



IECMSA - 2020

**9th INTERNATIONAL EURASIAN CONFERENCE
ON MATHEMATICAL SCIENCES AND APPLICATIONS**

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Book of Abstracts

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*9TH INTERNATIONAL
EURASIAN CONFERENCE
ON
MATHEMATICAL SCIENCES
AND
APPLICATIONS*



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FOREWORD

By

Prof. Dr. Murat TOSUN,

On behalf of the Organizing Committee

It is my great pleasure and honour to welcome you at the 9th International Eurasian Conference on Mathematical Sciences and Applications (IECMSA-2020) which has been organized in cooperation with Sakarya University and International Balkan University.

Unfortunately, in 2020 humanity has faced an unusual, dangerous challenge connected with the new COVID-19 and one impact of this virus has placed constraints on the ability of researchers to join a face-to-face meeting. As the health and safety of everyone is our priority, IECMSA will proceed with our annual gathering this year through a virtual conference, instead of an in-person event. The decision to hold IECMSA-2020 as a virtual conference on the original dates has appeared preferable to a postponed meeting face to face in Skopje, especially during this uncertain time. Thus, the virtual conference format will allow us to present our studies whilst still providing many of the benefits of a face to face meeting. Besides, virtual presentations will be more widely available, yielding a greater exposure to our studies.

Established since 2012, the series of IECMSA features the latest developments in the field of mathematics and applications. The previous conferences were held as follows: IECMSA-2012, Prishtine, Kosovo, IECMSA-2013, Sarajevo, Bosnia and Herzegovina, IECMSA-2014, Vienna, Austria, IECMSA-2015, Athens, Greece, IECMSA-2016, Belgrade, Serbia, IECMSA-2017, Budapest, Hungary, IECMSA-2018, Kyiv, Ukraine, and IECMSA-2019, Baku, Azerbaijan. These conferences gathered a large number of international world-renowned participants.

Now in IECMSA-2020, the scientific committee members and the external reviewers invested significant time in analyzing and assessing multiple papers, consequently, they hold and maintain a high

standard of quality for this conference. The scientific committee accepted 116 virtual presentations. Despite the effects of coronavirus, 136 participants are attending the conference from 23 different countries. The scientific program of the conference features keynote talks, followed by contributed presentations in two parallel sessions.

The conference program represents the efforts of many people. I would like to express my gratitude to all members of the scientific committee, external reviewers, sponsors and, honorary committee for their continued support to the IECMSA. I also thank the invited speakers for presenting their talks on current researches. Also, the success of IECMSA depends on the effort and talent of researchers in mathematics and its applications that have written and submitted papers on a variety of topics. So, I would like to sincerely thank all participants of IECMSA-2020 for contributing to this great meeting in many different ways. I believe and hope that each of you will get the maximum benefit from the conference.

Wish you all health and safety during this difficult time

Prof. Dr. Murat TOSUN

Chairman

On behalf of the Organizing Committee



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INVITED SPEAKERS



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GEOMETRIC DESIGN OF PATTERNS IN TRADITIONAL TURKISH ARCHITECTURE

Bayram Şahin ¹

ABSTRACT. In this talk, geometric structures of patterns in traditional Turkish architecture are presented. For this purpose, types of such patterns are examined and the construction process is given. In addition, the transformations used in the construction of such patterns and their relations with symmetry groups are discussed. An example of each type of patterns in traditional Turkish architecture is given and the building process is built with the support of the computer.

Keyword: Islamic ornament, traditional turkish architecture, symmetry group, geometric algorithm.

AMS 2010: 68U07

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STABILIZATION OF CONTROL SYSTEMS

Jean-Michel Coron ¹

ABSTRACT. A control system is a dynamical system that can be acted upon using controls. For these systems, a fundamental problem is the question of stabilization: is it possible to stabilize a given unstable equilibrium by using appropriate feedback laws? (Think of the classical experiment of a broomstick held on the tip of a finger.) On this problem, we present some old devices and pioneering works (Ctesibius, Watt, Maxwell, Lyapunov...), more recent results and an application to the regulation of the rivers La Sambre and La Meuse. We also give some results for the stabilization in finite time both in finite and infinite dimension.

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THE NEW TYPE OF THE STATISTICAL CONVERGENCE OF THE FUNCTIONS DEFINED ON THE TIME SCALE PRODUCT

Metin Başarır ¹

ABSTRACT. In this talk, we have introduced the concepts $(\lambda, \nu)_h^\alpha$ -density of a subset of the product time scale \mathbb{T}^2 and $(\lambda, \nu)_h^\alpha$ -statistical convergence of order α ($0 < \alpha \leq 1$) of Δ -measurable function f defined on the product time scale with the help of modulus function h and $\lambda = (\lambda_n)$, $\nu = (\nu_n)$ sequences. Later, we have discussed the connection between classical convergence, λ -statistical convergence and $(\lambda, \nu)_h^\alpha$ -statistical convergence. In addition, we have seen that f is strongly $(\lambda, \nu)_h^\alpha$ -summable on \mathbb{T} then f is $(\lambda, \nu)_h^\alpha$ -statistical convergent of order α .

Keyword: Time scale, statistical convergence, modulus function, lamda sequence, order alfa.

AMS 2010: 40A05, 47H10, 46A45.

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EFFECTIVE LOGICAL METHODS IN NONLINEAR ANALYSIS

Ulrich Kohlenbach ¹

ABSTRACT. During the last two decades a program of ‘proof mining’ emerged which uses tools from mathematical logic (so-called proof interpretations) to systematically extract explicit quantitative information (e.g. rates of convergence) from prima facie nonconstructive proofs (e.g. of convergence results). This has been applied particularly successful in the context of nonlinear analysis (see [3] for a recent survey).

In this talk we will outline the general background of this logic-based approach and indicate some recent applications in the context of convex optimization, fixed point theory, nonlinear semigroup theory and pursuit-evasion games. In particular, we will report on the recent extraction of a polynomial rate of convergence in Bauschke’s [1] solution of the zero-displacement conjecture [2], rates of convergence of asymptotic regularity for nonexpansive semigroups [4] and rates of convergence for the Lion-Man game [6] in geodesic spaces [5].

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ALGEBRA



IECMSA - 2020

NEW ASPECTS IN POLYGROUP THEORY

Andromeda Sonea ¹

ABSTRACT. The aim of this paper is to compute the commutativity degree in polygroup's theory, more exactly for the polygroup P_G and for extension of polygroups by polygroups, obtaining boundaries for them. Also, we have analyzed the nilpotency of $\mathcal{A}[\mathcal{B}]$, meaning the extension of polygroups \mathcal{A} and \mathcal{B} .

Keyword: Polygroup, commutativity degree, nilpotent polygroup

AMS 2010: 20N20

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IECMSA - 2020

GENERALIZATION OF QUASI-DISCRETE MODULES

Burcu Nişancı Türkmen ¹, Figen Eryılmaz ²

ABSTRACT. In this study, we define semi-ss-discrete modules and quasi-ss-discrete modules and some of the basic features of these modules are obtained. Let M be an ss-lifting module with finite internal exchange property, then we call M is a *semi-ss-discrete module*. If M is both π -projective and ss-supplemented module, then we call M is a *quasi-ss-discrete module*.

Keyword: (Quasi-)discrete modules, ss-supplement submodule.

AMS 2010: 16D10,16D40,16D60

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IECMSA - 2020

FINITELY E -SUPPLEMENTED MODULES

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ABSTRACT. Let M be an R -module. If every finitely generated essential submodule of M has a supplement in M or M have no finitely generated essential submodules, then M is called a finitely e -supplemented (or briefly fe -supplemented) module. In this work, some properties of these modules are investigated.

Keywords: Small submodules, radical, essential submodules, supplemented modules.

AMS 2010: 16D10, 16D80.

Results

Proposition 1. *Every f -supplemented module is fe -supplemented.*

Corollary 1. *Let M be an R -module and $L \ll M$. If M is f -supplemented, then M/L is fe -supplemented.*

Corollary 2. *Let M be an R -module and L be a finitely generated submodule of M . If M is f -supplemented, then M/L is fe -supplemented.*

Proposition 2. *Let M be a fe -supplemented R -module. If every nonzero finitely generated submodule of M is essential in M , then M is f -supplemented.*

Lemma 1. *Let M be a fe -supplemented R -module and N be a finitely generated submodule of M . Then M/N is fe -supplemented.*

Corollary 3. *Let M be a fe -supplemented R -module and N be a cyclic submodule of M . Then M/N is fe -supplemented.*

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Corollary 4. *Let $f : M \rightarrow N$ be an R -module epimorphism and $Ke f$ be finitely generated. If M is fe -supplemented, then N is also fe -supplemented.*

Corollary 5. *Let $f : M \rightarrow N$ be an R -module epimorphism with cyclic kernel. If M is fe -supplemented, then N is also fe -supplemented.*

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IECMSA - 2020

AMPLY COFINITELY \mathfrak{g} -RADICAL SUPPLEMENTED MODULES

Celil Nebiyev ¹

ABSTRACT. In this work, all rings have unity and all modules are unital left modules. Let M be an R -module. If every cofinite submodule of M has ample \mathfrak{g} -radical supplements in M , then M is called an amply cofinitely \mathfrak{g} -radical supplemented module. In this work some properties of amply cofinitely \mathfrak{g} -radical supplemented modules are investigated.

Keywords: \mathfrak{G} -small submodules, \mathfrak{g} -supplemented modules, cofinitely \mathfrak{g} -supplemented modules, \mathfrak{g} -radical supplemented modules.

AMS 2010: 16D10, 16D80.

Results

Proposition 3. *Every amply cofinitely \mathfrak{g} -radical supplemented module is cofinitely \mathfrak{g} -radical supplemented.*

Proposition 4. *Every amply cofinitely \mathfrak{g} -supplemented module is amply cofinitely \mathfrak{g} -radical supplemented.*

Proposition 5. *Every amply \mathfrak{g} -radical supplemented module is amply cofinitely \mathfrak{g} -radical supplemented.*

Proposition 6. *Every amply cofinitely Rad-supplemented module is amply cofinitely \mathfrak{g} -radical supplemented.*

Proposition 7. *Let M be an amply cofinitely \mathfrak{g} -radical supplemented R -module. If every nonzero submodule of M is essential in M , then M is amply cofinitely Rad-supplemented.*

Proposition 8. *Let M be an amply cofinitely \mathfrak{g} -radical supplemented R -module. If every nonzero submodule of M is essential in M , then M is cofinitely Rad-supplemented.*

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Proposition 9. *Every amply cofinitely supplemented module is amply cofinitely g -radical supplemented.*

Proposition 10. *Let M be an amply cofinitely g -radical supplemented R -module. Then every factor module of M is amply cofinitely g -radical supplemented.*

Proposition 11. *Let M be an amply cofinitely g -radical supplemented R -module. Then every homomorphic image of M is amply cofinitely g -radical supplemented.*

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$\oplus - e$ -SUPPLEMENTED MODULES

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ABSTRACT. Let M be an R -module. If every essential submodule of M has a supplement that is a direct summand of M , then M is called a $\oplus - e$ -supplemented module. In this work, some properties of these modules are investigated.

Keywords: Essential submodules, small submodules, supplemented modules, essential supplemented modules.

AMS 2010: 16D10, 16D80.

Results

Lemma 2. *Every $\oplus - e$ -supplemented module is $\oplus - e$ -supplemented.*

Corollary 6. *The finite direct sum of $\oplus - e$ -supplemented modules is $\oplus - e$ -supplemented.*

Proposition 12. *Let M be a $\oplus - e$ -supplemented module. If every nonzero submodule of M is essential in M , then M is $\oplus - e$ -supplemented.*

Lemma 3. *Every $\oplus - e$ -supplemented module is essential supplemented.*

Corollary 7. *Let $M = M_1 + M_2 + \dots + M_n$. If M_i is $\oplus - e$ -supplemented module for every $i = 1, 2, \dots, n$, then M is essential supplemented.*

Corollary 8. *Let M be a $\oplus - e$ -supplemented module. Then every finitely M -generated R -module is essential supplemented.*

Corollary 9. *Let R be a ring. If ${}_R R$ is $\oplus - e$ -supplemented, then every finitely generated R -module is essential supplemented.*

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Corollary 10. *Every factor module of a $\oplus - e$ -supplemented module is essential supplemented.*

Corollary 11. *Every homomorphic image of a $\oplus - e$ -supplemented module is essential supplemented.*

Corollary 12. *Let M be a $\oplus - e$ -supplemented module. Then $M/\text{Rad}M$ have no proper essential submodules.*

Lemma 4. *Let M be a distributive and $\oplus - e$ -supplemented R -module. Then every factor module of M is $\oplus - e$ -supplemented.*

Corollary 13. *Let M be a distributive and $\oplus - e$ -supplemented R -module. Then every homomorphic image of M is $\oplus - e$ -supplemented.*

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$\oplus - g$ -RAD-SUPPLEMENTED MODULES

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ABSTRACT. Let M be an R -module. If every submodule of M has a g -radical supplement that is a direct summand of M , then M is called a $\oplus - g - Rad$ -supplemented module. In this work, some properties of these modules are investigated.

Keywords: Essential submodules, small submodules, supplemented modules, essential supplemented modules.

AMS 2010: 16D10, 16D80.

Results

Lemma 5. *Every $\oplus - g - Rad$ -supplemented module is g -radical supplemented.*

Corollary 14. *Let M be a $\oplus - g - Rad$ -supplemented module. Then every factor module of M is g -radical supplemented.*

Corollary 15. *Let M be a $\oplus - g - Rad$ -supplemented module. Then every homomorphic image of M is g -radical supplemented.*

Lemma 6. *Let M be an R -module and $M = M_1 + M_2$. If M_1 and M_2 are $\oplus - g - Rad$ -supplemented, then M is g -radical supplemented.*

Corollary 16. *The finite sum of $\oplus - g - Rad$ -supplemented modules is g -radical supplemented.*

Corollary 17. *Let M be a $\oplus - g - Rad$ -supplemented module. Then every finitely M -generated module is g -radical supplemented.*

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Corollary 18. *Let R be a ring. If ${}_R R$ is $\oplus - g - \text{Rad}$ -supplemented, then every finitely generated R -module is g -radical supplemented.*

Proposition 13. *Every $\text{Rad} - \oplus$ -supplemented module is $\oplus - g - \text{Rad}$ -supplemented.*

Corollary 19. *The finite direct sum of $\text{Rad} - \oplus$ -supplemented modules is $\oplus - g - \text{Rad}$ -supplemented.*

Corollary 20. *Every \oplus -supplemented module is $\oplus - g - \text{Rad}$ -supplemented.*

Corollary 21. *The finite direct sum of \oplus -supplemented modules is $\oplus - g - \text{Rad}$ -supplemented.*

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ON ESSENTIAL g -SUPPLEMENTED MODULES

Celil Nebiyev ¹, Hasan Hüseyin Ökten ²

ABSTRACT. Let M be an R -module. If every essential submodule of M has a g -supplement in M , then M is called an essential g -supplemented (or briefly eg -supplemented) module. In this work, some properties of these modules are investigated.

Keywords: G -small submodules, generalized radical, essential submodules, g -supplemented modules.

AMS 2010: 16D10, 16D80.

Results

Lemma 7. *Every g -supplemented module is eg -supplemented.*

Corollary 22. *Every factor module of a g -supplemented module is eg -supplemented.*

Corollary 23. *The homomorphic image of a g -supplemented module is eg -supplemented.*

Corollary 24. *Let $M = M_1 + M_2 + \dots + M_n$. If M_i is g -supplemented for every $i = 1, 2, \dots, n$, then M is eg -supplemented.*

Corollary 25. *Let M be a g -supplemented module. Then every finitely M -generated module is eg -supplemented.*

Corollary 26. *Let R be a ring. If ${}_R R$ is g -supplemented, then every finitely generated R -module is eg -supplemented.*

Proposition 14. *Let M be an eg -supplemented module. If every nonzero submodule of M is essential in M , then M is g -supplemented.*

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Definition 1. Let M be an R -module and $X \leq M$. If X is a g -supplement of an essential submodule of M , then X is called an eg -supplement submodule in M .

Lemma 8. Let M be an R -module, V be an eg -supplement in M and $K \ll_g M$. Then $K \cap V \ll_g V$.

Corollary 27. Let M be an R -module, V be an eg -supplement in M and $K \leq V$. Then $K \ll_g M$ if and only if $K \ll_g V$.

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ON RELATIONS AMONG QUADRATIC MODULES

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ABSTRACT. Algebraic models of connected homotopy 3-types such as quadratic modules, 2-crossed modules, crossed squares and their relations are studied in various ways. In this work we obtain another natural equivalence for quadratic modules. That is we define functors between quadratic modules and our candidate category.

Keyword: Crossed module , functor, quadratic module.

AMS 2010: 18B40, 20L05, 18D05

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BINOMIAL TRANSFORMS OF THE HORADAM QUATERNION SEQUENCES AND ITS PROPERTIES

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ABSTRACT. Through this comprehensive study, we set out to apply the binomial transforms to Horadam quaternion. We present recurrence relation, generating function, Binet formula and some basic identities for the binomial sequence of Horadam quaternions. We gave new formulas for some identities of binomial transforms of Horadam quaternions by using Binet formula.

Keyword: Binomial transforms, binet formula, horadam quaternions.

AMS 2010: 11B37, 11B39, 11R52, 11B65.

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ON THE EXTENSIBILITY OF SOME PARAMETRIC FAMILIES OF $D(-1)$ -PAIRS TO QUADRUPLES IN THE RING $\mathbb{Z}[\sqrt{-t}]$, $t > 0$

Mirela Jukić Bokun ¹

ABSTRACT. Let R be a commutative ring. A set of m distinct elements in R such that the product of any two distinct elements increased by -1 is a perfect square is called a $D(-1) - m$ -tuple in R . The existence of positive integer solutions of the equation

$$x^2 - (p^{2k+2} + 1)y^2 = -p^{2l+1}, \quad l \in \{0, 1, \dots, k\}, k \geq 0, \quad (1)$$

where p is a prime, is closely related to the existence of some $D(-1)$ -quadruples in a certain ring. We discuss solubility of equation (1). By combining that result with other known results on the existence of Diophantine quadruples, we are able to prove results on the extensibility of some parametric families of $D(-1)$ -pairs to quadruples in the ring $\mathbb{Z}[\sqrt{-t}]$, $t > 0$.

Keyword: Pellian equation, quadratic field, Diophantine quadruple.

AMS 2010: 11D09, 11R11, 11J86.

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k -ORDER GAUSSIAN FIBONACCI MATRICES AND SOME APPLICATIONS

Süleyman Aydınüz¹, Mustafa Aşçı²

ABSTRACT. In this paper we introduce and study k -order Gaussian Fibonacci Coding theory. We give illustrative examples about coding theory. This coding theory is a method bound to the Q_k , R_k and $E_n^{(k)}$ matrices. This coding/decoding method is different from classical algebraic coding. k -order Gaussian Fibonacci Coding method depends on matrix multiplication and can be performed quickly and easily by today's computers. This method will not only ensures information security in data transfer but also has high correct ability. Consequently, this method aims to increase the reliability of information transfer by moving the coding theory to the complex space.

Keyword: Fibonacci numbers, gaussian fibonacci numbers, k-order gaussian fibonacci numbers, k-order gaussian fibonacci matrices, k-order gaussian fibonacci coding/decoding.

AMS 2010: 11Bxx, 11Txx

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IECMSA - 2020

ON INTEGRAL TRANSFORMS OF SOME SPECIAL FUNCTIONS

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ABSTRACT. In this study, known integral transforms such as Fourier and Hartley are studied and these integral transforms are studied in detail for bicomplex numbers. In addition, the properties of the bicomplex Hartley transformation were investigated. Also, the relation between Hartley and Fourier transform for bicomplex numbers is given.

Keyword: Bicomplex functions, fourier type integral transformations, integral transformations of special functions, integral transformations.

AMS 2010: 30G35, 44A15, 44A20.

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LOCALLY-ARTINIAN SUPPLEMENTED MODULES

Yavuz Şahin ¹, Burcu Nişancı Türkmen ²

ABSTRACT. In this paper, we introduce a notion of locally-artinian supplemented modules which is different from a notion of ss-supplemented modules and we study some properties of this module. We give a characterizations of this module over a left artinian ring.

Keyword: locally-artinian modules, supplement submodule, locally-artinian supplemented modules.

AMS 2010: 16D10, 16P20, 16D99

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ANALYSIS



IECMSA - 2020

COMMUTABLE MATRICES AND FUNCTIONAL COMMUTATIVITY OF COMPACT NORMAL OPERATORS

Abdelaziz Maouche ¹

ABSTRACT. A simple expression is established for an analytic family of commutable matrix-valued functions. Then a characterization of two by two functional commutative matrices is proven. In [2], Stuart Goff studied analytic hermitian function matrices which commute with their derivative on some real interval I , i.e, $A(t)A'(t) = A'(t)A(t)$ for all $t \in I$. He obtained as a main result that these matrices are functionally commutative on I , i.e.,

$$A(s)A(t) = A(t)A(s)$$

for all $s, t \in I$ [2], Theorem 3.6. Our aim is to further extend the result of Goff from matrices to the infinite-dimensional situation of compact normal operators on a separable Hilbert space. We study first analytic families of compact self-adjoint operators on a complex Hilbert space, which commute with their derivative on some real interval I . Our main result establishes that these operators must be functionally commutative on I , that is,

$$A(s)A(t) = A(t)A(s)$$

for all $s, t \in I$, extending the main result of [2] from the case of matrices to the infinite dimensional situation of operators on a Hilbert space.

Finally, it is shown that a family of analytic normal compact operators on a Hilbert space \mathcal{H} , which commute with their derivatives, must be functionally commutative.

Keyword: Commutable matrix valued-function, compact operator, functional commutativity, normal operator, self-adjont operator, riesz projection, spectral decomposition, analytic operator-valued function.

AMS 2010: Primary 47B15, Secondary 47A55

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IECMSA - 2020

ON THE SOLUTION OF THE FUNCTIONAL EQUATION ARISING IN MATHEMATICAL PSYCHOLOGY AND
THEORY OF LEARNING

Ali Turab ¹, Wutiphol Sintunavarat ²

ABSTRACT. In mathematical psychology and learning theory, the choice behavior model is the model that describes the spiritual process of thinking, which is concerned with the process of judging the merits of the numerous options and making the decision to determine one of them for action. This work intends to investigate such type of behavior and establish a general functional equation for it. The existence and uniqueness results of the solution to the proposed mathematical model are examined by using the fixed point tools.

Keyword: Functional equation, probability, fixed points, Banach contraction mapping principle.

AMS 2010: 30D05, 39B52, 47H10.

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FREDHOLM CRITERIA IN A C^* -ALGEBRA ACTING ON THE HARDY SPACE OF THE BI-DISC WITH
APPLICATIONS TO COMPOSITION OPERATORS

Beyaz Başak Eskişehirli ¹

ABSTRACT. In this work we give a Fredholm criteria for the operators in the C^* -algebra generated by certain Toeplitz operators and Fourier multipliers acting on the Hardy space of the bidisc. With help of the obtained results we also completely characterize the essential spectra of quasi-parabolic composition operators on the Hardy spaces of the bi-disc. This joint work with Uğur GÜL.

Keyword: Fredholm operator, hardy spaces, c^* -algebra, composition operators, essential spectra.

AMS 2010: 32A45, 47B33

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ON FACTORIZATION OF MULTILINEAR MAPS DEFINED ON SEQUENCE SPACES BY ZERO PRODUCT PRESERVATION

Ezgi Erdoğan ¹

ABSTRACT. In this presentation, we give a factorization theorem for multilinear operators acting in topological products of spaces of (scalar) p -summable sequences through a canonical map called *product*. This class of multilinear operators, that we call *product factorable maps*, coincides with the class of the *zero product preserving* operators. Due to the factorization, a product factorable multilinear map and the linear map appearing in the factorization share some good properties like compactness and summability. After presentation of these properties, we finish the presentation by giving some isomorphisms between spaces of linear and multilinear operators, and representations of some classes of multilinear maps as n -homogeneous orthogonally additive polynomials.

Keyword: Multilinear operators, factorization, zero product preserving map.

AMS 2010: Firstly 47H60, 47A68; Secondly 46B45, 46B42.

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FIXED POINTS OF GENERALIZED ORTHOGONAL L -SIMULATIVE CONTRACTION IN NON-ARCHIMEDEAN QUASI MODULAR METRIC SPACE AND APPLICATIONS

Ekber Girgin ¹, Mahpeyker Öztürk ²

ABSTRACT. In this study, by using the concept of cyclic (α, β) -admissible mapping, orthogonal set and L -simulation functions, we establish the existence and uniqueness of fixed point of a generalized orthogonal L -simulative contraction on non-Archimedean quasi modular metric spaces. Our results generalize and extend various comparable results in the existing literature. As an application, we acquire fixed point results in non-Archimedean quasi modular metric spaces with a graph.

Keyword: Orthogonal set, L -Simulation function, quasi modular metric, graph.

AMS 2010: 47H10, 54H25.

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ON SOME COUPLED FIXED POINT RESULTS IN ELLIPTIC VALUED \mathbf{b} -METRIC SPACES

Işıl Arda Kösal ¹, Mahpeyker Öztürk ²

ABSTRACT. The present work demonstrates the existence and uniqueness of coupled common fixed point for the mappings with the suitable properties and conditions in an elliptic valued \mathbf{b} -metric space. The obtained results include several generalizations, extension, and improvement of the recent fixed point theorems given in the literature.

Keyword: Elliptic valued b -metric space, coupled fixed point, rational expressions.

AMS 2010: 54H25, 47H10.

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STAR PARTIAL ORDER IN SPACES WITH AN INDEFINITE INNER PRODUCT

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ABSTRACT. The star partial order has been intensively investigated in the set of complex matrices. This partial order was defined by Drazin in [4]. Some of interesting properties can be found in [1, 7]. We present the extension of these results for matrices in indefinite inner product spaces. The characterization of that order in terms of matrices and their Moore-Penrose inverses are also given.

Keyword: Star partial order, indefinite inner product, Moore-Penrose inverse.

AMS 2010: 15A09, 15A63, 46C20.

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IECMSA - 2020

OPTIMAL L^∞ -ERROR ESTIMATE FOR THE IMPULSE CONTROL QUASI-VARIATIONAL INEQUALITY

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ABSTRACT. In this paper, we improve the result of [1] by deriving the optimal convergence order of the standard finite element approximation of the elliptic impulse control quasi-variational inequality (QVI). For that, we introduce a new method which combines, in both the continuous and discrete cases, the geometrical convergence of the Bensoussan-Lions iterative scheme with the concept of subsolutions for elliptic variational inequalities.

Keyword: Quasi-variational Inequality, finite elements, iterative scheme, subsolutions .

AMS 2010: 65N30, 65N15.

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IECMSA - 2020

ON GENERALIZED DEFERRED STATISTICAL CONVERGENCE OF DIFFERENCE SEQUENCES

Mikail Et ¹

ABSTRACT. In this paper, using the generalized difference operator Δ_m^n , we introduce the concepts of Δ_m^n -deferred statistical convergence and strong Δ_m^n -deferred Cesàro summability of real sequences. Additionally, some inclusion relations about Δ_m^n -deferred statistical convergence of and strong Δ_m^n -deferred Cesàro summability are given.

Keyword: Difference sequence, deferred cesàro mean, deferred statistical convergence.

AMS 2010: 40A05, 40C05, 46A45.

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IECMSA - 2020

ON ASYMPTOTICALLY LACUNARY STATISTICAL EQUIVALENT OF DIFFERENCE DOUBLE SEQUENCES

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ABSTRACT. In this study we introduce and examine the concepts of Δ_{θ}^m -asymptotically statistical equivalent and strong Δ_{θ}^m -asymptotically equivalent of double sequences. Also, we give some relations connected to these concepts.

Keyword: Asymptotically statistical equivalent, difference double sequence, lacunary sequence.

AMS 2010: 40A05, 40C05, 46A45

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FIXED POINTS OF MULTIVALUED ρ -NONEXPANSIVE MAPPINGS

Osman Alagöz ¹

ABSTRACT. In this work we study convergence of S iteration [3] that converges common fixed points of two multivalued ρ -nonexpansive mappings an modular function spaces. The given iteration is faster than Mann [3] and Ishikawa [2] iterations and is reduced to Mann iteration in a special case. So the findings generalize the results of Khan and Abbas [4]

Keyword: fixed point, modular function space, multivalued mapping.

AMS 2010: 47H09, 47H10.

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IECMSA - 2020

ON CERTAIN DIGITAL OPERATORS IN THE THEORY OF BOUNDARY VALUE PROBLEMS

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ABSTRACT. Let $\mathbb{T}^m = [-\pi, \pi]^m$, $h > 0$, $\tilde{A}(\xi)$, $\xi \in \mathbb{R}^m$ be a periodic function with basic cube of periods $h^{-1}\mathbb{T}^m$, $\tilde{A}(\xi) \in L_1(h^{-1}\mathbb{T}^m)$, $D \subset \mathbb{R}^m$ be a domain. We introduce a digital pseudo-differential operator

$$(A_d u_d)(\tilde{x}) = \sum_{\tilde{y} \in h\mathbb{Z}^m} \int_{h^{-1}\mathbb{T}^m} \tilde{A}(\xi) e^{i(\tilde{y}-\tilde{x}) \cdot \xi} u_d(\tilde{y}) d\xi h^m, \quad \tilde{x} \in D_d \equiv D \cap h\mathbb{Z}^m$$

which is defined for functions of a discrete variable $\tilde{x} \in h\mathbb{Z}^m$.

We study operator equations

$$A_d u_d = v_d \tag{1}$$

with appropriate boundary conditions.

To study the discrete equation (1) in a half-space we use a special factorization for the symbol $\tilde{A}(\xi)$

$$\tilde{A}(\xi) = \tilde{A}_+(\xi) \cdot \tilde{A}_-(\xi)$$

where the factors $\tilde{A}_\pm(\xi)$ admit a holomorphic continuation into half-strips

$$\Pi_\pm = \{z \in \mathbb{C} : z = \xi_m + i\tau, \xi_m \in [-h^{-1}\pi, h^{-1}\pi], \pm\tau > 0\}.$$

with respect to the last variable ξ_m under fixed $(\xi_1, \dots, \xi_{m-1}) \in h^{-1}\mathbb{T}^{m-1}$ and satisfy some estimates [1,2,3].

Such a representation can be constructed effectively and it fully determines a solvability picture for the equation (1).

If D is a half-space then we prove unique solvability for the equation (1) and related boundary value problems in corresponding discrete Sobolev–Slobodetskii spaces and construct finite dimensional approximations $u_{d,N}(\tilde{x})$ for the solution $u_d(\tilde{x})$. A rate of convergence under $N \rightarrow \infty$ is presented. Some comparison estimates are presented in [4].

Keyword: Digital pseudo-differential operator, periodic factorization, approximation rate.

AMS 2010: 35S15, 65T50

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IECMSA - 2020

LOCAL CONVERGENCE OF A FAMILY OF SAKURAI-TORII-SUGIURA TYPE SIMULTANEOUS METHODS WITH ACCELERATED CONVERGENCE

Petko D. Proinov ¹, Stoil I. Ivanov ²

ABSTRACT. This talk deals with the local convergence of a new family of iterative methods for finding all the zeros of a polynomial simultaneously. Such iterative methods are called *simultaneous methods*. One of the well-known simultaneous methods is due to Sakurai, Torii and Sugiura [1] (STS method). A comprehensive local and semilocal convergence analysis of the STS method can be found in [2]. There are different ways to increase the order of convergence of a simultaneous method. The most common way is to compose a simultaneous method with another iterative method (mostly with Newton, Halley and Weierstrass methods). This kind of methods are known as *simultaneous methods with correction*. The purpose of this work is to study the local convergence of the STS method with an arbitrary correction.

Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ over a normed field K and $\Phi : D \subset K^n \rightarrow K^n$. We define the following family of Sakurai-Torii-Sugiura type iterative methods:

$$x^{(k+1)} = x^{(k)} - \Delta(\Phi; x^{(k)}), \quad k = 0, 1, 2, \dots, \quad (1)$$

where the correction function $\Delta : D \subset \mathbb{K}^n \rightarrow \mathbb{K}^n$ is defined by

$$\Delta(\Phi; x) = (\Delta_1(\Phi; x), \dots, \Delta_n(\Phi; x)) \quad \text{with} \quad \Delta_i(\Phi; x) = \begin{cases} \frac{2L_i(\Phi; x)}{L_i(\Phi; x)^2 - F_i(\Phi; x)} & \text{if } f(x_i) \neq 0, \\ 0 & \text{if } f(x_i) = 0, \end{cases}$$

where $L_i(\Phi; x)$ and $F_i(\Phi; x)$ are defined as follows

$$L_i(\Phi; x) = \frac{f'(x_i)}{f(x_i)} - \sum_{j \neq i} \frac{1}{x_i - \Phi_j(x)} \quad \text{and} \quad F_i(\Phi; x) = \frac{f''(x_i)}{f(x_i)} - \left(\frac{f'(x_i)}{f(x_i)} \right)^2 + \sum_{j \neq i} \frac{1}{(x_i - \Phi_j(x))^2}.$$

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ABSTRACT. (Continuation) In this study, considering two large classes of iteration functions Φ , we obtain two local convergence theorems (with error estimates) about the methods (1).

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Keyword: Iterative methods, polynomial zeros, local convergence

AMS 2010: 65H04,12Y05

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IECMSA - 2020

LOCAL AND SEMILOCAL CONVERGENCE OF A TWO-POINT EHRlich-TYPE ROOT-FINDING METHOD

Petko D. Proinov¹, Milena D. Petkova²

ABSTRACT. In 1967, Ehrlich [1] constructed his famous iterative method for finding all the zeros of a polynomial simultaneously. In 1999, Tričković and Petković [2] introduced a two-point variant of Ehrlich's method with order of convergence $r = 1 + \sqrt{2}$. Let

$$f(z) = a_0 z^n + a_1 z^{n-1} + \dots + a_n$$

be a polynomial of degree $n \geq 2$ with coefficients in an algebraically closed normed field \mathbb{K} , and let $x^{(0)}, x^{(-1)} \in \mathbb{K}^n$ be two approximations of the zeros of f . Then the two-point Ehrlich-type method introduced in [2] can be defined by the following iteration

$$x^{(k+1)} = T(x^{(k)}, x^{(k-1)}), \quad k = 0, 1, 2, \dots, \quad (1)$$

where the iteration function $T: D \subset \mathbb{K}^n \times \mathbb{K}^n \rightarrow \mathbb{K}^n$ is defined by

$$T(x, y) = (T_1(x, y), \dots, T_n(x, y)) \quad \text{with} \quad T_i(x, y) = x_i - \frac{f(x_i)}{f'(x_i) - f(x_i) \sum_{j \neq i} \frac{1}{x_i - y_j}}.$$

In this talk, we provide a local as well as a semilocal convergence analysis for this method. In particular, we prove that the method (1) is convergent under the following computationally verifiable initial condition:

$$\max \left\{ \left\| \frac{W(x^{(0)})}{d(x^{(0)})} \right\|_\infty, \left\| \frac{W(x^{(-1)})}{d(x^{(-1)})} \right\|_\infty \right\} < \frac{8}{(3 + \sqrt{8n - 7})^2},$$

where the function $W: \mathcal{D} \subset \mathbb{K}^n \rightarrow \mathbb{K}^n$ (known as the Weierstrass correction) is defined by

$$W(x) = (W_1(x), \dots, W_n(x)) \quad \text{with} \quad W_i(x) = \frac{f(x_i)}{a_0 \prod_{j \neq i} (x_i - x_j)},$$

the function $d: \mathbb{K}^n \rightarrow \mathbb{R}^n$ is defined by $d(x) = (d_1(x), \dots, d_n(x))$ with $d_i(x) = \min_{j \neq i} |x_i - x_j|$, and $\|\cdot\|_\infty$ is the maximum norm in \mathbb{K}^n . Our approach is based on ideas developed in [3, 4].

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Keyword: Two-point iterative methods, polynomial zeros, local and semilocal convergence

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IECMSA - 2020

LOCAL CONVERGENCE OF A MODIFIED WEIERSTRASS METHOD FOR THE SIMULTANEOUS DETERMINATION OF POLYNOMIAL ZEROS

Plamena I. Marcheva¹, Stoil I. Ivanov²

ABSTRACT. Let f be a polynomial of degree $n \geq 2$ with coefficients in an arbitrary normed field $(\mathbb{K}, |\cdot|)$. In 1891, Weierstrass [2] introduced his famous iterative method for finding all zeros of f simultaneously. Recall that the *Weierstrass method* is defined in \mathbb{K}^n by the following iteration:

$$x^{(k+1)} = x^{(k)} - W_f(x^{(k)}), \quad k = 0, 1, 2, \dots, \quad (1)$$

where the *Weierstrass correction* $W_f: \mathcal{D} \subset \mathbb{K}^n \rightarrow \mathbb{K}^n$ is defined as follows

$$W_f(x) = (W_1(x), \dots, W_n(x)) \quad \text{with} \quad W_i(x) = \frac{f(x_i)}{a_0 \prod_{j \neq i} (x_i - x_j)} \quad (i = 1, \dots, n).$$

In 2016, Nedzhibov [1] constructed and studied the convergence of the following modification of the Weierstrass method:

$$x^{(k+1)} = T(x^{(k)}), \quad k = 0, 1, 2, \dots, \quad (2)$$

where the iteration function $T: D \subset \mathbb{K}^n \rightarrow \mathbb{K}^n$ is defined by

$$T(x) = (T_1(x), \dots, T_n(x)) \quad \text{with} \quad T_i(x) = \frac{x_i^2}{x_i + W_i(x)} \quad (i = 1, \dots, n).$$

The aim of this talk is to introduce a local convergence theorem that improves and complements all existing results about the *modified Weierstrass method* (2).

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Keyword: Iterative methods, polynomial zeros, local convergence

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IECMSA - 2020

THE REFINEMENTS OF LOCAL FRACTIONAL HILBERT-TYPE INEQUALITIES

Predrag Vuković ¹

ABSTRACT. In this paper we refine some known local fractional Hilbert-type inequalities in the sense that they interpolate Lebesgue norms of the local fractional Laplace transforms of the involved functions in the inequalities. As an application, main results are compared with some our previously known from the literature.

Keyword: Hilbert inequality, conjugate parameters, local fractional integral.

AMS 2010: 26D15.

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IECMSA - 2020

NONSTATIONARY WAVELET FRAME PACKETS IN WEIGHTED SOBOLEV SPACE

Raj Kumar ¹, Manish Chauhan ², Reena ³, Satyapriya ⁴

ABSTRACT. Important theorems and inequalities are established to help in construction of nonstationary wavelet frame packets in weighted Sobolev space $W_2^1(\mathbb{R})$. Finally, nonstationary wavelet frame packets are constructed in weighted Sobolev space $W_2^1(\mathbb{R})$ using splitting trick.

Keyword: Nonstationary wavelets, nonstationary wavelet packets, nonstationary wavelet frame packets, multiresolution analysis, Sobolev space.

AMS 2010: 42C40, 40A30, 46E35.

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IECMSA - 2020

THE NEW TYPE OF THE STATISTICAL CONVERGENCE OF THE FUNCTIONS DEFINED ON THE TIME SCALE PRODUCT

Rabia Savaş ¹

ABSTRACT. In this paper, we present new definitions for asymptotically equivalent functions. Two nonnegative measurable a real valued function $f(s)$ and $g(s)$ defined on $(1, \infty)$ are said to be asymptotically statistical equivalents of multiple L provided that for each $\varepsilon > 0$,

$$\lim_s \frac{1}{s} \left| \left\{ t \leq s : \left| \frac{f(t)}{g(t)} - L \right| \geq \varepsilon \right\} \right| = 0.$$

In this case, we denote this by $f \stackrel{FL}{\sim} g$. Moreover, definitions are used to examine the bivariate function transformation of asymptotically statistical equivalent measurable functions.

Keyword: Bivariate function transformation, measurable functions, rate of convergence, asymptotically equivalent.

AMS 2010: 40F02, 40G06.

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IECMSA - 2020

THE SPECTRA OF SUPERPOSITION OPERATORS GENERATED BY AN EXPONENTIAL FUNCTION

Sanela Halilović¹

ABSTRACT. We consider nonlinear superposition operator $F : l_p \rightarrow l_p$ generated by the function $f(s, u) = a(s) + b^{ku} - 1$, $b > 0$, $k \in \mathbb{R} \setminus \{0\}$. Here l_p ($1 \leq p \leq \infty$) are the spaces of sequences with the standard norm. There are various way for defining spectrum of nonlinear operators (see [1]). For the class $\mathfrak{C}(X)$ of all continuous operators F on Banach space X over \mathbb{K} the Rhodius resolvent set is given by:

$$\rho_R(F) = \{\lambda \in \mathbb{K} : \lambda I - F \text{ is bijective and } (\lambda I - F)^{-1} \in \mathfrak{C}(X)\}$$

and the Rhodius spectrum is the set $\sigma_R(F) = \mathbb{K} \setminus \rho_R(F)$. For continuously Fréchet differentiable operators, the Neuberger resolvent set is defined by

$$\rho_N(F) = \{\lambda \in \mathbb{K} : \lambda I - F \text{ is bijective and } (\lambda I - F)^{-1} \in \mathfrak{C}^1(X)\}$$

and the set $\sigma_N(F) = \mathbb{K} \setminus \rho_N(F)$ is called Neuberger spectrum of F .

We find the Rhodius and Neuberger spectra of this operator and conclude how these sets depend on constants b and k . Also we give some relations between the properties of an generating function $f(s, u)$ and the spectrum of its corresponding operator F .

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Keyword: Rhodius spectrum, Neuberger spectrum, superposition operator, nonlinear operator.

AMS 2010: 47J10, 47H30.

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IECMSA - 2020

P-MOMENT EXPONENTIALLY STABILITY OF SECOND ORDER DIFFERENTIAL EQUATIONS WITH EXPONENTIAL DISTRIBUTED MOMENTS OF IMPULSES

Snezhana Hristova ¹

ABSTRACT. Differential equations of second order with impulses at random moments are set up and investigated in this paper. The main characteristic of the studied equations is that the impulses occur at random moments which are exponentially distributed random variables. The presence of random variables in the ordinary differential equation leads to a total change of the behavior of the solution. It is not a function as in the case of deterministic equations, it is a stochastic process. It requires combining of the results in Theory of Differential Equations and Probability Theory. The initial value problem is set up in appropriate way. Sample path solutions are defined as a solutions of ordinary differential equations with determined fixed moments of impulses. P-moment exponential stability is defined and some sufficient conditions for this type of stability are obtained. The study is based on the application of Lyapunov functions.

AMS 2010: 34A37, 34D20.

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IECMSA - 2020

ON THE DETERMINATION OF THE JUMP BY CONJUGATE FOURIER-JACOBI SERIES

Samra Sadiković ¹

ABSTRACT. Conjugate Fourier-Jacobi series was introduced by B. Muckenhoupt and E. M. Stein [3] when $\alpha = \beta$, and by Zh.-K. Li [4] for general α and β . "Conjugacy" is an important concept in classical Fourier analysis which links the study of the more fundamental properties of harmonic functions to that of analytic functions and is used to study the mean convergence of Fourier series. We prove the equiconvergence related to conjugate Fourier-Jacobi series and differentiated Fourier-Jacobi series for functions of harmonic bounded variation. A jump of a such function is determined by the partial sums of its conjugate Fourier-Jacobi series. Also for *HBV* functions we give a new result on determination of jump discontinuities by the n -th order tails of the conjugate Fourier-Jacobi series.

Keyword: Conjugate Fourier-Jacobi series, determination of the jump, generalized bounded variation.
AMS 2010: 42A24, 42C10.

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CONSTRUCTION OF A RIESZ WAVELET BASIS ON LOCALLY COMPACT ABELIAN GROUPS

Satyapriya ¹, Raj Kumar ²

ABSTRACT. We have explored the concept of Riesz multiresolution analysis (Riesz MRA) on a locally compact Abelian group G , and have studied in detail, the methods of construction of a Riesz wavelet from the given Riesz MRA. We have proved that, if δ_α is the order of dilation, then precisely $\delta_\alpha - 1$ functions are required to construct a Riesz wavelet basis for $L^2(G)$. An example, supporting our theory and illustrating the methods developed, has also been discussed briefly.

Keyword: Lca groups, riesz basis, multiresolution analysis, order of dilation, refinement equation.

AMS 2010:42C40, 22B05.

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APPLIED MATHEMATICS



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MATHEMATICAL MODELING OF THE ENERGY SAVING PROBLEM IN THE PIPELINE

A. A. Adamov ¹, A. N. Satybaldina ²

ABSTRACT. Operating main pipelines can provide fault-free operation mode due to the reduction of operating pressure while being under conditions of constant increase of equipment wear, which, in turn, leads to the reduction of oil pipeline capacity. At the same time, pipeline transport should constantly increase capacity, because production volumes are growing annually [1].

The initial dependence for generalizing the experimental data on heat transfer is the general law of temperature distribution in a liquid, expressed by the differential equation of convective heat transfer, which is called the Fourier-Kirchhoff equation in a cylindrical coordinate system:

$$C_v w_x \frac{\partial T}{\partial x} = \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} \right) \quad (1)$$

for $r \in [0, r_0]$, $\varphi \in [0, 2\pi]$, $x \in [0, L]$, where r - current radius, φ - angle measured from the vertical, w_x - cross section average speed, x - coordinate characterizing the length of the pipe, r_0 - pipeline radius, L - the length of the pipe. Boundary conditions are accepted as:

$$T|_{x=0} = T_0(r, \varphi), \quad (2)$$

$$\frac{\partial T}{\partial r} \Big|_{r=0} = 0, \lambda \frac{\partial T}{\partial r} \Big|_{r=r_0} = -\alpha (T - T_w) \Big|_{r=r_0}, \quad (3)$$

$$T(r, \varphi, x) = T(r, \varphi + 2\pi, x). \quad (4)$$

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ABSTRACT. (Continuation) In this paper, we solved the problem of liquid transporting in underground pipeline. The Fourier-Kirchhoff equation is used for receiving solution of the problem. There are:

- 1) created a difference problem (1)-(4) by Alternating Direction Method [2,4];
- 2) solved a problem (1)-(4) by Cyclic Sweep Method [3];
- 3) constructed algorithm for computer program;
- 4) conducted the analyses of the received numerical solutions.

Keyword: The Fourier-Kirchhoff equation; the Pisman-Redford scheme; numerical solution; heat transfer equation; boundary conditions.

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IECMSA - 2020

A MODIFICATION OF THE FAST ALGORITHM FOR COMPUTING THE MOCK-Chebyshev NODES

B. Ali İbrahimođlu ¹

ABSTRACT. Polynomial interpolation with equidistant nodes is notoriously unreliable due to the Runge phenomenon, and also numerically ill-conditioned. By taking advantage of the optimality of the interpolation processes on Chebyshev nodes, the mock-Chebyshev subset interpolation is one of the best strategies to overcome the Runge phenomenon [1].

In the recent paper [2], we have presented a fast algorithm to compute the mock-Chebyshev nodes. In this study, we propose a modification of the algorithm by changing the function to compute the quotient of the distance between each pair of consecutive points and show that this modified algorithm is also fast and stable; and always gives a satisfactory grid with the complexity of the algorithm being $O(n)$. Some numerical experiments using the points obtained by the procedure are given to show the effectiveness of the proposed procedure. A discussion of bivariate generalization of the mock-Chebyshev nodes to the Padua nodes is also given.

Keyword: Interpolation, Runge phenomenon, mock-Chebyshev interpolation.

AMS 2010: 65D05, 41A10.

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IECMSA - 2020

A COMPARISON OF ROUGHNESS MODELS FOR MEAN FLOW SOLUTIONS OF THE EKMAN BOUNDARY LAYER FLOW

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ABSTRACT. Applying roughness on a surface of a rigid body moving through a fluid is an important technique to delay the onset of the turbulence [1, 4]. This present study aims to compare the effects of two fundamentally different roughness models on the mean flow solutions of the Ekman boundary layer flow. These particular models are called MW model [2] and YHP model [3]. Both models are used to investigate how consecutive increasing roughness levels initiate a divergence from the classic similarity solution of the Ekman flow over a flat disk. The results identified that the models lead to different velocity profiles for the mean flow solutions.

Keyword: Rotating-disk flow, ekman flow, surface roughness.

AMS 2010: 76E15

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IECMSA - 2020

ON THE STABILITY OF RELATED ROTATING FLOWS OF THE BEK SYSTEM OVER A ROUGH ROTATING DISK

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ABSTRACT. The BEK system refers to a family of boundary-layer flows driven by a differential rotation rate between an incompressible fluid and a disk immersed in it [1]. The characteristic flows in the system are the von Kármán, Ekman, and Bödewadt boundary-layer flows. On the other hand, there are infinitely many flows between these particular ones in which both the disk and fluid co-rotate with different angular velocities. These flows are characterized by Rossby number, Ro . The aim of this study is to establish a theoretical study exploring the convective instability properties of the flows of $0 < Ro < 1$ over a rough rotating disk. The roughness model used in the study is the YHP model suggested by Yoon, Hyun, and Park [2]. The findings can contribute to a better understanding of the transition of laminar flows via surface roughness [3].

Keyword: Rotating-disk flow, BEK system, surface roughness.

AMS 2010: 76E15

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IECMSA - 2020

INVERSE PROBLEMS OF HEAT AND MASS TRANSFER FOR FINDING DIFFUSION COEFFICIENT OF SOIL

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ABSTRACT. This paper studies the process of heat and mass transfer in soil. Physical Laws based on The Law of Conservation of Mass of dry air, steam and liquid water, as well as the Law of Conservation of Energy, are described in detail in the work of Lykov [1]. Works [2] and [3] are devoted to finding the coefficient of thermal conductivity and moisture characteristics of soil. In the present work we have derived the conjugate system of differential equations with partial derivatives. The boundary and initial conditions of the conjugate problem are defined. An iterative scheme for calculating the diffusion coefficient of soil moisture is derived by using the conjugate and direct system of differential equations, and measured values of temperature and moisture in the accessible area. The Dufort-Frankel Difference scheme is used for the discretization of continuous problems. The main advantages in terms of stability and accuracy of the Dufort-Frankel scheme are described in [4]. Numerical calculations are carried out by using MATLAB and compared with experimental data of other scientists.

Keyword: Inverse problems, conjugate problem, heat and mass transfer.

AMS 2010: 80A23, 65Q10.

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IECMSA - 2020

THE METHOD FOR FINDING THE SYSTEM OF THERMOPHYSICAL PARAMETERS FOR TWO-LAYERED STRUCTURE

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ABSTRACT. Nowadays, the construction market often receives a variety of new building materials. Often the thermophysical parameters of these materials are unknown or after a long operation of artificial structures under the influence of wind, moisture and the sun, the physical and chemical properties of the materials of the constituent structures change. As a result of which, all thermophysical parameters of composite materials become different. Under these conditions, long-term reliable prediction of the processes occurring in multilayer structures becomes impossible. Therefore, the development of methods for calculating all the thermophysical parameters of a multilayer medium and the automation of finding these parameters becomes an urgent task. The aim of this work is to develop methods for finding a complex of thermophysical parameters, to prove the stability and convergence of the developed methods, to compile and debug a software product designed to find all thermophysical parameters. As an experiment, two-layered rectangular construction is studied, which is affected by two different ambient temperatures on both sides. The developed methods are numerically implemented using optimization methods. The suitability of the developed methods for solving the inverse problem is established through the stability and convergence of the approximate method. The convergence and stability of the developed method are proved by the method of a priori estimates.

Keyword: Inverse problems, thermophysical parameters, iterative methods.

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IECMSA - 2020

A NOTE ON RING SOURCE OVER SEMI-INFINITE LINED AND PERFORATED DUCT

Burhan Tiryakioglu ¹

ABSTRACT. An analytical solution is presented for the problem of diffraction of acoustic waves emanating from a ring source by an infinite cylindrical duct. The part $z < l$ of the outer cylinder is coated with acoustically absorbing lining while the part $z > l$ is perforated. Applying the Fourier transform technique the mixed boundary value problem is described by a Wiener Hopf equation and then solved numerically. The present study can be used as a model for many engineering applications, such as noise reduction in exhaust systems, in ventilation systems, etc.

Keyword: Ring source, acoustic wave, perforated duct.

AMS 2010: 47A68, 42B10, 78A45.

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IECMSA - 2020

THE NEW TYPE OF THE STATISTICAL CONVERGENCE OF THE FUNCTIONS DEFINED ON THE TIME SCALE PRODUCT

Elif Güner ¹, Halis Aygün ²

ABSTRACT. The aim of this paper is to present the extension of a concept related to aggregation operators from spherical fuzzy sets to generalized spherical fuzzy sets. We first introduce Einstein sum, product and scalar multiplication for generalized spherical fuzzy sets based on Einstein t-norms and t-conorms. Then we give the generalized spherical fuzzy Einstein weighted averaging and generalized spherical fuzzy Einstein weighted geometric operators, namely generalized spherical fuzzy Einstein aggregation operators, constructed on these operations. After investigating some fundamental properties of these operators, we develop a model for generalized spherical fuzzy Einstein aggregation operators to solve a multiple attribute group decision-making problems. Finally, we give a numerical example to demonstrate that the developed method is suitable and effective for the decision process.

Keyword: Generalized spherical fuzzy number, Einstein aggregation operators, multi-criteria group decision making.

AMS 2010: 03E72, 62C86

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IECMSA - 2020

BLOW UP AND GROWTH OF SOLUTIONS TO A VISCOELASTIC PARABOLIC TYPE KIRCHHOFF EQUATIO

Erhan Piskin ¹, Fatma Ekinci ²

ABSTRACT. In this talk, we study a system of viscoelastic parabolic type Kirchhoff equation with multiple nonlinearities. We obtain a finite time blow up of solutions and exponential growth of solution with negative initial energy.

Keywords: Blow up, growth, parabolic equation.

AMS 2010: 35B40, 74H35, 35L05

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IECMSA - 2020

NONEXISTENCE OF GLOBAL SOLUTIONS FOR THE TIMOSHENKO EQUATION WITH DEGENERATE DAMPING

Erhan Piskin ¹, Fatma Ekinci ²

ABSTRACT. In this work, we consider a Timoshenko equations with degenerate damping term. We prove the nonexistence of global solutions with arbitrary positive initial energy. This result is extensions of earlier results.

Keywords: Nonexistence of solutions, timoshenko equation, degenerate damping.

AMS 2010: 35B40, 74H35, 35L05

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IECMSA - 2020

GLOBAL EXISTENCE AND GENERAL DECAY OF SOLUTIONS FOR COUPLED QUASILINEAR SYSTEM
WITH DEGENERATE DAMPING TERMS

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ABSTRACT. In this talk, we consider a system of two viscoelastic equations with degenerate damping terms. We establish global existence and general decay of solutions under suitable conditions.

Keywords: Blow up, growth, parabolic equation

AMS 2010: 35B40, 74H35, 35L05

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IECMSA - 2020

BLOW UP AND ASYMPTOTIC BEHAVIOUR OF SOLUTIONS FOR A KIRCHHOFF-TYPE EQUATION WITH DELAY AND VARIABLE-EXPONENTS

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ABSTRACT. In this work, we investigate a nonlinear Kirchhoff type equation with time delay and variable exponents. Firstly, we prove the blow up of solutions in a finite time. Then, by applying an integral inequality due to Komornik, we obtain the decay estimates result. Generally, time delays appear in many practical problems such as thermal, biological, chemical, physical and economic phenomena. Several physical phenomena such as flows of electro-rheological fluids or fluids with temperature-dependent viscosity, nonlinear viscoelasticity, filtration processes through a porous media and image processing are modelled by equations with variable exponents of nonlinearity.

Keywords: Blow up, nonlinear equation, delay, variable-exponent.

AMS 2010: 35B40, 74H35, 35L05

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IECMSA - 2020

DECAY AND NONEXISTENCE OF SOLUTIONS FOR A $p(x)$ -LAPLACIAN EQUATION WITH VARIABLE-EXPONENTS AND DELAY TERM

Erhan Pişkin ¹, Hazal Yüksekaya ²

ABSTRACT. In this work, we consider a nonlinear $p(x)$ -Laplacian equation with variable exponents and delay term. Under suitable conditions, we prove the blow up of solutions. We also, obtain the decay estimates result by applying an integral inequality due to Komornik. Time delays arise in many applications, for instance, chemical, physical, biological, thermal and economic phenomena. The problems with variable exponents arise in many branches in sciences such as nonlinear elasticity theory, electrorheological fluids and image processing.

Keywords: Blow up, delay term, variable-exponents.

AMS 2010: 35B40, 35B44, 35L05

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IECMSA - 2020

NONEXISTENCE OF SOLUTIONS TO A LOGARITHMIC NONLINEAR WAVE EQUATION WITH DELAY TERM

Erhan Pişkin ¹, Hazal Yüksekaya ²

ABSTRACT. In this work, we discuss a logarithmic nonlinear wave equation with delay term. We study the blow up of solutions in a finite time. The logarithmic nonlinearity appears naturally in inflation cosmology and supersymmetric field theories, quantum mechanics, and many other branches of physics such as nuclear physics, optics and geophysics.

Keywords: Blow up, wave equation, delay term.

AMS 2010: 35L05, 35L55, 35B44

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IECMSA - 2020

MATHEMATICAL BEHAVIOUR FOR A HIGHER-ORDER KIRCHHOFF-TYPE SYSTEMS WITH LOGARITHMIC NONLINEARITY

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ABSTRACT. In this present, our aim is to understand the characteristics of dynamical behaviour for system higher order Kirchhoff type equations with logarithmic nonlinearities. Based on the potential well method, the main ingredient of this study is to construct several conditions for initial data leading to the solution global existence in different case of energy functional. On the other hand, we estimate the decay rate of energy. The logarithmic nonlinearity is encountered naturally in quantum mechanics, inflation cosmolog, supersymmetric field theories, and a lot of different areas of physics such as, optics, geophysics and nuclear physics [2, 4]. These specific applications in physics attract a lot of mathematical scientists to study equation with logarithmic nonlinearity. The authors discussed the different mathematical behaviour of the different versions of equation and system with logarithmic nonlinearity, see [1, 3, 5, 6, 7].

Keywords: System of higher-order, kirchhoff type equation, logarithmic nonlinearity.

AMS 2010: 35G20, 35L55.

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IECMSA - 2020

COVID-19 AND THE FIBONACCI NUMBERS

Furkan Semih Dündar ¹

ABSTRACT. The World has been shaken by the appearance of a new type corona virus in December 2019 in the city of Wuhan, China. The virus has then spread around the Globe causing many infections and fatalities. In this paper we have given a simple model for the spread of virus in terms of Fibonacci numbers.

Keyword: Covid-19, Fibonacci numbers.

AMS 2010: 11B39.

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IECMSA - 2020

SECOND-ORDER GENERAL DIFFERENTIAL EQUATION FOR MULTI-LEVEL ASYMPTOTICS

Fatih Say ¹

ABSTRACT. Asymptotic representation of differential equations and integrals has attracted the increasing attention of many researchers, for example, see [1, 2, 3, 4, 5]. A delicate analysis of the second-order general equation leads to its asymptotic representation while the asymptotic parameter approaches zero. In this study, we will demonstrate an effective way of obtaining the asymptotic representations of the particular type of equations and provide an introduction to this.

Keyword: Asymptotic expansion, multi-level asymptotics, summability.

AMS 2010: 34E15, 34M30.

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IECMSA - 2020

SOME RECENT DEVELOPMENTS ON OPTIMAL SUCCESSIVE COMPLEMENTARY EXPANSION WITH NUMERICAL COMPUTATIONS

Fatih Say ¹

ABSTRACT. Recent remarkable works, especially in the last three decades, on singular perturbation theory have revealed many features for long-standing problems of the subject where their exact solutions cannot be found. Interpretation of singular differential equations or difference equations has been the subject of considerable research interest in both applied mathematics and theoretical physics. The successive complementary expansion method (SCEM) [1] is among the methods dealing with the interpretation of those equations, in as much detail as possible, whose behavior cannot be captured by traditional perturbative analysis. Recently introduced optimal SCEM for singular differential equations [2] provides the asymptotic behavior of solutions of such singular equations. In this talk, some recent advances on the optimal SCEM along with the numerical computation and efficiency of the method will be illustrated.

Keyword: Successive complementary expansion, asymptotic analysis.

AMS 2010: 34E05.

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IECMSA - 2020

NUMERICAL EXPERIMENTS WITH SPLINE COLLOCATION METHOD FOR 2D REACTION-DIFFUSION PROBLEM ON THE DIFFERENT TYPE MESHES

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ABSTRACT. Collocation with biquadratic C^1 -splines for a singularly perturbed reaction-diffusion problem in two dimension is studied. Second order a posteriori error estimation is obtained. Numerical results on the different layer-adapted meshes are analyzed.

Keywords: Reaction-diffusion problems, collocation method, supremum norm, singular perturbation and layer-adapted meshes.

AMS 2010: 65N15, 65N35, 65N50

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IECMSA - 2020

A GENERALIZED (3+1)-DIMENSIONAL KADOMTSEV-PETVIASHVILI EQUATION VIA THE MULTIPLE EXP-FUNCTION SCHEME

İlker Burak Giresunlu ¹

ABSTRACT. In this paper, multiple exp-function scheme is offered to construct exact multi-soliton solutions of nonlinear partial differential equations. To explain the effectiveness of the method, we have considered a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation. As a result, we get one-wave, two-wave and three-wave soliton solutions. Also, multi-wave solitons have been sampled by plotting the solutions.

Keyword: a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation, soliton solutions, multiple exp-function method, multiple wave solutions.

AMS 2010: 35C08, 83C15.

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IECMSA - 2020

ON POPOVSKI-LIKE METHODS FOR THE SIMULTANEOUS DETERMINATION OF POLYNOMIAL ZEROS

Ivan Petković ¹, Dorde Herceg ²

ABSTRACT. Algebraic polynomials and polynomial zeros are of great importance from theoretical as well as practical point of view so that a great attention has been devoted for decades to the design of numerical algorithms for finding polynomial zeros, see, e.g., [1]–[5], [7]. The aim of this paper is to present a new very efficient family of simultaneous methods for finding simple (real or complex) zeros of an algebraic polynomial. We focus on almost forgotten Popovski’s one-parameter family of third order method [6] from 1980. Starting from this family for finding a single zero, by a suitable transformation we construct a new one-parameter family of Popovski’s type for the simultaneous determination of all simple zeros of a polynomial. The order of convergence of new simultaneous methods is four, five or six, depending on the type of the used approximation. The great benefit of these methods is their high order of convergence obtained without any additional calculations of the given polynomial P and its derivatives P' and P'' , which points to a high computational efficiency of the proposed one-parameter root-finding methods.

Taking some specific values of the involved parameter, Popovski-like family generates simultaneous Halley-like method, Chebyshev-like method and Euler-like method. We employ computer algebra system *Mathematica* to perform convergence analysis and numerical experiments of the proposed family. Finally, using computer visualization of some particular methods from Popovski-like family and trajectories of Aberth’s type [1], we conjecture their globally convergent properties.

Keyword: Nonlinear equations, polynomial zeros, simultaneous methods, accelerated convergence.

AMS 2010: 65H05

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IECMSA - 2020

SOME GENERALIZATIONS OF AK MODEL WITH DATA ANALYSIS

Jelena Stanojević ¹, Katarina Kukić ², Nemanja Vuksanović ³

ABSTRACT. In this paper we briefly give one overview of the economic AK model, which is one simply modification of the Solow model of the increasing growth and we remark that solution in that case can be found with logistic equation. The main result in this paper is generalization of AK model, through three modifications of that model: modified AK model, AK model and government and simple chaotic AK model with increasing returns. Two of that modifications we can reduce to generalized logistic equation, which is mathematical generalization of the logistic equation. In the last section, we give data analysis with real data, estimate appropriate parameters of the models and we compared that two approaches: simple AK model and one of its generalization.

Keyword: logistic equation, Solow model, AK model, generalized logistic equation, modified AK model.

AMS 2010: 39A05, 39A60

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IECMSA - 2020

ON AN INITIAL AND NONLOCAL BOUNDARY CONDITION FOR A MIXED TYPE EQUATION

Khanlar R. Mamedov ¹, Veysel Kılınc ²

ABSTRACT. We consider the mixed type equation

$$(1 - sgn t) u_{tt} + (1 + sgn t) u_t - 2u_{xx} = 0, \quad (1)$$

in a rectangular domain

$$\mathcal{D} = \{x, t : 0 < x < t, -\alpha < t < \beta\},$$

where $\alpha > 0$, $\beta > 0$ are given reel numbers with the initial condition

$$u(x, -\alpha) = \psi(x), \quad 0 < x < 1, \quad (2)$$

and the nonlocal boundary conditions

$$u(0, t) = 0, \quad -\alpha \leq t \leq \beta, \quad (3)$$

$$\int_0^1 xu(x, t) dx = 0, \quad -\alpha \leq t \leq \beta. \quad (4)$$

It is encountered with parabolic - hyperbolic, elliptic- hyperbolic type equations in electromagnetic events, gas dynamics and similar non-homogenous processes [1-4]. The nonlocal boundary conditions show that physical process not only at the point but also at the whole object. The boundary conditions are examined in [5-7] and many other works for different mathematical physics equations.

In this work, mixed type equation (1) is considered. Firstly, the integral condition (4) is reduced to the nonclassical point boundary condition. We establish a uniqueness criterion for the solution constructed as the sum of Fourier series. The existence of the solution and the stability of the solution is shown.

Keyword: mixed type equation, stability, existence theorem, uniqueness theorem.

AMS 2010: Firstly, Secondly..

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IECMSA - 2020

RELAXED MANGSARIAN-FROMOVITZ CONSTRAINT QUALIFICATION IN PARAMETRIC PROGRAMMING

Leonid Minchenko ¹, Sergey Sirotko ², Aliaksandr Pashuk ³

ABSTRACT. We consider a weak constraint qualification, which is called by the relaxed Mangasarian-Fromovitz constraint qualification (RMFCQ) [1] and plays the same role as traditional regularity conditions but do not impose as strong requirements on the structure of the optimization problem as traditional conditions do.

Let us consider a parametric nonlinear programming problem $P(x)$:

minimize $f(x, y)$ subject to $y \in F(x) = i \in I_0$, where $I = \{1, \dots, s\}$ and $I_0 = \{s + 1, \dots, p\}$, functions $f, h_i : R^n \times R^m \rightarrow R$ are continuous together with their partial gradients $\nabla_y f(x, y), \nabla_y h_i(x, y)$.

It is known that the Mangasarian-Fromovitz constraint qualification (MFCQ) [2] at a point $y^0 \in F(x^0)$ implies the Aubin property [3] for the multivalued mapping F at a point $(x^0, y^0) \in \text{gph}F$.

Our goal is to show that this result is also valid under weaker assumptions.

Let $I(x, y) = \{i \in I \mid h_i(x, y) = 0\}$. Introduce the linearized cone $\Gamma(F(x), y)$ to the set $F(x)$ at a point $y \in F(x)$:

$$\Gamma(F(x), y) = \{\bar{y} \in R^m \mid \langle \nabla_y h_i(x, y), \bar{y} \rangle \leq 0 \ i \in I(x, y), \langle \nabla_y h_i(x, y), \bar{y} \rangle = 0 \ i \in I^0\}$$

and the index set

$$I^a(x, y) = \{i \in I(x, y) \mid \langle \nabla_y h_i(x, y), \bar{y} \rangle = 0 \ \forall \bar{y} \in \Gamma(F(x), y)\}.$$

We say that RMFCQ holds at a point $(x^0, y^0) \in \text{gph}F$ if there exists a neighborhood $V(x^0, y^0)$ such that $\text{rank}\{\nabla_y h_i(x, y) \mid i \in I^a(x^0, y^0)\} = \text{const}$ for all $(x, y) \in V(x^0, y^0)$.

Theorem 1. *Suppose that the mapping F is lower semicontinuous at $(x^0, y^0) \in \text{gph}F$ and satisfies RMFCQ at this point. Then F satisfies the Aubin property at the given point.*

Keyword: parametric optimization, constraint qualification, Aubin property.

AMS 2010: 90C30, 90C31.

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IECMSA - 2020

PREDICTION OF SHORT TIME-SERIES BASED ON THE SMART INTERPOLATION WITH CHEBYSHEV POLYNOMIALS

Loreta Saunoriene¹, Minvydas Ragulskis²

ABSTRACT. We introduce a one-step ahead time-series prediction technique based on Chebyshev polynomials and evolutionary algorithms. Chebyshev polynomials form a special class of polynomials [1] widely used in many areas of numerical analysis [1, 2, 3, 4]. Commonly, Chebyshev polynomials are interpolated within a nonuniform time-grid [1, 2] with higher density of the interpolation nodes at the ends of the interpolation interval what helps to decrease the effect of Runge's phenomenon [1] and leads to the smaller extrapolation errors. Interpolation within a non-uniform grid also ensures that the values of the time-series close to the present time-moment have more influence on the predicted future value than older ones. Additionally, we incorporate an internal smoothing procedure [5, 6] into the Chebyshev interpolation scheme what helps to reduce the influence of the noise on a predicted future value. The integration of the internal smoothing into the Chebyshev interpolation scheme requires a construction of novel non-standard cost-functions optimized employing evolutionary optimization algorithms. Finally, we demonstrate the effectiveness of the proposed forecasting algorithm via series of computational experiments with standard real-world time-series. We would like to acknowledge that this research was supported by the Research and Innovation Fund of Kaunas University of Technology (Tspredict, grant No. PP59/2011).

Keyword: Time-series prediction, chebyshev polynomials, evolutionary optimization.

AMS 2010: 62M10, 65D05, 90C27.

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IECMSA - 2020

STREAMLINE-DIFFUSION FINITE ELEMENT METHOD ON GRADED MESH FOR A SINGULARLY PERTURBED PROBLEM

Mirjana Brdar ¹, Ljiljana Teofanov ², Goran Radojev ³

ABSTRACT. In this paper we consider a singularly perturbed elliptic model problem with exponential and parabolic layers posed on the unit square. The problem is solved numerically by the streamline-diffusion finite element method using piecewise bilinear elements on a graded mesh. We give superconvergence property of the method in the induced streamline-diffusion norm with the appropriate choice of the streamline-diffusion parameter.

Keyword: convection-diffusion, singular perturbation, streamline -diffusion, graded mesh, stabilization parameter.

AMS 2010: 65N30, 76R99

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PYTHAGOREAN FUZZY MULTISSET IN ROBOTICS: A THEORETICAL FRAMEWORK

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ABSTRACT. Pythagorean Fuzzy Multisets(PFSMs) is Pythagorean fuzzy set in the framework of multiset. Supposing the sum of the degrees of membership and non-membership is greater than or equal to 1 at any level, then the concept of PFMS is appropriate to handling such scenario. PFMS is a soft computing technique. This soft computing technique could find expression in other multi-criteria decision-making (MCDM) problems.

In its simplest sense, the robot can be described as follows: A robot is a programmable automated machine which can interpret information from the physical environment in order to adapt its behaviour. It has the capacity to interact with the environment and carry out different functions accordingly. All robots have three types of components:

- Control System: such as the controller board
- Sensors: They can read information on the surrounding environment or the robot itself
- Actuators: They produce an effect in the environment for the robot.

When multiple robots are used for completing a task, the system is called a multi robot system.

In present work, the application of PFMS in robotics is investigated. The collaboration of robots was worked with PFMS. The scenario in this study is to explain the system created by robots that navigate and surveillance in a certain region through a central server, with PFMS.

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IECMSA - 2020

RISK ASSESSMENT OF COGNITIVE DEVELOPMENT OF EARLY CHILDHOOD CHILDREN IN QUARANTINE DAYS: A NEW AHP APPROACH

Murat Kirişci ¹, Nihat Topaç ², Musa Bardak ³

ABSTRACT. The world is faced with disasters caused by natural or human effects from time to time. The various political, economic, health, and social consequences of these disasters affect people for different periods of time.

In natural disasters and especially in epidemic diseases, some measures are taken to protect people from the negative effects of the situation. One of the measures that can be taken is quarantine.

The target audience of this study is children aged 5-6 in early childhood. Children of this age group are in the process of gaining skills in expressing their feelings during this period. In addition, the emotional responses of these children can be noticed by a careful observer or even an expert.

The purpose of this study is to evaluate the risks of the effects of quarantine status related to COVID-19 pandemic on cognition and behavior of children staying at home. A new AHP technique was used to assess the risks of the quarantine process in early childhood children.

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IECMSA - 2020

NUMERICAL SOLUTION OF THE POROELASTIC WAVE EQUATION USING FINITE ELEMENT METHOD

Marat Nurtas ¹, Fatima Tokmukhamedova ²

ABSTRACT. The problem of acoustics in porous media in three separated subdomains is studied. In each region different physical properties are assumed: geometry of the pore, viscosity of fluid places in the middle of the two elastic domains. In this task, firstly the solution of differential equations is considered. A mathematical model of these physical phenomena is described by the initial boundary-value problems for complex systems of differential equations in partial derivatives. Then these equations were solved using two numerical methods: finite element method (FEM) and the traditional finite difference method (FDM). Solutions allow to analyse wave propagation phenomena in porous media. The polynomial functions were used as the interpolation basis-test functions in order to get weak formulation for the finite element method. The numerical results of our simulation illustrate that this method is obviously effective, especially if we want research physical problems with complex domains in 2D and 3D spaces.

Keyword: Acoustics equation, poroelastic medium, finite difference method, finite element method, basis functions, mixed medium.

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EXISTENCE AND BLOW UP FOR A NONLINEAR PETROVSKY TYPE EQUATION WITH LOGARITHMIC NONLINEARITY

Erhan Pişkin ¹ Nazlı Irkil ²

ABSTRACT. In this paper, our aim is to work the initial boundary value problem of nonlinear viscoelastic Petrovsky-type equation with logarithmic nonlinearity. Firstly, we prove the local existence of weak solution by using Faedo- Galerkin's method and contraction mapping principle. Later, we derive the finite time blow-up results. The equation with the logarithmic source term is related with many branches of physics. In 1970, the working of Dafermos with viscoelastic term provide a basis to the different papers [3]. The importance of the viscoelastic properties of materials has been realized because of the rapid developments in rubber and plastic industry. Additionally, viscoelasticity influence part on working of biological phenomena. The other important property of viscoelastic material that return back to its original size after a impact force cut off [2]. The studies have intensified about analysis of solutions for a class of viscoelastic equation with logarithmic source term. We refer to work of see [1, 4, 5, 6].

Keywords: Existence, blow up, viscoelastic equation, logarithmic nonlinearity.

AMS 2010: 35L05, 35B40, 35B44.

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IECMSA - 2020

ON EXCELLENT SAFE PRIMARY NUMBERS AND ENCRYPTION

Nazlı Koca ¹, Serpil Halıcı ²

ABSTRACT. In this study, we first included the definition of perfectly safe prime numbers that we created. We then examined the RSA and Rabin cryptosystem, which are the techniques of implementing prime numbers in encrypting any message. Finally, we used these perfectly safe prime numbers, which were first defined, in RSA and Rabin's encryption methods.

Keyword: Prime numbers, distribution of primes, applications of prime numbers.

AMS 2010: 11A41, 11N05, 11N99.

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INVESTIGATION OF GOMPERTZ LAW THROUGH TEMPERED FRACTIONAL CASE

Ramazan Özarslan ¹

ABSTRACT. The main goal of this work is to analyze the effect of tempered fractional derivative on the Gompertz Law, used for determining biomass growth, fermentation, etc.. Furthermore, the results are compared with other fractional derivatives like Caputo and ψ -Caputo fractional derivatives.

Keyword: Gompertz law, laplace transform, tempered fractional calculus.

AMS 2010: 26A33, 74B15

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IECMSA - 2020

DETERMINING ALCOHOL CONCENTRATION IN HUMAN BODY WITH GENERALIZED FRACTIONAL DERIVATIVE

Ramazan Özarslan ¹

ABSTRACT. In this work, a two component system determines the concentration of alcohol in human body is considered within generalized fractional derivative. To obtain the solution of this two component system, a modified Laplace transform is used. Furthermore, two parameters are used for preserving the dimension of quantities. The results obtained are compared with Caputo fractional and integer order counterparts with some illustrations.

Keyword: Fractional differential systems, mathematical modeling, alcohol concentration, mathematical biology.

AMS 2010: 26A33, 92B05.

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IECMSA - 2020

ON A DIFFERENT METHOD FOR DETERMINING THE PRIMARY NUMBERS

Serpil Halıcı ¹, Hamit Cacur ², Nazlı Koca ³

ABSTRACT. In this study, firstly, ordinal numbers of the odd integers were defined to reach prime numbers, and a new prime number sieve was obtained with the help of these numbers. A general formula was given to examine this sieve. Then, by representing the ordinal numbers with the help of matrices, some properties of these numbers were also examined.

Keyword: Prime numbers, prime number sieves, applications of sieves methods.

AMS 2010: 11A41, 11N05, 11N35.

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INVESTIGATION OF JOINT DISTRIBUTION OF THE FIRST MOMENT OF SEMI-MARKOV RANDOM WALK
PROCESS CROSSING LEVEL A ($a > 0$) AND JUMP THROUGH IT

U. Y. Kerimova ¹

ABSTRACT. Using sequence of independent identically distributed random variables is constructed semi-Markov random walk process with positive tendency, negative jumps. The Laplace-Stieltjes transformation of a compatible distribution of the first reaching moment to level a ($a > 0$) and length of jump from the level of constructed process is obtained.

Keyword: random variables, process of semi-Markov random walk, Laplace-Stieltjes transformation.

AMS 2010: 60Jxx.

Introduction

The Laplace-Stieltjes transform and its properties were studied by Geetha K. V. and John J. K. [3]. In [2], the asymptotic method is used to find distributions of the process and its main boundary functionals. In [4] the Laplace transform of the distribution of the first moment of reaching the delaying screen at zero by the semi-Markov process is found. In [5] the first passage of the zero level of the semi-Markov process with positive tendency and negative jumps is found. The Laplace transform for the distribution of this random variable is defined. In [6], the asymptotic behavior of the first moment of crossing a certain level by a semi-Markov random walk is studied. In [7], the asymptotic behavior of the moment of reaching a specified level by a transient onedimensional random walk in a random environment with the delaying screen at zero, whose jumps take three values (-1 , 0 , and $+1$), is studied.

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This article is dedicated to investigation of the Laplace-Stieltjes transformation of joint distribution of the first moment of semi-Markov random walk process with positive tendency crossing level a ($a > 0$) and jump through it

Statement of the problem

Let a sequence of independent and identically distributed pairs of random variables $\{\xi_k, \varsigma_k\}_{k \geq 1}$, $k = \overline{1, \infty}$ defined on a probability space (Ω, F, P) such that ξ_k and ς_k are independent random variables and $\xi_k > 0$, $\varsigma_k > 0$. Using these random variables we will derive the following step processes of semi-Markov random walk:

$$X_z(t) = z + t - \sum_{i=1}^{k-1} \zeta_i, \text{ if } \sum_{i=1}^{k-1} \xi_i \leq t < \sum_{i=1}^k \xi_i, k = \overline{1, \infty}, z \geq 0,$$

$X_z(t)$ process is the (asymptotic) semi-Markov random processes with positive tendency and negative jump.

Let's include the τ_a random variable defined as below:

$\tau_a = \min \{k : X_z(t) \geq a\}$ $k=0,1,2,..$ and $\gamma_a = X(\tau_a) - a$, where τ_a is the first reaching moment to level a ($a > 0$) and is the length of jump from the level of constructed process:

$$\tau_a = \begin{cases} a - z & \text{if } z + \xi_1 > a; \\ \xi_1 + T(\omega) & \text{if } z + \xi_1 < a. \end{cases}$$

We need to find the evident form of the Laplace-Stieltjes transformation of joint distribution of τ_a and γ_a . Let us set Laplace-Stieltjes transformation of joint distribution of τ_a and random variables as:

$$L(\theta, \gamma | z) = E(e^{-\theta \tau_a} | X(0) = z), z \geq 0.$$

The main aim of this study is to express the Laplace-Stieltjes transformation of joint distribution of the first moment of semi-Markov random walk process with positive tendency crossing level a ($a > 0$) and jump through it by means of some probability characteristics of random variables ξ_k and ς_k .

By the additivity of the definite integral, we have

$$\begin{aligned} E\left(e^{-\theta \tau_a} / X_z(0) = (z)\right) &= \int_{\Omega} e^{-\theta \tau_a} P(d\omega / X(0) = z) \\ &= \int_{\{\omega: z + \xi_1 > a\}} e^{-\theta(a-z)} P(d\omega) \\ &+ \int_{\{\omega: z + \xi_1 < a\}} e^{-\theta(\xi_1 + T(\omega))} P(d\omega | X(0) = z + \xi_1 - \zeta_1) \end{aligned}$$

Applying the following change of variables $\xi_1 = t$, $\zeta_1 = y$ and $z + t - y = u \Rightarrow y = z + t - u$ we can find

$$\begin{aligned}
L(\theta|z) &= E\left(e^{-\theta \tau_a} / |X_z(0) = z\right) = e^{-\theta(a-z)} P\{\xi_1 > a - z\} \\
&+ L(\theta|0) \int_{t=0}^{a-z} e^{-\theta t} \int_{y=z+t}^{\infty} d_y P\{\zeta_1 < y\} d_t P\{\xi_1 < t\} \\
&+ \int_{t=0}^{a-z} e^{-\theta t} \int_{u=-\infty}^{z+t} L(\theta|u) d_u P\{\zeta_1 < z + t - u\} d_t P\{\xi_1 < t\}
\end{aligned}$$

Making substitutions, $u = y$, we obtain an integral equation of Laplace-Stieltjes transformation of joint distribution of the first moment of semi-Markov random walk process with positive tendency crossing level a ($a > 0$) and jump through it by means of some probability characteristics of random variables ξ_k and ζ_k .

$$\begin{aligned}
L(\theta|z) &= e^{-\theta(a-z)} P\{\xi_1 > a - z\} \\
&+ \int_{t=0}^{a-z} e^{-\theta t} \int_{y=z+t}^{\infty} d_y P\{\zeta_1 < y\} d_t P\{\xi_1 < t\} L(\theta|0) \\
&+ \int_{t=0}^{a-z} e^{-\theta t} \int_{y=0}^{z+t} L(\theta|y) d_y P\{\zeta_1 < z + t - \alpha\} d_t P\{\xi_1 < t\}.
\end{aligned}$$

Therefore we obtain an integral equation of Laplace-Stieltjes transformation of joint distribution of the first moment of semi-Markov random walk process with positive tendency crossing level a ($a > 0$) and jump through it by means of some probability characteristics of random variables ξ_k and ζ_k . This integral equation can be solved by the method of successive approximation. However, the obtained expressions are not suitable for applications. Therefore, we can restrict the class of walks in order to obtain the explicit form.

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EXACT SOLUTIONS OF A CONFORMABLE FRACTIONAL EQUATION VIA IMPROVED BERNOULLI SUB-EQUATION FUNCTION METHOD

Volkan Ala ¹, Ulviye Demirbilek ², Khanlar R. Mamedov ³

ABSTRACT. The aim of this study is to present several new exact solutions of a conformable fractional nonlinear partial differential equation. For this purpose, the improved Bernoulli sub-equation function method has been used. The obtained results are shown by way of the 3D-2D graphs and contour surfaces for suitable values. The results show that the proposed method is powerful and applicable for solving different types of conformable fractional partial differential equations.

Keywords: Conformable fractional derivative, improved bernoulli sub-equation function method.

AMS 2010: 35C08, 34K20, 32W50.

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THE NEW TYPE OF THE STATISTICAL CONVERGENCE OF THE FUNCTIONS DEFINED ON THE TIME SCALE PRODUCT

Victor Martinez-Luaces ¹

ABSTRACT. In this paper the mathematical models corresponding to a couple of chemical problems -chemical kinetics and mixing problems- are studied. Both situations can be represented by using directed graphs and multigraphs [1] and both of them -under certain conditions- are modeled by linear ODE systems, which associated matrices, so-called FOCKM and MP [2-3] have important similarities [4]. Moreover, several formulas and equations can be considered as analogous in both cases. As a consequence of the previous facts, some FOCKM and MP theorems have similar statements, while others need to be adapted to the corresponding chemical situation. These similarities and differences in mathematical models are deeply analyzed in the article.

Keywords: Chemical kinetics, mixing problems, linear ODE systems, directed graphs and multigraphs.
AMS 2010: 34A30, 05C20, 80A30.

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INVESTIGATION OF RANDOM ZIKA VIRUS TRANSMISSION WITH MODIFIED RANDOM DIFFERENTIAL TRANSFORMATION METHOD

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ABSTRACT. In this study, a deterministic model of Zika virus transmission is investigated under random conditions. The random model, obtained as a system of random differential equations, is analyzed by using the random differential transformation method (rDTM). The approximate solutions obtained with rDTM are modified with Laplace-Pade technique to achieve a better approximation. Numerical and simulation results show the improvement of the approximation through the use of modified random differential transformation method.

Keyword: Differential transformation method, random differential equation, simulation.

AMS 2010: 34F05, 92D30, 37M05

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IECMSA - 2020

AN ANALYTICAL APPROACH TO AN ELASTIC CIRCULAR ROD EQUATION

Zehra Pinar ¹

ABSTRACT. Recently, studies on the size-dependent dynamic models of small-scaled rods are arising, especially in optics. In this work, the generalized form of the nonlinear elastic circular equation is considered. Solutions obtained by different methods are discussed and illustrated in details. The considered studies are the special case of the elastic rod equation, such as magneto-electro circular equation, which has not been studied in the literature with the proposed methods. Therefore, for further analytical and numerical analyses for waves in such two-phase media, the obtained results could play important role.

Keyword: Analytical method, the elastic rod equation, travelling wave solutions.

AMS 2010: 35DXX, 35QXX, 35CXX .

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DISCRETE MATHEMATICS



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DISJUNCTIVE TOTAL DOMINATION SUBDIVISION NUMBER OF SOME GRAPHS

Canan Çiftçi ¹

ABSTRACT. It is often of interest to know how the value of a graph parameter is affected when a small change is made in a graph. Under the consideration that how many modifications influence the change of the domination number, some graph modification parameters are defined such as bondage number [1], reinforcement number [2] and domination subdivision number [3]. The concept of the subdivision was first introduced for domination and then extended for some variations of graph domination [5, 6, 7]. In this study, effects on the disjunctive total domination are concentrated on. A set S of vertices in a graph G is a disjunctive total dominating set of G if every vertex is adjacent to a vertex of S or has at least two vertices in S at distance two from it. The disjunctive total domination number is the minimum cardinality of such a set. Çiftçi and Aytaç [8] defined the disjunctive total domination subdivision number of a graph G as the minimum number of edges which must be subdivided (every edge in G is subdivided exactly once) to increase the disjunctive total domination number of G . In this study, the disjunctive total domination subdivision number of some graphs are determined.

Keyword: Domination, disjunctive total domination, disjunctive total domination subdivision.

AMS 2010: 05C69, 05C35.

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IECMSA - 2020

DISJUNCTIVE TOTAL BONDAGE NUMBER OF GRAPHS

Canan Çiftçi ¹, Aysun Aytac ²

ABSTRACT. In network design, as well as a parameter is significant to study, it is also significant to know the effects on the value of the parameter when a graph is modified for example by deleting a vertex or an edge or adding an edge. One of the graph parameters is domination. The bondage number [1, 2] is defined as the minimum number of edges that must be removed from a graph in order to increase the domination number. There are many variations of domination, one of which is disjunctive total domination [3]. A set S of vertices in a graph G is a disjunctive total dominating set of G if every vertex is adjacent to a vertex of S or has at least two vertices in S at distance two from it. The disjunctive total domination number is the minimum cardinality of such a set. In this study, we consider disjunctive total bondage which was first defined in [4] and we present some bounds for the disjunctive total bondage number of a graph.

Keyword: Domination, disjunctive total domination, disjunctive total bondage.

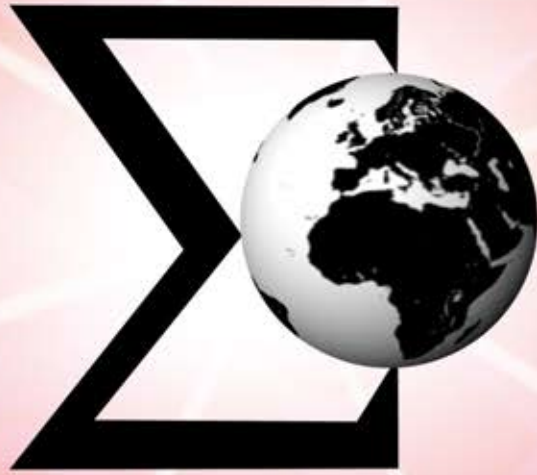
AMS 2010: 05C69.

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GEOMETRY



IECMSA - 2020

SOME CHARACTERIZATIONS FOR THE B-LIFT CURVE

Anıl Altınkaya ¹, Mustafa Çalışkan ²

ABSTRACT. In this study, firstly, the B-lift curve α_B of a curve α defined and then the Frenet vector fields T_B , N_B , B_B and the curvature κ_B and the torsion τ_B of the B-lift α_B of a curve α calculated. Finally these operators are compared with each other for curve α .

Keyword: B-lift, frenet formula, curvature, torsion.

AMS 2010: 51B20, 53A15, 53A04, 53A05.

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IECMSA - 2020

COMMUTATIVE OCTONION MATRICES

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ABSTRACT. In this article, commutative octonions the matrix representations of commutative octonions and their properties are described. Firstly, definitions and theorems are given for commutative octonion matrices using commutative quaternions matrices. Adjoint matrices, eigenvalues and eigenvectors of these matrices are investigated. Then the Gersgorin Theorem is proved using these eigenvalues and eigenvectors. Finally, the result that found in Gershgorin Theorem supported in the example.

Keyword: Commutative octonions, fundamental matrices, commutative octonion matrices.

AMS 2010: 15A27, 17A35.

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IECMSA - 2020

SINGULAR MAXIMAL TRANSLATION HYPERSURFACES IN LORENTZ-MINKOWSKI SPACE

Ayla Erdur ¹, Muhittin Evren Aydın ², Mahmut Ergüt ³

ABSTRACT. Let the pair $(\mathbb{R}^2, \langle \cdot, \cdot \rangle)$ denote the Euclidean 2-space, $\beta = \beta(s)$ a curve in \mathbb{R}^2 and $a \in \mathbb{R}^2$ a fixed unit vector. For some real constant α , the curve β is called α -catenary if the following holds

$$\kappa(s) = \alpha \frac{\langle n, a \rangle}{\langle \beta, a \rangle} \quad (1)$$

where κ and n are the curvature and unit principle normal vector field of β . By a change of coordinate we may take $a = (0, 1)$ and $\beta(s) = (s, \phi(s))$, $\phi : I \subset \mathbb{R} \rightarrow \mathbb{R}^+$. In this case, Eq. (1) writes

$$\frac{\phi''}{1 + (\phi')^2} = \frac{\alpha}{\phi}. \quad (2)$$

In case $\alpha = 1$ in Eq.(2), we have well known *catenary equation*. Physically, Eq. (2) defines a configuration in which a uniform chain, whose two ends are fixed and hanged under its own weight, is in balance with the effect of the gravitational field. So, a α -catenary actually minimizes potential energy under the influence of gravity force, in other words has the lowest center of gravity.

In this talk, by generalizing this property of the catenary, we are interested in the problem of characterizing spacelike translation hypersurfaces with the lowest center of gravity in the halfspace $x_{n+1} > 0$ in Lorentz-Minkowski space \mathbb{R}_1^{n+1} , which satisfy the following equation so called singular maximal hypersurfaces equation :

$$nH = \alpha \frac{\langle \xi, a \rangle_L}{\langle \phi, a \rangle_L}, n \geq 2, \quad (3)$$

where $a \in \mathbb{R}_1^{n+1}$ is fixed timelike unit vector, ξ and H the timelike Gauss map and the mean curvature of the smooth immersion ϕ of an oriented spacelike hypersurface M^n .

Keyword: α -catenary, singular maximal surface, translation graphs.

AMS 2010: Firstly 53A10, Secondly 53C42.

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ON CANAL SURFACES OBTAINED BY THE CURVES IN THE SPACE FORMS

Ali Uçum ¹

ABSTRACT. Canal surfaces was firstly investigated by Monge in 1850. A canal surface is defined as a surface formed as the envelope of a family of spheres whose centers lie on a space curve $C(t)$ with radius $r(t)$. If the radius $r(t)$ is constant, then the canal surface is called as pipe surface or tubular surface.

In [1] and [2], the author defines the Frenet equations of the curves on the sphere S^2 , hyperbolic sphere H^2 and lightlike cone.

In this paper, we consider the canal surfaces whose center curves are the curve on the sphere S^2 , in the hyperbolic space H^2 and lightlike cone. Also we classify the such curves with constant curvature and give the related examples for canal surfaces.

Keyword: Canal surfaces, tubular surfaces, hyperbolic curve, lightlike cone.

AMS 2010: 53A35, 53C42, 53C50.

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CLASSIFICATION OF FRAMED RECTIFYING CURVES IN EUCLIDEAN SPACE

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ABSTRACT. There are many studies on regular rectifying curves in classical differential geometry and important results have been obtained. As smooth curves with singular points, we consider framed curves in the Euclidean space. A framed curve in the 3-dimensional Euclidean space is a smooth space curve with a moving frame[8]. We study framed rectifying curves via the dilation of unit speed framed curves on the unit sphere S^2 in the Euclidean space E^3 . Also, the result of this dilation of framed curves is the framed rectifying curve or not. Classifications for this situation are given. Finally, we give some related examples with their figures.

Keyword: Framed curves, singular point, framed rectifying curves, framed spherical curves, dilation of framed curves.

AMS 2010: 53A04, 58K05, 58K030.

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SURFACES WITH CONSTANT MEAN CURVATURE ALONG A CURVE IN 3-DIMENSIONAL EUCLIDEAN SPACE

Ergin Bayram ¹, Hüsnü Çoşanoğlu ²

ABSTRACT. In this study, the sufficient conditions are obtained to find surfaces that pass through any given curve in 3-dimensional Euclidean space and whose mean curvature is constant along this curve. For this purpose, firstly, surfaces passing through the given curve are expressed parametrically with the help of the tangent vector field, the principal normal vector field and the binormal vector field of the Frenet frame of the given curve, and the so called marching scale functions which are real valued C^1 functions of two variables. The mean curvature of these surfaces along the given curve was calculated in terms of curvature and torsion of the given curve and, marching scale functions and their partial derivatives. Sufficient conditions are obtained to keep the mean curvature constant along the given curve. Some examples are given.

Keyword: Surface family, constant mean curvature, Euclidean 3-space.

AMS 2010: 53A04, 53A05.

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IECMSA - 2020

ON THE LIGHTCONE FRAME IN MINKOWSKI 3-SPACE

Ergin Bayram ¹

ABSTRACT. There are three types of curves in Minkowski 3-space, e.g. timelike, spacelike and lightlike curves. A mixed type curve is a regular curve, and there are both non-lightlike points and lightlike points in a mixed-type curve. Using lightcone frame for mixed-type curves we study some fundamental properties of surfaces.

Keyword: Mixed-type curves, the lightcone frame, Minkowski 3-space.

AMS 2010: 53A35, 53B30, 51B20.

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IECMSA - 2020

ON SLANT RULED SURFACES AND TANGENT BUNDLE OF UNIT 2-SPHERE

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ABSTRACT. In this study, firstly, new types of ruled surfaces called slant ruled surfaces generated by natural lift curves are defined by using E. Study mapping and the isomorphism between the unit dual sphere, DS^2 and the subset of tangent bundle of unit 2- sphere, $T\bar{M}$. Secondly, some characterizations for a regular ruled surface to be a slant ruled surface in Euclidean 3- space are denoted. Finally, an example is given to support the obtained results.

Keyword: Ruled surface, natural lift curve, slant ruled surface.

AMS 2010: 51A04, 53A25, 14J26.

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HOMOTHETIC MOTIONS VIA GENERALIZED TRICOMPLEX NUMBERS

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ABSTRACT. In this paper, we define the generalized tricomplex numbers and give some algebraic properties of them. By using the matrix representation of generalized tricomplex numbers, we define the homothetic motion on the hypersurface M in eight dimensional generalized linear space $\mathbb{R}_{\alpha\beta\gamma}^8$. It is shown that this is a homothetic motion. Also, it is found that the motion defined by a regular curve of order r and derivations curves on the hypersurface M has only one acceleration center of order $(r - 1)$ at every t -instant.

Keyword: Tricomplex number, generalized tricomplex numbers, homothetic motion.

AMS 2010: 53A05, 53A17.

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ALGEBRAIC TECHNIQUES FOR LEAST SQUARES PROBLEMS IN ELLIPTIC COMPLEX MATRIX THEORY AND APPLICATIONS

Hidayet Hüda Kösal ¹, Müge Pekyaman ²

ABSTRACT. In this study, we derive the expressions of the minimal norm least squares solution for the elliptic complex matrix equation $AX = B$ by using the real representation of RB matrices, and the Moore-Penrose generalized inverse. To prove the authenticity of our results and to distinguish them from existing ones, some illustrative examples are also given.

Keyword: Elliptic complex numbers, real representations, least squares problems.

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TUBULAR SURFACES ASSOCIATED WITH FRAMED BASE CURVES

Kemal Eren ¹, Önder Gökmen Yıldız ², Mahmut Akyiğit ³

ABSTRACT. The aim of this paper is to examine tubular surfaces with framed base curves, which play an important role in design to guide the studies on the tubular surface. We build the structure of the tubular surface which has singular points. In section 2 and 3, we give a brief exposition of framed base curves and framed surfaces, respectively. In the fourth section our main results are stated and proved. Moreover section 4 normal of the tubular surface, mean and Gauss curvatures are found and the characterization of the parameter curves on the surface are given. Finally, we have expressed the tubular surface with a framed base curve with an example.

Keyword: Tubular surfaces, frame base curve, geodesic curve, asymptotic curve, line of curvature, Gaussian and mean curvature

AMS 2010: 53A10, 53C50

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IECMSA - 2020

ON CURVE PAIRS OF TZITZEICA TYPE

Kemal Eren ¹, Soley Ersoy ²

ABSTRACT. The most important curve pairs in differential geometry are involute evolute, Bertrand and Mannheim curve pairs. In this study, for each of these special curve pairs, the condition of the conjugate of the original curve to be a Tzitzeica curve in Euclidean 3-space is formulated. Moreover, under consideration of the special states of the curvatures of the original curve of curve pairs, the condition of the conjugate curve to be a Tzitzeica curve is investigated. Especially, if a curve is a planar curve, circle or helix, it is found whether its conjugate satisfies the condition of being Tzitzeica curve.

Keyword: Tzitzeica curve, involute evolute curve, Bertrand curve, Mannheim curve

AMS 2010: 53A04, 53A05

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IECMSA - 2020

A NOTE ON D-HOMOTHEIC DEFORMATION ON ALMOST PARACONTACT METRIC MANIFOLDS

Mehmet Solgun ¹

ABSTRACT. In this work, we introduce the notion of D-homothetic deformation on almost paracontact metric manifolds. Besides, we show that the structure after the deformation is also almost paracontact metric structure. Moreover, we investigate the classes of the structures after deformation for some certain conditions.

Keyword: Almost paracontact structure, d-homothetic deformation, almost paracomplex structure.

AMS 2010: 53C15, 53C25, 53C50.

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IECMSA - 2020

FRENET CURVE COUPLES IN THREE DIMENSIONAL LIE GROUPS

Osman Zeki Okuyucu ¹

ABSTRACT. In this study, we examine the possible relations between the Frenet planes of given two curves in three dimensional Lie groups with left invariant metric. Also, we give some characterizations for these curves. Moreover, we introduce these notions for bi-invariant metric case.

Keyword: Curves in Lie groups, Curvatures, Frenet plane.

AMS 2010: 53A04, 22E15.

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IECMSA - 2020

ON THE MATRIX REPRESENTATION OF BEZIER CURVES AND DERIVATIVES IN E^3

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ABSTRACT. In this study we have examined, the coefficient matrix of a general Bezier curve. It's derivatives have been examined too with matrix form based on the control points in E^3 . Also a simple way has been given to find the control points of any 4th degree Bezier curve.

Keyword: Bezier curve, control points, matrix representation.

AMS 2010: 53A04, 53A05.

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IECMSA - 2020

MANNHEIM CURVES IN E^3 AND SPINORS

Tülay Erişir ¹

ABSTRACT. In this paper, the spinors with two complex components have been studied and the spinor representations of Mannheim curves in E^3 have been obtained. Firstly, the spinor representations of Frenet vectors of curve in three dimensional Euclidean space E^3 have been introduced. Moreover, the Mannheim curves corresponding two spinor with complex components have been chosen. So, the relations between the spinors corresponding to the Mannheim curves have been researched. Finally, an example which crosscheck to theorems throughout this study has been given.

Keyword: Spinors, mannheim curves.

AMS 2010: 11B39, 11R52.

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IECMSA - 2020

ABOUT LORENTZ TRANSFORMATIONS WITH ELLIPTIC BIQUATERNIONS

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ABSTRACT. In this study, the Lorentz transformations which are in accordant with special relativity have been examined for the first time with elliptic biquaternions. Since the elliptic biquaternions contain the complex structure, it is quite beneficial to examine with elliptic biquaternions the Lorentz transformations which one of the building blocks of relativistic physics via elliptic biquaternions. Therefore, as a result of relativistic transformation relation, it has been seen that the Lorentz transformations can be expressed with elliptic biquaternions and some special results have been given. In addition, matrix representations of obtained mathematical expressions are given. Thanks to the matrix representations of elliptical biquaternions, the property of commutativeness which is not valid for elliptic biquaternions has been eliminated and these representations provide easiness for relativistic transformation relation. In this context, the presented method in this article is very useful in many other areas of physics such as relativistic electromagnetism.

Keyword: Lorentz transformations, elliptic biquaternions.

AMS 2010: 11R52, 00A79

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IECMSA - 2020

A STUDY OF ELLIPTIC BIQUATERNIONIC ANGULAR MOMENTUM AND DIRAC EQUATION

Zülal Derin ¹, Mehmet Ali Güngör ²

ABSTRACT. In this article, we deal with the Dirac equation and angular momentum, which have an important place in physics, in terms of elliptic biquaternions. Thanks to the elliptic biquaternionic representation of angular momentum, we have expressed some useful mathematical and physical results. We obtained the solutions of the Dirac equation with elliptic Dirac matrices. Then, we expressed the elliptic biquaternionic rotational Dirac equation. This equation could be interpreted as the combination of rotational energy and angular momentum of the particle and antiparticle. Therefore, we also discuss the rotational energy momentum in the Euclidean space the elliptic biquaternionic form of the relativistic mass. Further, we expressed the spinor wave function with elliptic biquaternions. Accordingly, we also showed elliptic biquaternionic rotational Dirac energy-momentum solutions through this function.

Keyword: Dirac equation, elliptic biquaternions, angular momentum.

AMS 2010: 11R52, 00A79

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MATHEMATICS EDUCATION



IECMSA - 2020

TASKS ENRICHMENT, MODELING PROBLEMS AND INVERSE REFORMULATIONS: AN EXPERIENCE WITH PROSPECTIVE TEACHERS IN SPAIN

Victor Martinez-Luaces ¹, Jose Antonio Fernandez-Plaza ², Luis Rico ³

ABSTRACT. This article is devoted to study the enrichment of tasks by prospective teachers, focusing on the reformulation of direct problems. For this purpose, we worked with a population of 74 students of the Master's Degree in Teaching Secondary Education offered by the University of Granada, Spain [1-2]. In order to guide the participants, several examples of possible reformulations were provided all of them posed in an inverse form although the subjects were allowed to answer without any constraint, only focusing on producing richer proposals. Some prospective teachers were very creative, but the majority preferred to copy or to adapt the given examples [3], simply making small changes in the geometry and/or in the given conditions.

Keyword: Tasks enrichment, teaching training courses, inverse modeling problems.

AMS 2010: 97B50, 97D50, 97C70.

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STATISTICS



IECMSA - 2020

COMPARING APPROACHES FOR APPROXIMATING CONTINUOUS RANDOM DISTRIBUTIONS WITH APPLICATION IN RELIABILITY ENGINEERING

Alessandro Barbiero ¹, Asmerilda Hitaj ²

ABSTRACT. In many problems of applied probability, it is a common procedure to represent a continuous random variable through a finite number k of points, that is, approximating it with a discrete random variable. This may occur when the solution to a complex problem involving several random variables cannot be derived in an analytic closed-form; in this case, approximation through discretization is a valid alternative to Monte Carlo evaluation.

Many discretization techniques have been proposed so far, which possibly try to retain different features of the original continuous distribution. We can roughly classify them into two main broad classes. The techniques belonging to the first class try to match the first moments of the original distribution: among them, we find an early method based on the Gaussian quadrature [1], which, despite its name, can be applied to other distributions than the Gaussian and matches all the first $2k - 1$ moments; the techniques belonging to the second class try instead to preserve the cumulative distribution function (or, equivalently, the survival function) of the original distribution (see, e.g., [2]).

In this work, we revise several techniques, highlighting their pros and cons, and empirically investigate their performance through a comparative study applied to a well-known engineering problem, formulated as a stress-strength model, with the aim of assessing their feasibility and accuracy (in terms of mean absolute deviation) in recovering the value of the reliability parameter over a large array of artificial scenarios. The results overall reward a recently introduced method [3] as the best performer, which derives the discrete approximation as the solution to a constrained non-linear optimization, preserving the first two moments of the original distribution.

Keyword: Discrete approximation, Gaussian quadrature, reliability parameter, stress-strength model.

AMS 2010: 62-XX, 62E17

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IECMSA - 2020

INFERENCE FOR $P(X > Y)$ UNDER NON-IDENTICAL COMPONENT STRENGTHS BASED ON THE RAYLEIGH DISTRIBUTION

Çağatay Çetinkaya¹

ABSTRACT. The simple stress-strength reliability model contains a strength variable X and a stress variable Y which is exposed to it. Such a system will properly function when X exceeds Y and $R = P(X > Y)$ denotes to the system reliability. This model can be extended for the systems with two or more components and denoted by multicomponent stress-strength models [1]. However, these reliability models mostly have unrealistic assumptions such as they have independently and identically distributed components. Since these assumptions are not quite realistic in most cases due to different structures of the system components, Johnson [2] was introduced the system reliability with non-identical component strengths and Pandey et al.[3] generalized to more than two groups of components. Recent studies considered these reliability models under various cases.

On the other hand, as a special case of the two parameter Weibull distribution and mostly used distribution in reliability studies, Rayleigh distribution has significantly importance for modelling lifetimes which have increasing failure rates. There are many various studies based on Rayleigh distribution in the literature. These studies generally take part in project effort loadings modelling, life testing experiments, wind speed modelling, reliability analysis, communication theory, physical sciences, engineering, medical imaging science, applied statistics and clinical studies. Recently, multicomponent stress-strength reliability estimation under Rayleigh distribution was studied by Rao [4]. As a further study, in this study, we consider a stress-strength model with non-identical strength components under assumptions of Rayleigh distribution. In this purpose, we assume two different categories for strength variables of the system. Maximum likelihood estimation (MLE), method of moments (MOM) estimation and Bayesian estimation procedures are used for inference of this reliability problem. Further, corresponding asymptotic confidence intervals for the MLEs and MOM estimations and the highest posterior density (HPD) credible intervals for Bayesian estimation of reliability are obtained. Proposed estimation methods are compared numerically with simulation studies.

Keyword: Maximum likelihood, methods of moment estimation, rayleigh distribution, stress-strength.
AMS 2010: 62N05, 62F10, 78M05

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IECMSA - 2020

ORTHOGONAL MIXED MODELS AND PRIME BASIS FACTORIALS

Dário Ferreira ¹, Sandra S Ferreira ², Célia Nunes ³, João T Mexia ⁴

ABSTRACT. Prime basis factorial models have the advantage of allowing the simultaneous study of a larger number of interactions compared to classical approaches. However, the procedures for obtaining the sums of squares can be a bit cumbersome. In this work we present a recurrence way for obtaining the sums of squares, for cases with any number of factors and levels, even when it is neither a prime or a power of a prime. We illustrate the method with an application to real data, where the % of total deaths due to cancer, diabets and any circulatory system disease in three European countries, in three years, is compared. We compare our results with the ones obtained using ANOVA.

Keyword: ANOVA, inference, mixed models, prime basis factorial models.

AMS 2010: 62E20, 62F10, 62J10.

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ON MODELING TIME SERIES OF COUNTS USING INAR MODELS

Miroslav M. Ristić ¹, Aleksandar S. Nastić ², Predrag M. Popović ³, Petra N. Laketa ⁴

ABSTRACT. Modeling time series of counts is a topic that arise a lot of interest among researchers and practitioners in recent decades. Since each and every time series has its specificity, there is a necessity to define an appropriate model for the observed series. We will discuss the main approaches in modeling time series of counts, and present some of the recent results in this field of science. The models that we discuss are autoregressive and they are composed of two components: the survival and the innovation. The survival component is the autoregressive part of the model. It is defined by using the thinning operators. We will present models based on different types of thinning operators. Also, we will discuss how the choice of the innovation component influence on the model properties. We will give some comments regarding models dimensionality where we pay special attention on bivariate models. Stationary as well as non-stationary models will be presented, and we will demonstrate the practical aspect of these models on real life data series.

Keyword: Binomial thinning operator, negative binomial thinning operator, stationary model, non-stationary model.

AMS 2010: 62M10.

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CUMULANTS AND THEIR ESTIMATORS IN ADDITIVE MODELS

Patrícia Antunes ¹, Sandra Ferreira ², Dário Ferreira ³, João T Mexia ⁴

ABSTRACT. In this work we will show how to estimate the cumulants of order r of the components of the vectors of the random part of an additive model. We will see how to overcome the estimation problem of the fourth cumulant, through the use of pairs of observation vectors, in order to find an unbiased estimator for the square of variance. Note that this requirement can be discarded if we only want to estimate the cumulants up to the third order. In the particular case where the distributions of the vector components of the random part of the model have parameters of location and dispersion, we will no longer need pairs of models since the cumulant estimators are linear combinations of the central moments. Thus, we can estimate all cumulants of any order, including fourth order, which previously offered estimation problems. We will particularize for the case in which distributions of the vector components are Normal.

Keyword: Cumulants, estimators, mixed models, moments.

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IECMSA - 2020

CONFIDENCE REGIONS AND TESTS FOR NORMAL MODELS WITH ORTHOGONAL BLOCK STRUCTURE

Sandra S Ferreira ¹, Dário Ferreira ², Célia Nunes ³, João T Mexia ⁴

ABSTRACT. We emphasize the using of pivot variables to obtain confidence regions and, through duality, to test hypothesis for variance components, estimable functions and estimable vectors. In deriving confidence regions for the variance components and the estimable vectors we apply the Glivenko-Cantelli theorem and related results to samples of values of pivot variables. Moreover, for estimable vectors we consider families of samples in order to adjust confidence ellipsoids using a similar technique to least square adjustments of linear regressions that may bring some gain relatively to the previous one. A numerical example using real data set is presented to illustrate the methodology developed (confidence intervals [spheres] for estimable functions [vectors]). The nearness of the estimators and the modes is remarkable, validating the numerical results and allowing the safe use of induced densities.

Keyword: Confidence regions, estimable functions, mixed models, orthogonal block structure, variance components.

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EXAMINATION OF PARALLEL AND SERIES CONNECTED COMPONENTS UNDER REPAIRABLE PRINCIPLE

Yunus Gral ¹, Mehmet Grcan ²

ABSTRACT. Investigation of the reliability of technical systems is one of the application areas of stochastic processes. The reliability of a technical system is basically based on two main elements. The first is the connection type of the system, and the second is the distribution of the working times of the components consisting of the system. In this study, system signatures and their reliability will be calculated under the principle of repairable parallel and serial systems consisting of two components. Although there are a limited number of studies in the literature for repairable systems, there is no study on creating the signature of repairable systems. In technical systems where there is no repair principle, although the system signature has limited components, the technical systems working under the repair principle, have infinite components of the system signature. While creating the system signature, the probability that the working time of the component that is in the state of the system failure is greater than the repair time was defined as the parameter ξ . In the application part of the study, under the principle of repair, the system signature, and the reliability of the system were be successfully calculated.

Keyword: Technical systems, system signature, system reliability, repairable technical systems.

AMS 2010: 60H30.

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TOPOLOGY



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SUZUKI TYPE E -CONTRACTION VIA SIMULATION FUNCTIONS IN MODULAR b -METRIC SPACES

Abdurrahman Büyükkaya ¹, Mahpeyker Öztürk ²

ABSTRACT. This study aims to introduce Suzuki type E -contraction mappings with simulation functions in the frame of modular b -metric spaces. Also, some coincidence and common fixed point results are obtained for four mappings using the weakly compatibility property which these results are the extensions and improvements of the existing literature.

Keyword: Modular b -metric space, simulation function, suzuki type contraction, weakly compatible mappings.

AMS 2010: 54H25, 47H10.

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REDEFINING DISORIENTED KNOTS AND LINKS

İsmet Altıntaş¹², Hatice Parlatıcı³

ABSTRACT. The concept of disoriented knot was first introduced by Altıntaş [1] in 2018. In [1], a disoriented knot was defined as the embedding of a disoriented circle with two arcs into 3-dimensional space or 3-dimensional sphere. In this paper, we define a disoriented knot as an embedding of a disoriented circle with a $2n$ arcs into 3-dimensional space or 3-dimensional sphere, and provide diagrammatic methods such as disoriented Reidemeister moves and disoriented Gauss diagrams to create invariants of the diagrams of disoriented knots and links.

Keyword: Disoriented knot, disoriented connected sum, disoriented Reidemeister moves, disoriented Gauss codes, disoriented Gauss diagrams.

AMS 2010: 57M25, 57M27

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A SOFT SET APPROACH TO RELATION

Kemal Taşköprü¹

ABSTRACT. This paper presents a soft set approach to the relations via the soft elements. Here, we introduce soft relation on the collection of soft elements, give basic properties of them and some examples. Also, we investigate the relationships between soft relations and classical relations.

Keyword: Soft relation, order, equivalence.

AMS 2010: 54A05, 54A10, 54H25.

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ON SOFT PARTIAL METRIC

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ABSTRACT. In this paper, an introduction has been made to soft partial metric spaces via soft elements, which is a generalisation of the soft metric. Some properties of soft partial metric and the relationships of soft partial metric, classical metric and soft metric are investigated. Also, a generalised soft metric for non-Hausdorff soft topologies is proposed and a new approach that guides how to expand soft metric implements like the Banach theorem to such topologies is given.

Keyword: Soft set, soft element, soft partial metric.

AMS 2010: 54A05, 54A10, 54H25.

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