Konishi form factor at three loops in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

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We present the first results on the third order corrections to on-shell form factor (FF) of the Konishi operator in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory using Feynman diagrammatic approach in modified dimensional reduction ($\overline{DR}$) scheme. We show that it satisfies the KG equation in $\overline{DR}$ scheme while the result obtained in four dimensional helicity (FDH) scheme needs to be suitably modified not only to satisfy the KG equation but also to get the correct ultraviolet (UV) anomalous dimensions. We find that the cusp, soft and collinear anomalous dimensions obtained to third order are same as those of the FF of the half-BPS operator confirming the universality of the infrared (IR) structures of on-shell form factors. In addition, the highest transcendental terms of the FF of the Konishi operator are identical to those of half-BPS operator indicating the probable existence of deeper structure of the on-shell FF. We also confirm the UV anomalous dimensions of the Konishi operator up to third order providing a consistency check on the both UV and universal IR structures in $\mathcal{N} = 4$.

I. INTRODUCTION

The ability to accomplish the challenging job of calculating quantities is of fundamental importance in any potential mathematical theory. In quantum field theory (QFT), this manifests itself in the quest for computing the multiloop and multileg scattering amplitudes under the framework of perturbation theory. The fundamental quantities to be calculated in any gauge theory are the scattering amplitudes or the correlation functions. Recently, there has been a surge of interest to study the form factors (FFs) as they connect fully on-shell amplitudes and correlation functions. The FFs are a set of quantities which are constructed out of the scattering amplitudes involving on-shell states consisting of elementary fields and an off-shell state described through a composite operator. These are operator matrix elements of the form $\langle p_1^{\sigma_1}, \ldots, p_l^{\sigma_l} | \mathcal{O} | 0 \rangle$ where $\mathcal{O}$ represents a gauge invariant composite operator which generates a multiparticle on-shell state $| p_1^{\sigma_1}, \ldots, p_l^{\sigma_l} \rangle$ upon operating on the vacuum of the theory. p_i are the momenta and σ_i encapsulate all the other quantum numbers of the particles. More precisely, FFs are the amplitudes of the processes where the classical current or field, coupled through gauge invariant composite operator $\mathcal{O}$, produces some quantum state. Studying these quantities not only helps to understand the underlying ultraviolet and infrared structures of the theory, but also enables us to calculate the anomalous dimensions of the associated composite operator.

The Sudakov FFs ($l = 2$) in $\mathcal{N} = 4$ maximally supersymmetric Yang-Mills (SYM) theory [1,2] were initially considered by van Neerven in [3], almost three decades back, where a half-BPS operator belonging to the stress-energy supermultiplet, that contains the conserved currents of $\mathcal{N} = 4$ SYM, was investigated to 2-loop order:

$$\mathcal{O}_{\text{BPS}} = \phi_m^a \phi_n^a - \frac{1}{3} \delta_{mn} \phi_s^a \phi_s^a. \tag{1}$$

Very recently, this was extended to 3-loops in [4]. We will represent scalar and pseudoscalar fields by ϕ_m^a and χ_m^a, respectively. The symbol $a \in [1, N^2 - 1]$ denotes the SU(N) adjoint color index, whereas m, n stand for the generation indices which run from $[1, n_g]$. In $d = 4$ dimensions, we have $n_g = 3$. The sum over repeated index will be assumed throughout the paper unless otherwise stated. One of the most salient features of this operator is that it is protected by the supersymmetry (SUSY) i.e. the FFs exhibit no ultraviolet (UV) divergences but infrared (IR) ones to all orders in perturbation theory. In this article, our goal is to investigate the Sudakov FFs of another very important operator in the context of $\mathcal{N} = 4$ SYM, called the Konishi operator, which is not protected by the SUSY and consequently, exhibits UV divergences beyond leading order:

$$\mathcal{O}_K = \phi_m^a \phi_m^a + \chi_m^a \chi_m^a. \tag{2}$$

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The existence of UV divergences is captured through the presence of nonzero anomalous dimensions. This operator is one of the members of the Konishi supermultiplet and all the members of the multiplet give rise to same anomalous dimensions. The one and two loop Sudakov FFs of the Konishi operator were computed in [5] employing the on-shell unitarity method. In addition, the IR poles at 3-loop were also predicted in the same article using the universal behavior of those, though the finite part was not computed. In this paper, we calculate the full 3-loop Sudakov FFs using the Feynman diagrammatic approach. In the same spirit of the FFs in quantum chromodynamics (QCD), we examine the results in the context of the KG equation [6–9]. Quite remarkably, it has been found that the logarithms of the FFs satisfy the universal decomposition in terms of the cusp, collinear, soft and UV anomalous dimensions, exactly similar to those of QCD [10,11]. Except UV, which is a property of the associated operator, all the remaining universal anomalous dimensions match exactly with the leading transcendental terms of the corresponding ones in QCD upon putting $C_F = 2n_f T_f = C_A$. The quantities C_F and C_A are the quadratic Casimirs of the SU(N) gauge group in fundamental and adjoint representations, respectively. n_f is the number of active quark flavors and $T_f = 1/2$.

II. FRAMEWORK OF THE CALCULATION

The interacting Lagrangian encapsulating the interaction between off-shell state (J) described by $\mathcal{O}_{\mathrm{BPS}}$ or $\mathcal{O}_{\mathcal{K}}$ and the fields in $\mathcal{N} = 4$ SYM are given by

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{BPS}} = J_{\mathrm{BPS}}^{mn} \mathcal{O}_{\mathrm{BPS}}, \qquad \mathcal{L}_{\mathrm{int}}^{\mathcal{K}} = J_{\mathcal{K}} \mathcal{O}_{\mathcal{K}}. \tag{3}$$

We define the form factors at $\mathcal{O}(a^n)$ as

$$\mathcal{F}_f^{\rho,(n)} \equiv \frac{\langle \mathcal{M}_f^{\rho,(0)} | \mathcal{M}_f^{\rho,(n)} \rangle}{\langle \mathcal{M}_f^{\rho,(0)} | \mathcal{M}_f^{\rho,(0)} \rangle} \tag{4}$$

where, $n = 0, 1, 2, \ldots$ and a is the 't Hooft coupling [12]:

$$a \equiv \frac{g_{\mathrm{YM}}^2 C_A}{(4\pi)^2} (4\pi e^{-\gamma_E})^{-\frac{\epsilon}{2}}, \tag{5}$$

that depends on the Yang-Mills coupling constant g_{YM}, the loop-counting parameter and C_A. The quantity $|\mathcal{M}_f^{\rho,(n)}\rangle$ is the transition matrix element of $\mathcal{O}(a^n)$ for the production of a pair of on-shell particles $f\bar{f}$ from the off-shell state represented through ρ. For the case under consideration, we take $f = \phi_m^a = \bar{f}$, $\rho = \mathcal{K}$ and Bogomonlyi-Prasad-Sommerfeld (BPS) for $J_{\mathcal{K}}$ and J_{BPS}^{mn}, respectively. The full form factor in terms of the components (4) reads as

$$\mathcal{F}_f^{\rho} = 1 + \sum_{n=1}^{\infty} \left[a^n \left(\frac{Q^2}{\mu^2} \right)^{\frac{n\epsilon}{2}} \mathcal{F}_f^{\rho,(n)} \right]. \tag{6}$$

The transition matrix element also follows same expansion. The quantity $Q^2 = -2p_1.p_2$ and μ is introduced to keep the coupling constant a dimensionless in $d = 4 + \epsilon$ dimensions.

III. REGULARIZATION PRESCRIPTIONS

The calculation of the FFs in $\mathcal{N} = 4$ SYM theory involves a subtlety originating from the dependence of the composite operators on space-time dimensions d. Unlike the half-BPS operator $\mathcal{O}_{\mathrm{BPS}}$, the Konishi operator $\mathcal{O}_{\mathcal{K}}$ involves a sum over generation of the scalar and pseudoscalar fields and consequently, it does depend on d. The problem arises while making the choice of regularization scheme [5], which is necessary in order to regulate the theory for identifying the nature of divergences present in the FFs. Though the FFs of the protected operators are free from UV divergences in 4-dimensions, these do involve IR divergences arising from the soft and collinear configurations of the loop momenta.

For performing the regularization, there exist several schemes, the four dimensional helicity (FDH) [13,14] formalism is the most popular one where everything is treated in 4-dimensions, except the loop integrals that are evaluated in d-dimensions. In spite of its spectacular applicability, this prescription may fail to produce the correct result for the operators involving space-time dimensions [5], such as Konishi. However, this can be rectified and the rectification scenarios differ from one operator to another. According to the prescription prescribed in the article [5], in order to obtain the correct result for the Konishi operator, one is required to multiply a factor of $\Delta_{\mathcal{K}}^{\mathrm{BPS}}$ which is $\Delta_{\mathcal{K}}^{\mathrm{BPS}} = n_{g,\epsilon}/2n_g$ with the difference between the FFs of the Konishi and those of BPS i.e.

$$\mathcal{F}_f^{\mathcal{K}} = \mathcal{F}_f^{\mathrm{BPS}} + \delta \mathcal{F}_f^{\mathcal{K}}, \tag{7}$$

where, $\delta \mathcal{F}_f^{\mathcal{K}} = \Delta_{\mathcal{K}}^{\mathrm{BPS}} (\mathcal{F}_f^{\mathcal{K}} - \mathcal{F}_f^{\mathrm{BPS}})$. The second subscript of $n_{g,\epsilon}$ represents the dependence of the number of generations of the scalar and pseudoscalar fields on the space-time dimensions: $n_{g,\epsilon} = (2n_g - \epsilon)$. The prescription is validated through the production of the correct anomalous dimensions up to 2-loop. In this article, for the first time, this formalism is applied to the case of 3-loop FFs and is observed to produce the correct anomalous dimensions for the Konishi.

On the other hand, there exists another very elegant formalism, called modified dimensional reduction ($\overline{DR}$) [15,16], which is very much similar to the 't Hooft and Veltman prescription of the dimensional regularization and quite remarkably, it is universally applicable to all kinds of operators including the ones dependent on the space-time dimensions. In this prescription, in addition to treating everything in $d = 4 + \epsilon$ dimensions, the number of generations of the scalar and pseudo-scalar fields is considered as $n_{g,\epsilon}/2$ instead of n_g in order to preserve the $\mathcal{N} = 4$ SUSY throughout. The dependence on ϵ preserves SUSY

in a sense that the total number of gauge, scalar (n_g) and pseudo-scalar (n_g) degrees of freedom continues to remain 8. Within this framework, we have calculated the Konishi FFs up to 3-loop level and the results come out to be exactly same as the ones obtained in Eq. (7). This, in turn, provides a direct check on the earlier prescription. In the next section, we will discuss the methodology of computing the FFs.

IV. CALCULATION OF THE FORM FACTORS

The calculation of the FFs follows closely the steps used in the derivation of the 3-loop spin-2 and pseudoscalar FFs in QCD [17,18]. In contrast to the most popular method of on-shell unitarity for computing the scattering amplitudes in $\mathcal{N} = 4$ SYM, we use the conventional Feynman diagrammatic approach, which carries its own advantages in light of following the regularization scheme, to accomplish the job. The relevant Feynman diagrams are generated using QGRAF [19]. Indeed, very special care is taken to incorporate the Majorana fermions present in the $\mathcal{N} = 4$ SYM appropriately. For Konishi as well as half-BPS operator, 1631 number of Feynman diagrams appear at 3-loop order which include the scalar, pseudoscalar, gauge boson as well as Majorana fermions in the loops. The ghost loops are also taken into account ensuring the inclusion of only the physical degrees of freedom of the gauge bosons. The raw output of the QGRAF is converted to a suitable format for further calculation. Employing a set of in-house routines based on PYTHON and the symbolic manipulating program FORM [20], the simplification of the matrix elements involving the Lorentz, color, Dirac and generation indices is performed. In the FDH regularization scheme, except the loop integrals all the remaining algebra is performed in $d = 4$, whereas in $\bar{DR}$, everything is executed in $d = 4 + \epsilon$ dimensions. While calculating within the

framework of $\bar{DR}$, the factor of $1/3$ in the second part of $\mathcal{O}_{\text{BPS}}$, Eq. (1), should be replaced by $2/n_{g,\epsilon}$ to maintain its traceless property in d-dimensions.

The expressions involve thousands of apparently different 3-loop Feynman scalar integrals. However, they are expressible in terms of a much smaller set, called master integrals (MIs), by employing the integration-by-parts (IBP) [21,22] and Lorentz invariance (LI) [23] identities. Though, the LI are not linearly independent of the IBP [24], their inclusion however accelerates the procedure of obtaining the solutions. All the scalar integrals are reduced to the set of MIs using a *Mathematica* based package LITERED [25,26]. In the literature, there exists similar packages to perform the reduction: AIR [27], FIRE [28], REDUZE2 [29,30]. As a result, all the thousands of scalar integrals can be expressed in terms of 22 topologically different MIs which were computed analytically as Laurent series in ϵ in the articles [31–37] and are collected in the appendix of [38]. Using those, we obtain the final expressions for the 3-loop FFs of the $\mathcal{O}_{\text{BPS}}$ and $\mathcal{O}_{\mathcal{K}}$.

V. RESULTS OF THE FORM FACTORS

Employing the Feynman diagrammatic approach described in the previous section, we have first confirmed the form factor results for the $\mathcal{O}_{\text{BPS}}$ up to 3-loop level presented in [3,4] and for $\mathcal{O}_{\mathcal{K}}$ up to 2-loop [5]. In the present paper, we present only the ϵ expanded results for the $\mathcal{F}_\phi^{\mathcal{K},(i)}$, $i = 1$, 2, 3 [see Eq. (6)]. The exact results in terms of d and MIs are too long to present here and can be obtained from the authors. In order to demonstrate the subtleties involved in the choice of regularization scheme, we have expressed them in terms of δ_R which is unity in $\bar{DR}$ scheme and zero in FDH scheme.

$$\mathcal{F}_\phi^{\mathcal{K},(1)} = \frac{1}{\epsilon^2}\{-8\} + \frac{1}{\epsilon}\{12\} - 12 + \zeta_2 + \epsilon\left\{12 - \frac{7}{3}\zeta_3 - \frac{3}{2}\zeta_2\right\} + \epsilon^2\left\{-12 + \frac{7}{2}\zeta_3 + \frac{3}{2}\zeta_2 + \frac{47}{80}\zeta_2^2\right\} + \epsilon^3\left\{12 - \frac{31}{20}\zeta_5\right.$$
$$\left. - \frac{7}{2}\zeta_3 - \frac{3}{2}\zeta_2 + \frac{7}{24}\zeta_2\zeta_3 - \frac{141}{160}\zeta_2^2\right\} + \epsilon^4\left\{-12 + \frac{93}{40}\zeta_5 + \frac{7}{2}\zeta_3 - \frac{49}{144}\zeta_3^2 + \frac{3}{2}\zeta_2 - \frac{7}{16}\zeta_2\zeta_3 + \frac{141}{160}\zeta_2^2 + \frac{949}{4480}\zeta_2^3\right\}$$
$$+ \delta_R\left[-2 + 2\epsilon + \epsilon^2\left\{-2 + \frac{1}{4}\zeta_2\right\} + \epsilon^3\left\{2 - \frac{7}{12}\zeta_3 - \frac{1}{4}\zeta_2\right\} + \epsilon^4\left\{-2 + \frac{7}{12}\zeta_3 + \frac{1}{4}\zeta_2 + \frac{47}{320}\zeta_2^2\right\}\right],$$

$$\mathcal{F}_\phi^{\mathcal{K},(2)} = \frac{1}{\epsilon^4}\{32\} + \frac{1}{\epsilon^3}\{-96\} + \frac{1}{\epsilon^2}\{168 - 4\zeta_2\} + \frac{1}{\epsilon}\left\{-276 + \frac{50}{3}\zeta_3 + 24\zeta_2\right\} + 438 - 56\zeta_3 - 66\zeta_2 - \frac{21}{5}\zeta_2^2$$
$$+ \epsilon\left\{-681 - \frac{71}{10}\zeta_5 + 128\zeta_3 + 141\zeta_2 - \frac{23}{6}\zeta_2\zeta_3 + 15\zeta_2^2\right\} + \epsilon^2\left\{\frac{2091}{2} + \frac{84}{5}\zeta_5 - 314\zeta_3 + \frac{901}{36}\zeta_3^2 - \frac{519}{2}\zeta_2\right.$$
$$\left. + 26\zeta_2\zeta_3 - \frac{741}{20}\zeta_2^2 + \frac{2313}{280}\zeta_2^3\right\} + \delta_R\left[\frac{1}{\epsilon^2}\{16\} + \frac{1}{\epsilon}\{-28\} + 46 - 4\zeta_2 + \epsilon\left\{-73 + \frac{28}{3}\zeta_3 + 11\zeta_2\right\}\right.$$
$$\left. + \epsilon^2\left\{\frac{227}{2} - \frac{64}{3}\zeta_3 - \frac{47}{2}\zeta_2 - \frac{5}{2}\zeta_2^2\right\}\right],$$

$$\mathcal{F}_{\phi}^{\mathcal{K},(3)} = \frac{1}{\epsilon^6}\left\{-\frac{256}{3}\right\} + \frac{1}{\epsilon^5}\{384\} + \frac{1}{\epsilon^4}\{-960\} + \frac{1}{\epsilon^3}\left\{2112 - \frac{176}{3}\zeta_3 - 96\zeta_2\right\} + \frac{1}{\epsilon^2}\left\{-4368 + 312\zeta_3\right.$$

$$\left. + 504\zeta_2 + \frac{494}{45}\zeta_2^2\right\} + \frac{1}{\epsilon}\left\{8760 + \frac{1756}{15}\zeta_5 - 1056\zeta_3 - 1608\zeta_2 + \frac{170}{9}\zeta_2\zeta_3 - \frac{459}{5}\zeta_2^2\right\} - 17316 - \frac{1014}{5}\zeta_5$$

$$+ 3192\zeta_3 - \frac{1766}{9}\zeta_3^2 + 4158\zeta_2 - 195\zeta_2\zeta_3 + \frac{3789}{10}\zeta_2^2 - \frac{22523}{270}\zeta_2^3 + \delta_R\left[\frac{1}{\epsilon^4}\{-64\} + \frac{1}{\epsilon^3}\{160\}\right.$$

$$\left. + \frac{1}{\epsilon^2}\{-352 + 16\zeta_2\} + \frac{1}{\epsilon}\{728 - 52\zeta_3 - 84\zeta_2\} - 1460 + 176\zeta_3 + 268\zeta_2 + \frac{153}{10}\zeta_2^2\right], \tag{8}$$

where $\zeta_2 = \pi^2/6$, $\zeta_3 \approx 1.2020569$, $\zeta_5 \approx 1.0369277$, $\zeta_7 \approx 1.0083492$. The presence of the nonzero coefficients of δ_R signifies the shortcoming of the FDH scheme in case of Konishi operator. We observe that our results for $\delta\mathcal{F}_{\phi}^{\mathcal{K},(i)}$, $i = 1, 2, 3$ expressed in terms of d and MIs contain an overall factor $(6 - \delta_R\epsilon)/6$ explaining the necessity of correcting the results computed in FDH scheme by this factor advocated in [5].

VI. OPERATOR RENORMALIZATION

Though the $\mathcal{N} = 4$ SYM is UV finite i.e. neither coupling constant nor wave function renormalization is required, nevertheless the FFs of the composite unprotected operators, like Konishi, do involve divergences of the UV source which are captured by the presence of nonzero UV anomalous dimensions, γ^{ρ}. As a consequence, to get rid of the UV divergences, the FFs are required to undergo UV renormalization which is performed through the multiplication of an overall operator renormalization, $Z^{\rho}(a, \mu, \epsilon)$:

$$\frac{d}{d\ln\mu^2}\ln Z^{\rho} = \gamma^{\rho} = \sum_{i=1}^{\infty} a^i \gamma_i^{\rho}. \tag{9}$$

Since $\hat{a}_s = a_s(\mu_0/\mu)^{\epsilon}$, the solution to the above equation takes the simple form:

$$Z^{\rho} = \exp\left(\sum_{n=1}^{\infty} a^n \frac{2\gamma_n^{\rho}}{n\epsilon}\right). \tag{10}$$

The UV finite Konishi FFs are obtained as $[\mathcal{F}_f^{\mathcal{K}}]_R = Z^{\mathcal{K}}\mathcal{F}_f^{\mathcal{K}}$, whereas $[\mathcal{F}_f^{\text{BPS}}]_R = \mathcal{F}_f^{\text{BPS}}$. Since, this is a property of the associated composite operator, the γ^{ρ} and so Z^{ρ} are independent of the type as well as number of the external on-shell states. In the next section, we will discuss the methodology to obtain the γ^{ρ} for the Konishi type of operators in addition to discussing the IR singularities of the FFs.

VII. UNIVERSALITY OF THE POLE STRUCTURES

The FFs in $\mathcal{N} = 4$ SYM contain divergences arising from the IR region which show up as poles in ϵ. The associated pole structures can be revealed and studied in an elegant way through the KG-equation [6–9] which is obeyed by the FFs as a consequence of factorization, gauge and renormalization group invariances:

$$\frac{d}{d\ln Q^2}\ln\mathcal{F}_f^{\rho} = \frac{1}{2}[K_f^{\rho} + G_f^{\rho}]. \tag{11}$$

The Q^2 independent $K_f^{\rho}(a, \epsilon)$ contains all the poles in ϵ, whereas $G_f^{\rho}(a, Q^2/\mu^2, \epsilon)$ involves only the finite terms in $\epsilon \to 0$. Inspired from QCD [12,39,40], we propose the general solution to be

$$\ln\mathcal{F}_f^{\rho}(a, Q^2, \mu^2, \epsilon) = \sum_{j=1}^{\infty} a^j\left(\frac{Q^2}{\mu^2}\right)^{\frac{j\epsilon}{2}}\mathcal{L}_{f,j}^{\rho}(\epsilon) \tag{12}$$

with

$$\mathcal{L}_{f,j}^{\rho}(\epsilon) = \frac{1}{\epsilon^2}\left\{-\frac{2}{j^2}A_j\right\} + \frac{1}{\epsilon}\left\{\frac{1}{j}G_{f,j}^{\rho}(\epsilon)\right\} \tag{13}$$

where, $A = \sum_{j=1}^{\infty} a^j A_j$ are the cusp anomalous dimensions in $\mathcal{N} = 4$ SYM. The absence of the superscript ρ and subscript f signifies the independence of these quantities on the nature of composite operators as well as external particles. These are determined by looking at the highest poles of the $\ln F_f^{\rho}$ which are found to be

$$A_1 = 4, \qquad A_2 = -8\zeta_2, \qquad A_3 = \frac{176}{5}\zeta_2^2 \tag{14}$$

up to 3-loops which are consistent with the results presented in [41,42]. These are basically the highest transcendental parts of those of QCD [11,43–45]. The other quantities in Eq. (13), $G_{f,j}^{\rho}$ are postulated, like QCD [10,11], to satisfy

$$G_{f,j}^{\rho}(\epsilon) = 2(B_j - \gamma_j^{\rho}) + f_j + \sum_{k=1}^{\infty}\epsilon^k g_{f,j}^{\rho,k} \tag{15}$$

where, $B = \sum_{j=1}^{\infty} a^j B_j$ and $f = \sum_{j=1}^{\infty} a^j f_j$ are the collinear and soft anomalous dimensions in $\mathcal{N} = 4$ SYM which

are independent of the operators as well as external legs. For the $\mathcal{O}_{\mathrm{BPS}}$ and $\mathcal{O}_{\mathcal{K}}$, we obtain

$$\gamma_j^{\mathrm{BPS}} = 0, \quad \gamma_1^{\mathcal{K}} = -6, \quad \gamma_2^{\mathcal{K}} = 24, \quad \gamma_3^{\mathcal{K}} = -168 \quad (16)$$

up to 3-loop. For the Konishi operator, the results up to 2-loop are in agreement with the existing ones [46–48] and the 3-loop result also matches with previous computations [49,50]. By subtracting out the γ_j, we can only calculate the combination of $(2B_j + f_j)$. However, by looking at the similarities between A_j of QCD and $\mathcal{N} = 4$, we propose

$$B_1 = 0, \quad B_2 = 12\zeta_3, \quad B_3 = 16(-\zeta_2\zeta_3 - 5\zeta_5),$$

$$f_1 = 0, \quad f_2 = -28\zeta_3, \quad f_3 = \left(\frac{176}{3}\zeta_2\zeta_3 + 192\zeta_5\right) \quad (17)$$

which are essentially the highest transcendental parts of those of QCD [10,11,44]. The other process dependent constants, that are relevant up to 3-loop, in Eq. (15) are obtained as

$$g_{\phi,1}^{\mathrm{BPS},1} = \zeta_2, \qquad g_{\phi,1}^{\mathrm{BPS},2} = -\frac{7}{3}\zeta_3, \qquad g_{\phi,1}^{\mathrm{BPS},3} = \frac{47}{80}\zeta_2^2,$$

$$g_{\phi,1}^{\mathrm{BPS},4} = \frac{7}{24}\zeta_2\zeta_3 - \frac{31}{20}\zeta_5, \qquad g_{\phi,1}^{\mathrm{BPS},5} = \frac{949}{4480}\zeta_2^3 - \frac{49}{144}\zeta_3^2,$$

$$g_{\phi,2}^{\mathrm{BPS},1} = 0, \qquad g_{\phi,2}^{\mathrm{BPS},2} = \frac{5}{3}\zeta_2\zeta_3 - 39\zeta_5,$$

$$g_{\phi,2}^{\mathrm{BPS},3} = \frac{2623}{140}\zeta_2^3 + \frac{235}{6}\zeta_3^2, \qquad g_{\phi,3}^{\mathrm{BPS},1} = -\frac{12352}{315}\zeta_2^3 - \frac{104}{3}\zeta_3^2$$

$$(18)$$

for $\mathcal{O}_{\mathrm{BPS}}$. Similarly for the $\mathcal{O}_{\mathcal{K}}$, we get

$$g_{\phi,1}^{\mathcal{K},1} = g_{\phi,1}^{\mathrm{BPS},1} - 14, \qquad g_{\phi,1}^{\mathcal{K},2} = g_{\phi,1}^{\mathrm{BPS},2} + 14 - \frac{3}{2}\zeta_2, \qquad g_{\phi,1}^{\mathcal{K},3} = g_{\phi,1}^{\mathrm{BPS},3} - 14 + \frac{7}{4}\zeta_2 + \frac{7}{2}\zeta_3,$$

$$g_{\phi,1}^{\mathcal{K},4} = g_{\phi,1}^{\mathrm{BPS},4} + 14 - \frac{7}{4}\zeta_2 - \frac{141}{160}\zeta_2^2 - \frac{49}{12}\zeta_3, \qquad g_{\phi,1}^{\mathcal{K},5} = g_{\phi,1}^{\mathrm{BPS},5} - 14 + \frac{7}{4}\zeta_2 + \frac{329}{320}\zeta_2^2 + \frac{49}{12}\zeta_3 - \frac{7}{16}\zeta_2\zeta_3 + \frac{93}{40}\zeta_5,$$

$$g_{\phi,2}^{\mathcal{K},1} = g_{\phi,2}^{\mathrm{BPS},1} + 212 - 48\zeta_2, \qquad g_{\phi,2}^{\mathcal{K},2} = g_{\phi,2}^{\mathrm{BPS},2} - 556 + 164\zeta_2 + \frac{24}{5}\zeta_2^2 + 60\zeta_3,$$

$$g_{\phi,2}^{\mathcal{K},3} = g_{\phi,2}^{\mathrm{BPS},3} + 1170 - 377\zeta_2 - \frac{154}{5}\zeta_2^2 - 344\zeta_3 + 24\zeta_2\zeta_3 + 108\zeta_5,$$

$$g_{\phi,3}^{\mathcal{K},1} = g_{\phi,3}^{\mathrm{BPS},1} - 2936 + 504\zeta_2 + \frac{1224}{5}\zeta_2^2 - 648\zeta_3 + 720\zeta_5. \quad (19)$$

In a clear contrast to that of QCD, due to absence of the nonzero β-functions in $\mathcal{N} = 4$ SYM, all the higher poles vanish in Eq. (13). We observe that the leading transcendental terms in the operator dependent parts of the FFs of $\mathcal{O}_{\mathcal{K}}$ and $\mathcal{O}_{\mathrm{BPS}}$, namely $g_{\phi,j}^{\rho,k}$, coincide. This is indeed the case with QCD form factors when the color factors are chosen suitably.

VIII. FORM FACTORS BEYOND THREE LOOP

The KG equation (11) enables us to predict all the poles but constant term of the FFs at 4-loop. Expanding the results of the FFs of previous orders sufficiently high, using the A_4 [12,51–54], $(2B_4 + f_4)$ [54–56] denoted by α from [57] and $\gamma_4^{\mathcal{K}}$ from [58–61] we obtain $\mathcal{F}_\phi^{\mathcal{K},(4)}\big|_{\mathrm{poles}}$:

$$\mathcal{F}_\phi^{\mathcal{K},(4)}\big|_{\mathrm{poles}} = \frac{1}{\epsilon^8}\left\{\frac{512}{3}\right\} + \frac{1}{\epsilon^7}\{-1024\} + \frac{1}{\epsilon^6}\left\{\frac{10496}{3} + \frac{128}{3}\zeta_2\right\} + \frac{1}{\epsilon^5}\left\{-\frac{28928}{3} + \frac{1216}{9}\zeta_3 + 128\zeta_2\right\} + \frac{1}{\epsilon^4}\left\{\frac{72992}{3}\right.$$

$$\left. - \frac{3008}{3}\zeta_3 - \frac{5344}{3}\zeta_2 + \frac{40}{9}\zeta_2^2\right\} + \frac{1}{\epsilon^3}\left\{-\frac{176192}{3} - \frac{8656}{15}\zeta_5 + \frac{42064}{9}\zeta_3 + \frac{25024}{3}\zeta_2 - \frac{184}{3}\zeta_2\zeta_3 + \frac{4256}{15}\zeta_2^2\right\}$$

$$+ \frac{1}{\epsilon^2}\left\{\frac{416096}{3} + \frac{5072}{5}\zeta_5 - \frac{151648}{9}\zeta_3 + \frac{21706}{27}\zeta_3^2 - \frac{85408}{3}\zeta_2 + 736\zeta_2\zeta_3 - \frac{18488}{9}\zeta_2^2 + \frac{381908}{945}\zeta_2^3\right\}$$

$$+ \frac{1}{\epsilon}\left\{-\frac{973136}{3} - 4\alpha - \frac{536894}{63}\zeta_7 + \frac{160412}{15}\zeta_5 + \frac{409192}{9}\zeta_3 - \frac{18680}{9}\zeta_3^2 + \frac{254536}{3}\zeta_2 + \frac{33938}{45}\zeta_2\zeta_5\right.$$

$$\left. - \frac{14336}{3}\zeta_2\zeta_3 + \frac{67664}{9}\zeta_2^2 - \frac{14590}{27}\zeta_2^2\zeta_3 - \frac{333712}{315}\zeta_2^3\right\}, \quad (20)$$

where $\alpha = -(77.56 \pm 0.02)$. Explicit computation is required to get the constant terms. The exact matching of the highest transcendental terms of $\mathcal{O}_\mathcal{K}$ and $\mathcal{O}_{\mathrm{BPS}}$ at 4-loop holds true, similar to the previous orders.

To summarize, we have presented for the first time the complete third order and partial fourth order corrections to the on-shell form factor of the Konishi operator employing state-of-the-art techniques. The superiority of the Feynman diagrammatic approach, which is followed in this paper, over unitarity method in the context of Konishi type composite operators (which depend on the space-time dimensions) is demonstrated very transparently in our present paper. One of the main issues in computing the higher order corrections in $\mathcal{N} = 4$ SYM is related to the necessity of preserving the supersymmetry throughout the computation. In order to achieve that, one generally follows either of the two regularization schemes: $\overline{DR}$ and FDH. Through performing the computation explicitly in both the schemes, we have shown that the form factor in $\overline{DR}$ automatically provides the correct results, whereas, the results obtained in the FDH needs to be suitably modified. In addition, the modification in FDH is operator dependent and nontrivial. Hence, performing a computation in $\overline{DR}$ is much more preferable for unprotected operators like Konishi. In this paper, we have clearly demonstrated that in our approach it is much more elegant as well as easier to execute the calculation in $\overline{DR}$ scheme. Hence, even though the standard Feynman diagrammatic approach seems complex compared to the unitarity method in the context of $\mathcal{N} = 4$ SYM, it is evident, as a consequence of this work, that the standard method is very efficient and advantageous if it is employed in a systematic way. This is, indeed, a novel information to carry forward.

Moreover, using an independent methodology, we have computed the UV anomalous dimensions of the Konishi operator by exploiting the universal infrared structure of the scattering amplitudes through the KG equation. This prescription is much more general and it can be applied to any other operator to extract the anomalous dimensions. In the end, the principle of leading transcendentality in the context of non-BPS operator at three loop level is studied for the first time. This has the potential to unravel the underlying deeper connections between the amplitudes in SYM and those of QCD.

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