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Mixed QCD-electroweak corrections to on-shell Z production at the LHC

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ABSTRACT

We present the first complete calculation of mixed QCD-electroweak corrections to the production of on-shell Z bosons in hadron collisions and their decays to massless charged leptons. Our computation is fully differential with respect to final state QCD partons and resolved photons, allowing us to compute any infra-red safe observable pertinent to the $pp \to Z \to l^+l^-$ process in the approximation that the Z boson is on shell. Although mixed QCD-electroweak corrections are small, at about the per mill level, we observe that the interplay between QCD-QED and QCD-weak contributions is subtle and observable-dependent. It is therefore not possible to avoid computing one or the other if $\mathcal{O}(\alpha_{EW}\alpha_s)$ precision is desired.

The production of lepton pairs in hadron collisions $pp \to l^+l^-$ has played and continues to play an important role in the exploration of the inner workings of the Standard Model (SM) and in ongoing attempts to access physics beyond it. A seminal 1970 paper by Drell and Yan [1] pointed out the connection between a theoretical description of this process and the parton model of deep-inelastic scattering. This observation initiated the development of the quantitative theory of lepton pair production in hadron collisions [2–4] and encouraged its experimental exploration [5,6]. Subsequent theoretical developments in perturbative QCD and in the SM resulted in a continuously improving description of this process and provided a solid foundation for ambitious experimental studies aiming at measuring the SM parameters with high precision and at constraining New Physics.

Indeed, the production of lepton pairs at the LHC is the process from which the mass of the W boson is expected to be determined with an astounding precision of about 5 MeV [7,8]. This process is also very important for constraining parton distribution functions [9,10] and for determining the electroweak mixing angle [11,12]. Finally, it can be used to constrain higher-dimensional operators which parametrize deviations from the SM by studying the invariant mass distribution of the dilepton system at high $\mathcal{O}(1\ \text{TeV})$ invariant masses [13–16]. An obvious pre-requisite for the success of this challenging research program is the existence of a reliable theoretical description of all aspects of lepton pair production in hadron collisions.

A central role in providing such a description is played by perturbative calculations in the Standard Model. Currently, the fully-differential cross sections for dilepton production in hadron collisions are known through next-to-next-to-leading order (NNLO) in perturbative QCD [17–27] and through next-to-leading order (NLO) in the electroweak theory [28–37]. Recently the inclusive cross section of the process $pp \to \gamma^* \to l^+l^-$ has been computed through N^3LO in perturbative QCD [38]. Important steps in further increasing precision are the extension of this N^3LO result to the case of Z and W production and the calculation of the so-called mixed QCD-electroweak $\mathcal{O}(\alpha_{EW}\alpha_s)$ corrections. The latter class of corrections is the subject of the present paper.

The computation of mixed QCD-electroweak corrections is made complicated by the fact that they require broad technical expertise. Indeed, on the one hand, one has to compute two-loop three- and even four-point functions with massive internal and external particles yet, on the other hand, a detailed understanding of infra-red and collinear singularities and their regularization is also needed.

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There is quite a number of different physical aspects of dilepton pair production in hadron collisions that gets reflected in technical complexities of theoretical computations. This implies that by choosing a suitable *physical problem*, one may scale the technical complexity up or down. Indeed, if one focuses on QCD-electroweak corrections to the full production of dileptons at e.g. high invariant masses, two-loop virtual corrections involve box diagrams that depend on several mass scales. The corresponding master integrals have been computed recently [39–41] and the scattering amplitudes are still not available. On the contrary, if one focuses on on-shell Z or W production, the cross-talk between production and decay stages of the process is suppressed[1] [43,44] so that the most complicated two-loop contributions one has to consider are two-loop corrections to the $q\bar{q}' \to Z(W)$ vertex. Similarly, if one considers the production of on-shell Z bosons, the regularization of infra-red and collinear singularities in mixed contributions simplifies since NNLO-like emissions of a photon and a gluon can only happen in the production stage. In this sense, the treatment of infra-red and collinear singularities is very similar to what happens when computing NNLO QCD corrections to Z boson production.

Thanks to these significant technical simplifications, it is quite natural that physical results for mixed QCD-electroweak corrections to $pp \to l^+l^-$ started to appear in the context of on-shell Z boson production. In Ref. [45] it was pointed out that a simple modification of colour factors in an analytic result for NNLO QCD corrections to the total cross section of $pp \to l^+l^-$ [46–48] allows one to obtain mixed QCD-QED corrections to the total cross section of dilepton production. In Ref. [49] some of us performed a fully-differential computation of these QCD-QED corrections to Z production and decay into a pair of massless leptons adapting the soft-collinear subtraction scheme [50] developed for NNLO QCD computations to describe mixed QCD-QED effects. Similar calculations were reported in Refs. [51,52] within the q_T slicing framework.

The next natural step is to extend these results to include mixed QCD-*weak* corrections to the description of on-shell Z production and its subsequent decay to a pair of massless electrons. A first step in this direction was done in [53]. The main of focus of the current paper is to consider the process $pp \to Z \to e^+e^- + X$ at full $\mathcal{O}(\alpha_{EW}\alpha_s)$, in the approximation that the Z boson is on shell and electrons are massless. From the phenomenological point of view, the knowledge of mixed corrections to on-shell Z production is perhaps not extremely interesting but the calculation of these corrections is a good starting point for the analysis of the much more interesting case of the W boson production.

Since, by definition, weak corrections include exchanges of massive gauge bosons, mixed QCD-weak corrections do not contribute to genuine NNLO infra-red and collinear divergences; all such divergences reside in mixed QCD-QED corrections which have already been studied in Ref. [49]. Hence, from a technical point of view, the inclusion of mixed QCD-weak corrections requires the computation of one- and two-loop mixed QCD-weak contributions to e.g. $q\bar{q} \to Z + g$ and $q\bar{q} \to Z$ amplitudes as well as their renormalization.

We computed two-loop QCD-electroweak corrections to the $q\bar{q} \to Z$ vertex using standard techniques and found agreement with available results in the literature [54].[2] We extracted the ingredients required for the two-loop mixed QCD-electroweak renormalization from Ref. [55].[3] We obtained one-loop weak corrections to $q\bar{q} \to Z + g$ and related partonic channels numerically using the OpenLoops package [56–58]. In OpenLoops scalar integrals are provided by [59,60]. The renormalization of weak corrections is performed in the G_μ scheme[4]; the strong coupling constant is renormalized in the $\overline{\text{MS}}$ scheme. Numerically, we use $G_F = 1.16639 \times 10^{-5}\ \text{GeV}^{-2}$, $M_Z = 91.1876$ GeV, $M_W = 80.398$ GeV, $M_t = 173.2$ GeV and $M_H = 125$ GeV as input parameters. With this setup, we obtain $1/\alpha = 132.338$ for the fine-structure constant. We use the NNLO NNPDF3.1luxQED [62–64] parton distribution functions for *all* numerical computations. The value of the strong coupling constant is provided as part of the PDF set; numerically it reads $\alpha_s(M_Z) = 0.118$.

We also employ standard kinematic selection criteria by requiring that the transverse momenta of the two leptons satisfy $p_{t,l} > 24(16)$ GeV for the harder (softer) lepton. The rapidities of the two leptons should satisfy $-2.4 < y_l < 2.4$. Finally, since we neglect lepton masses, we have to *define* photons and leptons in a way that is robust against the collinear splittings $e \to e + \gamma$. To this end, leptons and photons are clustered into "lepton jets" provided that the angular distance $R_{e\gamma} = \sqrt{(y_e - y_\gamma)^2 + (\varphi_e - \varphi_\gamma)^2}$ between e and γ is smaller than 0.1 [65]. The reconstructed dilepton system is required to have an invariant mass greater than 50 GeV. For all results reported below, we choose the renormalization scale of the strong coupling constant and the factorization scale in parton distributions to be $\mu_R = \mu_F = M_Z/2$. Additional results for the scale choice $\mu_R = \mu_F = m_Z$ are shown in the Appendix.

We compute the production of the Z boson in the narrow width approximation. We find it convenient to re-write the differential cross section as

$$\mathrm{d}\sigma_{pp\to e^+e^-} = \mathrm{Br}(Z \to e^+e^-)\, \mathrm{d}\sigma_{pp\to Z} \frac{\mathrm{d}\Gamma_{Z\to e^+e^-}}{\Gamma_{Z\to e^+e^-}}, \tag{1}$$

factoring out the branching fraction $\mathrm{Br}(Z \to e^+e^-)$. We *do not* perform a perturbative expansion of $\mathrm{Br}(Z \to e^+e^-)$. In what follows, we will consider ratios of cross sections and kinematic distributions, so the branching ratio drops from our results.[5] All other contributions in Eq. (1) are expanded in powers of α_{EW} and α_s. For further details, the reader should consult Ref. [49].

To present our results, we expand the cross section of the process $pp \to Z \to e^+e^-$ in series in α_s and α_{EW}

$$\begin{aligned}\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{QCD}} + \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{EW}} \\ + \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{QCD-QCD}} + \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{QCD-EW}} + \ldots\end{aligned} \tag{2}$$

The new result that we describe in this paper is the mixed QCD-electroweak contribution $\mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{QCD-EW}}$. This contribution is the sum of QCD-QED and QCD-weak corrections; in what follows we will show these contributions separately.

We find it convenient to quote ratios of NLO electroweak and NNLO contributions to the NLO QCD differential cross section. Hence, we define

$$\mathrm{d}\Delta^i = \frac{\mathrm{d}\sigma^i}{\mathrm{d}\sigma_{\mathrm{LO}} + \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{QCD}}}, \tag{3}$$

where $i \in \{\mathrm{EW}, \mathrm{QCD-EW}, \mathrm{QCD-QCD}\}$, and EW can be further split in QED and *weak*. Furthermore, we will also show corrections to the production stage by themselves.

We begin by discussing corrections to the total (inclusive) cross section where no restrictions on the kinematics of the final state

[1] A concise discussion of physical reasons behind this suppression can be found in Ref. [42].

[2] We note that we do not include the finite part of two-loop contributions involving exchanges of virtual top quarks. At one-loop these contributions amount to about 10 percent of the full one-loop weak virtual correction.

[3] We note that there is a small typo in Eq. (5.4) of this reference.

[4] See e.g. Ref. [61] for a review.

[5] We note that mixed QCD-electroweak corrections to $\mathrm{Br}(Z \to e^+e^-)$ can be extracted from Ref. [66].

Table 1
Corrections to the total cross section of $pp \to Z \to e^+e^-$ in the narrow width approximation at the 13 TeV LHC. See text for further details.

Type	Inclusive	Cuts	Cuts (production)
$\Delta_{\rm NLO}^{\rm QED}$	$+2.3 \times 10^{-3}$	-5.3×10^{-3}	$+2.2 \times 10^{-3}$
$\Delta_{\rm NLO}^{\rm weak}$	-5.5×10^{-3}	-5.0×10^{-3}	-5.0×10^{-3}
$\Delta_{\rm NLO}^{\rm EW}$	-3.2×10^{-3}	-1.0×10^{-2}	-2.8×10^{-3}
$\Delta_{\rm NNLO}^{\rm QCD-QCD}$	$+1.3 \times 10^{-2}$	$+5.8 \times 10^{-3}$	$+5.8 \times 10^{-3}$
$\Delta_{\rm NNLO}^{\rm QCD-QED}$	$+5.5 \times 10^{-4}$	-5.9×10^{-3}	$+1.4 \times 10^{-4}$
$\Delta_{\rm NNLO}^{\rm QCD-weak}$	-1.6×10^{-3}	-2.1×10^{-3}	-2.1×10^{-3}
$\Delta_{\rm NNLO}^{\rm QCD-EW}$	-1.1×10^{-3}	-8.0×10^{-3}	-2.0×10^{-3}

particles are applied. The corresponding results for the 13 TeV LHC are shown in the second column of Table 1. We observe that NNLO QCD corrections exceed mixed QCD-electroweak ones by almost one order of magnitude. Interestingly, mixed corrections are dominated by weak ones; they are larger than mixed QCD-QED corrections by almost a factor of three. Moreover, there is a cancellation between QCD-QED and QCD-weak corrections so that the combined QCD-electroweak effect is about one permille. Note that since we factorize the branching ratio $\mathrm{Br}(Z \to e^+e^-)$, corrections to the decay have no bearing on the inclusive cross section so that results in Table 1 can be regarded as corrections to the inclusive process $pp \to Z$. We note that mixed QCD-electroweak corrections do not make an appreciable change to the scale uncertainty, which is still dominated by NNLO QCD contributions. For this reason, an assessment of how the present calculation reduces theoretical uncertainties on the Z boson production cross section will strongly depend on the quality of available QCD predictions. Hence, the completion of N^3LO QCD calculations for this class of processes becomes even more relevant.

The results change significantly when cuts to final state leptons are applied, see third column in Table 1. First, NNLO QCD corrections decrease so strongly that mixed QCD-electroweak contributions become very relevant. This is the consequence of an accidental cancellation between $q\bar{q}$ and qg channels that appears to be quite dramatic once fiducial cuts are applied and the renormalization and factorization scales $\mu = M_Z/2$ are chosen. For example, using a scale $\mu = M_Z$ results in a significantly larger NNLO QCD correction relative to the $\mu = M_Z/2$ values, see Table 2 in the Appendix. Returning to Table 1, among electroweak corrections the change mostly concerns QED corrections which flip sign relative to the inclusive case and increase by an order of magnitude. The latter issue is well-known since QED corrections to Z decays appear to be quite unstable for the set of fiducial cuts defined earlier. However, to the best of our knowledge a thorough study of how to ameliorate this situation has not been done yet. The change in sign of the QED corrections implies that instead of a cancellation between QED and weak contributions occurring in the inclusive cross section, they add up in the case of the fiducial one. As the consequence, the QCD-electroweak corrections *exceed* the NNLO QCD corrections in this case.[6]

It is also useful to show the results for mixed corrections to the production stage only, considering decays of Z bosons in the leading-order approximation; this removes the dependence of the result on kinematic constraints on the leptons that are not well-described in perturbation theory. The corresponding results are shown in the fourth column of Table 1. It follows from this table that if we consider corrections to the production stage *only*, the behaviour of individual contributions looks better but when corrections are put together, mixed NNLO contributions turn out to be only thirty percent smaller than the NLO ones. The reason for this seems to be the smallness of the NLO corrections, caused by a partial cancellation between QED and weak ones inherent to the G_μ scheme, rather than an abnormal enhancement of the NNLO mixed QCD-electroweak contributions.

We turn to the discussion of kinematic distributions. In Fig. 1 relative corrections to the rapidity and transverse momentum of the reconstructed dilepton system are shown for the QCD-QED, QCD-weak and QCD-electroweak contributions. Left panes describe corrections to the full process that includes production and decays of Z bosons; in the right panes we show corrections to the production stage only. NNLO QCD corrections rescaled by a factor $1/10$ are also shown there, to put the relevance of other contributions into perspective. Similar to the inclusive case, we observe that *weak corrections* are often not negligible when compared to QED corrections and, in case of production, they are actually the dominant ones. At the same time, we also observe that the relative importance of NNLO QCD and mixed corrections depends on the observable and kinematic range. For example, in the central rapidity region NNLO QCD corrections are somewhat smaller than the mixed ones but the situation becomes opposite at large rapidities. Similarly, NNLO QCD corrections at large $p_{t,ll}$ are dominant whereas at smaller values of the transverse momenta NNLO QCD and mixed QCD-electroweak contributions may be comparable.

In Fig. 2 we show two distributions that depend on kinematic features of individual leptons. In the upper panes, we present the transverse momentum distribution of the hardest lepton; in the two lower ones we show the distribution in the Collins-Soper angle θ^* [67], in the rapidity window $0.6 < |y_{ll}| < 1.2$. This angle can be computed from lepton momenta in the laboratory frame using the following formula

$$\cos\theta^* = \frac{\mathrm{sgn}(p_{z,l^+l^-})(P_{l^-}^+ P_{l^+}^- - P_{l^-}^- P_{l^+}^+)}{\sqrt{m_{l^+l^-}^2 \left(m_{l^+l^-}^2 + p_{t,l^+l^-}^2\right)}}, \tag{4}$$

where $P_i^\pm = E_i \pm p_{i,z}$. Studies of the $\cos\theta^*$ distribution at the LHC allow for a precise determination of the weak mixing angle.

The major features of distributions shown in Fig. 2 are similar to what we have seen already in Table 1 and Fig. 1. When corrections to production and decay are included, mixed QCD-QED corrections play an important, sometimes the dominant role; when only corrections to the production stage are considered, weak effects become more pronounced than QED ones. In the case of the $\cos\theta^*$ distribution, weak and QED corrections have similar magnitude even in the case when full corrections to the $pp \to Z \to l^+l^-$ process are considered. As is well-known, the spikes in corrections to $p_{t,l}$ distributions are caused by an interplay of cuts on lepton momenta and the leading-order kinematic boundary $p_{t,l} < M_Z/2$. Not surprisingly, they are much more pronounced when QED corrections to decays are included.

Conclusions. We have presented the first complete computation of mixed QCD-electroweak corrections to the production of on-shell Z bosons in hadron collisions and their subsequent decay to a pair of massless electrons. We find that mixed corrections are about a few permille. The only exceptions are QCD-QED corrections to the inclusive process and QCD-QED corrections to the production stage – both at the inclusive level and in the fiducial region – which are smaller. However, corrections strongly depend on the imposed kinematic constraints and, in general, do not follow a clear hierarchy that would allow an approximate but reliable

[6] We emphasize again that this result strongly depends on the choice of the renormalization and factorization scales used to compute NNLO QCD corrections. As can be seen in Table 2, a scale choice $\mu = M_Z$ results in a NNLO QCD correction which is roughly a factor of two larger than the mixed QCD-electroweak corrections, with these kinematic cuts applied.

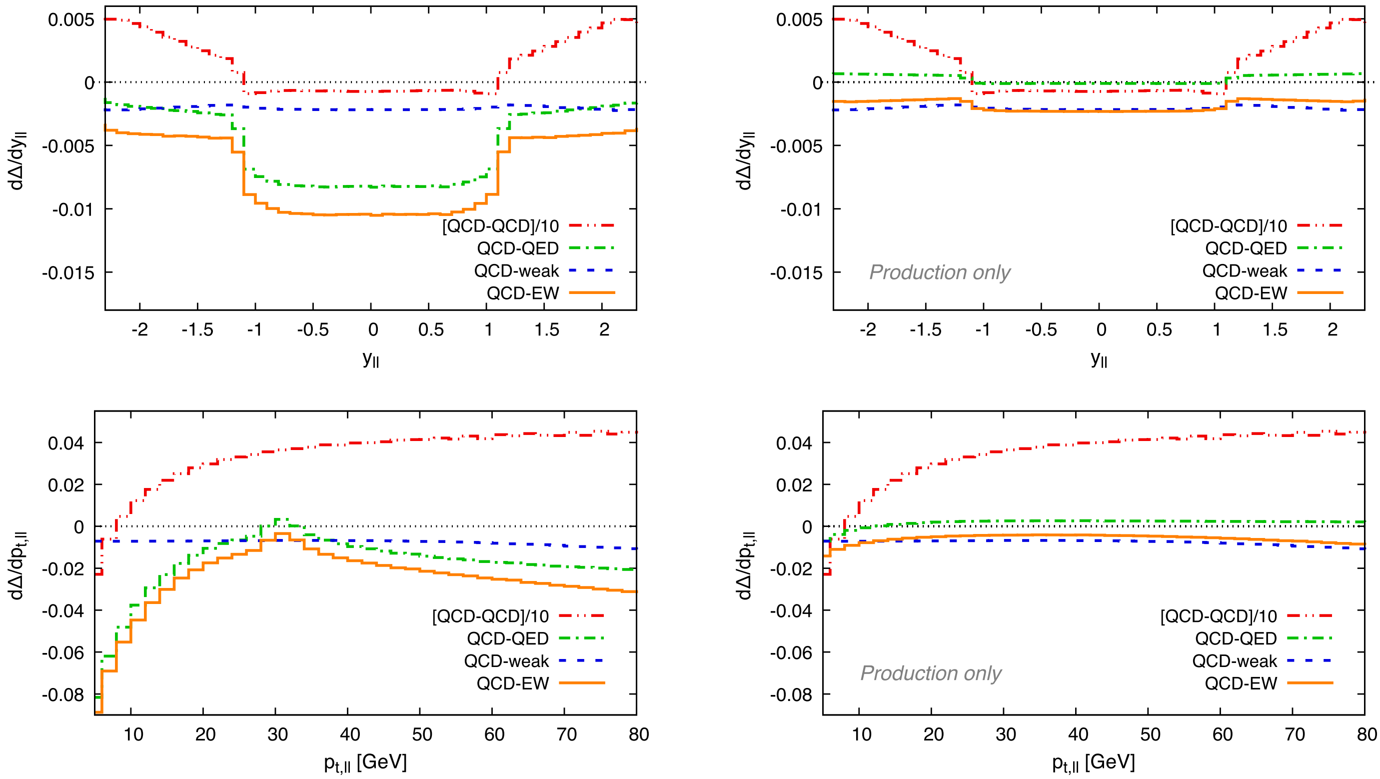

Fig. 1. Mixed QCD-electroweak corrections to dilepton rapidity and transverse momentum distributions at the 13 TeV LHC. Left pane includes corrections to both production and decay whereas right pane includes corrections to the production stage only. See text for details.

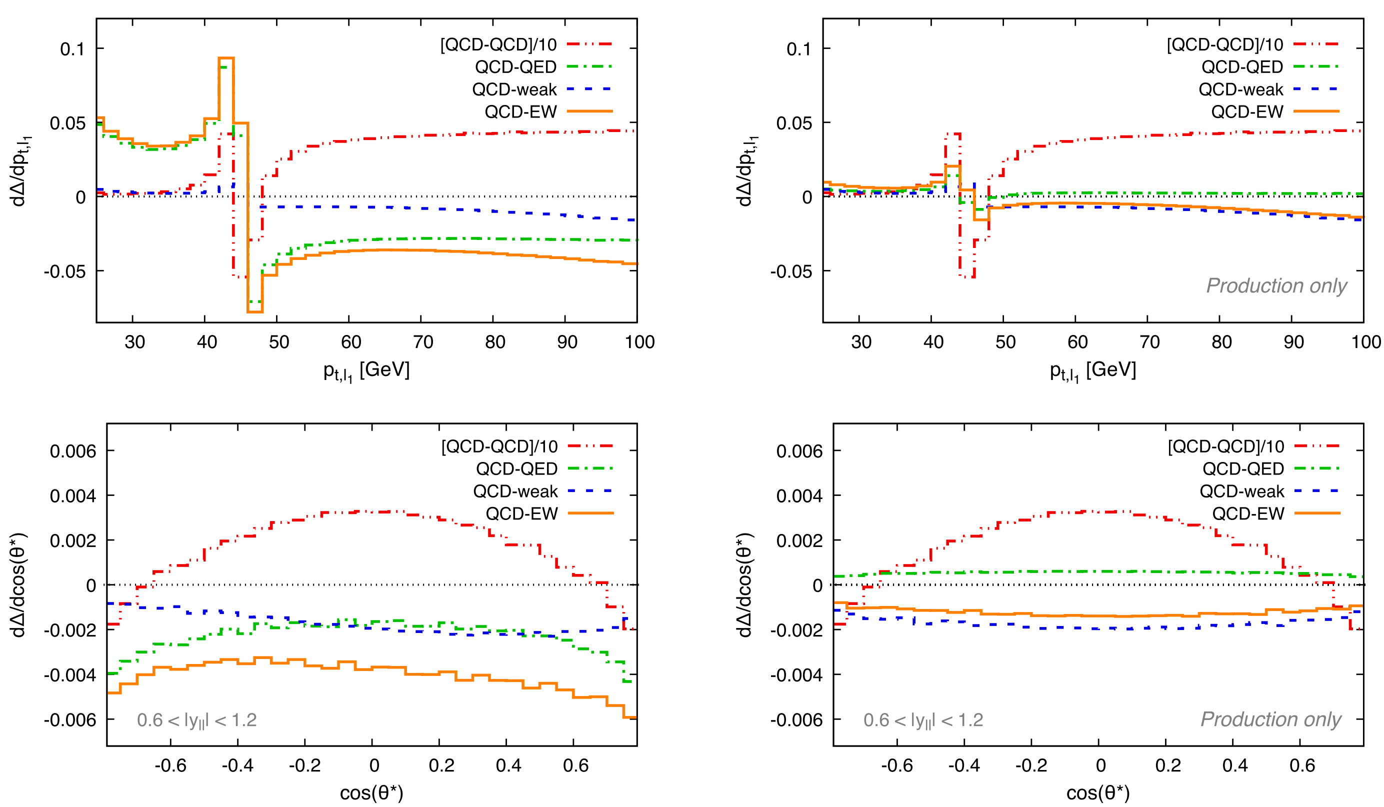

Fig. 2. Mixed QCD-electroweak corrections to distributions of the hardest lepton transverse momentum and Collins-Soper angle θ^* at the 13 TeV LHC. Left pane includes corrections to both production and decay whereas right pane includes corrections to the production stage only. See text for details.

treatment of them. As we mentioned in the introduction, given the smallness of these mixed corrections, the Z boson case is, perhaps, not very interesting phenomenologically. However, an ambitious goal of extracting the mass of the W boson from the LHC data

Table 2
Same as Table 1, but with $\mu_R = \mu_F = M_Z$.

Type	Inclusive	Cuts	Cuts (production)
$\Delta_{\rm NLO}^{\rm QED}$	$+3.1 \times 10^{-3}$	-5.5×10^{-3}	$+3.0 \times 10^{-3}$
$\Delta_{\rm NLO}^{\rm weak}$	-6.2×10^{-3}	-5.8×10^{-3}	-5.8×10^{-3}
$\Delta_{\rm NLO}^{\rm EW}$	-3.1×10^{-3}	-1.1×10^{-2}	-2.9×10^{-3}
$\Delta_{\rm NNLO}^{\rm QCD-QCD}$	-6.3×10^{-3}	-1.2×10^{-2}	-1.2×10^{-2}
$\Delta_{\rm NNLO}^{\rm QCD-QED}$	$+2.9 \times 10^{-4}$	-5.2×10^{-3}	-1.5×10^{-4}
$\Delta_{\rm NNLO}^{\rm QCD-weak}$	-9.2×10^{-4}	-1.3×10^{-3}	-1.3×10^{-3}
$\Delta_{\rm NNLO}^{\rm QCD-EW}$	-6.4×10^{-4}	-6.5×10^{-3}	-1.5×10^{-3}

with very high precision calls for a complete computation of mixed QCD-electroweak correction to the W production process. We look forward to this interesting challenge.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

In the main text we presented results for the scale choice $\mu_R = \mu_F = M_Z/2$. For completeness, we present results for the scale choice $\mu_R = \mu_F = M_Z$ in Table 2 above.

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