R&D networks with heterogenous firms

Lorenzo Zirulia
CESPRI, Bocconi University, Milan.
lorenzo.zirulia@unibocconi.it

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Abstract: This paper models the formation of R&D networks in an industry where firms are technologically heterogenous, extending previous work by Goyal and Moraga (2001). While remaining competitors in the market side, firms share their R&D efforts on a pairwise base, to an extent that depends on their technological capabilities. First, we consider a four firms' industry. In the class of symmetric networks, the complete network is the only pairwise stable network, although not necessarily profit or social welfare maximizing. Then, we extend the analysis to asymmetric structures in a three firms' industry. Only the complete and the partially connected networks are possibly stable, but which network is stable depends on the level of heterogeneity and technological opportunities. The complete and partially connected networks are also the possible welfare and aggregate profit maximizing networks, but social and private incentives do not generally coincide. Finally, we consider the notion of strongly stable networks, where all the possible deviations by coalitions of agents are allowed. It turns out that in the four firms' case, the complete network is very rarely strongly stable, while in the three firms' case the partially connected network where two firms in different technological group are linked is, for a large subset of the parameter space, the only strongly stable network.

Keywords: Strategic alliances, networks, research and development, technological complementarities.
JEL Codes: D21, D43.

1. Introduction

There is significant evidence that technological agreements among firms are becoming increasingly popular (Hagedoorn, 2002). Especially in high tech industries (e.g.., ICT and biotechnology), firms more and more collaborate in the technological domain, under different forms, ranging from joint R&D to the exchange of knowledge through cross licensing agreements.

Several scholars in different disciplines have tackled the issue of explaining theoretically the phenomenon. Initially put forth by sociologists, but promptly accepted in the business literature, the *network* perspective has recently gained a prominent role. In a sociological perspective, the overall network emerging from the alliances in an industry matters because typically the position of a firm in the network is associated with variables like power, status and access to information. These variables, in turn, affect firm's performance (Powell *et al.*, 1996).

Recently, economists have shown interest in the formation of economic and social networks, and have developed formal tools to address this issue (Jackson and Wolinski, 1996). R&D networks represent a natural application of such tools, and they have been studied by Goyal and Moraga (2001), Goyal and Joshi (2003) and Goyal *et al.* (2004).

This paper belongs to this last stream of literature. It extends previous work considering the role of technological heterogeneity. The issue of technological complementarity has been often mentioned by the empirical literature as an important motive for firms to enter into collaborative agreements. In high tech industries, innovation is more and more complex and building on several technological fields. This is the case in pharmaceuticals, after the new discoveries in molecular biology in the mid 1970s, and in microelectronics, where innovation hinges on competences in fields as different as solid physics, construction of semiconductor manufacturing and testing equipment, and programming logic. Firms cannot possess all the relevant knowledge required to innovate and therefore they look for partners having complementary capabilities to face

an increased rate in the introduction of new products and processes, to monitor new opportunities and enter new markets, to sustain long-lasting competitive advantage.
Based on the MERIT-CATI database on world wide technological agreements (Hagedoorn, 1993), among the alliances formed in the period 1980-1989 technological complementarity is cited as a key motivation in 35% of alliances in biotechnology, 38% in new materials technology, 41% in the industrial automation sector and 38% in the software industry. In the sample considered by Mariti and Smiley (1983), technological complementarity constitutes the motivation of 41% cooperative agreements.

In the economic literature, there is a consolidated tradition of models of R&D cooperation (D'Aspremont and Jacquemin, 1988, Kamien *et al.*, 1992). These models usually identify R&D spillovers as the factor that can make cooperation among firms welfare improving, and in that respect they have a strong policy orientation. This literature analyzes cooperation occurring at the industry-wide level (Suzumura, 1992), or comparing exogenously given coalitions (Katz, 1986).
The literature on endogenous coalitions (i.e. partition of firms) in oligopolistic industries (Bloch, 1995) can be considered an extension allowing for strategic consideration on the cooperative side. In this paper, we consider networks of R&D collaborations, which is at the same time more restrictive (because we allow exclusively coalitions of two firms) and less restrictive (because we do not require transitivity in the collaborative relations).

The paper is structured as follows. Section 2 describes the model, focusing on the extensions to the existing literature. Section 3 is concerned with symmetric networks. We first characterize the effect of different degrees of cooperative activity on R&D investments and production costs. Then, we consider the issue of stability of different network structures in a four firms industry, and their properties in terms of aggregate profits and social welfare. In section 4, we extend the analysis to asymmetric networks in a three firms industry. This leads us to consider a situation where the distribution of technological capabilities in the industry is asymmetric. As in section 3, we study the stability of the different network structures, and their properties in terms of aggregate profits and social welfare. In section 5, we introduce a refinement to the notion of stability used in the previous sections, which provides some interesting economic insights. Section 6 concludes.

2. The model

Informally, the model can be described as follows. We consider n firms in an industry, producing a homogenous good. In the product market, firms compete *à la* Cournot, i.e. choosing quantities. Before market competition, firms can engage in an R&D activity in order to reduce their unit cost of production. Firms can share their efforts on a bilateral basis, and this information sharing is what we define as collaboration. Firms are assumed to be heterogeneous from the technological point of view (for sake of simplicity, firms are divided in two groups). Suppose for instance that heterogeneity comes from different firms' specializations in the range of technological or scientific fields that are required for innovation. Technological heterogeneity has an impact on the consequences of collaboration: information sharing is assumed to be more effective for firms with different technological capabilities, due to the existence of technological complementarities between them.

Formally, we deal with a three-stage game Γ, which coincides with the one presented in Goyal and Moraga (2001). In the first stage, firms can form collaborative links, which give raise to a well specified R&D network. Given the network structure, firms choose non-cooperatively their R&D effort. Given the level of R&D efforts, the cost function of each firm is determined. Finally, given costs, firms compete in the market.

Let $N = \{1,..,n\}$ be the set of firms. Firms are identif ied by an index $r = 1,2$, which corresponds to the technological group a firm belongs to. $N^r \subseteq N$ represents the set of firms of group r. The R&D network resulting from the first stage is denoted by g. When we write $ij \in g$, this implies that there is a collaborative link between i and j. We define $N_i(g) = \{ j \in N \setminus \{i\} : ij \in g \}$ as the set of firms having a collaborative link with i. Assume that firm i belongs to the technological group r. We can write $N_i(g) \equiv N_i^r(g) \cup N_i^{3-r}(g)$, that is we can partition the set of firms collaborating with i

in the sets of firm belonging to the same technological group, $N_i^r(g) = \{j \in N^r \setminus \{i\} : ij \in g\}$ and to the other technological group, $N_i^{3-r}(g) = \{j \notin N^r : ij \in g\}$. Also, we indicate with $n_i(g) = |N_i(g)|$ the cardinality of the set of partners for firm i in g, and similarly for $n_i^r(g)$ and $n_i^{3-r}(g)$.

If g is the network resulting from the first stage, we denote with $\Gamma(g)$ the corresponding subgame. In such a subgame, firms fix their level of R&D expenditures correctly anticipating the Cournot outcome of the last stage. Firm i's action in this stage is given by $e_i \in [0, \bar{c}]$, where e_i is the effort put by firm i in the R&D activity. The cost associated to e_i is given by $C(e_i) = e_i^2$. Consequently, $e = (e_i)_{i \in N}$ is the action profile of $\Gamma(g)$.

With respect to Goyal and Moraga (2001), we modify the formulation of collaboration effects. Their paper strictly follows the representation of R&D activity that is standard in the literature on R&D collaboration and spillovers. Kamien *et al.* (1992) summarize the approach as follows:

"The R&D process (…) is supposed to involve trial and error. Put another way, it is a multidimensional heuristic rather than a one-dimensional algorithmic process. The individual firm's R&D activity does not involve following a simple path. If this were the case, the only spillover potential would be from the firm that had somehow forged ahead in the execution of the algorithm to the laggards. However, in an R&D process involving many possible paths and trial and error, it is unlikely that individual firms will pursue identical activities. Indeed it is reasonable for each firm to pursue several avenues simultaneously, the differences among the firms being in the greater emphasis each places on one over the others. The spillover effect in this vision of the R&D process takes the form of each firm learning something about the other's experience. This information, which may become available through deliberate disclosure or leak out involuntarily (e.g, at scientific conferences), enables a firm to improve the efficiency of its R&D process by concentrating on the more promising approaches and avoiding the others"

This view of R&D as a trial and errors process implies that the dimension of the space that firms can explore in their efforts is high, and firms are not "constrained" in their exploration. This derives from the hypothesis that, when information sharing is complete, duplication of efforts are completely eliminated. This assumption is justified because the focus is on the effects of different degrees of R&D appropriability on the desiderability of R&D collaboration.

In this paper we propose a different interpretation. We do not consider the issue of R&D appropriability and we do not consider the degree of information sharing as a variable of choice. We assume that the capacity of other firms' R&D to be a substitute of a firm's R&D depends on the technological specialization of firms. We assume that the area of the technological space firms can explore that is constrained by their technological specialization. Firms are characterized by "competences", which implies a process of search which is necessarily local (Nelson and Winter, 1982). Whenever firms belong to the same technological group, the probability that firms pursue the same path increases. If firms are heterogeneous in their technological capabilities, this creates possible opportunities for complementarities as the result of information sharing. Since we consider cost reducing R&D, we formalize the argument assuming that the fraction of R&D effort of firm j that is able to reduce firm's i costs when i and j cooperate is $\overline{b}$ if firms belong to different technological groups, and $\underline{b}$ if firms belong to the same technological group, with $1 \geq \overline{b} \geq \underline{b}$. The case discussed in Goyal and Moraga implies $\overline{b} = \underline{b} = 1$.

Two remarks are needed. First, when both $\overline{b}$ and $\underline{b}$ are high, information sharing is effective, independently of technological groups. In other words, the likelihood of effort duplication is low, or, in terms of our interpretation, firms have "naturally" several possible paths to follow. As long as an economic interpretation is concerned, we can relate this to a situation where the technological space that firms can explore is particularly rich. For that reason, when discussing our results about stability, aggregate profits and social welfare, we will refer to the notions of technological heterogeneity (measured by $\overline{b} - \underline{b}$) and technological opportunities (measured by the values of $\overline{b}$ and $\underline{b}$).

Second, the literature on the economics of innovation has argued theoretically and showed empirically the important role played by absorptive capacity (Cohen and Levinthal, 1989): in order to evaluate and absorb fully the outcomes from cooperative ventures, firms need to have pre-existing capabilities in those scientific or technological fields. Then, even if a firm may lack the knowledge possessed by another firm, it can fail in absorbing it. For our model, this implies that b can be more properly seen as the product of two parameters: g, which captures the extent to which a firm possesses knowledge that is not possessed by the other firm (with $\overline{g} > \underline{g}$); and a, which captures the extent to which a firm can actually learn by the experience of the other firm, due to absorptive capacity (with $\overline{a} < \underline{a}$). According to this interpretation, we are assuming that the first effect prevails, in the sense that $\overline{g}\,\overline{a} > \underline{g}\,\underline{a}$.

Given the R&D investments *e*, the unit cost of production for $i \in N$ is determined by:[1]

$$c_i(g,e) = \bar{c} - e_i - \sum_{j \in N_i^{r}(g)} \underline{b} e_j - \sum_{j \in N_i^{3-r}(g)} \overline{b} e_j \tag{1}$$

Finally, given the costs $c_i(g,e)$, firms compete in the market choosing quantities. $q_i(g,e) \in [0, A]$ denotes the action taken by firm i at this stage. The inverse demand function is linear: $p = A - \sum_{i \in N} q_i(g,e)$. In the Cournot-Nash equilibrium, quantities are given by:

$$q_i(g,e) = \frac{A - nc_i(g,e) + \sum_{j \neq i} c_j(g,e)}{n+1} \tag{2}$$

[1] In line with Goyal and Moraga (2001), we assume that there are no *indirect* effects from link formation. This admittedly strong assumptions implies that a firm can exclude other firms from the returns of its R&D investment if information sharing is not explicitly agreed (say, because knowledge is embodied in machineries or protected by patents).

Net profits are given by:

$$\Pi_i(g,e) = (q_i(g,e))^2 - C(e_i) \tag{3}$$

In the next sections, we will analyze the social welfare property of the different networks. In order to do that, we introduce the following social welfare function:[2]

$$W(g,e) = \sum_{i\in N} \Pi_i(g,e) + \tfrac{1}{2} Q(g,e)^2 \tag{4}$$

This is in the spirit of "second best" (Goyal and Moraga, 2001): we assume that for given network structure efforts are still chosen non-cooperatively and quantities are those resulting from the Cournot-Nash equilibrium.

3. Symmetric networks

This section focuses on symmetric networks. Networks are symmetric when all the firms are equivalent in terms of connections (i.e. they have the same number of links inside and outside their technological group). With technologically homogenous firms, a symmetric network is characterized by a single value k identifying the number of links that any firm has. Goyal and Moraga define k as the degree of collaborative activity. Given the assumption of heterogeneous firms, however, our notion must change accordingly. In our case, a symmetric network is identified by a pair $k \equiv (k^r, k^{3-r})$, corresponding to the number of links that a representative firm has within and outside its technological group respectively, i.e. $n_i^r(g) = k^r$ and $n_i^{3-r}(g) = k^{3-r}$ $\forall i \in N$. We maintain the convention of calling this vector the degree of collaborative activity, and we indicate with g^k the symmetric network with degree of collaborative activity $k \equiv (k^r, k^{3-r})$. We can define a partial ordering over symmetric networks: $k_1 > k_2$ if $k_1^r \geq k_2^r$ and $k_1^{3-r} \geq k_2^{3-r}$, where at least one inequality is strict.

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[2] The second term represents consumer surplus, given the hypothesis of linear demand function with a 45° slope.

For the notion of symmetric network to be meaningful, we must restrict our attention to cases where N is given by two *equal* size groups of firms in even number. In this section we choose to concentrate and completely characterize the results for the case with $n=4$. Some results can be extended to generic n, but the complete analysis is quite difficult to obtain (also Goyal and Moraga, in their simpler framework, limit themselves to partial results).[3]

Given the network g and other firms' investments, the representative firm i maximizes $\Pi_i(g,e)$ in e_i subject to $e_i \in [0,\bar{c}]$. We need to consider five types of firms: a) firm i; b) k^r firms linked to firm i and belonging to its technological group (subscript *lr*); c) k^{3-r} firms linked to i and belonging to a different technological group (subscript *l3-r*); d) $\frac{n}{2}-k^r-1$ firms that are not linked to firm i and belong to its technological group (subscript *mr*); e) $\frac{n}{2}-k^{3-r}$ firms that are not linked to i and belong to the other technological group (subscript *m3-r*). This results in a specific cost structure for each type of firm:

$$c_i(g^k)=\bar{c}-e_i-\bar{b}k^{3-r}e_{l3-r}-\underline{b}k^r e_{lr} \tag{5a}$$

$$c_{lr}(g^k)=\bar{c}-e_{lr}-\sum_{j\in N_{lr}^{3-r}(g^k)}\bar{b}\,k^{3-r}\,e_j-\sum_{j\in N_{lr}^{r}(g^k)}\underline{b}k^r e_j \tag{5b}$$

$$c_{l3-r}(g^k)=\bar{c}-e_{l3-r}-\sum_{j\in N_{l3-r}^{3-r}(g^k)}\bar{b}\,k^{3-r}\,e_j-\sum_{j\in N_{l3-r}^{r}(g^k)}\underline{b}k^r e_j \tag{5c}$$

$$c_{mr}(g^k)=\bar{c}-e_{mr}-\sum_{j\in N_{mr}^{3-r}(g^k)}\bar{b}\,k^{3-r}\,e_j-\sum_{j\in N_{mr}^{r}(g^k)}\underline{b}k^r e_j \tag{5d}$$

[3] As explained by Goyal and Moraga (2001), it is difficult to generalize in the study of asymmetric networks. All the set of direct and *indirect* connections determines the maximization problem the firm has to solve. For each asymmetric network, one needs to solve a different system of first order conditions, in which the possibility of invoking symmetry may be limited. As we will see in section 3.1, the study of asymmetric networks is required to apply the definition of pairwise stability.

$$c_{m3-r}(g^k) = \bar{c} - e_{m3-r} - \sum_{j \in N^{3-r}_{m3-r}(g^k)} \bar{b}\, k^{3-r}\, e_j - \sum_{j \in N^{r}_{m3-r}(g^k)} \underline{b}\, k^r e_j \qquad (5e)$$

Plugging (5a-5e) into firm i's profit function and deriving with respect to e_i we obtain the following first order condition:

$$\frac{\partial \Pi_i}{\partial e_i} \equiv 2q_i(g,e)[n - k^r \underline{b} - k^{3-r}\bar{b}] - 2e_i = 0 \qquad (6)$$

Invoking symmetry across all firms, we impose $e_i = e_{lr} = e_{l3-r} = e_{mr} = e_{m3-r} = e(g^k)$. Rearranging the first order condition, we obtain the equilibrium effort:

$$e(g^k) = \frac{(A - \bar{c})(n - \underline{b}k^r - \bar{b}k^{3-r})}{(n+1)^2 - (n - \underline{b}k^r - \bar{b}k^{3-r})(1 + \underline{b}k^r + \bar{b}k^{3-r})} \qquad (7)$$

Plugging (7) into (5a), one obtains the unit cost of production for the representative firm:

$$c(g^k) = \frac{\bar{c}(n - \underline{b}k^r - \bar{b}k^{3-r}) - A(n - \underline{b}k^r - \bar{b}k^{3-r})(1 + \underline{b}k^r + \bar{b}k^{3-r})}{(n+1)^2 - (n - \underline{b}k^r - \bar{b}k^{3-r})(1 + k^r\underline{b} + k^{3-r}\bar{b})} \qquad (8)$$

It is interesting to study how effort levels and unit costs in equilibrium vary in different symmetric networks. In other words, varying the network g^k, we study the equilibrium values $e(g^k)$ and $c(g^k)$ in the corresponding subgame. The next proposition summarizes the results:

Proposition 1: *there exists a negative relation between the degree of collaborative activity and the equilibrium effort. Furthermore, the effort is decreasing in* $\bar{b}$ *and* $\underline{b}$ *.*
There exists a non monotonic relation between the unit cost of production and the degree of collaborative activity. In particular, the unit cost is initially declining in the

degree of collaborative activity and then possibly increasing. The complete network is cost minimizing for sufficiently low $\overline{b}$ *and* $\underline{b}$.

The level of equilibrium effort is declining in the level of collaboration for two reasons. The first one is a "duplication" effect: since firms take advantage of R&D by other firms, they tend to reduce their efforts in order to save on the R&D costs. The second effect is due to the existence of competition among firms. Forming new links, firms share their effort with more firms, making them stronger competitors. This reduces the firms' incentives to invest in R&D.

The negative effect on efforts when b is high is intuitive. In our interpretation, high b means a low "probability" that two firms will pursue the same path in the research activity. For given R&D efforts, the cost reduction (both for the firm and its collaborators) is increasing in b . This makes both the duplication and the competition effect stronger and results in a more significant reduction in $e(g^k)$.

The *a-priori* ambiguous relation between the degree of collaborative activity and costs comes from two effects that go in opposite direction: the increase in collaborative activity reduces the effort, but a firm can benefit from the research activities of more firms.

Computations show that, for $\overline{b}$ and $\underline{b}$ sufficiently low (i.e., when the negative effect of an increase of k on $e(g^k)$ is moderate), the positive effect prevails and costs are minimized in a complete network.

3.1 Stability

In this paragraph, we focus on the stability of different symmetric network structures. From now on, we consider the case $n=4$. This allows us to obtain a full characterization of the results. We will verify the stability of six (symmetric) networks, since k^r can take value in the set $\{0,1\}$ and k^{3-r} in the set $\{0,1,2\}$.

Plugging $n = 4$ and equilibrium efforts, costs and quantities in the profit function yields:

$$\Pi(g_k) = \frac{(A-c)^2(25-(4-\overline{b}k^{3-r}-\underline{b}\,k^r)^2)}{(25-(4-\overline{b}k^{3-r}-\underline{b}\,k^r)(1+\overline{b}k^{3-r}+\underline{b}\,k^r))^2} \quad (9)$$

The notion of stability that is used is the notion of pairwise stability introduced by Jackson and Wolisnki (1996). In the definition we denote with $g-ij$ the network obtained by removing ij from g, and with $g+ij$ the network obtained by adding ij to g.

Pairwise stability: *A network g is pairwise stable if and only if for all* $i, j \in N$

(i) *If* $ij \in g$ *, then* $\Pi_i(g) \geq \Pi_i(g-ij)$ *and* $\Pi_j(g) \geq \Pi_j(g-ij)$

(ii) *If* $ij \notin g$ *and* $\Pi_i(g+ij) > \Pi_i(g)$*, then* $\Pi_j(g) < \Pi_j(g+ij)$

The definition implies that both agents need to agree to form a link, while they can unilaterally sever it. This notion of stability is the weakest one can think of, since it allows a single link to be modified: firms cannot simultaneously form and/or sever more than one link. Consequently, the set of stable networks is the largest, compared with set of stable networks resulting from stricter notions of stability; nevertheless, such a set is relatively small in all the cases we will consider (a singleton in the case of symmetric networks in a four firms industry), so that pairwise stability constitutes a useful solution concept. In section 5, we will consider an alternative, stricter notion of stability, strong stability.

The following proposition summarizes the results. The sketch of the proof is in appendix:

Proposition 2*: for every strictly positive* $\overline{b}$ *and* $\underline{b}$*, the complete network is the only stable network.*

Proposition 2 strictly follows the result by Goyal and Moraga (2001) They show that for generic n, the empty network is not stable, while the complete network is always stable. It can be shown that this result holds also in our model. They also show that for $n = 4$, the complete network is the only symmetric stable network, as it is the case here.

Then, no matter what are the degrees of technological opportunities and technological heterogeneity, firms have always the incentive to "destabilize" a symmetric network different from the complete network, forming a new link. Starting from a situation in which firms are symmetric, firms which form a new link can create an asymmetric market structure by sharing their R&D effort. In all the cases this leads to some reduction in costs, even if links occur between firms in the same technological group, for which information sharing may be not very effective. The complete network is stable because in this case, by definition, it is not possible to form new links, and firms do not find convenient to sever one of their links, weakening their competitive position.

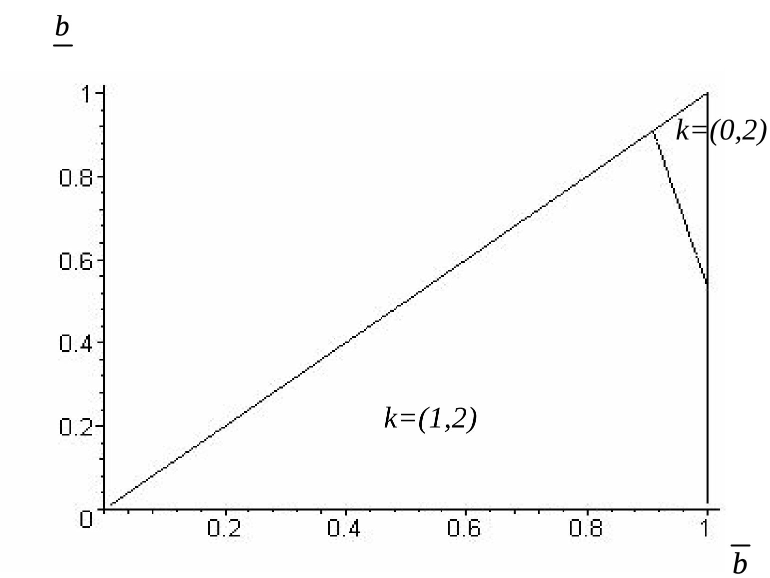
3.2 Aggregate profits

In this section, we consider the behavior of different symmetric networks in terms of aggregate profits. We try to assess the relation between the incentive for individual firms to form collaborative links and what is desirable for them *collectively*. Since in symmetric networks all firms obtain the same level of profits, it is sufficient to compare equilibrium profits for the all possible network structures (denoted with $\Pi(g^k)$, where the subscript is omitted for symmetry), in the range of all conceivable values of $\overline{b}$ and $\underline{b}$. Proposition 3 summarizes the results.

Proposition 3: *define* $H_1(\overline{b},\underline{b}) = \Pi(g^{(1,2)}) - \Pi(g^{(0,2)})$. *For all* $\overline{b}$ *and* $\underline{b}$ *such that* $H_1(\overline{b},\underline{b}) > 0$, *the complete network maximizes aggregate profits. Otherwise, a network in which all the firms are linked with and only with the firms of the other technological group (*$k^r = 0, k^{3-r} = 2$*) maximizes aggregate profits. In economic terms, the complete network is optimal for firms collectively when technological opportunities are not "too high".*

Figure 1 summarizes graphically proposition 3. This figure represents the set of possible values of parameters, $\{(\underline{b},\overline{b}) \mid (\underline{b},\overline{b}) \in [0,1]\times[0,1] \wedge \overline{b} \geq \underline{b}\}$, and it indicate the areas the parameter space for which a particular network of degree $k \equiv (k^{r}, k^{3-r})$ is profit maximizing. The following figures must be read in a similar way.

Figure 1: profit maximizing symmetric networks in four firms industry



Firms' private incentive towards link formation can be aligned or excessive with respect to their collective incentive to form links. In fact, for a very significant area in the parameter space, the complete network maximizes aggregate profits.

The increase in the degree of collaborative activity affects net profits in equilibrium through two channels: gross profits and through R&D costs. The effect on gross profit is ambiguous, reflecting the behavior of unit cost (in a symmetric network, $\Pi(g^{k}) = \left(\frac{(A - c(g^{k})}{5}\right)^{2} - e(g^{k})^{2}$); while R&D costs are decreasing in k. For a large subset of the parameter space, the complete network maximizes aggregate profits: the net effect of increasing network density is always positive. In case of (very) high technological opportunities, the negative effects of an increase in the degree of collaborative activity are more pronounced. The situation, then, resembles a prisoner's dilemma. While firms would collectively prefer a lower degree of collaboration, individually they have the incentive to destabilize a symmetric network in order to alter market structure in their favour. This results in a Pareto dominated situation.

3.3 Welfare Analysis

While the previous section has considered the collective incentives for firms to form collaborative links, this section takes into account social welfare, as defined by equation (4).

Proposition 4 *: define*

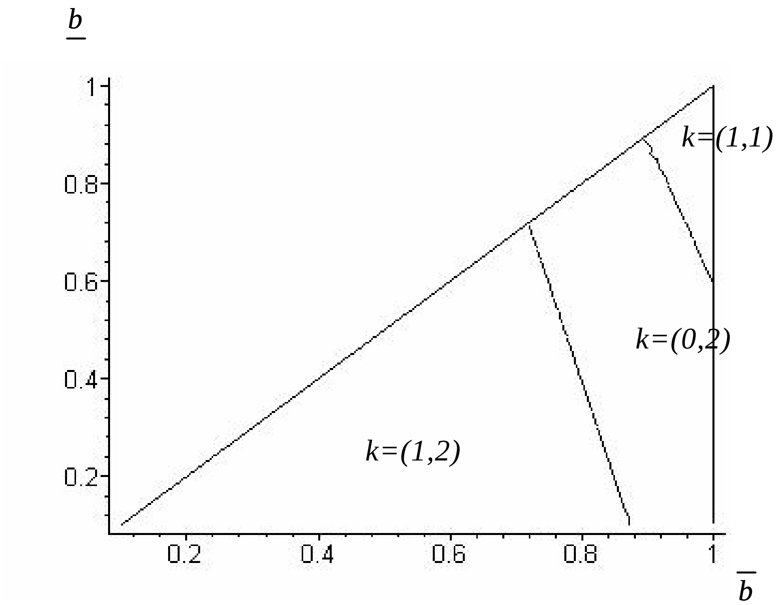
$H_2(\overline{b},\underline{b}) = W(g^{(1,1)}) - W(g^{(0,2)})$

$H_3(\overline{b},\underline{b}) = W(g^{(0,2)}) - W(g^{(1,2)})$

It can be shown that $H_2(\overline{b},\underline{b}) > 0$ *implies* $H_3(\overline{b},\underline{b}) > 0$, and $H_3(\overline{b},\underline{b}) < 0$ *implies* $H_2(\overline{b},\underline{b}) < 0$.

For all $\overline{b}$ *and* $\underline{b}$ *such that* $H_2(\overline{b},\underline{b}) > 0$, *the network where all firms have one link inside and one link outside their technological group (*$k^r = 1, k^{3-r} = 1$*) is welfare maximizing. For all* $\overline{b}$ *and* $\underline{b}$ *such that* $H_3(\overline{b},\underline{b}) > 0 > H_2(\overline{b},\underline{b})$, *the network where all firms have two links outside and zero link inside their technological group (*$k^r = 0, k^{3-r} = 2$*) is welfare maximizing. Finally, if* $H_3(\overline{b},\underline{b}) < 0$, *the complete network is welfare maximizing.*

Figure 2: Welfare maximizing symmetric networks



When technological opportunities are low, the complete network is welfare maximizing, and social interests and firms' private incentives coincide. Social welfare depends on the degree of collaborative activity through its effect on profits and through the total quantity produced, which determines consumer surplus and it is inversely related to the unit cost of production. When technological opportunities are low, the net effect of an increase in the degree of collaborative activity is always positive, and maximal information sharing is optimal. When technological opportunities increase, a less dense network becomes more desirable from a social point of view, because the negative effects from an increased degree of collaboration are higher than in the previous case. Although $k = (0,2)$ and $k = (1,1)$ are equally dense, the latter is socially preferred for very high technological opportunities. This happens because this structure minimizes the negative effects of an increase of k on equilibrium effort: as shown by Proposition 1, such a negative effect is higher when b is higher, and so it can socially optimal to "substitute" a link outside firms' technological group with a link inside firms' technological group.

Finally, it is worth noting that the area of the parameter space for which welfare is maximized by a complete network is included in the area of the parameter space for which aggregate profits are maximized by a complete network. In other words, when the complete network is social welfare maximizing, it is also profit maximizing, but the converse is not true. This is because, when considering social welfare, one needs to add the possibly negative effect that an increase in k generates for consumer surplus, through the reduction of total quantity produced due to higher production costs.

3.4 Discussion

The analysis of symmetric networks has shown that the results of Goyal and Moraga in terms of stability are not significantly modified by introducing a role for technological opportunity and technological heterogeneity: the complete network is the only symmetric stable network, independently from $\bar{b}$ and $\underline{b}$. Firms have always the incentive to alter a symmetric architecture (resulting in an asymmetric market structure) by forming a new link, whenever this is possible.

With respect to networks that maximize aggregate profits and social welfare, we do not find that individual incentives towards link formation are necessarily excessive, as in Goyal and Moraga. Actually, the complete network maximizes aggregate profits for a large set of parameters, while, if technological opportunities are sufficiently low, it is optimal also both from the society point of view to have maximal information sharing.

A more specific role for technological heterogeneity is clearly seen comparing the results about pairwise stability and social welfare. There is an area of the parameter space, where both technological heterogeneity and technological opportunities are high, in which it is socially optimal that information sharing occurs only when it is more effective, that is among firms in different technological groups. However, firms aiming at capturing strategic positions in the network (and consequently a competitive advantage in the industry) have the incentive the share their efforts with firms in their same technological group, which is detrimental in terms of the "collective" incentives to invest in R&D. This leads to a network which is denser that the social optimum.

4. Asymmetric networks

The analysis in section 3 has restricted the attention only to symmetric network. In this section we extend the analysis to the properties of asymmetric networks. We will develop the simplest case of $n=3$. This will lead us to consider a situation where technological groups have different size. We shall assume that firm 1 belongs to group 1, while firms 2 and 3 belong to group 2.

Technological groups that are asymmetric in size represent an interesting case because we can study if and how the firm in the smaller group (which possesses technological capabilities that are unique in the context of the industry) can exploit this situation and obtain an advantageous position in the network and in the market.

We need to compare six typologies of networks:

1. The empty network, denoted with $\varnothing$. In this case all the firms gain in equilibrium the same profit, which we indicate with $\Pi_1^{\varnothing}$.
2. The partially connected network of type 1, where there is one link between firm 1 and one firm in the other technological group (say firm 2). This network is denoted with $p1$, and we indicate with Π_1^{p1}, Π_2^{p1} and Π_3^{p1} profits in equilibrium for firm 1,2 and 3 respectively.
3. The partially connected network of type 2, where there is one link between the two firms in the same technological group. This network is denoted with $p2$, and equilibrium profits are Π_1^{p2} and Π_2^{p2} for firm 1 and firm 2 respectively (the positions of firms 2 and 3 are symmetric).
4. The star network of type 1, where firm 1 is the hub (i.e. it is connected both with firm 2 and firm 3) and firm 2 and firm 3 are the spokes (they are connected only to firm 1). This network is denoted with $st1$, and equilibrium profits are Π_1^{st2} and Π_2^{st2} for firm 1 and firm 2 respectively (again, the positions of firms 2 and 3 are symmetric).
5. The star network of type 2, where say firm 2 is the hub and the remaining firms are the spokes. This network is denoted with $st2$, and we indicate with Π_1^{st2}, Π_2^{st2} and Π_3^{st2} profits in equilibrium for firm 1, 2 and 3 respectively.

6. The complete network, denoted with *c*. We indicate with Π_1^c and Π_2^c equilibrium profits for firm 1 and 2 respectively (the positions of firms and 3 are symmetric).

4.1 Stability

The next proposition summarizes the results about stability. Goyal and Moraga shows that two kinds of structures are possibly stable, when spillovers outside collaboration are absent as in our model: the partially connected network and the complete network.

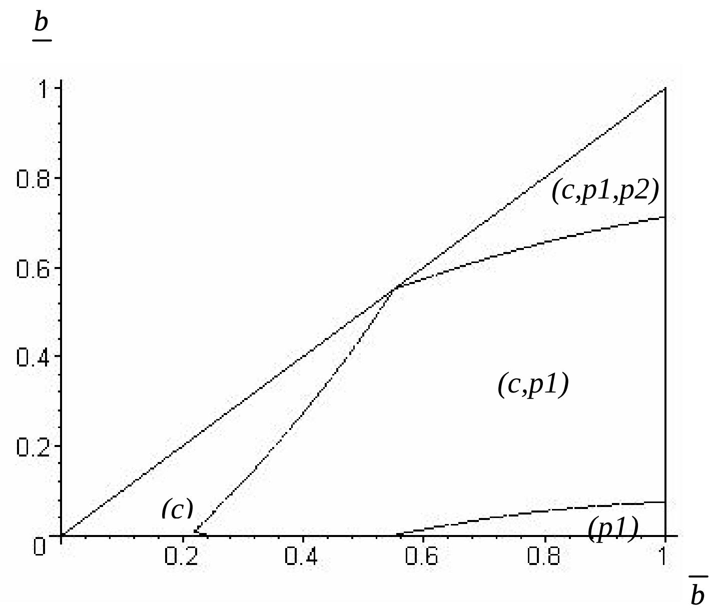
Proposition 5: *the complete network is stable unless technological heterogeneity is very high. There exists a function* $H_4(\overline{b}, \underline{b}) = \Pi_1^c - \Pi_1^{st2}$ *such that, for any value of* $\overline{b}$ *and* $\underline{b}$ *satisfying* $H_4(\overline{b}, \underline{b}) \geq 0$, *the complete network is stable.*

The partial network of type 1 is stable unless technological opportunities are low and heterogeneity is limited. There exists a function $H_5(\overline{b}, \underline{b}) = \Pi_2^{p1} - \Pi_2^{st2}$ *such that for any value of* $\overline{b}$ *and* $\underline{b}$ *satisfying* $H_5(\overline{b}, \underline{b}) > 0$, *the partial network of type 1 is stable.*

The partial network of type 2 is stable if heterogeneity is limited. There exists a function $H_6(\overline{b}, \underline{b}) = \Pi_2^{p2} - \Pi_2^{st2}$ *such that for any value of* $\overline{b}$ *and* $\underline{b}$ *satisfying* $H_6(\overline{b}, \underline{b}) > 0$, *the partial network of type 2 is stable.*

Figure 3 summarizes the results about stability in the parameter space.

Figure 3: stability in the three firms industry



Introducing firms' heterogeneity does not impact on the types of networks that are possibly stable, but the stability of different network structures *does* depend on $\bar{b}$ and $\underline{b}$.

Star networks are never stable. In particular firm 1 will not use its "special" position to become the hub of a star. The star of type 1 is not stable because of two possible deviations.

First, given the existence of a link between firm 1 and firm 2, firm 1 and firm 3 never agree in maintaining a collaborative link. Firm 1 is willing to form a link for low $\bar{b}$ ($\bar{b}$ <0.35). In this case, given that the opportunity of avoiding duplication of efforts is limited, firm 1 does not find the strategy of an exclusive alliance with firm 2 attractive, and it would rather collaborate also with firm 3. At the same time, firm 3 is willing to cooperate with 1 only when $\bar{b}$ is sufficiently high ($\bar{b}$ >0.48). Forming an alliance with 1, firm 3 obtains access to firm 1's R&D effort, but it makes firm 1 even stronger. It turns out that the first effect prevails for $\bar{b}$ high.

A second profitable deviation is given by firm 2 and firm 3 forming a link. In this case they can make their position stronger in market competition vis-à-vis firm 1, by sharing their R&D efforts.

In partially connected network of type 1, the position of firm 1 is not "special", in the sense that it obtains the same level of profit as the firm it is connected with. However, whenever heterogeneity is above a minimum threshold (such that we are not in the range in which the partially connected network of type 1 is stable) firm 1 can obtain the maximum industry profit in any stable network.

Firms in the relatively "crowded" technological group, instead, show more variability in the profits associated to stable networks.

4.2 Aggregate profits

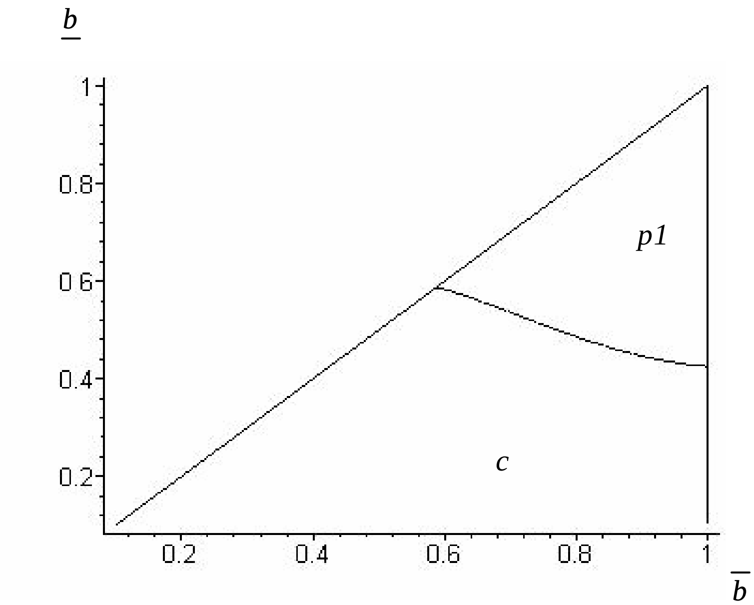
This section considers how aggregate profits vary as a function of the network. In this case it is necessary to sum the profits of the three firms, since it is not possible to talk about a representative firm in the industry (apart from the special case of the empty network)

Define: $\Pi(g) = \Pi_1(g) + \Pi_2(g) + \Pi_3(g)$, with $g \in \{c, p1, p2, st1, st2, \varnothing\}$.

Proposition 6*: when the technological opportunities are sufficiently high, the partially connected network of type 1 maximizes profits; otherwise the complete network does. There exists a function* $H_7(\bar{b}, \underline{b}) = \Pi(c) - \Pi(p1)$ *such that, for any value of* $\bar{b}$ *and* $\underline{b}$ *satisfying* $H_7(\bar{b}, \underline{b}) > 0$ *, the complete network maximizes aggregate profits.*

Figure 4 summarizes the results.

Figure 4 : profit maximizing networks in three firms industry



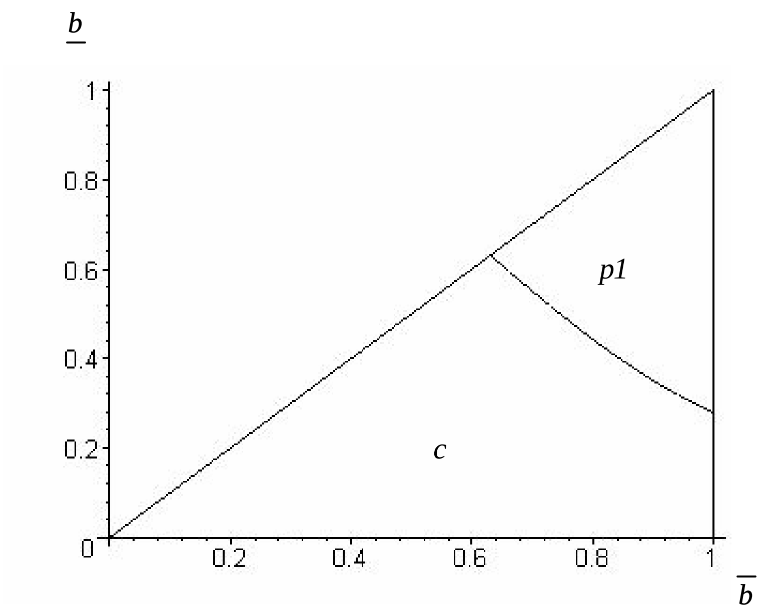
When technological opportunities are high (in particular, when information sharing between technologically heterogeneous firms is effective), allied firms have a strong incentive to invest in R&D and waken the position of the remaining firm in market competition. Then, their costs are low, and their profits high. Although unevenly distributed, aggregate profits in the partially connected network turn out to be higher than in the complete network.

4.3 Social Welfare

Finally, we consider the social welfare properties of networks in a three firms' industry.

Proposition 7: *social welfare is maximized by a partially connected network of type 1 whenever technological opportunities are sufficiently high. Otherwise the complete network maximizes social welfare. There exists a function* $H_8(\overline{b},\underline{b}) = \Pi(c) - \Pi(p1)$ *such that, for any values of* $\overline{b}$ *and* $\underline{b}$ *satisfying* $H_8(\overline{b},\underline{b}) > 0$, *the complete network maximizes social welfare.*

Figure 5: welfare maximizing networks in the three firms industry



It is interesting to notice how firms and social interests substantially coincide (closer inspection reveals that there is a small portion of the parameter space for which these do not coincide). Firms that alter market structure in their favour invest more in R&D, and this reflects in a cost reduction which is beneficial also to consumers.

4.4 Discussion

The properties of asymmetric networks in terms of stability are consistent with previous results in the literature (Goyal and Moraga, 2001; Goyal and Joshi, 2003) and with the emphasis on asymmetric structures that one can find in the firms' coalition literature (Bloch, 1995).

Technological heterogeneity does not impact on the architectures that are possibly stable, but that stability of different network structures *does* depend on $\overline{b}$ and $\underline{b}$. Very intuitively, the partially connected network of type 1 is the only stable network when heterogeneity is very significant, while if technological opportunities are limited, the partially connected networks (both of type 1 and 2) are not stable.

Firm 1, which is the unique firm belonging to its technological group, does not gain a prominent role in any stable networks. Nevertheless, it obtains the highest profit in the complete network and it can be excluded in pairwise stable networks only in the limited range of parameters where technological opportunities are high and technological heterogeneity is low.

Comparing networks that are pairwise stable and networks maximizing aggregate profits and social welfare, one can observe that in general at least one stable network (if the set of stable networks is not a singleton) is efficient, from firms' and social point of view. The exception is the range in which the partially connected network of type 1 is the only stable network, where profits and social welfare are maximized by a complete network.

5 Strong stability in symmetric and asymmetric networks

In this section, we apply a stronger notion of stability to the two cases studied in the sections 3 and 4. As we said, pairwise stability is a weak notion of stability, because it considers as admissible only a small set of deviations. In particular, it does not allow for coordinated actions of agents that form or sever more than one link. In contexts where the number of agents is small, it seems plausible that agents can arrange more complex deviations, to which a network must resist to be considered as stable. [4]

The notions we will use is the notion of strongly stable networks, discussed in Jackson and van den Nouweland (2003) and Dutta and Mutuswami (1997). In words, a network is strongly stable if there are no coalitions of players that by forming or severing links can strictly increase the payoff of the members of the coalition, where members of the coalition can add links only among them, but they can sever links with all the agents in the network.

[4] In an alternative approach, one could consider network formation as a noncooperative game, in line with Myerson (1991). Firms simultaneously propose the subset of agents they want to be connected with, and links are formed only when the proposals are reciprocated. However, Nash equilibrium is too weak as a solution of concept, due to the coordination problem that arises for the required double coincidence of wants for the formation of a link. The refinement of undominated Nash equilibrium, which is sometimes used in the literature (Goyal and Joshi, 2003), is not of particular help here, because only the empty set as a strategy is weakly dominated for all the parameters values. In the four firms industry, all the symmetric networks can be sustained as Nash equilibrium of the link formation game, and all the symmetric networks but the empty network can be sustained as undominated Nash equilibrium for some range of the parameters. Finally, it is worth noting that the notion of strongly stable networks we discuss in the text coincides with the notion of strong Nash equilibrium in the link formation game.

Formally, strong stability is defined as follows:

Strong stability*: define $S \subseteq N$ as a coalition in N. A network g' is obtainable from g via S if:*

(i) $ij \in g'$ and $ij \notin g$ implies $\{i, j\} \subset S$.

(ii) $ij \in g$ and $ij \notin g'$ implies $\{i, j\} \cap S \neq \emptyset$.

A network g is strongly stable if there are no coalitions S and network g' obtainable from g via S for which $\Pi_i(g') > \Pi_i(g)$, for all $i \in S$.

This definition of stability is strict, and consequently the existence of strongly stable networks is not guaranteed. When existing, strongly stable networks have nice properties. In particular, strongly stable networks are by definition Pareto efficient. The definition of strong stability that we use here (which is taken from Dutta and Mutuswami, 1997) does not imply pairwise stability as defined in section 3 (which is the original definition by Jackson and Wolinski, 1996): in the former, establishing a new link is an admissible deviation only if both firms are strictly better off; in the latter, one agent can be weakly better off. However, the implication does not hold only for parameters values that constitute the borders between areas of stability of different network structures.[5]

5.1 Strong stability in the four firms' industry

In the case of four firms, only one symmetric network turns out to be pairwise stable. Then, we simply need here to verify if (and when) the complete network, which is always pairwise stable, is also strongly stable.

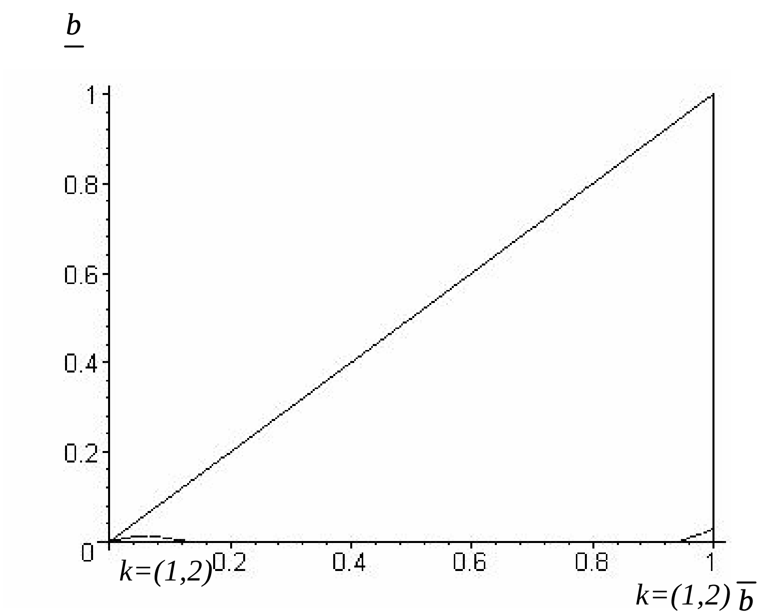
The results are summarized in proposition 8. In proving Proposition 3, we will refer to a particular asymmetric structure, the *triangle* (denoted with *tr*), where we have a fully connected component of three firms (say 1, 3 and 4, with 3 and 4 belonging to the same

[5] We will show in the next subsections why is preferable to adopt this version of strong stability. Another alternative would be to modify the definition of pairwise stability, again with minor differences.

technological group) and one firm (firm 2) is isolated. In equilibrium, profits are $\Pi_1(tr), \Pi_2(tr)$ and $\Pi_3(tr)$ (the positions of firm 3 and 4 are symmetric).

Proposition 8*: the complete network is almost never strongly stable, except that for very low technological opportunities or very high technological heterogeneity. There exists a function $H_9(\overline{b}, \underline{b}) = \Pi_3(tr) - \Pi(g^{(1,2)})$ such that, for all the values of $\overline{b}$ and $\underline{b}$ for which $H_9(\overline{b}, \underline{b}) > 0$ the complete network is not strongly stable.*

Figure 6: strongly stable networks in the four firms industry



Proposition 8 is very close to a non-existence result: for a largely predominant subset part of the parameter space, the complete network is not strongly stable, so that there are no symmetric networks that are strongly stable. Nevertheless, the result has interesting economic implications for the nature of the coalition and the deviation that turns out to be profitable. Except that for a very limited small area in the parameter space, three firms have the incentive to sever jointly their links towards the fourth firm, creating an asymmetric market structure where three, "networked" firms have a dominant position in the product market. In particular, while firm 1 (which is the only firm in its technological group to have connections) always prefers to be in the triangle network, firm 2 and firm 3 do for the range of parameters shown in the figure. Furthermore, when $\overline{b}$ and $\underline{b}$ are sufficiently high, the isolated firms is forced out of the market $(q_2 = 0)$.

This result is interesting because it confirms the importance of asymmetric network structures, as shown by the three firms' analysis, and the role played by collaborative ventures in creating *ex post* asymmetries in *ex ante* symmetric situations. A natural question then is when the triangle network turns out to be pairwise stable. It can be shown that for a significant range of parameters (in particular, when technological opportunities are high or technological heterogeneity is high) the triangle network is not pairwise stable because connected firms prefer to form the link with the isolated firm.[6] This leads towards the formation of a complete network, where profits for such firms are generally lower. Although the model is purely static, it suggests a dynamic story in which firms have the *private* incentive to form very dense networks, but then they have the *"collective"* incentive to sever the links towards one firm, to exclude it from the network and create an asymmetric market structures. This has two consequences: it suggests instability of cooperative ventures, and a cycle in alliances formation. Both aspects are consistent with empirical evidence (Kogut, 1988; Hagedoorn, 2002).

5.2 Strong stability in the three firms' industry

In the case of three firms, three structures turn out to be pairwise stable: the complete network, the partially connected network of type 1 and the partially connected network of type 2.

Proposition 9 summarizes the results about strong stability in the three firms' industry.

Proposition 9*: the partially connected network of type 2 is never strongly stable. The partially connected network of type 1 is always strongly stable, when is pairwise stable. The complete network is strongly stable only when technological opportunities are low. There exists a function $H_{10}(\overline{b}, \underline{b}) = \Pi_1(p1) - \Pi_1(c)$ such that the complete network is strongly stable for all values of $\overline{b}$ and $\underline{b}$ for which $H_{10}(\overline{b}, \underline{b}) < 0$.*

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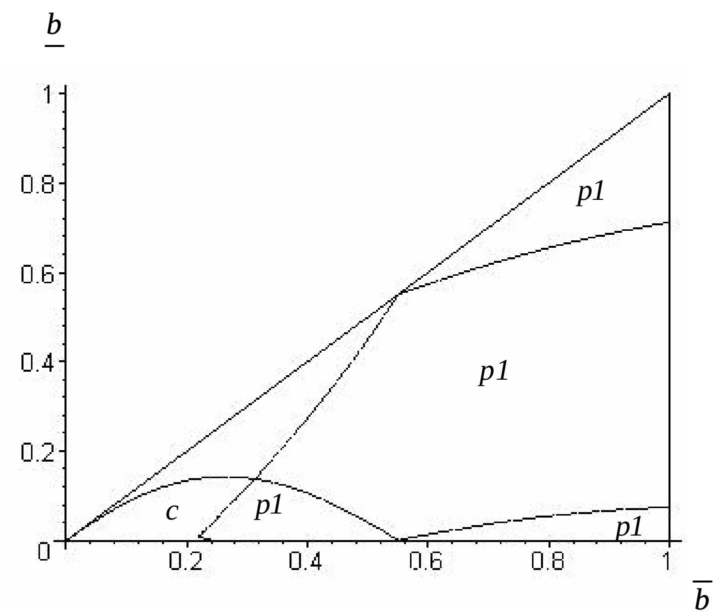
[6] The graphical representation of pairwise stability for the triangle network is reported in the appendix.

Figure 7: strongly stable networks in the three firms industry



In the three firms' case, the complete network, when it is pairwise stable, is very often not robust to a deviation by two firms in different technological group (say firm 1 and 3), which form a coalition and sever jointly the link with firm 2. Apart a small area where technological opportunities are low, profits of connected firms in *p1* are higher than the profits of firm 1 in a complete network.[7]

A partially connected network of type 2 is never strongly stable because firms 2 and 3 have the incentive to substitute their current partner with firm 1.[8]

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[7] The firm in the other technological group always gains a higher profit in the partially connected network of type 1.

[8] The same emphasis on the partially connected network is obtained if one refers to a dynamic model of network formation (Watts, 2001; Jackson and Watts, 2002).

Consider the following algorithm for network formation, adapted from Watts (2001). Start from the empty network at *t=0*, and suppose to be in the range of parameters where the partially connected network of type 1 is pairwise stable. From then on, each period a pair of firms is drawn. The two firms can form a link between them, if not existing, or sever the link, if already existing. The agreement is required only to form a new link. Firms form and sever links on the basis of comparison with profits associated with the existing network structure. Firms are myopic: they do not consider the effect of their decision on subsequent choices. The process continues until a stable network is reached. Then firms invest in R&D and market competition occurs.

It is straightforward to see that, under this algorithm, the complete network can emerge only for relatively small class of histories. In particular, apart the consecutive revision of the same link, the complete network emerges only if the sequence is 23-13-12 or 23-12-13. Instead, the partially connected network

The partially connected network of type 1 is always strongly stable, because there are no deviations that can make any pair of agents strictly better off. Moreover, firm 2 has no incentive to move to a complete network either.[9]

The analysis of strongly stable networks clearly points out the partially connected of type 1 as a natural solution for the process of network formation. This is interesting for several reasons.

First, on the empirical side, the special role played by this asymmetric network is consistent with the empirical analysis that underlies the motive of altering market structure as an important rationale for interfirm technological agreements (Hagedoorn, 1993). Also the results from the analysis of the four firms' case are in line with this evidence.

Second, the firm in group 2 that "succeeds" in forming the link obtains an advantage in terms of profits, a gain this is increasing in $\vec{b}$. This leads naturally to consider the strong competition occurring between the two firms in the larger technological group. There are two ways to tackle this issue. First, one can take the model as it is and solve the problem of multiple equilibria invoking a role for "historical accidents" and path-dependence, in a way that is similar to the one in Zirulia (2004). "Random" events (like social contacts or geographical proximity) leads one firm in group 2 to form a link with 1, with long lasting effects on firms' performance. It is interesting to observe that some business scholars (for instance, Gulati *et al.*, 2000) have underlined the importance for firms to "rush" and form alliances with the "right" partners in the early phases of technological or industrial cycles. Our simple model is consistent with this view. The second solution is to explicitly model such a competition, supposing for instance a role for side payments that allows firm 1 to exploit its strong bargaining power. If side-payments are allowed, we can expect that the firm excluded by the network would "undercut" the other firm, transferring part of the surplus of being connected to firm 1.

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of type 1 is immediately obtained whenever the first two firms forming a link are firms 1 and 2 or firms 2 and 3.

[9] If one uses a notion of strong stability where agents in a deviating coalition may be weakly better off, the partially connected network of type 1 is never strongly stable, because the coalition of 1, which is indifferent between the two partners, and the excluded partner from the network is winning.

In this view, firm 1 would exploit the “scarcity” of its technological resources in terms of performance also under this architecture.

Third, in terms of policy, we can observe how the partially connected network is welfare maximizing only when technological opportunities are high. There is a significant area in the technological space (with high technological heterogeneity) where welfare is maximized by the complete network. If technological opportunities are not too high, a dense network is not detrimental to R&D efforts, and consequently it has beneficial effects on consumer surplus. However, firms have the incentive to alter market structure in their favor, excluding one firm from the network. In this case, there is possibly room for public intervention to favor industry-wide cooperation.

6. Conclusions and plan for future work

The goal of this paper was to extend the analysis of R&D network formation in a setting when technological heterogeneity among firms is considered. First (Section 3 and 4), the results were derived in terms of pairwise stability, aggregate profits and social welfare associated with different network structures. We wanted to consider the robustness of Goyal and Moraga’s results to a modification that seems empirically relevant. We consider two classes of networks. First, we consider symmetric networks in a four firms industry. The complete network is always the only symmetric stable network. Firm have always the incentive of altering the market structure adding a new link, when network is not complete. Aggregate profits and social welfare are also maximized by a complete network, if technological opportunities are not too high, so that private and social incentives are aligned in these cases. Otherwise, less dense networks are optimal from firms’ and society point of view. In the class of asymmetric networks, for which the analysis has been performed in the case of three firms, technological heterogeneity matters. Only the complete and the partially connected networks are possibly stable, but which network is stable actually depends on the level of heterogeneity and technological opportunities. Firms belonging to the smaller technological group (having unique technological resources) obtain a special position in the industry, since they can guarantee the maximum profits in the industry in every stable network. The complete and partially connected networks are also the possible welfare and aggregate profit maximizing networks, but social and private incentives do not generally coincide. When technological opportunities are high, the partially connected network involving two firms of different technological groups is pairwise stable and it maximizes aggregate profits and social welfare.

In section 5, we consider the refinement of strong stability, where all the possible deviations by coalitions of agents are allowed. It turns out that, in the four firms' case, the complete network is very rarely strongly stable, because a coalition of three firms has the incentive to isolate the fourth firm and create an asymmetric market structure. In the three firms' case, the partially connected network where two firms in different technological group are linked is for a large subset of parameter space the only strongly stable network.

In this paper we made a number of restrictive assumptions. In particular, we considered the role of technological heterogeneity independently from the nature and intensity of competition and we kept the assumptions of homogenous good and Cournot competition. Furthermore, we consider a simple representation of technological heterogeneity, allowing only for two types of firms. For the future, we plan to develop a model where firms are located in a technological space that affects both the intensity of competition and the effects of information sharing, and study the stability and efficiency properties of the networks as a function of firms' localization.

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References

d'Aspremont, C. and Jacquemin, A. (1988), "Cooperative and noncooperative R&D in duopoly with spillovers", *American Economic Review*, 78,1133-1137.

Bloch, F. (1995), "Endogenous structures of Association in Oligopolies" *RAND Journal of Economics*, 26,537-556.

Cohen, W. and Levinthal, D. (1989), "Innovation and learning: the two faces of Research and development", *The Economic Journal*, 99, 569-596.

Dutta, B. and Mutuswami, S. (1997) , "Stable networks", *Journal of Economic Theory*, 76,322-344.

Goyal, S. and Joshi, S. (2003), "Networks of Collaboration in oligopoly, *Games and Economic behavior*,43, *57-85*.

Goyal, S and Moraga, J. (2001), "R&D Networks", *RAND Journal of Economics*, 32, 686-707.

Goyal, S. and Konovalov, A. and Moraga, J. (2004), "Hybrid R&D", mimeo.

Gulati, R Nohria, N. and Zaheer, A. (2000), "Strategic networks", *Strategic management Journal* 21, 203-215.

Hagedoorn, J. (1993), "Understanding the rationale of strategic technology partnering: inter-organizational modes of cooperation and sectoral differences" *Strategic management Journal*, 14, 371-385.

Hagedoorn, J. (2002), "Inter-firm R&D partnerships: an overview of major trends and patterns since 1960", *Research Policy*, 31,477-492.

Jackson, M. and Watts, A. (2002), "The evolution of social and economic networks" *Journal of economic theory*, 106, 265.-295.

Jackson, M. and Wolinski, A. (1996), "A Strategic Model of Social and Economic Networks", *Journal of Economic Theory*, 71, 44-74.

Jackson, M. and van den Nouweland, A. (2003), "Strongly Stable Network", *Games and Economic Behavior*, forthcoming.

Kamien, M. Mueller, M. and Zang, I. (1992), "Research joint ventures and R&D cartels", *American Economic Review*, 82, 1293-1306.

Katz, M. (1986), "An analysis of cooperative research and development", *RAND Journal of Economics*, 17, 527-543.

Kogut, B. (1988) "Joint ventures: theoretical and empirical perspectives", *Strategic Management Journal*, 9, 319-332.

Mariti, P and Smiley, R. (1983), "Cooperative agreements and the organization of industry", The Journal of Industrial Economics, 31, 437-451.

Myerson, R. (1991) "Game Theory: Analysis of Conflict". Harvard University Press.

Nelson, R. and Winter, S. (1982), "An evolutionary theory of economic change", Harvard University Press, Cambridge (MA).

Powell, W.W., Koput, K.W. and Smith-Doerr, L. (1996), "Interorganizational collaboration and the locus of innovation: networks of learning in biotechnology" *Administrative Science Quarterly*, 41, 116-145.

Suzumura, K.(1992), "Cooperative and noncooperative R&D in an oligopoly with spillovers" American Economic Review, 82, 1307-1320.

Watts, A. (2001)"A dynamic model of network formation", *Games and economic behavior*, 34, 331-341.

Zirulia, L. (2004) "The evolution of R&D networks", MERIT-Infonomics Research Memorandum, n°007.

Appendix

Proof of Proposition 1:

The proposition immediately derives from the following expression. Symmetric expression holds for k^{3-r}:

$$e(g^{(k^r,k^{3-r})})-e(g^{(k^r+1,k^{3-r})})=$$

$$\frac{(A-\bar{c})\underline{b}(n-k^{3-r}\overline{b}-k^r\underline{b})(n-k^{3-r}\overline{b}-k^r\underline{b}+\underline{b})}{[(n+1)^2-(n-k^{3-r}\overline{b}-k^r\underline{b})(1+k^{3-r}\overline{b}-k^r\underline{b})][(n+1)^2-(n-k^{3-r}\overline{b}-k^r\underline{b}+\underline{b})(1+k^{3-r}\overline{b}-k^r\underline{b})}>0$$

$$c(g^{(k^r,k^{3-r})})-c(g^{(k^r+1,k^{3-r})})=$$

$$\frac{(A-\bar{c})(n+1)^2\underline{b}(n-2k^{3-r}\overline{b}-2k^r\underline{b}-\underline{b}-1)}{[(n+1)^2-(n-k^{3-r}\overline{b}-k^r\underline{b})(1+k^{3-r}\overline{b}-k^r\underline{b})][(n+1)^2-(n-k^{3-r}\overline{b}-k^r\underline{b}+\underline{b})(1+k^{3-r}\overline{b}-k^r\underline{b})}$$

which is positive only for $n-2k^{3-r}\overline{b}-2k^r\underline{b}+\underline{b}+1>0$

and finally,

$$\frac{\partial e_i}{\partial \underline{b}}=\frac{-(A-\bar{c})k^r[(n+1)^2-(n-k^{3-r}\overline{b}-k^r\underline{b})(n+k^{3-r}\overline{b}+k^r\underline{b}]}{(n+1)^2-(n-k^{3-r}\overline{b}-k^r\underline{b})(1+k^{3-r}\overline{b}-k^r\underline{b})}<0$$

Proof of Proposition 2: Pairwise stability of symmetric networks in the four firms industry

We report here a sketch of the proof of this proposition. All the computations and the relevant plots have been performed with the help of the software Maple, and they are available upon request (to: lorenzo.zirulia@unibocconi.it).

We assume, without loss of generality, that firm 1 and firm 2 belong to the same technological group 1, and firm 3 and 4 belong to the technological group 2. Then, the procedure is as follows:

- For each network (apart from isomorphic networks) one need to consider all the deviations that are considered in the notion of pairwise stability;
- this yields unit cost as a function of efforts for each firm, and consequently profit function;
- the first order conditions for representative firms (i.e. firms playing the same role in the network) are computed;
- the system of first order conditions is solved, invoking symmetry of effort for firms playing the same role in the network;
- equilibrium efforts are computed, and plugged into the profit function of deviating firms;
- equilibrium profits from the deviation and equilibrium profits in the symmetric network under consideration are compared.

The complete network is stable

In this case, the only deviation one needs to take into account is when two firms sever one link. It can be shown that independently from b, such a deviation is not profitable.

The empty network is not stable

In this case, the possible deviations are those where two firms form a link. It can be shown that for any strictly positive value of b, such a deviation is profitable. Furthermore, if $b > 3/2 - 1/2\sqrt{5}$, the solution is a corner solution where, for one isolated firm, *e=0* and *q=0.*

The network $k^r = 1, k^{3-r} = 1$ *is not stable*

In this case, the deviation in which two firms belonging to different technological group, say firm 1 and 4, form a link is profitable.

The network $k^r = 0, k^{3-r} = 1$ *is not stable*

In this case, the deviation in which two firms belonging to different technological group, say firm 1 and 4, form a link is profitable.

The network $k^r = 1, k^{3-r} = 0$ *is not stable*

In this case, the deviation in which two firms belonging to different technological group, say firm 1 and 4, form a link is profitable.

The network $k^r = 0, k^{3-r} = 2$ *is not stable*

In this case, it can be shown that the deviation in which two firms belonging to the same technological group, say firm 1and 2, form a link is profitable.

Proof of Proposition 5: Pairwise stability in the three firms industry

In this case, we need to take into account six types of structures, and studying the incentives of firms to move from one structure to other by forming or severing links.

Without loss of gener ality we assume that firm 1 belongs to technological group 1, while firm 2 and 3 belong to technological group 2. Computations show that:

The empty network is never stable

Any pair of firms has the incentive to form a link and moving to a partially connected network of type 1 or 2, for any strictly positive value of b .

A star network of type 1 is never stable

For any strictly positive value of $\underline{b}$, firm 2 and firm 3 find convenient to form a link, and transform the star 1 in a complete network.

A star network of type 2 is never stable

Expect that for high $\overline{b}$ and low $\underline{b}$, firm 1 would prefer to form a link with firm 3 (which is always willing to form such a link) and make the star network of type 2 a complete network. Furthermore, except that for very low values both of $\overline{b}$ and $\underline{b}$, firm 2 (supposed to be the hub in the star) wants to sever the link with firm 3 and make the network a partially connected network of type 1. It can be shown that the area in the parameter spaces for which the two deviations are not profitable do not intersect, so that there is always a profitable deviation.

A complete network is stable unless $\overline{b}$ is very high and $\underline{b}$ is very low.

There is a range of values (as reported in the paper) for which firm 1 would prefer to severe the link say with 3 and make the network a star of type 2. Firm 3 is never willing to sever such a link, while it is never profitable for firm 2 and firm 3 to sever their link.

A partially connected network of type 1 is stable unless technological opportunities are low and technological heterogeneity is limited.

In this case firm 1 and firm 3 never agree on forming the link between them (there are no values of $\underline{b}$ for which the double coincidence of wants hold). Firm 2 and firm 3 agree on forming a link between them (making the network a star of type 2) for the range of values of $\overline{b}$ and $\underline{b}$ specified in the paper. Indeed, firm 3 is always willing to form such a link.

A partially connected network of type 2 is stable if technological opportunities are high and technological heterogeneity is limited.

Firm 2 and 3 are never willing to sever their existing link. While firm 1 always agrees on forming a link with say firm 2, firm 2 gives its consent only for the range shown in the paper.

Proposition 9: Pairwise stability of the triangle network

Figure 8: Pairwise stability of the triangle network