Health and Economic Development: Evidence from non-OECD Countries

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Abstract

This paper studies the empirical relationship between a country's health and its GDP dynamics in low- and middle-income countries. We employ a semi-parametric technique, which combines mixed panel data models and cluster analysis to account for unobserved heterogeneity, which is an important source of estimation bias in growth regressions. We estimate a version of Mankiw, Romer and Weil (1992) augmented with human capital, in the form of both education and health. Our estimates show that population's health, here proxied by the life expectancy at birth, has a positive, sizable, and statistically significant effect on both the level and the growth rate of the real per capita *GDP.*

***Keywords**: Health; Education and Human Capital; Economic Development and Growth; Finite Mixture Models; Classification.*

***JEL codes**: I15; I25; J24; O41; C14.*

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1 Introduction

Following to COVID-19 pandemic outbreak, health policies are a topic of renewed interest and lively debate among policy makers and scholars. Less developed economies are comparatively more exposed to the pandemic emergency, due to their weakness in terms of health structures. The consequent risk is an additional slow down along their development path, since health is one of the most important driver of economic growth. Using a simple theoretical framework, this paper provides a model and a robust empirical evidence on the close link between a country's health and its economic prosperity.

We build on Mankiw, Romer and Weil (1992), which convincingly provides evidence that: i) a standard Solow (1956) model, augmented with the inclusion of the accumulation of human capital, can provide a better understanding of the international differences in income *per capita*, and ii) the existing disparities in saving rates, education, and population change can account for most of the cross-country variations in living standards. We extend their empirical model in order to answer two questions which are not explicitly addressed in their paper, thus remaining open for discussion. First, what is the role of health in determining a country's living standards (measured by the level of real *per capita* GDP) and its economic development (measured by the growth rate of real *per capita* GDP)? Second, what is the role of health in explaining cross-country differences in economic development?

Since human capital can appear both in the form of education/schooling and in the form of health, we start our analysis by presenting a simple *augmented* version of the Solow model which accounts for this. Then we take our model to the data, using a large panel of low- and middle-income countries. To empirically assess the interplay between GDP, physical capital, education and health, four well-known econometric issues must be tackled: (i) measurement error (what is the correct measure for the health status?), (ii) omitted variable (both the level and the growth rate of GDP could be affected by other variables not included in our model), (iii) simultaneity between regressors and response variable, (iv) heterogeneity in country-specific effects of human capital on GDP. All these issues are at the root of endogeneity bias and

unobserved heterogeneity bias. Each of them, indeed, may produce correlation between the estimated residuals and regressors (see e.g. Griliches and Hausman, 1986; Davidson and McKinnon,1993, Wooldrige, 2010): the higher the correlation, the starker the bias in the significance of the estimated coefficients. Several estimators have been proposed to solve this problem, such as the Two Stage Least Squares, the dynamic Generalized Method of Moments (GMM) or the Two Stage GMM with Instrumental Variables. In this paper, we follow an alternative route and propose a flexible Bivariate Finite Mixture model, which, as we show below, in our case, performs better than both OLS and GMM. The key feature of this econometric model is the inclusion of a latent term in the estimated equation. The latent term is distribution free and cluster-specific (e.g., Rabe-Hecksc 2004, Aitkin Rocci, 2002). A further advantage is that this approach allows for a posterior classification, such that within each cluster the classical homogeneity assumption still holds. In this way, we are able to study the role that health-differences have in explaining international income-differences. Notably, our cluster analysis complements the existing classifications, which are mainly obtained through efficiency analysis (health output maximization or cost minimization).[1]

Our study reveals that, at least for non-OECD countries, aggregate health – here proxied by life expectancy at birth, as it is standard in the macroeconomic literature – positively affects both the level and the growth rate of real *per capita* GDP. In the baseline Bivariate Finite Mixture model, a one-year increase in life expectancy raises the long-run level of *per capita* GDP by 5%. Hence, the link between overall health status and economic development appears to be rather substantial across countries. Importantly, this effect is robust to changes in our econometric strategy and/or the inclusion of other explanatory variables in our regressions.

Literature review This paper is a contribution to the empirical literature which quantifies the direct and indirect effects of health on living standards and GDP growth

[1]Notice that, despite our empirical model can not identify *per se* any causal relationship between population's health and GDP level/growth rate, which are, however somehow provided by the *augmented* Solow model, it is able to capture the mutual dependency between covariates and response variables, assuming that such a dependency may vary across countries.

in low- and middle-income countries. By looking only at the direct effects of health on income, Weil (2007) finds that they are not particularly sizable: a 5 years increase in life expectancy would increase labor productivity by 3.6% and output *per capita* by the same amount at the steady state. To have a raw idea of what these figures might imply, note that along the 2010's Preston curve an increase in life expectancy of 5 years would be associated with a doubling of output per capita. In line with Weil (2007), Ashraf et al. (2009) estimate that an increase in life expectancy from 40 to 60 years would raise GDP *per capita* in the long-run by only 15%, and, for the first 30 years after such an increase, output *per capita* would be lower than if life expectancy had not improved at all.

The size of the indirect effects from a better health seems instead remarkable. Hurd, McFadden and Gan (1998) find that increased expectation of longevity leads to greater household's wealth in the United States. Lee et al. (2000) argue that rising life expectancy accounts for the boom in savings in Taiwan since the 1960s. Bloom, Canning, and Graham (2003) find a positive effect of life expectancy on national savings, using cross-country data. Zhang and Zhang (2005) construct a three-period overlapping-generations model showing that rising longevity reduces fertility and enhances savings and schooling investment, even though these effects are empirically small. Bleakley and Lange (2009), and Jayachandran and Lleras-Muney (2009) provide robust evidence that higher life expectancy increases educational attainments at the individual level. [2] The recent regression results of Madsen (2016) clearly show that, since 1870, health has been highly influential for economic growth in 21 OECD countries because it affects not only human capital investment, but also ideas-production.

Our paper is also related to the strand of literature which focuses on health's effects across different sample-compositions. Weil (2007 and 2005) suggests that health's positive effect on GDP is stronger across poor countries. For rich countries, instead, the existing empirical evidence is mixed. For a sample of 31 high income countries over the period 1995-2010, Bucci, Carbonari and Trovato (2019) obtain estimates for the coefficient of life expectancy ranging from 0.399 to 0.458. For a panel of countries over

[2] See also de la Croix and Licandro (1999), Kalemli-Ozcan, Ryder and Weil (2000), Boucekkine, de la Croix and Licandro (2002 and 2003), Chakraborty (2004), Cervellati and Sunde (2005), and Soares (2005).

the period 1960-1990, Bloom, Canning, and Sevilla (2004) find that a one-year increase in population's life expectancy contributes to an increase of 4% in aggregate GDP (an effect that the same authors reckon as extraordinary large). Cervellati and Sunde (2011) and Hansen and Lönstrup (2015) document a strong and robust positive causal effect of life expectancy on *per capita* GDP in countries which had already experienced the onset of the demographic transition by 1940. Acemoglu and Johnson (2007) find that life expectancy has a positive impact on aggregate GDP but a negative one (despite often not statistically significant) on *per capita* and per worker GDP, for a panel of 47 countries over the period 1940-1980. They also find that health improvements have a negative causal effect on economic growth.

Outline The paper is organized as follows. After deriving the *augmented* Solow model (Section 2), we develop the econometric analysis, present the main results and show how unobserved heterogeneity can help in explaining differences across countries (Section 3). Then, we discuss our main findings along with a comparison with other contributions closest to ours (Section 4). Section 5 concludes.

2 The *augmented* Solow model

As in Barro (2013), we assume that production at time t takes the following Cobb-Douglas form:

$$Y_t = K_t^{\alpha} E_t^{\beta} H_t^{\gamma} \left(A_t L_t\right)^{1-\alpha-\beta-\gamma} \quad \text{with } \alpha, \beta, \gamma > 0 \quad \text{and } 0 < \alpha + \beta + \gamma < 1 \qquad (1)$$

where K, E, H, L and A denote physical capital, human capital in the form of education, human capital in the form of health, raw labor and the exogenous labor-augmenting technological progress, respectively. In equation (1), the contribution to total real GDP of raw labor, human capital in the form of education and human capital in the form of health (as reflected, respectively, by the elasticities $1-\alpha-\beta-\gamma$, β, and γ) is potentially dissimilar across each other and different from that of physical capital, as well. For the sake of simplicity, the total of labor input (L) is also assumed to correspond to total population. The dynamics of the size of population and the level of technology are exogenous and obey, respectively to $L_t = L_0 e^{nt}$ and $A_t = A_0 e^{gt}$.

At each date t, the amount of effective labor is $A_t L_t$, and grows at rate $(n+g)$. Physical capital, human capital in the form of education and human capital in the form of health are three reproducible factor inputs. The economy-wide budget constraint is:

$$Y_t = K_t^{\alpha} E_t^{\beta} H_t^{\gamma} \left(A_t L_t\right)^{1-\alpha-\beta-\gamma} = C_t + I_{Kt} + I_{Et} + I_{Ht} \tag{2}$$

Thus, the same production function applies to physical capital, education, health, and consumption: once produced, one unit of output can interchangeably be transformed, instantaneously and without costs, into units of consumption, physical capital, human capital in the form of schooling, and human capital in the form of health.

Let now $k_t \equiv \frac{K_t}{A_t L_t}$, $e_t \equiv \frac{E_t}{A_t L_t}$ and $h_t \equiv \frac{H_t}{A_t L_t}$ define the variables K_t, E_t and H_t per unit of effective labor. The production function in intensive form is given by:

$$y_t \equiv \frac{Y_t}{A_t L_t} = k_t^{\alpha} e_t^{\beta} h_t^{\gamma} \tag{3}$$

Let now s_k, s_e and s_h denote, respectively, the exogenous fractions of total income invested in physical capital, education and health, with $s \equiv s_k + s_e + s_h$ being the total saving rate of the economy. We assume that these saving rates are time invariant. The evolution of the three capital stocks is given by:

$$\dot{k}_t = s_k y_t - (n+g+\delta) k_t \tag{4}$$

$$\dot{e}_t = s_e y_t - (n+g+d) e_t \tag{5}$$

$$\dot{h}_t = s_h y_t - (n+g+d) h_t \tag{6}$$

We continue to follow Barro (2013, p. 353) in assuming that the exogenous depreciation rate of physical capital ($\delta > 0$) differs from the exogenous depreciation rate of education and health ($d > 0$).

Eqs. 4-6 imply that the economy converges to a steady state equilibrium (defined

by $\dot{k}_t = \dot{e}_t = \dot{h}_t = 0$) in which:

$$h^* = \left[\frac{s_k^{\alpha} s_e^{\beta} s_h^{1-\alpha-\beta}}{(n+g+d)^{1-\alpha}(n+g+\delta)^{\alpha}} \right]^{\frac{1}{1-\alpha-\beta-\gamma}} \tag{7}$$

$$e^* = \left[\frac{s_k^{\alpha} s_h^{\gamma} s_e^{1-\alpha-\gamma}}{(n+g+d)^{1-\alpha}(n+g+\delta)^{\alpha}} \right]^{\frac{1}{1-\alpha-\beta-\gamma}} \tag{8}$$

$$k^* = \left[\frac{s_e^{\beta} s_h^{\gamma} s_k^{1-\beta-\gamma}}{(n+g+d)^{\beta+\gamma}(n+g+\delta)^{1-\beta-\gamma}} \right]^{\frac{1}{1-\alpha-\beta-\gamma}} \tag{9}$$

After some algebraic steps, it is possible to show that at the steady state the relation linking the level of *per capita* income, to (some of) the exogenous variables of the model and, more importantly, to the level of health, h^*, is represented by:[3]

$$\begin{aligned} \ln\left(\frac{Y_t}{L_t}\right)^* &= \ln A_0 + gt + \left(\frac{\beta}{1-\alpha-\beta}\right)\ln(s_e) + \left(\frac{\alpha}{1-\alpha-\beta}\right)\ln(s_k) \\ &- \left(\frac{\beta}{1-\alpha-\beta}\right)\ln(n+g+d) - \left(\frac{\alpha}{1-\alpha-\beta}\right)\ln(n+g+\delta) + \\ &+ \left(\frac{\gamma}{1-\alpha-\beta}\right)\ln(h^*) \end{aligned} \tag{10}$$

Following Mankiw, Romer and Weil (1992), it is also easy to show that the growth of *per capita* income, along the transition, is a function of the determinants of the ultimate steady state and the initial level of income, i.e

$$\ln\left(\frac{Y_t/L_t}{Y_0/L_0}\right) = \zeta \ln\left(\frac{Y_t}{L_t}\right)^* - \zeta \ln\left(\frac{Y_0}{L_0}\right) \qquad (\lambda > 0) \tag{11}$$

where Y_0/L_0 is the *per capita* income at some initial date, $\zeta \equiv \left(1 - e^{-\lambda t}\right)$ and λ indicates the speed of conditional convergence toward the steady state. Plugging (10) into (11) yields:

$$\begin{aligned} \ln\left(\frac{Y_t/L_t}{Y_0/L_0}\right) &= \zeta\left(\frac{\beta}{1-\alpha-\beta}\right)\ln(s_e) + \zeta\left(\frac{\alpha}{1-\alpha-\beta}\right)\ln(s_k) - \\ &- \zeta\left[\left(\frac{\beta}{1-\alpha-\beta}\right)\ln(n+g+d) + \left(\frac{\alpha}{1-\alpha-\beta}\right)\ln(n+g+\delta)\right] + \\ &+ \zeta\left(\frac{\gamma}{1-\alpha-\beta}\right)\ln(h^*) - \zeta\ln\left(\frac{Y_0}{L_0}\right) + \zeta\ln A_0 + gt \end{aligned} \tag{12}$$

[3]See Bucci, Carbonari and Trovato (2019) for the derivation of equation (10).

3 Empirical analysis

Table 1: Descriptive statistics: mean, standard deviation, min, median and max.

Variable	**Mean**	**St. Dev.**	**Min**	**Median**	**Max**
5-years avg. *per capita* GDP growth rate	2.165	4.422	-37.493	2.296	29.617
life expectancy at birth	64.97	9.5	31.96	68.04	81.95
log of *per capita* real GDP	8.005	0.969	4.959	7.977	10.496
log of the ratio real domestic investment to GDP	-1.883	0.621	-4.386	-1.777	-0.472
log of HC index	0.478	0.313	0.000	0.468	1.000
log of $(g + n + \delta)$	-2.679	0.212	-7.634	-2.630	-1.742

Data Our sample consists of 72 non-OECD, non-oil countries along the period 1995-2014 (3,203 observations). The data are from the Penn World Table 8.1 (PWT hereafter) and the World Bank. The variables taken into account are real GDP, physical capital, population, education and life expectancy at birth. We measure the population growth rate as the average rate of growth of the working-age population, where the working age is defined as 15 to 65. As a measure of the theoretical variable s_k we use the average share of real investment (including government investment) on real GDP. The human capital index (HC, provided by PWT) and the life expectancy at birth (provided by the World Bank) proxy s_e and s_h, respectively.[4] For simplicity, we assume $d = \delta$, i.e. human and physical capital have the same depreciation rate. Summary statistics are provided in Table 1.

Econometric strategy The econometric part of the paper is aimed at i) assessing quantitatively the relative contribution of health on living standards and real GDP growth, and ii) quantify the cross-country differences in long-run income and growth, taking into account the dependence between GDP and health. We start by employing OLS Fixed Effects (FE) and then GMM estimators to deal with the reverse causation

[4]The HC index is based on the average years of schooling from Barro and Lee (2013) and an assumed rate of return to education, based on Mincer equation estimates. Alternative measures for population health are the health adjusted life expectancy, the adult mortality rate or child mortality. Data series for these variables, however, are available only for shorter duration and/or with respect to a limited number of countries.

between the level of real *per capita* GDP and country's health status (see Weil, 2014; Tamakoshi and Hamori, 2015; and Linden and Ray, 2017). Since our aim is to show that these regression models are not able to solve the bias due to the correlation between residuals and regressors, in the following paragraph, for the sake of brevity we restrict our attention only on the regression for the level of GDP.[5] Then, we present the flexible Bivariate Finite Mixture model (BFMM, hereafter), which allows for parameter heterogeneity among countries with similar fundamentals (see Alfò and Trovato, 2004; Alfò, Trovato and Waldmann 2008; Owen, Videras and Davis, 2009; Ng and Mclachlan, 2014; Yu, Malley and Ghosh, 2014; Lu, Huang and Zhu, 2016; Alfò, Carbonari and Trovato, 2020).[6] Moreover, through this estimation procedure, we are able to provide a cluster analysis, i.e. we sort countries into groups based on the homogeneity of the conditional joint distribution of their income levels and life expectancies with respect to the estimated unobservable factors.[7]

OLS and GMM The empirical counterpart of the theoretical equation (10) is given by:

$$\ln(y)_{it} = a_1 + \beta_1 \ln(s_e)_{it} + \beta_2 \ln(s_k)_{it} + \beta_3 \ln(n+g+\delta)_{it} + \beta_4 \ln(s_h)_{it} + \nu_{it} \qquad (13)$$

As stressed above, due to the endogeneity of life expectancy (s_h), we can get incon-

[5]Results for the growth equation (12) are available upon request.

[6]Notice that measurement error, omitted variable and varying parameters may be additional source of unobserved heterogeneity (and thus, model mis-specification).

[7]Consider the case of varying parameters among sample and suppose that the influence of x_i on the response, y_i, is country specific. In this case, $\beta_i = \beta + u_i$ where u_i is the country specific effect for subject $i = 1, \ldots, N$, with $E(u_i) = 0$, and β is the OLS estimator, capturing the average effect of x_i on y_i. Formally:

$$y_i = \alpha + (\beta + u_i)x_i + \epsilon_i$$

If we ignore the country specific heterogeneity and estimate the model with a homogeneous estimator (e.g. OLS), we get:

$$\begin{aligned} y_i &= \alpha + \beta x_i + (\epsilon_i + u_i x_i) \\ &= \alpha + \beta x_i + \tilde{\epsilon}_i \end{aligned}$$

As the classical endogeneity bias, the variable x_i is correlated with the error term $\tilde{\epsilon}_i$.

sistent estimates for β_4. A possible solution is to to use IV regressions (both two stage or GMM) for panel data, in which the instruments are the intercept and a vector of instruments correlated with the suspected endogenous variable and uncorrelated with the gaussian error. According to Lewbel (1997 and 2012), we estimate equation (13) using as instruments some transformations of the covariates and response. Such transformations are useful when there is no available additional data or when it is not possible to set a model to correlate instrument with unobserved variables. Here, the choice of the regressors is driven by our *augmented* Solow model. Table 3 reports the estimates of equation (13) for the OLS FE model and for three different specifications for the GMM with Continuous Updating Estimator (CUE): GMM1 includes only the Lewbel (2012) instruments, GMM2 includes only the lagged (from $t-1$ to $t-3$ values of covariates) while GMM3 presents both Lewbel's instruments and the lagged variables. All models are estimated controlling for time and subject's correlation and are estimated with robust standard errors.

Results for the OLS FE model and GMM models, which are more robust for heteroscedasticity (e.g. see Kleibergen, 2005, Caner, 2010, Baum et al., 2012), are not univocal. Once we correct for the endogeneity of life expectancy, we can observe that the effect of investment rate (s_k) and human capital (s_e) are not statistically different from zero. GMM1 and GMM2 deliver the same estimated parameter for β_4 while GMM3 estimates a parameter for life expectancy quite similar to that obtained from OLS FE. For the OLS FE model all the estimated parameters seem to be in line with the standard literature on growth, for the models GMM1 and GMM2 only the parameter for population growth and the rate of depreciation ($n+g+\delta$) is significant while for GMM3 is significant also that for human capital (s_e). Several issues emerge with respect to the GMM models. Estimates are sensitive to the change of the selected instruments, this indicating the presence of a possible model uncertainty problem, i.e. uncertainty about the actual model we have selected to estimate equation (13). The Sagan, Hansen and Jensen's test for the orthogonality and endogeneity of instruments does not reject the assumption that the instruments are valid, i.e. uncorrelated with the error term. The Hansen J-statistic shows that, once we include instruments in our regression, we can consider the life expectancy (s_h) as orthogonal. The under identifi-

Table 2: Panel Instrumental Variable Results

	OLS FE	**Panel GMM CUE 1**	**Panel GMM CUE 2**	**Panel GMM CUE 3**
log of $(g+n+\delta)$	-0.300***	-0.950***	-0.678***	-0.390***
	(0.041)	(0.254)	(0.193)	(0.080)
log of investment rate (s_k)	0.036**	0.041	0.028	0.008
	(0.014)	(0.057)	(0.0559)	(0.0540)
log of HC index (s_e)	0.4427***	-0.201	-0.120	0.567***
	(0.035)	(0.316)	(0.336)	(0.182)
log of life expectancy $(s_h)_{-1}$	0.474***	1.268***	1.267***	0.380*
	(0.039)	(0.384)	(0.440)	(0.220)
controlled for Time and Subjects	YES	YES	YES	YES
R-squared	0.3721	0.3548	0.2814	0.3678
Number of individuals	3203	3203	2986	2986
		Underidentification tests		
Kleibergen-Paap LM $\chi^2(3)$		13.456	14.70	15.45
(P-value)		0.0025	0.0021	0.016
		Weak-instrument-robust inference		
Kleibergen-Paap Wald F		8.32	14.03	12.65
$Stock-Yogo$ critical values				
10% maximal LIML size		5.44	6.46	4.45
15% maximal LIML size		3.87	4.36	3.34
20% maximal LIML size		3.30	3.69	2.87
		Overidentification test		
Sargan-Hansen-Jensen		4.23	2.454	7.97
(P-value)		0.402	0.293	0.158
		Orthogonality Statistics for *life expectancy*		
Hansen J statistics		0.90	2.093	7.01
(P-value)		0.34	0.143	0.402
		Test for Normal Residuals		
Shapiro Francia (P-value)	0.006	0.004	0.007	0.004
Shapiro Wilk (P-value)	0.000	0.000	0.000	0.000
Instrumets		q vector as in Lebwel (1997 and 2012)	Lagged covariates and trend variable	q vector as in Lebwel (1997 and 2012) and lagged covariates.

cation test suggests that we may reject the assumption of not identified model. Finally, looking at the weak of instrument test, we can reject the assumption of a small correlation between instruments and covariates (see the LIML maximum critical values). To sum up, these tests, though significant, do not help us to discriminate the best model to describe the relationship between *per capita* GDP, human capital and health status. Figure A1 shows that the residual are still informative, meaning that the assumptions about their orthogonality and homogeneity do not hold. Observations are clusterized, some unobserved heterogeneity is still present. This is confirmed by the Shapiro Wilk and the Shapiro Francia tests (that are robust for heteroscedasticity), which reject the assumption of Gaussian residuals. Consequently, all the models presented in Table 3, regardless of both the estimator (FE or GMM) or the instruments employed, are not able to correct the parameters and standard errors bias due to the correlation between residuals and covariates.

BFMM To avoid uncertainty about instruments and to allow for a country-specific effects, we modify the empirical model as follows. We assume that the dependence between the endogenous variables and regressors is not the same for all countries. Therefore, we introduce a mixture model to explain the existing heterogeneity among countries and to deliver a cluster analysis. The mixture model is obtained as the non-parametric estimation of a model involving two correlated random effects and it leads to a weighted sum of bivariate distributions. This allows to capture the country-specific effect. The advantage of this model is twofold. First, it allows to correct the bias between residuals and covariates. Second, it permits to group countries within homogeneous clusters where cluster specific homogeneity implies unbiased standard errors and more reliable estimates. The model requires a local independence assumption, i.e. there exists independence among variables given the random effects. This does not mean that the model requires independence marginally. However, the cluster memberships do not vary over time. The belonging to a specific cluster is based on the maximum a posteriori criterion (MAP), i.e. the country is assigned to the cluster showing the highest posterior probability. This can be done using the output of the (E step of the) EM algorithm, which we describe in Appendix A.

Following Linden and Ray (2017), we assume that real GDP levels and life expectancy are jointly correlated in some points in time.[8] Three main parameters are involved in the distributions of our flexible BFMM: location, scale and shape. Let y_{itj} be continuous variables corresponding to two ($j = 1, 2$) outcomes observed over n ($i = 1, 2, \ldots, n$) countries and time t ($t = 1, 2, \ldots, T$), with parameters $\boldsymbol{\theta}_{itj} = (\theta_{itj1}, \theta_{itj2}, \theta_{itj3})$. Since we are interested in understanding how much health affects GDP level and its dynamics, and viceversa, we run two alternative models: one, labelled BFMM$_Y$, in which the outcomes will be the level of the real *per capita* GDP and the aggregate level of health and one, labelled BFMM$_g$, in which the outcomes will be the real *per capita* GDP 5-years growth rate and the aggregate level of health. Aggregate health will be proxied by life expectancy at birth.

Let $\mathbf{x}'_{itj} = (1, x_{itj1}, \ldots, x_{itjP_j})$ and $\mathbf{z}'_{itj} = (1, z_{itj1}, \ldots, z_{itjQ_j})$ two sets of covariates, which can vary over outcomes. To account for potential heterogeneity among countries a matrix of correlated random effects is introduced, where each row is given by $\mathbf{u}_i = (\mathbf{u}_{i1}\mathbf{u}_{i2})$. It follows that the likelihood function can be written as

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} \left\{ \int_U \prod_{j=1}^{2} \prod_{t=1}^{T} f(y_{itj} \mid \mathbf{u}_{ij}, \mathbf{x}_{itj}, \mathbf{z}_{itj}) b(\mathbf{u}_i) d\mathbf{u}_i \right\} \tag{14}$$

where $f(\cdot)$ is a generic probability density function and U is the support for $b(\mathbf{u}_i)$, the bivariate distribution density of $\mathbf{u}_i$, with $E(\mathbf{u}_i) = 0$. The presence of random effects makes the parameter estimation not always feasible due to the presence of multidimensional integrals. However, if the multivariate random variable follows a multivariate normal distribution, different approaches exist in literature to approximate it. Nevertheless, the normality assumption may result to be too strong. A more flexible approach is to adopt a non parametric maximum likelihood approach, without defining a specific parametric distribution for $\mathbf{u}_i$. This leads to a bivariate finite mixture model (see Lindsay, 1983). Formally, random effects can be approximated by a discrete distribution of $C_j \leq n$ support points associated to $p_{c_1 c_2}$ mass joint probabilities attached to locations

[8]Notice that the flexible BFMM allows to deal with non-trivial correlation structure. For instance, omitted covariates may affect both real GDP and aggregate health. It is well known that when responses are correlated (in our case, real GDP level and life expectancy), the univariate approach is less efficient than the multivariate one.

$(\mathbf{u}_{i1} = \mathbf{u}_{c_1}, \mathbf{u}_{i2} = \mathbf{u}_{c_2})$ for $c_j = 1, \ldots, C_j$ as follows

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} \left\{ \sum_{c_1=1}^{C_1} \sum_{c_2=1}^{C_2} p_{c_1 c_2} \prod_{j=1}^{2} \prod_{t=1}^{T} f(y_{itj} \mid \mathbf{u}_{i1} = \mathbf{u}_{c_1}, \mathbf{u}_{i2} = \mathbf{u}_{c_2}, \mathbf{x}_{itj}, \mathbf{z}_{itj}) \right\} \tag{15}$$

where $p_{c_1 c_2} = \Pr(\mathbf{u}_{i1} = \mathbf{u}_{c_1}, \mathbf{u}_{i2} = \mathbf{u}_{c_2})$ is the joint probability associated to each pair of locations $(\mathbf{u}_{c_1}, \mathbf{u}_{c_2})$. In other words, the bivariate integral is approximated by a bivariate weighted sum. By the definition of weighted sum, it follows that the weights have to be positive and have to satisfy the following constraints: both univariate and bivariate weights should sum to 1, i.e.$\sum_{c_1}^{C_1} p_{c_1} = \sum_{c_2}^{C_2} p_{c_2} = \sum_{c_1 c_2} p_{c_1 c_2} = 1$, $p_{c_1} = \Pr(\mathbf{u}_{i1} = \mathbf{u}_{c_1}) = \sum_{c_2}^{C_2} p_{c_1 c_2}$ and $p_{c_2} = \Pr(\mathbf{u}_{i2} = \mathbf{u}_{c_2}) = \sum_{c_1}^{C_1} p_{c_1 c_2}$, respectively.

The number of support points (and thus the number of mixture components) may, in principle, be different among outcomes. It leads to a finite mixture model with $C_1 \times C_2$ components, where each of the C_1 locations are matched with each of the C_2 locations of the second component.

Finite Mixture models overcome the issues, in observational studies, of OLS and GMM with reference to confounding and measurement error. Recalling equations (10) and (12) and their corresponding empirical log-likelihoods (14), the BFMM$_Y$ and the BFMM$_g$ can be written respectively as

$$\begin{aligned} E\left[\ln(y)_{it,j=1} | u_{i1}, X_{it}\right] &= a_{i1} + \beta_1 \ln(s_e)_{it} + \beta_2 \ln(s_k)_{it} + \beta_3 \ln(n+g+\delta)_{it} \quad (16) \\ &+ \beta_4 \ln(s_h)_{it-1} \\ E\left[\ln(s_h)_{it,j=2} | u_{i2}, X_{it}\right] &= a_{i2} + a_1 \ln(y)_{it-1} \quad (17) \end{aligned}$$

and

$$\begin{aligned} E\left[\gamma_{it,j=1} | u_{i1}, X_{it}\right] &= a_{i1} + \xi_0 \ln(y)_{it-1} + \xi_1 \ln(s_e)_{it} + \xi_2 \ln(s_k)_{it} + \xi_3 \ln(n+g+\delta)_{it} \\ &+ \xi_4 \ln(s_h)_{it-1} \quad (18) \\ E\left[\ln(s_h)_{it,j=2} | u_{i2}, X_{it}\right] &= a_{i2} + a_2 \ln(y)_{it-1} \quad (19) \end{aligned}$$

where X_{it} is the vector of covariates for country i at time t, while $\ln(y)_{it}$ is log of the *per capita* GDP, γ_{it} is its 5 years average growth rate, a_{i1} and a_{i2} are the two random intercepts estimating the country specific unobserved (or unmeasured) characteristics,

affecting the relationship between response variables and $\ln(s_h)_{it}$ via the locations u_i and u_j in equation (14).

Table 3: Estimates

	$\mathbf{BFMM}_Y$	s.e.	$\mathbf{BFMM}_g$	s.e.
GDP level & GDP growth				
Intercept				
a_{01}	6.194	0.105	23.780	1.653
a_{02}	7.001	0.140	20.343	1.542
a_{03}	7.624	0.106	26.620	1.774
a_{04}	5.546	0.100		
a_{05}	6.658	0.102		
log of investment rate (s_k)	0.116	0.012	1.542	0.137
log of HC index (s_e)	0.433	0.034	0.878	0.372
log of ($g + n + \delta$)	-0.460	0.037	0.419	0.434
log of life expectancy $(s_h)_{-1}$	0.445	0.040	3.413	0.403
GDP_{-1}			-2.431	0.163
$\ln(\sigma)$	-0.992	0.014	1.351	0.013
Life expectancy (s_h)				
a_{01}	19.592	1.721	14.588	1.218
a_{02}	8.942	0.065	19.585	1.198
a_{03}	3.074	1.571	3.066	1.245
a_{04}	14.595	1.360	8.934	1.248
GDP_{-1}	6.010	0.155	6.011	0.144
$\ln(\sigma)$	1.681	0.014	1.681	0.013

Note: in $BFMM_Y$ and $BFMM_g$ the number of intercepts, in both regressions, depends on the number of univariate locations. Instrumented variable: life expectancy$_{-1}$.

Table 3 reports results for the two Bivariate Finite Mixture Models, BFMM_Y, for the levels of *per capita* GDP, and BFMM_g, for the rates of growth of GDP.[9] The results strongly support our *augmented* version of the Solow model with education and health.[10] Both BFMM_Y and BFMM_g rely on the assumption that one specific source of unobserved heterogeneity bias is due to the bivariate relationship between observed income (levels and growth rates) and life expectancy. This source of unobserved heterogeneity may affect the significance of the estimated parameters. Following equation (14), we set the intercept for country i as $a_i = a + u_i$, where latent term u_i has an unspecified random discrete distribution with $E(u_i) = 0$. Since $\mathbf{u_i}$ is country specific we can group countries, with same latent term, in cluster for which the standard OLS homogeneity assumption holds.

In BFMM_Y, the coefficients of s_k, s_e and $(n+g+\delta)$ are in line with the literature and our OLS estimates. The estimated elasticity of output with respect to physical capital is relatively low (0.116), while that of human capital is relatively high (0.433), although still in line with the microeconomic literature on private returns from schooling (see e.g Arnold et al., 2011). Human capital is also found to be an important factor for growth (0.878).[11] The contribution of aggregate health is positive both on the level (0.445) and on the growth (3.413) of *per capita* income, i.e. a one-year increase in life expectancy raises the long-run level of *per capita* GDP by 5%. Notice that despite the regression for life expectancy in the two models is the same, the estimates are different. This is due to the fact that, within each component, we have a *weighted* regression in

[9] In clusterwise regressions, the standards errors are obtained by the bootstrap method based on 500 samples.

[10] In the two systems of equations presented here, human capital in the form of education/schooling appears as a control only in equations (16) and (18). We run regressions, available upon request, in which it appears even in the two equations for life expectancy with no significant change in our results. Given the importance of education, as a productive input in the *augmented* Solow model, we also estimate a three-equation model with human capital added as a third response variable. In this case, however, we obtain less accurate estimates.

[11] As a robustness check, we run our regression using the average years of education in working age population, as an alternative proxy for human capital. Qualitatively, our results do not change. Estimates are available upon request.

which the univariate weights are obtained as a marginalization of the bivariate posterior probabilities. The posterior probabilities involve both responses, i.e. life expectancy and *per capita* GDP in BFMM$_Y$, and life expectancy and *per capita* GDP growth in BFMM$_g$.

Since the number of groups are determined by how many different latent terms exist for the sample, we choose the optimal number of support points ($\mathbf{u_i}$) following the Bayesian information criterion (BIC). Table A4 shows BIC values for the two multivariate models (BFMM$_Y$ and BFMM$_g$). Such values reject for both models the hypothesis of no clustering in favor of:

i) a BFMM$_Y$ containing 5 clusters with respect to the level of *per capita* income and 4 clusters with respect to life expectancy, and

ii) a BFMM$_g$ containing 3 clusters with respect to the growth rate of *per capita* income and 4 clusters with respect to life expectancy.

Tables A5 and A7 present our classifications while Tables A6 and A8 report descriptive statistics for each cluster in BFMM$_Y$ and BFMM$_g$, respectively. Figures A4-A11 show the patterns of GDP, levels and growth, and life expectancy over time across countries. In the BFMM$_Y$, we identify five clusters with respect to GDP levels and four with respect to life expectancy. The cluster K_1=3 (Argentina, Barbados, Botswana, Brazil, Bulgaria, Costa Rica, Croatia, Cyprus, Latvia, Lithuania, Malaysia, Malta, Mauritius, Panama, Serbia and South Africa) is the one in which the unobserved factors that affect aggregate income are the strongest (i.e., a_{03}=7.624). Not surprisingly, this cluster is the one with the highest *per capita* GDP, with a cluster mean of 9.23 (see the last column in Table A6). Analogously, the cluster K_2=1 (Albania, Argentina, Armenia, Belize, Bulgaria, China, Costa Rica, Croatia, Cyprus, El Salvador, Jamaica, Jordan, Malta, Myanmar, Panama, Paraguay, Romania, Serbia, Sri Lanka, Tajikistan and Thailand) is the one in which the unobserved factors that affect life expectancy are the strongest (i.e., a_{01}=19.592). Tandon et al. (2000), who produce a rankings-based comparison of the efficiency of the health care system of 191 countries, list many of the countries included in the cluster K_2=1 among the most efficient ones: Malta (7th), Cyprus (24th), Costa Rica (36th), Croatia (43rd), Jamaica (53rd) and Albania (55th).

Using a different statistical technique, Kumbhakar (2010) provides a classification in which Jamaica, China, Sri Lanka and Armenia appear among the top 10 countries, ranked by efficiency in health. This cluster is also the one, between those identified by our proxy for aggregate health, with the highest *per capita* GDP, with a cluster mean of 8.62 (see the bottom of Table A6).

In the BFMM$_g$, we identify three clusters with respect to GDP growth and four with respect to life expectancy. For all the countries in the sample, we find that aggregate health has positive impact on growth, with an elasticity of *per capita* GDP growth rate on life expectancy equal to 3.413. Looking at the clusters' composition, some interesting analogies with the classification provided by BFMM$_Y$ emerge. For instance, the cluster K_1=3 (Argentina, Barbados, Botswana, Brazil, Bulgaria, Croatia, Cyprus, Latvia, Lithuania, Malaysia, Maldives, Malta, Mauritius, Panama, Romania Serbia and Thailand) is the one in which the unobserved factors that affect the GDP growth are the strongest (i.e., a_{03}=26.620) but also the one with the highest average *per capita* GDP growth rate, 3.99% (see the last column in Table A8). A final point that is worth mentioning is that in BFMM$_g$, the unobserved factors that affect life expectancy are particularly strong (a_{02}=19.585) in K_2=2 (Albania, Argentina, Armenia, Belize, Bulgaria, China, Costa Rica, Croatia, Cyprus, El Salvador, Jamaica, Jordan, Malta, Myanmar, Panama, Paraguay, Romania, Serbia, Sri Lanka, Tajikistan, Thailand), which is also the cluster with the highest average life expectancy, 71.05 (see the bottom of Table A8).

Finallly figures A2 and A3 show that the within groups residuals for the models BFMM$_Y$ and BFMM$_g$ are not informative anymore (compared to the OLS and GMM results), meaning that the assumptions about their normality, orthogonality and homogeneity hold. Observations are not clusterized. This is confirmed by the Shapiro Wilk and the Shapiro Francia tests, which reject the assumption of Gaussian residuals.[12]

[12]We do not produce the residual plots for life expectancy in the two models, since the variable is needed only for solving the endogeneity issue, thus reducing the bias in the estimation.

4 Discussion

The models presented in the previous section explain cross-country income and growth differences with the cross-country differences in the capital output ratios and life expectancy, conditional on the estimated country-specific level of technology. We deal with endogeneity using a two-step GMM model. To account for unobserved heterogeneity we run two BFMMs. Despite we do not formally test any causality, the *augmented* Solow model can be used as a guidance to discuss our empirical results.

Our econometric analysis reveals that, for a large sample of low- and middle-income economies, population's health positively and significantly affects both the level and the growth rate of *per capita* income. The positive impact on income level is consistent with the *augmented* Solow model, in which the typical capital "dilution effect", due to the increase in population induced by a better aggregate health, is offset by the increase in productivity arising from healthier workers. The size of the impact that we document is quite large and is mainly due to the fact that we focus on a sample of non-OECD/non-oil countries. Qualitatively, the result is in line with Bucci, Carbonari and Trovato (2019), Bloom, Canning, and Sevilla (2004), Cervellati and Sunde (2011) and Hansen and Lönstrup (2015) while it contrasts Acemoglu and Johnson (2007). There are (at least) two possible, not mutually exclusive, explanations for the discrepancy between our and Acemoglu and Johnson's results: the different period considered and the econometric design employed in the two studies. Acemoglu and Johnson exploit the drop in mortality from specific infectious diseases, due to the international epidemiological transition, as an instrument for the change in life expectancy. This identification strategy makes use of the fact that the mortality rate from these diseases was exogenous in 1940, because no treatment, medication, or vaccines were available before that time. Starting from 1980, instead, all these diseases can be treated or prevented in all countries, due to medical advances. After regressing *per capita* income growth on the increase in life expectancy between 1940 and 1980, Acemoglu and Johnson report a positive but non-significant effect of increased life expectancy on aggregate GDP and a positive and significant effect on population growth. The overall impact on *per capita* GDP is found to be negative (which means that countries

that experienced larger exogenous health improvements saw lower gains in *per capita* income). The authors ascribe their findings to the fact that increases in health result mainly in large increases in population. In turn, the capital-dilution effect associated to a faster population growth reduces income *per capita* at the steady state. Therefore, improved health finally lowers *per capita* income. Notably, the Acemoglu and Johnson (2007)'s methodology has been questioned, as it regresses economic growth against health improvements without including initial health in the model. As such, the negative correlation between health improvements and economic growth shown in their data may simply be the consequence of the fact that countries starting with better health economically grow faster (while experiencing smaller improvements in health) than those starting with lower initial health conditions (but experimenting larger health enhancements during the transition).[13] In our multivariate set-up, we tackle this issue by using a one-period lag for life expectancy on the RHS of the equation (18).

The evidence of a positive effect of health on economic conditions for low- and middle-income countries provides a rationale for international health aid. Especially in low income countries, health aid is found to have a positive and significant effect on health outcomes, i.e. an increase in life expectancy (Arndt et al., 2015) and a decrease in infant mortality (Mishra and Newhouse, 2009). In particular, health aid is found to be more effective at improving health outcomes for countries with higher domestic health expenditure or a more efficient public sector (e.g. Gyimah-Brempong, 1992).[14]

[13]To study this possibility, Aghion, Howitt and Murtin (2011) and Bloom, Canning, and Fink (2014) include initial health in the Acemoglu and Johnson (2007)'s regressions and find that, indeed, the negative causal effect vanishes. More specifically, Aghion, Howitt and Murtin (2011) combine the Mankiw et al. (1992)'s approach (whereby output growth is correlated with the rate of improvement in human capital) with the Nelson and Phelps (1966)'s approach (whereby a higher level of health should spur growth by facilitating technological innovation), and look at the joint effect of health (level and accumulation) on economic growth. After running cross-country growth regressions over the period 1960-2000, they show that the level and the accumulation of health have significant positive effects on *per capita* income growth. Moreover, they find a weaker relationship between health and growth over the contemporary period in OECD countries. According to them, this result is explained by the fact that only gains in life expectancy below 40 years are significantly correlated with *per capita* income growth.

[14]Chunling et al. (2010) warn that a potential substitution may occur between international health aid

Improving the effectiveness and/or increasing *sic et simpliciter* the public resources allocated to health programs can increase life expectancy. From a theoretical standpoint, it is immediate to see that if the increase of this specific type of public expenditure occurs – keeping the size of public sector unchanged – the accumulation of physical capital turns to be unaffected while possible gains for the accumulation of human capital, in the form of health, can emerge. Through this channel, therefore, such a public policy can generate higher GDP and faster growth in low-income countries. Despite the available evidence on this potentially virtuous link is mixed[15], when we include public expenditure on health as an explicative variable on the RHS of equations (17) and (19), we find that it positively affects both the level and the growth rate of GDP. The elasticity of life expectancy with respect to such expenditure is 0.695 in BFMM_Y and 0.733 in BFMM_g.[16]

5 Concluding remarks

There are two alternative approaches to estimate the effect of health on economic growth. The first is to calibrate the size of the effects of health at the aggregate level, using estimates from microeconomic studies. The second is to estimate the aggregate relationship directly, using macroeconomic data. We follow the second route. Building on Mankiw, Romer and Weil (1992), we argue that international differences in income

and domestic expenditure on health. Studying a sample of developing countries they find that the presence of programs aimed at providing Development Assistance for Health (DAH) to countries has a negative effect on domestic government spending on health, while having a positive and significant effect on domestic non-governmental health spending.

[15]Studying a sample of Sub-Saharan African countries, Novignon et al. (2012) find that health expenditure significantly improves life expectancy, and reduces death and infant mortality rates. Barenberg, Basu and Soylu (2017) find similar results using Indian data in the periods 1983-1984 and 2011-2012. For a large sample of developing countries, Baldacci et al. (2008) explore the channels through which social spending can affect human capital and GDP growth. They find that health spending has a positive and significant impact on human capital, and thus supports higher growth. Ssozi and Amlani (2015) find that, although health expenditure in Sub-Saharan Africa has substantially increased since 2000, it has had a low impact on both life expectancy and infant mortality.

[16]For the sake of brevity we do not report these regressions, which, however, are available upon request.

per capita are best understood using an *augmented* Solow growth model in which output is produced from physical capital, raw labor, human capital in the form of education, and human capital in the form of health. The model predicts that the long-run level of *per capita* GDP and its growth rate (along the transition path) are both positively affected by the level of aggregate health. We test these predictions by using data from a sample of low- and middle-income countries, along the period 1995-2014. As it is standard in this literature, life expectancy at birth has been used as a proxy for population's health. To take into account the unobserved heterogeneity, we estimate a flexible Bivariate Finite Mixture Model, which incorporates the restrictions provided by the *augmented* Solow model.

Our estimates document a sizable effect of health on living standards (*per capita* GDP level) and economic development (*per capita* GDP growth rate). In the baseline model (i.e. the BFMM_y), a one-year increase in life expectancy raises the long-run level of per capita GDP by 5%. A reverse positive channel from GDP on life expectancy is also confirmed. Our analysis also reveals the relevance of heterogeneity and the clustering of countries according to outcomes (life expectancy and GDP level/growth) clearly matters. Finally, despite we are not able to distinguish between the effects of different types of health investments, our study provides an argument in favor of an increase in the size of public national or international health plans, which may improve aggregate welfare in low- and middle-income countries not only *directly*, by producing better health conditions, but also *indirectly*, by positively affecting aggregate productivity.

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Appendix

A EM algorithm

The parameter estimates are carried out through the EM (Expectation-Maximization) algorithm (Dempster et al., 1977) based on the following complete-data log-likelihood:

$$\ell_c(\boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{k_1=1}^{K_1} \sum_{k_2=1}^{K_2} w_{k_1 k_2} \left\{ \ln(p_{k_1 k_2}) + \ln(f_{i k_1 k_2}) \right\}, \tag{20}$$

where $w_{k_1 k_2}$ ia a dummy variable assuming value 1 if unit (country) i is in component k_1 and k_2 at the same time, 0 otherwise, and

$$f_{i k_1 k_2} \equiv f_{i k_1} f_{i k_2} = \prod_{t=1}^{T} f(y_{it1} \mid u_{k_1}, \mathbf{x}_{it1}, \mathbf{z}_{it1}) f(y_{it2} \mid u_{k_2}, \mathbf{x}_{it2}, \mathbf{z}_{it2})$$

In the E-step (Expectation step) we compute the posterior probabilities for each unit i to belong jointly to the k_1-th and k_2-th components of the mixture, that is

$$\hat{w}_{i k_1 k_2} = \frac{p_{k_1 k_2} f_{i k_1 k_2}}{\sum_{k_1 k_2} p_{k_1 k_2} f_{i k_1 k_2}}$$

with $k_1 = 1, \ldots, K_1$ and $k_2 = 1, \ldots, K_2$. The marginal posterior probabilities can be easily derived as

$$\hat{w}_{i k_1} = \textstyle\sum_{k_2} \hat{w}_{i k_1 k_2} \quad \hat{w}_{i k_2} = \textstyle\sum_{k_1} \hat{w}_{i k_1 k_2}.$$

In the M-step (Maximization step), we maximize 20 with respect to model parameters, exploiting its separability. A close-form solution is available for $\hat{p}_{k_1 k_2}$, that is $\hat{p}_{k_1 k_2} = \sum_{i=1}^{n} \hat{w}_{k_1 k_2} n^{-1}$.

The maximization over the remaining parameters can be carried out using a standard maximization routine. Nevertheless, the algorithm may be trapped at a local maximum and, consequently, may fail to reach a global maximum. A simple way to overcome the issue is to run the EM algorithm from multiple random starting points for a number of steps, then pick the one with the highest likelihood, and continue the EM from the selected point until convergence. To determine the value of K_1 and K_2 we use the Bayesian Information Criterion (BIC).[17]

[17] The BIC is largely used in cluster analysis because it allows to compare models with different parametriza-

Table A4: Model choice based on the minimization of BIC

BFMM$_Y$		**BFMM**$_g$	
(K_1, K_2)	**BIC**	(K_1, K_2)	**BIC**
(2,2)	5046.226	(2,2)	18234.78
(2,3)	4906.209	(2,3)	18094.51
(2,4)	4865.328	(2,4)	18063.07
(2,5)	4894.884	(2,5)	18093.29
(3,2)	4074.441	(3,2)	18211.47
(3,3)	3925.361	(3,3)	18067.44
(3,4)	3892.534	(3,4)	**18043.98**
(3,5)	3933.105	(3,5)	18080.03
(4,2)	3477.886	(4,2)	18212.51
(4,3)	3345.62	(4,3)	18072.59
(4,4)	3314.203	(4,4)	18057.01
(4,5)	3349.274	(4,5)	18101.12
(5,2)	3349.274	(5,2)	18272.87
(5,3)	3158.749	(5,3)	18131.41
(5,4)	**3133.159**	(5,4)	18120.7
(5,5)	3176.276	(5,5)	18171.94

Note: 16 different models (K_i, K_j) *have been considered with* $i = 1, \ldots, 5$ *and* $j = 1, \ldots, 5$.

tion, different numbers of components, or both (see Fraley and Raftery, 1998).

Table A5: BFMM$_Y$: estimated clusters within the combinations of groups (K_i, K_j, $i = 1, 2, 3, 4, 5$ and $j = 1, 2, 3, 4$) and marginally (K_i, $i = 1, 2, 3, 4, 5$ in the last column; K_j, $j = 1, 2, 3, 4$ in the last row)

	$K_2 = 1$	$K_2 = 2$	$K_2 = 3$	$K_2 = 4$	
$K_1 = 1$	China, El Salvador Tajikistan	Benin, Ghana Kenya, Mauritania Senegal, Zimbabwe	Sierra Leone, Zambia	Bangladesh, India Mongolia, Pakistan	Bangladesh, Benin, China El Salvador, Ghana, India Kenya, Mauritania, Mongolia Pakistan, Senegal, Sierra Leone Tajikistan, Zambia, Zimbabwe
$K_1 = 2$	Belize, Jamaica Jordan ,Romania Thailand	Botswana	Namibia, Swaziland	Dominican Republic, Fiji Guatemala, Maldives Nicaragua, Peru Tunisia, Ukraine	Belize, Botswana, Dominican Republic Fiji, Guatemala, Jamaica Jordan, Maldives, Namibia Nicaragua, Peru, Romania Swaziland, Thailand, Tunisia Ukraine
$K_1 = 3$	Argentina, Bulgaria Costa Rica, Costa Cyprus, Malta Panama, Serbia		South Africa	Barbados, Brazil Latvia, Lithuania Malaysia, Mauritius	Argentina, Barbados, Brazil Bulgaria, Costa Rica, Croatia Cyprus, Latvia, Lithuania Malaysia, Malta, Mauritius Panama, Serbia, South Africa
$K_1 = 4$	Myanmar	Burkina Faso, Burundi Cambodia, Ethiopia Haiti, Lesotho Liberia, Madagascar Mozambique, Rwanda Togo	Malawi, Mali Niger	Nepal	Burkina Faso, Burundi Cambodia, Ethiopia, Haiti Lesotho, Liberia, Madagascar Malawi, Mali, Mozambique Myanmar, Nepal, Niger Rwanda, Togo, Uganda
$K_1 = 5$	Albania, Armenia Paraguay, Sri Lanka			Honduras, Morocco Philippines	Albania, Armenia, Honduras Morocco, Paraguay, Philippines Sri Lanka
	Albania, Argentina Armenia, Belize Bulgaria, China Costa Rica, Croatia Cyprus, El Salvador Jamaica, Jordan Malta, Myanmar Panama, Paraguay Romania, Serbia Sri Lanka, Tajikistan Thailand	Benin, Botswana Burkina Faso, Burundi Cambodia, Ethiopia Ghana, Haiti Kenya, Lesotho Liberia, Madagascar Mauritania, Mozambique Rwanda, Senegal Togo, Uganda Zimbabwe	Malawi, Mali Namibia, Niger Sierra Leone South Africa Swaziland, Zambia	Bangladesh, Barbados Brazil, Dominican Republic Fiji, Guatemala Honduras, India Latvia, Lithuania Malaysia, Maldives Mauritius, Mongolia Morocco, Nepal Nicaragua, Pakistan Peru, Philippines Tunisia, Ukraine	

Table A6: BFMM_Y: income and health

			$K_2=1$	$K_2=2$	$K_2=3$	$K_2=4$	
$K_1=1$	GDP	Mean	7.68	7.65	7.38	7.61	7.61
		Median	7.60	7.62	7.24	7.52	7.58
		St.Dev	0.71	0.27	0.39	0.51	0.46
	life exp	Mean	65.80	54.38	44.57	59.20	56.24
		Median	67.67	55.58	44.55	59.76	56.87
		St.Dev.	6.52	6.02	6.51	6.36	8.80
$K_1=2$	GDP	Mean	8.60	8.31	8.65	8.52	8.55
		Median	8.62	8.74	8.64	8.49	8.55
		St.Dev.	0.49	1.06	0.29	0.44	0.51
	life exp	Mean	69.43	57.04	55.36	64.78	64.62
		Median	69.95	57.33	55.63	66.22	67.01
		St.Dev.	3.90	4.57	4.31	7.16	7.44
$K_1=3$	GDP	Mean	9.24	-	9.10	9.22	9.22
		Median	9.23	-	9.06	9.35	9.23
		St.Dev.	0.58	-	0.15	0.57	0.55
	life exp	Mean	73.65	-	57.59	69.21	70.61
		Median	73.93	-	57.46	70.32	71.81
		St.Dev.	3.75	-	2.90	4.31	5.90
$K_1=4$	GDP	Mean	7.02	6.88	6.84	6.95	6.89
		Median	6.88	6.90	6.89	6.90	6.90
		St.Dev.	0.67	0.40	0.30	0.33	0.40
	life exp	Mean	58.50	50.21	45.19	53.94	50.01
		Median	58.87	49.48	45.89	53.77	49.40
		St.Dev.	4.99	7.15	7.20	10.27	7.88
$K_1=5$	GDP	Mean	8.30	-	-	8.12	8.21
		Median	8.28	-	-	8.07	8.20
		St.Dev.	0.43	-	-	0.32	0.39
	life exp	Mean	70.32	-	-	63.97	67.25
		Median	70.29	-	-	65.23	68.46
		St.Dev.	3.39	-	-	6.53	6.05
	GDP	Mean	8.57	7.21	7.69	8.38	
		Median	8.62	7.16	7.26	8.37	
		St.Dev	0.85	0.61	0.93	0.79	
	life exp	Mean	70.04	51.92	48.98	64.11	
		Median	71.05	52.24	49.49	65.46	
		St.Dev.	5.72	7.06	8.09	7.67	

Note: means, medians and standard deviations of GDP and life expectancy within the combinations of groups (K_i, K_j, $i=1,2,3,4,5$ and $j=1,2,3,4$) and marginally (K_i, $i=1,2,3$ in the last column; K_j, $j=1,2,3,4$ in the last row).

Table A7: BFMM$_g$: estimated clusters within the combinations of groups (K_i, K_j, $i = 1, 2, 3$ and $j = 1, 2, 3, 4$) and marginally (K_i, $i = 1, 2, 3$ in the last column; K_j, $j = 1, 2, 3, 4$ in the last row)

	$K_2 = 1$	$K_2 = 2$	$K_2 = 3$	$K_2 = 4$	
$K_1 = 1$	Dominican Republic, Fiji Guatemala, Mongolia Morocco, Pakistan Peru, Philippines Tunisia, Ukraine	Albania, Armenia Belize, China Costa Rica, El Salvador Jamaica, Jordan Myanmar, Paraguay Sri Lanka	Namibia, Sierra Leone South Africa, Swaziland Zambia		Albania, Armenia, Belize China, Costa Rica, Dominican Republic El Salvador, Fiji, Guatemala Jamaica, Jordan, Mongolia Morocco, Myanmar, Namibia Pakistan, Paraguay, Peru Philippines, Sierra Leone, South Africa Sri Lanka, Swaziland, Tunisia Ukraine, Zambia
$K_1 = 2$	Bangladesh, Honduras India, Nepal Nicaragua	Tajikistan	Malawi, Mali Niger	Benin, Burkina Faso Burundi, Cambodia Ethiopia, Ghana Haiti, Kenya Lesotho, Liberia Madagascar, Mauritania Mozambique, Rwanda Senegal, Togo Uganda, Zimbabwe	Bangladesh, Benin, Burkina Faso Burundi, Cambodia, Ethiopia Ghana, Haiti, Honduras India, Kenya, Lesotho Liberia, Madagascar, Malawi Mali, Mauritania, Mozambique Nepal, Nicaragua, Niger Rwanda, Senegal, Tajikistan Togo, Uganda, Zimbabwe
$K_1 = 3$	Barbados, Brazil Latvia, Lithuania Malaysia, Maldives Mauritius	Argentina, Bulgaria Croatia,Cyprus Malta, Panama Romania, Serbia Thailand		Botswana	Argentina, Barbados, Botswana Brazil, Bulgaria, Croatia Cyprus, Latvia, Lithuania Malaysia, Maldives, Malta Mauritius, Panama, Romania Serbia, Thailand
	Bangladesh, Barbados Brazil, Dominican Republic Fiji, Guatemala Honduras, India Latvia, Lithuania Malaysia, Maldives Mauritius, Mongolia Morocco, Nepal Nicaragua, Pakistan Peru, Philippines Tunisia, Ukraine	Albania, Argentina Armenia, Belize Bulgaria, China Costa Rica, Croatia Cyprus, El Salvador Jamaica, Jordan Malta, Myanmar Panama, Paraguay Romania, Serbia Sri Lanka, Tajikistan Thailand	Malawi, Mali Namibia, Niger Sierra Leone South Africa Swaziland, Zambia	Benin, Botswana Burkina Faso Burundi, Cambodia Ethiopia, Ghana Haiti, Kenya Lesotho, Liberia Madagascar, Mauritania Mozambique, Rwanda Senegal, Togo Uganda, Zimbabwe	

Table A8: BFMM$_g$: GDP growth and health

			$K_2=1$	$K_2=2$	$K_2=3$	$K_2=4$	
$K_1=1$	GDP growth	Mean	2.68	3.05	1.36	-	2.57
		Median	2.58	2.79	1.79	-	2.49
		St.Dev.	3.27	4.56	4.39	-	4.12
	life exp.	Mean	63.85	68.43	51.36	-	63.26
		Median	64.48	69.66	52.88	-	65.24
		St.Dev.	6.42	6.14	7.72	-	9.11
$K_1=2$	GDP growth	Mean	1.48	-1.98	0.36	0.76	0.81
		Median	1.49	2.38	0.51	1.04	1.11
		St.Dev.	3.33	9.75	3.28	4.83	4.56
	life exp.	Mean	59.57	67.38	45.19	51.63	52.65
		Median	60.07	67.51	45.89	51.82	52.48
		St.Dev.	8.99	2.46	7.20	7.06	8.65
$K_1=3$	GDP growth	Mean	3.32	4.12	-	6.79	3.99
		Median	3.78	4.18	-	5.99	4.16
		St.Dev.	3.95	3.50	-	4.22	3.83
	life exp.	Mean	68.58	72.34	-	57.04	69.76
		Median	70.11	72.50	-	57.33	71.20
		St.Dev.	5.49	4.29	-	4.57	6.23
	GDP growth	Mean	2.56	3.38	0.98	1.08	
		Median	2.53	3.32	1.38	1.21	
		St.Dev.	3.55	4.44	4.02	4.99	
	life exp.	Mean	64.11	70.04	48.98	51.92	
		Median	65.46	71.05	49.49	52.24	
		St.Dev	7.67	5.72	8.09	7.06	

Note: means, medians and standard deviations of GDP growth rate and life expectancy within the combinations of groups (K_i, K_j, $i=1,2,3$ and $j=1,2,3,4$) and marginally (K_i, $i=1,2,3$ in the last column; K_j, $j=1,2,3,4$ in the last row).

Figure A1: Residual plots for the models OLS and GMM.

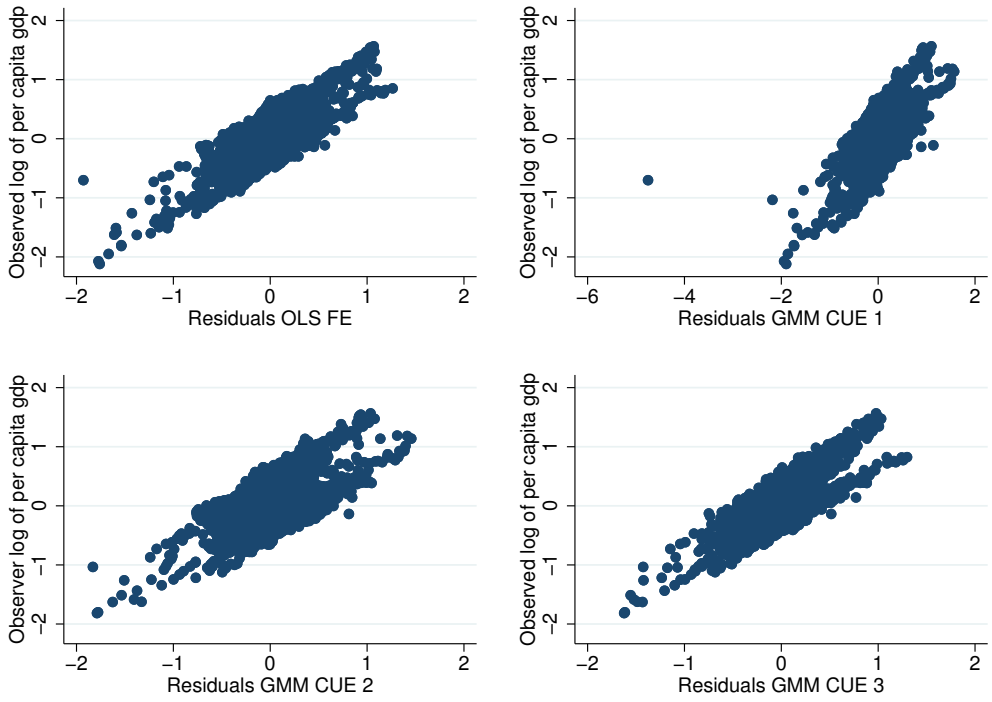


Figure A2: Residual plots for the models BFMM_Y.

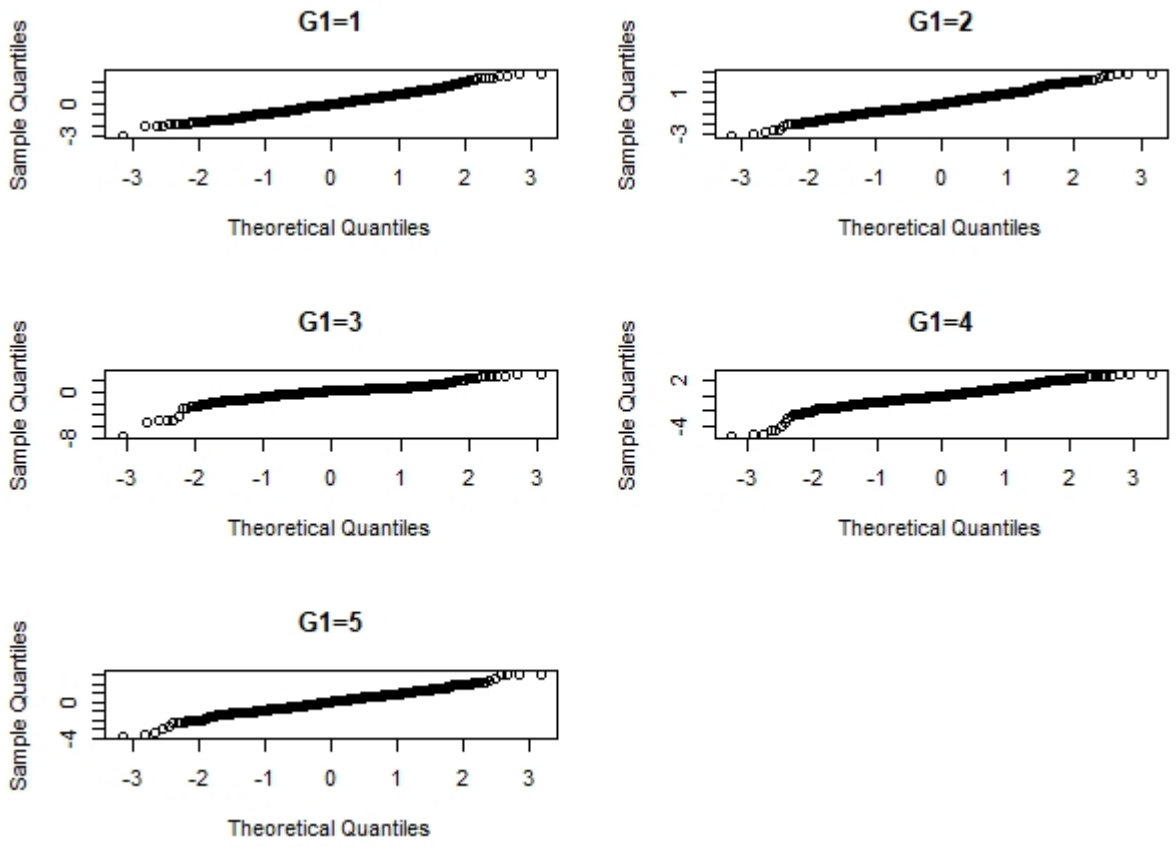
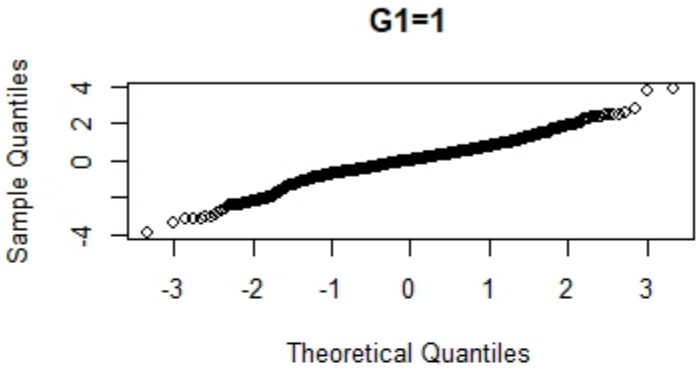
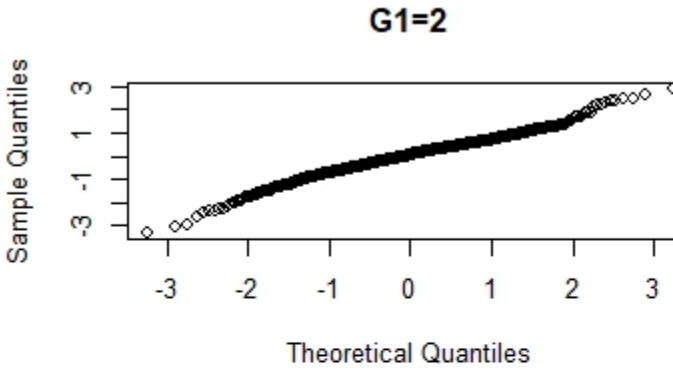


Figure A3: Residual plots for the models BFMM$_g$.





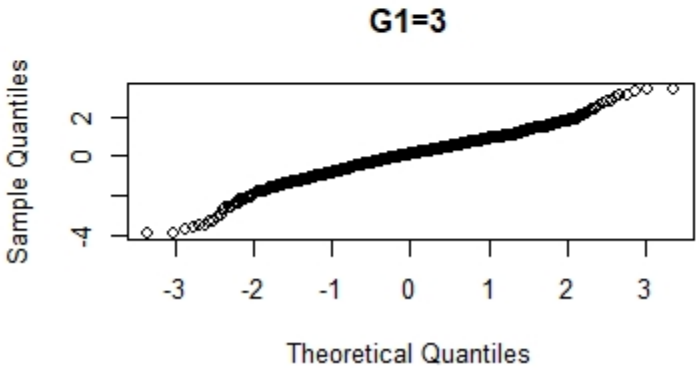


Figure A4: GDP over time across countries. The color of the solid line (GDP) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red), $K_1 = 3$ (green), $K_1 = 4$ (blue) and $K_1 = 5$ (light blue).

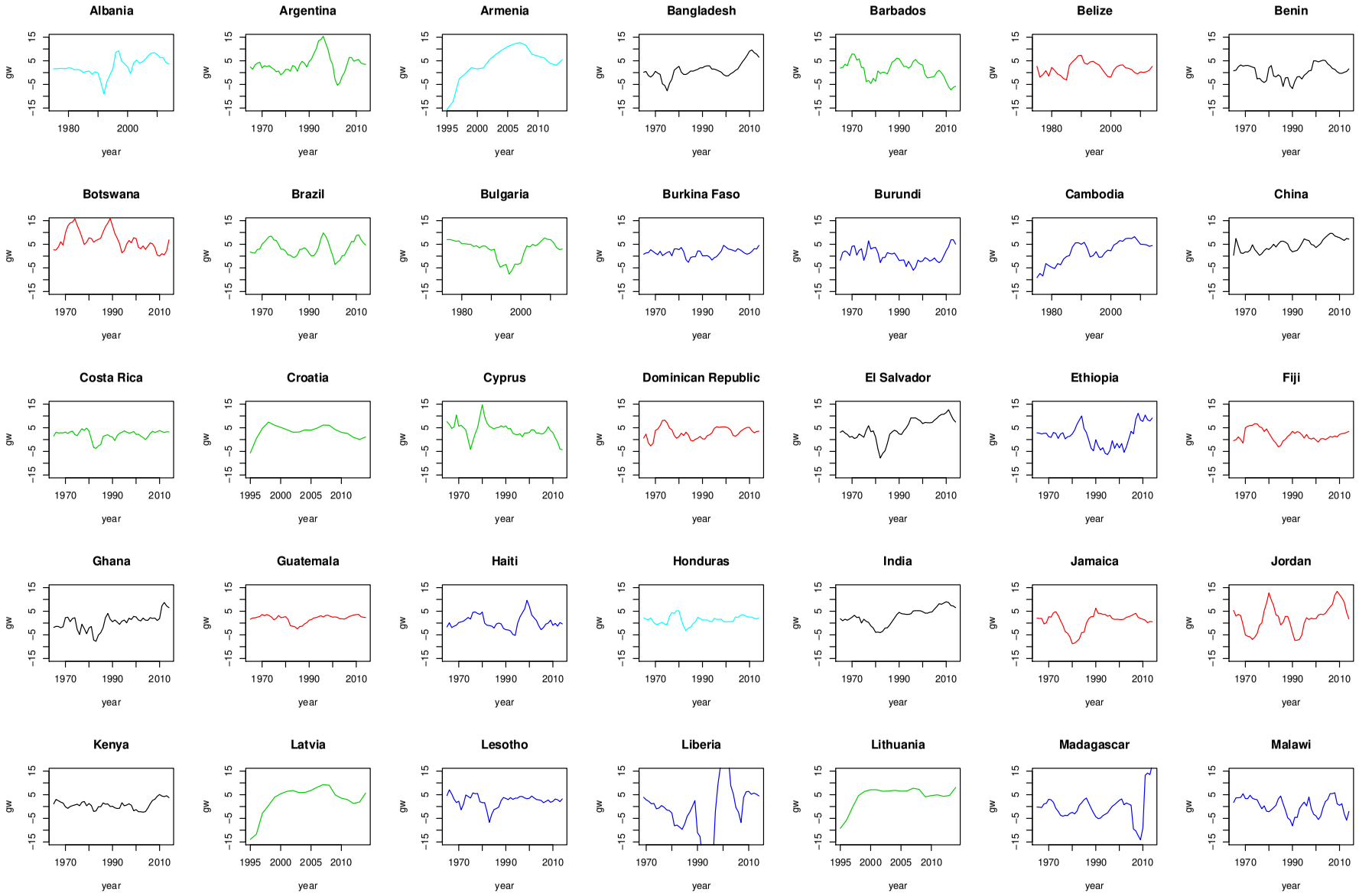


Figure A5: GDP over time across countries. The color of the solid line (GDP) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red), $K_1 = 3$ (green), $K_1 = 4$ (blue) and $K_1 = 5$ (light blue).

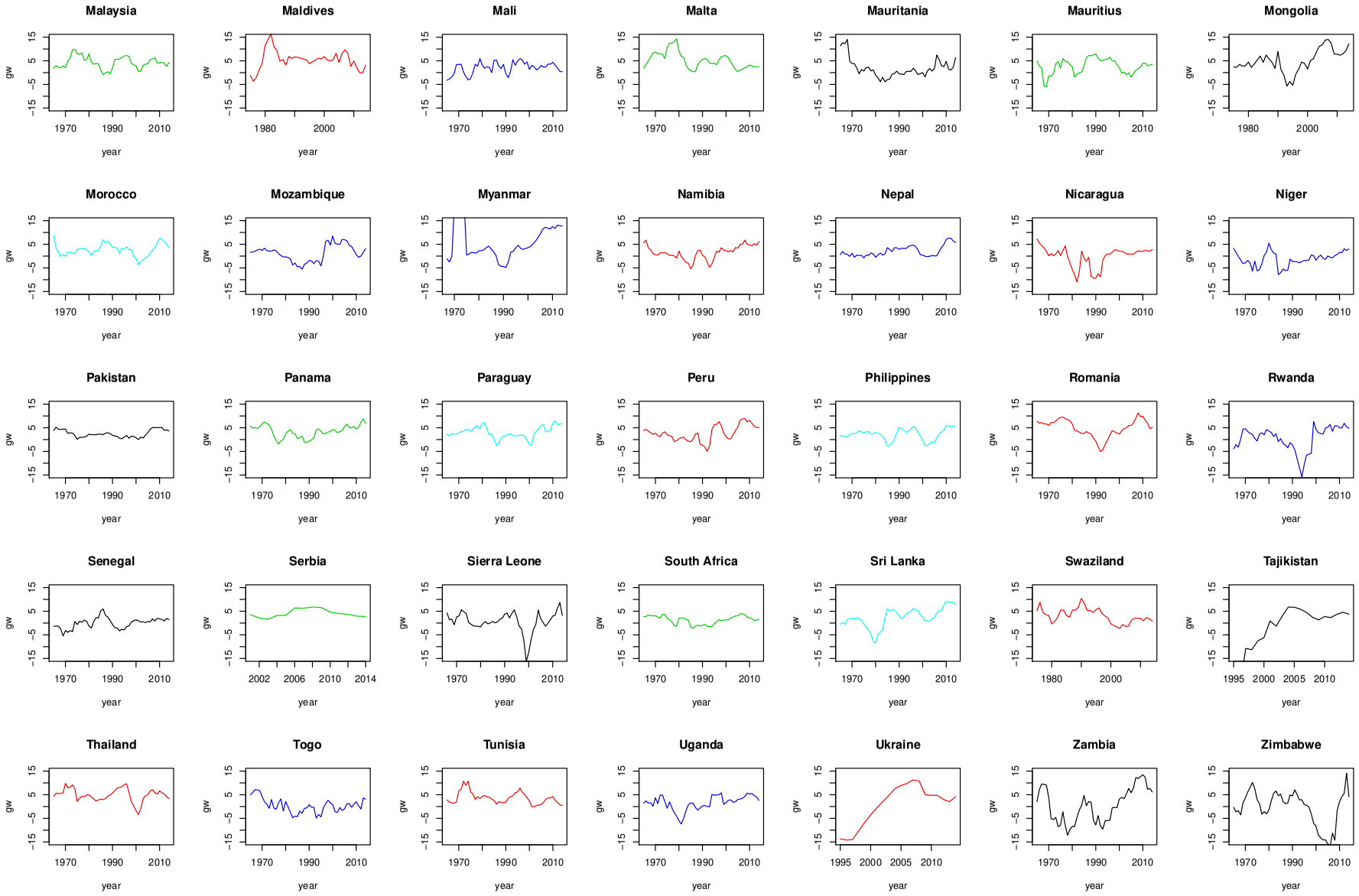


Figure A6: `life exp.` over time across countries. The color of the dashed line (`life exp.`) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red), $K_1 = 3$ (green) and $K_1 = 4$ (blue).

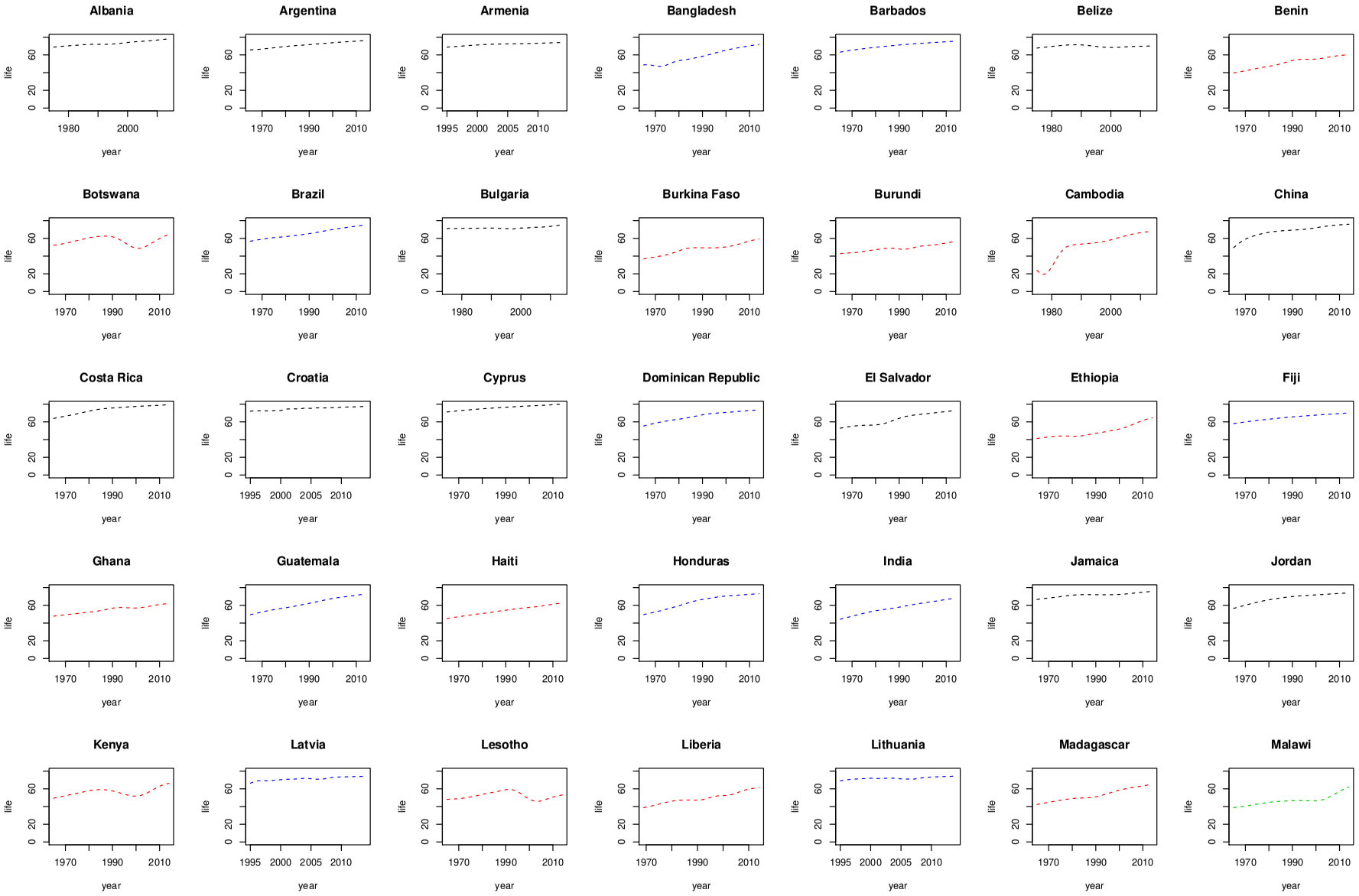


Figure A7: `life exp.` over time across countries. The color of the dashed line (`life exp.`) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red), $K_1 = 3$ (green) and $K_1 = 4$ (blue).

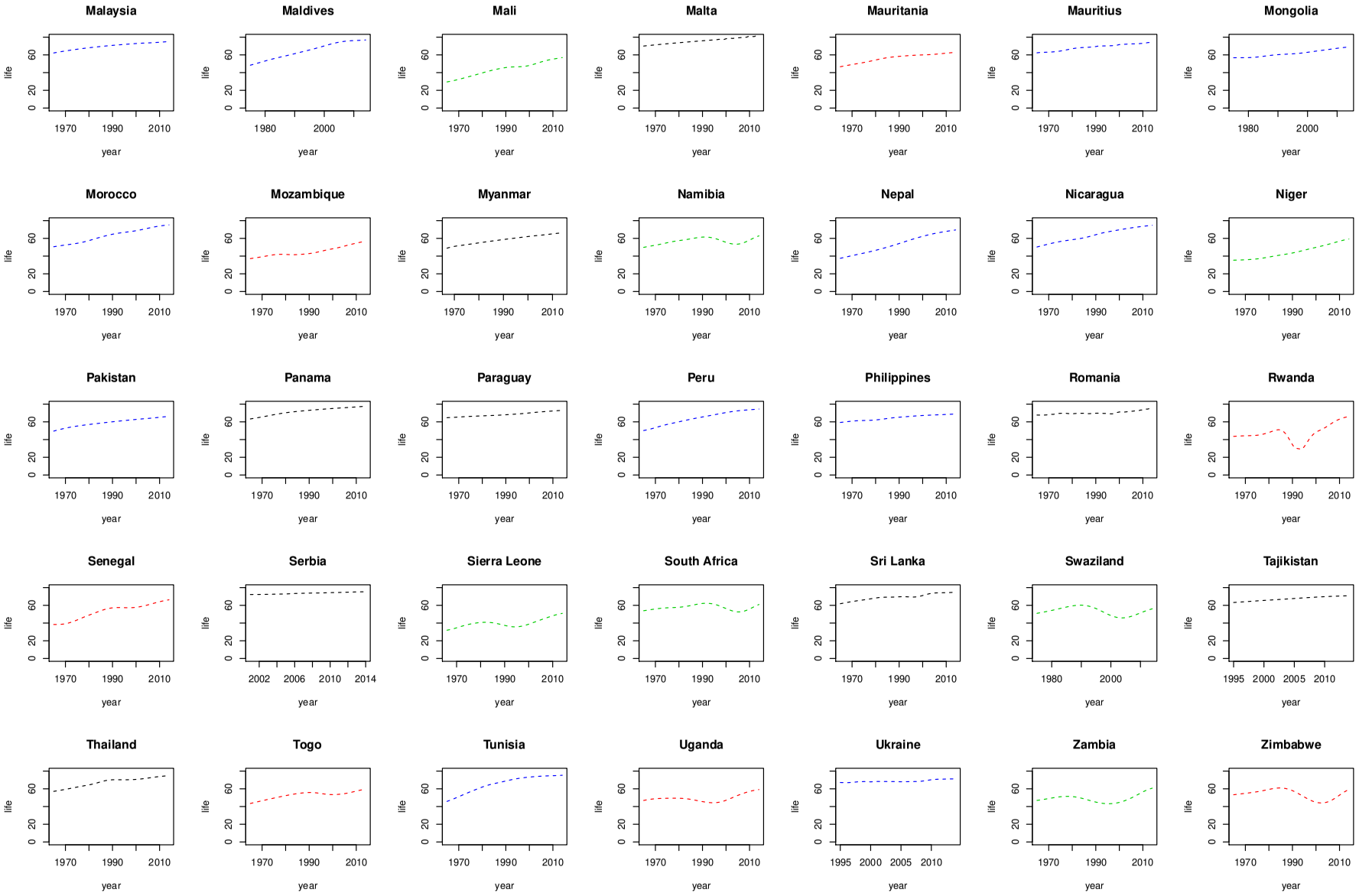


Figure A8: `GDP growth` over time across countries. The color of the solid line (`GDP growth`) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red) and $K_1 = 3$ (green).

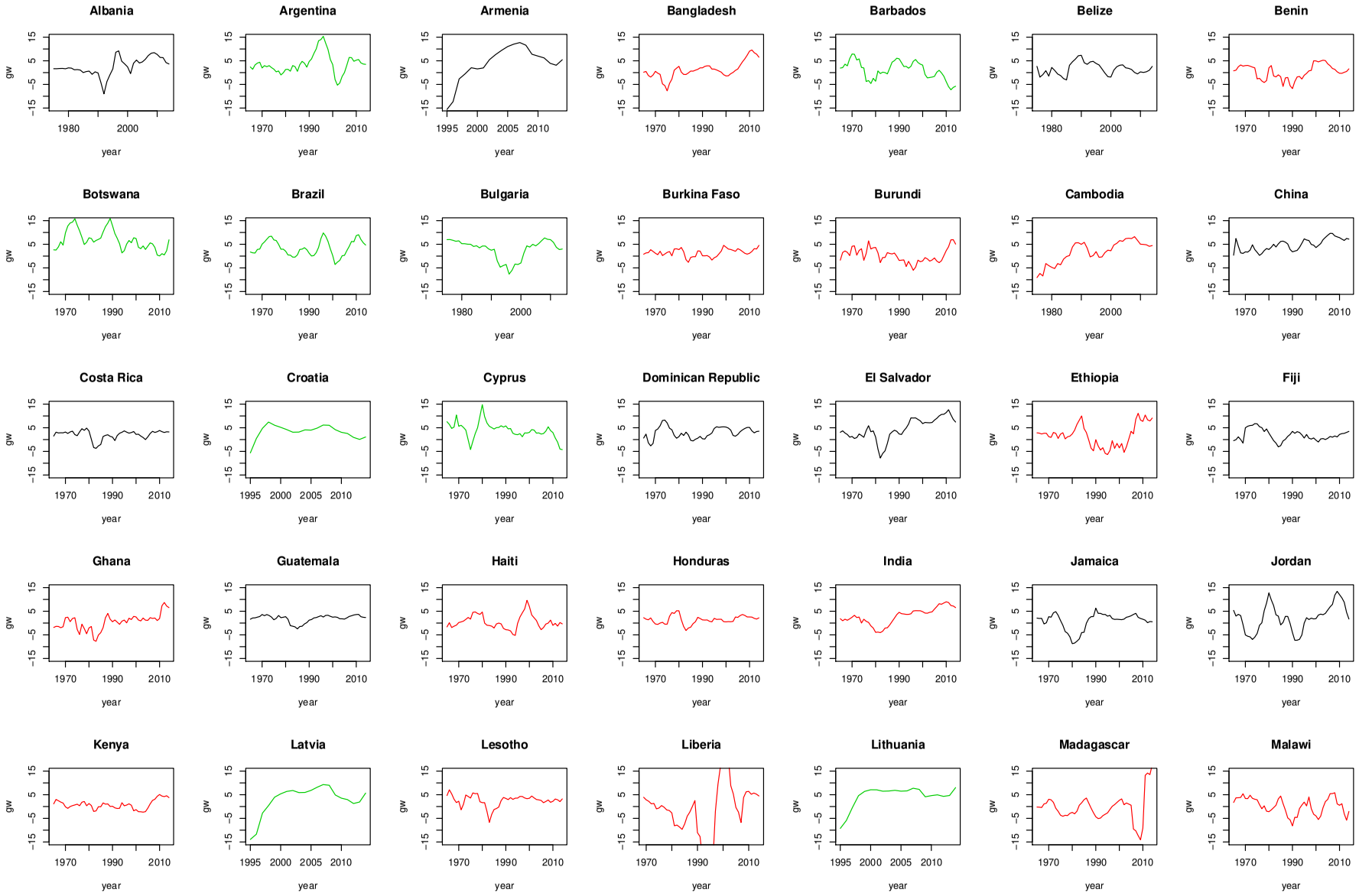


Figure A9: `GDP growth` over time across countries. The color of the solid line (`GDP growth`) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red) and $K_1 = 3$ (green).

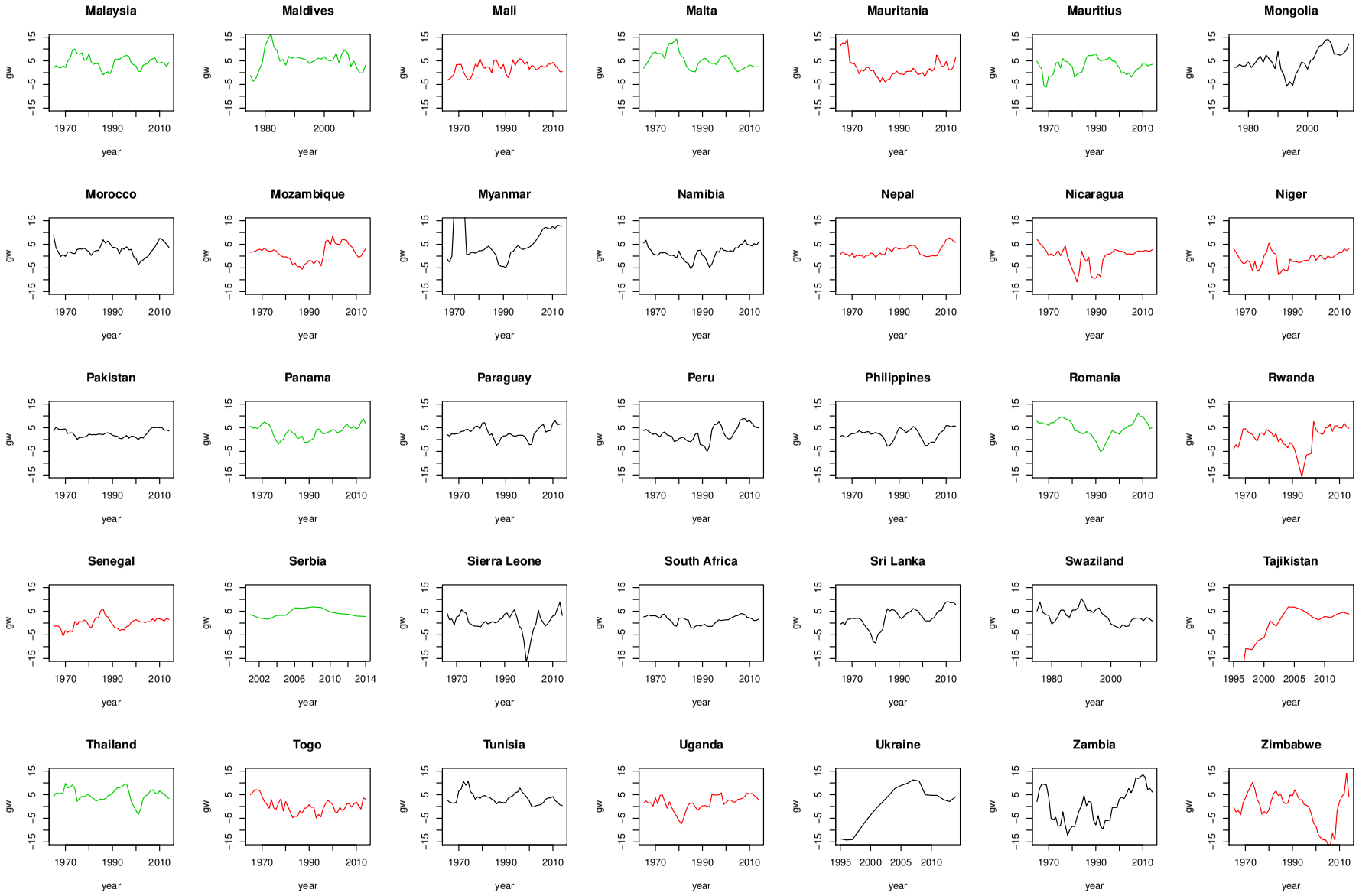


Figure A10: `life exp.` over time across countries. The color of the dashed line (`life exp.`) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red), $K_1 = 3$ (green) and $K_1 = 4$ (blue).

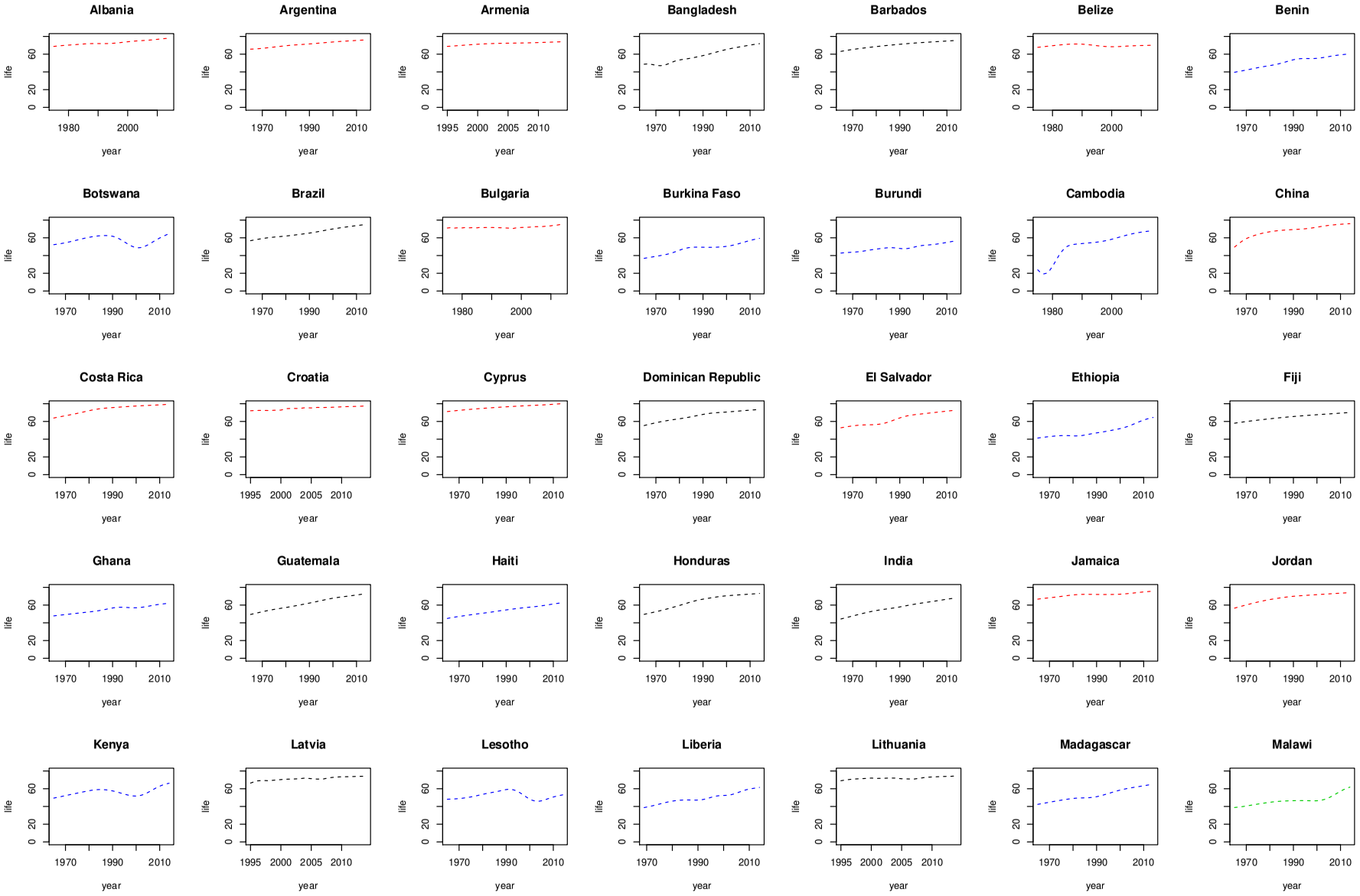


Figure A11: `life exp.` over time across countries. The color of the dashed line (`life exp.`) represents the belonging of the country to $K_1 = 1$ (black), $K_1 = 2$ (red), $K_1 = 3$ (green) and $K_1 = 4$ (blue).