

# Polytropic models of filamentary interstellar clouds – I

## Structure and stability

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### ABSTRACT

The properties of filamentary interstellar clouds observed at sub-millimetre wavelengths, especially by the *Herschel Space Observatory*, are analysed with polytropic models in cylindrical symmetry. The observed radial density profiles are well reproduced by negative-index cylindrical polytropes with polytropic exponent  $1/3 \lesssim \gamma_p \lesssim 2/3$  (polytropic index  $-3 \lesssim n \lesssim -3/2$ ), indicating either external heating or non-thermal pressure components. However, the former possibility requires unrealistically high gas temperatures at the filament’s surface and is therefore very unlikely. Non-thermal support, perhaps resulting from a superposition of small-amplitude Alfvén waves (corresponding to  $\gamma_p = 1/2$ ), is a more realistic possibility, at least for the most massive filaments. If the velocity dispersion scales as the square root of the density (or column density) on the filament’s axis, as suggested by observations, then polytropic models are characterised by a uniform width. The mass per unit length of pressure-bounded cylindrical polytropes depends on the conditions at the boundary and is not limited as in the isothermal case. However, polytropic filaments can remain stable to radial collapse for values of the axis-to-surface density contrast as large as the values observed only if they are supported by a non-isentropic pressure component.

**Key words:** ISM: clouds – instabilities

### 1 INTRODUCTION

The filamentary structure of molecular clouds has recently received considerable attention, especially thanks to the high sensitivity and dynamic range of images obtained at sub-millimetre wavelengths by the *Herschel Space Observatory*. The observed filaments typically represent enhancements by a factor of  $\sim 10^2$  in volume density (or by a factor of  $\sim 10$  in column density) with respect to the ambient medium of the molecular cloud, extending over  $\sim$  pc scales and often forming complex networks (André et al. 2010, Molinari et al. 2010). From a theoretical point of view, the origin of interstellar filaments still needs to be fully understood. It is debated whether filaments are stagnation regions formed either at the intersections of planar shocks (Padoan et al. 2001), or by the collapse and fragmentation of self-gravitating gaseous sheets (Burkert & Hartmann 2004) perhaps mediated by magnetic fields (Miyama, Narita & Hayashi 1987a,b; Nagai, Inutsuka & Miyama 1998; Van Loo, Keto & Zhang 2014), or they are long-lived features of the flow resulting from hierarchical fragmentation (Gómez & Vázquez-Semadeni 2013),

or they are formed by turbulent shear and maintained coherent by magnetic stresses (Hennebelle 2013). Remarkably, the observed properties of individual filamentary clouds are well characterised and rather uniform, at least for filaments located in nearby molecular clouds (mostly in the Gould’s Belt, see André 2013 for a review). Their density profiles in the radial direction, perpendicular to the filament’s axis, are characterised by a flat-density inner part of size  $\sim \varpi_{\text{flat}}$  and a power-law envelope extending to an outer radius  $\sim 10 \varpi_{\text{flat}}$ , where the filaments merge with the surrounding ambient medium. The size of the flat-density region appears to have a uniform value  $\varpi_{\text{flat}} = (0.03 \pm 0.02)$  pc despite a variation of the central column density  $N_c$  of about 3 orders of magnitude, from  $\sim 10^{20}$  to  $\sim 10^{23}$  cm<sup>-2</sup> (Arzoumanian et al. 2011, hereafter A11).

A convenient parametrisation of the radial density profile that reproduces these basic features is the softened power-law profile

$$\rho(\varpi) = \frac{\rho_c}{[1 + (\varpi/\varpi_{\text{flat}})^2]^{\alpha/2}}, \quad (1)$$

where  $\rho_c$  is the central density and  $\alpha$  is a parameter. If  $\alpha = 4$ , eq. (1) is an exact solution of the equation of hydrostatic equilibrium for a self-gravitating isothermal cylin-

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der, hereafter referred to as the Stodólkiewicz–Ostriker density profile (Stodólkiewicz 1963, Ostriker 1964a). In this case  $\varpi_{\text{flat}} = (2a^2/\pi G\rho_c)$ , where  $a$  is the isothermal sound speed. The isothermal cylinder has infinite radius, but finite mass per unit length,

$$\mu_{\text{iso}} = \frac{2a^2}{G} = 16.5 \left( \frac{T}{10 \text{ K}} \right) M_{\odot} \text{ pc}^{-1}. \quad (2)$$

However, the power-law slope  $\alpha$  measured in a sample of filaments in the IC5146, Aquila and Polaris clouds, mapped by the *Herschel* satellite, is significantly different from  $\alpha = 4$ : on average,  $\alpha = 1.6 \pm 0.3$  (A11). Thus, the possibility that the gas in these filaments obeys a non-isothermal equation of state, and the implications of relaxing the hypothesis of thermal support, should be explored.

A fundamental difference between the behaviour of *isothermal* spherical and cylindrical interstellar clouds with respect to gravitational collapse was pointed out by McCrea (1957): while for a spherical cloud of given mass and temperature there is a maximum value of the external pressure for which an equilibrium state is possible (the Bonnor-Ebert criterion), a cylindrical cloud can be in equilibrium for any value of the external pressure, provided its mass per unit length is smaller than a maximum value. This led McCrea (1957) to conclude that filamentary (or sheet-like) clouds must first break up into fragments of roughly the same size in all directions before gravitational collapse (and therefore star formation) can take place. However, as we will argue in Sect. 3.1, filamentary clouds are unlikely to be thermally supported, and their radial density profiles are well reproduced by assuming an equations of state “softer” than isothermal. In this case, as shown by Viala & Horedt (1974a) the behaviour of cylindrical clouds with respect to gravitational instability becomes essentially analogous to that of spherical clouds (see Sect. 4).

The goal of this paper is to analyse the radial density profiles of filamentary clouds, their stability with respect to collapse, and to derive from the observed properties some conclusions on the relative importance of various mechanisms of radial support (or confinement) of these clouds. In particular, we analyse the stability of filamentary clouds following ideas explored in theoretical studies of spherical clouds by McKee & Holliman (1999) and Curry & McKee (2000), stressing the need for non-isentropic models to account for the observed large density contrasts. As in the case of spherical polytropes, the stability properties of polytropic cylinders depend on the polytropic exponent  $\gamma_p$ , that characterises the spatial properties of the filament, and on the adiabatic exponent  $\gamma$ , that determines the temporal response of the cloud to adiabatic perturbations.

The paper is organised as follows: in Sect. 2 we analyse the radial density profiles of filamentary clouds on the basis of polytropic cylindrical models; in Sect. 3 we compare the role of thermal and non-thermal pressure in supporting the cloud against its self-gravity; the stability to radial collapse of filaments of increasing mass per unit length under fixed external pressure is analysed in Sect. 4; finally, in Sect. 5 we summarise our conclusions. In this paper we focus on unmagnetised filaments. However, we consider particular forms of the equation of state that may simulate the effects of a large-scale or wavelike magnetic field on the cloud’s

structure. Magnetised models are presented in a companion paper (Toci & Galli 2014b, hereafter Paper II).

## 2 RADIAL DENSITY PROFILES OF FILAMENTARY CLOUDS

### 2.1 Basic equations

Neglecting magnetic fields, the structure and evolution of a self-gravitating filament is governed by the force equation

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla V - \frac{1}{\rho}\nabla p, \quad (3)$$

and Poisson’s equation

$$\nabla^2 V = 4\pi G\rho, \quad (4)$$

where  $\mathbf{u}$  is the gas velocity,  $V$  is the gravitational potential, and  $p$  is the gas pressure. The left-hand side term in eq. (3) represent the effects of dynamical motions on the momentum balance. These include the laminar and turbulent flows associated to the formation of the filament and/or produced by the gravitational field of the filament itself. In a cylindrical coordinate system with the  $z$  axis along the filament’s axis and the  $\varpi$  axis in the radial direction, assuming azimuthal symmetry ( $\partial/\partial\varphi = 0$ ), and neglecting rotation, the radial component of the left-hand side of eq. (3) reads

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = \left( u_{\varpi} \frac{\partial u_{\varpi}}{\partial \varpi} + u_z \frac{\partial u_{\varpi}}{\partial z} \right) \hat{\mathbf{e}}_{\varpi}. \quad (5)$$

The first term in eq. (5) represents a ram-pressure compressing the filament. If the filament is building mass by accretion from the surrounding medium, then  $u_{\varpi}$  is negative and decreases inward ( $u_{\varpi} = 0$  by symmetry on the filament’s axis). If  $u_{\varpi}$  becomes subsonic inside the filament, where  $\mathbf{u}$  is expected to be mostly parallel to the filament’s axis (as e.g. in the simulations of Gómez & Vázquez-Semadeni 2013), then the internal pressure dominates over the accretion ram pressure. Accretion ram-pressure can be neglected in the central parts of a filament, although it may play a role in the envelope<sup>1</sup>. Thus, a description of the structure of filamentary clouds in terms of hydrostatic equilibrium models does not necessarily imply that the velocity field is zero everywhere. Of course the velocity term in eq. (3) cannot be ignored during the growth of the varicose (or sausage) gravitational instability when significant radial and longitudinal gas flows can occur (see e.g. Gehman, Adams & Watkins 1996). These motions may lead to the formation of dense prestellar cores as observed e.g. in the SDC13 infrared dark cloud (Peretto et al. 2014).

### 2.2 Isothermal models

Observations of limited spatial extent of intensity profiles have been successfully modelled with isothermal cylinders.

<sup>1</sup> However, for a Larson-Penson type of accretion,  $u_{\varpi}$  approaches a constant value at large radii (Kawachi & Hanawa 1992) and the accretion ram-pressure drops to zero. The second term in eq. (5) is negligible if the accretion velocity  $u_{\varpi}$  does not change significantly along the filament, and vanishes in cylindrical symmetry ( $\partial/\partial z = 0$ ).

For example, radial density profiles derived from molecular line emission in L1517 (Hacar & Tafalla 2011), and from the 850  $\mu\text{m}$  emission in the filamentary dark cloud G11.11-0.12 (Johnstone et al. 2003) are compatible with the Stodółkiewicz-Ostriker density profile up to  $\sim 0.2$  pc. Fischera & Martin (2013b) have successfully modelled the surface brightness profiles of 4 filaments observed by *Herschel* in the IC5146 region with truncated isothermal cylinders, limiting their analysis to  $\sim 1'$  radial distance from the emission peak on both sides of the filament (corresponding to 0.13 pc at the distance of 460 pc). Over this radial extent, the column density profiles obtained by eq. (1) with  $\alpha = 2$  or  $\alpha = 4$ , or by a gaussian profile, are all indistinguishable (see e.g. Fig. 4 of A11). The large dynamic range allowed by the *Herschel* Space Observatory has made possible to map the sub-millimetre emission of interstellar filaments up to the radial distances from the filament's axis where the structures merge with the ambient medium ( $\sim 0.4$  pc for B211/213, Palmeirim et al. 2013;  $\sim 1$  pc for IC5112, A11). At large radii, deviation of the observed density profiles from the Stodółkiewicz-Ostriker profile become evident, and the observations are generally not well reproduced by isothermal cylinders. First, as already mentioned in Sect. 1, the density profiles at large radii are characterised by power-law exponents  $\alpha$  close to  $\sim 2$ , rather than 4; second, the mass per unit length is in some cases larger than the maximum value allowed for an isothermal cylinder (eq. 2). These aspects will be considered in the following sections.

### 2.3 Polytropic models

A more general class of hydrostatic models for filamentary clouds is represented by polytropic cylinders (Ostriker 1964a, Viala & Horedt 1974a), in which the gas pressure (arising from thermal or non-thermal motions) is parametrised by a polytropic equation of state,

$$p = K\rho^{\gamma_p}, \quad (6)$$

where  $\rho$  is the gas density,  $K$  a constant and  $\gamma_p$  is the polytropic exponent. The constant  $K$  is a measure of the cloud's entropy (for an isothermal gas  $K = a^2$ , where  $a$  is the isothermal sound speed). The polytropic exponent is usually written as  $\gamma_p = 1 + 1/n$ , where the polytropic index  $n$  can take values in the range  $n \leq -1$  or  $n > 0$  (the range  $-1 < n < 0$  corresponds to negative values of  $\gamma_p$  and is therefore unphysical). For  $0 \leq \gamma_p \leq 1$  ( $n \leq -1$ ) polytropic cylinders have infinite radii and infinite mass per unit length, whereas for  $\gamma_p > 1$  ( $n > 0$ ) the density and pressure become zero at some finite radius and therefore have finite masses per unit length. For  $\gamma_p = 1$  ( $n \rightarrow -\infty$ ) the gas is isothermal, whereas for  $\gamma_p \rightarrow 0$  ( $n = -1$ ) the equation of state becomes "logatropic",  $p \propto \ln \rho$ . This latter form was first used by Lizano & Shu (1989) to model the non-thermal support in molecular clouds associated to the observed supersonic line widths (see also McLaughlin & Pudritz 1996, 1997). Logatropic cylinders have infinite radius and infinite mass per unit length (Gehman et al. 1996, Fiege & Pudritz 2000). Negative index polytropes ( $0 \leq \gamma_p < 1$ ) were first proposed as models for thermally-supported interstellar clouds heated by an external flux of photons or cosmic rays (Viala 1972, Shu et al. 1972, de Jong et al. 1980). On the other hand, Maloney (1988) interpreted the polytropic

temperature  $T \propto (p/\rho)^{1/2}$  as a measure of the contribution of non-thermal (turbulent) motions to the support of the cloud. In this case negative index polytropes reproduce the observed increase of non-thermal line width with size observed in molecular clouds (McKee & Holliman 1999, Curry & McKee 2000).

With the equation of state (6), the equation of hydrostatic equilibrium eq. (3) with the advective term set equal to 0 reduces to the standard cylindrical Lane-Emden equation

$$\frac{1}{\xi} \frac{d}{d\xi} \left( \xi \frac{d\theta}{d\xi} \right) = \pm \theta^n, \quad (7)$$

for the non-dimensional density  $\theta$  and radius  $\xi$  defined by

$$\varpi = \varpi_0 \xi = \left[ \frac{\mp(1+n)K}{4\pi G \rho_c^{1-1/n}} \right]^{1/2} \xi, \quad \rho = \rho_c \theta^n. \quad (8)$$

In eq. (7) and (8), as well as in the rest of the paper, the upper (lower) sign is for  $0 \leq \gamma_p < 1$  ( $\gamma_p > 1$ ), and the subscripts "c" and "s" denote values at the center (axis of the cylinder) and at the surface of the filament, respectively. Numerical and analytical solutions of eq. (7) with boundary conditions  $\theta = 1$  and  $d\theta/d\xi = 0$  at  $\xi = 0$  have been obtained by Viala & Horedt (1974b) for  $0 < \gamma_p < 1$ , by Stodółkiewicz (1963) and Ostriker (1964a) for  $\gamma_p = 1$ , and by Ostriker (1964a) for  $\gamma_p > 1$ . The mass per unit length  $\mu$  is

$$\mu = 2\pi \int_0^{\varpi_s} \rho \varpi d\varpi = \mp \frac{(1+n)K \rho_c^{1/n}}{2G} \xi_s \theta'_s, \quad (9)$$

where eq. (7) has been used to simplify the integral.

In order to compare different models for the radial density profiles, it is necessary to normalise the radial coordinate  $\varpi$  to the same length scale. To the lowest order in a series expansion for small radii, the density profile of polytropic filaments is  $\rho(\varpi) \approx \rho_c(1 - \varpi^2/\varpi_{\text{core}}^2 + \dots)$ . The "core radius"  $\varpi_{\text{core}}$  is

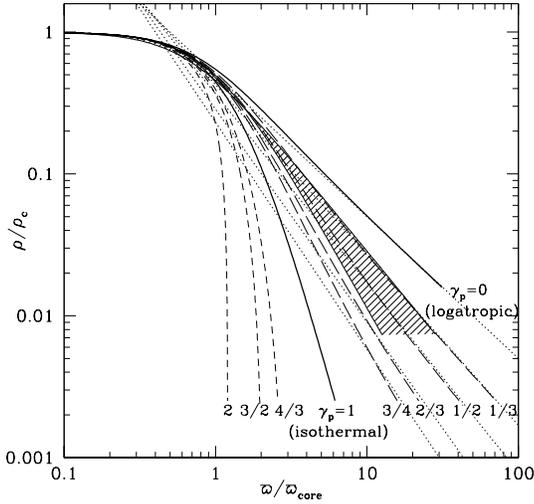
$$\varpi_{\text{core}} = \frac{2\varpi_0}{\sqrt{\mp n}} = \left( \frac{1+n}{n} \right)^{1/2} \frac{\sigma_c}{(\pi G \rho_c)^{1/2}}, \quad (10)$$

where  $\sigma_c = (p_c/\rho_c)^{1/2}$  is the velocity dispersion on the filament's axis<sup>2</sup>. For the observed value  $\sigma_c \approx 0.26 \text{ km s}^{-1}$  (Arzoumanian et al. 2013, hereafter A13), using the fiducial value  $n_c \approx 2 \times 10^4 \text{ cm}^{-3}$ , and setting  $n = -2$  the core radius is

$$\varpi_{\text{core}} \approx 0.047 \left( \frac{\sigma_c}{0.26 \text{ km s}^{-1}} \right) \left( \frac{n_c}{2 \times 10^4 \text{ cm}^{-3}} \right)^{-1/2} \text{ pc}. \quad (11)$$

Fig. 1 compares the density profiles of various cylindrical polytropes of positive and negative index, as function of radius normalised to  $\varpi_{\text{core}}$ . The longitudinally averaged density profiles of the filaments in IC5146, given by eq. (1) with  $\alpha = 1.6 \pm 0.3$  (A11), are well reproduced by cylindrical polytropes with  $1/3 \lesssim \gamma_p \lesssim 2/3$  ( $-3 \lesssim n \lesssim -3/2$ ) at least over the observed radial extent of the filaments (from  $\varpi \approx 0.1 \varpi_{\text{core}}$  to  $\varpi \approx 10 \varpi_{\text{core}}$ ). Overall, the single value

<sup>2</sup> A comparison with an analogous series expansion of the softened power-law profile (eq. 1) leads to the identification  $\varpi_{\text{core}} = (2/\alpha)^{1/2} \varpi_{\text{flat}}$ . Since  $\alpha \approx 2$ ,  $\varpi_{\text{core}} \approx \varpi_{\text{flat}}$ .



**Figure 1.** Radial density profiles (normalised to the central density  $\rho_c$ ) of polytropic cylinders with  $\gamma_p = 2, 3/2, 4/3$  ( $n = 1, 2$  and  $3$ , *short-dashed* lines, from left to right) and  $\gamma_p = 1/3, 1/2, 2/3$  and  $3/4$  ( $n = -3/2, -2, -3$  and  $-4$ , *long-dashed* lines, from right to left). The *thick solid* lines show the density profiles of an isothermal ( $\gamma_p = 1$ , or  $n = \pm\infty$ ) and a logatropic ( $\gamma_p = 0$ , or  $n = -1$ ) cylinder. *Dotted* lines are the singular solutions given by eq. (12). The radius is normalised to the core radius  $\varpi_{\text{core}}$  defined by eq. (10). The hatched area corresponds to the observed mean density profile of filaments in IC5146, given by eq. (1) with  $\alpha = 1.6 \pm 0.3$ .

$\gamma_p \approx 1/2$  ( $n = -2$ ) provides a good fit to the data, at least for this sample of filaments. The implications of these results are discussed in Sect. 3.2.

#### 2.4 Power-law behaviour at large radius

In addition to regular solutions, eq. (7) also allows singular (or scale-free) solutions for  $0 \leq \gamma_p < 1$  ( $n < -1$ ), characterised by a power-law behaviour intermediate between  $\rho \propto \varpi^{-1}$  (for  $\gamma_p = 0$ ) and  $\rho \propto \varpi^{-2}$  (for  $\gamma_p \rightarrow 1$ ), given by

$$\rho(\varpi) = \left[ \frac{(1-n)^2 \pi G}{-(1+n)K} \right]^{n/(1-n)} \varpi^{2n/(1-n)}, \quad (12)$$

(Viala & Horedt 1974a). The mass per unit length of the scale-free models is

$$\begin{aligned} \mu(\varpi) &= (1-n)\pi \left[ \frac{(1-n)^2 \pi G}{-(1+n)K} \right]^{n/(1-n)} \varpi^{2/(1-n)} \\ &= (1-n)\pi \varpi^2 \rho(\varpi), \end{aligned} \quad (13)$$

and approaches the constant value  $\mu \rightarrow a^2/G$  if  $\gamma_p \rightarrow 1$ .

The scale-free solutions are plotted in Fig. 1 along with the regular solutions. As shown by Fig. 1, scale-free solutions represent the asymptotic behaviour around which the regular solutions oscillate with decreasing amplitude for  $\varpi \rightarrow \infty$ . This asymptotic behaviour is the same for both polytropic spheres and cylinders. However, while a spherical singular solution exists also for  $\gamma_p = 1$  (the singular isothermal sphere), this does not happen in cylindrical geometry<sup>3</sup>. In fact, whereas for spheres the amplitude of the oscillatory component decreases as  $r^{-1/2}$  for  $\gamma_p = 1$ , for cylinders it approaches instead a constant value for  $\gamma_p \rightarrow 1$ , and the period of the oscillation becomes infinite: the isothermal cylinder converges to the singular solution eq. (12) only at infinite radius. Thus, if a quasi-isothermal filamentary cloud goes through an evolutionary stage independent of the initial and boundary conditions, yet still far from the ultimate equilibrium state (an intermediate asymptotic, Barenblatt 1979), a radial density profile closer to  $\rho \propto \varpi^{-2}$  rather than  $\rho \propto \varpi^{-4}$  should be expected. This happens, for example, in the self-similar collapse solutions of quasi-isothermal filaments by Kawachi & Hanawa (1998).

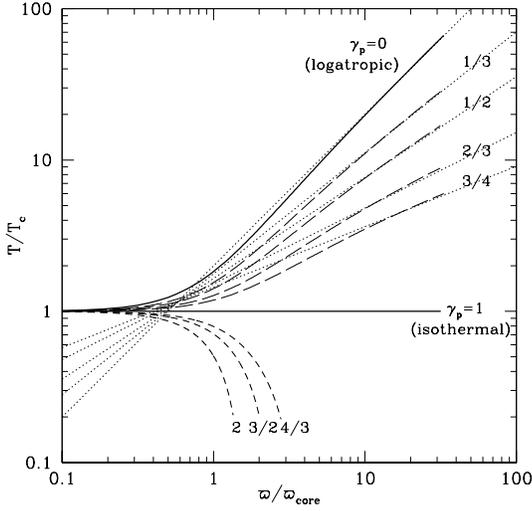
### 3 SUPPORT AGAINST GRAVITY

#### 3.1 Thermal support

The deviations of the observed radial behaviour of the density from an isothermal Stódkiewicz-Ostriker profile has been interpreted as an indication of temperature gradients increasing outwards, resulting in a larger thermal pressure gradient with respect to an isothermal gas (Recchi, Hacar & Palestini 2013). This possibility is supported by the presence of significant radial gradients in the dust temperature profiles derived from radiative transfer modelling of the infrared emission (see e.g. Stepnik et al. 2003). In particular, the dust temperature  $T_d$  increases outward from  $\sim 10$ – $12$  K on the axis to  $\sim 14$  K at  $\varpi \approx 0.5$  pc in the B211 filament (Palmeirim et al. 2013), and to  $\sim 18$  K in the L1506 filament (Ysard et al. 2013). Similar (or larger) gradients are expected in the gas temperature  $T_g$  as well: in fact, while some mild coupling of the dust and gas temperatures is possible at the typical densities on the filament’s axis ( $\sim 10^4$  cm<sup>-3</sup>), in general the gas is expected to be significantly hotter than the dust in the outer regions (see e.g. Galli, Walmsley & Gonçalves 2002; Keto & Caselli 2008).

The polytropic models presented in Sect. 2.3 make possible to quantify the magnitude of the gradient in the gas temperature needed to reproduce the observed density profiles. Fig. 2 shows the radial behaviour of polytropic temperature  $T \propto (p/\rho)^{1/2}$  for the same models shown in Fig. 1. For the range of polytropic exponents that reproduce the density profiles of filaments in IC5146, the polytropic temperature increases by a factor  $\sim 5$ – $12$  from the filament’s axis to the boundary, assumed to be located at  $\rho_s \approx 10^{-2} \rho_c$  or  $\varpi_s \approx 10 \varpi_{\text{core}}$  (corresponding to a radius of about 1 pc, where the filaments merge with the ambient medium). If the polytropic temperature is identified with the gas temperature, this implies a temperature at the filament’s surface

<sup>3</sup> Conversely, for a logatropic equation of state, a singular solution exist in cylindrical geometry but not in spherical geometry.



**Figure 2.** Radial profiles of the polytropic temperature  $T$  (normalised to the central temperature value  $T_c$ ) of polytropic cylinders with values of  $\gamma_p$  (or  $n$ ) as in Fig. 1. The *thick solid* lines show the temperature profiles of an isothermal ( $\gamma_p = 1$ , or  $n = \pm\infty$ ) and a logatropic ( $\gamma_p = 0$ , or  $n = -1$ ) cylinder. *Dotted* lines correspond to the singular solutions given by eq. (12). The radius is normalised to the core radius  $\varpi_{\text{core}}$  as in Fig. 1

$T_s \sim 70\text{--}170$  K, assuming a central temperature  $T_c = 14$  K (A11). Such high temperatures are very unlikely. The observed gradients of gas temperature in prestellar cores are much shallower (Crapsi et al. 2007), in agreement with the predictions of theoretical models. Therefore, alternatives to thermal pressure must be sought.

### 3.2 Non-thermal support

Turbulence and magnetic fields, either large-scale or wave-like, can contribute to the pressure supporting the filament. If approximated as isotropic pressure components, their effects can be modelled with appropriate polytropic laws. For example, in the limit of small amplitude, small wavelength, and negligible damping, Alfvén waves behave as a polytropic gas with  $\gamma_p = 1/2$  (Walén 1944), a value consistent with the observations, as shown in Sect. 2.3. Thus, the filamentary clouds observed by *Herschel* may be supported radially by non-thermal motions associated to Alfvénic “turbulence”, i. e. a superposition of hydromagnetic waves (Fatuzzo & Adams 1993, McKee & Zweibel 1995).

If small-amplitude Alfvén waves (modelled with a  $\gamma_p = 1/2$  polytropic law) dominate the pressure, observed molecular transitions should be characterised by a non-thermal

line width increasing roughly by a factor  $\sim 3$  from the axis to the filament boundary, following approximately a  $\rho^{-1/4}$  (or  $\varpi^{1/3}$ ) dependence at large radial distances. However the available data do not allow any firm conclusion to be drawn on the magnitude and spatial distribution of non-thermal motions inside filamentary clouds. Hacar & Tafalla (2011) find that in L1517 the non-thermal line width of molecular transitions like  $\text{C}^{18}\text{O}$  and  $\text{SO}$  is everywhere subsonic ( $\sigma_{\text{nt}} < a$ ) and very uniform, typically  $\sigma_{\text{nt}} = 0.1 \pm 0.04$  km s $^{-1}$  across the sampled region. Li & Goldsmith (2012) find that the velocity dispersion on the axis of the B213 filament is slightly supersonic ( $\sigma_{\text{nt}} \approx 0.3$  km s $^{-1}$ ). Millimetre line studies indicate that self-gravitating filaments have intrinsic, suprathreshold linewidths  $\sigma_{\text{nt}} \gtrsim a$  (A13). In the massive filament DR21 ( $N_c \approx 10^{23}$  cm $^{-2}$ ,  $\mu \approx 4 \times 10^3 M_\odot$  pc $^{-1}$ ) Schneider et al. (2010) find that the velocity dispersion increases towards the filament’s axis, where it reaches  $\sigma_{\text{nt}} \approx 1$  km s $^{-1}$  (see their Fig. 18), whereas condensations in the filaments are characterised by lower velocity dispersions. Further observations should explore the spatial distribution of non-thermal motions in filamentary clouds and the correlation (if any) of  $\sigma_c$  with  $\rho_c$ .

As shown in Sect. 2, negative-index cylindrical polytropes with appropriate values of  $\gamma_p$  reproduce the observed radial density profiles of filaments and predict a core radius  $\varpi_{\text{core}} \propto \sigma_c / \rho_c^{1/2}$ . This result is consistent with the observed uniformity of filament widths if  $\sigma_c$  scales as the square root of the central density,  $\sigma_c \propto \rho_c^{1/2}$ . Since the central column density is  $N_c \propto \rho_c \varpi_{\text{core}}$ , it follows that  $\sigma_c \propto N_c^{1/2}$ , since  $\varpi_{\text{core}}$  is constant. Observationally, filaments with central column densities above  $\sim 10^{22}$  cm $^{-2}$  follow this trend (A13). Theoretically, the relation  $\sigma \propto \rho^{1/2}$  seems to characterise the behaviour of the turbulent pressure during the relaxation processes leading to virialization in a strongly self-gravitating collapse flow, according to the numerical simulations of Vázquez-Semadeni, Cantó & Lizano (1998). This could be an indication that, at least in the more massive filaments, the gas in the central parts is still undergoing turbulent dissipation (perhaps following accretion, Hennebelle & André 2013). Numerical simulations and analytic considerations show that the polytropic exponent of magnetohydrodynamic turbulence depends on the dominant wave mode via the Alfvén Mach number  $M_A$ , ranging from  $\gamma_p \approx 1/2$  at low  $M_A$ , where the slow mode dominates, to  $\gamma_p \approx 2$  at large  $M_A$ , where the slow and fast mode are comparable (Passot & Vázquez-Semadeni 2003). Thus, a picture of magnetohydrodynamic turbulence in terms of small-amplitude Alfvén waves is clearly an oversimplification.

## 4 RADIAL STABILITY OF POLYTROPIC FILAMENTS

Cylindrical polytropes are known to be unstable to longitudinal perturbations of wavelength larger than some critical value. This varicose (or sausage) gravitational instability (Ostriker 1964b; Larson 1985; Inutsuka & Miyama 1992; Freundlich, Jog & Combes 2014) and its magnetic variant (Nagasawa 1987; Nakamura et al. 1993; Gehman et al. 1996; Fiege & Pudritz 2000b) produces the fragmentation of a filamentary cloud in a chain of equally spaced dense cores, as observed in some cases, and represents therefore a promising

mechanism for star formation. However, it is important to assess first the conditions for radial stability. i.e. with respect to collapse to a line mass, in analogy with the Bonnor-Ebert stability criterion for spherical polytropes. For observed filaments, stability considerations are usually based on a comparison with the mass per unit length of the isothermal cylinder,  $\mu_{\text{iso}}$  (eq. 2). However, as mentioned in the Introduction, the stability properties of an isothermal cylinder are different from those of polytropic cylinders with  $\gamma_p < 1$ , essentially because its mass per unit length approaches the finite value  $\mu^{\text{iso}}$  as the radius of the cylinder increases to infinity, whereas for  $\gamma_p < 1$  the mass increases with radius. As a consequence, an isothermal filament is always radially stable: if the pressure  $p_s$  exerted over an isothermal cylinder with fixed  $\mu < \mu_{\text{iso}}$  is gradually increased, the filament contracts, reducing its radius  $\varpi_s$  and core radius  $\varpi_{\text{core}}$  as  $p_s^{-1/2}$ , and increasing its central density  $\rho_c$  as  $p_s^{-1}$ , but otherwise maintaining the same shape of the density profile. Conversely, for  $0 \leq \gamma_p < 1$ , also cylindrical polytropes become unstable if the external pressure becomes larger than some critical value. The instability extends to  $\gamma_p = 4/3$  for spheres (the classical Bonnor-Ebert instability) but not for cylinders.

The stability of polytropic cylindrical clouds to radial perturbations can be determined by solving the equation of radial motion for small perturbations about equilibrium, first derived for spherical clouds by Eddington (1926). For cylindrical clouds it becomes (Breyse, Kamionkowski & Benson 2014)

$$\frac{d^2 h}{d\varpi^2} + \frac{3 - 4q}{\varpi} \frac{dh}{d\varpi} + \left[ \frac{\omega^2}{f^2} + 8 \left( \frac{1}{\gamma} - 1 \right) q \right] \frac{h}{\varpi^2} = 0, \quad (14)$$

where  $h = \delta\varpi/\varpi$  is the relative amplitude of the perturbation,  $\omega$  is the frequency of the oscillations,  $\gamma$  is the adiabatic exponent, and we have defined

$$q \equiv \frac{G\mu\rho}{2p} = -\frac{(1+n)\xi\theta'}{4\theta}, \quad (15)$$

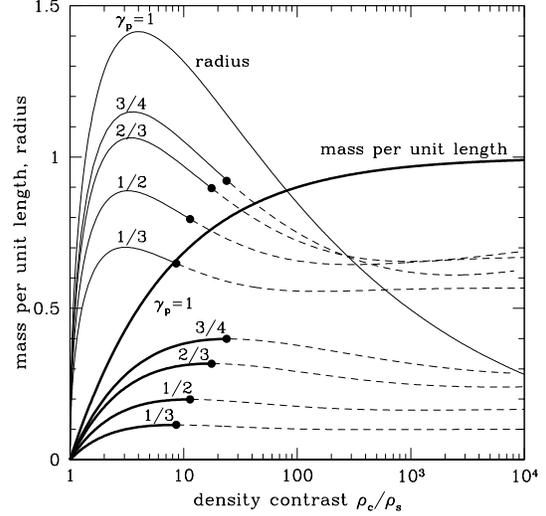
and

$$f \equiv \frac{1}{\varpi} \left( \frac{\gamma p}{\rho} \right)^{1/2} = \frac{(4\pi G \rho_c)^{1/2}}{\xi} \left( \frac{\mp \gamma \theta}{1+n} \right)^{1/2}. \quad (16)$$

In deriving eq. (14), the simplifying assumption has been made that the perturbations occur adiabatically,  $\delta p/p = \gamma \delta \rho/\rho$ . It is important to notice that the adiabatic exponent  $\gamma$  determining the response of the cloud to small perturbations is not necessarily equal to the polytropic exponent characterising the equilibrium structure discussed in Sec. 2. Only if the perturbation occurs on a time much longer than the characteristic time for internal redistribution of entropy, the adiabatic exponent  $\gamma$  is equal to  $\gamma_p$  (see examples and discussion in Sect. 4.2).

#### 4.1 Isentropic filaments

We first consider isentropic clouds, in which the entropy is both spatially uniform and constant during an adiabatic perturbation, and set  $\gamma = \gamma_p$ . To determine the condition of marginal stability, we set  $\omega = 0$  and we solve eq. (14) with the boundary condition  $dh/d\xi = 0$  at  $\xi = 0$  in order for  $h$  to remain finite on the axis (since eq. 14 is linear and homogeneous, the value of  $h$  at  $\xi = 0$  is arbitrary). For



**Figure 3.** Mass per unit length  $\mu$  (thick curves) and radius  $\varpi_s$  (thin curves) of cylindrical polytropes bounded a fixed external pressure as function of the density contrast  $\rho_c/\rho_s$ . The cases shown are, from bottom to top,  $\gamma_p = 1/3, 1/2, 2/3, 3/4$  and 1 ( $n = -1.5, -2, -3, -4$  and  $\infty$ ). Dots on each curve indicate critical points. The stable and unstable parts of each sequence are shown by *solid* and *dashed* curves, respectively. The radius  $\varpi_s$  is in units of  $[p_s/(4\pi G \rho_s^2)]^{1/2}$ , the mass per unit length  $\mu$  in units of  $2p_s/G\rho_s$ .

any fixed value of the polytropic exponent  $\gamma_p$ , the critical point  $\xi_{\text{cr}}$  can be found determining the radius at which the Lagrangian variation in the pressure at the boundary vanishes,

$$\left( \frac{\delta p}{p} \right)_{\xi=\xi_{\text{cr}}} = -\gamma \left( 2h + \varpi \frac{dh}{d\varpi} \right)_{\xi=\xi_{\text{cr}}} = 0. \quad (17)$$

If  $\xi > \xi_{\text{cr}}$ , the filament is unstable to radial collapse. At the critical point the density contrast is  $(\rho_c/\rho_s)_{\text{cr}} = \theta_{\text{cr}}^{-n}$  and the mass per unit length is

$$\mu_{\text{cr}} = q_{\text{cr}} \left( \frac{2p_s}{G\rho_s} \right), \quad (18)$$

where

$$q_{\text{cr}} = -\frac{(1+n)\xi_{\text{cr}}\theta'_{\text{cr}}}{4\theta_{\text{cr}}}. \quad (19)$$

The values of  $\xi_{\text{cr}}$ ,  $(\rho_c/\rho_s)_{\text{cr}}$ , and  $q_{\text{cr}}$  for different polytropes are listed in Table 1. For the same value of the ratio  $p_s/\rho_s$ , the marginally stable configuration with the largest mass per unit length is the isothermal filament with  $\gamma_p = 1$ , for which  $q_{\text{cr}} = 1$  and  $\mu_{\text{cr}} = \mu_{\text{iso}}$ . At the opposite end, the logatropic

**Table 1.** Critical points for isentropic cylindrical polytropes.

$n$	$\gamma_p$	$\xi_{cr}$	$(\rho_c/\rho_s)_{cr}$	$q_{cr}$
-1	0	6.62	6.05	0
-1.01	0.0099	6.59	6.10	0.0272
-1.5	1/3	5.52	8.61	0.115
-2	1/2	4.93	11.4	0.199
-3	2/3	4.28	17.6	0.317
-4	3/4	3.92	23.9	0.399
-5	0.8	3.68	32.8	0.459
-10	0.9	3.13	80.0	0.626
-20	0.95	2.76	228	0.752
-30	0.967	2.60	441	0.812
-40	0.975	2.50	701	0.846
$-\infty$	1	$\infty$	$\infty$	1

filament with  $\gamma_p = 0$  has  $q_{cr} = 0$ . Thus, for fixed values of the surface pressure and density, filaments with increasingly “softer” equations of state can support less and less mass per unit length, as in the case of spherical polytropes (McKee & Holliman 1999).

Fig. 3 shows the radius and the mass per unit length of cylindrical polytropes with various values of  $\gamma_p$  between  $\gamma_p = 1/3$  and 1 (from  $n = -3/2$  to  $-\infty$ ) as function of  $\rho_c/\rho_s$  and the position of the critical point on both sets of curves. The stability properties of polytropic filaments with  $0 \leq \gamma_p < 1$  for the same value of the entropy parameter  $K$  and the ratio  $p_s/\rho_s$  are qualitatively similar: increasing  $\rho_c/\rho_s$  the filament first expands then contracts, until the filament becomes unstable when  $\rho_c/\rho_s$  becomes larger than the critical value listed in Table 1. Equilibria also exist above this critical value, but they are unstable to radial collapse. The instability occurs for increasingly larger values of  $\rho_c/\rho_s$  when  $\gamma_p$  increases (for  $\gamma_p = 1$ , the critical point is at  $\xi_{cr} = \infty$ ).

The problem here is that several filamentary clouds observed by *Herschel* have mass per unit length in excess of  $\mu_{iso}$ , if the latter is computed with  $a^2$  corresponding to the measured central temperature  $\sim 10$  K. Although the fact that prestellar cores are preferentially found in filaments with  $\mu > \mu_{iso}$  is considered a signature of gravitational instability (see, e.g., André et al. 2010), it is difficult to justify the formation by accretion (or by other processes) of isothermal filaments with mass per unit line larger than  $\mu_{iso}$ . Consider for example the evolution of an isothermal filament with  $\mu < \mu_{iso}$ , bounded by an external constant pressure  $p_s = a^2 \rho_s$ , slowly increasing its mass per unit length while keeping its temperature uniform and constant with time. As  $\mu$  increases, the filament becomes more and more centrally condensed, its density contrast  $\rho_c/\rho_s$  increasing as  $(1 - \mu/\mu_{iso})^{-2}$ . At the same time, the flat core region shrinks as  $(1 - \mu/\mu_{iso})$ , and the outer radius first expands then contracts as  $[(1 - \mu/\mu_{iso})(\mu/\mu_{iso})]^{1/2}$  (Fischera & Martin 2013a). As  $\mu \rightarrow \mu_{iso}$ , the filament approaches a delta-like line mass of zero radius and infinite density on the axis. During this evolution the filament is subject to the varicose instability and can fragment into a chain of cores, but can never reach a stage with  $\mu > \mu_{cr}$ . This is not the case for non-isothermal filaments. In fact, it is reasonable to expect that in actual filaments the ratio  $p_s/\rho_s$  in eq. (18) is much larger than  $a^2$ , the value for isothermal gas. If filamentary clouds are pres-

sure confined,  $p_s$  must be equal to the pressure exerted on the filament by the surrounding intercloud medium, where turbulent motions are likely to dominate the pressure.

## 4.2 Non-isentropic filaments

If filamentary clouds are well described by cylindrical polytropes with  $1/3 \lesssim \gamma_p \lesssim 2/3$  as shown in Sect. 2, their density contrast cannot be larger than  $\rho_c/\rho_s = 8.61\text{--}17.6$  (see Table 1) or they would collapse to a line mass. However, the observations summarised in Sect. 2 indicate that the density contrasts measured by *Herschel* are of the order of  $\sim 100$ . As in the case of spherical clouds, this limitation is alleviated if the cloud is non-isentropic ( $\gamma \neq \gamma_p$ ).

While the assumption of isentropy has been made in most studies of polytropes, McKee & Holliman (1999) and Curry & McKee (2000) showed that it is not generally valid for molecular clouds. In fact, a significant contribution to the pressure supporting the cloud against its self-gravity may be provided by non-thermal components whose behaviour is not isentropic: for example, small-amplitude Alfvén waves have  $\gamma_p = 1/2$  and  $\gamma = 3/2$  (McKee & Zweibel 1995). In general, non-isentropic polytropes remain stable for larger density contrasts than isentropic clouds. In practice, the analysis must be limited to values of  $\gamma > \gamma_p$  since polytropes with  $\gamma < \gamma_p$  are convectively unstable according to the Schwarzschild criterion.

To obtain the critical point  $\xi_{cr}$  of non-isentropic cylindrical polytropes, eq. (14) is solved for a fixed  $\gamma_p$  and arbitrary  $\gamma > \gamma_p$ . The results are shown in Table 2, listing the values of  $\xi_{cr}$ ,  $(\rho_c/\rho_s)_{cr}$  and  $q_{cr}$  for polytropes with  $\gamma_p = 1/3$ ,  $1/2$  and  $2/3$  for various values of  $\gamma$ . As for the case of spherical polytropes, the critical points moves to larger and larger values of the density contrast  $\rho_c/\rho_s$  as  $\gamma$  increases. At a threshold value  $\gamma_\infty$ , the critical point reaches  $\xi_{cr} = \infty$  and the density profile approaches that of a singular polytropic cylinder. The value of  $q_{cr} = q_\infty$  at this point can be easily determined substituting eq. (12) into eq. (14),

$$q_\infty = \frac{\gamma_p}{2(2 - \gamma_p)}. \quad (20)$$

The threshold value of the adiabatic exponent,  $\gamma_\infty$  can also be found analytically. For a singular polytropic cylinder, eq. (14) with  $\omega = 0$  has constant coefficients, and the characteristic equation has two real and negative roots if  $\gamma$  is larger than

$$\gamma_\infty = \gamma_p(2 - \gamma_p), \quad (21)$$

corresponding to  $h$  exponentially decreasing with  $\xi$ . The values of  $q_\infty$  and  $\gamma_\infty$  for  $\gamma_p = 1/3$ ,  $1/2$  and  $2/3$  are also listed in Table 2. Non-isentropic polytropes are more stable than their isentropic counterparts as they can support larger centre-to-surface density contrasts. For  $\gamma > \gamma_\infty$ , polytropic filaments are unconditionally stable for any  $\rho_c/\rho_s$ .

Fig. 4 summarises the stability properties of cylindrical polytropes in the  $\gamma_p\text{--}\gamma$  plane. In cylindrical geometry the polytropic exponent  $\gamma_p = 1$  is a critical value that plays the same role of  $\gamma_p = 4/3$  for spherical polytropes: while spheres with  $\gamma_p > 4/3$  are unconditionally stable to small perturbations, cylinders become stable already for  $\gamma_p > 1$  (McCrea 1957, Larson 2005). The analysis presented in this Section extends the study of the gravitational instability to

**Table 2.** Stability of non-isentropic cylindrical polytropes.

$\gamma$	$\gamma_p = 1/3$		
	$\xi_{cr}$	$(\rho_c/\rho_s)_{cr}$	$q_{cr}$
1/3	5.52	8.61	0.115
0.4	10.3	20.2	0.111
$\gamma_\infty = 5/9$	$\infty$	$\infty$	$q_\infty = 1/10$

$\gamma$	$\gamma_p = 1/2$		
	$\xi_{cr}$	$(\rho_c/\rho_s)_{cr}$	$q_{cr}$
1/2	4.93	11.4	0.199
0.6	10.7	38.0	0.188
0.7	124	1096	0.163
$\gamma_\infty = 3/4$	$\infty$	$\infty$	$q_\infty = 1/6$

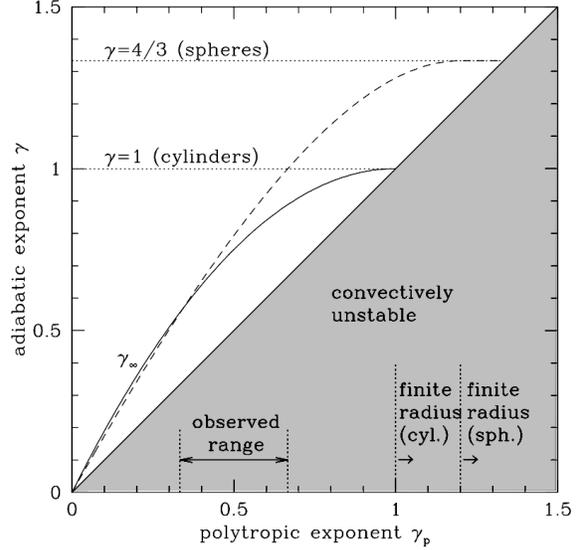
  

$\gamma$	$\gamma_p = 2/3$		
	$\xi_{cr}$	$(\rho_c/\rho_s)_{cr}$	$q_{cr}$
2/3	4.28	17.6	0.317
0.7	5.18	25.2	0.316
0.8	14.1	153	0.282
$\gamma_\infty = 8/9$	$\infty$	$\infty$	$q_\infty = 1/4$

non-isentropic clouds determining the threshold value  $\gamma_\infty$  for the stability of polytropic cylinders as function of the polytropic exponent  $\gamma_p$ . For example, for  $\gamma_p = 1/2$ , a value consistent with the observed radial density profiles of filamentary clouds, the threshold value for stability (from eq. 21) is  $\gamma_\infty = 3/4$ . Notice that for a “soft” equation of state the stability condition is about the same for cylinders and spheres: for  $\gamma_p \ll 1$ , a first-order approximation gives  $\gamma_\infty \approx 2\gamma_p$  for cylinders and  $\gamma_\infty \approx (16/9)\gamma_p$  for spheres. For larger values of  $\gamma_p$ , cylinders are intrinsically more stable than spheres in the  $\gamma_p$ - $\gamma$  plane. A pressure-bounded isothermal cylinder, for example, is always stable with respect to an arbitrary increase in the external pressure, whereas an isothermal sphere is not. For the range of polytropic exponents allowed by the observations of the radial density profiles ( $1/3 \lesssim \gamma_p \lesssim 2/3$ , see Sect. 2), the stability properties of cylindrical and spherical clouds are very similar.

## 5 CONCLUSIONS

The typical core-envelope structure and the uniformity of the observed properties of filamentary molecular clouds suggests that their main physical characteristics can be analysed with polytropic models in cylindrical symmetry. Isothermal models fail to reproduce the observed power-law behaviour of the density at radii larger than the core radius, and cannot explain the existence of filaments with mass per unit length larger than the limiting value for an isothermal cylinder. Conversely, the observed radial density profiles of filamentary clouds are well reproduced by negative-index cylindrical polytropes with  $1/3 \lesssim \gamma_p \lesssim 2/3$  ( $-3 \lesssim n \lesssim -3/2$ ) indicating either outward-increasing temperature gradients, or the presence of a dominant non-thermal contribution to the pressure. In the former case, however, the predicted gas temperature at the filament’s surface would be unrealistically high. Non-thermal support, perhaps in the form of a superposition of small-amplitude Alfvén waves (for which  $\gamma_p = 1/2$ ) is an attractive possibility. In addition, the mass per unit length of negative-index



**Figure 4.** Stability properties of cylindrical and spherical polytropes in the  $\gamma_p$ - $\gamma$  plane. Polytropes in the  $\gamma < \gamma_p$  region (shaded) are convectively unstable. On the line  $\gamma = \gamma_p$ , polytropic cylinders (spheres) are isentropic, and become unstable at some finite  $(\rho_c/\rho_s)_{cr}$  if  $\gamma < 1$  ( $\gamma < 4/3$ ). Above the curve labelled  $\gamma_\infty$  (dashed for spheres) cylindrical polytropes are unconditionally stable even for  $\rho_c/\rho_s = \infty$ . Cylindrical (spherical) polytropes have finite radii for  $\gamma_p > 1$  ( $\gamma_p > 6/5$ ) as indicated by the vertical dotted lines. The stability properties of spherical polytropes are from McKee & Holliman (1999).

polytropes is not limited, but depends on the pressure and density at the surface, if the filaments are pressure confined by the ambient medium.

Negative-index cylindrical polytropes have uniform width, as observed, if the central velocity dispersion  $\sigma_c$  is proportional to the square root of the central density  $\rho_c$  (or the central column density  $N_c$ ), a relation that seems to be satisfied at least by the most dense filaments (A13) and has been found in numerical simulations of self-gravitating collapse flows (Vázquez-Semadeni et al. 1998).

Outside the core radius, the density profile of polytropic filaments has often a power-law behaviour and carries important information on the cloud’s thermodynamics and equation of state. Irrespective of geometry, both spherical and cylindrical polytropes converge at large radii to the same power-law behaviour in radius with a slope equal to  $-2/(2 - \gamma_p)$ , that approaches  $-2$  for a quasi isothermal gas. However, for cylinders, this power-law behaviour is approached at increasingly larger radii for  $\gamma_p \rightarrow 1$  (at infinite radius for  $\gamma_p = 1$ ).

Pressure-bounded polytropic cylinders with  $1/3 \lesssim \gamma_p \lesssim 2/3$  can support a mass per unit length as large as observed depending on the conditions at the surface. However, their density contrast cannot be larger than about a factor of 10–20 if they are isentropic. Like their spherical counterparts (McKee & Holliman 1999), non-isentropic cylinders remain stable at larger density contrasts (in principle even infinite)

with respect to adiabatic pressure perturbations. Since magnetic fields and turbulence (modelled here in the very simplified framework of Alfvén waves) behave as non-isentropic pressure components, isentropic (and, in particular, isothermal) models are inadequate to study the structure and the stability properties of filamentary clouds.

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