Endogenous Growth of Open Collaborative Innovation Communities [*]

A Supply-Side Perspective

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Abstract

This paper develops a formal model of the interaction between the Open *Collaborative* Innovation Community and the alternative, *competitive*, institutional setting generating innovation, that we call *Technology*. The presence of spillovers both across and within each institution generates *multiple equilibria*, each associated with a different degree of development of the open communities. We study the stability of equilibria and find that *initial conditions* are crucial in determining whether open communities are doomed to fail or, instead, are able to grow endogenously and prosper in the long-run. Importantly, in our model such endogenous mechanism is triggered by supply-side factors, as captured in the structure of the researchers' motivations. In a comparative statics analysis, we finally show that the Community's innovative performance depends, not only on its own characteristics (such as the level of protection of the community-produced knowledge), but also on the institutional characteristics of Technology. We discuss the managerial and policy implications of our findings.

Keywords: open collaborative communities, knowledge externalities, innovation policy, free and open source.

JEL Classification: O31, L86, L88.

1   Introduction

The open model of knowledge production has been recently gaining momentum in market economies. Knowledge-intensive communities (David and Foray, 2003) are typically characterized by a large number of members who produce and reproduce knowledge in a "public" (often virtual) space in which new information and communication technologies (ICT) are intensively used to codify and transmit knowledge. In this paper, we focus on Baldwin and von Hippel's (2011) version of knowledge-intensive communities, referred to as *Open Collaborative Innovation Communities*. In such communities, agents collaboratively develop and openly distribute knowledge without direct external funding or rents assured by the usual Intellectual Property Rights environment (O'Mahony, 2003).

One of the most prominent examples of open collaborative innovation communities, in terms of economic and social impact, is the free and open source software community (FOSS). In this community, a large number of individuals, spread all over the world, cooperate online to create software and release it openly through the Internet (David and Rullani, 2008, Gonzalez-Barahona et al., 2008). Anyone can enter the production process and report bugs, propose patches, cooperate with other developers on existing software, or launch new projects. Moreover, thanks to the license scheme adopted by the community (mostly the General Public License, GPL), no one can appropriate the jointly developed software. Openness in this case is preserved via *copyleft*, i.e., via licenses that prevent the appropriation and force the subsequent developers of the original code to redistribute the improved software under the original open terms.

The aim of this paper is to build a formal model of open collaborative innovation communities to evaluate the conditions under which such communities can be generated and are able to grow endogenously and prosper in the long-run. Our modeling strategy is inspired by Carraro and Siniscalco (2003). In the model, N researchers have to take two sequential decisions. First, they choose between a *closed* mode of knowledge production based on copyright and patenting (what Dasgupta and David 1987, 1994, call *Technology* or T), and an open collaborative innovation community (more simply, a *Community* or C). While in Technology researchers gain from economic rents, in Community openness hinders the possibility to obtain directly a monetary reward. However, benefits of different nature may attract the researcher, ranging from *individual* motives (e.g., signaling one's ability, reputation, fun, own-use of the produced knowledge) to *social* motives (being involved in the social dimension of Community).

Secondly, once inside either T or C, members decide how much research effort to exert. Solving these two decision problems (entry and effort choice), we are able to characterize the equilibrium distribution of the N researchers across T and C.

In the model, we consider both the *inter-group* and *intra-group* knowledge externalities that participation in either T or C generates. In line with previous literature, we suppose that Community generates (both *inter-group* and *intra-group*) positive externalities, while Technology generates (both *inter-group* and *intra-group*) negative externalities (we further discuss these issues in Sections 2 and 3). This complex interaction across the two institutional settings is responsible for equilibrium multiplicity. In particular, there is a *threshold* in terms of the number of Community members below which communities are doomed to fail and above which communities are able to grow *endogenously* and establish themselves as leading actors in the production of innovation. As a result, initial conditions may turn out to be crucial in determining the prosperity of the open model of knowledge production.

While the presence of a threshold size has been already recognised in the literature about communities (e.g., Bonaccorsi and Rossi, 2003), the explanation has always been based on *demand*-side factors. This paper is the first one that concentrates on *supply*-side factors (namely, the structure of the researchers' motivations) as key determinants for the existence of the threshold. In particular, our focus is on two key supply-side features of the model: (i) the *social* dimension in the researchers' motivation to join C and (ii) the level of openness protection associated with the innovation produced in C, which governs the strength of positive knowledge externalities from C to T.

We study how these two aspects affect the threshold. On the one hand, the strength of social motivation is a key determinant of the Community's development. On the other hand, communities whose ability to defend the openness from the appropriation of Technology researchers is stronger, are more likely to grow and successfully endure. This happens because protection mechanisms (such as licenses) preserving openness, limit Technology's capability to exploit the positive Community's spillovers, thus reducing its attractiveness while triggering endogenous Community growth. These peculiar features of the model are particularly useful to inform managers and project leaders on the strategies to create communities around their projects. Initial steps to generate a social environment and an organization able to attract and motivate participants are much more important than any future efforts, as remaining below the threshold at the beginning means being doomed to remain small and eventually disappear, while being above means triggering an endogenous and self-reinforcing growth. We return to this

issue in the conclusions.

Finally, after defining the concept of *innovativeness* in terms of the total flow of innovations generated by each institutional setting, we carry out a comparative statics analysis to study the impact of changes in the strength of social motivation and in openness protection policy on both C's and T's innovativeness. We find that an increase in the attractiveness of Community in terms of a stronger social motivation may actually *reduce* its innovativeness - the reason being that larger positive spillovers from Community increase the level of investments of Technology researchers, which in turn negatively affects incentives to join the Community and thus may, in principle, more than offset the direct positive effect on Community's innovativeness. An analogous reasoning applies when investigating the effect of the Community's ability to protect openness: a stronger openness protection policy has an ambiguous effect on Community's innovativeness -in particular, the effect is positive only *if* total efforts in Technology decrease. Such ambiguities allow us to highlight an interesting and non-obvious point: in both cases, the overall effect on Community's innovativeness crucially depends on the institutional features of the alternative setting, Technology. We return to this point at the end of the paper when discussing the policy implications.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the formal model. Section 4 presents the main findings. Section 5 concludes with a few managerial and policy implications.

2 Related Literature

Our paper directly relates to two main streams of literature developing formal models of the open source Community as an exemplary form of the open collaborative innovation model. A first stream deals with conditions for individuals to contribute, thus focusing on the supply-side of the market. Johnson (2002), Baldwin and Clark (2006), and Bitzer and Schroder (2005), for instance, consider open source software as a public good and develop a game-theoretic model of contribution by self-interested individuals, while Reisinger et al. (2014) focus on contribution by profit-maximizing firms. Gambardella and Hall (2006) and Johnson (2006), instead, consider how the competition of the free and open source community with IPR-based system attracts developers. Landini (2012) develops a game of system design, where a group of agents is responsible for the choice of property rights regime, while a second group is responsible for designing

technology (degree of modularity). He shows that multiple equilibria exist, and that open and close systems may coexist. Our work relates to this literature, in particular to Gambardella and Hall (2006), who also include positive externalities from researchers operating in an open environment and allow the benefit from participation in the community to depend on its size. We, however, move away from Gambardella and Hall (2006) in at least two crucial respects. First, we build on the Community of Practice (CoP) literature (Wenger, 1998) to explicitly analyze the role of social motivations in explaining the relative attractiveness of the Community Model (on the role of social motivations see also Amin and Cohendet (2004) and Bagozzi and Dholakia (2006)). Secondly, we take full account of both *inter-group* and *intra-group* spillovers in both institutional settings.

A second stream of related literature deals with competition between proprietary and open source software on the final market for software (Casadesus-Masanell and Llanes (2011), Di Gaetano (2014), Sacks (2015) among others). In particular, Casadesus-Masanell and Ghemawat (2006) develop a dynamic duopoly model between a profit-oriented firm and an open source community. Economides and Katsamakas (2006) consider the two-sided competition between proprietary and open source platforms, with particular attention paid to the incentives for complementary good production. Lanzi (2009) jointly considers product differentiation, lock-in and network externalities, and consumers' experiences in software use and implementation. Llanes and de Elejalde (2013) consider competition between for profit open source and proprietary firms, in a context where both the market structure and the development model are endogenous. Dalle and Jullien (2003) and Bonaccorsi and Rossi (2003) take a technology diffusion perspective to study the conditions under which open source software can overcome an existing and dominant proprietary software. Finally, Mustonen (2003) bridges demand side and supply side assuming that developers are heterogeneous in their productivity and that more productive developers choose the open source model, whereas our model shows that the two institutions can coexist even when the heterogeneity of individuals is ruled out. Our model draws from this literature and extends it thanks to its linkage with the supply-side stream of literature presented above. To the best of our knowledge, we are the first to establish and investigate this link between the supply side and the demand side in the analysis of open collaborative innovation communities.

3 The Theoretical Framework

Our formal analysis builds on the model by Carraro and Siniscalco (2003, CS henceforth). A population of N researchers is active in a given field of research. Researchers are assumed to be risk-neutral and *identical* in terms of both preferences and productivity. They exert effort to produce knowledge, and they can do this in two institutional settings: Technology (T) or Community (C). We assume that researchers, before choosing how much effort to exert, choose which institution they intend to belong, based on an expected payoff comparison. Participation in one institution is exclusive. Technology and Community differ both in their payoff structures, capturing different motivations, and in the nature of externalities within and between institutions. We now characterize the payoff functions in the two institutional settings, starting with Technology.

3.1 The Payoff Function in Technology

Economic return (R^T) constitutes the main source of motivation to join Technology: new knowledge is kept secret, embodied in patents, or protected by copyright law (Dasgupta and David, 1994). Moreover, the knowledge produced by a researcher that chooses this institution has a negative impact on the probability of any others' success in knowledge creation, both in Technology and in Community, since the limits imposed by property rights or secrecy reduce the space for further innovations. This effect is one of technological competition, namely that for discovering innovation opportunities in a given technological space.[1]

Formally, we define the payoff function from participating in Technology as

$$\Pi_i^T \equiv Pr_i^T \left(x_i^T, X_{-i}^T, \beta X^C \right) R^T - c_i^T(x_i^T) \tag{1}$$

where x_i^T is the individual effort of researcher i in institution T, X_{-i}^T and X^C represent, respectively, the sum of efforts of all researchers in T (excluding i) and in C, and $Pr_i^T(\cdot)$ is the probability of innovation (successful production of knowledge) in T. The product $Pr_i^T \left(x_i^T, X_{-i}^T, \beta X^C \right) R^T$ is the expected revenue associated with the entrepreneurial activity (or the expected wage for employed software developers).

The probability of innovation is increasing in effort at decreasing rate, i.e. $\partial Pr_i^T / \partial x_i^T > 0$ and $\partial^2 Pr_i^T / \partial \left(x_i^T \right)^2 < 0$ (that is to say, researchers' marginal productivities are pos-

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[1]The model does not include *product market competition*. Product market competition will also affect innovation efforts, but in different ways. See Section 4 for more on this point.

itive and decreasing in effort). The individual cost of effort is, instead, increasing and convex in effort, i.e. $\partial c_i^T / \partial x_i^T > 0$ and $\partial^2 c_i^T / \partial \left(x_i^T \right)^2 > 0$. These assumptions are standard and guarantee that each researcher always exerts positive (and finite) effort in equilibrium.

In line with the previous literature, spillovers from Community are assumed positive $(\partial Pr_i^T / \partial X_i^C > 0)$. On the other hand, spillovers from Technology are assumed negative $(\partial Pr_i^T / \partial X_{-i}^T < 0)$. $\beta \in [0, 1]$ is a policy parameter capturing the strength of spillovers from Community to Technology. Its size (inversely) depends on how strong openness is protected from exclusive appropriation (O'Mahony, 2003). In open source, for instance, licenses may limit the rights of others to use the code as inputs for their productions of "closed software", thus reducing spillovers from Community to Technology. In creative industries, Creative Commons licenses may have the same effect.

Finally, we assume that individual efforts in Technology are strategic complements with total efforts in Community $(\partial Pr_i^T / \partial x_i^T X_i^C > 0)$ and strategic substitutes with total efforts in Technology $(\partial Pr_i^T / \partial x_i^T X_{-i}^T < 0)$. Intuitively, the larger is the total effort in Community, the larger are the opportunities for a researcher in Technology to recombine and build upon the knowledge produced in Community; while, the larger is the total effort in Technology, the smaller are these opportunities due to stronger technological competition, and so the lower is the marginal return from individual effort.

3.2   The Payoff Function in Community

Benefits from joining free and open source communities are of both individual and social type (Nuvolari and Rullani, 2007). Let us analyze them in order. Among the main *individual* benefits from participating in C we may recall (i) reputation and peer regard (Bezroukov,1999a, 1999b; Dalle and David, 2005) (ii) status and signaling one's talent (Roberts et al., 2006; Lerner and Tirole, 2002, Bitzer et Al. 2016), (iii) own-use (von Hippel, 1988; von Hippel, 2001; Franke and Shah, 2003; von Hippel and von Krogh, 2003; Bessen, 2006) and (iv) fun (Raymond, 1998a; 1998b; Torvalds and Diamond, 2001; Lakhani and Wolf, 2005). In addition, the desire to learn from others is also a fundamental incentive to join the collective production of code (von Hippel and von Krogh, 2003; Ghosh et al., 2002).

Surveys and empirical studies that measure the relative importance of individual motivations – such as the FOSS-EU survey (Ghosh et al., 2002), the Boston Consulting

Group survey (Lakhani et al., 2002), Bonaccorsi and Rossi (2006) and David and Shapiro (2008) – show that own-use-related incentives and psychological motivations such as fun are the most important drivers, together with learning, while reputation, signaling and possible monetary gains are marginal (Lakhani and Wolf, 2005). Since individual motives are not at the core of our analysis, in the model we group them all in a single parameter (k^C).[2]

The *social* dimension, which instead is central in our contribution, has been analyzed along several dimensions, such as gift economy (Raymond, 1998a), epistemic communities (Amin and Cohendet, 2004; Lin, 2003, 2004a, 2004b; Mateos-Garcia and Steinmueller, 2008) and Communities of Practice (Wenger, 1998).

The CoP perspective can be particularly useful to describe in detail the social processes at work in the free and open source community. Following Wenger (1998), the payoff structure must account for two factors directly related to socially-induced motivations: the communitarian activity and the degree of personal involvement in it. The *communitarian activity* (denoted by Y) is the production activity undertaken by the Community. In the case of the free and open source software, for instance, the communitarian activity is software development. The subjective effect of that activity on the individual's payoff function is then mediated by his or her *degree of personal involvement* (denoted by function $e\left(\cdot\right)$). For example, in the free and open source case, the development of GNU/Linux (the most famous open source operating system) has a greater effect on the payoff of a developer who "believes" in the GNU/Linux project compared to the payoff based on the simple usefulness of GNU/Linux as software program.

Personal involvement is endogenous to the development of the Community. Shah (2006) describes the evolution in developers' motivations as follows: "... a need for software-related improvements drives initial participation. The majority of participants leave the community once their needs are met, however, a small subset remains involved. For this set of developers, motives evolve over time and participation becomes a hobby" (p. 1000). Among the possible explanations for this process, the author also identifies the hypothesis that the "interaction with the community leads to a shift in the individual's identity and self-perception" (Shah, 2006, p. 1011). This is the perspective taken by Bagozzi and Dholakia (2006), who write: "Initial partici-

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[2]The reader interested in this dimension of analysis may further refer to Krishnamurthy and Tripathi (2009), Sauermann and Cohen (2010), Krishnamurthy et Al. (2014), Belenzon and Schankerman (2015).

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pation by novice users is driven by specific task-oriented goals .... But over time, as the user comes to form deeper relationships with other [free and open source community] members, the community metamorphosizes into a friendship group and a social entity with which one identifies" (p. 1111). Therefore, when a community grows, it becomes stronger not only "quantitatively" (e.g., it produces more software), but also "qualitatively", determining a higher average rate of the personal involvement of its core members: this reasoning justifies our hypothesis that function $e(\cdot)$ grows with the size of the Community.

A third aspect related to the social dimension of open collaborative innovation communities refers to *scale costs* (denoted by function $C(\cdot)$), that is, to costs related to the scale of the Community. First of all, $C(\cdot)$ includes coordination costs, i.e., capturing the increasing difficulties of organizing the work of an ever larger group of collaborators (Comino et Al., 2007). Moroever, as any group of people who collaborate, the Community is expected to be subject to free riding episodes. The group must then create some rules and enforcing mechanisms to sustain cooperation and avoid free riding (O'Mahony, 2003). Monitoring others' behavior, spreading information about it, discovering rule violation, and punishing free riding are costly activities, which probably increase with the scale of the Community. Finally, abundant experimental evidence, partly inspired by Social Identity Theory (Tajfel and Turner, 1985) suggests that larger groups are more prone to loosen identitarian ties among their members (see, among others, Brewer and Kramer (1986), Hornsey and Jetten (2004), Hogg et Al. (2017) and references therein). In fact, other things equal, an ever larger Community might translate into weaker committment and participation by its members. A function $C(\cdot)$ increasing in Community's size captures exactly this diminishing degree of identification for larger communities.

Taking all these aspects into account, we can define the payoff function from participating in Community as[3]

$$\Pi_i^C \equiv Pr_i^C \left(x_i^C, X_{-i}^C, X^T \right) k^C - c_i^C(x_i^C) + \alpha e(n) Y \left(x_i^C, X_{-i}^C, X^T \right) - C(n) \qquad (2)$$

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[3]In reality, the motivational differences between Technology and Community can be much more blurred than those implied by the comparison between (1) and (2). A researcher that works for a firm but embedded in the scientific debate with his or her colleagues from other firms can reach the same social motivation as an open source developer. Likewise, the latter can find a job in an open source-based company and receive a monetary incentive similar to that of the former. However, we seek to grasp the inner difference between the two institutions, and thus we magnify the differences in the payoff functions they offer to the researchers.

where n $(N - n)$ denotes the (endogenous) number of researchers in Technology (Community), while x_i^C is the individual effort of researcher i in institution C. X_{-i}^C and X^T represent, respectively, the sum of efforts of all researchers in C (excluding i) and in T. $Pr_i^C(\cdot)$ is the probability of innovation in C. k^C is the "prize" obtained from successful innovating, and captures all the different motivational dimensions at the individual level. The same hypotheses as those under Technology hold about the effect of effort on the probability of innovation and on individual costs: (i) $\partial Pr_i^C / \partial x_i^C > 0$, $\partial^2 Pr_i^C / \partial \left(x_i^C \right)^2 < 0$ and (ii) $\partial c_i^C / \partial x_i^C > 0$, $\partial^2 c_i^C / \partial \left(x_i^C \right)^2 > 0$.

As for spillovers within and across institutions, following our previous discussion we assume that Technology's spillovers are always negative, i.e. $\partial Pr_i^C / \partial X^T < 0$, while Community's spillovers are always positive, i.e. $\partial Pr_i^C / \partial X_{-i}^C > 0$. Moreover, individual efforts in Community are strategic complements with total efforts in Community and strategic substitutes with total efforts in Technology, i.e. $\partial Pr_i^C / \partial x_i X^T < 0$ and $\partial Pr_i^C / \partial x_i X_{-i}^C > 0$: in words, the larger is the total effort in Community (Technology), the larger (smaller) are the opportunities for an individual researcher in Community to recombine and build upon them.

The other terms in (2) capture the social dimension as defined above. $Y(\cdot)$ is the communitarian activity, $e(\cdot)$ measures the degree of personal involvement in Community, $C(\cdot)$ denotes the scale costs, and the parameter $\alpha > 0$ captures the relative strength of social motivations.[4] The product of Y with e is meant to capture the two main processes described by CoP theory, *Engagement* and *Legitimate Peripheral Participation* (Lave and Wenger, 1991; Wenger, 1998). Community members, while collaborating to undertake the communitarian activity, get involved in a process of reciprocal influence which alters their personal involvement.

We naturally assume that the communitarian activity (i) responds positively to the total effort in Community and negatively to the total effort in Technology, i.e. $\partial Y / \partial X_{-i}^C > 0$, $\partial Y / \partial X^T < 0$; (ii) is weakly increasing (and concave) in the individual effort (so that we even allow it to have a negligible impact value), that is, $\partial Y / \partial x_i^C \geq 0$, $\partial^2 Y / \partial \left(x_i^C \right)^2 \leq 0$. Moreover, in producing communitarian value, individ-

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[4]There are three main differences between our formalization of the payoff function and that in CS. First, in CS the role of the State in Science leads to the presence of a fixed salary. Second, in CS the social dimension of the institution (degree of personal involvement, communitarian activity and coordination costs) is not considered. Third, differently from them, we model explicitly the policy dimension in the magnitude of spillovers from Community to Technology (β), which allows us to perform comparative statics exercises.

ual efforts are supposed to be weak strategic complements with total efforts in Community $(\partial Y/\partial x_i X_{-i}^C \geq 0)$ and weak strategic substitutes with total efforts in Technology $(\partial Y/\partial x_i X^T \leq 0)$. As usual, scale costs are assumed increasing and convex in the size of Community $(N - n)$, that is, $\partial C(n)/\partial n < 0$, $\partial^2 C(n)/\partial n^2 > 0$ (with $C(N) = 0$). Finally, and in line with the previous discussion, we suppose $\partial e(n)/\partial n < 0$, that is to say: the degree of personal involvement is monotonically increasing in the size of Community.

3.3 Definition of Equilibrium and Stability

Researchers' interaction is represented as a two-stage non-cooperative game: in the first stage, each researcher decides whether to enter into Technology or Community, while in the second stage, after observing n, each agent decides simultaneously his or her effort level. The game is solved backwards, computing the optimal effort of each researcher given N and n. Then, the analysis moves to the first stage, where researchers choose the institution that predicts correctly the outcome, and reap the payoffs associated with their choices.

We restrict our attention to pure strategy Nash equilibria in which n^* researchers choose Technology and the remaining $N - n^*$ choose Community. Furthermore, we consider only symmetric equilibria in terms of efforts within each institution. Consequently, we define $\Pi^T(n)$ and $\Pi^C(n)$, the reduced-form payoffs in the first stage for a researcher choosing Technology or Community, as a function of the number of researchers in Technology. Following CS and established coalition formation theory (e.g. D'Aspremont et al., 1983; Yi, 1997), we define the Nash equilibrium as the size of Technology $n^* \in (0, N)$ that satisfies the following two conditions:[5]

$$\Pi_i^T(n^*) \geq \Pi_i^C(n^* - 1) \tag{3}$$

$$\Pi_i^T(n^* + 1) \geq \Pi_i^C(n^*) \tag{4}$$

Condition (3) implies that, at equilibrium, researchers in Technology do not have the incentive to move to Community, and (4), symmetrically, implies that researchers in Community do not have the incentive to move to Technology. If N is large enough,

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[5]In addition, it must be that $\Pi^T(n^*) > 0$ and $\Pi^C(n^*) > 0$: each researcher prefers to join one of the two institutions rather than getting an outside option normalized to 0. We shall assume that this is always true in our model.

so that n can be in fact treated as a continuous variable, the determination of an interior equilibrium n^* can be approximated by the condition

$$\Pi^T(n^*) = \Pi^C(n^*) \tag{5}$$

which we will use on in the next section.

We interpret (Nash) equilibria as the stationary states of a dynamic adjustment process in which individuals may change over time the institution they belong to, taking as given the choice of others, that is, assuming given institutions' size. Keeping in mind such adjustment process, we are able to discuss the steady-state community size and its stability properties.

We use the standard notion of equilibrium stability. An equilibrium is (locally) stable if there is a neighborhood of n^* such that any n in such a neighborhood converges to n^*. In other words, an allocation of researchers between Technology and Community is stable if (sufficiently small) exogenous shocks in institution size do not move the equilibrium away (permanently) from the initial configuration. Formally, an equilibrium n^* is stable if and only if:[6]

$$\frac{d\Pi^T(n^*)}{dn} - \frac{d\Pi^C(n^*)}{dn} < 0. \tag{6}$$

4   Findings

We solve the game through backwards induction. We first determine the equilibrium efforts in the second stage of the game for a given allocation of researchers in Technology and Community (Subsection 4.1). We then proceed backwards to analyze the first stage decision (Subsection 4.2). We finally characterize equilibria and their stability properties (Subsection 4.3).

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[6]In order to see why (6) must hold, suppose that the adjustment process (in continuous time) is represented by the differential equation $dn/dt = F(n) \equiv g\left(\Pi^T(n) - \Pi^C(n)\right)$, with g being a positive constant affecting the speed of adjustment. A state n^* is stationary if $F(n^*) = 0$, while local stability requires $F'(n) < 0$, which corresponds to

$$g\left(\frac{d\Pi^T(n)}{dn} - \frac{d\Pi^C(n)}{dn}\right) < 0.$$

We also observe that, given our notion of stability, all equilibria are stable in CS set-up.

4.1 Decision in the Second Stage

In the second stage of the game, each researcher, either in Technology or Community, chooses the effort that maximizes his or her payoff given n and the effort choices of the other researchers. The first order conditions for payoff maximization in Technology and Community are, respectively, given by:

$$\frac{\partial \Pi_i^T}{\partial x_i^T} = \frac{\partial Pr^T\left(x_i^T, X_{-i}^T, \beta X^C\right)}{\partial x_i^T} R^T - \frac{\partial c_i^T(x_i^T)}{\partial x_i^T} = 0 \tag{7}$$

and

$$\frac{\partial \Pi_i^C}{\partial x_i^C} = \frac{\partial Pr^C\left(x_i^C, X_{-i}^C, \beta X^T\right)}{\partial x_i^C} k^C - \frac{\partial c_i^C(x_i^C)}{\partial x_i^C} + \alpha e(n)\frac{\partial Y\left(x_i^C, X_{-i}^C, X^T\right)}{\partial x_i^C} = 0 \tag{8}$$

Since we are interested in symmetric Nash equilibria, the equilibrium efforts in Technology and Community (as a function of n), denoted by $\hat{x}^T(\cdot)$ and $\hat{x}^C(\cdot)$, are implicitly defined by:

$$\frac{\partial Pr^T\left(\hat{x}^T(n^*), (n^*-1)\hat{x}^T(n^*), \beta(N-n^*)\hat{x}^C(n^*)\right)}{\partial x_i^T} R^T - \frac{\partial c^T(\hat{x}^T(n^*))}{\partial x_i^T} = 0 \tag{9}$$

$$\frac{\partial Pr^C\left(\hat{x}^C(n^*), (N-n^*-1)\hat{x}^C(n^*), n^*\hat{x}^T(n^*)\right)}{\partial x_i^C} k^C - \frac{\partial c_i^C(\hat{x}^C(n^*))}{\partial x_i^C} +$$
$$\alpha e(n)\frac{\partial Y\left(\hat{x}^C(n^*), \hat{x}^C(n^*), \hat{x}^T(n^*)\right)}{\partial x_i^C} = 0. \tag{10}$$

As proven in Appendix A, an increase in the size of Technology reduces individual effort in both Technology and in Community. i.e., $\partial \hat{x}^T(n^*)/\partial n < 0$ and $\partial \hat{x}^C(n^*)/\partial n < 0$. The intuition is straightforward. An increase in the Technology's size (that is, a higher n) is associated with higher negative spillovers from Technology (a stronger *competition* effect) and lower positive spillovers from Community (a weaker *cooperation* effect), which in turn negatively affects both Technology's and Community's payoffs and and thus lowers the equilibrium research efforts in both institutions, $\hat{x}^T(n^*)$, $\hat{x}^C(n^*)$.

When we look at total efforts in each institution, it is clear that total efforts in Community, defined as $\hat{X}^C(n) \equiv (N-n)\hat{x}^C(n)$ are decreasing in n, i.e., $d\hat{X}^C(n)/dn < 0$ As for total efforts in Technology, defined as $\hat{X}^T(n) \equiv n\hat{x}^T(n)$, in Appendix A we prove that the positive effect on the *extensive* margin (that is, the increase in n) dominates the negative effect on the *intensive* margin (the decrease in $\hat{x}^T(n)$), so that total effort in Technology is also always increasing in group size, i.e., $d\hat{X}^T(n)/dn > 0$.

4.2   Decision in the First Stage

We are now ready to analyze the decision in the first stage. Substituting for the effort functions, $\hat{x}^T(n^*)$ and $\hat{x}^C(n^*)$ into the payoff functions, we obtain the reduced-form payoffs (depending on n only) used for comparison in the first stage:

$$\Pi_i^T(n) = Pr^T\left(\hat{x}^T, \hat{X}_{-i}^T(n), \beta\hat{X}^C(n)\right) R^T - c^T(\hat{x}^T) \tag{11}$$

$$\begin{aligned}\Pi_i^C(n) = Pr^C\left(\hat{x}^C, \hat{X}_{-i}^C(n), \hat{X}^T(n)\right) k^C - c^C(\hat{x}^C)+ \\ \alpha e(n)Y\left(\hat{x}^C, \hat{X}_{-i}^C(n), \hat{X}^T(n)\right) - C(n)\end{aligned} \tag{12}$$

In Appendix B we show that (i) (11) is decreasing in n; (ii) (12) has an inverted-U shape in n. Both results have a clear intuition. The payoff function in Technology is decreasing in the size of this group because more researchers in Technology (and thus fewer researchers in Community) imply more competition within Technology and lower positive spillovers from Community, and thus a lower probability for individual innovation. As for the payoff function in Community, an increase in its size (lower n) has a positive effect on researchers' payoff for three reasons: 1) larger positive intra-group spillovers; 2) smaller negative inter-group spillovers from Technology; 3) higher value of the communitarian activity. On the other hand, when communities grow larger and larger, they incur ever higher scale costs (as these are increasing and convex in Community's size). This negative effect of group size will then ultimately prevail for large enough communities.

4.3   Equilibrium Analysis

Given the reduced-form payoffs (11) and (12) and the notions of equilibrium and stability (5) and (6), we are now ready to characterize the equilibria, together with their stability properties. Along the lines of CS, the equilibrium analysis is performed through a graphical representation (Figure 1).

<div align="center">INSERT FIGURE 1 ABOUT HERE</div>

In Figure 1, we represent the most interesting and natural situation in which $\Pi_i^T(0) > \Pi_i^C(0)$ and $\Pi_i^T(N) > \Pi_i^C(N)$.[7]   In words, when either everybody works

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[7]We discuss the two alternative situations in which either of the two inequalities does not hold in a footnote at the end of this subsection.

in Community and no one in Technology ($n = 0$), or *viceversa* ($n = N$), the payoff associated with joining Technology is higher than in Community. The first inequality -$\Pi_i^T(0) > \Pi_i^C(0)$- is totally intuitive and due to the presence of scale costs in Community and of decreasing marginal returns in joining Technology. The second inequality - $\Pi_i^T(N) > \Pi_i^C(N)$ - captures the idea that, when Community does not exist (or is very small), the incentives to found (and/or join) one are modest, because its crucial *social* elements - that is, the communitarian activity and the degree of personal involvement - tend to increase and *self-reinforce* with the size of the Community itself.

Recalling that an interior equilibrium n^* satisfies $\Pi^T(n^*) = \Pi^C(n^*)$, Figure 1 shows the existence of two interior equilibria, n_1^* and n_2^*, with $n_1^* < n_2^*$. Since $d\Pi^T(n_1^*)/dn - d\Pi^C(n_1^*)/dn < 0$ and $d\Pi^T(n_2^*)/dn - d\Pi^C(n_2^*)/dn > 0$, it turns out that n_1^* is stable, while n_2^* is unstable. The instability of n_2^* implies that any shock that "perturbates" the system by increasing n moves it towards $n^* = N$, which is then also a stable equilibrium (*corner* solution). As a result, there are two possible stable equilibria, one in which Technology and Community co-exist and Community is relatively large, and one in which all researchers are in Technology and Community does not exist. The "small Community" equilibrium, instead, is unstable. From an empirical point of view, equilibrium n_1^* is consistent with the evidence in the software industry, where similar competing products are offered under proprietary and open regimes. Notably, this result has been obtained with *ex-ante symmetric* researchers, and it is the outcome of the endogenous mechanisms within and across the two alternative institutions.

In the dynamic interpretation we previously suggested, the unstable equilibrium (n_2^*) constitutes a *threshold* that divides the realm of small communities, which are doomed to disappear over time, from the set of communities that are able to grow fast and large. In each one of those spaces, the dynamics of the model shows a sort of bandwagon effect. If a Community, for whatever reason, is able to grow enough and overcomes the threshold, then it grows endogenously up to size $N - n_1^*$, which in a sense expresses the full potential of a Community. The system is then characterized by path-dependence (David, 1985).

The importance of the initial conditions has been already recognised in the literature on the free and open source community (e.g., Bonaccorsi and Rossi, 2003; Bitzer and Schröder, 2005).[8] The novelty in our "Critical Mass" argument for free and open source

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[8] Notice that this approach takes into account the quantitative aspect of free and open source community growth, but not its qualitative side. When communities grow, their social space becomes more complex and their forms of participation and governance structures are placed under pressure.

development is that it is not based on demand factors (as, for instance, in Bonaccorsi and Rossi, 2003) but, instead, on the structure of the developers' motivation and thus on, among others, the social forces described in the Community of Practice literature (Wenger, 1998).

Finally note that the relevance of the initial Community's size to predict its subsequent development finds some indirect empirical support in the literature on free and open source community. On the one hand, larger projects/communities act as more powerful attractors for new members (David and Shapiro, 2008); on the other hand, smaller communities have a higher chance of becoming inactive (Zirpoli et Al. 2013). This empirical evidence seems to support the narrative proposed in this paper, where the growth of relatively larger communities tends to be strong and self-reinforcing while small communities are more likely to vanish.[9]

4.4 Discussion

Our findings emphasize the importance of the *initial conditions* for n in determining the ultimate equilibrium size of the Community. Community can be initially attractive through a series of different processes. First, communities may become economically relevant filling an unfilled market, either creating one *ex novo* or providing the conditions to fill an established one (Bonaccorsi and Rossi, 2003). The definition of "market", of course, must be interpreted in a wide sense, so that not only the product is important, but also the model of production—in the free and open source case, allowing users to be part of the process. The simple existence of a Community attracts all the individuals interested in that market. Thus, the more the Community responds

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The case of Debian is a clear example of the radical transformation needed to make a growing project able to bear the challenges determined by its own growth (Mateos-Garcia and Steinmueller, 2008; O'Mahony and Ferraro, 2007; Sadowski et al., 2008).

   [9]Let us close this section hinting at two special cases, alternative to that depicted in Figure 1. The first (probably unrealistic) case occurs when $\Pi_i^T(0) < \Pi_i^C(0)$, which implies the existence of a single interior, unstable, equilibrium n^* and of two corner solutions ($n^* = 0$ and $n^* = N$) as stable equilibria: here the large Community equilibrium involves all researchers. This scenario occurs when scale costs are low enough to allow Community to grow unbounded. The other special case is the one in which $\Pi_i^T(N) < \Pi_i^C(N)$. In this situation, there exists a unique interior, stable, equilibrium n^*. Co-existence of Technology and Community emerges as the only possible configuration. This scenario coincides with that identified in CS, where the *social* side of Science (the institution "competing" with Technology in their model) is not considered. A more detailed analysis of these scenarios is available upon request.

to such unfilled gaps, the more attractive it becomes to interested individuals.

In addition, social mechanisms of *identity* can all attract new potential members and trigger the self-reinforcing growth described in the model as a movement from a Community below the threshold to one above the threshold that is able to grow endogenously. Communities, as other institutions, cover a particular space of social interaction. They provide members with a specific interaction environment, ruled by implicit laws and grounded in peculiar identities, i.e., structures of meanings, principles, and values. One of the debates around which the free and open source community is structured concerns the concepts of free and open source software (Dahlander, 2007). This debate shapes the environment in which developers act, defining rules (from rules against free-riding to recognition by peers, O'Mahony, 2003), meanings (what "openness" means), values (whether software should be always free), and visions of the world (whether all the produced knowledge should be free). Such interaction contributes to build the "identity" of the Community. Non-members interested in this debate and sensitive to such an identity are then attracted by the Community, and may eventually become members.[10]

Another mechanism can also be activated by *trust building*, which in small communities can lead to a common language, established rules, and improved efficiency at the organizational level. Community of Practice theory (Lave and Wenger, 1991; Wenger, 1998) suggests that, even in their initial phases, communities become structured in a series of concentric circles. Inner circles connect "senior members", possibly the founders of the Community, who perform a great share of the work and guide the Community. Other outer circles group together individuals who are less involved in the Community, namely those who participate to a lower degree the further is the circle

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[10]Our model excluded heterogeneity among agents, which was instead adopted by other authors in the same field (Mustonen, 2003). Researchers' heterogeneity would indeed influence the dynamic interpretation of the model in that, for any given characteristics of the "average" researcher, higher heterogeneity would favour the constitution of a Community. Initially, the Community is set up by people with a relatively higher interest in the activity (that is, a higher k^C) and in the "vision" it embodies (captured by a higher value of α), an interest high enough to make them bear the costs connected with the small size of the Community. The Community then can be created and developed, even if it links only a few individuals. The simple existence of a Community would then make it more rewarding for other individuals to join, thus triggering Community growth and making the threshold more likely to be overcome. In this description, it is easy to recognise the actual evolution of the free and open source Community (Bonaccorsi and Rossi, 2003; Bitzer and Schröder, 2005). Notice that this argument follows the logic of the so-called threshold models (Granovetter, 1978).

from the centre (Crowston and Howison, 2006). The process from outer circles to inner circles is a learning process (von Krogh et al., 2003): new members joining the community engage in a series of activities with senior members, getting to know about the community in more depth and letting the community acquire knowledge about them. This process of reciprocal learning, termed *Legitimate Peripheral Participation* by Lave and Wenger (1991), triggers the process of the negotiation of meanings discussed earlier, affecting the identities of the new members, of the other members involved in the activity, and of the community as a whole. The gradual acceptance of new members into the community increases the level of trust, where belonging to inner circles also means being recognised as more trustworthy. This implies that, when member i starts to engage in the communitarian activity, increasingly interacting with the inner circle and acquiring more trust, he or she begins to perceive the community as a trustworthy environment. Thus, the fact that j also belongs to the inner circle is taken by i as a signal of j's trustworthiness. Therefore, j's potential free-riding behavior is perceived by i as an almost irrelevant exception, and i reduces his or her monitoring and punishing activities, decreasing scale costs. This maps the results obtained by Bagozzi and Dholakia (2006), who, as already noted, find that "the community metamorphosizes into a friendship group and a social entity with which one identifies" (p. 1111). Legitimate Peripheral Participation creates trust, and this in turn increases the payoff from participating in Community and makes it more attractive for potential members, thereby fuelling Community growth.

Our equilibrium analysis also provides insights into the role of inter-group *spillovers* in the comparative development of the two alternative institutions. In the case of open source, for instance, the role of licenses has been at the centre of a lively debate since the beginning (e.g., Lerner and Tirole, 2005; Comino et al., 2007). In the model, tools that protect openness and prevent appropriation such as strict licenses (e.g., the GPL) are captured by lower values of β, since they reduce the positive externalities from Community to Technology without affecting intra-group externalities. This has the consequence of decreasing the payoff from Technology $(d\Pi_i^T(n)/d\beta = R^T \cdot (\partial Pr^T/\partial X^C) \cdot X^C > 0)$ and leading towards an equilibrium in which Community is larger and/or enlarging the basin of attraction of such equilibrium. Therefore, instruments that protect openness (e.g., the GPL) are fundamental for enhancing the sustainability of Community, creating the conditions for its endogenous growth (Gambardella and Hall, 2006). Furthermore, when set at the initial stage of Community, such instruments may signal attention to openness protection and can

help to attract individuals that care about it and about the ideological component of the Community.[11]

4.5 Comparative Statics Analysis on Innovativeness

Thus far, we have limited our attention to the size of groups that choose each institution at equilibrium. However, from a social point of view, it is the performance of institutions that matters. We now assess the performance in terms of the expected number of innovations in the institution, i.e., its *innovativeness*. We define innovativeness for Technology as

$$I^T (n^*) \equiv n^* \cdot Pr^T \left(\hat{x}^T (n^*), (n^* - 1) \hat{x}^T (n^*), \beta (N - n^*) \hat{x}^C (n^*) \right) \qquad (13)$$

and innovativeness for Community as

$$I^C (n^*) \equiv (N - n^*) \cdot Pr^C \left(\hat{x}^C (n^*), (N - n^* - 1) \hat{x}^C (n^*), n^* \hat{x}^T (n^*) \right). \qquad (14)$$

In particular, we are interested in the effects on innovativeness in Technology and Community of changes in the two main parameters of our model, i.e., α -measuring the strength of social motivations- and β -measuring the strength of knowledge externalities from Community to technology, under the control, at least in part, of the policy maker regulating the degree of openness protection of the knowledge produced in open communities.

In Appendix C we show that (i) an increase in α has an ambiguous effect on Technology's innovativeness, while it increases Community's innovativeness only *if* total efforts in Technology decrease (otherwise the effect is ambiguous);[12] (ii) a *decrease* in

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[11] Although a fully-fledged analysis of this point is outside the scope of this paper, our model also hints at the effects of patenting software code on the viability of open source communities. As a first effect, an increase in the strength of IPRs in Technology implies an increase in the Technology's payoff via a higher economic return R^T. On the other hand, stronger IPRs limit the scope of the innovative activity (constraining the "field" in which research could be exploited without violating them), thus reducing the probability of innovation in both Technology and Community. While in Technology this effect may be offset by the increase in R^T, the effect on the Community's payoff is unambiguously negative, in that it raises the threshold above which Communities start growing endogenously, and it generates equilibria characterized by smaller stable Communities. This result is consistent with the concerns about extending the rights of software producers to patent their code in Europe (Torvalds and Cox, 2003).

[12] Intuitively, an increase in α makes Community more attractive: a larger Community increases

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β (that is, a stronger policy of openness protection) has an ambiguous effect on Technology's innovativeness, while it raises Community's innovativeness only *if* total efforts in Technology decrease (otherwise the effect is ambiguous).[13]

The reason why these results are interesting is exactly their ambiguity. In fact, the ultimate effect on the Community's degree of innovativeness of the two parameters α and β (which concern the functioning of Community) crucially depends on Technology, whose institutional characteristics then contribute to determine the overall level of innovativeness of the system. Moreover, the direction of this effect hinges upon the strength of the impact that an increase in the number of researchers in Technology has on the individual research effort $(\partial \hat{x}^T(n)/\partial n)$. While in our model it is $\partial \hat{x}^T(n)/\partial n < 0$, recent empirical literature on the relationship between (product market) competition and innovation suggests a more complex (non-linear) relation between the two (Aghion and Griffith, 2005). Extending our model in this direction would probably enrich the set of results and would resolve the ambiguity of the analysis carried out in this subsection: a $\partial \hat{x}^T(n)/\partial n > 0$ would imply that both an increase in α (a higher importance of communitarian social processes) and a decrease in β (a higher capability to protect openness) positively affect the Community's innovativeness.

5   Concluding Remarks

In this paper, we have developed a model where Open Collaborative Innovation Communities are compared with Technology (Dasgupta and David, 1994) in their ability to attract researchers. In particular, attention is paid to the social nature of the community institution, as captured by the degree of personal involvement, the product of

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positive intra-group externalities, with a positive effect on innovativeness. On the Technology side, its size reduction has an ambiguous effect on innovativeness because on one side, there are fewer researchers; on the other side, the effort exerted in Technology by each individual increases. Therefore, the effect on total effort is ambiguous. From the point of view of Community, if the total effort in Technology decreases, this reinforces the direct positive effect of the increase in α. Otherwise, the overall impact of this increase on Community's innovativeness is ambiguous.

[13]Intuitively, a decrease in β makes Technology less attractive, and thus contributes to reduce its size. The impact on Technology's innovativeness is, however, ambiguous because the effort exerted in Technology by each individual may increase. On the other hand, the size of the Community increases, each individual exerts higher effort, and intra-group positive externalities are stronger. If total effort in Technology decreases, then the expected number of innovations in Community unambiguously increases; otherwise, the overall effect on Community is ambiguous.

the communitarian activity, and the scale costs (Wenger, 1998).

We confirm the presence of a threshold size for community, below which it can only remain small and eventually disappear, and above which it is pushed by internal forces to grow large. However, in contrast to all previous literature that focused on the final market for knowledge products (demand side), we highlight the economic forces working on the supply side (the input market where institutions compete for knowledge workers). This perspective enriches the debate around the effectiveness of the two alternative institutional environments (Community Vs. Technology) to promote the innovative performance of the system.

5.1   Managerial Implications

This conclusion is important for firms. It suggests that, when firms decide to initiate a community around their innovation processes, they cannot adopt a step-by-step procedure. Such a "real option-like" approach, where firms gradually increase their investments by deciding in each subsequent step whether and how to foster community growth and solidity, does not square with our findings that emphasize the importance of the initial conditions and of overcoming the initial threshold size. In contrast, we describe the community's development as an endogenous and self-reinforcing process. The lesson is that firms need to invest a lot in planning and realizing the initial phase of community development, and in gathering an initial core group of members that is large enough to place the community beyond the threshold size. These initial crucial steps will then trigger an endogenous growth process, attracting researchers from outside and enlarging the payoffs for those already in the community. Firms can then compensate for the larger expenses this strategy calls for in the initial phase with lower control and support costs in the take-off stage. This mechanism has wide implications for managers and project leaders because it speaks against the diffused wisdom that community growth (i) can be treated as a gradual process and (ii) should be closely attended by the firm. We claim here instead that an important and careful investment at the beginning would be enough to generate endogenous growth later on. For instance, Spaeth et al. (2010) show that, in the case of the Eclipse development process, not only did IBM release the source code, but its employed contributors played a fundamental role in fuelling the growth of the community in its starting phase. This sort of "preemptive generosity", as the authors call it, is the strategy our model indicates as the most effective one.

5.2 Policy Implications

Our model has also allowed us to evaluate the role of the two main drivers of community development - the strength of social motivation (α) and the openness protection policy (β) - in affecting its overall level of innovativeness. We have shown that a deeper social motivation (higher α) and/or a stronger openness protection policy (lower β) both foster innovativeness in Community only if the research effort in Technology is not too sensitive to the number of individuals in the institution, i.e., to the level of competition. The policy implication is worth emphasizing: to spur the Community's level of innovativeness, the policy-maker also needs to take into account the institutional design of the alternative institution, that is, of Technology. For instance, if the regulatory background that describes IPR and related markets for technology (Arora et al., 2001) are designed in ways that protect the effort of researchers from high-level competition, then Community's innovativeness is more likely to respond positively to openness protection and to stronger social motivations. While this paper does not aim at investigating how Technology should be designed to maximize innovativeness in the overall system, it however indicates to policymakers that (i) the interrelations between Community and Technology are tight and subtle and hence that, (ii) to obtain positive results on one side, action is also needed on the other side. In other words, designing markets crucially affects non-market social bodies, and *viceversa*. Neglecting such links risks causing unintended policy effects.

5.3 Further Research

While the model is suggestive of several forms of interaction between the two institutional modes of Community and Technology, its stylised form gives several opportunities for potentially useful extensions. First, product market competition (including the issue of pricing and product differentiation) could be modeled explicitly, both in Technology and in Community. Second, the role of firms in Community could be considered, removing explicitly the assumption that participation in one institution is exclusive. Finally, the value of innovation (rather than the probability of innovation) could be made endogenous. We leave these model extensions for future research.

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A Group Size and Individual/Total Effort in T and C

Let us start with individual effort. Applying the implicit function theorem on (9) and (10), we obtain, respectively

$$\frac{\partial \hat{x}^T}{\partial n} = -\frac{\left(\frac{\partial Pr^T}{\partial x^T \partial X^T_{-i}} \hat{x}^T - \frac{\partial Pr^T}{\partial x^T X^C} \hat{x}^C\right) R^T}{\frac{\partial^2 Pr^T}{\partial (x^T)^2} R^T - \frac{\partial^2 c^T}{\partial (x^T)^2}},$$

$$\frac{\partial \hat{x}^C}{\partial n} = -\frac{\left(\frac{\partial Pr^C}{\partial x^C \partial X^T_{-i}} \hat{x}^T - \frac{\partial Pr^C}{\partial x^C X^C} \hat{x}^C\right) k^c + \frac{\partial e}{\partial n} \frac{\partial Y}{\partial x^C} + e(n) \left(\frac{\partial Y}{\partial x^C \partial X^T_{-i}} \hat{x}^T - \frac{\partial Y}{\partial x^C X^C} \hat{x}^C\right)}{\frac{\partial^2 Pr^C}{\partial (x^C)^2} k^C - \frac{\partial^2 c^C}{\partial (x^C)^2} + e(n) \frac{\partial^2 Y}{\partial (x^C)^2}}.$$

Concerning the former, we notice that the denominator is negative since $\partial^2 Pr^T / \partial \left(x^T\right)^2$ is negative while $\partial^2 c^T / \partial \left(x^T\right)^2$ is positive by assumption. As for the numerator, $\partial Pr^T / \partial x^T \partial X^T_{-i}$ is negative and $\partial Pr^T / \partial x^T X^C$ is positve by assumption. Therefore $\partial \hat{x}^T / \partial n$ is negative.

Concerning the latter, the denominator is negative since $\partial^2 Pr^C / \partial \left(x^C\right)^2$ and $\partial^2 Y / \partial \left(x^C\right)^2$ are negative by assumption, while $\partial^2 c^C / \partial \left(x^C\right)^2$ is positive. As for the numerator, $\partial e / \partial n$ is negative, while $\partial Y / \partial x^C$ is positive; $\partial Pr^C / \partial x^C \partial X^T_{-i}$ and $\partial Y / \partial x^C \partial X^T_{-i}$ are negative by assumption, while $\partial Pr^C / \partial x^C X^C$ and $\partial Y / \partial x^C X^C$ are positive. Therefore $\partial \hat{x}^C / \partial n$ is negative.

We now turn to total efforts. Total efforts in Technology are defined as $\hat{X}^T(n) \equiv n\hat{x}^T(n)$. Hence, it is $\partial \hat{X}^T(n) / \partial n \equiv \hat{x}^T(n) + n \partial \hat{x}^T(n) / \partial n$. This expression is positive whenever $\hat{x}^T(n)/n + \partial \hat{x}^T(n)/\partial n > 0$, that is, whenever the average effort is higher than the marginal effort, which is always the case given that marginal effort is decreasing in n.

Proving that $\hat{X}^C(n)/n < 0$ is straightforward given that, in $\hat{X}^C(n) \equiv (N-n)\hat{x}^C(n)$, the two terms $(N - n)$ and $\hat{x}^C(n)$ are both decreasing in n.

B Group Size and Payoff Function in T and C

Differentiating (11) w.r.t. n, we obtain

$$\frac{d\Pi^T_i(n)}{dn} = \left(\frac{\partial Pr^T}{\partial X^T_{-i}} \frac{dX^T_{-i}}{dn} + \frac{\partial Pr^T}{\partial X^C} \frac{dX^C}{dn}\right) R^T,$$

which is lower than zero given that $\partial Pr^T / \partial X^T_{-i} < 0$, $dX^T_{-i}/dn > 0$, $\partial Pr^T / \partial X^C > 0$ and $dX^C/dn < 0$.

On the other hand, differentiating (12) w.r.t. n, we obtain

$$
\begin{aligned}
\frac{d\Pi^C_i(n)}{dn} & = \left(\frac{\partial Pr^C}{\partial X^C_{-i}} \frac{dX^C_{-i}}{dn} + \frac{\partial Pr^C}{\partial X^T} \frac{dX^T}{dn} \right) k^C \\
& \quad + \alpha \left[\frac{de}{dn} Y + e \cdot \left(\frac{\partial Y}{\partial X^C_{-i}} \frac{dX^C_{-i}}{dn} + \frac{\partial Y}{\partial X^T} \frac{dX^T}{dn} \right) \right] - \frac{dC}{dn}
\end{aligned}
$$

Our hypotheses ensure that the first two addend are both negative: in fact, an increase in the Technology's size implies both a lower *economic* value (the first addend) and a lower *social* value of the research endevour in Community. However, dC/dn is also negative by hypothesis: in fact, an increase in the Technology's size also implies lower scale costs in Community. The overall sign is then ambiguous. We now prove that, if there exists a unique solution to the equation $d\Pi^C_i(n)/dn = 0$, that solution is a global maximizer, denoted by $n^{\max}$. When $n \in (n^{\max}, N)$, Community's (convex) scale costs are relatively low, and hence, net benefits from its size *increase* are positive, that is, $d\Pi^C_i(n)/dn < 0$. When $n \in (0, n^{\max})$, Community's scale costs are relatively high and predominant, and hence net benefits from from its size *decrease* are positive, that is, $d\Pi^C_i(n)/dn > 0$. As a result, $\Pi^C_i(n)$ is increasing up to $n^{\max}$ and then decreasing. That is to say, function $\Pi^C_i(n)$ has an inverted-U shape.

C Effects of α and β on Innovativeness in T and C

To determine the effect of the variation of a parameter on the expected number of innovations we proceed as follows. We calculate i) the derivative of n^* w.r.t. the parameter; ii) the derivative of the individual and total efforts in each institution w.r.t. the parameter; iii) the derivative of the expected number of innovations w.r.t. the parameter. Furthermore, we observe that, since we focus on stable equilibria, condition (6) must hold.

Consider case (i) first. Regarding the effect of α on the equilibrium size of the Technology group, applying the implicit function theorem to the equilibrium condition yields

$$
\frac{dn^*}{d\alpha} = - \frac{e(n)Y}{\frac{d\Pi^T(n^*)}{dn} - \frac{d\Pi^C(n^*)}{dn}} < 0.
$$

The effect of α on $\hat{x}^C$ is obtained as the sum of the direct impact of the parameter variation through the first order condition, and the indirect impact due to the variation

in the number of individuals in the institution. Then, by applying the implicit function theorem to equation (8), we obtain:

$$\frac{d\hat{x}^C}{d\alpha} = -\frac{e(n)\frac{\partial V}{\partial \hat{x}^C}}{\frac{\partial^2 Pr^C}{\partial(\hat{x}^C)^2} - \frac{\partial^2 c^C}{\partial(\hat{x}^C)^2}} + \frac{\partial \hat{x}^C}{\partial n}\frac{dn^*}{d\alpha} > 0.$$

In the first term (the direct effect), our hypotheses ensure that the denominator is negative and the numerator is positive. The second term (the indirect effect) is negative (see Appendix A). The effect of total investment in Community is given by

$$\frac{d\hat{X}^C}{d\alpha} = (N - n^*)\frac{d\hat{x}^C}{d\alpha} - \frac{dn^*}{d\alpha}\hat{x}^C > 0.$$

As for the effect on the Technology side, we note that only an indirect effect exists. By applying the implicit function theorem to (7), we obtain

$$\frac{d\hat{x}^T}{d\alpha} = -\frac{\frac{\partial \hat{x}^T}{\partial n}\frac{dn^*}{d\alpha}}{\frac{\partial^2 Pr^T}{\partial(\hat{x}^T)^2} - \frac{\partial^2 c^T}{\partial(\hat{x}^T)^2}}$$

which is positive since the denominator is positive and $\partial \hat{x}^T/\partial n$ is negative (Appendix A). In terms of total effort, we obtain

$$\frac{d\hat{X}^T}{d\alpha} = n^*\frac{d\hat{x}^T}{d\alpha} + \frac{dn^*}{d\alpha}\hat{x}^T,$$

whose sign is ambiguous since $d\hat{x}^T/d\alpha$ is positive, while $dn^*/d\alpha$ is negative.

Considering the results so far, we finally get

$$\begin{aligned}\frac{d(N - n^*)Pr^C}{d\alpha} &= -\frac{dn^*}{d\alpha}Pr^C + (N - n^*)\frac{dPr^C}{d\alpha} \\ &= -\frac{dn^*}{d\alpha}Pr^C + (N - n^*)\left(\frac{\partial Pr^C}{\partial \hat{x}^C}\frac{d\hat{x}^C}{d\alpha} + \frac{\partial Pr^C}{\partial \hat{X}^C_{-i}}\frac{d\hat{X}^C_{-i}}{d\alpha} + \frac{\partial Pr^C}{\partial \hat{X}^T}\frac{d\hat{X}^T}{d\alpha}\right)\end{aligned}$$

While the first term is positive, the term within brackets has an ambiguous sign since $d\hat{X}^T/d\alpha$ has an ambiguous sign (the other terms are positive). Then, the effect of α on the innovativeness in the Community group is overall ambiguous, unless $d\hat{X}^T/d\alpha$ is negative, which would imply $d(N - n^*)Pr^C/d\alpha > 0$ since $\partial Pr^C/\partial \hat{X}^T < 0$. As for the effect on Technology, we obtain

$$\begin{aligned}\frac{dn^*Pr^T}{d\alpha} &= \frac{dn^*}{d\alpha}Pr^T + n^*\frac{dPr^T}{d\alpha} \\ &= \frac{dn^*}{d\alpha}Pr^T + n^*\left(\frac{\partial Pr^T}{\partial \hat{x}^T}\frac{d\hat{x}^T}{d\alpha} + \frac{\partial Pr^T}{\partial \hat{X}^T_{-i}}\frac{d\hat{X}^T_{-i}}{d\alpha} + \frac{\partial Pr^T}{\partial \hat{X}^C}\frac{d\hat{X}^C}{d\alpha}\right)\end{aligned}$$

which has an ambiguous sign since $dn^*/d\alpha < 0$ and, within brackets, the first and the third term are positive, while the second term is ambiguous.

Now consider case (ii). Applying the implicit function theorem to (5), we obtain the effect of β on the equilibrium size of the Technology group as

$$\frac{dn^*}{d\beta} = \frac{R^T X^C \frac{\partial Pr^T}{\partial \beta}}{\frac{d\Pi^T(n^*)}{dn} - \frac{d\Pi^C(n^*)}{dn}} > 0$$

since the denominator is negative in a stable equilibrium, while the numerator is positive given that $\partial Pr^T/\partial \beta > 0$ by hypothesis.

As for the effect of β on $\hat{x}^T$, this is obtained as the sum of the direct impact of the parameter variation through the first order condition, and the indirect impact due to the variation in the number of individuals in the institution. Therefore, by applying the implicit function theorem to equation (7), we obtain

$$\frac{d\hat{x}^T}{d\beta} = \frac{\partial \hat{x}^T}{\partial \beta} + \frac{\partial \hat{x}^T}{\partial n} \frac{dn^*}{d\beta} = -\frac{R^T X^C \frac{\partial^2 Pr^T}{\partial \hat{x}^T \partial \beta}}{\frac{\partial^2 Pr^T}{\partial (\hat{x}^T)^2} - \frac{\partial^2 c^T}{\partial (\hat{x}^T)^2}} + \frac{\partial \hat{x}^T}{\partial n} \frac{dn^*}{d\beta}.$$

In the first term (direct effect), the denominator is positive since, by assumption, $\partial^2 Pr^T/\partial \left(\hat{x}^T\right)^2$ is negative and $\partial^2 c^T/\partial \left(\hat{x}^T\right)^2$ is positive. $R^T X^C \left(\partial^2 Pr^T/\partial \hat{x}^T \partial \beta\right)$ is positive, $\left(\partial \hat{x}^T/\partial n\right) \cdot (dn^*/d\beta)$ (the indirect effect via n^*) is negative, since $dn^*/d\beta > 0$ and $\partial \hat{x}^T/\partial n < 0$ (Appendix A). Therefore, the overall sign is ambiguous, with $d\hat{x}^T/d\beta$ being positive if the direct effect prevails.

Computing $d\hat{X}^T/d\beta$ we obtain

$$\frac{d\hat{X}^T}{d\beta} = n^* \frac{d\hat{x}^T}{d\beta} + \frac{dn^*}{d\beta} \hat{x}^T,$$

which is positive if $d\hat{x}^T/d\beta > 0$, while it has an ambiguous sign in the opposite case.

On the Community side, we note that only an indirect effect exists. Applying the implicit function theorem on (8), and computing afterwards the derivative of total effort in Community, yields

$$\frac{d\hat{x}^C}{d\beta} = -\frac{\frac{\partial \hat{x}^C}{\partial n} \frac{dn^*}{d\beta}}{\frac{\partial^2 Pr^C}{\partial (\hat{x}^C)^2} - \frac{\partial^2 c^C}{\partial (\hat{x}^C)^2}} < 0,$$

$$\frac{d\hat{X}^C}{d\beta} = (N - n^*) \frac{d\hat{x}^C}{d\beta} - \frac{dn^*}{d\beta} \hat{x}^C < 0.$$

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since $\partial \hat{x}^C / \partial n < 0$, $\partial^2 Pr^C / \partial \left(\hat{x}^C \right)^2 < 0$ and $\partial^2 c^C / \partial \left(\hat{x}^C \right)^2 > 0$ by assumption.

Considering the results so far, we finally get the impact of β on the expected number of innovations in Technology as

$$
\begin{aligned}
\frac{dn^* Pr^T}{d\beta} &= \frac{dn^*}{d\beta} Pr^T + n^* \frac{dPr^T}{d\beta} \\
&= \frac{dn^*}{d\beta} Pr^T + n^* \left(\frac{\partial Pr^T}{\partial \hat{x}^T} \frac{d\hat{x}^T}{d\beta} + \frac{\partial Pr^T}{\partial \hat{X}^T_{-i}} \frac{d\hat{X}^T_{-i}}{d\beta} + \beta \frac{\partial Pr^T}{\partial \hat{X}^C} \frac{d\hat{X}^C}{d\beta} + X^C \frac{\partial Pr^T}{\partial \beta} \right)
\end{aligned}
$$

which has an ambiguous sign. The first term is positive. Within brackets, the first two terms, capturing the effect of a change in β on Technology, have ambiguous signs, which turn out to be positive if the direct effect prevails. The third term is positive and the fourth negative, capturing the idea that an increase in β increases the spillovers towards Technology for given total effort in Community, but also reduces such effort via a reduction in $N - n^*$.

As for the impact of β on the expected number of innovations in Community, we obtain

$$
\begin{aligned}
\frac{d \left(N - n^* \right) Pr^C}{d\beta} &= -\frac{dn^*}{d\beta} Pr^C + \left(N - n^* \right) \frac{dPr^C}{d\beta} \\
&= -\frac{dn^*}{d\beta} Pr^C + \left(N - n^* \right) \left(\frac{\partial Pr^C}{\partial \hat{x}^C} \frac{d\hat{x}^C}{d\beta} + \frac{\partial Pr^C}{\partial \hat{X}^C_{-i}} \frac{d\hat{X}^C_{-i}}{d\beta} + \frac{\partial Pr^C}{\partial \hat{X}^T} \frac{d\hat{X}^T}{d\beta} \right)
\end{aligned}
$$

whose sign is ambiguous. However, if $dX^T/d\beta$ is positive, the expression above is unambiguously negative since all the addends are negative.

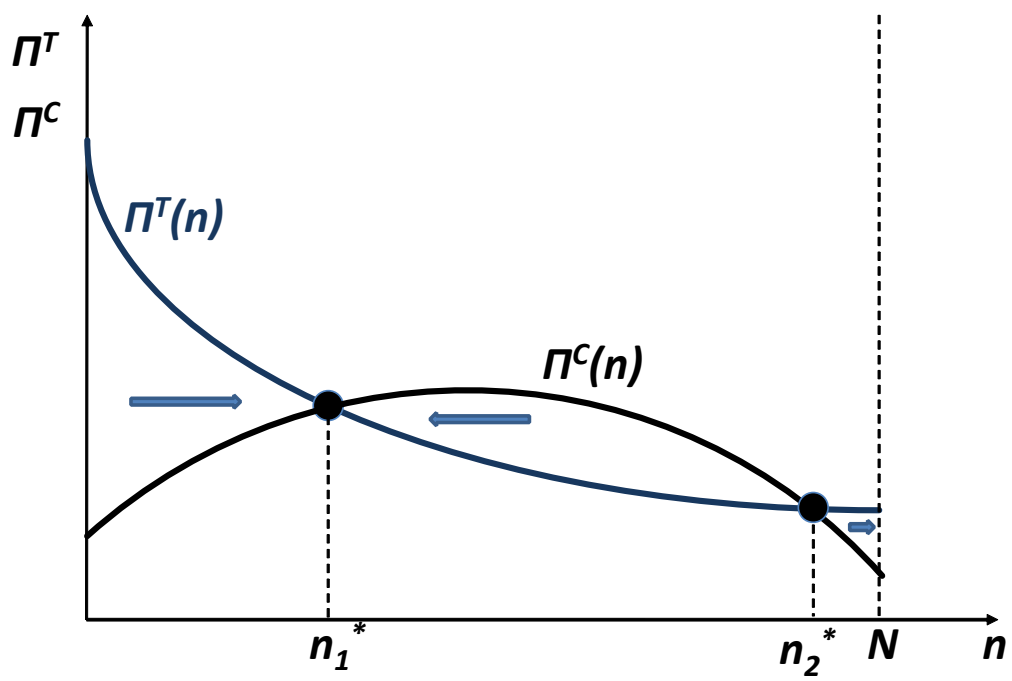Figure 1: Equilibria