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RESEARCH ARTICLE

The phase shift between potential and kinetic energy in human walking

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ABSTRACT

It is known that mechanical work to sustain walking is reduced, owing to a transfer of gravitational potential energy into kinetic energy, as in a pendulum. The factors affecting this transfer are unclear. In particular, the phase relationship between potential and kinetic energy curves of the center of mass is not known. In this study, we measured this relationship. The normalized time intervals α, between the maximum kinetic energy in the sagittal plane (E_k) and the minimum gravitational potential energy (E_p), and β, between the minimum E_k and the maximum E_p, were measured during walking at various speeds (0.5–2.5 m s^{-1}). In our group of subjects, α=β at 1.6 m s^{-1}, indicating that, at this speed, the time difference between E_p and E_k extremes is the same at the top and the bottom of the trajectory of the center of mass. It turns out that, at the same speed, the work done to lift the center of mass equals the work to accelerate it forwards, the E_p–E_k energy transfer approaches a maximum and the mass-specific external work per unit distance approaches a minimum.

KEY WORDS: Human locomotion, Walking, Energy transfer in walking

INTRODUCTION

Contrary to flying and swimming, during which the body can slide against the surrounding medium, legged terrestrial locomotion is hindered by the fixed point of contact that the foot makes with the ground at each step (Tucker, 1973). When, during the step, the point of contact is in front of the center of mass, the body decelerates forwards, owing to the link between the center of mass and the point of contact. This results in a loss of kinetic energy of forward motion:

$$\Delta E_{kf} = 0.5M(V_{f,max}^2 - V_{f,min}^2), \tag{1}$$

where M is the mass of the body and $V_{f,max}$ and $V_{f,min}$, respectively, are the maximum and minimum values of the forward velocity of the center of mass attained at each step. In order to maintain a constant average step speed, the loss in kinetic energy taking place at each step must be restored by reaccelerating the body forwards, and this requires work and energy expenditure.

Fortunately, as in pole vaulting, the mechanical energy remaining after the impact on the ground (Donelan et al., 2001, 2002a,b), and the work done during double contact by one leg against the other (Bastien et al., 2003), is stored during the lift (S_v) of the body as an increase in gravitational potential energy, $\Delta E_p = M\boldsymbol{g}S_v$ (where $\boldsymbol{g}$ is the acceleration of gravity). After the lift, the body 'falls forwards', restoring E_{kf} at the expense of E_p. In this way, some mechanical energy is conserved by a pendulum mechanism. Contrary to a pendulum, however, the two energy changes in walking are not exact mirror images of each other, with the consequence that mechanical work must be done by the muscles to maintain the motion of the center of mass in the sagittal plane.

The effectiveness of the pendulum mechanism during human walking has been evaluated by measuring the fraction of the total mechanical energy changes of the center of mass that is recovered as a result of the transduction between E_p and E_{kf} (Cavagna et al., 1976; Heglund et al., 1982; Cavagna et al., 2000; Massaad et al., 2007):

$$\text{Recovery} = (W_v + W_f - W_{ext})/(W_v + W_f) = 1 - W_{ext}/(W_v + W_f), \tag{2}$$

where W_v represents the amplitude of the E_p curve, i.e. the positive work (W_v=ΔE_p) done at each step to lift the center of mass; W_f=ΔE_{kf} represents the amplitude of the E_{kf} curve; and W_{ext} is the total positive external work actually done in each step to maintain the motion of the center of mass in the sagittal plane. In a frictionless pendulum, W_{ext}=0 and recovery=1.

In order to optimize the recovery of mechanical energy, the kinetic and the gravitational potential energy curves must have the same shape, be equal in amplitude and be opposite in phase, as in a pendulum. This study aimed to describe the phase relationship between gravitational potential energy E_p and the total kinetic energy E_k=E_{kf}+E_{kv}, where $E_{kv} = 0.5M(V_v^2)$ is the instantaneous kinetic energy of vertical motion (V_v is the vertical velocity of the center mass). To this aim, the time shift between the maximum kinetic energy and the minimum gravitational potential energy, and the time shift between the minimum kinetic energy and the maximum potential energy, were simultaneously measured, for the first time.

MATERIALS AND METHODS

Subjects and experimental procedure

The experiments were performed on five untrained male subjects (Table 1). Informed consent was obtained from all subjects. The study protocol was approved by the Ethics Committee of the University of Milan.

A total of 292 walks were analyzed during experiments performed on different days. Body mass values refer to the average of all days.

The subjects walked at different speeds across a 4 m×0.5 m force platform sensitive to the fore–aft (F_f) and vertical (F_v) components of the force impressed on it by the feet; the lateral component of the force was not considered. The characteristics of the platform were as described by Cavagna (1975). The platform had its surface at the level of the floor and was set 30 m from the beginning of a corridor 50 m long so that the subjects had plenty of space to reach a constant

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Table 1. Characteristics of the human experimental subjects

Subject	Age (years)	Mass (kg)	Height (m)
LB	23	69.7	1.80
PB	50	77.9	1.79
ML	42	80.2	1.82
MP	31	71.1	1.80
RG	43	80.8	1.75

speed over the platform. The speed range considered in this study was from 0.5 m s^{-1} to 2.5 m s^{-1}. Two photocells set 2–3 m apart at the shoulder level of a subject were used to measure the average walking speed (V_f); we analyzed the steps recorded within the two photocells. Because the displacement of the center of mass within the body and the tilting of the trunk (Fenn, 1930) are small in comparison with the distance between the photocells, V_f, as measured, should not differ appreciably from the average forward speed of the center of gravity. The subjects wore gym shoes and walked over the platform on several different days. The time in which both feet were on the ground (t_{dc}) was determined using a small transmitter, carried at the waist, connected by wires to metal gauze patches glued to the soles of the shoes; for all subjects we used a complete heel–lateral–toe metal gauze, which is the most reliable to determine touch down and lift-off. When both feet were on the ground, the circuit was closed through the metallic surface of the platform and the transmitter operated, thus giving rise to a square signal at the output of a receiver (Cavagna et al., 1976). The t_{dc} signal was acquired together with the F_f and F_v tracings, and the photocell circuit output by an A/D board (PCI-MIO-16E-4, National Instruments, Austin, TX, USA), with a sample rate of 500 Hz. Acquired data were stored in memory for subsequent analysis.

Analysis of force platform records

A custom LabVIEW software (version 7.1, National Instruments) (available from M.A.L. upon request) was used to calculate the forward and vertical velocities of the center of mass of the body, every 2 ms, by integrating the F_f and F_v tracings. The kinetic energies of forward and vertical motion (E_{kf} and E_{kv}) were calculated from the forward and vertical velocities. The potential energy (E_p) was calculated by integrating vertical velocity as a function of time to yield vertical displacement and multiplying this by the body weight. The total mechanical energy was calculated as E_{cg}=E_{kf}+E_{kv}+E_p. The positive external mechanical work was obtained by adding the increments in E_{cg} over an integer number of steps. The procedure involved in using force platforms records to measure external mechanical work has been described in detail by Cavagna, (1975). More specifically, the platform measures the vertical and horizontal forces during locomotion (fig. 1 of Cavagna, 1975), the integration of which yields the vertical and horizontal velocity changes. From the velocity changes, one can work out the vertical and horizontal kinetic energy, and through further integration of the vertical velocity, the center of mass displacement is calculated (fig. 2 of Cavagna, 1975), from which the potential energy can be worked out (fig. 3 of Cavagna, 1975). Once total kinetic energy and potential energy are determined, thus giving the total mechanical energy of the system, external work is calculated from the variation in it (fig. 3 of Cavagna, 1975).

In a perfect steady walk at flat level, the ratio between positive (W^+) and negative (W^-) work should be equal to one. In the 292 walks selected for analysis, W_v^+/W_v^- =1.01±0.14, W_k^+/W_k^-=1.03±0.14 and W_{ext}^+/W_{ext}^-=1.06±0.28 (means±s.d., N=292).

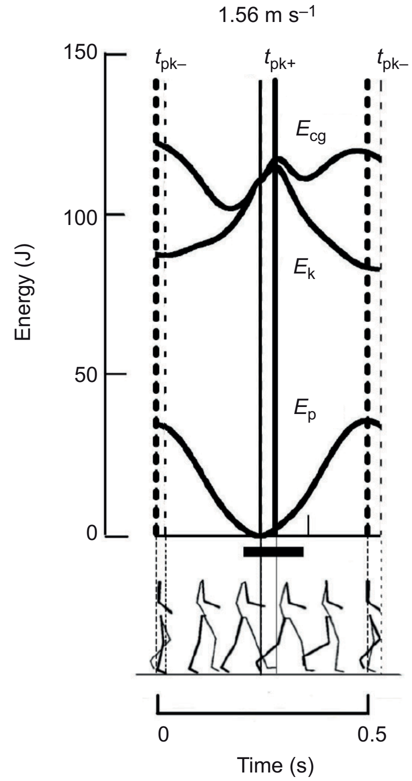

Fig. 1. Experimental tracings for human during a walking step. The walking step period (τ) lasts 0.5 s at a speed of 1.56 m s^{-1}. Curves show (from bottom to top) the gravitational potential energy of the center of mass (E_p), the kinetic energy (E_k=E_{kv}+E_{kf}) in the sagittal plane, and the total mechanical energy (E_{cg}=E_p+E_{kv}+E_{kf}). The thick horizontal line below the records indicates the time of double contact when both feet are on the ground. t_{pk+} is the time interval between the maximum E_k (thick solid vertical line) and minimum E_p (thin solid vertical line) during double contact. t_{pk-} is the time interval between minimum E_k (thin dashed vertical lines) and maximum E_p (thick dashed vertical lines) during single contact. The stick figures at the bottom show the position of the limbs during the step (subject, ML).

The phase shift α between the E_k and E_p curves during double contact was calculated as

$$\alpha = 360\text{deg}\ t_{pk+}/\tau, \tag{3}$$

where τ is the step period and t_{pk+} is the difference between the time at which E_k=E_{kf}+E_{kv} is maximum and the time at which E_p is minimum (solid vertical lines in Figs 1 and 2, thicker lines refer to maxima, thinner lines refer to minima). Positive values of α (maximum E_k following the minimum E_p) indicate that positive work is done during double contact to attain the maximum kinetic energy E_k and to lift the center of mass from its lowest point (Fig. 1). Negative values of α (maximum E_k preceding the minimum E_p; top record in Fig. 2) indicate that negative work is done during double contact to decelerate forward while attaining the lowest value of the vertical displacement. In conclusion, positive α values indicate positive work done during double contact, whereas negative values indicate negative work.

The phase shift β between the E_k and E_p curves during single contact was calculated as

$$\beta = 360\text{deg}\ t_{pk-}/\tau, \tag{4}$$

where t_{pk-} is the difference between the time at which E_k is minimum and the time at which E_p is maximum (dashed vertical lines in Figs 1 and 2, thicker lines refer to maxima, thinner lines refer to minima). Positive values of β (minimum E_k following the

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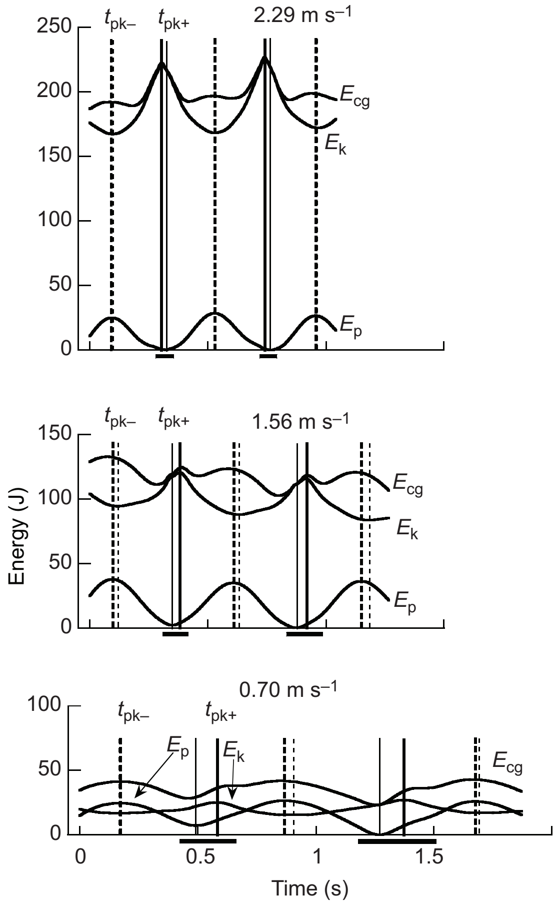

Fig. 2. Experimental tracings for human at three walking speeds increasing from bottom to top. The second step of the middle tracing is amplified in Fig. 1 (subjects, from top to bottom: LB, ML and PB). These representative curves show qualitative records of the average data shown in Fig. 3.

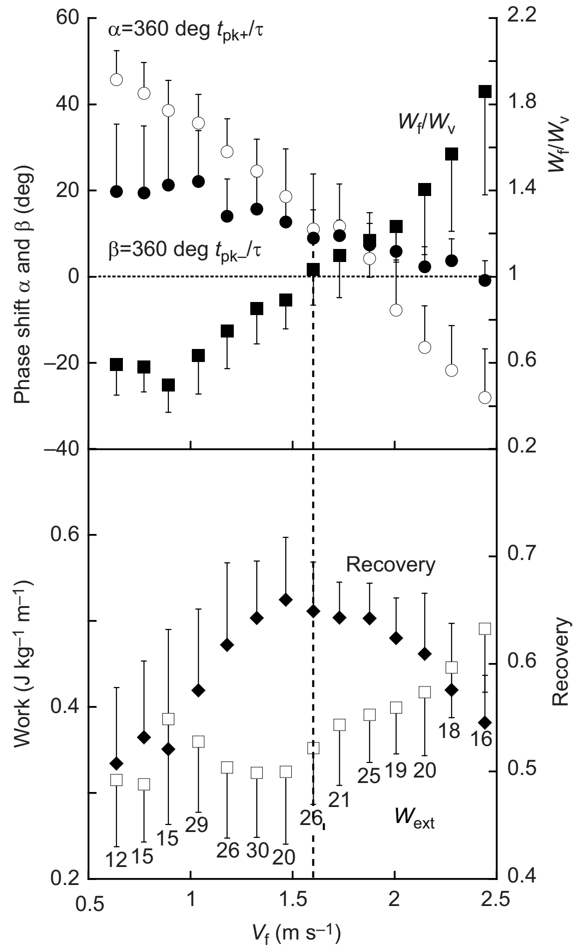

Fig. 3. Phase shifts, the ratio between forward and vertical work, the recovery and the mass-specific external work per unit distance, as functions of walking speed in human. Upper panel: the phase shifts α (open circles) and β (filled circles) and the ratio between forward and vertical work, W_f/W_v (filled squares; W_f=ΔE_{kf} and W_v=ΔE_p), are plotted as a function of the walking speed. Note that α=β when W_f/W_v=1 (1.6 m s^{-1}) as indicated by the vertical dashed line. Lower panel: the recovery (filled diamonds) and the mass-specific external work per unit distance (W_{ext}; open squares) are plotted as a function of the walking speed. Note that recovery and W_{ext} attain a maximum and a minimum, respectively, at about the same speed, at which α=β and W_f/W_v=1 [note that $W_f = \Delta E_{kf} = 0.5M(V^2_{f,max} - V^2_{f,min})$, where M is the body mass, $V_{f,max}$ and $V_{f,min}$, respectively, are the maximum and minimum values of the forward velocity of the center of mass attained at each step, and ΔE_p=M $\boldsymbol{g}$ S_v (where $\boldsymbol{g}$ is the acceleration of gravity and S_v is the vertical displacement of the body center of mass at each step)]. The data collected as a function of walking speed were grouped in ∼0.14 m s^{-1} intervals as follows: 0.55<0.69 m s^{-1}, 0.69<0.83 m s^{-1}…2.36<2.5 m s^{-1}. The data points and the vertical segments represent the mean±s.d. in each of the above speed intervals. The values below the symbols indicate the number of steps analyzed in each of the above speed intervals. Subjects walked over the platform on several different days. The units on the ordinate refer to work in joules/(body mass and distance traveled during the step).

maximum E_p) indicate that negative work is done to decelerate forward while beginning the downward fall. In conclusion, positive β values indicate negative work done during midstance, whereas negative values indicate positive work.

It should be noted that the two curves of E_p and E_k have somewhat different shapes, so they are not symmetric (e.g. specular); for this reason, it is indeed tricky to define a single phase shift for the entire curves. However, the phase shift α that we define and study throughout this work is uniquely determined by its energetic role; namely, to be positive to accelerate forward and lift the body and vice versa. For this to be true, the only features of the E_p and E_k curves that are necessary to exploit are the neighborhoods of the extrema, i.e. the minimum E_p and the maximum E_k. Fortunately, these extrema are unique: E_p has just one minimum and E_k has just one maximum, the positions of which are compared with each other (and vice versa for the phase shift β).

In Fig. 3, the data collected as a function of walking speed were grouped in ∼0.14 m s^{-1} (0.5 km h^{-1}). The data points and the vertical segments represent the mean±s.d. in each speed interval. The values below the symbols indicate the number of items used to calculate the mean.

RESULTS

Fig. 1 represents typical experimental records of the mechanical energy changes of the center of mass of the body: E_p, to sustain vertical displacement; E_k=E_{kf}+E_{kv}, the forward and vertical velocity changes; and E_{cg}, the combined displacement in the sagittal plane. As described in the Materials and Methods, the time intervals t_{pk+} and t_{pk-} are defined by the horizontal distance between the solid and dashed vertical lines, respectively. The position of the limbs during the step is shown by the stick figures at the bottom of the figure. The thick horizontal bar indicates the time of double contact. As shown, t_{pk+} takes place during double contact and t_{pk-} during single contact.

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Fig. 2 shows records obtained at three walking speeds, increasing from bottom to top (part of the middle record is amplified in Fig. 1). It can be seen that, on average, t_{pk+} is greater than t_{pk-} at 0.70 m s^{-1} and that $t_{pk+}\approx t_{pk-}$ at 1.56 m s^{-1}, whereas at 2.29 m s^{-1}, t_{pk-} is nil and t_{pk+} is reversed.

The upper panel of Fig. 3 shows α, β and the ratio between forward and vertical work (W_f/W_v) as a function of speed. It appears that both α and β decrease with speed: α from positive to negative values, whereas β is always positive and reduces to zero at the highest speeds. β is less than α up to 1.6 m s^{-1} where it equals α (dashed vertical line). In addition, the ratio between work to sustain forward speed changes of the center of gravity (W_f) and its vertical displacement (W_v) increases with speed, attaining unity at the same speed at which α=β.

The bottom panel of Fig. 3 shows external work and recovery (Eqn 2) as a function of speed. Figs 1 and 2 show the energy tracings of one individual subject; hence, there is no need for normalizing the energy, which can be measured directly in joules. However, in Fig. 3, we average over all different subjects, who have different body masses and have traveled for different total distances during their walk; for this reason, we must normalize the energy expenditure (J) by body mass (kg) and distance traveled (m), thus obtaining the mass-specific external work done per unit distance, W_{ext}. The recovery attains a maximum of 0.66 at a speed of 1.46 m s^{-1}, in agreement with Cavagna et al. (1983). The minimum W_{ext} that is relevant for the energetics of the system is the local one attained at intermediate speed, and not the absolute minimum at 0.6 m s^{-1}, which is an artifact caused by the dragged motion of the limbs at very low speed. The local minimum W_{ext} and the maximum recovery are attained at a speed slightly less (0.14 m s^{-1}) than the speed at which α=β and W_k=W_v (dashed line).

Fig. 4 shows that the time of upward displacement (t_{up}) approximately equals that of downward displacement (t_{down}) for most walking speeds, except at speeds less than 0.9 m s^{-1}, at which t_{up}>t_{down}, and at speeds greater than ∼1.9 m s^{-1}, at which t_{down}>t_{up}. At the 'optimal speed' (dashed vertical line) t_{up}=t_{down}, the time of single contact equals ∼75% of τ.

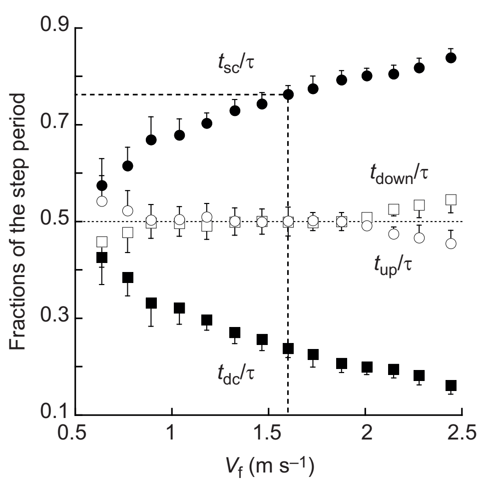

Fig. 4. Fractions of the step period in human as a function of the walking speed. The fractions of the step period during single contact t_{sc}/τ (filled circles), double contact t_{dc}/τ (filled squares), downward displacement t_{down}/τ (open squares) and upward displacement t_{up}/τ (open circles) of the center of gravity, are plotted as a function of the walking speed. Note that at the optimal speed (∼1.6 m s^{-1}), t_{down}/τ=t_{up}/τ and the time of single contact is ∼75% of the total step period (dashed lines). Other indications are as in Fig. 3.

DISCUSSION

The most relevant feature of this study derives from the measurement of the phase shifts α and β between E_p and E_k. In particular, it is interesting that β=α≈10 deg at the speed (∼1.6 m s^{-1} in our group of subjects) at which W_f=W_v, the mass-specific external work per unit distance W_{ext} approaches a minimum and the recovery approaches a maximum. It follows that, at this optimal speed, the E_p and E_k curves are not exactly in opposite phase but are shifted by the same amount (β=α) at the top and the bottom of the trajectory of the center of mass. Furthermore, at this speed, the lift duration (t_{up}) equals the fall duration (t_{down}), whereas t_{up}>t_{down} at low speeds and t_{down}>t_{up} at high speeds (Fig. 4).

Given that the step length is the most natural scale of walking, we find it convenient to express the shifts α and β (measured in units of degrees) with two corresponding shifts (measured in units of lengths), by multiplying α and β by the step length, and dividing by 360 deg. Because the step length at the optimal speed was found to be, on average, 0.82 m, the shifts α (from the maximum E_k to the minimum E_p), at the bottom of the trajectory, and β (from the minimum E_k to the maximum E_p), at the top of the trajectory (Fig. 1), correspond in both cases to a length 0.82×10/360=0.023 m.

At the lowest speed, at which step length is, on average, 0.51 m, the shift during double contact at the bottom of the trajectory, from the maximum E_k to the minimum E_p (α≈46 deg in Fig. 3), corresponds to a length 0.51×46/360=0.065 m, whereas during single contact, at the top of the trajectory, from the minimum E_k to the maximum E_p (β≈20 deg in Fig. 3), it corresponds to a length 0.51×20/360=0.028 m.

At the highest speed, at which step length is, on average, 1 m, the shift during double contact, at the bottom of the trajectory, from the maximum E_k to the minimum E_p (α≈−28 deg in Fig. 3), corresponds to a length 1.00×(−28)/360=−0.078 m, whereas the shift during single contact at the top of the trajectory, from the minimum E_k to the maximum E_p (β≈−1 deg), corresponds to a length 1.00×(−1)/360=−0.002 m.

The expression 'optimal speed' should, in principle, refer to the speed at which the mass-specific metabolic energy expenditure per unit distance is at the minimum. This is not the case in the present study, as we define the optimal speed as that of the minimum mechanical energy expenditure, W_{ext}. Several studies showed that other mechanical work must contribute to the metabolic energy output: (1) the internal work as a result of the kinetic energy changes of the limbs relative to the center of mass (Cavagna and Kaneko, 1977), (2) the work to redirect the center of mass velocity from one inverted pendulum to the other (Donelan et al., 2001, 2002a,b), (3) the internal work done by one leg against the other during double contact (Bastien et al., 2003) and (4) the internal work done against frictional losses (Minetti et al., 2020). Furthermore, the optimal speed determined by the minimum W_{ext} differs between subjects (Fig. 3 shows the average of five subjects). The interesting (and new) finding here is that, at this speed, the time difference between E_p and E_k extremes is the same at the top and the bottom of the trajectory of the center of mass, i.e. E_p and E_k are shifted by the same amount at the top and bottom of the trajectory of the center of mass. This (together with the minimum external work and the maximum recovery) could prove to be true, independent of the absolute value of the optimal speed attained by different subjects. Further studies are required to determine how the optimal speed is changed in different groups of subjects (training, age, etc.).

In conclusion, the present study shows a new feature of the mechanics of walking: the shift between potential and kinetic energy of the center of mass being equal at the bottom and at the top of its trajectory; this occurs when the work done in forward and vertical direction are equal, the gravitational–kinetic energy transfer approaches a maximum and the mass-specific external mechanical work done per unit distance approaches a minimum (optimal walking speed). It must be pointed out that the optimal speed might differ between subjects according to their training.

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Competing interests
The authors declare no competing or financial interests.

Author contributions
Conceptualization: G.A.C.; Methodology: G.A.C., M.A.L.; Software: G.A.C., M.A.L.; Data curation: G.A.C., M.A.L.; Writing - original draft: G.A.C.; Writing - review & editing: G.A.C., M.A.L.; Visualization: G.A.C.; Supervision: G.A.C.

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