Physics Letters B 798 (2019) 134951

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Low-virtuality photon transitions $\gamma^* \to f\bar{f}$ and the photon-to-jet conversion function

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ARTICLE INFO

Article history:
Received 10 July 2019
Received in revised form 5 September 2019
Accepted 16 September 2019
Available online 20 September 2019
Editor: B. Grinstein

ABSTRACT

The calculation of electroweak corrections to processes with jets in the final state involves contributions of low-virtuality photons leading to jets in the final state via the singular splitting $\gamma^* \to q\bar{q}$. These singularities can be absorbed into a photon-to-jet "fragmentation function", better called "conversion function", since the physical final state is any hadronic activity rather than an identified hadron. Using unitarity and a dispersion relation, we relate this $\gamma^* \to q\bar{q}$ conversion contribution to an integral over the imaginary part of the hadronic vacuum polarization and thus to the experimentally known quantity $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$. Therefore no unknown non-perturbative contribution remains that has to be taken from experiment. We also describe practical procedures following subtraction and phase-space-slicing approaches for isolating and cancelling the $\gamma^* \to q\bar{q}$ singularities against the photon-to-jet conversion function. The production of Z+jet at the LHC is considered as an example, where the photon-to-jet conversion is part of a correction of the order α^2/α_s relative to the leading-order cross section.

1. Introduction

The experimental precision for scattering processes at the LHC and future colliders requires the inclusion of electroweak (EW) corrections in theoretical predictions. The mixing of EW and QCD corrections gives rise to additional complications. Since in general the leading-order (LO) matrix elements receive contributions of different orders in the strong and electromagnetic coupling constants, a complete tower of NLO corrections appears, as, e.g., discussed for several LHC processes in Refs. [1–5]. Moreover, the EW corrections to hadron collider processes involve contributions from the photon content of the proton, which should be calculated with photon parton distribution functions (PDFs) based on the LUXqed recipe of Refs. [6,7]. The photon PDF absorbs infrared singularities associated with virtual photons coupling to initial-state particles. The corresponding singularities related to final-state particles can be treated by using *fragmentation functions* [8]. These are required, in particular, in processes involving photons and/or jets in the final state, as, e.g., discussed in Refs. [9,10] for $\mathrm{W}+\mathrm{jet}/\gamma$ production at the LHC and for jet production in $\mathrm{e}^+\mathrm{e}^-$ annihilation in Ref. [11].[1]

Beside their direct production, jets can be initiated by EW mechanisms, in particular via splittings of EW gauge bosons $V \to f\bar{f}'$. For the massive gauge bosons $V = \mathrm{W}, \mathrm{Z}$ those additional jets mostly result from resonant W/Z bosons, i.e. from process classes that are not directly related to the "mother process" $ab \to C + \mathrm{jet}$ (where C is any multi-particle final state) and can be treated separately in a fully perturbative manner. On the other hand, most mechanisms for gluonic jet production, $ab \to C + \mathrm{g}$, have a direct counterpart in photon production, $ab \to C + \gamma$, which in turn leads to jet production via possible splittings $\gamma^* \to q\bar{q}$ one order higher in perturbation theory. If the resulting quark- or antiquark-initiated jets are very close, i.e. nearly collinear, they are merged to one jet by the jet algorithm, so that the resulting event topology contributes to $ab \to C + \mathrm{jet}$. This contribution is infrared singular in the collinear limit and develops non-perturbative parts,

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[1] Alternatively, final-state photons and jets may be isolated by geometrical cuts that are designed to attribute infrared-singular contributions to the jets, such as so-called *Frixione isolation* [12].

https://doi.org/10.1016/j.physletb.2019.134951
0370-2693/

since the integration over the virtuality of the intermediate photon reaches down to the mass scale of the light hadrons (pions etc.) which is of the order of Λ_{QCD}. By virtue of the KLN theorem [13] this singularity resulting from real EW corrections to $ab \to C + \mathrm{jet}$ could be cancelled by adding the virtual EW corrections to $ab \to C + \gamma$ production, similar to the infrared-safe combination of real and virtual QCD corrections in the overlap region of one- and two-jet production. In experimental analyses, however, the photon production process is often separated from the corresponding jet production process. Hence, the collinear singularity from the low-virtuality limit in the $\gamma^* \to q\bar{q}$ splitting and its accompanying non-perturbative contribution do not cancel in cross-section predictions. Proceeding as in the similar case of identified hadron production, we absorb the singularity and the non-perturbative contribution into a "fragmentation function" $D_{\gamma\to\mathrm{jet}}$, which is rather called *conversion function* in the following, because a jet is not an identified hadron.

In the context of EW corrections to LHC processes the fragmentation functions of quarks and gluons into photons have been used [9–11]. These have been introduced in Ref. [8] and measured by the ALEPH experiment in photon-plus-jet production at the Z pole [14]. Later, the issue of describing the separation of photons and jets in high-energy collisions via fragmentation functions and their connection to EW corrections was briefly outlined in Ref. [2] in the context of the calculation of EW NLO corrections to hadronic dijet production. Here, photon jets are defined as usual using the photon fragmentation functions $D_{i\to\gamma}$. Then, using the hadron-parton-duality unitarity condition, hadronic jets are defined as jets that are not photon jets in accordance with the procedure used in Ref. [9].

The photon-to-jet conversion function $D_{\gamma\to\mathrm{jet}}$ has not received much attention in the literature so far, since its effect, being of EW origin, is quite small. Counting the mother process $ab \to C + \mathrm{g}$ as $\mathcal{O}(1)$, the contribution involving $D_{\gamma\to\mathrm{jet}}$ is suppressed by the coupling factor $\alpha^2/\alpha_\mathrm{s}$. Nevertheless, this contribution might compete in size with next-to-next-to-leading-order (NNLO) QCD or next-to-leading-order (NLO) EW corrections, which involve the relative coupling factors α_s^2 and α, respectively. The simplest hadronic processes that get contributions from $D_{\gamma\to\mathrm{jet}}$ are photon-plus-jet and Z-plus-jet production. More complicated processes that require such contributions are dijet production, dijet production in association with a vector boson, and vector-boson scattering (VBS). For the last process the contribution of $D_{\gamma\to\mathrm{jet}}$ is actually an $\mathcal{O}(\alpha_\mathrm{s})$ correction to the EW VBS process, while it is still of $\mathcal{O}(\alpha^2/\alpha_\mathrm{s})$ relative to the LO contribution to vector-boson-pair + 2 jet production via strong interactions. In Ref. [5], the NLO QCD and EW corrections to WZ scattering at the LHC, i.e. to the EW channel in $\mathrm{pp} \to 3\ell\nu + 2\,\mathrm{jets} + X$, were calculated, treating the collinear $\gamma^* \to q\bar{q}$ contribution with the method described in this paper.

A lepton collider offers better possibilities to measure the photon-to-jet conversion function. In photon-plus-jet production away from the Z resonance peak both the quark-to-photon fragmentation function and the photon-to-jet conversion function contribute at LO. At LEP this process has only been investigated on the Z pole, where the contribution of $D_{\gamma\to\mathrm{jet}}$ is strongly suppressed. Another possibility is offered by Z-boson-plus-jet production at lepton colliders which receives its leading SM contribution exclusively from the photon-to-jet conversion function and might be suited for a measurement thereof. This study could be ideally carried out at some future $\mathrm{e}^+\mathrm{e}^-$ collider with high luminosity above the Z resonance.

This paper is organized as follows: In Section 2 we calculate the contribution of low-virtuality photon transitions to fermions in perturbation theory. In Section 3 we use a dispersion relation to express the non-perturbative contribution to the photon-to-jet transition by the hadronic vacuum polarization. This result is used in Section 4 to derive an approximate result for the photon-to-jet conversion function. In Section 5 we provide an illustrative numerical application of the photon-to-jet conversion function for Z+jet production at the LHC. Our conclusions are presented in Section 6.

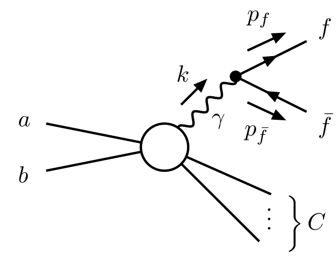

Fig. 1. Generic diagram for the $\gamma^* \to f\bar{f}$ splitting contribution to the cross section for the process $ab \to C + \mathrm{jet}$.

2. Low-virtuality photon transitions $\gamma^* \to f\bar{f}$—perturbative calculation

In perturbative calculations of scattering matrix elements, contributions appear where a virtual photon splits into a fermion–antifermion pair. If the virtuality of the photon becomes small this gives rise to large or singular contributions that require a dedicated treatment. Fig. 1 illustrates the leading-order (LO) $\gamma^* \to f\bar{f}$ splitting contribution to the cross section for the process $ab \to C + \mathrm{jet}$. The definition of the (anti)fermion and photon four-momenta p_f, $p_{\bar f}$, and $k = (p_f + p_{\bar f})$ can also be found there. In the phase-space region of low photon virtuality k^2, the contribution to the squared matrix element $|\mathcal{M}_{ab\to Cf\bar f}(p_f, p_{\bar f})|^2$ asymptotically factorizes into the squared matrix element $|\mathcal{M}_{ab\to C\gamma}(\tilde{k})|^2$ for a real photon and a radiator function describing the asymptotic behaviour for $k^2 \to 0$ (see, e.g., Ref. [15]). Fully differentially, spin correlations between the photon and the $f\bar f$ state build up. But after averaging the splitting process over the azimuthal angle ϕ_f around the collinear axis $\vec{k}$, the factorization takes the simple form

$$\langle|\mathcal{M}_{ab\to Cf\bar f}(p_f, p_{\bar f})|^2\rangle_{\phi_f} \underset{k^2\to 0}{\sim} N_{\mathrm{c},f}\, Q_f^2 e^2\, h_{f\bar f}(p_f, p_{\bar f})\, |\mathcal{M}_{ab\to C\gamma}(\tilde{k})|^2, \tag{2.1}$$

where

$$h_{f\bar f}(p_f, p_{\bar f}) = \frac{2}{(p_f + p_{\bar f})^2} \times \left[1 - \frac{2}{1-\epsilon}\left(z(1-z) - \frac{m_f^2}{(p_f + p_{\bar f})^2}\right)\right] \tag{2.2}$$

and $N_{\mathrm{c},f}$ is the colour multiplicity of fermion f, i.e. $N_{\mathrm{c,lepton}} = 1$ and $N_{\mathrm{c,quark}} = 3$. In this asymptotic limit, the virtuality k^2 is of the same order as the square of the light-fermion mass m_f, which is assumed to be much smaller than any relevant scale of the process. For heavy fermions, the splitting is not enhanced by a singularity since $(p_f + p_{\bar f})^2 > 4m_f^2$. In (2.2), both the deviation $\epsilon = (4-D)/2$ of the number D from the four space–time dimensions and the non-vanishing fermion mass m_f are kept. Results in dimensional regularization (DR) for massless fermions or in mass regularization (MR) in four dimensions can be obtained upon setting $m_f = 0$ or $\epsilon = 0$, respectively. The energy ratio

$$z = \frac{p_f^0}{k^0} \tag{2.3}$$

controls how the photon momentum k is shared between f and $\bar{f}$ in the collinear limit, and the modified photon momentum $\tilde{k}$ is the on-shell limit ($\tilde{k}^2 = 0$) reached by $k = p_f + p_{\bar{f}}$ for $k^2 \to 0$ in DR or $k^2 \to 4m_f^2$ in MR, where m_f serves just as a regularization parameter.

In Ref. [15], both dipole subtraction functions and the cross-section contributions in phase-space slicing (defined by a small cut $\Delta\theta$ on the opening angle between f and $\bar{f}$) were derived, using the phase-space factorization described in Sects. 5.1.1 and 5.2.1 of Ref. [16]. Using the same techniques, it is straightforward to derive the (perturbative) cross-section contribution of the low-virtuality phase-space region defined by the cut

$$4m_f^2 < k^2 < \Delta s \tag{2.4}$$

on the $f\bar{f}$ invariant mass k^2, which is bounded from below by the mass threshold for $f\bar{f}$ production. The cut parameter Δs is smaller than any relevant energy scale $Q^2 \gg \Delta s$ of the mother process, but $\Delta s \gg 4m_f^2$ in the case of mass regularization, where m_f plays merely the role of a regulator. The result for the phase-space integral of the squared matrix element is

$$\begin{aligned}&\int\limits_{k^2<\Delta s} \mathrm{d}\Phi_{Cf\bar{f}}\, |\mathcal{M}_{ab\to Cf\bar{f}}(\Phi_{Cf\bar{f}})|^2 \\ &= N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{2\pi}\int \mathrm{d}\tilde{\Phi}_{C\gamma}\,|\mathcal{M}_{ab\to C\gamma}(\tilde{k})|^2 \\ &\quad\times\int\limits_0^1 \mathrm{d}z\,\Theta_{\mathrm{cut}}\Big(p_f = z\tilde{k},\, p_{\bar{f}} = (1-z)\tilde{k}\Big)\,\mathcal{H}_{f\bar{f}}(\Delta s, z),\end{aligned} \tag{2.5}$$

which is valid up to terms that are suppressed by the factor $\Delta s/Q^2 \ll 1$. For DR and MR the functions $\mathcal{H}_{f\bar{f}}$ are given by

$$\begin{aligned}\mathcal{H}^{\mathrm{DR}}_{f\bar{f}}(\Delta s, z) = {}& -P_{f\gamma}(z)\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{\Delta s z(1-z)}\right)\right] \\ &+ 2z(1-z),\end{aligned} \tag{2.6}$$

$$\mathcal{H}^{\mathrm{MR}}_{f\bar{f}}(\Delta s, z) = -P_{f\gamma}(z)\ln\left(\frac{m_f^2}{\Delta s z(1-z)}\right) + 2z(1-z), \tag{2.7}$$

with the $\gamma \to f\bar{f}$ splitting function

$$P_{f\gamma}(z) = (1-z)^2 + z^2 \tag{2.8}$$

and μ denoting the reference mass scale of DR. The step function Θ_{cut} is equal to 1 if an event passes all cuts on the momenta p_f and $p_{\bar{f}}$, and 0 otherwise. If the complete z range is integrated over, we obtain

$$\begin{aligned}&\int\limits_{k^2<\Delta s} \mathrm{d}\Phi_{Cf\bar{f}}\, |\mathcal{M}_{ab\to Cf\bar{f}}(\Phi_{Cf\bar{f}})|^2 \\ &= N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{2\pi}\int \mathrm{d}\tilde{\Phi}_{C\gamma}\,|\mathcal{M}_{ab\to C\gamma}(\tilde{k})|^2 H_{f\bar{f}}(\Delta s),\end{aligned} \tag{2.9}$$

with

$$H^{\mathrm{DR}}_{f\bar{f}}(\Delta s) = -\frac{2}{3}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{\Delta s}\right)\right] - \frac{10}{9}, \tag{2.10}$$

$$H^{\mathrm{MR}}_{f\bar{f}}(\Delta s) = -\frac{2}{3}\ln\left(\frac{m_f^2}{\Delta s}\right) - \frac{10}{9}. \tag{2.11}$$

As a technical remark, we note that this collinear singularity (which does not overlap with a soft singularity) obeys the simple correspondence $(4\pi\mu^2)^\epsilon/[\epsilon\Gamma(1-\epsilon)] \leftrightarrow \ln(m_f^2)$ between the singular terms in DR and MR.

The result of this section can be used to include the low-virtuality region in a full phase-space integration perturbatively as in any phase-space slicing approach. Then, the analytical dependence of the low-virtuality contribution (2.9) on the small cut parameter Δs is cancelled by the implicit dependence of the remaining phase-space integral on Δs, which emerges in the numerical integration, which can be performed for $\epsilon = 0$ and $m_f = 0$.

3. Low-virtuality photon transitions $\gamma^* \to f\bar{f}$—calculation via dispersion relation

The result of the previous section cannot be used directly to evaluate the low-virtuality contribution to the $ab \to Cf\bar{f}$ cross section if f corresponds to quarks. For low virtualities the hadronic contributions cannot be calculated within perturbation theory as signalled by the logarithmic quark-mass dependence in MR. The low-virtuality contribution to the integral $\int \mathrm{d}\Phi_{Cf\bar{f}}\,|\mathcal{M}_{ab\to Cf\bar{f}}|^2$ can, however, be evaluated via a dispersion relation and eventually related to the running electromagnetic coupling $\alpha(Q^2)$, which is known from low-energy data on $\mathrm{e}^+\mathrm{e}^- \to f\bar{f}$, including in particular the case where the $f\bar{f}$ states refer to hadrons.

The starting point of this procedure is to rewrite the asymptotic formula for the squared matrix element in the form

$$\begin{aligned}&\langle|\mathcal{M}_{ab\to Cf\bar{f}}(p_f, p_{\bar{f}})|^2\rangle_{\phi_f} \\ &\underset{k^2\to 0}{\widetilde{\quad}}\ |\mathcal{M}_{ab\to C\gamma}(\tilde{k})|^2 \times \frac{\langle|\mathcal{M}_{\gamma^*\to f\bar{f}}(k^2)|^2\rangle}{(k^2)^2},\end{aligned} \tag{3.1}$$

where the azimuthal average on the l.h.s. can be traded for a photon spin sum and average in $|\mathcal{M}_{ab\to C\gamma}|^2$ and $\langle|\mathcal{M}_{\gamma^*\to f\bar{f}}|^2\rangle$ on the r.h.s., respectively. Note that the spin-averaged squared matrix element $\langle|\mathcal{M}_{\gamma^*\to f\bar{f}}|^2\rangle$ depends only on the virtuality k^2 and on the splitting variable z, but not on the full momenta p_f and $p_{\bar{f}}$ anymore. Taking into account a phase-space factorization over the virtuality k^2, we get

$$\begin{aligned}&\int\limits_{k^2<\Delta s} \mathrm{d}\Phi_{Cf\bar{f}}\, |\mathcal{M}_{ab\to Cf\bar{f}}(p_f, p_{\bar{f}})|^2 \\ &\underset{k^2\to 0}{\widetilde{\quad}}\ \int \mathrm{d}\tilde{\Phi}_{C\gamma}\,|\mathcal{M}_{ab\to C\gamma}(\tilde{k})|^2 \times F_f(\Delta s)\end{aligned} \tag{3.2}$$

with

$$F_f(\Delta s) = \int\limits_{k^2<\Delta s}\frac{\mathrm{d}k^2}{2\pi(k^2)^2}\int \mathrm{d}\Phi_{f\bar{f}}\,\langle|\mathcal{M}_{\gamma^*\to f\bar{f}}(k^2)|^2\rangle. \tag{3.3}$$

The phase-space integral over the squared $\gamma^* \to f\bar{f}$ off-shell matrix element is related to the imaginary part of the transverse part of the photon self-energy, $\Sigma^{\gamma\gamma}_{\mathrm{T},f}(k^2)$, via well-known cut equations,

$$\int \mathrm{d}\Phi_{f\bar{f}}\,\langle|\mathcal{M}_{\gamma^*\to f\bar{f}}(k^2)|^2\rangle = 2\,\mathrm{Im}\{\Sigma^{\gamma\gamma}_{\mathrm{T},f}(k^2)\}, \tag{3.4}$$

where the subscript f in $\Sigma^{\gamma\gamma}_{\mathrm{T},f}$ indicates that only cuts through "f-loops" (intermediate states involving the fermion flavour f) are taken into account. Thus, we get

$$F_f(\Delta s) = \frac{1}{\pi}\int\limits_{s'<\Delta s}\mathrm{d}s'\,\frac{\mathrm{Im}\{\Sigma^{\gamma\gamma}_{\mathrm{T},f}(s')\}}{s'^2}. \tag{3.5}$$

Since $\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s)/s$ is an analytic function in the complex s plane apart from the positive real axis, real and imaginary parts are related by the dispersion relation

$$\frac{\mathrm{Re}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s)\}-s\Sigma_{\mathrm{T},f}^{\prime\gamma\gamma}(0)}{s^2}=\frac{1}{\pi}\,\mathrm{Re}\int\limits_{4m_\pi^2}^{\infty}\mathrm{d}s'\,\frac{\mathrm{Im}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s')\}}{s'^2(s'-s-\mathrm{i}0)},\qquad(3.6)$$

where $\Sigma_{\mathrm{T},f}^{\prime\gamma\gamma}(0)=\mathrm{d}\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s)/\mathrm{d}s|_{s=0}$ is a real quantity. Note that we have used $\Sigma_{\mathrm{T},f}^{\gamma\gamma}(0)=0$ because of electromagnetic gauge invariance and the fact that $\mathrm{Im}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s)\}$ vanishes for s values below the lightest hadronic threshold ($s<4m_\pi^2$, m_π = pion mass) because of causality. The running electromagnetic coupling

$$\alpha(s)=\frac{\alpha(0)}{1-\Delta\alpha(s)},\qquad \Delta\alpha(s)=\sum_f\Delta\alpha_f(s),\qquad(3.7)$$

comes into play via its relation to the real part of $\Sigma_{\mathrm{T},f}^{\gamma\gamma}$ (see, e.g., Ref. [17]),

$$\Delta\alpha_f(s)=\Sigma_{\mathrm{T},f}^{\prime\gamma\gamma}(0)-\frac{\mathrm{Re}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s)\}}{s}.\qquad(3.8)$$

Note that up to this point all arguments hold to any order (only the identification of contributions by a flavour f would deserve clarification beyond NLO). In the following we restrict the analysis, however, to NLO contributions in the self-energy, which corresponds to the LO splitting contribution. The quantity $\Delta\alpha_{\mathrm{had}}=\sum_q\Delta\alpha_q$ is extracted [17,18] (see also references therein) from low-energy data on the ratio $R=\sigma(\mathrm{e}^+\mathrm{e}^-\to\text{hadrons})/\sigma(\mathrm{e}^+\mathrm{e}^-\to\mu^+\mu^-)$ and will be used to evaluate $F_{\mathrm{had}}(\Delta s)=\sum_q F_q(\Delta s)$. To this end, we choose $s=M_\mathrm{Z}^2\gg\Delta s$, for which $\Delta\alpha_{\mathrm{had}}(s)$ is quoted in the literature, and split the dispersion integral of (3.6) into a non-perturbative ($4m_\pi^2<s'<\Delta s$) and a perturbative part ($\Delta s<s'<\infty$),

$$\begin{aligned}\Delta\alpha_f(M_\mathrm{Z}^2)&=-\frac{M_\mathrm{Z}^2}{\pi}\int\limits_{4m_\pi^2}^{\Delta s}\mathrm{d}s'\,\frac{\mathrm{Im}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s')\}}{s'^2(s'-M_\mathrm{Z}^2)}\\&\quad-\frac{M_\mathrm{Z}^2}{\pi}\,\mathrm{Re}\int\limits_{\Delta s}^{\infty}\mathrm{d}s'\,\frac{\mathrm{Im}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s')\}}{s'^2(s'-M_\mathrm{Z}^2-\mathrm{i}0)}\\&=\frac{1}{\pi}\int\limits_{4m_\pi^2}^{\Delta s}\mathrm{d}s'\,\frac{\mathrm{Im}\{\Sigma_{\mathrm{T},f}^{\gamma\gamma}(s')\}}{s'^2}-N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{3\pi}\,\ln\left(\frac{\Delta s}{M_\mathrm{Z}^2}\right)+\ldots,\end{aligned}\qquad(3.9)$$

where the non-perturbative part is accurate up to power corrections of $\mathcal{O}(M_{\mathrm{had}}^2/M_\mathrm{Z}^2)$ with hadron masses $M_{\mathrm{had}}\lesssim 5\,\mathrm{GeV}$ and the perturbative part up to two-loop corrections. Thus, we get for $Q^2\gg\Delta s\gg 4m_f^2$ the approximation

$$F_f(\Delta s)=\Delta\alpha_f(M_\mathrm{Z}^2)+N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{3\pi}\,\ln\left(\frac{\Delta s}{M_\mathrm{Z}^2}\right).\qquad(3.10)$$

Summing over the light quarks (u, d, s, c, b), this yields the hadronic contribution

$$F_{\mathrm{had}}(\Delta s)=\Delta\alpha_{\mathrm{had}}^{(5)}(M_\mathrm{Z}^2)+\sum_q\frac{Q_q^2\alpha}{\pi}\,\ln\left(\frac{\Delta s}{M_\mathrm{Z}^2}\right),\qquad(3.11)$$

where the superscript in $\Delta\alpha_{\mathrm{had}}^{(5)}(M_\mathrm{Z}^2)$ refers to five active light quark flavours. This is certainly sufficient to evaluate the $\mathcal{O}(\alpha^2/\alpha_s)$ corrections induced by the transitions $\gamma^*\to$ hadrons at low photon virtualities to any jet production cross section at the LHC. A recent fit to data [18] gives the result

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_\mathrm{Z}^2)=(276.11\pm1.11)\times10^{-4}.\qquad(3.12)$$

To make contact with the fully perturbative calculation of the previous section, we recall the perturbative NLO expression for $\Delta\alpha_f(s)$ in MR,

$$\Delta\alpha_f(s)=N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{3\pi}\left[\ln\left(\frac{|s|}{m_f^2}\right)-\frac{5}{3}\right],\qquad(3.13)$$

which leads to the perturbative result for $F_f(\Delta s)$,

$$\begin{aligned}F_f^{\mathrm{pert,MR}}(\Delta s)&=N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{3\pi}\left[\ln\left(\frac{\Delta s}{m_f^2}\right)-\frac{5}{3}\right]\\&=N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{2\pi}\,H_{f\bar f}^{\mathrm{MR}}(\Delta s),\end{aligned}\qquad(3.14)$$

in agreement with the result (2.11) of the previous section. The corresponding result in DR obviously reads

$$\begin{aligned}F_f^{\mathrm{pert,DR}}(\Delta s)&=N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{3\pi}\left[\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(-\frac{1}{\epsilon}+\ln\left(\frac{\Delta s}{\mu^2}\right)\right)-\frac{5}{3}\right]\\&=N_{\mathrm{c},f}\,\frac{Q_f^2\alpha}{2\pi}\,H_{f\bar f}^{\mathrm{DR}}(\Delta s).\end{aligned}\qquad(3.15)$$

We conclude this section by a side comment on the cancellation of the considered singularities as a consequence of the KLN theorem if photons are considered democratically [8] as possible initiators of jets just like any QCD parton. In this case, the cross section for $ab\to C+\gamma$ becomes part of the $ab\to C+\mathrm{jet}$ cross section. Adding the contribution from the $\gamma^*\to f\bar f$ splitting to the NLO EW cross section for $ab\to C+\gamma$, adds the contribution $\Delta\alpha(Q^2)$ to the relative EW corrections to this process, where Q^2 is some high scale typical for the process (such as M_Z^2). Since $\Delta\alpha(Q^2)$ involves perturbatively ill-defined mass logarithms of the light quarks, the EW input parameter scheme should be chosen in such a way that those quark-mass logarithms cancel in the EW correction. If the electromagnetic coupling factor α originating from the outgoing on-shell photon is taken as the fine-structure constant $\alpha(0)$ ($\alpha(0)$ scheme), the quark-mass logarithms in the charge renormalization constant and in the photon wave-function renormalization constant cancel, so that the additional logarithms in $\Delta\alpha(Q^2)$ stemming from the photon conversion would remain. If, however, the respective factor α is effectively taken at some high scale, as, e.g., in the $\alpha(M_\mathrm{Z}^2)$ or G_μ schemes [19–21], the $\Delta\alpha(Q^2)$ contribution from the photon conversion cancels. In other words, adding the $\gamma^*\to f\bar f$ splitting contribution to the EW correction to the process $ab\to C+\gamma$ effectively replaces the coupling factor $\alpha(0)$ for the emitted photon by $\alpha(Q^2)$ for some high scale like $Q^2=M_\mathrm{Z}^2$.

4. The photon-to-jet conversion function $D_{\gamma\to\mathrm{jet}}$

The common treatment of singular splitting processes associated with the final state, in which perturbative and non-perturbative contributions to cross sections arise, makes use of the concept of fragmentation functions. In the case of the splitting $\gamma^*\to q\bar q$ at low photon virtualities, this means that the NLO cross section for $ab\to Cq\bar q$ receives a perturbative (pert) contribution, as calculated above, and a conversion (conv) contribution,

$$\sum_q \mathrm{d}\sigma_{ab\to Cq\bar{q}}(k^2<\Delta s) = \sum_q \mathrm{d}\sigma^{\mathrm{pert}}_{ab\to Cq\bar{q}}(k^2<\Delta s) + \mathrm{d}\sigma^{\mathrm{conv}}_{ab\to C+\mathrm{jet}}, \tag{4.1}$$

where

$$\mathrm{d}\sigma^{\mathrm{pert}}_{ab\to Cq\bar{q}}(k^2<\Delta s) = \mathrm{d}\sigma^{\mathrm{LO}}_{ab\to C\gamma}\, F^{\mathrm{pert}}_q(\Delta s),$$
$$\mathrm{d}\sigma^{\mathrm{conv}}_{ab\to C+\mathrm{jet}} = \mathrm{d}\sigma^{\mathrm{LO}}_{ab\to C\gamma} \int_0^1 \mathrm{d}z\, D^{\mathrm{bare}}_{\gamma\to\mathrm{jet}}(z), \tag{4.2}$$

and F^{pert}_q refers to F^{DR}_q (3.15) or F^{MR}_q (3.14) for $f=q$. Here $D^{\mathrm{bare}}_{\gamma\to\mathrm{jet}}(z)$ is the "bare" $\gamma\to$ jet conversion function, which depends on the variable z describing the fraction of the photon momentum $\tilde{k}$ transferred to one of the jets ($p_{\mathrm{jet}} = z\tilde{k}$). The bare conversion function contains singular contributions so that the sum in (4.1) is non-singular. Extracting the singular contribution from $D^{\mathrm{bare}}_{\gamma\to\mathrm{jet}}(z)$ at some factorization scale μ_F requires a "factorization scheme", for which we take the $\overline{\mathrm{MS}}$ scheme following common practice,

$$D^{\mathrm{bare,DR}}_{\gamma\to\mathrm{jet}}(z) = D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F}) + \sum_q N_{\mathrm{c},q}\frac{Q_q^2\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_\mathrm{F}^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}P_{f\gamma}(z), \tag{4.3}$$

$$D^{\mathrm{bare,MR}}_{\gamma\to\mathrm{jet}}(z) = D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F}) + \sum_q N_{\mathrm{c},q}\frac{Q_q^2\alpha}{2\pi}\ln\left(\frac{m_q^2}{\mu_\mathrm{F}^2}\right)P_{f\gamma}(z). \tag{4.4}$$

In DR, it is just the $1/\epsilon$ pole with the usual prefactors that is subtracted; in MR we have adjusted the finite contributions accompanying the singular part ($\propto \alpha\ln m_q$) to define the same "renormalized conversion function" $D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F})$ as in DR. To get a handle on the non-perturbative contributions to $D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F})$, it would be desirable to exploit empirical information. This would, however, require an extremely accurate differential measurement of a jet production cross section (with low jet invariant mass) and of its corresponding prompt-photon counterpart, i.e. experimental information that is not available at present. We can, however, make use of the results of the previous section to at least get non-perturbative information on $D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F})$ for the case where the full z range is integrated over. Comparison of (3.2) with (4.1)–(4.2) leads to the identification

$$F_{\mathrm{had}}(\Delta s) = \sum_q F^{\mathrm{pert}}_q(\Delta s) + \int_0^1 \mathrm{d}z\, D^{\mathrm{bare}}_{\gamma\to\mathrm{jet}}(z). \tag{4.5}$$

Taking the perturbative result for the conversion function either in DR (4.3) or MR (4.4), and using (3.11) and (3.14) or (3.15) for the integrated renormalized conversion function, we get

$$\int_0^1 \mathrm{d}z\, D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F}) = \Delta\alpha^{(5)}_{\mathrm{had}}(M_\mathrm{Z}^2) + \sum_q N_{\mathrm{c},q}\frac{Q_q^2\alpha}{3\pi}\left[\ln\left(\frac{\mu_\mathrm{F}^2}{M_\mathrm{Z}^2}\right)+\frac{5}{3}\right]. \tag{4.6}$$

Note that this z-integral of $D_{\gamma\to\mathrm{jet}}$ is sufficient to evaluate the cross-section contribution $\mathrm{d}\sigma^{\mathrm{conv}}_{ab\to C+\mathrm{jet}}$ of (4.2) with (4.3) or (4.4).

The z-dependence of $D_{\gamma\to\mathrm{jet}}$ is not provided by the approach employed in this paper, but would require a model for the hadronization of the low-virtuality photon into jets. At least we can make the following statement on the z-dependence of the conversion function,

$$D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F}) = \Delta\alpha^{(5)}_{\mathrm{had}}(M_\mathrm{Z}^2) + \sum_q N_{\mathrm{c},q}\frac{Q_q^2\alpha}{2\pi}\left[\ln\left(\frac{\mu_\mathrm{F}^2}{M_\mathrm{Z}^2}\right)+\frac{5}{3}\right]P_{f\gamma}(z) + g(z), \tag{4.7}$$

with $g(z)$ denoting a function that integrates to $0=\int_0^1 \mathrm{d}z\, g(z)$. To reproduce the correct integral over z and thus the correct cross-section contribution, we can simply set $g(z)\equiv 0$,

$$D_{\gamma\to\mathrm{jet}}(z,\mu_\mathrm{F}) = \Delta\alpha^{(5)}_{\mathrm{had}}(M_\mathrm{Z}^2) + \sum_q N_{\mathrm{c},q}\frac{Q_q^2\alpha}{2\pi}\left[\ln\left(\frac{\mu_\mathrm{F}^2}{M_\mathrm{Z}^2}\right)+\frac{5}{3}\right]P_{f\gamma}(z), \tag{4.8}$$

in which the non-perturbative z-dependence is approximated by a constant reproducing the correct z-integral.

An example for the use of $D_{\gamma\to\mathrm{jet}}$ in some cross-section prediction for the LHC is discussed in the next section.

5. An example: photon-to-jet conversion function in $\mathrm{pp}\to\ell^+\ell^- + \mathrm{jet} + X$

In this section we focus on the application of the above formalism to $\mathrm{pp}\to\ell^+\ell^-\mathrm{j}+X$. We consider the leading-order (LO) cross section at order $\mathcal{O}(\alpha_\mathrm{s}\alpha^2)$. The contributions featuring the conversion function are part of the corresponding real radiation process $\mathrm{pp}\to\ell^+\ell^-\mathrm{jj}+X$ at order $\mathcal{O}(\alpha^4)$ where all QCD partons are quarks. Some representative Feynman diagrams for this channel are shown in Fig. 2. While the two quark–quark-induced t-channel diagrams on the left of Fig. 2 dominate the $\mathcal{O}(\alpha^4)$ contributions, the conversion function only shows up in quark–antiquark-induced s-channel diagrams such as the third diagram of Fig. 2. Moreover, there are channels with no photon-to-quark conversion at all, as shown in the last diagram of Fig. 2.

The numerical study is carried out in the set-up of Ref. [22], where the EW corrections of order $\mathcal{O}(\alpha_\mathrm{s}\alpha^3)$ were computed. We first reproduce the input parameters and the event selection for completeness and then turn to numerical results.

The simulations are performed for the LHC at 14 TeV with the SM input parameters chosen as

$$\begin{aligned} G_\mu &= 1.16637\times10^{-5}\,\mathrm{GeV}^{-2}, & \alpha_\mathrm{s}(M_\mathrm{Z}) &= 0.1202,\\ M_\mathrm{W}^{\mathrm{OS}} &= 80.398\,\mathrm{GeV}, & \Gamma_\mathrm{W}^{\mathrm{OS}} &= 2.141\,\mathrm{GeV},\\ M_\mathrm{Z}^{\mathrm{OS}} &= 91.1876\,\mathrm{GeV}, & \Gamma_\mathrm{Z}^{\mathrm{OS}} &= 2.4952\,\mathrm{GeV}. \end{aligned} \tag{5.1}$$

Leptons are considered massless.

Throughout the article, the complex-mass scheme [23] is used along with the G_μ scheme for α. The on-shell (OS) widths and masses of the W and Z bosons are converted into pole values using [24]

$$\begin{aligned} M_V &= M_V^{\mathrm{OS}}/\sqrt{1+(\Gamma_V^{\mathrm{OS}}/M_V^{\mathrm{OS}})^2},\\ \Gamma_V &= \Gamma_V^{\mathrm{OS}}/\sqrt{1+(\Gamma_V^{\mathrm{OS}}/M_V^{\mathrm{OS}})^2}, \end{aligned} \tag{5.2}$$

leading to the input values

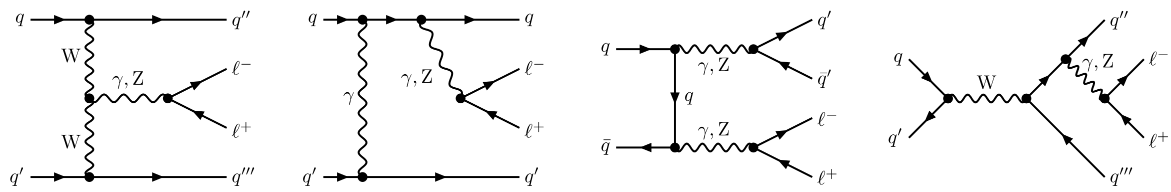

Fig. 2. Some representative Feynman diagrams for $qq \to \ell^+\ell^- qq$.

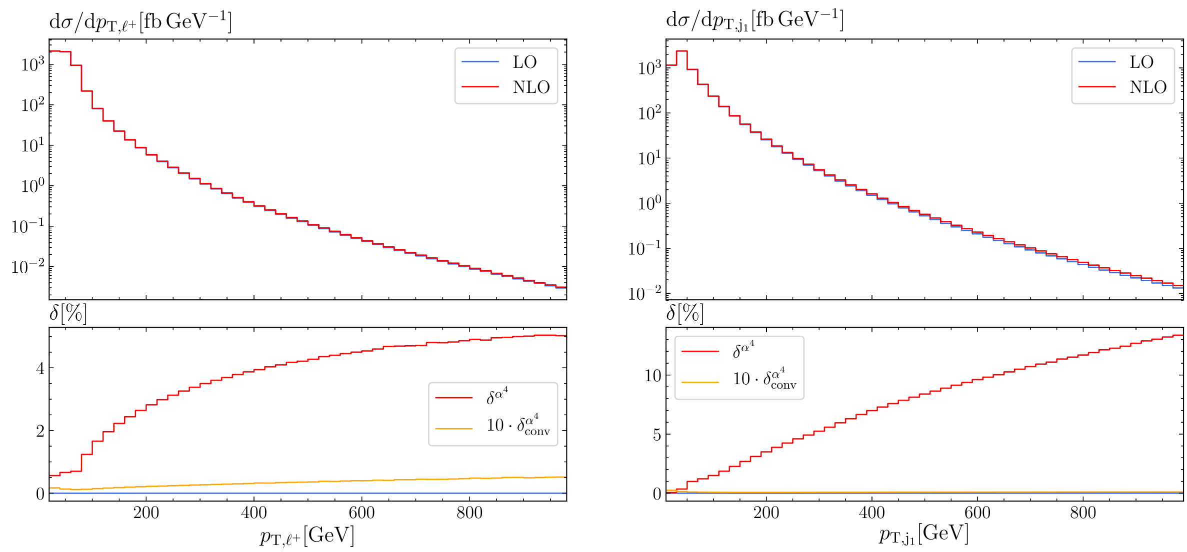

Fig. 3. Differential distributions for LO [order $\mathcal{O}(\alpha_s\alpha^2)$] and corrections of order $\mathcal{O}(\alpha^4)$ from $pp \to \ell^+\ell^- jj + X$ at the 14 TeV LHC in the transverse momentum of the antilepton (left) and of the hardest jet (right). The upper panels display the absolute predictions, while the lower panels show the relative corrections of order α^4 and its contribution from the photon conversion function.

$$M_W = 80.370\ldots \text{ GeV}, \qquad \Gamma_W = 2.1402\ldots \text{ GeV},$$
$$M_Z = 91.153\ldots \text{ GeV}, \qquad \Gamma_Z = 2.4943\ldots \text{ GeV}. \qquad (5.3)$$

The MSTW2008NLO PDF set [25] is used as provided by LHAPDF [26], while the factorization and renormalization scales are set to the Z-boson mass.

The recombination of QCD partons is done with the k_T-algorithm with $R = 0.5$. The event selection for the numerical analysis is defined as:

1. Jets are required to have transverse momentum p_T larger than $p_{T,jet}^{cut} = 25$ GeV. At least one of them (not necessarily the hardest jet) is required to have rapidity y smaller than $y_{max} = 2.5$.
2. The event must have two charged leptons of opposite sign with transverse momenta $p_{T,\ell} > 25$ GeV and rapidity $y_\ell < 2.5$.
3. The dilepton invariant mass is required to fulfil $M_{\ell\ell} > 50$ GeV.
4. The leptons must be isolated, i.e. $R_{\ell jet} > 0.5$ is required for all jets.

For the simulations, we consider only one lepton family. In Table 1, we report on the integrated cross section defined in the fiducial region specified above. The relative corrections of order $\mathcal{O}(\alpha^2/\alpha_s)$ are about half a per cent. For reference, the EW corrections have been found in Ref. [22] to amount to a few per cent and the photon-induced contributions at order $\mathcal{O}(\alpha^3)$ to be at the level of 0.1%. The present findings are in agreement with expectations based on naive power counting of couplings combined with the fact that the $\mathcal{O}(\alpha^4)$ contributions receive some enhancement owing t-channel diagrams in quark–quark channels where one of the quarks goes into the forward direction (see left two diagrams in Fig. 2). The contribution of the conversion function is only 0.013%. Besides the suppression of this contribution by the factor α^2/α_s there is an additional suppression due to the fact that it only features partonic channels with quark–antiquark initial states (see third diagram in Fig. 2).

Table 1
Cross sections at LO [order $\alpha_s\alpha^2$] and corrections of order α^4 from the real radiation process $pp \to \ell^+\ell^- jj + X$ at the 14 TeV LHC. The contribution $\delta^{\alpha^4}_{conv}$ of the conversion function is separately shown for a factorization scale $\mu_F = M_Z$. The digits in parentheses indicate the integration error.

$\sigma^{\alpha_s\alpha^2}$ [pb]	σ^{α^4} [pb]	δ^{α^4} [%]	$\delta^{\alpha^4}_{conv}$ [%]
122.414(7)	0.77116(5)	0.63	0.013%

In Fig. 3, the differential distributions in the transverse momentum of the antilepton and the, according to p_T ordering, hardest jet are presented. The corrections δ^{α^4} to the transverse momentum of the antilepton increase rather smoothly from nearly 0% at the minimum transverse momentum of 25 GeV up to about 5% at 1 TeV. For the transverse momentum of the hardest jet, the corrections increase more strongly and reach more than 10% at 1 TeV. This general trend can be explained by the behaviour of the PDFs of the dominant channels. While the LO contributions [order $\alpha_s\alpha^2$] are dominated by partonic channels with gluons and quarks in the initial state, the contributions of the order α^4 involve channels with two valence quarks in the initial state. The decrease of the gluon PDFs with increasing momentum fraction x (required by increasing scattering energy) causes an enhancement of the relative corrections. The contribution $\delta^{\alpha^4}_{conv}$ of the conversion function defined in Eqs. (4.2) and (4.8) with $\mu_F = M_Z$ is below 0.05% for all considered distributions.

6. Conclusion

The calculation of electroweak corrections to processes with jets in the final state involves contributions of low-virtuality photons leading to jets in the final state. Such contributions are typically small but contain infrared singularities, calling for a practical prescription for their treatment. These singularities can be absorbed into the photon-to-jet conversion function, which is similar to a fragmentation function for identified hadrons. In this letter, we have used the well-known hadronic contributions to the vacuum polarization to derive an approximative expression for the photon-to-jet conversion function. We have illustrated how this can be used in a practical calculation of electroweak corrections to Z+jet production at the LHC.

The effect of the photon-to-jet conversion function is typically small for processes at hadron colliders. Therefore, our recipe is certainly sufficient for the consistent calculation of electroweak corrections to processes at the LHC and the next generation of hadron colliders.

A measurement of the photon-to-jet conversion function might be possible at future high-luminosity lepton colliders in photon-plus-jet or Z-boson-plus-jet production above the Z-boson resonance.

Acknowledgements

The authors thank the organizers of the Les Houches Workshop “Physics at TeV Colliders”, 2019, where this work was completed, for their kind hospitality and the splendid organization of the workshop. AD acknowledges financial support by the German Federal Ministry of Education and Research (BMBF) under contract no. 05H18WWCA1 and the German Research Foundation (DFG) under reference number DE 623/6-1. SD and CS acknowledge support by the state of Baden-Württemberg through bwHPC and the DFG through grant no. INST 39/963-1 FUGG and grant DI 784/3. MP is supported by the European Research Council Consolidator Grant NNLOforLHC2. CS is supported by the European Research Council under the European Unions Horizon 2020 research and innovation Programme (grant agreement no. 740006).

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