achieve the highest possible sample completeness, and postponed a discussion of its purity to a later stage of this work. This selection leaves us with 49, 34, 25, and 25 members to be used to calibrate the FP for A2744, AS1063, MACS J0416, and MACS J1149, respectively. Fig. 10.5 shows the CM relation for those members.

We calibrated the FP using the representation presented in Eq. (1.34) and adopted in Chapter 6, with R_e and σ_0 measured in kpc and km s^{-1} , respectively. The calibration required us to connect the measured values of σ from the weighted MUSE cubes to the corresponding value of σ_0 . Jorgensen et al. (1995) found that a relation between the values of σ_{ap} for different apertures can be obtained as $\sigma_{ap}(R) = \sigma_{ap}(R') (R/R')^{-0.04}$. More recently, Zhu et al. (2023) showed that this approach is too simple, and that the slope value α of the power-law correction $\sigma_{ap}(R) = \sigma_{ap}(R') (R/R')^\alpha$ depends on the r -band absolute magnitude, the $g - i$ colour, and Sérsic index.

Bright red galaxies with high n , as expected for massive elliptical galaxies, show lower values of α , compatible with Jorgensen et al. (1995), but α grows to positive values for bluer, fainter galaxies, with a lower n value. The galaxies that we use to calibrate the FP have been selected for colour and Sérsic index, but we extended the velocity dispersion measurement to fainter galaxies, which have a higher value of α according to Zhu et al. (2023). At the redshift of our cluster sample, we considered the F105W, F814W, and F606W HFF bands as good approximations of the i , r , and g rest-frame bands, respectively, and assign α to all the selected cluster members. We could thus take the measured σ values, obtain from the weighted MUSE spectra, and use the correction by Zhu et al. (2023) to measure σ_0 . As we show in Fig. 10.6, we find the measured value of σ obtained from the weighted spectrum is, on average, approximately equivalent to the LOS velocity dispersion measured within an aperture equal to the effective radius R_e ($\sigma_{ap}(R_e)$), where $\sigma_{ap}(R)$ is defined in Eq. (1.23). We measured $\sigma_{ap}(R_e)$ using the velocity dispersion measured by extracting the non-weighted MUSE spectra within an aperture with a radius $R_{ap} = 0.8''$ (as in Bergamini et al. 2019) and applying the correction by Zhu et al. (2023) $\sigma_{ap}(R_e) = \sigma_{ap}(0.8'') (R_e/0.8'')^\alpha$.

In summary, the spectral weighing procedure allows us to probe a uniform physical relative size for all cluster galaxies without changing the aperture, while increasing the spectral S/N at the same time. We can then measure the central velocity dispersion σ_0 assuming that the velocity dispersion σ measured from the weighted spectra is, in good approximation, equivalent to $\sigma_{ap}(R_e)$, which can then be corrected to measure σ_0 with the relation

$$\sigma_0 = \sigma_{ap}(R_e) \left(\frac{R_e/8}{R_e} \right)^\alpha = \sigma \left(\frac{1}{8} \right)^\alpha. \quad (10.3)$$

Unlike in Chapter 6, we calibrated the FP for a sample of clusters at a different redshift. In order to compare the FP parameters for them, we measured SB_e using the absolute magnitude of the members, as

$$\text{SB}_e = -0.4\mu_e - 4.57 - 5 \log(D_L), \quad (10.4)$$

where 4.57 is the Sun magnitude in the F814W band, D_L is the cluster luminosity distance, and μ_e is

$$\mu_e = m_{814} + 5 \log(R_e) + 2.5 \log(2\pi) - 10 \log(1+z), \quad (10.5)$$

where R_e is measured in arcseconds. The resulting SB_e value is thus measured in absolute magnitudes per square arcsecond and includes a correction for cosmological dimming.

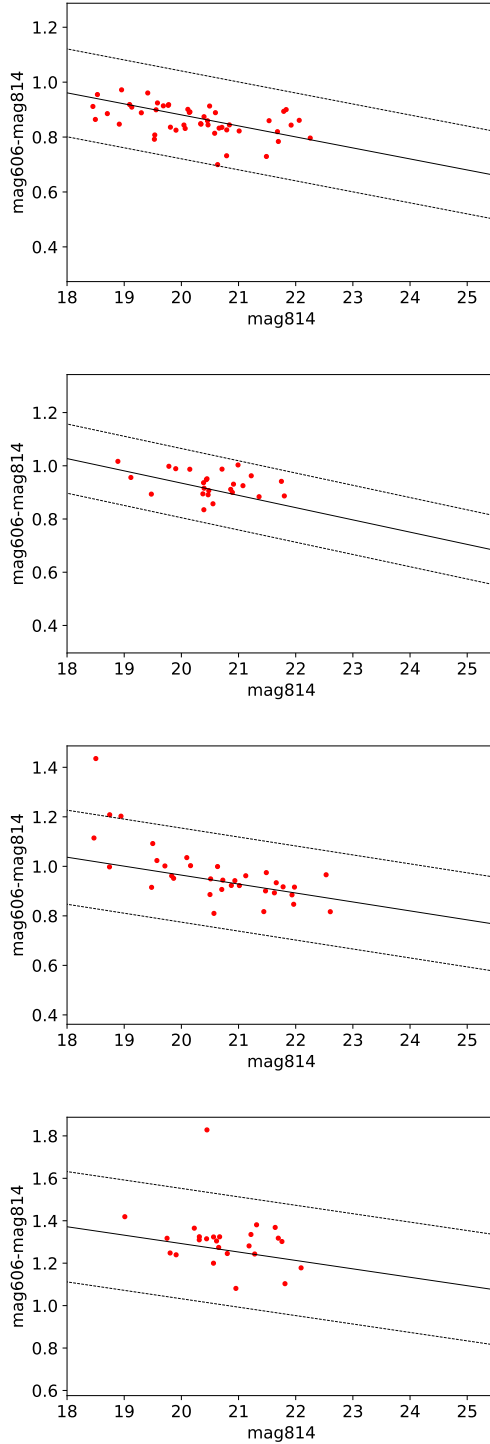


Figure 10.5: Cluster members on which we base the calibration of the FP plotted around the CM relations for the four clusters included in our sample. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. Solid and dashed lines indicate the best-fit relation and a 1σ scatter (2σ for A2744).

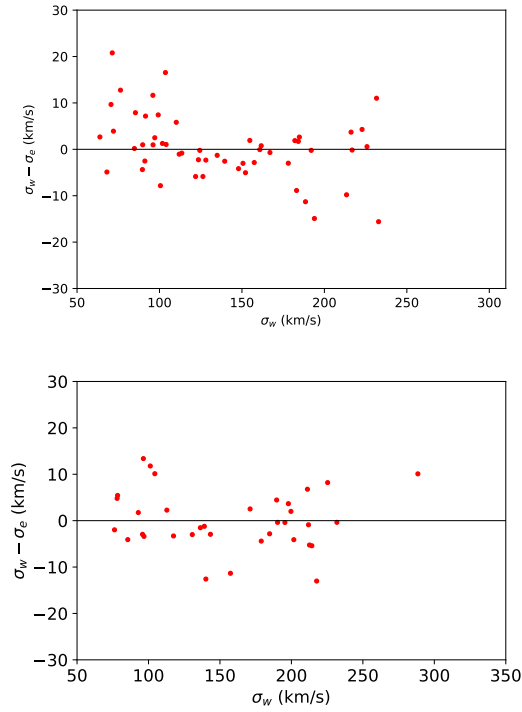


Figure 10.6: Comparison between the values of the velocity dispersion σ measured using weighted MUSE spectra and $\sigma_{ap}(R_e) = \sigma_e$. We show here the comparison for the cluster members of A2744 (top panel) and of MACS J0416 (bottom panel) for which we are able to measure the velocity dispersion.

As in Chapter 6, we calibrated the plane using `Ltsfit`. In Figs. 10.7 and 10.8, we present two different projections of the calibrated FP relations for all four clusters, and the member used for its calibration. Figure 10.8 is especially useful to evaluate the accuracy of the FP in predicting the value of σ_0 from the member structural parameters. The three FP parameters α , β , and γ are defined as in Eq. (1.34).

As clear from Fig. 10.8, the samples of measured σ_0 values adopted to calibrate the FP have significantly different lower limits, with those included in A2744 pushing below $50 \text{ km s}^{-1} \approx \log \sigma_0 (\text{km s}^{-1}) = 1.7$, while the samples for MACS J1149 and AS1063 do not push below 100 km s^{-1} , due to the shallower MUSE observations and to the higher redshift of the former. In the cases of A2744 and MACS J0416, we can verify whether a FP relation calibrated only with galaxies with a velocity dispersion value $\sigma_0 > 100 \text{ km s}^{-1}$ can be extrapolated to predict σ_0 values below this threshold. In Fig. 10.9, we show that this is the case: the measured values of $\sigma_0 < 100 \text{ km s}^{-1}$ still fall close to the FP relations calibrated for the two clusters using exclusively members with $\sigma_0 > 100 \text{ km s}^{-1}$. Similarly, in Fig. 10.10 we show, for the same clusters, that red galaxies with $n < 2.5$ also appear to follow the FP closely. This suggests that the calibrated FP relations can be used to predict, in good approximation, the σ_0 values for all the red member galaxies down to 50 km s^{-1} .

Finally, we used the FP relations calibrated for the four clusters to measure the value of σ_0 for all the red cluster members falling within the MUSE pointings analysed for which we have measured the structural parameters (Tortorelli et al. 2023). As discussed, we limited our studies to predicted σ_0 values larger than 50 km s^{-1} . We present the resulting velocity dispersion functions in Fig. 10.11. The results presented in this Thesis are still preliminary, and further work will be made to test their reliability and to extend the sample we include. We provide some details on how we intend to develop this study in the final Outlook Chapter and we postpone the discussion of the Astrophysical implications of our work to a paper focused on its results.

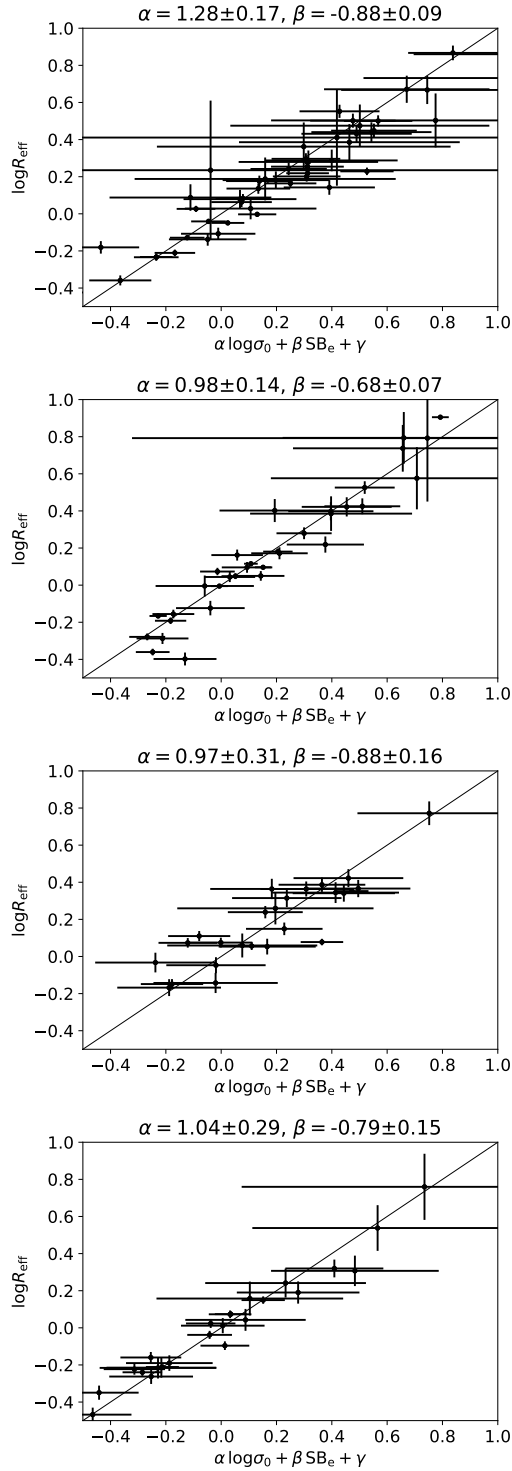


Figure 10.7: FP relations for the four clusters included in our sample in the projection $(\log R_e, \alpha \log \sigma_0 + \beta \text{SB}_e + \gamma)$. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. The solid lines indicate the best-fit FP relation, the black dot the observed members used for the plane calibration and the observational uncertainty. We report the values of the α and β parameters.

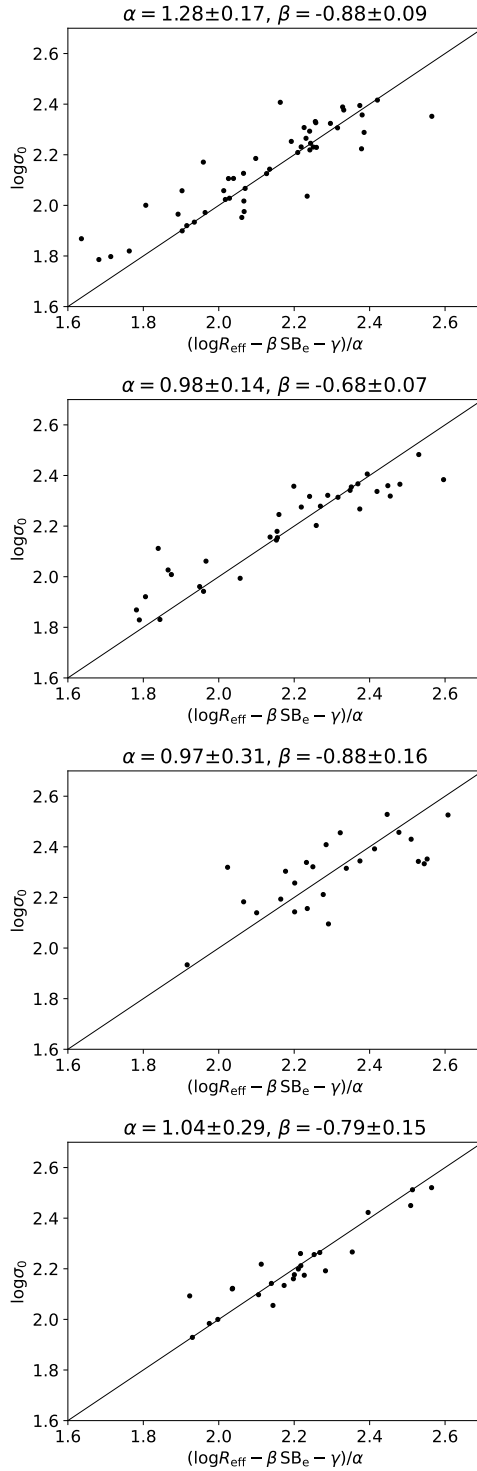


Figure 10.8: FP relations for the four clusters included in our sample in the projection $(\log \sigma_0, (\log R_e - \beta \text{SB}_e - \gamma)/\alpha)$. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. The solid lines indicate the best-fit FP relation, the black dot the observed members used for the plane calibration. We report the values of the α and β parameters.

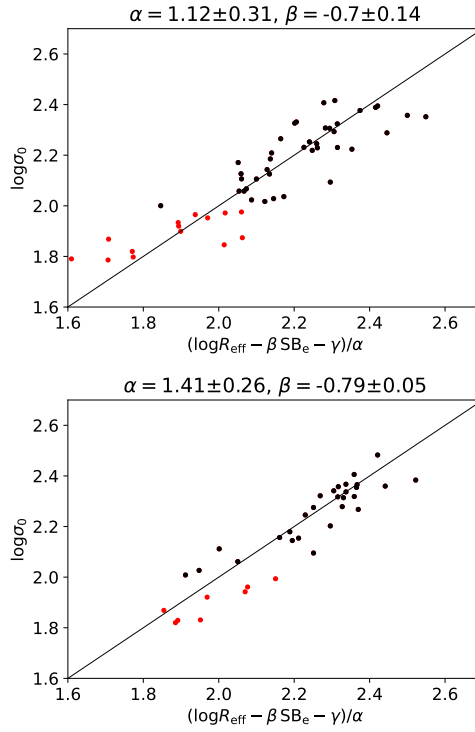


Figure 10.9: Distribution of the members with $\sigma_0 < 100 \text{ km s}^{-1}$ around the FP. The relations for the clusters A2744 (top panel), and MACS J0416 (bottom panel) have been calibrated including only the selected members with $\sigma_0 > 100 \text{ km s}^{-1}$. The solid lines indicate the best-fit FP relation, the black dot the members used for the plane calibration, the red dot those that were excluded as their $\sigma_0 < 100 \text{ km s}^{-1}$. We report the values of the α and β parameters.

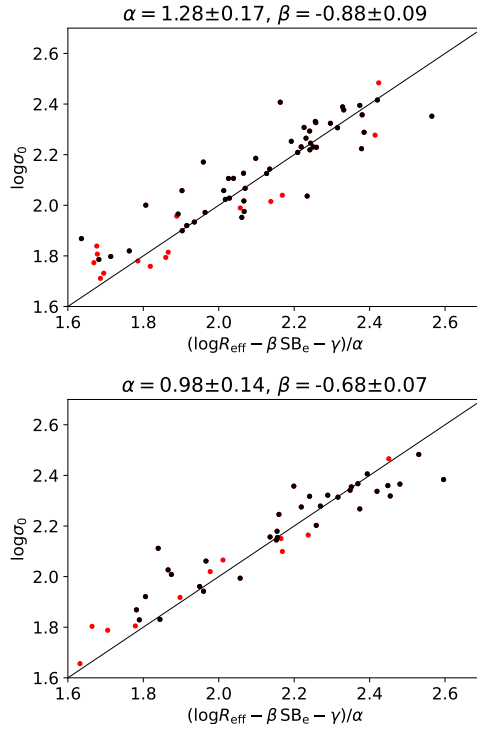


Figure 10.10: Distribution of all the red members of the clusters A2744 (top panel), and MACS J0416 (bottom panel) around the FP relation. The solid lines indicate the best-fit FP relation, the black dot the members used for the plane calibration, the red dot those that were excluded as their $n < 2.5$. We report the values of the α and β parameters.

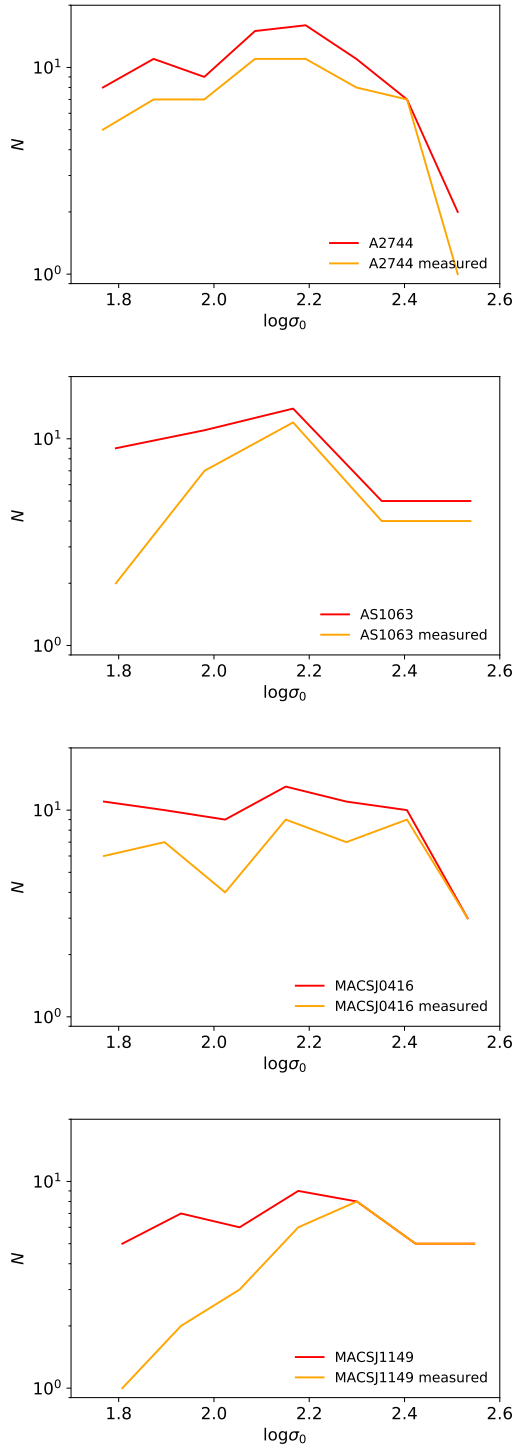


Figure 10.11: Velocity dispersion function of the four clusters included in our sample. From top to bottom: A2744, AS1063, MACS J0416, and MACS J1149. The red lines indicate the σ_0 function for all the red cluster members included in our morphological analysis. The orange lines indicate the σ_0 function for all the red cluster members for which we have a reliable velocity dispersion measurement.

Summary, conclusions and outlook

Summary of the main results and conclusions

In this Sub-section, we briefly summarise the main results and conclusions of this Thesis. At first we recap the findings of the studies on which the Thesis is based, and then we look more broadly at the general picture of our contribution to the state of the art of the field, and point out the unresolved issues evidenced by our work.

In this Thesis, we performed a deep, multi-wavelength study of the cores of SL galaxy clusters, with a special focus on the morphology, kinematics, and mass structure of their cluster members. The main findings of our study are summarised as follows.

Improved strong lensing modelling of galaxy clusters using the Fundamental Plane: in Chapters 6 and 7, we presented the first SL model of a galaxy clusters (AS1063) in which VLT/MUSE data allowed for the determination of the velocity dispersion of the cluster members through a newly-calibrated FP. With this novel technique, we could measure the velocity dispersion of the members with more realistic and complex scaling laws than previous works, breaking the degeneracy with the truncation radius and constraining the structure and mass function of the cluster sub-haloes down to a total mass $M \approx 10^{10} M_{\odot}$ with unprecedented accuracy. Compared to earlier work on the same cluster, our model allowed for the inclusion of some scatter on the relation between the total mass and the velocity dispersion of the cluster members, and for lower statistical uncertainty on the value of some model parameters. Taking advantage of a new estimate of the stellar mass of all cluster members from the HST multi-band data, we measured the cumulative two-dimensional mass profiles out to a radius of 350 kpc for all baryonic and DM components of the cluster.

Investigating the discrepancy in sub-halo compactness between observed and simulated galaxy clusters: we took advantage of the improved accuracy in the description of the mass structure of the cluster members allowed by the new SL modelling techniques for a comparison between the observed and simulated cluster galaxies. In Chapter 7, we presented a parallel between the compactness of the members of AS1063, as predicted by our new SL model, and of sub-haloes extracted from a suite of re-simulations of the Dianoga hydrodynamical set. Specifically, we probed the impact of the simulation set-up on the sub-halo compactness by testing a higher mass particle resolution and a different implementation of the sub-grid feedback mechanisms. On one hand, the parametrisation of the structure of the cluster galaxies based on the FP allows for a more accurate description of their properties, but does not significantly alleviate the compactness offset. On the other hand, we also found that

none of the proposed alternative simulation set-ups is enough to fully reconcile their discrepancy with observations.

Insights into the mass structure of cluster galaxies with galaxy-scale strong lensing: in Chapters 8 and 9, we took advantage of galaxy-scale SL events within clusters to probe the mass structure of several cluster galaxies without the adoption of any scaling relation, thus avoiding any bias they might introduce. We built galaxy-scale SL models for three lens galaxies within the massive clusters MACS J0416 and MACS J1206. We measured the values of the truncation radius and of the stellar-to-total mass fraction within the effective radius for the three members. We independently confirmed that the FP relation provides a better description of the compactness of the cluster sub-haloes, while power-law scaling relations cannot accurately describe their compactness over the full total mass range considered. Finally, we reported good agreement between our predicted values of the stellar-to-total mass fraction within the effective radius and those of early-type galaxies from the Sloan Lens ACS Survey. Our work significantly extends the regimes of the current samples of lens galaxies towards the mass range that will be probed by the Euclid, Rubin, and James Webb Telescopes.

Towards the calibration of the velocity dispersion function of the quiescent members of the Hubble Frontier Fields clusters: with the objective of a comprehensive study of the velocity dispersion function of the early-type members of the five HFF clusters for which deep MUSE data is available, we measured the LOS central stellar velocity dispersion for more than 200 members of four clusters, and we proposed some methods to complete the function based on the FP. A statistical study of the velocity dispersion function of the cluster members has the potential to yield competitive insights on their structure and compactness.

Impact of the results of this Thesis

We now weigh on the impact of the methods and results of this Thesis on the state-of-the-art research on SL galaxy clusters, and on their importance for the broader field. The techniques and findings presented in this Thesis have significantly contributed to several works developed within and outside the research group in which they have been developed. Our two key messages are summarised below.

Enhancing the SL modelling of galaxy clusters with the observed stellar kinematics of their member galaxies: as discussed, the novel parametrisation of the mass structure of the cluster galaxies presented in Chapters 6 and 7, based on the FP, has led to a significant improvement in the accuracy of their description. The FP is a more complex and tighter scaling law than the Faber-Jackson law, and allowed us to break the $\sigma - r_t$ degeneracy, allowing us a meaningful investigation of their compactness. This was confirmed by modelling three GGSL systems within clusters (see Chapters 8 and 9). In light of our results, the FP has been adopted as a basis for several SL models, even developed outside our group (Beauchesne et al. 2024). Furthermore, the light-weighted extraction of the spectra presented in Chapter 10 provides a simple tool to increase the size of the sample of cluster members for which we can measure a value of the stellar velocity dispersion, and a natural way to estimate its average value within the effective radius. In light of its effectiveness and simplicity, this method is currently being adopted in several cluster-scale SL studies, now in preparation, to obtain kinematic priors on the mass structure of cluster galaxies.

Confirming the discrepancy between the observed and simulated compactness of cluster galaxies: in this Thesis, we presented some tests of the reported discrepancy between the compactness of observed and simulated sub-haloes. In Chapter 7, we found that our improvement in the description of the sub-halo component of galaxy clusters does not reconcile the compactness problem, and that cluster galaxies have a very compact mass distribution profile, with a value of r_t of only a few times larger than their effective radius. Following our study, further works to investigate the mass distribution of cluster galaxies with GGSF systems have been performed (Galan et al. 2024), confirming their extremely compact structure. In conclusion, the issue on the predicted mass structure of cluster sub-halo persists and will require further tests and improvements to both SL models and simulations of galaxy clusters, and possibly points to potential shortcomings in our baryon modelling, or even potential limitations of the Λ CDM framework. In addition to addressing the compactness discrepancy, the work presented in Chapters 6 and 9 significantly extends the study of early-type galaxies to fainter masses and lower magnitudes, finding that cluster galaxies have a stellar-to-total mass fraction within the effective radius when compared to their analogues in the field, suggesting that the tidal truncation they undergo does not significantly affect their inner structure (i.e. within R_e).

The significance of our studies can be strengthened by an increase in the size of the sample of systems on which they are performed, so far limited to a few HFF clusters. A morpho-kinematic study of a larger sample of galaxies within several clusters would allow us to better characterise the influence of some assumptions required for our work on its final results. For instance, it would allow us to investigate the impact of the approximated description of the stellar kinematics with the Jeans isotropic equation in spherical symmetry, or to probe whether our findings on the structure of cluster galaxies might significantly depend on the larger-scale properties of the cluster – e.g. its relaxation state – or might show gradients within the different regions of the same cluster. To achieve this objective, as we suggest in the next sub-section, we will significantly extend the sample of cluster members included in our study presented in Chapter 10 towards regions of the cluster that are, in projection, more distant from its core, and to higher-redshift lensing clusters. Furthermore, as anticipated, the persistent discrepancy affecting the compactness of cluster galaxies may provide an interesting test bed for the implementation of improved models of baryonic feedback, or alternative DM models, such as SIDM (see e.g. Mastromarino et al. 2023) within cosmological hydrodynamical simulations. In the next sub-section, we list immediate developments to broaden the scope of our results and test their limitations.

Future outlook

The techniques presented in the present Thesis for the SL modelling of galaxy clusters and for the study of the morphological and kinematic properties of their quiescent cluster members can be further developed and extended, providing a basis for a number of projects that can provide a natural extension for our research. In particular, I am currently addressing or intend to dedicate my research to the issues listed below.

The velocity function of quiescent cluster members at $z \sim 0.3 - 0.5$: in Chapter 10, we have presented our preliminary steps to calibrate the velocity dispersion function of the quiescent member galaxies of the HFF clusters with deep MUSE pointings. We still need to extend the sample of measured stellar velocity dispersions to the members of Abell 370, so far not included in our study. We will further test our pro-

cedure to complete the function to all quiescent members based on the FP relation. Finally, we will include the regions of the cluster core covered by HFF photometry but outside of the MUSE pointings: a measurement of the structural parameters of these members with `MORPHOFIT` will be required as a preliminary step.

The sub-halo mass function of SL clusters at $z \sim 0.3 - 0.5$ down to a total mass $M \approx 10^{10} M_{\odot}$: in Chapters 6 and 7 and in Granata et al. (2022), we have shown how an accurate description of the mass structure of the cluster sub-haloes, down to a total mass of $M \approx 10^{10} M_{\odot}$, can be obtained by enhancing SL with more accurate scaling relations based on the observed stellar kinematics of the cluster members. The velocity dispersion measurements obtained to calibrate the velocity function of the four HFF clusters can be used to build a new generation of SL models based on the FP for all of them, achieving unprecedented constraints on the sub-halo mass function of a sample of SL clusters at $z \sim 0.3 - 0.5$.

Comparing the sub-halo velocity and mass functions of SL clusters with cosmological hydrodynamical simulations to test the Λ CDM model: as a following step, the procedure described in the previous points can also be applied to all of the CLASH clusters with 3 hours or more of MUSE exposure time, for a total of 9 clusters in the range $z = 0.23 - 0.57$, and then – given the relatively narrow redshift range of the selected sample of galaxy clusters – the same procedure can be applied to earlier cosmic times, to trace the evolution of clusters and of the galaxies they host. There are several higher- z SL clusters in the RELICS sample (Coe et al. 2019), and deep MUSE data are already available for two of them at $z \sim 0.85$ (‘El Gordo’ and CLJ0152.7–1357). Furthermore, our programme for 7 hours of MUSE observations of the core of the cluster SPT–CLJ2011–5228 ($z = 1.06$, see Collett et al. 2017). The work will be reliably extended at higher z with the same data and pipelines, as the wavelength range of MUSE includes several absorption features that can be used for σ measurements up to a cluster redshift of $z \lesssim 1.1$. This work will broaden the current redshift range at which accurate measurement of the sub-halo mass function of galaxy clusters are available. Finally, the predicted mass and velocity dispersion function and the mass structure of the cluster sub-haloes can be compared with high-resolution cosmological hydrodynamical simulations, as done in Chapter 7. This is the ideal tool to shed light on the interplay between DM and the baryonic processes driving galaxy formation across cosmic time.

Enhancing time-delay cosmography in SL clusters with cosmic chronometers: the exquisite sample of spectra of cluster galaxies extracted from deep MUSE observations allows for a study of their spectral absorption and emission features. Their observed spectra are representative of an old, massive, and passively evolving galaxy population, and show several absorption lines or spectral features (such as the 4000 Å break) that are commonly used to estimate their age and to control the intrinsic age-metallicity degeneracy. Given that quiescent cluster members are a coeval population of passive member galaxies, they can be used as cosmic chronometers, whose study can measure the differential time evolution of the Universe, providing us with measurements of the values of the cosmological parameters (Jimenez & Loeb 2002; Stern et al. 2010; Moresco et al. 2012). As we show in Bergamini et al. (2024), the constraining power of time-delay cosmography in galaxy clusters, which makes use of the measured time delays between the multiple images of time-varying sources strongly lensed by a cluster can be augmented by the study of the cosmic chronometers hosted by the cluster. The proposed combined method allows for a robust and accurate measurement of the value of the Hubble constant, and provides valuable

information on the other cosmological parameters (e.g. the present-day cosmological densities of matter Ω_m and dark energy Ω_{de} , and the dark-energy equation of state parameter w) thanks to the complementarity between the two different probes in breaking parameter degeneracies. This is a promising idea that can be developed refining the pipelines developed for this Thesis.

Complementing the new galaxy-scale strong lensing surveys with observed kinematics: in the next few years, the population of currently known galaxy-scale strong lenses will be boosted by orders of magnitude (from a few hundreds up to more than 100,000) by the upcoming wide and deep surveys at the Euclid and Vera Rubin (LSST) Telescopes, allowing us to systematically extend the current studies on early-type lens galaxies to lower masses and fainter magnitudes. The 4MOST Strong Lensing Spectroscopic Survey collaboration (4SLSL, Collett et al. 2023) will provide spectroscopic follow-up for a large sample of lens candidates, enabling the measurement of the LOSVD for about 5,000 confirmed lenses. Within 4SLSL, I am developing a pipeline, which I will present in an upcoming paper, to simulate realistic 4SLSL observations of the population of lens galaxies that will be unveiled by Euclid and LSST. I will contribute to the optimisation of the observing strategy and to the selection of the targets, and measure the values of the velocity dispersion for the lens galaxies observed by 4MOST. The combination of SL and dynamics will allow us to break the degeneracies on the determination of the total mass distribution of galaxies (e.g. the mass-sheet, the mass-slope, and the mass-anisotropy degeneracies) and to dissect the DM and stellar components. This is the key to obtaining, through a Bayesian hierarchical pipeline (Sonnenfeld et al. 2019), the normalisation of the stellar initial mass function (IMF) and the central slope of the DM mass density profile of early-type lens galaxies. Addressing these open questions will test some of our hypotheses on galaxy formation and evolution, providing us with insights on star formation, baryonic feedback, and on the nature of the DM particle.

Appendices

Velocity dispersion measurements from simulated MUSE spectra of cluster members

In this Appendix, we test the reliability of the kinematic measurements performed with `pPXF` on a set of simulated MUSE spectra of member galaxies that reproduce the cluster member spectra studied in this work.

These simulations are crucial to study:

- the required S/N that guarantees a reliable velocity dispersion measurement;
- the presence of a bias on the recovered velocity dispersion value due to the spectral sampling of MUSE;
- the impact of spectral noise on the recovered velocity dispersion values, in order to properly account for it within the error budget.

Here and in the main body of the Thesis, we refer to the S/N ratio measured as an average over the studied wavelength range of the ratio between the observed spectrum and its residuals with respect to the best-fit found by `pPXF`, computed within 150 Å bins.

A.1 Simulated MUSE spectra of cluster members

We generate simulated spectra of cluster galaxies using our MUSE observations as a reference and model. Specifically, we start from the best-fit template combination that `pPXF` finds to describe the spectra of the selected MACS J0416 member sample presented in Chapter 10 for which we obtain reliable velocity dispersion measurements extracting their spectra from the weighted MUSE cube. The choice of MACS J0416 is only dictated by the fact that it sits at an intermediate redshift within our sample, but any cluster could be used with the same procedure. The best-fit star template combination will therefore be a “bare” galaxy spectrum, with $\sigma = 0$. Its resolution and sampling will be equal to our X-Shooter templates ($R \approx 9700$ and $\Delta\lambda = 0.15$ Å).

These bare spectra are used as a basis to create 10000 simulated spectra of cluster members. For each spectrum, one of the 57 bare spectra based on a MACS J0416 member galaxy is randomly chosen. To avoid effects of the finite, albeit very small, sampling of the templates to affect our simulations, we re-sample then on a ten times finer axis $\Delta\lambda = 0.015$ Å. The over-sampled spectrum is then convolved with a Gaussian LOSVD, determined by a given LOS velocity v and velocity dispersion σ . The distributions of v , σ , and S/N values are chosen following Bergamini et al. (2019). The spectra are then shifted to a redshift $z = 0.4$, degraded to the resolution of MUSE of 2.6 Å FWHM, and re-sampled to the MUSE pixelscale of 1.25 Å. Finally, we add a random noise to the

Table A.1: Number of simulated spectra N of cluster members in six bins of velocity dispersion, and values intervals for σ , and S/N of the simulated spectra.

Number of spectra	$\sigma(\text{km s}^{-1})$	S/N
1000	[0.1 – 250]	[1 – 100]
1000	[10 – 150]	[1 – 30]
1000	[10 – 150]	[1 – 15]
2000	[45 – 250]	[1 – 100]
2000	[45 – 250]	[1 – 20]
3000	[50 – 250]	[1 – 15]

spectrum to ensure a given S/N average value. The values of v are chosen randomly between -50 and 50 km s^{-1} , while the values of σ and S/N are chosen in the intervals reported in Table A.1.

The mock galaxy spectra are fitted in the same rest-frame wavelength range of $[3700\text{--}5400] \text{ \AA}$ and the same `pPXF` set-up presented in Chapter 5. As a first test, we study the difference between the measured velocity dispersion σ_m and the input value of σ . As clear from Fig. A.1, the average offset is positive and goes to zero for $\sigma \approx 300 \text{ km s}^{-1}$. On average, for $\sigma > 30 \text{ km s}^{-1}$, it is never larger than a few kilometres per second, while it can be significantly larger for lower σ values. In general, we will consider this as a minimum σ value that we can reliably measure, provided that the S/N is high enough. We fit its value as a function of the measured velocity dispersion with the relation

$$\sigma(\text{km s}^{-1}) = \sigma_m(\text{km s}^{-1}) - \sigma_{\text{corr}}(\text{km s}^{-1}), \quad (\text{A.1})$$

where σ_m is the measured value of σ_{LOS} measured in km s^{-1} and σ_{corr} is a correction bound to σ_m by

$$\sigma_{\text{corr}}(\text{km s}^{-1}) = (3.32 - 1.23 \times 10^{-2} \sigma_m + 4.82 \times 10^{-6} \sigma_m^2) (\text{km s}^{-1}). \quad (\text{A.2})$$

As a following step, we test the impact of the spectral noise on the uncertainty on the velocity dispersion value. Both the simulations presented here and the fits of observed cluster members show that for $S/N < 10$ the fit often completely fails, or struggles to recover the observed shape of the absorption features. As such, we will disregard cluster members with a spectral $S/N < 10$ for our velocity dispersion measurements. Even for $S/N > 10$, however, the noise determines random offsets between the recovered and true values of the velocity dispersion. During the LOSVD fit, `pPXF` determines the statistical uncertainty on the best-fit value of σ required to fit the stellar templates to the observed spectrum. We will increase this value by summing in quadrature a further contribution from the random offsets of σ determined by the spectral noise. To estimate this contribution, we use our simulated spectra to study the relative offsets between the measured and observed values of σ as a function of S/N . We divide the simulations in S/N bins and take the standard deviation of the difference between the measured and input velocity dispersion in every bin, which we indicate as $\delta\sigma$. As shown in Fig. A.2, the relative offset $\delta\sigma/\sigma$ is typically smaller than 5% for $S/N > 10$, and is on average slightly positive due to the undersampling effects discussed previously. A polynomial fit of the relative uncertainty $\delta\sigma/\sigma$ as a function of the value of S/N gives the following

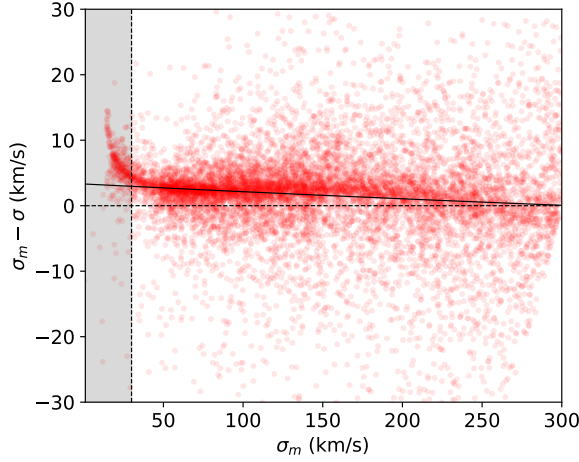


Figure A.1: Difference between the measured and true values of σ , as a function of the true value, for the 10000 simulated MUSE spectra of cluster members described in Table A.1. We also report the polynomial fit that we use in this Thesis to correct the measured σ values for this effect. We mark the threshold of $\sigma = 30 \text{ km s}^{-1}$ with a vertical line.

relation

$$\delta\sigma_{\text{noise}}/\sigma = 7.62 \times 10^{-2} - 3.28 \times 10^{-3}(S/N) + 5.30 \times 10^{-5}(S/N)^2 - 2.72 \times 10^{-7}(S/N)^3, \quad (\text{A.3})$$

where σ is the measured value of the LOS velocity dispersion provided by pPXF. As such, the contribution of the noise to the absolute uncertainty will be $\delta\sigma_{\text{noise}} = (\delta\sigma_{\text{noise}}/\sigma)\sigma$, and the final estimate of the absolute uncertainty on the velocity dispersion measurement will be $\delta\sigma = \sqrt{\delta\sigma_{\text{noise}}^2 + \delta\sigma_{\text{pPXF}}^2}$, where $\delta\sigma_{\text{pPXF}}$ is the velocity dispersion uncertainty estimated by pPXF during the fit of the observed spectrum to the best-fit template combination.

Finally, we test the impact of the optimisation set-up chosen for this work and presented in Chapter 5. We find that the accuracy in recovering the value of the LOSVD of the simulated spectra presented in Table A.1 is not significantly affected by the choice of a different degree for the additive polynomials adopted for the spectral fit (see Eq. (5.8)) nor improved by the inclusion of multiplicative polynomials (see, also, Eq. (5.8)). We verify that masking some pixels of the wavelength range, which is necessary to avoid including contaminated regions of the observed spectra, does not bias the measured values of the velocity dispersion, although we notice an increase of the scatter on the recovered values when the CaII H and K lines are not included in the spectral fit.

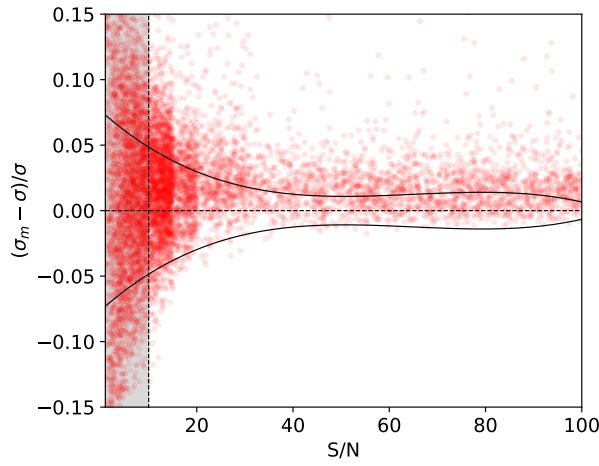


Figure A.2: Relative difference between the measured and true values of σ , as a function of the spectral S/N , for the 10000 simulated MUSE spectra of cluster members. We mark the threshold of $S/N = 10$ with a vertical line.

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