Self-Synchronization and Dissipation-Induced Threshold in Collective Atomic Recoil Lasing

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Networks of globally coupled oscillators exhibit phase transitions from incoherent to coherent states. Atoms interacting with the counterpropagating modes of a unidirectionally pumped high-finesse ring cavity form such a globally coupled network. The coupling mechanism is provided by collective atomic recoil lasing, i.e., cooperative Bragg scattering of laser light at an atomic density grating, which is self-induced by the laser light. Under the rule of an additional friction force, the atomic ensemble is expected to undergo a phase transition to a state of synchronized atomic motion. We present the experimental investigation of this phase transition by studying the threshold behavior of this lasing process.

Lasing mediated by collective atomic recoil (CARL) has been predicted as the analogue to free-electron lasing [1] and observed recently [2]. In the experiment an ensemble of cold atoms couples to the two counterpropagating modes of a unidirectionally pumped high-finesse ring cavity and collectively scatters photons between the modes. A collective instability leads to self-amplification and to exponential gain for a light mode on one hand and atomic bunching on the other. In the absence of energy dissipation for the kinetic degrees of freedom, the gain must remain transient. The introduction of a friction force for the kinetic energy of the atoms permits, however, a steady-state operation of CARL at a self-determined frequency (viscous CARL) [3].

CARL represents a system of coupled oscillators. Collective dynamics in networks of weakly coupled systems is a very general phenomenon. The richness of this paradigm is illustrated by the large amount of examples, ranging from physical systems, like arrays of Josephson junctions or phase-locked lasers, to biological systems like cardiac pacemaker cells or chorusing crickets [4]. Kuramoto, Strogatz, and others [5,6] reported synchronization of ensembles of coupled oscillators operating at different frequencies or being subject to stochastic noise, provided their number and their coupling strength exceeds a critical value. That is, cooperative action starts beyond a threshold value of the coupling strength after crossing a nonequilibrium phase transition.

In the present Letter, we investigate the threshold behavior of the CARL as an example of a phase transition of the Kuramoto type [5–7]: If, for viscous CARL, the dissipation (or cooling) mechanism is limited to finite temperatures by some diffusion process, a phase transition occurs in the atomic density distribution when the system is pumped at threshold. Consequently, a minimum

pump power is necessary to start CARL lasing, just like in ordinary lasers. However, in extension to the original Kuramoto model, the collective frequency is self-determined. We report on the observation of such a threshold and characterize it in terms of the theoretical model presented in Ref. [8].

The optical layout of our experiment has been outlined in Ref. [2]. A titanium-sapphire laser is tightly phase-locked to one of the two counterpropagating modes of a ring cavity. The amplitude decay rate of the ring cavity is $\kappa = (2\pi)22$ kHz. In the following, we label the modes by their complex field amplitudes scaled to the field per photon $\alpha_\pm$. The intracavity light power is then $P_\pm = \hbar\omega\delta|\alpha_\pm|^2$, where $\delta = 3.5$ GHz denotes the free spectral range of the cavity. The phase dynamics of the two counterpropagating cavity modes is monitored via the beat signal between the two outcoupled beams: Any displacement of the standing wave inside the ring cavity is translated into an amplitude variation of the observed interference signal, $P_{beat} = \hbar\omega\delta|\alpha_+ + \alpha_-|^2$.

The contrast of the standing wave P_{beat} is weak as compared to the pump power (a few %). Assuming α_+ real and $|\alpha_-| \ll \alpha_+$, the probe beam power can be related to the contrast of the beat signal $\Delta P_{cont} = 4\hbar\omega\delta\alpha_+|\alpha_-|$,

$$P_- = \frac{\Delta P_{cont}^2}{16 P_+}. \tag{1}$$

In our experiment, we load ^{85}Rb atoms from a standard magneto-optical trap into the optical dipole potential, which is generated by a TEM$_{00}$ mode of the unidirectionally pumped ring cavity and tuned to the red of the D_1 line. Typically 2×10^6 atoms are trapped and form a basically homogeneous 4 mm long cloud along the cavity axis around the waist of the cavity field. The cloud

reaches a peak density of about 2×10^9 cm^{-3} and a temperature of a few 100 μK.

A genuine problem of the CARL is the following: In the absence of damping for the external degrees of freedom the CARL process continuously accelerates the atomic center of mass [2,9]. Even though the acceleration decreases because the Doppler-shifted CARL frequency eventually drops out of the cavity resonance, it never reaches a stationary value. In fact, being focused on studies of transient phenomena, the original CARL model [1] does not consider relaxation of the translational degrees of freedom. On the other hand, standard methods of optically cooling atoms are based on controlled dissipation, e.g., optical molasses. Close to resonance the motion of atoms in an optical molasses is well described by a friction force. In our experiment, we harness this dissipation mechanism and subject the dipole-trapped atomic cloud to an optical molasses. We use the laser beams of the magneto-optical trap and tune them 50 MHz below the cooling transition (D_2, $F = 3 \rightarrow F' = 4$). In this situation, the beat signal oscillations quickly reach a stable equilibrium frequency between $\Delta\omega/2\pi = 100$ kHz and 170 kHz, which corresponds to an atomic velocity of seven to 13 cm/s.

In order to observe a threshold behavior in experiment, we adiabatically ramp up and down the intensity of the pump laser. The CARL radiation is monitored by recording the time evolution of the beat frequency of the counterpropagating modes. The curves shown in Fig. 1(a) represent frequency spectra obtained by Fourier transforming the beat signal restricted to successive time-intervals. The peaks'' locations then denote the instantaneous frequency shift of the probe beam, and their heights reflect the standing wave's contrast. The pronounced dependence of the CARL frequency on the pump intensity revealed by the series of Fourier spectra is emphasized in Fig. 1(b). The intensity of the CARL radiation shown in Fig. 1(c) decreases with the pump intensity, but more important is the fact that the curve exhibits a minimum pump intensity required to initiate CARL lasing. Just like in common lasers, only if the energy fed to the system exceeds the losses, the laser emits coherent radiation.

The experimental observations may be discussed in various ways. The model of Ref. [2] to describe the impact of optical molasses on CARL consisted of simply adding a friction force, proportional to a coefficient γ_{fr}, to the equations governing the atomic dynamics. This procedure certainly represents a coarse simplification. For example, it predicts that the atoms quickly bunch under the influence of the molasses and are cooled until the temperature of the cloud is $T = 0$, and it denies the presence of any threshold. In reality, the molasses temperature is limited by diffusion in momentum space, i.e., heating. To account for this heating, one may supplement the dynamic equations for the trajectories of individual atoms
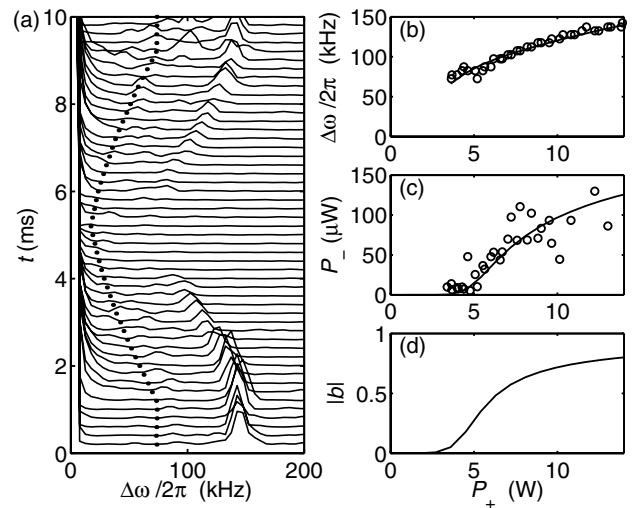

FIG. 1. (a) Sectionwise Fourier-transform of the interference signal P_{beat} with the pump laser power being ramped up and down. The dotted line is proportional to the pump laser power. (b) Dependence of the CARL frequency on the intracavity pump power. (c) Dependence of the probe field intensity on the pump power. The CARL laser threshold is around $P_+ = 4$ W intracavity power. The fitted curves are based on a Fokker-Planck theory outlined in [8]. (d) Calculated bunching parameter. The parameters are $\gamma_{fr} = 4\kappa$, $N = 10^6$, $\Delta_a = -(2\pi)1.7$ THz, and $T = 200$ μK.

[Ref. [2], Eq. (1)] with a stochastically fluctuating Langevin force, $\xi_n(t)$ with $\langle \xi_n(t) \rangle = 0$ and $\langle \xi_n(t)\xi_m(\tau) \rangle = 2\gamma_{fr}^2 D \delta_{mn} \delta(t - \tau)$, where the diffusion coefficient $D = \sigma^2/\gamma_{fr}$ is proportional to the atoms' equilibrium temperature, which is related to the Maxwell-Gaussian velocity spread by $\sigma \equiv 2k\sqrt{k_B T/m}$:

$$\ddot{\theta}_n = 4\varepsilon i U_0 \alpha_+ (\alpha_- e^{-i\theta_n} - \alpha_-^* e^{i\theta_n}) - \gamma_{fr}\dot{\theta}_n + \xi_n. \quad (2)$$

Here we defined $\theta_n \equiv 2kx_n$ as the position of the nth atom along the optical axis normalized to the optical wavelength and assumed the pump laser stabilized on resonance with the cavity α_+ is set real and constant. N is the total atom number and $\varepsilon \equiv \hbar k^2/m$ is twice the recoil frequency shift. The coupling strength U_0 (or single-photon light shift) is related to the one-photon Rabi-frequency g and to the laser detuning from resonance by $U_0 \equiv g^2/\Delta_a$. The functional dependence of the quantity α_- on the order (or bunching) parameter $b \equiv |b|e^{i\psi} \equiv N^{-1}\sum_m e^{i\theta_m}$ is determined by the differential equation

$$\dot{\alpha}_- = -\kappa\alpha_- - iNU_0\alpha_+ b. \quad (3)$$

An alternative to simulating trajectories of individual atoms is to calculate the dynamics of distribution functions. Particularly adequate to the problem of diffusion in momentum space induced by optical molasses is a Fokker-Planck approach. Here the thermalization of the

atomic *density* distribution P towards an equilibrium between cooling and heating is described by the interplay of friction and diffusion [7,8]. In the limit of strong viscous damping, where we may adiabatically eliminate the atomic momenta by setting $\ddot{\theta}_n = 0$, the Fokker-Planck equation associated to the Langevin Eq. (2) reads:

$$\frac{\partial P}{\partial t} = \frac{4i\varepsilon U_0 \alpha_+}{\gamma_{th}} \frac{\partial(\alpha_-^* e^{i\theta} - \alpha_- e^{-i\theta})P}{\partial \theta} + D \frac{\partial^2 P}{\partial \theta^2}, \quad (4)$$

and the bunching parameter is given by $b = \int_0^{2\pi} P e^{-i\theta} d\theta$. The solid lines fitted to the data in Fig. 1 are calculated by numerical integration of this Fokker-Planck equation. Figure 1(d) shows the calculated evolution of the *bunching*. Apparently, the bunching vanishes below the CARL threshold and tends toward one as the pump power is increased. The threshold behavior of the radiation mode is thus intrinsically connected to atomic self-organization. As has already been realized by Kuramoto [5], the Fokker-Planck equation predicts the occurrence of a phase transition. An alternative approach developed by Bonifacio and Verkerk [10] studies the evolution of the atomic *phase-space* distribution described by the Vlasov equation towards equilibrium with a single rate γ_{fr}, and Javaloyes *et al.* [7] found that the Vlasov approach leads to a phase transition.

Reference [8] pointed out that the threshold should depend on various parameters like the atom number, the coupling strength, the friction coefficient, and also on the atomic temperature. However, a proper scaling [8] shows that the threshold is ruled by only two independent quantities. Analytic expressions for the threshold conditions are obtained from a linear stability analysis of the Fokker-Planck equations. As shown in Ref. [8], by introducing the CARL parameter $\rho \equiv (NU_0^2 \alpha_+^2 / 2\varepsilon^2)^{1/3}$, the condition for lasing is given by

$$(\kappa \sigma^2 / \gamma_{fr})(\sigma^2 + \kappa \gamma_{fr})^2 \leq (2\varepsilon\rho)^6. \quad (5)$$

In the good cavity limit $\kappa \gamma_{fr} \ll \sigma^2$, we may neglect the second term in the bracket. At threshold, where the equality in the above equation holds, the CARL parameter becomes $\rho_{thr} = (\kappa/\gamma_{fr})^{1/6}(\sigma/2\varepsilon)$. Therefore the threshold pump power $P_{thr} = \hbar\omega\delta\alpha_{+,thr}^2$ follows with

$$\alpha_{+,thr}^2 = (\frac{\kappa}{\gamma_{fr}})^{1/2} \frac{\sigma^3}{4\varepsilon NU_0^2}. \quad (6)$$

The threshold power and the threshold CARL frequency are related by [8] $\Delta\omega_{thr}^3 = (2\varepsilon\rho_{thr})^3 \kappa/\gamma_{fr}$. As a consequence, the CARL frequency is independent of the atom-field coupling,

$$\Delta\omega_{thr} = \sigma(\kappa/\gamma_{fr})^{1/2}. \quad (7)$$

Figures 2(a) and 2(b) show measurements of the threshold pump power as a function of the laser detuning and the total atom number. Both experimental curves
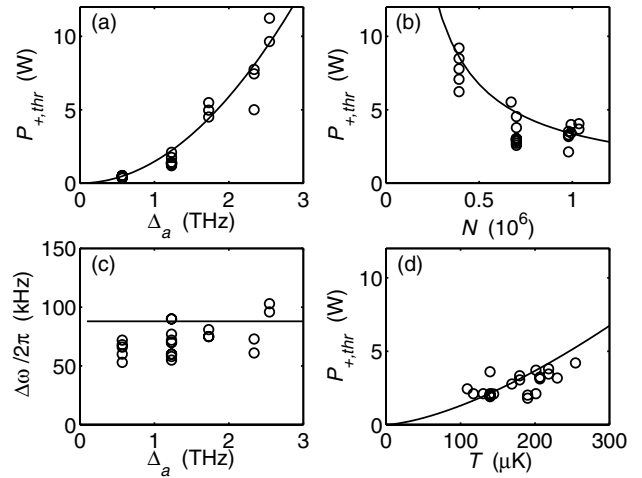

FIG. 2. Pump power threshold measured for various laser detunings (a) and atom numbers (b) determined from curves like those shown in Fig. 1. (c) CARL frequency as a function of the laser detuning. (d) Pump power threshold plotted as a function of the temperature derived from the threshold CARL frequency according to Eq. (7).

have been obtained from the same data set $P_{+,thr}(N, \Delta_a)$ scaled either to an arbitrarily chosen reference atom number $N = 10^6$ [curve (a)] or to a reference laser detuning $\Delta_a = -(2\pi)1.5$ THz [curve (b)] using the relationship (6). The fitted curves are obtained from the same Eq. (6) by adjusting the temperature to $T = 200$ μK and the friction coefficient to $\gamma_{fr} \approx 4\kappa$. Figure 2(c) demonstrates that the CARL frequencies measured at threshold do apparently not depend on the coupling strength. The horizontal line indicates the frequency corresponding to the temperature $T = 200$ μK according to Eq. (7). On the other hand, theory predicts a dependence of the threshold power on the atomic temperature: Hotter atomic clouds have a broader velocity distribution σ and exhibit a higher threshold power. Even though the temperature data shown in Fig. 2(d) seem to confirm this trend, they are too uncertain to be used to improve the fits of Figs. 2(a) and 2(b).

The steady-state operation of our system is ensured by the optical molasses. As soon as the molasses is turned off, the equilibrium is lost and the atoms and the standing-wave accelerate each other, *provided the atoms are bunched.* Figure 3(a) demonstrates the acceleration process. The instantaneous CARL frequency and intensity are again obtained from a sequence of Fourier spectra taken over successive time periods. The evolution of the probe power calculated with Eq. (1) is shown in Fig. 3(b). At times $t < 0.5$ ms, when the molasses is present, the CARL frequency is fixed at 170 kHz. As soon as the molasses is switched off, the probe's amplitude diminishes while its frequency detuning from the cavity resonance increases [11]. The acceleration indicates the occurrence of bunching induced by the optical molasses.
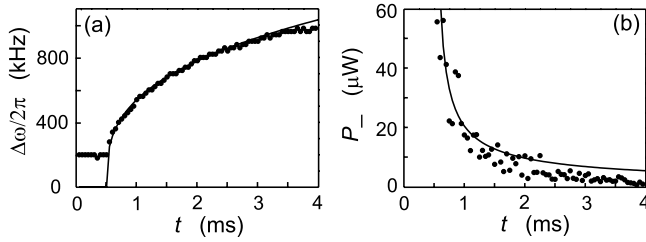
FIG. 3. (a) Evolution of the CARL frequency after molasses has been switched off at time $t = 0.5$ ms. (b) CARL power calculated via Eq. (1). The fits are based on the theoretical formulas Eqs. (2) and (3) of Ref. [2].

The self-organization of the atomic density distribution out of a homogeneous cloud into a spatially ordered arrangement spontaneously breaks translational symmetry and represents the strongest signature of CARL action [1].

The probe field α_- depends on the location of all atoms in a collective way. Consequently, Eq. (2) describes a *mean-field* type dependence of the location of every single atom on all the others. This fact gets particularly transparent in the limit of strong viscous damping, $\ddot{\theta}_n = 0$, if we furthermore assume that at steady state the optical standing wave propagates with a constant amplitude at a constant velocity. This condition is formulated by $\dot{\alpha}_- = i\omega_{ca}\alpha_-$, where the probe beam frequency shift $\omega_{ca} = \omega_{ca}(b)$ depends on the atomic bunching. With these approximations we substitute the solution of Eq. (3) into Eq. (2) and obtain, defining the coupling constant $K \equiv 8\varepsilon N U_0^2 \alpha_+^2 \omega_{ca}/[\gamma_{fr}(\omega_{ca}^2 + \kappa^2)]$ and restricting ourselves to the good cavity limit $\kappa \ll \omega_{ca}$,

$$\dot{\theta}_n = \xi_n/\gamma_{fr} + K|b|\sin(\psi - \theta_n). \tag{8}$$

The Eq. (8) describes the dynamics of N coupled oscillators with fictitious frequencies $\omega_n = \xi_n/\gamma_{fr}$. This system, which has been investigated by Kuramoto [5], predicts the synchronization of those oscillators over time, whose frequencies satisfy $\omega_n \leq K|b|$. The analogy between our viscous CARL and an ensemble of self-synchronizing harmonic oscillators resides in the following correspondences: The phases of the oscillators are represented by the positions of atoms. Synchronization of the oscillators corresponds to bunching of the atoms. The role of friction is to provide a steady atomic center-of-mass velocity to which the individual atomic velocities may lock. In the case of CARL and unlike for the Kuramoto model, the collective oscillation frequency is self-determined. Diffusion as a source of disorder rules the phase transition by competing with the dynamical coupling. While atomic ordering is sustained by feeding the energy of a pump laser to the system, excess entropy is carried away by diffusion. In this respect the observed nonequilibrium phase transition is similar to the formation of dissipative structures in extended systems.

To conclude, we point out that the viscous CARL is representative for a vast class of systems. Under the rule of optical molasses the coupled field-atom system behaves like an ensemble of weakly coupled oscillators. Despite the fact that the details of the molasses dynamics are complicated, its impact on our CARL is fairly well described by two constants, the friction and the diffusion coefficient. Although the CARL dynamics is essentially classical, we deal with microscopic particles. This bears the possibility of transferring the system to the quantum regime and thus to studying the coupling of large ensembles of quantum oscillators. Furthermore, classical networks of dynamical systems in general depend much on the details of how the coupling is realized. In contrast, the coupling in our system is generated by the fundamental interaction between atoms and light, which is very well understood and even controllable by experiment, e.g,. via the tunable friction force or the laser detuning. Because it is mediated by a delocalized object, i.e., a standing light wave, the coupling is instantaneous (no retardation effects) and truly uniform (every atom couples with the same strength to all its neighbors). The large number of oscillators ensures the validity of the thermodynamic limit.

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[11] Note that the pump laser locking servo ($\propto$ 1 MHz bandwidth) is faster than the dynamics of the cavity modes.