

# Information optimized multilayer network representation of high density electroencephalogram recordings

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## 2 ABSTRACT

3 High-density electroencephalography (hd-EEG) provides an accessible indirect method to  
4 record spatio-temporal brain activity with potential for disease diagnosis and monitoring. Due to  
5 their highly multidimensional nature, extracting useful information from hd-EEG recordings is a  
6 complex task. Network representations have been shown to provide an intuitive picture of the  
7 spatial connectivity underlying an electroencephalogram recording, although some information is  
8 lost in the projection. Here, we propose a method to construct multilayer network representations  
9 of hd-EEG recordings that maximize their information content and test it on sleep data recorded  
10 in individuals with mental health issues. We perform a series of statistical measurements on the  
11 multilayer networks obtained from patients and control subjects and detect significant differences  
12 between the groups in clustering coefficient, betweenness centrality, average shortest path length  
13 and parieto occipital edge presence. In particular, patients with a mood disorder display a  
14 increased edge presence in the parieto-occipital region with respect to healthy control subjects,  
15 indicating a highly correlated electrical activity in that region of the brain. We also show that  
16 multilayer networks at constant edge density perform better, since most network properties are  
17 correlated with the edge density itself which can act as a confounding factor. Our results show  
18 that it is possible to stratify patients through statistical measurements on a multilayer network  
19 representation of hd-EEG recordings. The analysis reveals that individuals with mental health  
20 issues display strongly correlated signals in the parieto-occipital region. Our methodology could

21 be useful as a visualization and analysis tool for hd-EEG recordings in a variety of pathological  
22 conditions.

23 **Keywords:** high density electroencephalogram (hd-EEG); multilayer networks; bipolar disorder; maximum information

## 1 INTRODUCTION

24 Recent developments in neuroscience are giving rise to an increasing amount of data on the functioning  
25 of the brain at different scales, from molecular processes at the level of single neurons to macroscopic  
26 signals encompassing the whole brain, as in electroencephalogram (EEG) or functional magnetic resonance  
27 imaging (fMRI). Despite the trove of accumulating data, disentangling the complexity of brain function is  
28 still a largely open issue. A particularly important goal is to develop tools that are able to extract useful  
29 information from brain activity measurements on individual subjects in order to identify potential network  
30 dysfunction and support diagnosis (Bassett, 2021).

31 It is becoming increasingly clear that brain activity is strongly interconnected and hierarchically organized,  
32 requiring a sophisticated mathematical description to infer its underlying properties from measurements.  
33 The emerging field of *network neuroscience* is advocating the use of networks descriptions for a statistical  
34 analysis of brain functions at multiple spatio-temporal scales (Bassett and Sporns, 2017). As in many  
35 other applications, a network representation can be derived by suitably thresholding the covariance matrix  
36 of the signal recorded at different locations (Masuda et al., 2018) with sophisticated methodologies to  
37 chose an optimal threshold (De Vico Fallani et al., 2017) or using singular value decomposition of  
38 the multidimensional signal (Worsley et al., 2005). A typical feature of many complex networks that  
39 appears promising to describe the hierarchical brain organization is the small-world topology involving at  
40 the same time small-scale local clusters and long-range connections between distant areas (Bassett and  
41 Bullmore, 2006). Networks provide a visual representation of brain connectivity (Rubinov and Sporns,  
42 2010), but extracting robust statistical information from brain network is a challenging task. Measures  
43 at the intersection between neuroscience and complexity theory have emerged such as topological data  
44 analysis (Phinyomark et al., 2017) or multivariate auto-regressive models (Astolfi et al., 2007).

45 EEG recordings have attracted a wide interest for many years in the study of brain function due to the  
46 relative simplicity in which spatially localized time dependent data can be acquired through non-invasive  
47 instrumentation. EEG data are conventionally analyzed by sampling time depended signals into different  
48 frequency bands at different locations on the scalp and then looking for specific signatures in each band.  
49 For instance, resting state EEG in patients diagnosed with First Episode Psychosis and Bipolar Disorder  
50 revealed a general trend of increased delta (0.5-4 Hz) and theta (4-8 Hz) activity, and a decrease in  
51 alpha (8-13 Hz) activity (Clementz et al., 1994). Resting state EEG of bipolar patients has also been  
52 studied using complex network analysis in (Kim et al., 2013), yielding differences from healthy control  
53 subjects across several network measures such as clustering coefficient or characteristic path length. More  
54 recently, machine learning combined with complex network analysis was used to classify non-epileptic and  
55 epileptic EEG signals (Gao et al., 2020). Network analysis was also performed for EEG signals recorded in  
56 Alzheimer Disease patients during cognitive tasks and resting state (Das and Puthankattil, 2020), revealing  
57 a higher betweenness centrality in patients compared to controls subjects.

58 Since EEG signals are highly multidimensional, considering their dependence on time, location and  
59 frequency band, a projection into a single network may overshadow some essential feature of the system.  
60 To overcome this limitation, multilayer networks have been recently proposed as a promising tool to study  
61 the dynamics of brain activity, reducing the information loss due to the projection into a single network

62 (Muldoon and Bassett, 2016; De Domenico, 2017). A multilayer network can be seen as an interconnected  
63 set single-layer networks where each layer represents a particular dimension of the original signal (Aleta  
64 and Moreno, 2019; Bianconi, 2018). In the context of EEG we can assign distinct layers to different time  
65 windows and/or different frequency bands and assign each electrode to a node in each single-layer network.  
66 For example, a time-based multilayer complex network analysis was performed on EEG recordings in  
67 patients with epilepsy (Leitgeb et al., 2020). The central issues in multilayer network based methods for  
68 EEG signal is to find a representation that minimizes information loss and introduce suitable statistical  
69 tools to extract readable information from the networks.

70 In this paper, we propose a multilayer network representation of EEG signals that maximize the  
71 information content and apply it to a set of sleep EEG data from patients diagnosed with First Episode  
72 Psychosis (FEP) or Bipolar Disorder (BD) and compared with control subjects. We then use a set of  
73 network measures and show that it is easier to reliably stratify patients from control subjects when using  
74 network representations with constant edge densities.

## 2 RESULTS

### 75 2.1 Maximization of total information change over time

76 Sleep hd-EEG recordings from 12 FEP, 7 BD patients, and 13 control subjects were analyzed, see  
77 Methods for details and Fig. 1. Raw data are extremely fine-grained: the sampling frequency of 500Hz  
78 during an average of 8 hours of sleep, multiplied by the 64 electrodes that comprise the EEG headset  
79 yields approximately, 1,000,000,000 measurements per patient. Clearly, these measurements are not all  
80 independent of each other, but they encode information that spans several sleep phases and brain regions.  
81 Therefore, we aim at finding a satisfactory compromise between compression and information.

82 To do so, we process the raw sleep EEG records through our pipeline as described in Methods in detail  
83 and illustrated in Fig. 1. The first step is to remove artifacts from the data. Eye-movement artifacts are  
84 well known to influence raw sleep EEG data. To mitigate their impact on our results, we use a fast linear  
85 regression model to correct for eye movements, see Methods for details and Fig. 1B: in this illustrative  
86 figure, the horizontal electro-oculogram potential (HEOG) well correlated with channels AF8, F7, FP2  
87 and FP1 in the top plot. After the correction step (bottom plot in pannel B), this dependence was almost  
88 completely eliminated. After splitting the signal into different frequency bands (see Methods for details),  
89 we compute time- and band-specific electrode-to-electrode correlations of the form  $C_{ij}^b(t)$ , represented as a  
90 heatmap in Fig. 1C. Finally, we construct time-varying multilayer networks using an innovative strategy  
91 that takes into account the whole dataset (and not each time snapshot individually), maximising the total  
92 amount of information contained in the time-varying dataset. Figure 1D offers a visual representation of  
93 the final output we obtain after processing the raw EEG data: a set of time-varying multilayer networks,  
94 where different layers correspond to different frequency bands, network nodes represent electrodes and  
95 edges represent high EEG correlations.

96 Networks offer a simplified and effective representation of interactions between nodes, but deciding the  
97 correlation threshold beyond which edges are added to the network is a nontrivial subject. In order to make  
98 an informed choice, here we introduce the Integrated Jensen-Shannon Divergence (IJSJ),

$$\mathcal{I}(\theta) = \sum_{t=1}^{\theta} D(\rho_{t-1}, \rho_t), \quad (1)$$

99 a measure of the total information change over time, computed as the sum of the Jensen-Shannon  
 100 divergence of each epoch with respect to the previous one. Here  $\rho_t$  are the density matrices associated to  
 101 each network in the framework of spectral entropies (Domenico and Biamonte, 2016), see Methods for  
 102 details. The value of  $\mathcal{I}$  depends on  $\theta$  in non-trivial ways, but the limit cases are clear: if  $\theta$  is too low (high),  
 103 all edges are present (absent) at all time steps, so there is no information change over time and thus  $\mathcal{I} = 0$   
 104 for both  $\theta = 0$  and  $\theta = 1$ . It is only for intermediate values of the correlation threshold  $\theta$  that the sequence  
 105 of multilayer networks can display richer temporal variations, yielding a higher information change. This  
 106 can be clearly seen in Fig. 2 panels (A, B, C), which show the value of  $\mathcal{I}$  as a function of  $\theta$  for one BD, one  
 107 FEP and one control example. As anticipated,  $\mathcal{I}(\theta) = 0$  for both  $\theta = 0$  and  $\theta = 1$ , with a clear maximum  
 108 at around  $\theta \sim 0.7$  for most frequency bands.

## 109 2.2 Fixed-threshold and fixed-density networks

110 We implement two strategies to choose the optimal correlation threshold  $\theta^*$  from the analysis of the  
 111 information content quantified by IJSD. In the first approach, we set a global absolute value for the  
 112 correlation threshold, while in the second approach that value is relative to each network and chosen to  
 113 maintain a constant edge density, keeping only the interactions with highest absolute correlation. In both  
 114 cases, the adjacency matrices can be build as

$$A_{ij}^b(t) = \begin{cases} 1 & \text{if } |C_{ij}^b(t)| \geq \theta^* \\ 0 & \text{else} \end{cases} \quad (2)$$

115 that is, we place edges for both large positive and large negative correlations.

116 The optimal correlation threshold  $\theta^*$  for fixed-threshold networks is computed as the average of the band-  
 117 and patient-specific optimal values that result from optimizing each case separately,

$$\theta^* = \langle \theta_{b,p}^* \rangle_{b,p} \quad (3)$$

$$\theta_{b,p}^* = \operatorname{argmax}_{\theta \in [0,1]} \mathcal{I}_{b,p}(\theta) \quad (4)$$

118 where  $\mathcal{I}_\theta(b, p)$  denotes the IJSD of patient  $p$  at frequency band  $b$ . In other words, for each patient  $p$  we  
 119 compute a band-specific optimal threshold  $\theta_{b,p}^*$ . The group averages and variability of these are shown in  
 120 Fig.2. Taking the average of all  $\theta_{b,p}^*$ , we reach an overall value of  $\theta^* = 0.72$ , shown as a black solid line in  
 121 Fig.2. Overall, the figure shows that a single global threshold can reasonably accommodate for the band-  
 122 and patient-specific optimal values.

123 The second approach consists in keeping the same fraction of edges in all networks, yielding what we  
 124 call *fixed-density* networks. The optimal density value in this case is set so that it coincides with the average  
 125 density of the fixed-threshold networks. This second approach takes into account that different patients,  
 126 time point or bands might have different intrinsic correlation levels, and presents additional advantages  
 127 from the network analysis point of view.

## 128 2.3 Network edge presence shows differences between groups

129 We investigate the group differences between BD and control patients, as well as between FEP and control  
 130 patients. To do so, we need to condense the information contained in our multilayer and time-varying  
 131 networks into simpler summary statistics. A simple yet useful measure in this case is what we coin as *edge*

132 *presence*, which is the fraction of time an edge is present (that is,  $A_{ij}^b(t) = 1$ ) during one full EEG sleep  
 133 session. Formally,

$$P_{ij}^b = \left\langle A_{ij}^b(t) \right\rangle_t \quad (5)$$

134 Figure 3 shows the group differences of  $P_{ij}^b$  for each edge  $(i, j)$  and each band when comparing BD  
 135 patients with controls (panels A, C), as well as FEP patients with controls (panels B, D). This analysis is  
 136 shown both for fixed-density networks (A, B) and for fixed-threshold networks (C, D). In both cases we see  
 137 differences in the parieto-occipital area, but the signal is stronger for fixed-density networks. If we focus  
 138 on Fig. 3A, for instance, we see that BD patients tend to have a lower edge presence in the parieto-occipital  
 139 area (strong blue edges). Notice that we employ a colorbar that goes from red to transparent to blue, so that  
 140 edges that do not have strong differences are effectively not drawn. Overall, the figure shows important  
 141 differences in the parieto-occipital area, with a similar but stronger signal for fixed-density networks.

## 142 **2.4 Parieto-occipital correlations and clustering measures differ between groups**

143 Motivated by the results shown visually in Fig. 3, we construct a parieto-occipital (PO) specific measure.  
 144 Selecting the 18 electrodes of that region (see methods for details), we compute the difference of PO  
 145 presence between the PO area and the rest.

$$P_{PO} = \left\langle P_{ij}^b \right\rangle_{(ij) \in PO} - \left\langle P_{ij}^b \right\rangle_{(ij) \notin PO} \quad (6)$$

146 Additionally, we also consider the average clustering coefficient, the average shortest path length and  
 147 the betweenness centrality as measures related to clustering and information navigability as candidates to  
 148 better quantify the differences that we see in Fig. 3.

149 Figure 4 shows boxplots of these four measures comparing, BD and FEP patients with control subjects.  
 150 Statistically significant differences are marked with a star, see Methods for details. Panels (A, B) show that  
 151 for bands 1 to 4 (that is, between 1Hz and 16Hz), FEP patients have a higher clustering coefficient when  
 152 compared to controls, while panels (C, D) shows some significant results on the same range of frequencies  
 153 for the average shortest path length, both for BD and for FEP patients. Turning to betweenness centrality,  
 154 panel E shows that when using fixed-threshold networks, FEP patients significantly differ from controls  
 155 in bands 2, 3 and 4 (2Hz to 16Hz) Interestingly, when looking at the parieto-occipital relative presence  
 156 (panels G, H), we observe a different pattern of marked differences between BD and control patients for  
 157 lower frequency bands, 0.5Hz to 4Hz. This is consistent with the fact that the more standard network  
 158 measures used in panels A to F treat all nodes under the same footing, independently of the brain region  
 159 they correspond to, while PO presence is a tailor-made measure, specifically designed to capture the visual  
 160 results of Fig. 3 taking into account the location of parieto-occipital electrodes.

## 161 **2.5 Network measures correlate with edge density**

162 It is interesting to ask if the network measures shown in Fig. 4 are correlated with network edge density,  
 163 for the case of fixed-threshold networks. Fig. 5 shows how indeed edge density is a strong driver of average  
 164 clustering coefficient, average shortest path length and betweenness centrality for all patient groups, but  
 165 not of parieto-occipital presence. This is consistent with the fact that, by construction,  $P_{PO}$  is a relative  
 166 difference of two averages taken on the same network.

### 3 DISCUSSION

167 Hd-EEG represents an attractive method to study brain function by providing non-invasive spatio-temporal  
168 measurements of brain activity with possible applications to disease diagnosis and monitoring. While it  
169 is relatively easy to obtain large amount of data from individual subjects, extracting useful information  
170 from hd-EEG recordings is a challenging task. Hd-EEG only provides an indirect far-field measurement  
171 of the underlying electrical activity and is intrinsically subject to noise. Furthermore, hd-EEG recordings  
172 typically involve noisy signals recorded in parallel through different electrodes for long time periods so  
173 that even the mere visualization of the data is complex.

174 Network representations have been shown in the past to provide a useful tool to highlight the connectivity  
175 and spatio-temporal correlation of brain activity as revealed from EEG or other measurements such as fMRI.  
176 Due to the complexity of hd-EEG recordings, multilayer networks are more appropriate to represent the  
177 data since they provide separate visualization for potentially crucial features of EEG signals such as the  
178 frequency band and/or the time dependence. An effective network representation of hd-EEG recordings  
179 should be able to extract most of the relevant information from the signal cross-correlation. To address this  
180 issue, we use the IJSD to quantify information content in the multilayer network (Domenico and Biamonte,  
181 2016) and adjust correlation threshold parameters to maximize it. In this way, we obtain a multilayer  
182 networks that maximizes the information content of the underlying hd-EEG recordings and test it on a set  
183 of EEG data obtained from patients with mental health issue, as well as healthy control subjects.

184 Statistical analysis on the resulting multilayer networks reveals a number of distinguishing topological  
185 features between patients and the control group. In particular, observed differences in parieto-occipital edge  
186 presence appear to be particularly relevant. These results indicate a stronger correlation of EEG signals in  
187 that area for BD patients with respect to control subjects, a feature that warrants further study and could  
188 potentially be used as a diagnostic tool.

189 An important issue in our analysis is that most statistical indicators crucially depend on the density  
190 of edges present in the network. To discount this effect, we constructed and analyzed constant-density  
191 multilayer networks. While our analysis only considers pairwise correlations, future work could also extend  
192 our analysis to the study of interactions between groups of nodes (Battiston et al., 2020).

193 We applied our strategy to a particular set of EEG recordings from patients with mental disorders, but the  
194 methodology could readily be generalized and applied to a variety of pathological conditions. It would  
195 be interesting for instance to use our multilayer network approach to predict the response of individual  
196 patients to specific drugs. Finally, the analysis of EEG signals could be enriched by measuring at the same  
197 time other physiological signals, such as heartbeat or respiration adding further layers to the network, in  
198 the spirit of the emerging field of network physiology (Bashan et al., 2012; Bartsch et al., 2015; Ivanov  
199 et al., 2016).

### 4 METHODS

#### 200 4.1 Data

201 Hd-EEG recordings were obtained from San Paolo Hospital in Milano. In particular, the dataset consists  
202 of sleep EEG recordings from 12 FEP patients (8 males and 4 females, mean age  $21.0 \pm 3.77$ ), 7 BD  
203 patients (3 males and 4 females, mean age  $34.57 \pm 7.09$ ), and 13 control subjects (6 males and 7 females,  
204 mean age  $25.61 \pm 10.64$ ). All participants underwent an in-laboratory sleep hd-EEG recording with a  
205 64-electrode Easycap net designed to enhance electrode contact with the scalp (BrainAmp, Brain Products  
206 GmbH, Gilching, Germany). The night of the recording, all subjects were accommodated in a sleep suite

207 and allowed to sleep within 1 hour of their usual bedtime. All subjects were recorded throughout the  
 208 night and until they woke up naturally the next morning. Table S1 shows the average length of recording  
 209 sessions and total sleep time for each participant group. The headset has 64 unipolar electrodes positioned  
 210 following the standard 10-20 system, and include two channels that record eye movements (one for vertical  
 211 movements and one for horizontal movements). All recordings had a sampling frequency of 500Hz. Data  
 212 was provided in anonymized form as pairs of *.set* and *.fdt* files.

## 213 4.2 Data preprocessing

214 Our preprocessing pipeline transforms the raw EEG recordings into correlation tensors of the form  $C_{ij}^b(t)$ ,  
 215 with  $(i, j)$  denoting an edge between electrodes  $i$  and  $j$ ,  $b$  a specific frequency band, and  $t$  a 30-second  
 216 epoch. The steps we carry are as follows:

- 217 1. **Epochs division:** divide the raw signal into epochs of approximately 30 seconds, see below for details,  
 218 obtaining a signal  $S_i(t)$  for electrode  $i$  and epoch  $t$ .
- 219 2. **Artifact correction:** apply eye-movement correction.
- 220 3. **Bands division:** divide the corrected signal into seven frequency bands. This gives a signal  $S_i^b(t)$  with  
 221  $b \in \{0, \dots, 6\}$ .
- 222 4. **Correlation analysis:** compute electrode-to-electrode Pearson correlations, obtaining a correlation  
 223 tensor of the form  $C_{ij}^b(t)$ .

224 *Epochs division:* We divide EEG recordings into epochs of around 30 seconds following Aboalayon et al.  
 225 (2016). To be precise, each epoch has a length of  $2^{14}$  raw time points which, at a sampling frequency of  
 226 500Hz, corresponds to 32.768 seconds. This choice is particularly convenient because pure powers of two  
 227 allow for faster discrete Fourier transform calculations.

228 *Artifact correction:* Following Gratton et al. (1983), we correct for eye-movements using a linear  
 229 regression equation of the form

$$Y = XB \quad (7)$$

230 where  $Y$  corresponds to the EEG data (62 channels in our case),  $X$  corresponds to the eye-movement data  
 231 (2 channels in our case), and  $B$  is the regression coefficient matrix to be determined. Solving for  $B$  via  
 232 least squares, the corrected signal  $X^*$  is computed as

$$X^* = (X - YB)^T \quad (8)$$

233 *Bands division:* We use seven frequency bands, numbered from 0 to 6 throughout the manuscript, which  
 234 logarithmically interpolate the 0.5 to 64 Hz range typical of brain waves.

- 235 • Band 0: [0.5, 1] Hz.
- 236 • Band 1: [1, 2] Hz.
- 237 • Band 2: [2, 4] Hz.
- 238 • Band 3: [4, 8] Hz.
- 239 • Band 4: [8, 16] Hz.
- 240 • Band 5: [16, 32] Hz.
- 241 • Band 6: [32, 64] Hz.

### 242 4.3 Correlation analysis

243 We use the Pearson correlation coefficient to measure the strength and direction of dependence between  
244 the signals  $x_i, x_j$  recorded by two electrodes  $i, j$ ,

$$C_{ij} = \frac{\text{cov}(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}. \quad (9)$$

245 Repeating this measurement for each band  $b$  and timepoint  $t$ , we get a full correlation tensor  $C_{ij}^b(t)$ .

### 246 4.4 Jensen-Shannon Divergence

247 We use Jensen-Shannon Divergence (JSD) as a distance measure between networks, in the framework of  
248 spectral entropies (Domenico and Biamonte, 2016). For a pair of networks with density matrices  $\rho$  and  $\sigma$ ,  
249 the JSD is defined as

$$J(\rho||\sigma) = S\left(\frac{\rho + \sigma}{2}\right) - \frac{1}{2}[S(\rho) + S(\sigma)], \quad (10)$$

250 where  $S(\rho)$  is the spectral entropy of the network,

$$S(\rho) = \log_2 Z + \frac{\tau}{\ln 2} \text{Tr}[L\rho], \quad (11)$$

251 with  $L$  denoting the Laplacian,  $\tau$  diffusion time and the density matrix  $\rho$  defined as

$$\rho = \frac{e^{-\tau L}}{Z}, \quad Z = \text{Tr}(e^{-\tau L}) \quad (12)$$

### 252 4.5 Network measures

253 *Parieto-occipital edge presence:* The parieto-occipital area is mapped to the following electrodes: P7, P5,  
254 P3, P1, PZ, P2, P4, P6, P8, PO7, PO3, PO4, PO8, O1, OZ, O2, and IZ. From this list, the parieto-occipital  
255 presence is computed as explained in the main text, mainly the difference of average presence between  
256 nodes in the parieto-occipital area and the rest.

257 *Clustering Coefficient:* We use the standard definition of clustering coefficient,

$$c_i = \frac{2 \cdot t_i}{k_i \cdot (k_i - 1)} \quad (13)$$

258 as implemented in the networkx python library (Hagberg et al., 2008), where  $t_i$  is the number of triangles  
259 in which node  $i$  is involved and  $k_i$  is the node degree. Averaging over all nodes, we define the clustering  
260 coefficient of the network as

$$c = \frac{1}{N} \sum_{i=1}^N c_i \quad (14)$$

261 *Betweenness Centrality:* We use the convention of Brandes (2008), which defines a node-dependent  
262 quantity as follows:

$$c_B(i) = \frac{2}{(N-1)(N-2)} \sum_{j,k \in V} \frac{\sigma(j, k|i)}{\sigma(j, k)} \quad (15)$$



263 where  $\sigma(j, k|i)$  is the number of shortest path that connect nodes  $j$  and  $k$  that passes through  $i$  and  
 264  $\sigma(j, k|i) = 0$  if  $i = j, k$ .  $\sigma(j, k)$  is the total number of shortest path connecting  $j$  and  $k$  and  $\sigma(j, k) = 1$  if  
 265  $j = k$ . By convention the fraction  $\frac{\sigma(j, k|i)}{\sigma(j, k)}$  is considered zero if both elements are zero. We then average  
 266 over all nodes to get a single measure for each network:

$$\text{BC} = \frac{1}{N} \sum_{i=1}^N c_B(i). \quad (16)$$

267 *Average Shortest Path Length:* We start from the standard definition of average shortest path length  
 268 (ASPL) for a connected graph  $G$

$$a_G = \frac{1}{N \cdot (N - 1)} \sum_{i \neq j} d(i, j). \quad (17)$$

269 where  $d(i, j)$  is defined as the length of the shortest path connecting two nodes  $i, j$ . If  $i$  and  $j$  belong to two  
 270 different connected components  $d(i, j)$  is said to be infinite, while  $d(i, j) = 0$  if  $i = j$ .

271 In our setting, networks can have more than one connected component, and we do not want to limit  
 272 ourselves to the largest connected component as important information could be missed. Hence we employ  
 273 a weighted version of the ASPL,

$$wa_G = \frac{\sum_{c=1}^{n_c} a_c \cdot w_c}{\sum_{c=1}^{n_c} w_c} \quad (18)$$

274 where  $n_c$  is the number of connected components with more than 2 nodes and  $w_c = N_c \cdot (N_c - 1)$ ,  $N_c$   
 275 is the number of nodes of component  $c$ . This formulation takes into account the ASPL of all nodes but  
 276 effectively gives more weight to the larger components.

## 277 4.6 Statistical Analysis

278 Group differences are assessed with a two-sided  $T$ -test without assuming equal variances between  
 279 groups, as implemented in the `ttest_ind` function from the `scipy` Python scientific library. Cases marked as  
 280 significant (\*) in Figure 4 correspond to a  $P$ -value below 0.05.

## 281 4.7 Ethical Approval

282 Data from the SPINDLE-1 study, approved by the Milan Area A Interhospital Ethics Committee (Approval  
 283 n. 22864). All participants signed an informed consent for participation in the SPINDLE-1 study.

## CONFLICT OF INTEREST STATEMENT

284 The authors declare that they have no competing interests.

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## AUTHOR'S CONTRIBUTIONS

286 FFC and BS analyzed the data. A'DA, FD, SS, MC performed experimental recording. SZ and CAMLP  
 287 coordinated the project. FFC, SZ and CAMLP wrote the paper.

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## DATA AVAILABILITY STATEMENT

290 Source code and data to reproduce the figures is available at [https://github.com/](https://github.com/ComplexityBiosystems/multilayer-eeeg)  
291 [ComplexityBiosystems/multilayer-eeeg](https://github.com/ComplexityBiosystems/multilayer-eeeg)

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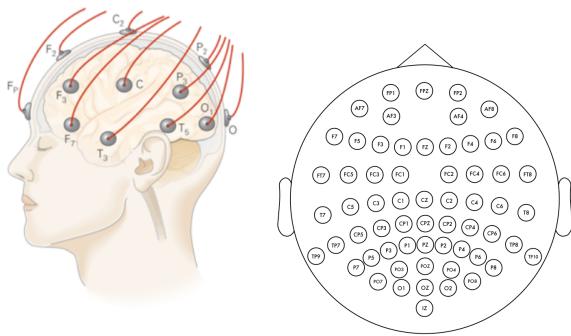
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	FEP	BD	HEALTHY CONTROL
RECORDING TIME (mean $\pm$ S.D.)	431.02 $\pm$ 136.94	526.06 $\pm$ 44.84	489.22 $\pm$ 42.44
TSLEEP TIME (mean $\pm$ S.D.)	300.02 $\pm$ 115.75	351.77 $\pm$ 102.38	361.47 $\pm$ 73.92

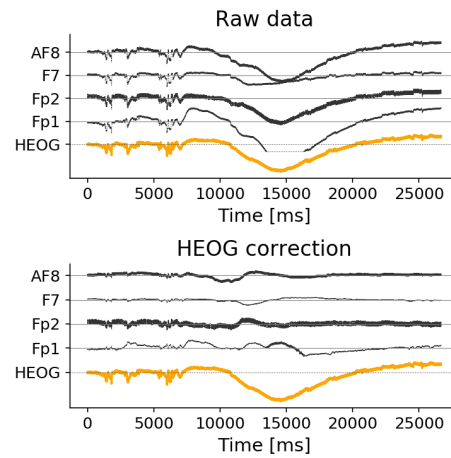
**Table 1.** Recording time and sleep time. Sleep time is obtained by visual scoring according to the American Academy of Sleep Medicine (AASM) Manual for the Scoring of Sleep and Associated Events (Berry, R. B., Brooks, R., Gamaldo, C. E. & Susan, M. 2012). All values expressed in minutes.

### A Data Acquisition



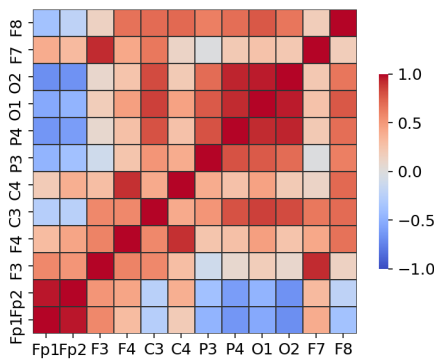
7 BD, 12 FEP, 13 Control  
 ~8h of sleep EEG recording  
 64 electrodes, 500Hz sampling

### B Data preprocessing



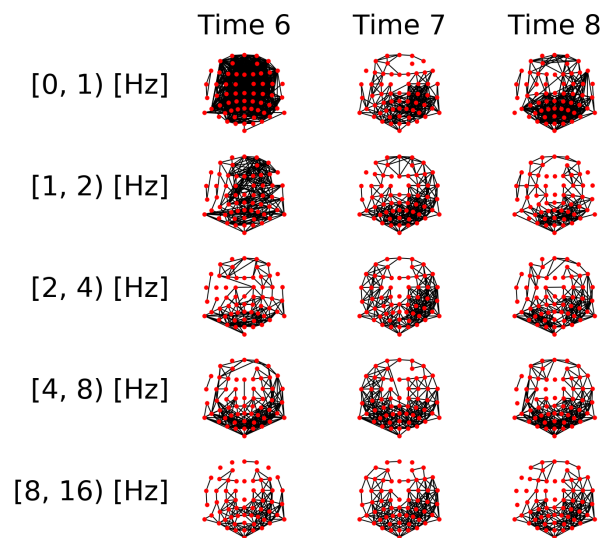
Data cleaning and eye artefact correction.

### C Data transformation

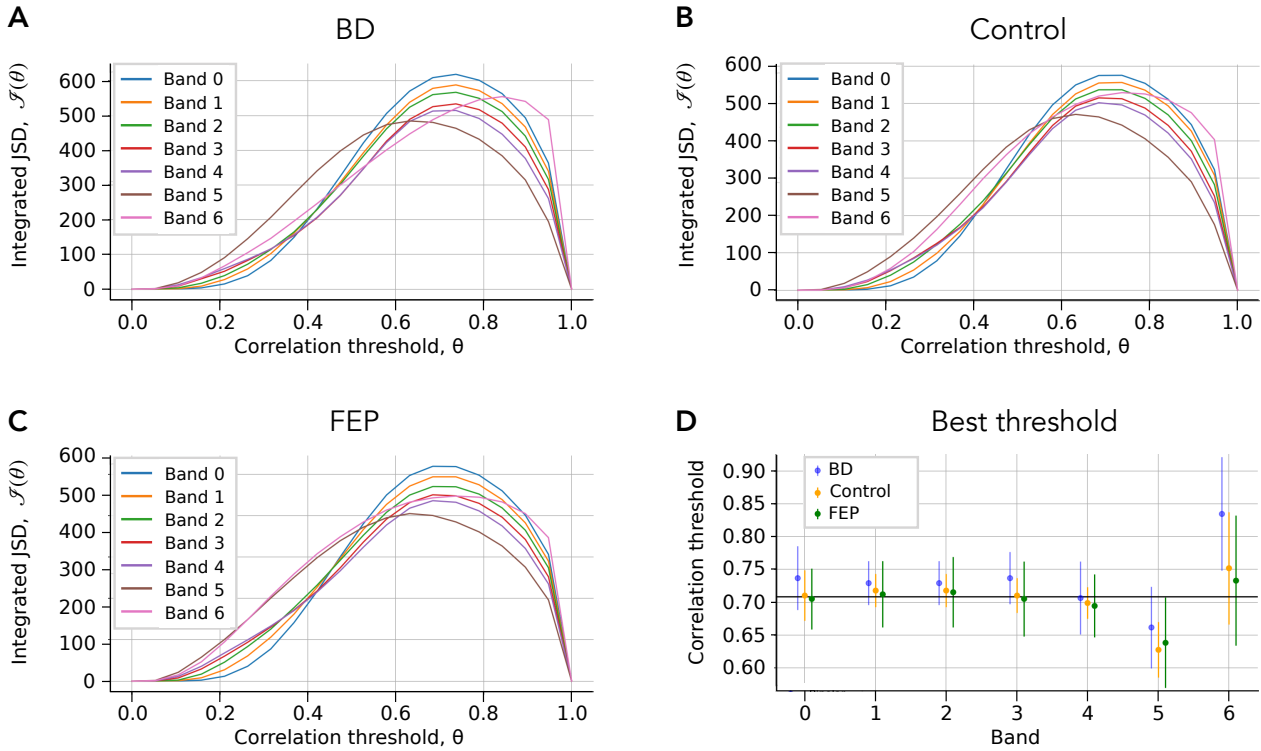


Band and time-specific electrode-to-electrode correlations.

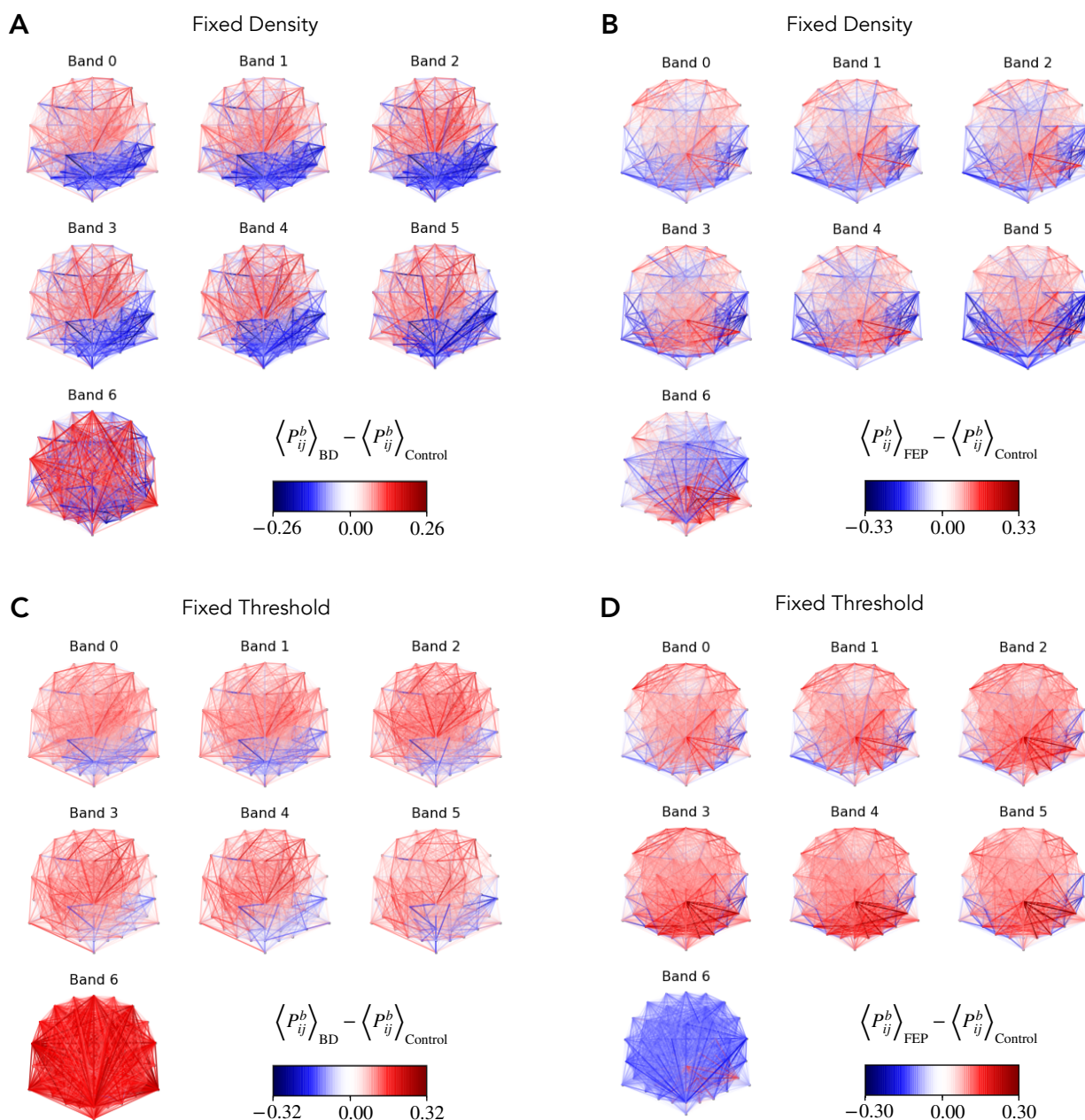
### D Multilayer Network



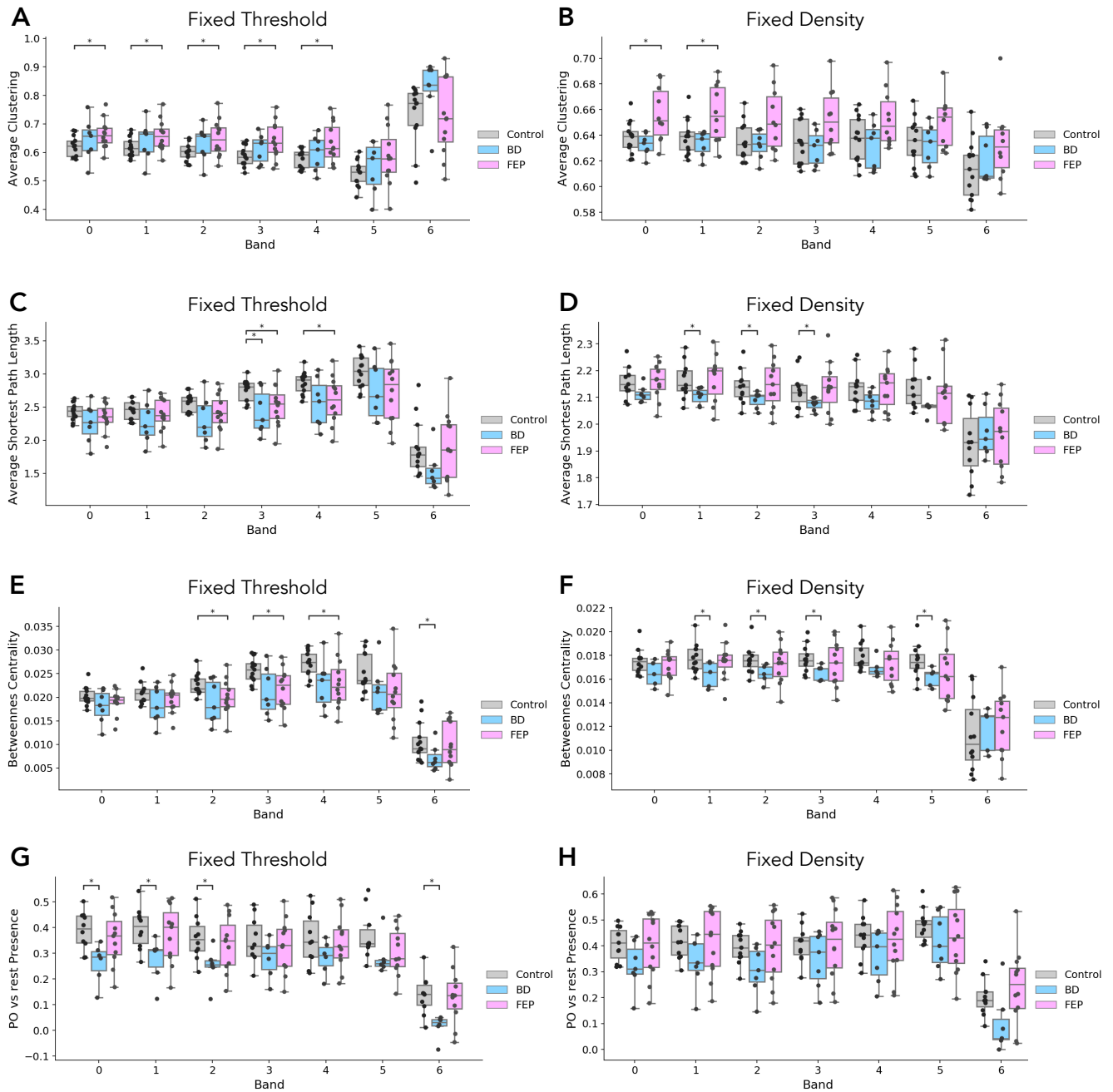
**Figure 1. Overview of the method.** (A) Short description of the dataset, see Methods for complete details. (B) Example of the eye-artifact correction method, showing the correlation of the signal from electrodes AF8, F7, Fp1 and Fp2 (black lines) with horizontal eye movements (HEOG, orange line) before (Raw data panel) and after (HEOG correction panel) the eye-artifact correction method. (C) Example of an electrode-to-electrode correlation matrix, depicted as a heatmap. Correlation goes from -1 (blue shading) to +1 (red shading). Correlations are both time- and band-specific. (D) Example of the resulting band- and time-dependent multilayer networks, where nodes represent electrode and edges represent high correlations, see Methods for the details of the different thresholding procedures.



**Figure 2. Choice of correlation thresholds.** (A, B, C) CHECK WHICH PATIENT. Integrated Jensen-Shannon divergence (JSD) as a function of the correlation threshold  $\theta$ , for each band (colored lines), for BD (A), Control (B) and FEP (C) patients. The panel shows a consistent maximum of the integrated JSD at around  $\theta = 0.7$ . (D) Threshold that maximizes the integrated JSD. The errorbars correspond to the average over different patients. The overall chosen best threshold is marked as a solid horizontal black line, see Methods for details. The panel shows that a single correlation threshold value can accommodate all patient groups and frequency bands.

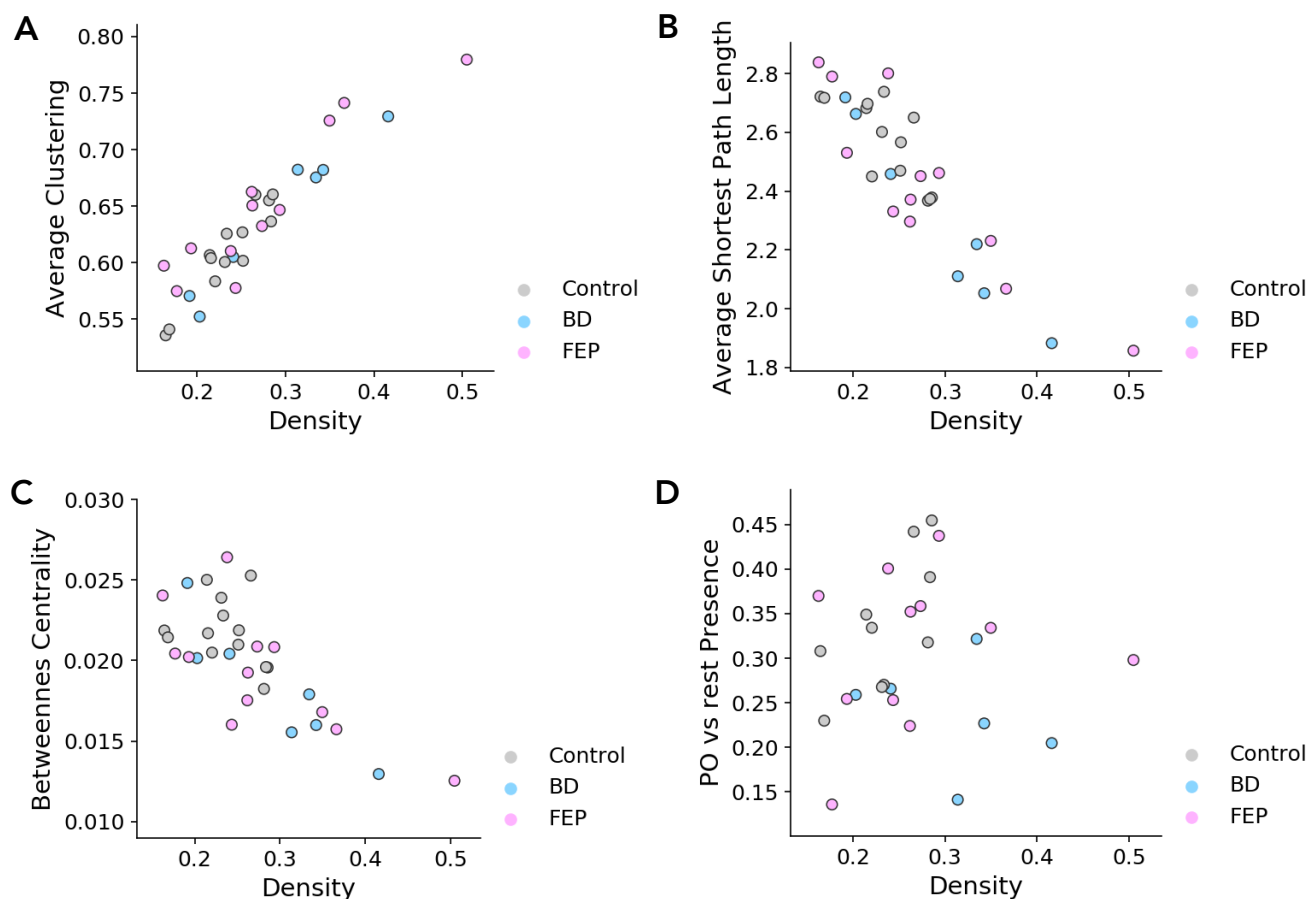


**Figure 3. Network edge presence highlights differences between groups.** Multilayer EEG fixed-density (A, B) or fixed-threshold (C, D) networks, with edges colored according to the average BD (A, D) or FEP (C, D) presence minus the corresponding average value of control patients. Edge presence is a measure of the fraction of time an edge is active, see Methods for details. The four panels use a divergent colormap that is blue for negatives values, red for positive values, and becomes gradually transparent as values approach zero. Overall, the figure visually shows clear differences between BD and control patients, and between FEP and control patients.



**Figure 4. EEG network measures evidence differences between groups.** Boxplots of average clustering (A, B), average shortest path length (C, D), betweenness centrality (E, F) and parieto-occipital presence (G, H) for control (gray), BD (blue) and FEP (pink) patients. Panels in the left column correspond to fixed-threshold networks, while panels in the right column correspond to fixed-density networks.





**Figure 5. Fixed-threshold networks yield measures that correlate with edge density.** Scatter plots of average clustering (A), average shortest path length (B), betweenness centrality (C) and parieto-occipital presence (D) vs network density, for fixed-threshold networks. Overall, the figure shows that all measures except parieto-occipital presence correlate with network density.