Simple Wave Breaking Depth Index Formula for Regular Waves

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ABSTRACT

A simple formula to determine the wave breaking depth index for regular waves is proposed. In the literature, there are several formulas for determining the breaker depth index, which over the years have gradually evolved by improving the advanced measurement tools. The proposed formula has been obtained by means of dimensional analysis and incomplete self-similarity (Barenblatt 1978), and it has been calibrated and verified using a large breaker depth index database yield fairing good predictions for a wide range of wave conditions and beach slopes. The application of some existing formulas for determining the breaker depth index has been examined using previous published laboratory data comparing with the new formula.

INTRODUCTION

The process of wave breaking on a beach is both one of the most dramatic visually and one of the most important physically for the wave motion and for the development of the nearshore currents (Svendsen, 2006).

The wave breaking phenomenon has been studied for several years, and many research contributions have been published. The following literature review contains a rich database that has been used in the current study.

Goda (1970), using Goda (1964) and Goda et al. (1966) data and also all data from Iversen (1951), Kishi and Iohara (1958), Mitsuyasu (1962), Toyoshima et al. (1968), Bowen et al. (1968) and Galvin (1969) (see Table 1), presented a series of graphical curves of breaker depth index. Goda (1974) based on Goda (1970) findings, presented an empirical formula (cited also in the CIRIA, CUR, CETMEF, 2007, *The Rock Manual)* to determine the breaker depth index γ as it follows:

$$\gamma = \frac{H_b}{h_b} = \frac{A}{\left(h_b \big/ L_0 \right)} \left\{ 1 - \exp\left[-1.5\,\pi \frac{h_b}{L_0} \left(1 + 15 s^{4/3} \right) \right] \right\} \qquad (1)$$

where H_b = wave height, h_b = water depth at the incipient breaking, A = 0.17 (Goda, 1974), L_0 = deep water wavelength, and $s = \tan \alpha$ (Fig. 1). As it appears, the term h_b/L_0 has been used both inside and outside of the exponential function.

Rattanapitikon and Shibayama (2000) have recommended a modification of the slope effect term of $(1 + 15s^{4/3})$ into $(1.033 + 4.71s - 10.46s)$. Goda (2010) based on this recommendation and re-examination of experiments on breaker depth index and using a part of the same dataset (shown in Table 1) obtained a new form of the formula that it is expressed as:

$$\frac{H_b}{h_b} = \frac{A}{\left(h_b/L_0\right)}\left\{1-\exp\left[-1.5\,\pi\,\frac{h_b}{L_0}\left(1+11s^{4/3}\right)\right]\right\} \qquad (2)$$

Kamphuis (1991) classified the breaker index formulas in four different types. By assigning the best-fitting value between formulas and his laboratory data, he obtained the correlation coefficients for the following types of breaker index formulas:

a) $H_b/h_b = f_1\,(0)$ = constant $\qquad R^2 = 0.69$

b) $H_b/h_b = f_2\,(h_b/L_0$ or $h_b/L_b)$ $\qquad R^2 = 0.67$

c) $H_b/h_b = f_3\,(s)$ $\qquad R^2 = 0.84$

d) $H_b/h_b = f_4\,(s,\, h_b/L_0$ or $h_b/L_b)$ $\qquad R^2 = 0.88$

Based on obtained correlations coefficients, he suggested to include both the parameters of beach slope and relative water depth in the breaker index formula.

Camenen and Larson (2007) using a large dataset including experimental data from 22 published sources containing a wide range of wave conditions and beach slopes, presented a new formula to predict the breaker depth index and breaker type comparing with six existing breaker depth index formulas. They showed that the modified Goda (1970) formula, proposed by Rattanapitikon and Shibayama (2000), improves the results of the original formula for slopes up to 0.1 but gives overestimated results for smaller slopes.

Liu et al. (2011) compared the results of Goda (2010) and Ostendorf and Madsen (1979) formulas with the measured data for a total number of 1193 cases reported in literature. They showed that Ostendorf and Madsen (1979) formula is fairly good even for cases of very steep slopes, but for milder slopes, it is not accurate as good as Goda (2010) formula.

Recently, Saprykina et al. (2017), using laboratory and field experiments data, investigated the influence of properties of nonlinear wave transformations and type of wave breaking on the breaking index; they showed that, if the relative part of the wave energy in the frequency range of the second nonlinear harmonic is more than 35%, the value of the breaking index can be taken as a constant equal to 0.6.

The main objective of the present study is to find a simple formula to determine the breaker wave depth index by means of dimensional analysis and incomplete self-similarity (Barenblatt 1978) considering the both variables beach slope and relative water depth as parameters.

METHODOLOGY

Dimensional analysis

Generally, a power-law relationship between certain variables y and x appears in mathematical modelling of various phenomena in engineering of the form of $y = a\, x_1^b\, x_2^c \ldots x_n^z$ where a, b, c, and z are constants. A very common view is that these power-law relations are nothing more than the simplest approximations to the available experimental data, having no special advantages over other approximations but as a matter of fact, these types of relationships always reveal the "self-similarity" of the phenomenon which means reproducing itself on different time and space scales. "Self-similarity" allows to reduce a problem in mathematical physics of a phenomenon. Self-similar solutions always limit problems where the governing variables are equal to either zero or infinity. Significant importance is the analysis of incomplete self-similarity in fluid dynamics, where solving a complete mathematical formulation of the problem is very difficult and sometimes impossible, therefore the comparison of similarity laws with experimental data is of decisive importance in estimating the character of the self-similarity (Barenblatt 1978).

A dimensional analysis has been conducted using the Π-Buckingham theorem (Barenblatt 1987), in which it is assumed that the main parameters to determine the breaker depth index ($\gamma = H_b/h_b$) are:

$$f(H_b, h_b, h, \rho_s, \rho, g, L_0, C_b, T, \sin\alpha) = 0 \quad (2)$$

where f = functional symbol; h = water depth; ρ_s = bed material density; ρ = water density; g = gravitational acceleration; C_b = wave celerity at the incipient breaking; T = wave period; and $\sin\alpha$ = beach slope ($0° \le \alpha \le 90°$) (Fig. 1).

According to the Π-Buckingham theorem (Barenblatt 1987), Eq. (2) can be expressed in a non-dimensional form as it follows:

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \Pi_7) \quad (3)$$

Eq. (3) in the same order of the non-dimensional parameters can be written as:

$$\frac{H_b}{h_b} = f'\left(\frac{T^2 g}{h}, \frac{h_b}{h}, \frac{T}{h} C_b, \frac{\rho_s}{\rho}, \frac{L_0}{h}, \sin\alpha\right) \quad (4)$$

where f' = functional symbol; since all data used from existing database from literature are related to the experiments with a fixed bed, the relative density $G_s = \frac{\rho_s}{\rho}$ is not considered as parameter to find the new formula.

Tomasicchio and Kurdistani (2019) considering $C = \frac{g \cdot T}{2\pi} \tanh\left(2\pi \frac{h}{L}\right)$ for intermediate wave condition and using incomplete self-similarity (Barenblatt 1987) proposed the non-dimensional wave parameter ω:

$$\omega = \frac{1}{2\pi}\tanh\left(2\pi\frac{h}{L}\right) \quad (5)$$

Assuming $C_b = \frac{g \cdot T}{2\pi}\tanh\left(2\pi\frac{h_b}{L_0}\right)$ as the wave celerity at the breaking point and combining Π_2 and Π_4, the wave parameter ω_b for the incipient breaking as the character of the self-similarity will be obtained:

$$\left(\frac{\Pi_4}{\Pi_2}\right) = \frac{\frac{T\,C_b}{h}}{\frac{T^2\,g}{h}} = \frac{C_b}{g\,T} = \frac{1}{2\pi}\tanh\left(2\pi\frac{h_b}{L_0}\right) \quad (6)$$

$$\omega_b = \frac{1}{2\pi}\tanh\left(2\pi\frac{h_b}{L_0}\right) \quad (7)$$

Another non-dimensional group showing influence of the water depth at the breaking point and the wave length, can be obtained as follows:

$$\left(\frac{\Pi_3}{\Pi_6}\right) = \frac{\frac{h_b}{h}}{\frac{L_0}{h}} = \frac{h_b}{L_0} \quad (8)$$

As it is shown in Eq. (7), h_b/L_0 is already a part of ω_b and Eq. (4) can be rewritten as it follows:

$$\frac{H_b}{h_b} = f''(\omega_b, \sin\alpha) \quad (9)$$

where f'' = functional symbol;

The results of the multivariate regression of adopted data from Goda (1970) in accordance with Goda (1974), lead to an exponential function of ω_b. Therefore, Eq. (9) can be expressed as:

$$\gamma = \frac{H_b}{h_b} = a\,(1+\sin\alpha)^i\, e^{k\omega_b} \quad (10)$$

where a, i and k are constants; This means that the wave similarity character ω_b as a function of h_b/L_0 is the main parameter and γ has a numerical trend with ω_b that increasing ω_b toward infinity, decreases γ toward zero and the beach slope changes the trend steepness. Table 2 represents some existing formulas for the wave breaking depth index. This table shows that the term $\tanh(2\pi h_b/L_b)$ of ω_b has been used in several existing wave breaking index formulas like Miche (1944), Battjes and Janssen (1978), Ostendorf and Madsen (1979), Battjes and Stive (1985), and Kamphuis (1991) while ω_b independent of these formulas and by means of dimensional analysis and incomplete self-similarity has been found and can be considered as the self-similar character for wave breaking depth index.

THE PROPOSED EMPIRICAL EQUATION

Calibration

To calibrate and obtain the mathematical form of Eq. (10), twelve sets of data have been adopted from literature (Goda, 2010) containing different beach slopes and varying breaking wave conditions that are listed in Table 1. Using multivariate regression for the adopted data range ($0.001 \leq \omega_b \leq 1$), Eq. (10) could be reworked as it follows: ($R^2 = 0.79$)

$$\frac{H_b}{h_b} = 0.75\,(1+\sin\alpha)^5\, e^{-3.4\omega_b} \qquad (11)$$

All adopted values of breaker depth index γ have been compared with the Goda (2010) equation and Eq. (11) in Fig. 2. Comparison shows a fairly good agreement between the proposed equation and Goda (2010) equation.

The laboratory data of the breaker depth index γ as a function of ω_b have been compared with Eq. (11) in Fig. 3(a-e). These figures indicate that for s = 1/9, 1/10, 1/12 (Fig. 3a), s = 1/15, 1/17, 1/20 (Fig. 3b), s = 1/30 (Fig. 3c), s = 1/50 (Fig. 3d), and s = 1/100, 1/200 (Fig. 3e), Eq. (11) has a fairly good agreement with all observed data within the 20% of deviation and increasing ω_b decreases γ.

Verification

Eq. (11) has been verified using a new dataset including laboratory and field experiments data on the wave breaker depth index recently published by Saprykina et al. (2017). Fig. 4(a-f) compares the results from Eq. (11) and other existing formulas with measured data by Saprykina et al. (2017). For different used formulas, Table 3 presents root mean square error E_{rms} (%) defined as:

$$E_{rms} = 100\sqrt{\frac{\Sigma\left(\gamma_{b(formula)} - \gamma_{b(\exp.)}\right)^2}{\Sigma\gamma_{b(\exp.)}{}^2}} \qquad (12)$$

Fig. 5 shows that Saprykina et al. (2017) covers a wide range beach slopes from s = 1/23 to s = 1/100, and all data confirm agreement with Eq. (11) within the 30% of deviation. As a consequence, Eq. (11) appears to be independent of the scale effect influence.

CONCLUSIONS

By means of dimensional analysis and incomplete self-similarity a simple equation to determine the wave breaking index has been obtained presenting a new non-dimensional wave similarity character ω_b. The wave similarity character ω_b as a function of h_b/L_0 is the main parameter and γ has a numerical trend with ω_b; the beach slope changes the trend steepness. The proposed equation appears to be independent of the scale effect influence. It has been calibrated and verified using different datasets with different model scales. For the range of adopted data ($0.001 \leq \omega_b \leq 1$), the results from the proposed formula show a fair good agreement with the Larson and Kraus (1989) formula and Goda's formula (2010).

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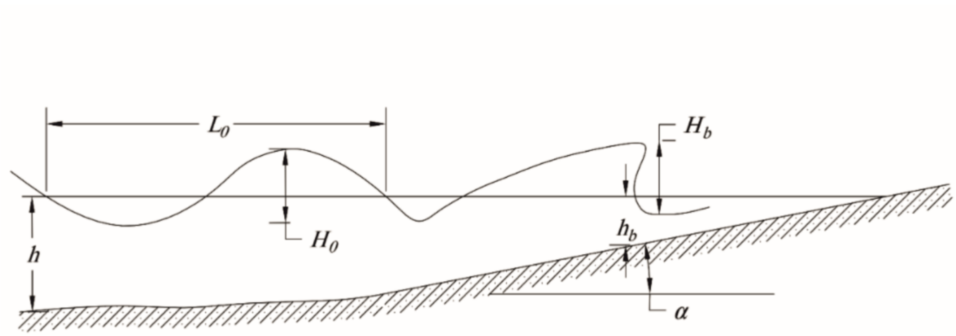

Fig. 1. Breaking wave parameters.

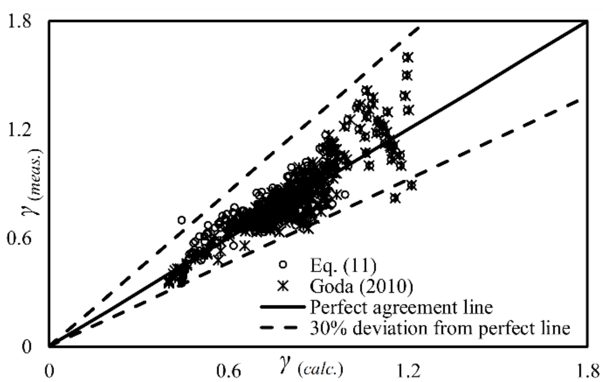

Fig. 2. Comparison of the proposed Eq. (11) with Goda (1970) equation.

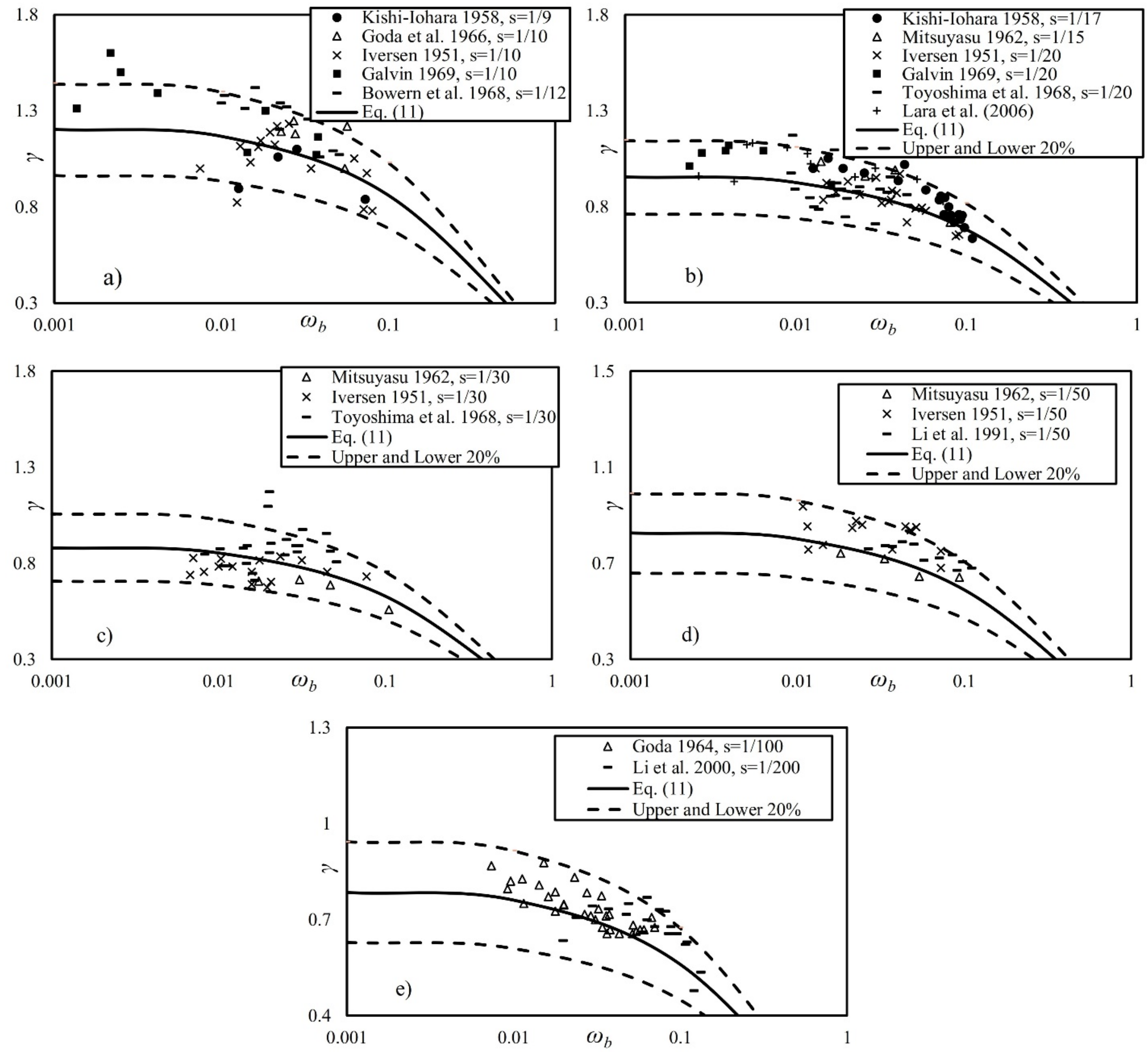

Fig. 3. Comparison of γ as a function of ω_b for different beach slope with Eq. (11).

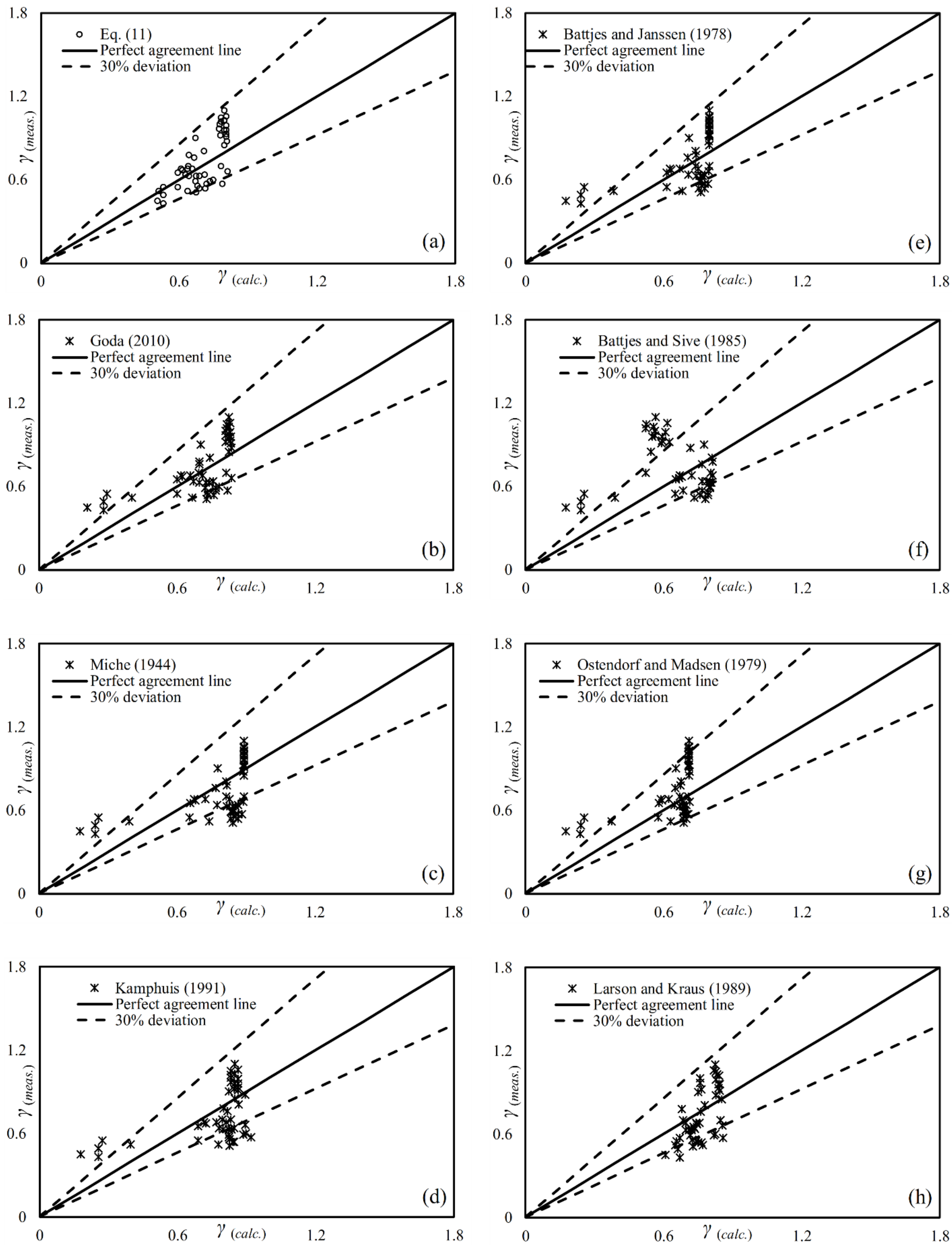

Fig. 4. Comparison of Eq. (11) and other existing formulas using Saprykina et al. (2017) data.

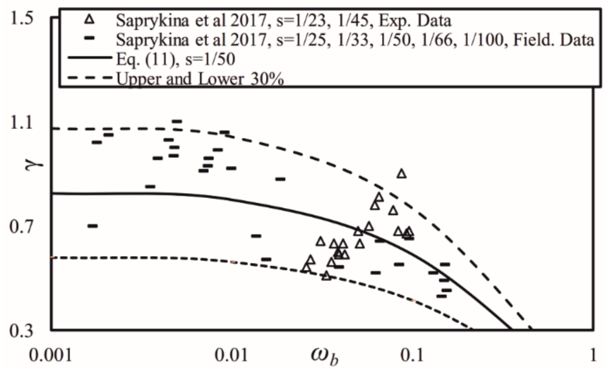

Fig. 5. Verification of Eq. (11), using Saprykina et al. (2017) data.

Table 1. Summary of breaker depth index dataset (Goda, 2010)

Author	Beach slope	Range of Wave period T (s)	Range of Breaking wave height H_b (m)	Range of Breaking depth h_b (m)	No. data
Iversen (1951)	1/10 1/20 1/30 1/50	0.80 – 2.50 0.74 – 2.24 1.49 – 2.65 0.90 – 2.65	0.049 – 0.122 0.043 – 0.128 0.053 – 0.127 0.055 – 0.121	0.043 – 0.137 0.049 – 0.162 0.070 – 0.155 0.065 – 0.156	15 19 15 13
Kishi and Iohara (1958)	1/9 1/17	0.9 – 2.0 0.65 – 2.0	0.070 – 0.106 0.050 – 0.125	0.079 – 0.100 0.065 – 0.135	4 21
Mitsuyasu (1962)	1/15 1/30 1/50	1.02 – 2.57 1.02 – 2.57 1.02 – 2.57	0.104 – 0.150 0.096 – 0.111 0.098 – 0.141	0.124 – 0.145 0.153 – 0.206 0.177 – 0.190	4 4 4
Goda (1964)	1/100	2.30 – 7.30	0.417 – 0.931	0.603 – 0.1250	32
Goda et al. (1966)	1/10	1.36 – 2.24	0.140 – 0.215	0.110 – 0.180	6
Toyoshima et al. (1968)	1/20 1/30	1.84 – 3.04 1.94 – 3.75	0.062 – 0.408 0.119 – 0.500	0.073 – 0.610 0.131 – 0.616	22 44
Bowen et al. (1968)	1/12	0.82 – 2.27	0.044 – 0.130	0.042 – 0.097	11
Galvin (1969)	1/10 1/20	1.00 – 6.00 2.00 – 6.00	0.038 – 0.150 0.093 – 0.176	0.039 – 0.120 0.100 – 0.182	8 6
Li et al. (1991)	1/30 1/50	–	–	–	10 11
Li et a. (2000a)	1/200	–	–	–	19
Lara et al. (2006)	1/20	1.20 – 4.00	0.067 – 0.185	0.068 – 0.195	12

Table 2. List of examined existing breaker depth index formulas.

Researchers	Formula
Miche (1944)	$\frac{H_b}{h_b} = 0.142\left(\frac{L_b}{h_b}\right)\tanh\left(\frac{2\pi\, h_b}{L_b}\right)$
Battjes and Janssen (1978)	$\frac{H_b}{h_b} = 0.14\left(\frac{L_b}{h_b}\right)\tanh\left(\left(\frac{0.8}{0.88}\right)\frac{2\pi\, h_b}{L_b}\right)$
Ostendorf and Madsen (1979)	$\frac{H_b}{h_b} = 0.14\left(\frac{L_b}{h_b}\right)\tanh\left((0.8+5\,s)\frac{2\pi\, h_b}{L_b}\right) \qquad \mathrm{m} \le 0.1$
Battjes and Stive (1985)	$\frac{H_b}{h_b} = 0.14\left(\frac{L_b}{h_b}\right)\tanh\left(\left[0.5+0.4\tanh\left(33\left(\frac{H_0}{L_0}\right)\right)\right]\frac{2\pi\, h_b}{0.88L_b}\right)$
Larson and Kraus (1989)	$\frac{H_b}{h_b} = 1.14\left(\frac{\mathrm{s}}{\sqrt{\mathrm{H}_0/\mathrm{L}_0}}\right)^{0.21}$
Kamphuis (1991)	$\frac{H_b}{h_b} = 0.127\left(\frac{L_b}{h_b}\right)\exp(4\,s)\tanh\left(\frac{2\pi\, h_b}{L_b}\right)$
Goda (2010)	$\frac{H_b}{h_b} = \frac{A}{\left(h_b/L_0\right)}\left\{1-\exp\left[-1.5\,\pi\,\frac{h_b}{L_0}\left(1+11s^{4/3}\right)\right]\right\}$

Table 3. Root mean square error (E_{rms}) for different used formulas using Saprykina et al. (2017) data.

Formula	E_{rms} %
Proposed Eq. (11)	18.7
Miche (1944)	24.2
Battjes and Janssen (1978)	22.6
Ostendorf and Madsen (1979)	25.1
Battjes and Stive (1985)	35.4
Larson and Kraus (1989)	21.2
Kamphuis (1991)	25.0
Goda (2010)	20.4