Electoral Competition with Strategic Voters

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Abstract

A recent literature has found a positive relationship between the disproportionality of the electoral system and the convergence of parties' positions. Such a relationship depends crucially on the assumption that voting is sincere. We show that, when voters are players in the game and not simply automata that vote for their favorite party, two policy-motivated parties always take extreme positions in equilibrium.

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1. Introduction

The policies announced before an election is called are crucial to determine the electoral outcome and the resulting policy outcome. How voters are expected to behave and how the electoral outcome translates into actual political power can influence parties' positioning choice. In particular, the mapping from the distribution of votes to the distribution of power varies according to the electoral system. For instance, under majoritarian rule the winning party takes all the political power, while under proportional representation each party's power share coincides with her share of votes. In this paper we analyze the electoral competition between two policy-motivated parties when they expect voters to vote strategically and under multiple power-sharing rules.

The political process consists of three stages. First, a leftist and a rightist party choose a policy they commit to, then voters vote, and finally the policy outcome is implemented. The outcome function is a weighted average of parties' positions, where weights are given by the corresponding power shares. This implies that parties potentially face a trade-off. The more moderate is the policy that they choose, the higher is their share of votes and their relative political power, but such a weight will be assigned to a less extreme position. Since the power share depends in turn on the electoral system, one may expect this to influence the net effect of the two forces. As the proportionality decreases, indeed, the incentive to obtain more votes than the opponent intensifies, because the vote share translates into a higher power share.

Some recent papers have explored such a conjecture and found results in this direction. In particular, a positive relationship between the disproportionality of the electoral system and the convergence of parties' positions has

been found for the case of two parties with mixed motives (Saporiti, 2014) and of policy-motivated parties (Matakos et al., 2016). These studies assume that voters vote sincerely, that is, they just vote for their favorite party.

This work is the first study in this context of party electoral competition under *strategic voting*; that is, voters are players in the game. To characterize voters' strategic behavior we borrow the results of De Sinopoli and Iannantuoni (2007), who analyze it under purely proportional rule and multiple parties. They show that, as the number of voters goes to infinity, in equilibrium basically voters split in two and only the two extremist parties take votes. This result guarantees an unambiguous interpretation of strategic voting in a game with a continuum of voters in which, in principle, nobody can affect the outcome. In fact, such a game can be seen as limit of finite games and, therefore, voters' strategic behavior is precisely identified by the limit of their equilibria.

The basic spatial model is described in Section 2. In Section 3 we analyze the voting subgame, while in Section 4 we study the electoral stage.

2. Preliminaries

Let the one-dimensional *policy space* be represented by the closed interval $\mathbb{X} = [0,1]$. Parties L and R announce simultaneously their platforms $x_L, x_R \in \mathbb{X}$, to which they are committed. Knowing these positions, every voter votes for a party, and given the electoral outcome a policy is implemented. Each party j is purely policy-motivated, that is, she cares only about the implemented policy, and has single-peaked preferences characterized by an ideal point $\theta_j \in \mathbb{X}$, with $\theta_L < \theta_R$.

The implemented policy, or *policy outcome*, is a function of parties' platforms and vote shares, and of the degree of power sharing. This is captured by the parameter of the Tullock "contest success function", which allows to embed

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different electoral systems ranging from proportional representation to majoritarian rule.[1] Let v be the vote share of party L. The policy outcome $\hat{x}$ is given by

$$\hat{x}_\gamma(x_L, x_R) = \frac{v^\gamma}{v^\gamma + (1-v)^\gamma} x_L + \frac{(1-v)^\gamma}{v^\gamma + (1-v)^\gamma} x_R, \quad (1)$$

where $\gamma \geq 1$. In a purely proportional system the weight of each party's policy is the corresponding share of votes, i.e. $\gamma = 1$. As γ increases, the weight of the party who gets the majority of the votes increases, up to the limit case ($\gamma \to \infty$) in which she takes all the power and her proposed policy is implemented. Thus, higher values of γ correspond to lower degrees of power sharing.

The electorate consists of a continuum of voters of unit mass, whose bliss points are distributed over $[0,1]$ according to a continuously differentiable and strictly increasing distribution function F.[2] Each voter i has single-peaked preferences with bliss point $\theta_i \in \mathbb{X}$, which can be represented by a real-valued utility function $u(\hat{x}, \theta_i)$ that is continuously differentiable in $\hat{x}$.[3]

The whole game is a three-stage process and we solve it by backward induction. The last stage consists simply in the implementation of the policy outcome according to (1).

3. The voting subgame

Let us analyze the second stage in which voters, having observed the announced platforms x_L and x_R, vote strategically for one of the two parties. To this end, we can extend to our framework the results in De Sinopoli and Iannantuoni (2007), who study this kind of game under proportional rule and with multiple parties assuming that their positions are exogenous.

Consider two announced policies $x_A < x_B$ and a finite value of γ.[4] Let the *cutpoint outcome* x^* be the unique policy implemented when all the voters to its left vote for party A and all the voters to its right vote for party B, i.e., the unique solution of

$$x^* = \frac{F(x^*)^\gamma}{F(x^*)^\gamma + (1-F(x^*))^\gamma} x_A + \frac{(1-F(x^*))^\gamma}{F(x^*)^\gamma + (1-F(x^*))^\gamma} x_B.$$

De Sinopoli and Iannantuoni (2007) prove that, for $\gamma = 1$, x^* is the unique equilibrium outcome of the game with a continuum of voters when this is seen as limit of

games with a finite electorate, where voters are strategic.[5] In any mixed-strategy equilibrium of finite games, indeed, all the voters to the left of the cutpoint outcome vote for party A and all the voters to its right vote for party B, except for a neighborhood that shrinks as the number of voters increases. They also remark that this result remains true for any continuous and monotonic transformation of the purely-proportional outcome function. In fact, the function needs just to be continuous in the parties' vote shares and satisfy an ordinal assumption that, in the two-party case, reduces to strict increasingness in the rightist party's vote share. It is easy to see that our outcome function satisfies these assumptions.

Thus, take a sequence of finite games as the number of voters goes to infinity, whose associated sequence of bliss point distributions converges to the function F. Each game in this sequence has two particular voters, the rightmost one such that, in any equilibrium, she and all the voters to her left vote for party A, and the leftmost one such that, in any equilibrium, she and all the voters to her right vote for party B. The assumptions on the distribution function imply that the two corresponding sequences of these voters' bliss points converge to the same limit point, which is precisely the cutpoint outcome associated to F. This allows to fully characterize strategic voting in the game with a continuum of voters. Therefore, we can conclude that x^* is the unique equilibrium outcome of the voting subgame whenever parties' announced policies differ. Obviously, when the two policies are equal, every strategy profile is a Nash equilibrium also for a finite electorate, but the equilibrium outcome is unique and coincides with them.

4. The electoral stage

We study now the two-party electoral competition. An immediate consequence of the assumption that $\theta_L < \theta_R$ and of the form of the outcome function is that, in every equilibrium, $x_L < x_R$. Otherwise, at least one party has the incentive to deviate from her proposed platform in order to induce an outcome closer to her favorite one.

We can characterize the equilibrium for any fixed (and finite) value of γ. First, we show that the equilibrium outcome is strictly increasing in both parties' proposed platforms, independently of the (non-null) degree of power sharing. That is, by offering a less extreme policy, the higher power share that a party gets does never balance the loss from assigning this weight to a more moderate position, and the implemented policy is always moved in the direction of the change. The implication for the equilibrium of the game is that parties' positions will always be extreme.

Given the degree of power sharing γ, let $x^*_\gamma(0,1)$ be the equilibrium outcome of the voting subgame when party L's

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[1] See Herrera et al. (2014); Saporiti (2014); Matakos et al. (2016).

[2] A continuum of voters is assumed also in Saporiti (2014) and Matakos et al. (2016).

[3] The assumptions on the distribution function and on the voters' utility function are those needed in De Sinopoli and Iannantuoni (2007) to properly characterize strategic voting in a game with a continuum of players. We refer to their work for intuitions and for an exhaustive discussion of those hypothesis.

[4] The general notation for policies is meant to allow both the case $x_L < x_R$ and $x_L > x_R$.

[5] We talk about the *unique* equilibrium of a game with a continuum of players with slight abuse of terminology.

proposed platform is 0 and party R's proposed platform is 1.

Proposition 1. *If $\theta_L < x_\gamma^*(0,1) < \theta_R$, then $(x_L, x_R) = (0,1)$ is the unique Nash equilibrium. If $x_\gamma^*(0,1) \le \theta_L < \theta_R$, then the unique equilibrium is $(x_L, x_R) = (\tilde{x}_L, 1)$ with $\tilde{x}_L$ such that $x_\gamma^*(\tilde{x}_L, 1) = \theta_L$. If $\theta_L < \theta_R \le x_\gamma^*(0,1)$, then the unique equilibrium is $(x_L, x_R) = (0, \tilde{x}_R)$ with $\tilde{x}_R$ such that $x_\gamma^*(0, \tilde{x}_R) = \theta_R$.*

Proof. We first prove that, for every γ and $x_L < x_R$, x^* is strictly increasing in x_L and x_R. Let $P(x) = \frac{F(x)^\gamma}{F(x)^\gamma + (1 - F(x))^\gamma}$. We have $x^* = P(x^*)x_L + (1 - P(x^*))x_R$. Taking the partial derivatives with respect to x_L and x_R of both sides of this expression and rearranging, we have

$$\frac{\partial x^*}{\partial x_L} = \frac{P(x^*)}{1 + P'(x^*)(x_R - x_L)},$$

and

$$\frac{\partial x^*}{\partial x_R} = \frac{1 - P(x^*)}{1 + P'(x^*)(x_R - x_L)}.$$

When $x^* \notin \{0,1\}$ and for every γ, both expressions are strictly positive, since $P(\cdot)$ is strictly increasing and $x_R - x_L > 0$.

Now, let (x_L, x_R) be an equilibrium such that $\theta_L < x_\gamma^*(x_L, x_R) < \theta_R$. Obviously, party L would prefer the outcome to be more to the left, while party R would prefer it to be more to the right. Given the previous result, the only equilibrium of this kind is necessarily $(0, 1)$.

Then, let $x_\gamma^*(0,1) \le \theta_L < \theta_R$ (an analogous and symmetric argument applies to prove the last case). Clearly, $(\tilde{x}_L, 1)$ such that $x_\gamma^*(\tilde{x}_L, 1) = \theta_L$ is a Nash equilibrium, as party L gets her ideal policy implemented and party R cannot induce a more rightward outcome.[6] To see that this equilibrium is unique, note first that if party R's position is 1 then party L has a unique best reply. Thus, suppose that there exists another equilibrium $(\bar{x}_L, \bar{x}_R)$ with $\bar{x}_R \ne 1$. By the above result, this can be the case only if $x_\gamma^*(\bar{x}_L, \bar{x}_R) \ge \theta_R$. But then party L has a unique best reply, viz. $\bar{x}_L = 0$. Hence, we would have $x_\gamma^*(0, \bar{x}_R) \ge \theta_R$ and, by assumption and given that $\partial x^*/\partial x_R > 0$ for every γ, also $x_\gamma^*(0, \bar{x}_R) < x_\gamma^*(0, 1) \le \theta_L < \theta_R$, a contradiction. $\square$

Thus, when there is some power sharing, parties' positions always diverge in equilibrium. According to how voters' bliss points are distributed, either the two parties choose extreme platforms and the equilibrium outcome is between θ_L and θ_R, or one party's choice is extreme and the other party can induce her favorite policy by choosing, however, a more extreme one. The reason why, differently from the case of sincere voting, the "centrifugal" force is always dominant is that voters' strategic behavior counterbalances parties' incentive to converge to obtain a higher power share. Precisely, some strategic voters will vote for their less favorite party in order to bring the policy outcome closer to their bliss point. Hence, when voters are strategic, a party who proposes a more moderate policy gains less votes (and so less power) than when voters are sincere, and these votes are not enough to induce policy convergence.

The above results hold for finite values of γ. Consider finally the limit case of the winner-take-all ($\gamma \to \infty$), in which there is no power sharing. Since the policy outcome is the winner's ideal policy and there are just two parties, voting strategically coincides with voting sincerely. In this case the basic insight of Hotelling model survives, even if parties care only about the enacted policy rather than about winning the election. In fact, each party has the incentive to move towards the other, because now the change in the power share is very abrupt. It follows that in equilibrium parties' positions converge (we refer to Osborne, 1995, Proposition 2).

5. Conclusions

This initial study of party electoral competition with strategic voters has shown that, for any positive degree of power sharing, positions of parties always diverge in equilibrium when their preferences depend solely on the implemented policy. This suggests that recent results in favor of a positive relationship between electoral rule disproportionality and policy convergence rely crucially on the assumption that voting is sincere. From an applied standpoint, it seems then worthwhile exploring whether the same can be stated for more general parties' preferences, that is, depending also on the vote/power shares.

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[6] Note that there exists a unique x_L such that $x_\gamma^*(x_L, 1) = \theta_L$, since $\partial x^*/\partial x_L > 0$ for every γ.