ABSTRACT BOOK

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Gaussian quadrature for non-Gaussian distributions

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Abstract

Many problems of operations research or decision science involve continuous probability distributions, whose handling may be sometimes unmanageable; in this case, some form of approximation is used. When constructing a k-point discrete approximation of a continuous random variable, moment matching, i.e., matching as many moments as possible of the original distribution, is the most popular technique. It is well-known that a k-point discrete random variable, which is characterized by $2k$ values, the k support points $(x_1, x_2, \ldots, x_k)$ and the corresponding k probabilities $(p_1, p_2, \ldots, p_k)$, can preserve up to the first $2k-1$ moments of the continuous random variable. The values of the x_i and p_i which lead to the exact matching of all the first $2k-1$ moments can be obtained by solving the corresponding non-linear system of $2k$ equations in the $2k$ unknowns:

$$\sum_{i=1}^{k} p_i \cdot x_i^r = m_r, \quad r = 0, 1, \ldots, 2k-1,$$

where m_r is the r-th raw moment of the continuous random variable, and for $r = 0$ we have the trivial requirement that the p_i must sum up to 1. This can be done by resorting to the so-called Gaussian quadrature procedure (originally developed by Gauss in the nineteenth century) and solving for the roots of an orthogonal polynomial [1] or for the eigenvalues of a real symmetric tridiagonal matrix [2, 3]. The moment-matching discretization has been widely applied to the Gaussian distribution and more generally to symmetric distributions, for which the procedure considerably simplifies. Despite the name, Gaussian quadrature can be theoretically applied to any continuous distribution (having the first $2k-1$ moments finite), but not much interest has been shown in the literature so far. In this work, we will consider some examples of asymmetric distributions defined over the positive real line (namely, the gamma and the Weibull, for which expressions for the integer moments are available in closed form) and show possible practical issues of the moment-matching procedure. Comparison with alternative discretization techniques will be discussed. An application to a real problem will be provided for illustrative purposes.

Keywords: discrete approximation, moments, random distribution.
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