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ESSAY ON INVESTMENT AND THE ELASTICITY OF SUBSTITUTION

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Introduction

Standard models used to analyze business-cycle fluctuations rely on the Cobb-Douglas production function. In their paper, Cantore et al. (2015) (CLPY in the following) analyze if and to what extent the use of a Constant Elasticity of Substitution (CES) production function improves a standard medium-size Dynamic Stochastic General Equilibrium (DSGE) model. Extending a model similar to Christiano et al. (2005) and Smets and Wouters (2007) with the CES production function, they show that allowing for non-unitary elasticity of substitution (i.e., non-Cobb-Douglas production function) between inputs has a significant improvement in terms of second moments and posterior marginal likelihood.

Our research builds upon the results of CLPY and is carried out in two chapters, each inspecting the implications of CES production function on a different definition of investment; in particular, the first chapter considers investment in physical capital, the second investment in firm creation.

The focus on CES production function is motivated by the following considerations. Despite the Cobb-Douglas functional form has proved useful because of its ease of tractability, some major drawbacks emerge. We summarize these in three points; first, the Cobb-Douglas implicitly assumes constant factor shares: even though in the long-run the assumption can be reasonably retained, there is mounting evidence that at business cycles level factor shares do fluctuate (among many others, Blanchard et al. (1997), Jones et al. (2003), McAdam and Willman (2008)). Secondly, assuming that output is produced through a Cobb-Douglas formulation implies that the inputs involved in the production process have a unitary degree of substitution, whereas a considerable body of research suggests lower values of substitutability (Chirinko (2008), Klump et al. (2012)). Third, unitary elasticity of substitution, by construction, neglects the existence of factor-biased technical change: nonetheless Acemoglu (2009), chap. 15, claims that neutral technical change is hardly admissible over business cycle frequencies.

An alternative specification that has risen in the economic literature is the CES production function. This specification solves the aforementioned flaws of the Cobb-Douglas by allowing for non-unitary elasticity of substitution among inputs and endogenous factor shares; moreover, by construction, input-specific technological progresses don't collapse

to the Hicks-neutral, or Total Factor Productivity, one.

The aim of the first chapter is threefold. First, we want to determine which feature of the CES production function - and to what extent - drives the results of CLPY: the presence of input-specific shocks or the possibility of non-unitary elasticity of substitution and endogenous factor shares.

The second aim is to disentangle the relative importance of two similar disturbances: the capital-augmenting shock and the marginal efficiency of investment (MEI) shock; in the following we'll prove that the two have a close relationship, given the capital law of motion. Closely related to this topic, the third issue we deal with is the "comovement puzzle" generated by MEI disturbances. As Ascari et al. (2016) argue, the negative comovements implied by shocks to the marginal efficiency of investment are at odds with US data. Standard DSGE models which consider MEI shocks predict negative - although weak - comovements between consumption and investment (and implicitly output) in response to such disturbance, whereas US data exhibit strong positive correlation. We want to see if the aforementioned features of the CES function can help in overcoming this puzzle.

We find that the CES production function is better for data explanation and confirm the established importance for the business cycle of the MEI shock. This importance is well beyond that of the capital-augmenting shock; a CES model with capital-augmenting shock performs poorly in terms of model estimation against a CES model with MEI shock. When the model considers both shocks, capital-specific technology has virtually no relevance for the business-cycle movements of the considered variables, while MEI still accounts for a considerable part of the observed variance.

The second definition of investment we consider, analyzed in the last chapter, is that of investment in firm establishment. Two important contribution on the importance of product creation for the business cycle are the ones by Bernard et al. (2010) and Broda and Weinstein (2010): they both find - at different levels of business aggregation - that a non-negligible fraction of output value is represented by new products; moreover, the latter demonstrate the procyclicality of new business formation at business-cycle fluctuations. Following Bilbiie et al. (2012) (BGM), in the second part of this thesis we build a model

of endogenous product creation, modifying their benchmark modelization with a CES production function. The model does not consider product creation within the firm for the sake of simplicity, but takes - in the spirit of BGM - the broader view of producer entry and exit as creation and destruction of product varieties. We then consider the general implications of different degrees of input substitutability with respect to the baseline RBC model in Bilbiie et al. (2012), with a focus on the two investment shocks on investment in new firm creation.

We show that another comovement puzzle emerges for both shocks, and its presence is related to the existence of investment adjustment costs.

1 Chapter 1

Abstract

We look at the determinants of the results in Cantore et al. (2015) of the superiority of the Constant Elasticity of Substitution (CES) production function over the standard Cobb-Douglas. The main features that distinguish the former from the latter are the non-unitary degree of input substitutability and the presence of input-specific shock: we investigate which of the two plays a major role. Related to this objective, we also look at the capital-productivity shock as a means of solving the comovement puzzle implied by investment shocks.

We find that the relative advantage of CES production functions mainly comes from the possibility of non-unitary elasticities of substitution and endogenous factor shares. A Bayesian estimation of the model with both investment and capital-augmenting shocks suggests that the latter has virtually no role in explaining the observed volatility of key variables. Moreover, we show that the presence of investment shocks makes autocorrelation and second moments better than under the capital-productivity specification.

1.1 Introduction to the chapter

In their influential papers, Justiniano et al. (2010) and Justiniano et al. (2011) proved the paramount importance of the Marginal Efficiency of Investment (MEI) shock for explaining the observed oscillations in US economic data, mainly output, consumption and investment.

As the literature (and the authors themselves) acknowledge, modeling an economy with such kind of shock comes at a cost in terms of correlations among variables. In particular, the comovement puzzle generated by the MEI disturbance makes consumption counter-cyclical on impact, whereas the empirical literature finds positive - and strong - positive correlation between the two, as well as between consumption and investment.

We propose another way to look at this puzzle by adopting a more microfounded production function, the Constant Elasticity of Substitution one. The superiority of the CES function over the standard Cobb-Douglas was first highlighted in Cantore et al. (2014) and empirically tested in Cantore et al. (2015). In particular, we want to exploit the feature of truly input-specific shocks that comes with the CES function, to see if a

capital-productivity shock is able to solve this "Barro-King curse" (cfr Barro and King (1984)). Our argument is made clear in Figure 1, where we plot the impulse response functions (IRFs) of the benchmark model in CLPY to a MEI and a capital-augmenting shock, calibrating the elasticity parameter to imply gross complementarity between inputs (i.e., $\sigma < 1$). Note that the latter shock correctly captures the controversial comovements that arise in response to the former.

Starting from this observation, the core of our analysis looks at the drivers of the results in CLPY; it is not clear, indeed, which feature of the CES production function - and to what extent - drives its superiority over the Cobb-Douglas specification: the presence of input-specific shocks or the possibility of non-unitary elasticity of substitution (and implicitly endogenous factor shares).

To analyze these questions, we'll rely on Bayesian techniques. Indeed, applying Bayes' rule to a dynamical model delivers a channel to validate business-cycle models through data: by modifying *ad hoc* the CLPY model when can thus look at each of the aforementioned matters in turn.

We proceed as follows: first, we build a model like in CLPY; this is what will be done in the next section. We then estimate three versions of it, turning on and off different shocks in order to obtain the "best" model in term of Bayesian odds and posterior-implied moments: this is the topic of Section 1.3. The results of this step, displayed in Section 1.4, will allow us to draw conclusions on the first aim of the paper. Moreover, by looking at variance decompositions, we can establish which shocks is more relevant for business-cycle fluctuations. This will determine if the capital-augmenting shock has a stronger explanatory power with respect to the MEI one, thus answering to the second and third point of our research questions.

We argue that the relative advantage of CES production functions does not come from the existence of a input-specific shocks, but mainly from the possibility of non-unitary elasticities of substitution and endogenous factor shares. This also is a point in favor of model specifications which embed MEI shock. Indeed, its presence makes autocorrelation and second moments better than under the capital-productivity specification.

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Related literature This chapter relates to two strands of the DSGE literature: one on the definition of investment shocks and one on the importance of CES production function.

Greenwood et al. (1988) were the first to theoretically analyze the implications of generating output fluctuations from shocks to the marginal efficiency of investment (MEI), assuming a vintage capital structure. By comparing the simulated second moments generated by the model with actual US data the authors are able to mimic the data's features. Note that they model the representative utility such that the marginal rate of substitution between consumption and labor depends on the latter only: this makes the intertemporal substitution effect vanish.

In a subsequent paper, Greenwood et al. (1997) proved that investment-specific technological change was able to explain the fall in equipment prices and the rise in the equipment-to-GNP ratio in US postwar data.

By relying on structural VAR evidence, Fisher (2006) showed that investment specific shocks in an economy modeled like Greenwood et al. (1997) accounts for the majority of business cycle variations of hours and output. Both papers assume that in equilibrium, technological improvements in the production of investment goods should be reflected in their relative price.

In their contribution, Justiniano et al. (2011) model a multi-sectoral, Smets-and-Wouters-like (Smets and Wouters (2007)) economy, in which production of consumption, capital and investment goods is decentralized into separate sectors. This involves the coexistence of both MEI and IST technological change in the capital accumulation process, as shocks residing in different sectors. Then, having disentangled these two shocks implies that their individual role for business cycle fluctuations can be obtained: the authors use Bayesian methods in order to obtain a posterior distribution for the shocks, concluding that the MEI shock is responsible for about 60 to 85 percent of the variance of output, hours and investment, while the contribution of the IST shock is negligible.

However, the model also predicts that investment and consumption are negatively correlated and that output growth and consumption growth are weakly correlated, even if with a positive sign.

Ascari et al. (2016) try to solve what they refer to as the "Barro-King curse" (cfr Barro and King (1984)), that is, the impossibility for shocks other than productivity ones to cor-

rectly generate the business cycle comovements found in data. Augmenting a benchmark medium-scale new Keynesian model with firm networking and trend in output growth makes it able to capture the right correlation among variables.

Our model differs from this literature in the way in which production is modeled: we allow for different "qualities" of input and for input-specific technological change through the implementation of a Constant Elasticity of Substitution (CES) production function.

Cantore et al. (2014) were the first to investigate the implications of assuming degrees of substitutability between capital and labor different from unity in DSGE modeling. The comparison exercise was made possible through the *normalization* procedure of the production function: as argued by the seminal contributions of de La Grandville (1989), Klump and de La Grandville (2000) and León-Ledesma et al. (2010), such procedure is necessary if one wants to compare production function which are only distinguished by the value of the elasticity of substitution. Moreover, normalization ensures that the parameter of the production function are deep, and it improves the empirical identification by accounting for bias in technical change (León-Ledesma et al. (2010)).

Putting the CES production function in a Bayesian estimation framework, Cantore et al. (2015) find that this functional form performs empirically better than the standard Cobb-Douglas function in terms of marginal likelihood, with an estimated elasticity of substitution of 0.18. The main reason they argue as responsible for such result is that allowing - with a $\sigma \neq 1$ - for endogenous factor shares helps explaining data. They rely on the same model developed by Smets and Wouters (2007) augmented with a wholesale and a retail sector, Calvo prices and wages, CES production function, adjustment costs of investment and variable capital utilization. In order to make our contribution comparable we will adopt this same model.

1.2 Derivation of the model

The structure of the economy follows Cantore et al. (2015).

1.2.1 Consumption goods

We assume a two-sector production side of the economy. Perfectly competitive firms produce the final good Y by taking as input the intermediate goods, $Y(j)$, $j \in [0, 1]$,

through the CES aggregator

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

Where θ is the demand elasticity of substitution among intermediate goods.

The final good is purchased by households at the unit price P, and it's used either for consumption or investment.

Maximizing profits for the final firms, given the zero-profits condition, we obtain the standard first-order conditions for Dixit-Stiglitz aggregator: the price index is

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{1}$$

and the demand function for the intermediate good is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \tag{2}$$

1.2.2 Intermediate consumption goods

Each j firm in the intermediate sector combines labor and capital services through a CES production function y_f and is subject to fixed costs of production F, so that its output is:

$$y_t(j) = \left[\alpha_k \left(k_{s,t}(j) Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}} + \alpha_l \left(l_t(j) Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - F \tag{3}$$

Where $\sigma \in (0, +\infty)$ is the elasticity of substitution between capital and labor, $Z_{h,t}$ is the h-augmenting shock, α_k and α_l are the so-called distribution parameters. When the elasticity of substitution is less than one, the production function implies gross complementarity among inputs, whereas when it is above unity inputs are substitutes. The CES functional form nests the Cobb-Douglas when $\sigma = 1$, and the Leontief as $\sigma \to 0$.

One of the disadvantages of adopting the CES function as in Equation 3 is that its non-linearity makes the parameters α_k and α_l do not correspond to input shares and moreover their values depend on the value of σ. This generates problems of calibration, as these share are not actually observed, and of comparison of production functions that differ only in their elasticity of substitution, as the production parameters are functions of σ. One way to solve this problem is to *normalize* the production function by expressing it in terms of deviation from a $t = 0$ steady state; it can be proved that this ensures that changing σ does not affect the steady state ratios and factor shares of the model, as

showed in Cantore and Levine (2012).

Then the normalized CES production function used in the intermediate sector is given by:

$$y_{f,t}(j) = y_{f,0} \left[\alpha_0 \left(\frac{k_{s,t}(j)}{k_0} Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{l_t(j)}{l_0} Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

We assume that the labor-augmenting technical progress is non-stationary and characterized by a unit root; consequently, only this disturbance has a long-run impact on productivity. This is done to make the model consistent with the Neoclassical growth theory which suggests that, for an economy to possess a steady state with growth and constant factor shares, either the elasticity of input substitution must be one (Cobb-Douglas technology), or technical change must be labor-augmenting (cfr. Uzawa (1961)'s theorem of BGP).

This implies that non-stationary variables will be detrended by the common trend Z_l. Moreover, in the log-linearized version of the model, we define the stationary log of the first difference as $\hat{A}_t$.

The cost-minimization problem in real terms yields, defining mc as the Lagrange multiplier of the optimization and integrating to obtain aggregate variables:

$$w_t = (1-\alpha_0) mc_t \left(\frac{Y_{f,t}}{L_t} \right)^{\frac{1}{\sigma}} \left(\frac{Y_{f,0}}{L_0} Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}}$$

$$r_{k,t} = \alpha_0 mc_t \left(\frac{Y_{f,t}}{K_{s,t}} \right)^{\frac{1}{\sigma}} \left(\frac{Y_{f,0}}{K_0} Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}}$$

We assume that the sector operates in monopolistic competition, and is subject to a Calvo-pricing mechanism with indexation, so that price readjustment are not immediate; each period a constant fraction $1 - \xi_p$ of firms can optimally choose its pricing to $P^\star$. If the price is not optimized, then it follows an indexation rule on past inflation; thus, $\xi_p \in [0,1]$ can be thought of as a price stickiness index. This implies that the price a firm can charge in period t is

$$P_t(j) = \begin{cases} P_t^*(j), & \text{for } (1-\xi_p) \text{ firms} \\ \pi_{t-1}^{\iota_p} P_{t-1}(j), & \text{for } \xi_p \text{ firms} \end{cases}$$

Where ι_p is the indexation parameter.

Since there is a chance that the firm will be stuck with it price of multiple periods, the price problem becomes dynamic. Following CLPY, we assume a time varying markup of

prices over marginal costs, $\mu_t = \frac{\theta}{\theta-1} Z_{pmk,t}$, where $Z_{pmk,t}$ is the price markup shock AR(1) process.

1.2.3   Households

The utility function is separable in labor and consumption and accounts for the presence of external habits in consumption. The representative k household maximizes her discounted future stream of utility

$$E_t \sum_{s=0}^{\infty} \beta^s Z_{b,t+s} \left[log(C_{t+s} - hC_{t+s-1}) - \chi \frac{L_{t+s}^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right] \tag{5}$$

where Z_b is an intertemporal preference shock, h controls for the degree of habits on past consumption levels, χ is a parameter on the disutility of labor and ϕ is the Frisch elasticity parameter.

Households own capital stock, K, which is subject to the following law of motion

$$K_t = (1-\delta)K_{t-1} + Z_{i,t} \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \tag{6}$$

Where δ is the depreciation rate of capital, S represents quadratic investment-adjustment costs, and the conversion of investment into capital stock is subject to an "investment productivity" shock, Z_i. Note that, given that capital stock is predetermined at time t, if one substitutes the capital law of motion in the production function, the MEI disturbance has a time lag with respect to the capital-specific one.

Given the capital stock, the representative household chooses how much to rent as capital services, K_s, to the intermediate firms:

$$K_{s,t} = u_t K_{t-1} \tag{7}$$

Capital utilization has a cost, equal to $P_t a(u_t)$. Following the literature, we assume that in steady state $u = 1$, $a(1) = 0$, $\psi \equiv \frac{a''(1)}{a'(1)}$.

Moreover, the representative household owns government bonds, B, with a nominal interest rate equal to R, pays taxes t, receives a net cash flow form her portfolio of state-contingent securities, q, and the nominal profits Π_n coming from firms'ownership. The nominal budget constraint for the individual household is

$$P_t c_t(k) + P_t i_t(k) + t_t + b_t(k) \leq$$

$$(1 + R_{t-1})b_{t-1} + q_t(k) + \Pi_{n,t} + W_t(k)l_t(k) + R_{k,t}u_t(k)k_{t-1} - P_t a\left(u_t\right)k_{t-1}$$

Lastly, we assume that wages are sticky; that is, each household is a monopolistic supplier of specialized labor $l_t(k)$. Similar to the production side of the economy, here specialized labor types are combined by competitive employment agencies into a homogeneous labor input to be sold in production according to the CES aggregator

$$L_t = \left[\int_0^1 L_t(j)^{\frac{\mu-1}{\mu}} dj \right]^{\frac{\mu}{\mu-1}}$$

The choice of the optimal wage is subject to a Calvo lottery with indexation, as for intermediate firms. We introduce a wage markup shock, $Z_{wmk,t}$.

The remaining choices of the representative household are: present consumption, hours worked, investment, government bonds, capital utilization and capital stock. These define the following FOCs in nominal terms:

$$C_t : P_t \lambda_t^n = \frac{Z_{b,t}}{C_t - hC_{t-1}} - \frac{\beta h Z_{b,t+1}}{C_{t+1} - hC_t}$$

$$L_t : \lambda_t^n W_t^n = b_t \varphi L_t^\nu$$

$$B_t : \lambda_t^n = \beta \lambda_{t+1}^n (1 + R_t)$$

$$K_t : \zeta_t = \beta \left[\lambda_{t+1}^n \left(R_{k,t+1} u_{t+1} - P_{t+1} a(u_{t+1}) \right) + \zeta_{t+1}(1 - \delta) \right]$$

$$u_t : r_{k,t} = a'$$

$$I_t : - \lambda_t^n P_t^I +$$

$$+ \zeta_t Z_{i,t} \left[1 - \left[\frac{\psi_K}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \psi_K \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \right] +$$

$$- \beta \zeta_{t+1} \left\{ -Z_{i,t+1} \left[\psi_K \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \right\} = 0$$

Where λ_t^n is the Lagrange multiplier.

1.2.4 Government and Monetary policy

The government finances its budget deficit by issuing short term bonds. Public spending is determined exogenously as a time-varying fraction of GDP:

$$G_t = \left(1 - \frac{1}{g_t} \right) Y_t, \tag{8}$$

where the government speding shock g_t follows the stochastic process

$$\log g_t = (1 - \rho_g)\log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}$$

Monetary policy is set according to

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{1+\pi_t}{1+\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_X}\right]^{1-\rho_R} \varepsilon_{m,t} \tag{9}$$

Where variables without a time subscript are taken at the steady state. The interest rate responds to deviations of inflation and output from their steady states and is *smoothed* according to its previous-period deviation from steady state. $\varepsilon_{m,t}$ is a white noise that accounts for shocks to the monetary policy instrument.

1.2.5 Shocks

The full model just described has 8 shocks. These are: capital-augmenting, labor-augmenting, price markup, intertemporal preferences, marginal efficiency of investment, wage markup, monetary policy, government spending. Note that the estimated versions of the model will have seven shocks, as the MEI or the capital-augmenting ones will be shut down accordingly.

With the exception of the monetary policy and the labor-augmenting shock, all of them follow AR(1) processes.

1.2.6 Equilibrium

By considering firms and households' symmetry, it can be shown that aggregating the real budget constraint yields the following accounting identity

$$Y_t = C_t + I_t + G_t + a(u_t)K_{t-1}$$

In equilibrium the aggregate price index is

$$(1+\pi_t)^{1-\epsilon_p} = \xi_p (1+\pi_{t-1})^{\iota_p(1-\epsilon_p)} + (1-\xi_p)(1+\pi_t^*)^{1-\epsilon_p}$$

While the aggregate wage expression is:

$$w_t^{1-\epsilon_w} = \xi_w(1+\pi_{t-1})^{\iota_w(1-\epsilon_w)} w_{t-1}^{1-\epsilon_w}(1+\pi_t)^{\epsilon_w-1} + (1-\xi_w)w_t^{*1-\epsilon_w}$$

1.3 Bayesian methodology

1.3.1 Data

We use the same dataset provided by Cantore et al. (2015) to ensure comparability of our results: the set of observables follows Smets and Wouters (2007) but is extended to

2008QII.

In detail, the observables are log GDP growth, log consumption growth; log investment growth; log of hours worked; log real wage growth; inflation; federal funds rate.

The measurement equations, mapping model's state variables into the observed ones, are, in log linear terms:

$$
\begin{bmatrix}
\Delta logGDP_t \\
\Delta logC_t \\
\Delta logI_t \\
logL_t \\
\Delta log\frac{W_t}{P_t} \\
\pi_t \\
R_t
\end{bmatrix}
=
\begin{bmatrix}
\hat{y}_t - \hat{y}_{t-1} + \hat{A}_t + \gamma \\
\hat{c}_t - \hat{c}_{t-1} + \hat{A}_t + \gamma \\
\hat{\hat{i}}_t - \hat{\hat{i}}_{t-1} + \hat{A}_t + \gamma \\
\hat{l}_t + \bar{L} \\
\hat{w}_t - \hat{w}_{t-1} + \hat{A}_t + \gamma \\
\hat{\pi}_t + \bar{\pi} \\
\hat{R}_t + \bar{R}
\end{bmatrix}
$$

Where we define in general terms $\bar{x}$ as the steady state value of x, and where γ is the trend in the labor-augmenting technology.

1.3.2 Estimation

Before presenting the results, we introduce the reader to the chosen procedures behind next section's outcomes.

We want to investigate if, and to what extent, the relative strength of CES formulation with respect to the standard CD one comes from the existence of input-specific technical changes and/or non-unitary degree of input substitutability. Thus, we'll proceed as follows; we define four models, that differ in the way σ and either the MEI or Z_k shocks are defined: in the first model we use the same model presented in section 1.2 and calibrate the elasticity of substitution to one, implicitly assuming a Cobb-Douglas specification (note that, by definition, the input-specific disturbances collapse to a TFP shock); in the second model we leave σ in the set of estimated parameters and do not consider the capital-specific technology; in the third specification we then turn off the MEI shock and take into account Z_k; the last model features both shocks.

For each model, we use Bayesian methods of estimation in order to pin down the values of structural parameters implied by data. In a nutshell, conditional on distributional assumptions for the exogenous shocks, the model specifications generate likelihood functions: these can be used to update prior beliefs about the structural parameters of the

model, obtaining a posterior distribution for each parameter, on which the researcher can then infer the model's ability of capturing the cyclical features of the data. Using Bayes' theorem, the posterior distribution of the set of parameters, θ, given T observations of the variable Y can be defined as

$$p(\theta|Y^T) = \frac{L(Y^T|\theta) \cdot p(\theta)}{\int L(Y^T|\theta) \cdot p(\theta)d\theta} \propto L(Y^T|\theta) \cdot p(\theta)$$

Where the first term on the right-hand side is the likelihood density given model parameters, $p(\theta)$ is the set of prior distribution of each parameter, while the denominator after the equal sign is the unconditional density.

After obtaining the posterior mode as the maximizing value of the likelihood given the priors, we generate samples of the unobserved distribution through the Metropolis-Hastings algorithm; in particular, we run two chains of 125,000 draws each generated by a Random Walk Metropolis-Hastings algorithm: the first 30% observations are considered as *burn-in* and thus discarded. After each parameter value is pinned down to a distribution, key statistics can be extracted from the estimated models and compared to the ones of the time series.

Following CLPY, as an additional exercise we compute the Bayesian factors of the four models, computing the unconditional density of each particular model h in the set of alternative models m_h, $p(Y|m_h) = \int L(Y^T|\theta, m_h)p(\theta|m_h)d\theta$, and use it to obtain a measure of model's probability, assessing which one explain best data.

We use the same prior distribution as in CLPY.

1.4   Results

Table 1 gives the estimated values of the elasticity of substitution for the three CES models; we assumed as prior for σ a Gamma distribution with mean 1 and standard deviation 0.5, and obtained estimated values which are in line with the results by CLPY of $\sigma \approx 0.18$. A first difference emerges for the model with only the capital-augmenting shock, CES-Z_k, which displays the lowest estimated elasticity, at 0.145.

Table 2 contains the log data density of the four estimated models and their respective probabilities computed through Bayes' factors. The numbers are in line with CLPY, but overall we find a slightly better result for the CES model with only the MEI shock, whereas they find that it is the model with both disturbances which performs better;

note, however, that the distance between the two models' densities is very small and thus cannot be indicative of a better performance of one model over the other.

Our original contribution comes up in the third line of the table: it appears that the model featuring the CES function and the capital-specific technology performs even worse that the one with the Cobb-Douglas specification; this is an interesting results because one would expect that, by allowing σ to be different from unity, the resulting model would be more realistic and thus preferred by data in terms of likelihood. This is not the case and - given that the only difference between the model in the second line and the one considered here is only in the presence of either investment or capital-specific technology - we ascribe this outcome to the importance of marginal efficiency of investment for models of business-cycle, confirming the results brought about in Justiniano et al. (2011)[1]. In the following we investigate the determinants of these results by comparing each model's implied characteristics with the ones of the time series.

We now turn to the analysis of summary statistics of the estimated models; this allows us to assess the absolute fit of each model with respect to data. In Table 3 we report second-moment validation, displaying variance, first-period autocorrelation and output cross-correlation for each estimated modelization. Consider the variance, displayed in the second and fifth column: all models imply high variances with respect to the ones in data (we emphasize in bold the values which are closer to the true ones, without considering the full model, CES-Z_k-MEI, for the purpose of this exercise). In relative terms, every model is able to capture the higher volatility of investment and employment, and the MEI model is the one that gets closer to the true value, and we find that all models capture the relative lower variance of consumption with respect to output - although it is still CES-MEI which implies a more realistic value. The superiority of CES-MEI is evident also for hours worked and wages volatility. Note that CES-Z_k is the worst performer, especially in terms of hours worked, partly explaining what we think is the cause of the results in the previous paragraph.

The remaining columns display period-1 autocorrelation and output cross-correlation for

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[1]Note, however, that our modelization of the economy is different from their paper. We do not distinguish between the investment-specific technology (IST) shock and the MEI one; the implications will be clear when we 'll look at the variance decomposition of the shocks, as we'll find less importance of MEI shocks over, for example, the technology shock.

each variables; as in CLPY, no model is able to qualitatively capture the cross-correlation of hours and wage with output, and all models are able to match the negative correlation of inflation and output. Overall, we conclude that the MEI model is the best in terms of moments analysis, whereas the model without the investment-productivity shock is the worst. We elaborate on this point in the following paragraph.

As an additional means of validation, we extend the previous analysis, plotting in Figure 2 the correlogram of the estimated models (solid lines) against the autocorrelations implied by data (dashed line), up to order 10. A qualitative inspection reveals that, overall, we are able to match the decaying autocorrelation structure of data, but some differences emerge. Although the models based on CLPY render the same results, with CES-MEI being the closest to the data-implied values especially on higher lags, the dynamics of the autocorrelations implied by CES-Z_k for hours, inflation and interest rate indicate that this model is again the worst for moment matching.

We now look at the major sources of business-cycle fluctuations in our models. To do so, we compute the variance decomposition of each observable implied by each shock in the CES models; these are reported in Tables 4 to 6. In line with Justiniano et al. (2010), the first table assigns a minor role for the MEI shock for output, hours and wages; this is expected as our models do not distinguish between Investment-Specific technology shock (IST) and MEI. Extending the model and dataset to account for this difference, as they did in Justiniano et al. (2011), will probably overturn this result. Note, however, that the Uzawa's theorem poses an obstacle to this procedure, as in order to have a BGP the model must either have a unitary elasticity of substitution, or permanent technical change must be only labor-augmenting. As in JPT (2011) the IST shock differs from the MEI in being a permanent shock, we would have to change the CES specification to allow for $\sigma = 1$ in the long run. We discuss this in our conclusions.

In Table 5 we consider the CES-Z_k specification; with respect to the previous table, the capital-augmenting shock becomes the major source of fluctuation for consumption. Moreover, the influence of the labor-specific shock is much lower. We point out that this model predicts a major role of the price markup disturbances: looking at the third column of the table, one can see that these capture almost all of the variation in employment and

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more than 60% of that in investment. This, we argue, is an undesirable result, especially in light of the evidence in Ferroni et al. (2019) of non-significance of the markup-kind of shocks.

Table 6 presents the full model with both disturbances. It appears that the role for capital-augmenting shocks is null under this specification: indeed, the variance decomposition is almost identical to the CES-MEI model, and assigns a value close to zero to ε_k. This indicates the paramount importance of investment shocks with respect to capital-specific ones in capturing the business-cycle fluctuations. With respect to the conclusions in Cantore et al. (2015) we argue that, although we do find that when Z_k is absent it is Z_i which captures capital-biased technological progress, the analysis of variance decomposition proves that this productivity shock has not much importance, overall.

1.5 Conclusions

Our research adds to the recent DSGE literature on the Constant Elasticity of Substitution production function, by building on a major empirical outcome of Cantore et al. (2015): using Bayesian estimation techniques, they find that data significantly push for this functional form, instead of the standard Cobb-Douglas one.

In this chapter we inspect the mechanism underlying their results, disclosing which are the determinants of CES function's superiority against the Cobb-Douglas. This determinants, we argue, can be shrunk in two: the advantage either comes from the implication of endogenous input shares or from the definition of input-specific productivity shock. These are indeed the main differences that arise - with respect to the Cobb-Douglas functional form - when one assumes the CES. If it is the case that both features are important, we want to be able to point out which one drives the outcome, as such analysis has not been carried out yet and we think that the implications for business-cycle modelization are interesting.

The analysis is particularly attractive when one considers the close relationship between the capital productivity shock and the Marginal Efficiency of Investment one. Assessing which of the two is more relevant in a medium-size model can disentangle their contribution to the business-cycle and help overcoming the so-called *Barro-King curse*, as defined by Ascari et al. (2016), in more straightforward way.

Viewing the paper from the perspective of the PhD thesis, this study can be considered as the first chapter of the research and will help choosing, in the following chapter, the best framework for a medium-size model that accounts for both non-unitary degree of input substitutability and firm dynamics.

Turning to the results obtained in Section 1.4, we conclude that: *a)* the relative advantage of the CES production function mainly comes from the possibility of non-unitary elasticity of substitution among inputs and the resulting endogeneity of input shares, rather than from input-specific technological change; indeed, in a log likelihood race the latter provides the worst result when compared to a model with MEI disturbances, and this holds true also against a Cobb-Douglas production function. *b)* In a full-model with both capital-augmenting and MEI shock, the former contributes to an unimportant fraction of the business-cycle, thus the role for the MEI shock is paramount for any short-run economic model. *c)* Based on these outcomes, we are not able to solve the comovement puzzle implied by shock to the marginal efficiency of investment; even though the capital-augmenting disturbance moves the variables in the desired direction, its importance over the business-cycle is virtually null.

We leave few directions open for future research. In the light of the results in Justiniano et al. (2011), considering an extended model in which the effect of investment shocks can be directed toward IST or MEI may change the results of point *b)* in the previous paragraph. Another direction we leave open is the analysis of permanent shocks to inputs' productivity and/or IST processes; this is linked to a recent branch of the literature, proposed by León-Ledesma and Satchi (2018), where the CES production function is modeled so as to meet Uzawa's steady state growth theorem (Uzawa (1961)); in particular, input the input elasticity of substitution assumes gross complementarity at business-cycle frequencies, whereas its value goes to unity in the long run, allowing for permanent shock other than the Harrod-neutral one.

Figure 1: Benchmark model with CES production function. IRFs for a 1% increase in the capital-augmenting shock and MEI shock; $\sigma = 0.4$.
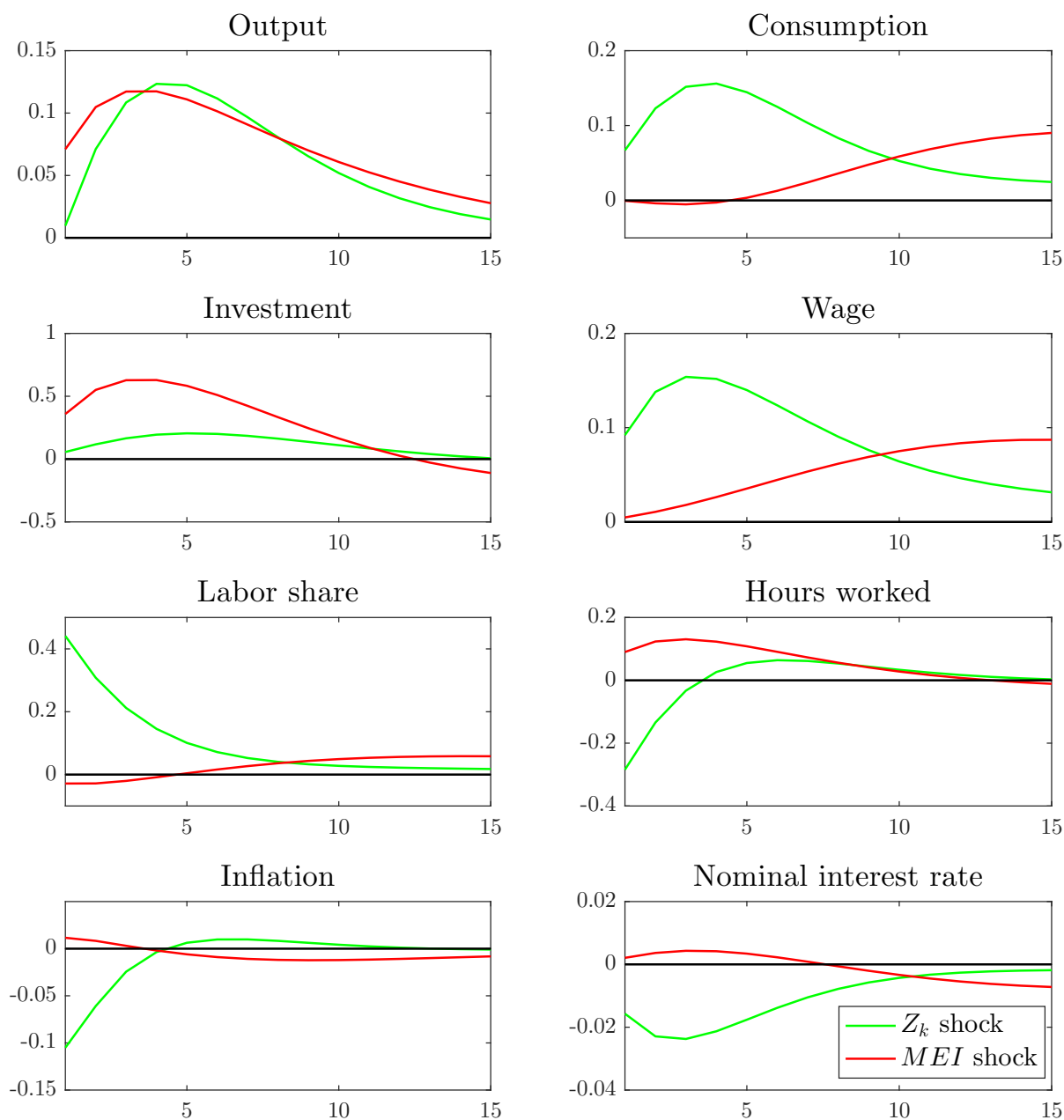
Figure 2: Correlogram of observable variables in the estimated baseline models and data.
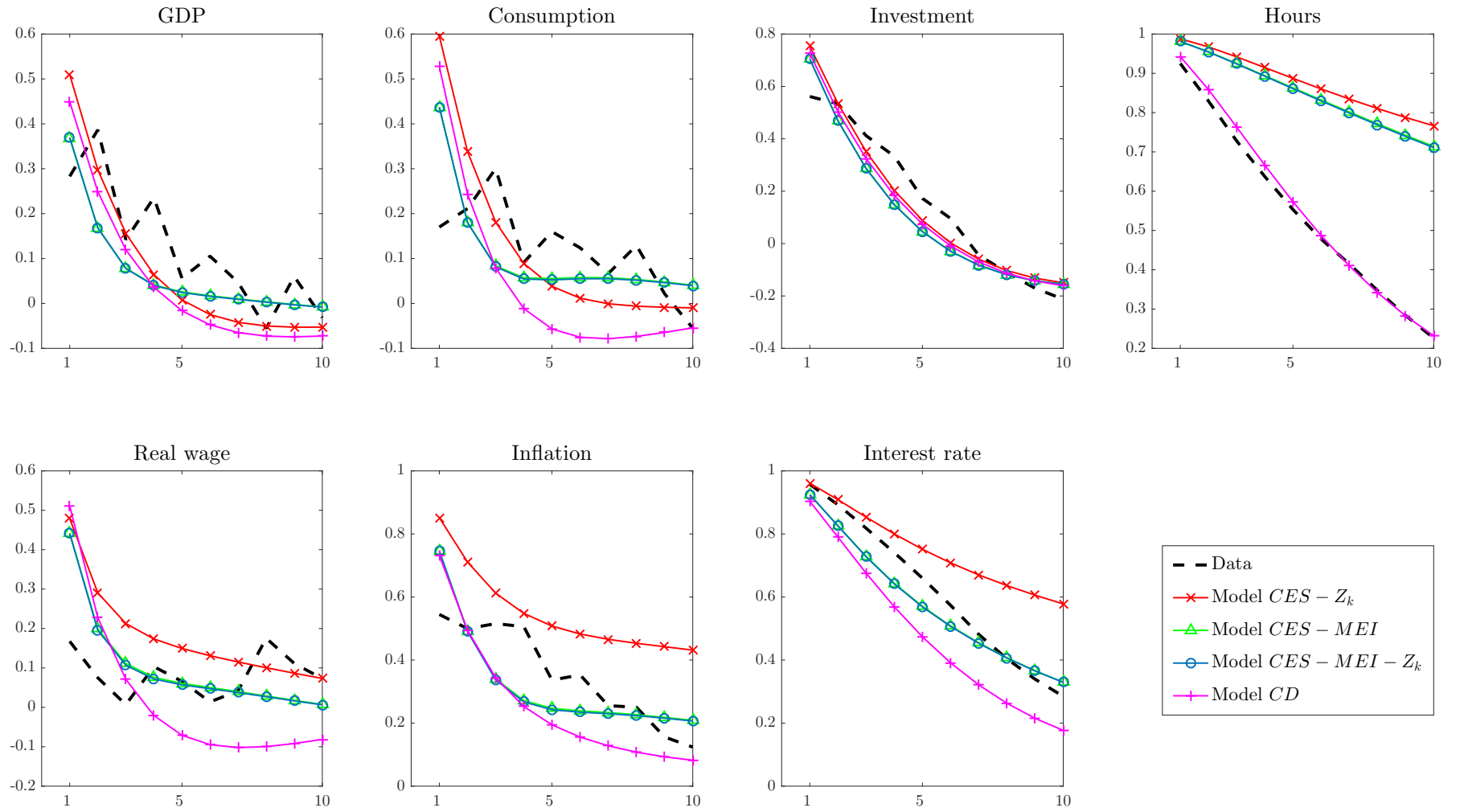
Table 1: Estimated values of the elasticity of substitution (at the mode).

Model	σ
CES-MEI-Z_k	0.153
CES-MEI	0.15
CES-Z_k	0.145

Table 2: Marginal likelihood comparison.

Model	Posterior log marginal density	Bayesian odds
CES-MEI-Z_k	-528.90	0.48
CES-MEI	-528.83	0.52
CES-Z_k	-575.21	0.00
CD	-543	0.00

Table 3: Moments implied by the models' posteriors and SW dataset. In bold are the values closer to the ones of the time series.

Variable	Variance	Autocorr ($t=1$)	Corr $\Delta(logGDP_t)$	Variance	Autocorr ($t=1$)	Corr $\Delta(logGDP_t)$
Data				**CD**		
$\Delta(logGDP_t)$	0.34	0.28	1	0.78	0.50	1
$\Delta(logC_t)$	0.28	0.17	0.62	0.50	0.53	0.70
$\Delta(logI_t)$	3.03	0.56	0.64	5.52	0.77	0.55
$\Delta(logw_t)$	0.44	0.17	-0.11	0.83	0.51	0.16
$logH_t$	5.90	0.93	-0.25	**8.33**	**0.94**	0.14
π_t	0.06	0.54	-0.12	**0.12**	**0.73**	-0.20
$R_{n,t}$	0.37	0.96	0.22	0.15	0.90	-0.24
CES-MEI				**CES-Z_k**		
$\Delta(logGDP_t)$	**0.74**	**0.37**	1	1.24	0.51	1
$\Delta(logC_t)$	**0.46**	**0.44**	**0.61**	0.75	0.59	0.73
$\Delta(logI_t)$	**5.33**	**0.47**	**0.62**	6.27	0.75	0.69
$\Delta(logw_t)$	**0.58**	**0.44**	**0.14**	0.76	0.29	0.22
$logH_t$	17.23	0.98	**0.07**	49.52	0.99	0.07
π_t	0.16	0.74	-0.005	0.27	0.85	**-0.12**
$R_{n,t}$	0.24	0.92	**-0.03**	**0.42**	**0.96**	-0.09
CES-Z_k-MEI						
$\Delta(logGDP_t)$	0.75	0.37	1			
$\Delta(logC_t)$	0.46	0.44	0.61			
$\Delta(logI_t)$	5.36	0.70	0.62			
$\Delta(logw_t)$	0.58	0.44	0.14			
$logH_t$	17.17	0.98	0.06			
π_t	0.16	0.74	-0.008			
$R_{n,t}$	0.24	0.92	-0.04			

Table 4: Unconditional variance decomposition (in %) for the benchmark model with MEI shock.

Series/shock	ε_l	ε_{pmk}	ε_M	ε_G	ε_i	ε_b	ε_{wmk}
$\Delta(logGDP_t)$	19.43	17.61	2.74	15.74	6.72	5.89	31.87
$\Delta(logC_t)$	15.37	14.11	3.65	11.93	11.43	14.57	28.95
$\Delta(logI_t)$	11.88	8.63	0.17	0.16	59.71	4.13	15.32
$\Delta(logw_t)$	13.01	40.62	1.55	1.66	9.16	3.18	30.83
$logH_t$	0.78	16.67	0.16	5.46	4.74	0.34	71.86
π_t	6.12	16.10	14.65	1.15	22.25	20.28	19.45
$R_{n,t}$	7.97	9.15	5.97	1.33	34.23	21.42	19.92

Table 5: Unconditional variance decomposition (in %) for the benchmark model with Z_k shock.

Series/shock	ε_l	ε_k	ε_{pmk}	ε_M	ε_G	ε_b	ε_{wmk}
$\Delta(logGDP_t)$	8.76	8.67	38.93	3.36	17.19	15.15	7.96
$\Delta(logC_t)$	4.96	31.04	19.34	2.21	4.40	32.99	5.06
$\Delta(logI_t)$	14.72	2.27	66.79	3.48	0.06	2.06	10.63
$\Delta(logw_t)$	7.07	15.05	26.39	0.08	0.53	0.22	50.66
$logH_t$	0.64	1.97	88.73	0.73	1.93	1.33	4.67
π_t	3.19	38.82	40.35	3.23	1.40	5.22	7.79
$R_{n,t}$	3.98	39.92	33.09	7.67	2.20	6.02	7.13

Table 6: Unconditional variance decomposition (in %) for the benchmark model with both Z_k and MEI shocks.

Series/shock	ε_l	ε_k	ε_{pmk}	ε_M	ε_G	ε_i	ε_b	ε_{wmk}
$\Delta(logGDP_t)$	19.34	0.02	17.46	2.72	15.76	6.76	5.99	31.95
$\Delta(logC_t)$	15.43	0.05	14.01	3.63	11.68	11.20	14.81	29.19
$\Delta(logI_t)$	11.74	0.00	8.54	0.17	0.15	59.97	4.13	15.29
$\Delta(logw_t)$	13.10	0.08	40.64	1.60	1.60	8.90	3.32	30.76
$logH_t$	0.77	0.00	16.28	0.16	5.43	4.69	0.35	72.32
π_t	6.03	0.18	16.10	14.60	1.13	21.99	20.41	19.57
$R_{n,t}$	7.86	0.06	9.18	6.04	1.34	33.79	21.54	20.20

2 Chapter 2

Abstract

We build a business-cycle model that incorporates both endogenous variation in the number of products and different degrees of input subsititutability. We find that a comovement puzzle emerges when the values of the elasticity of subsution are close to the ones that are estimated in the literature. In particular, both the capital-augmenting and the investment-specific shocks generate a counter-cyclical variation in business creation that is not matched in US data. Using Global Sensitivity Analysis techniques, we prove that the existence of the puzzle is subject to the presence of investment adjustment costs, which affects the intertemporal choice of the representative household between investing in physical capital or in business creation.

2.1 Introduction to the chapter

In most cases, business-cycle models assume a fixed number of producers and products. This is at odds with the stylized fact of procyclical variations in the number of competitors (Jaimovich and Floetotto (2008)): as Figure 3 shows, net entry is strongly procyclical and comoves with real profits. Moreover, Bilbiie et al. (2007) demonstrate that net entry is correlated to profits, and that the former leads GDP and profit expansions. In a successive work Bilbiie et al. (2012), henceforth BGM, introduce endogenous firm dynamics in a standard Real Business Cycle (RBC) model, showing that variations in the number of producers-products are important sources of fluctuations, a result in line with the empirical contributions on firm dynamics, and that a model with endogenous product variety performs as well as a standard RBC model in terms of simulated second moments. Following this literature, in recent years Lewis and Poilly (2012) and Lewis and Stevens (2015) modified the baseline model in BGM with a number of extensions in order to fit a medium-scale DSGE model with firm dynamics.

Adopting the CES production function in a slightly modified BGM-RBC model, we introduce input-specific shocks and look at the impulse response functions generated by the model: these have different implications in terms of firm entry. The exercise considers such model both in absence and in presence of investment adjustment costs, which have a major impact in driving investment and thus firm entry. In our knowledge, we are the

first to carry out this analysis.

In particular, in this chapter we build on the analysis presented in Chapter 1 of investment choice and input substitutability, providing theoretical implications for firm dynamics of assuming different degrees of input substitutability. This model determines that capital-augmenting technological shocks can lead to an outflow of investment in new firm towards physical capital and consumption, thus reducing the total number of firm in the system -a result at odds with the literature on endogenous firm entry. Using Monte Carlo Filtering (MCF) techniques, we demonstrate that the result depends on the presence of investment adjustment costs. We show that also the MEI shock is subject to this comevement puzzle, as long as no adjustment costs are considered.

The chapter is organized as follows. Section 2.2 presents the model equation and its equilibrium. In section 2.3 the calibration procedure is presented, along with the key dynamic responses of each shock: Global Sensitivity Analysis (GSA) will be used to find which parameters drive the response of entry. Section 2.4 concludes.

Related literature   The empirical facts that motivates the implementation of endogenous product variety in DSGE models mainly come from two contributions; the first one is by Bernard et al. (2010): they are the first to measure product creation and destruction within firms in the US economy. Their results can be summarized in three key facts: they find that 94% of product creation occurs within pre-existing plants, that 68% of the firm sample changes their product set within a 5-years period, and that the value of new products in total output is about 33%.

The second contribution is by Broda and Weinstein (2010), who measure product creation and destruction at the finest level of disaggregation, by looking at purchases of products with bar codes by a representative sample of US consumers. Their results confirm the ones in Bernard et al. (2010); moreover, their contribution highlights the strong procyclicality of product creation in the short run. Together, these two papers call for a major role of firm dynamics at business-cycle fluctuations.

Bilbiie et al. (2012) introduce endogenous firm creation in a standard RBC model. In this settings, each firm produces a single variety of goods, and endogenous creation of products occurs when prospective entrants decides to start producing, subject to a sunk cost of entry. Their modelization allows markups to be dependent on the degree of "competi-

tiveness" (i.e., number of varieties) in the economy, by adopting the *translog* preferences definition defined in Feenstra (2003). Based on the dynamic response of the economy after a technology shock, they conclude that variations in the number of producers is an important transmission mechanism, driving output and the intertemporal choice of the household. As an empirical means of validation, looking at the simulated time series of the model compared with US data, their model performs at least as well as traditional RBC models, with the additional feature of capturing the empirical relationship between entry and profits.

Lately, Lewis and Stevens (2015) estimated a medium-size DSGE model with firm creation and translog preferences. They report that a 1% increase in the number of firms lowers markup by 0.17%, and this endogenous change play a non-negligible role for the variation in inflation: as more firms enter the production sector, competitive pressures is higher and this pushes desired markups and inflation to be lower.

2.2 The benchmark model

The model follows the RBC modelization presented in BGM.

2.2.1 Household preferences

The economy in our model is composed of a unit mass of identical households indexed by $h \in (0,1)$. The representative household supplies L_t hours of work each period t in a competitive labor market, for which she is paid a nominal wage rate W_t; moreover, she supplies K_t units of capital to the representative firm to be used as input in production: the rental rate of such capital is defined as r_k. Prices and wages are assumed to be flexible.

The representative household wants to maximize her expected lifetime utility at time t

$$\mathbf{E}_t \sum_{s=0}^{\infty} \beta^s U_{t+s}$$

where β is the subjective discount factor. Utility takes the following functional form, positive in consumption C and negative in hours worked

$$E_t \sum_{s=0}^{\infty} \beta^s Z_{b,t+s} \left[\frac{(C_{t+s} - hC_{t+s-1})^{1-\sigma_C}}{1-\sigma_C} - \chi \frac{L_{t+s}^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right], \quad \chi > 0, \; \phi \geq 0 \qquad (10)$$

where ϕ is the Frisch elasticity, Z_b is an intertemporal preference shock, h accounts for external habit formation in consumption, and σ_C is the degree of relative risk aversion.

Given our hypothesis of endogenous product variety, at each moment in time t, only a subset $N_t \in \Omega$ of goods is available, where Ω represents the set of all conceivable goods. Then, the basket of goods consumed C_t is defined as a CES aggregator over the continuum of available goods,

$$C_t(h) = \left(\int_\omega c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution among goods, and we indicated the single variety as $\omega \in \Omega$.

The consumption-based price index is defined as $P_t = \left(\int_\omega p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$; household's demand for each individual good is then

$$c_t(\omega, h) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t(h)$$

In the following, given households' symmetry, we'll abstract from the representative household omitting the index h, as the intertemporal choices will be the same.

2.2.2 Endogenous product variety

As in Bilbiie et al. (2012), every period is characterized by a defined number of incumbent firms, N_t, each producing a different variety $\omega \in \Omega_t$, where N_t is the mass of Ω at time t, and an unbounded mass of prospective entrants: I assume that startups starts producing one period after they are created. Moreover, there is a probability δ for both kind of firms to be hit by a death shock, thus exiting production.

Prospective entrants must bear a sunk-cost equal to f_e effective labor units. This implies that setting up a firm requires a cost of:

$$f_e Z_{l,t} l_{e,t} \tag{11}$$

Where $Z_{l,t}$ is the labor-specific shock. This shock enters also in the production function of the good sector. We can define the "technology" for new goods as

$$N_{e,t} = Z_{l,t} \frac{L_{e,t}}{f_e} \tag{12}$$

where $L_{e,t} = N_{e,t} l_{e,t}$. The unit cost of setting up a new firm is then

$$\frac{w_t l_{e,t}}{Z_{l,t} \frac{l_{e,t}}{f_e}} \Rightarrow w_t \frac{f_e}{Z_{l,t}} \tag{13}$$

Prospective firms decide to start producing if their discounted expected future stream of profits at time t - that is, their post-entry value $v_t(\omega)$ - is greater than, or at least equal to, the entry cost they have to bear. Considering the assumption on entry costs, such free-entry condition is:

$$v_t(\omega) = w_t \frac{f_e}{Z_{l,t}}$$

The law of motion for the number of firms can be defined as:

$$N_{t+1} = (1 - \delta)(N_t + N_{e,t}) \tag{14}$$

2.2.3 Firm optimization

There is a continuum of monopolistically competitive firms, each producing a different variety ω. Production requires capital and labor: note that in order to distinguish between labor used in creating new firm and labor used in production, we define the former as L_e and the latter as L_c, where $L_c + L_e = L$.

The two inputs are used according to the normalized CES production technology:

$$y_{f,t}(\omega) = y_{f,0} \left[\alpha_0 \left(\frac{k_t(\omega)}{k_0} Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_0) \left(\frac{l_{c,t}(\omega)}{l_{c,0}} Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Firms face demand for their output from consumers in terms of consumption and investment goods. So total demand for the output of firm ω is

$$y_{d,t}(\omega) \equiv c_t(\omega) + i_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} (C_t + I_t)$$

Where $C_t + I_t$ is total output.

The firm chooses labor and capital in order to minimize costs, subject to the production function. Defining mc_t as the lagrangian multiplier, the first order conditions are:

$$w_t = (1 - \alpha_0)mc_t \left(\frac{y_t}{l_{c,t}} \right)^{\frac{1}{\sigma}} \left(\frac{y_0}{l_{c,0}} Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}}$$

$$r_{k,t} = \alpha_0 mc_t \left(\frac{y_t}{k_t} \right)^{\frac{1}{\sigma}} \left(\frac{y_0}{k_0} Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}}$$

The real profit function is

$$d_t = \frac{p_t}{P_t} y_t^D - w_t l_{c,t} - r_{k,t} k_t \tag{15}$$

Optimal pricing yields $p_t = \frac{\theta}{\theta-1} mc_t P_t$, where $\frac{\theta}{\theta-1} = \mu$ is the price markup charged by the firm; this is fixed in this setting, but as BGM show, it can be made endogenous and

dependent on the number of competitors when one adopts translogarithmic preferences as in Feenstra (2003). Dividing by the price index P yields optimal price in terms of the relative price, ϱ:

$$\varrho_t = \mu mc_t \tag{16}$$

Given symmetry, the consumption-based price index is $P_t = N_t^{\frac{1}{1-\theta}} p_t$, or in terms of the relative price

$$\varrho_t = N_t^{\frac{1}{\theta-1}} \tag{17}$$

Equation 17 defines the *love for variety* parameter. An increase in the number of firms implies that the relative price of each individual good increases: when there are more firms, a dollar buys more consumption utility and the price index falls.

It is easy to show that capital and labor demands can be rewritten in aggregate terms as:

$$w_t = \frac{(1-\alpha_0)}{\mu} \left(\frac{\varrho_t}{\varrho_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_{c,t}}\right)^{\frac{1}{\sigma}} \left(\frac{Y_0}{L_{c,0}} Z_{l,t}\right)^{\frac{\sigma-1}{\sigma}} \tag{18}$$

$$r_{k,t} = \frac{\alpha_0}{\mu} \left(\frac{\varrho_t}{\varrho_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} \left(\frac{Y_0}{K_0} Z_{k,t}\right)^{\frac{\sigma-1}{\sigma}} \tag{19}$$

Aggregating the production function by the number of firms yields.

$$Y_t = \frac{\varrho_t}{\varrho_0} Y_0 \left[\alpha_0 \left(\frac{K_t}{K_0} Z_{k,t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L_{c,t}}{L_{c,0}} Z_{l,t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{20}$$

Finally, substituting the FOCs for input choices in Equation 15 allows us to simplify the definition of individual profits:

$$d_t = \left(1 - \frac{1}{\mu}\right) \frac{Y_t}{N_t}$$

2.2.4 Household intertemporal optimization

Households hold shares in a mutual fund of firms: following Bilbiie et al. (2012), we define x_t as the share in the mutual fund held by the representative household entering period t. Such fund pays a profit in each period equal to the total profit of all firms that produce in that period, $P_t N_t d_t$. In period t, the representative household buys x_{t+1} shares in a mutual fund of $N_t + N_{e,t}$ firms: those already operating at time t and the new entrants. The exogenous exit assumption implies that only $1 - \delta$ of all firms will produce and pay dividends at time $t + 1$. The date t price of a claim to the future profit stream of the mutual fund is equal to the nominal price of claims to future firm profits, $P_t v_t = V_t$.

The representative household enters period t with nominal bond holdings $B_{N,t}$ and mutual fund share holdings x_t. She receives gross interest income on bond holdings, dividend income on mutual fund shares and the value of selling its initial share position, plus labor income and the rental rate on units of capital lent to the firms.

The households allocates these resources among purchases of bonds and shares to be carried into next period, consumption and investment. Then the period budget constraint in nominal terms is

$$B_{N,t+1} + V_t(N_t + N_{e,t})x_{t+1} + P_t C_t + P_t I_t = \tag{21}$$

$$= (1 + i_{t-1})B_{N,t} + (D_t + V_t)N_t x_t + R_{k,t}K_t + W_t L_t \tag{22}$$

where D_t is nominal dividends, equal to firms' profits ($D_t = P_t d_t$).

Capital accumulation is given by

$$K_{t+1} = (1 - \delta_K)K_t + I_t\left[1 - \frac{\psi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]Z_{i,t} \tag{23}$$

Changes in the capital stock are costly, and subject to a convex adjustment cost, with $\psi_K > 0$. The term Z_i accounts for the MEI shock.

The investment good I is assumed to be a composite made of the aggregator function seen for consumption goods: $i_t(\omega) = \left(\frac{p_t(\omega)}{P_t}\right)^{-\theta}I_t$. Now the household's problem is to choose C_t, x_{t+1}, B_{t+1}, L_t, I_t, K_{t+1} in order to maximize 10 under the constraint given by 21 and 23:

$$\underset{C_t,\ x_{t+1},\ B_{t+1},\ L_t,\ I_t,\ K_{t+1}}{\text{maximize}} \quad U_t = Z_{b,t}\left[\frac{(C_t - hC_{t-1})^{1-\sigma_C}}{1 - \sigma_C} - \chi\frac{L_t^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}}\right]$$

$$\text{subject to} \quad B_{t+1} + v_t(N_t + N_{e,t})x_{t+1} + C_t + I_t =$$

$$= B_t(1 + r_t) + (d_t + v_t)N_t x_t + r_t^K K_t + w_t L_t,$$

$$K_{t+1} = (1 - \delta_K)K_t + I_t\left[1 - \frac{\psi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]Z_{i,t}$$

The fist order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow \frac{Z_{b,t}}{(C_t - hC_{t-1})^{\sigma_C}} - \frac{\beta h Z_{b,t+1}}{(C_{t+1} - hC_t)^{\sigma_C}} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial x_{t+1}} = 0 \Rightarrow v_t = \beta(1 - \delta)\frac{\lambda_{t+1}}{\lambda_t}(d_{t+1} + v_{t+1})$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \Rightarrow \lambda_t = \beta\left(\frac{1 + r_t}{C_{t+1}}\right)$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = 0 \Rightarrow w_t = \frac{\chi}{\lambda_t} L_t^{\frac{1}{\phi}}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = 0 \Rightarrow \lambda_t = \zeta_t Z_{i,t} \left[1 - \frac{\psi^K}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \psi^K \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] +$$

$$+ \beta Z_{i,t+1} \psi^K \zeta_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Rightarrow \zeta_t = \beta \frac{1}{C_{t+1}} r_{k,t+1} + \beta \zeta_{t+1} (1 - \delta_K)$$

Where in the Euler equation on shares I used the law of motion in Equation 14 to simplify the notation.

The equilibrium conditions, $B_{t+1} = B_t = 0$ and $x_{t+1} = x_t = 1 \forall t$ yields the aggregate accounting identity:

$$C_t + I_t + N_{e,t} v_t = w_t L_t + r_{k,t} K_t + N_t d_t \tag{24}$$

Table 7 summarizes the main equilibrium conditions of the model. Because of the way they have been defined, the stock of capital K and the number of varieties N are both predetermined at t.

2.3   Calibration and impulse response functions

Before turning to the analysis of impulse responses we must point out a common issue that emerges when adopting models with product variety. As Bilbiie et al. (2012) point out, when we discuss model properties with respect to empirical evidence, it is important to acknowledge that data collected on relevant variables don't include the so-called variety effect, that is, the effect of changes in the range of available products. For this reason, in our analysis we focus also on variables deflated by a data-consistent price index, to "clean" such variables from the variety effect. This implies that when looking at the IRFs that the models generate, we define, for each relevant variable X_t in units of the consumption basket, a data consistent variable $X_{R,t} = \frac{X_t}{\varrho_t}$.

Calibration of the model follows from the interpretation of periods as quarters, and is based on Bilbiie et al. (2012). The probability of the firm-exit shock equals $\delta = 0.025$ to match a 10% annual production destruction rate consistent with the empirical literature, and the preference parameter is $\theta = 3.8$. β, the discount rate, equals 0.99, implying a 4 percent annualized interest rate. The parameter controlling for investment adjustment costs, ψ_K, equals 2.5. The entry cost f_e is set to unity, as it does not affect the dynamics

of the IRFs. We set the value of steady state output and total labor - corresponding to their *normalized* - values to unity, $Y = L = 1$.

The capital share in steady state equals $\frac{0.2}{\mu}$. The autoregressive parameters of the shocks are set to 0.9.

Since we are interested in comparing the IRFs under different assumption on the "quality" of inputs (either gross complements or gross substitutes), we calibrate the elasticity of input substitution to be $\sigma = (0.4, 1, 1.4)$, where by definition the parametrization $\sigma = 1$ corresponds to the Cobb-Douglas production function. Note, however, that we'll omit this last specification when plotting the IRFs since they are known.

We moreover define data consistent return of investment in new firms as $r_{R,t+1}^E = \frac{v_{R,t+1}+d_{R,t+1}}{v_{R,t}}$ and investment in new firms as $I_{R,t}^E = v_{R,t}N_{e,t}$.

2.3.1 Impulse response functions

Figures 4 to 7 plot the dynamics (percent deviations from steady state) after a 1 percent increase in each of the error term of the AR processes for the shocks. Remember that Z_L is the only "aggregate" shock, as it symmetrically affects the technology in both the production and the firm-creation sector, so that we would expect to find a dynamic response similar to the one in BGM.

Consider Figure 4. As labor productivity increases, the business environment becomes more attractive because of increased demand and higher future profits. The free-entry mechanism ensures that the ex-ante value of each variety remains always equal to the marginal cost of entry. Firm entry increase on impact, slowly returning to the steady-state as the individual demand decreases: the hump-shaped response of N is due to its definition of predetermined state variable and to the presence of the death shock δ.

Consider the optimal allocation of the productivity increase between the three sectors: consumption C, investment in physical capital I, and investment in entry N_e. In order to understand the dynamics involved, remember that the price of a share (given by the value of a firm, v) and its payoff (the dividends obtained from such firm) determine altogether the return on a share, that is, the return to entry. On impact r_{t+1}^E (evaluated ex ante the investment decision) is positive for the Cobb-Douglas and gross-substitute case since present value is low compared to its future value and profits are expected to be higher. Then, the household decides each period how much of the productivity increase

to consume and to intertemporally transfer through investments; because of the presence of investment adjustment costs, the fraction of resources invested in physical capital I is less than the one in new firms $I^E = vN_e$. This is mirrored by the evolution of the stock of capital and of new products.

Note that as on impact new firms are created, there is an outflow of workers form the production sector L_c to the new variety creation L_e, leading to an overall increase in total labor with respect to the steady state. Real wage increases accordingly to the productivity shock, so that households decide to work more in order to have a higher level of consumption. As demand starts decreasing - this is due to love of variety, remember that $y_t = Y_t/(\varrho_t N_t)$ -, firm value and profits reduce and the opportunity cost of investment (in terms of foregone consumption that the household can enjoy because of more product variety) rises: this leads investment in startups to lower. The reallocation of labor pushes workers into production, so that the overall number of firms returns to its steady state. As we would expect, the response of the input shares symmetrically depends on the size of the elasticity of substitution. As biased technical change predicts, the labor-augmenting shock favors the labor share only in the case of gross-substitutability of inputs. The dynamic response of labor share under imperfect complementarity resemble the empirical finding documented in Rios-Rull and Santaeulalia-Llopis (2010): a productivity innovation produces a reduction of labor share on impact, making it countercyclical, but it also produces a long-lasting subsequent increase of labor share that overshoots its long-run average and slowly returning to its steady state.

The shock to the productivity of capital is plotted in Figure 5.

The productivity shock spurs expectations of profits as the demand rises over its steady state value; the qualitative response of real wage and firm value push the return to investment in startups to be lower on impact. Biased technical change implies that the dynamics of inputs shares mirror the ones of the labor-augmenting shock.

The difference in this scenario regards the response of firm entry under gross complementarity among inputs; on impact the shock lowers N_e, favoring labor in production. The consequence is that immediately after the shock the total number of varieties decreases. This is a key feature of the model.

This result is at odds with the key empirical findings we outlined at the beginning of the

chapter.

As this deserves a deeper investigation, in the next section I rely on Global Sensitivity Analysis as a means to identify which parameters drive such dynamics. I anticipate here that the investment adjustment cost parameter ψ_K and the elasticity of substitution are responsible for the differences in the intertemporal allocation of resources by the household: the definition of a threshold for the response functions allows us to gather some theoretical insights on the underlying mechanism of investment decisions.

Global Sensitivity Analysis   Global sensitivity analysis (GSA) technique consists, broadly speaking, in the process of recalculating outcomes under alternative assumptions to determine the impact of a variable on a mathematical system (in our case, the set of equations defining the general equilibrium), "helping to make the model structure and properties more transparent to the analyst" (Ratto (2008)).

In the following I'll rely on such techniques to measure under which calibration the relevant parameters of the model are able to capture certain firm dynamics. In particular, the Monte Carlo Filtering (MCF) procedure will be applied to define regions of the parameters' mapping which result in established IRFs for the number of entrants. I anticipate that the results will show the key role of investment adjustment costs in driving the intertemporal redistribution of resources.

A brief explanation of the methodology follows. First, a multi-parameter Monte Carlo simulation is performed, sampling model parameters $(X_1, \ldots, X_k)$ from prior ranges and propagating parameter values through the model. Then a categorization is defined for each MC model realization as either within (B) or outside ($\overline{B}$) the target region: note that in our case the target region is the positive response on impact of the number of new varieties.

Given the full set of N Monte Carlo runs, one obtains a subset of parametrization inside the target region, $(X_i|B)$, of generic size n, and a subset outside the target region, $(X_i|\overline{B})$, of size $\overline{n}$, where $n + \overline{n} = N$. The two sub-samples will come from different unknown probability density functions: $f_n(X_i|B)$ and $f_{\overline{n}}(X_i|\overline{B})$.

In order to identify the parameters that mostly drive the DSGE model into the target behaviour, the distributions f_n and $f_{\overline{n}}$ are compared with standard statistical tests, for each parameter independently. If for a given parameter X_i the two distributions are

significantly different, than it is a key factor in driving the model behavior. If the two distributions are not significantly different, than X_i is unimportant and any value in its predefined range is likely to fall either inside or outside the target region.

Figure 6 plots the GSA applied to the impulse response of entry after the capital-augmenting shock; here only the parameters that are statistically significant by the Smirnov test are displayed. Consider first the graphs on the main diagonal: the blue lines show the cumulative distribution function of $f_n(X_i|B)$, where the behavior B is a positive (≥ 0) response on impact of N_e. The larger the distance between the CDF under the different behaviors, the more likely is the parameter to produce different sign response for different calibrations: the slope of the CDF is the marginal contribution of a rise in the parameter in object in leading the response. Looking at the first graph, that shows the influence of σ, we see that for values of the elasticity of substitution greater than unity the contribution of the parameter is greater in driving the positive response of N_e: that is, gross substitutability is a leading factor for the positive response of entry. The graphs off the main diagonal show the combined influence of both σ and ψ_K for product creation: note that they are symmetrical. When no investment adjustment cost is considered ($\psi_K = 0$) households will disinvest from new firm under any value of σ: this leads to an outflow of workers from the entry sector towards production; as ψ_K increases, the sign response of entry depends on the definition of inputs' substitutability: if capital and labor are gross substitutes ($\sigma < 1$), firm entry decreases, leading to an overall decline in the number of total firms.

To explain this results, I plot in the next section of this chapter the same model with no investment adjustment costs. Consider Figure 9: note that in the absence of investment adjustment costs the capital-augmenting shock generates negative movements in firm dynamics for every calibration of σ. When σ is less than unity, the intertemporal allocation of resources favors investment in physical capital and consumption over investment in new firms even if the rental rate after the shock diminishes: indeed the implication of biased technical change pushes the real wage to rise - which makes households more likely to consume a greater part of their income -this explains the differences in the response of both wage rate and consumption under different calibration of σ-, so that also the marginal cost of producing a new variety is higher, discouraging firm dynamics.

If instead increasing investment in new capital is costly, then the previous dynamics

holds only in the gross-substitute input case with a capital-augmenting shock, where the opposite dynamics of the remuneration of inputs makes entry too costly in terms of wage.

The last shock to be analyzed is the one to the efficiency of investment, plotted in Figure 7.

The dynamic responses of the shock, with the exception of firms, are in line with the previous work by Justiniano et al. (2010). As investing in physical capital is now more efficient, households will prefer to transfer resources towards investments. The observation of decline in consumption reflects this intertemporal allocation of the increased efficiency. By definition, lower consumption increases the marginal utility of income: the representative household will raise his labor supply against a fixed labor demand; hours and output - as total investments enter the definition of GDP - rise, while the real wages fall.

The implication for firm dynamics is that the shock Z_i has - independently from the value of the elasticity of substitution - a positive impact on the number of new varieties. Note that the quantitative response of N_e is much lower than in the case of productivity shocks, so that - as one would expect - the shock favors investment in physical capital over investment in startups.

In the capital-augmenting/MEI shock race, the latter is favored in this theoretical analysis, given the positive comovement generated between entry and profits. In the next section we'll show that this result holds as long as investment adjustment costs are considered. In Figure 8 we argue that changing the investment adjustment cost parameter generates a new comovement puzzle for the MEI shock; indeed, for lower adjustment costs, the response of entry is negative on impact. Note that there seems to be a threshold for this mechanism at $\sigma \approx 1.5$: higher values for the elasticity of substitution always imply positive investment in startups. This is an important result, as, to the best of our knowledge, this new *comovement puzzle* has never been mentioned in the literature.

2.3.2 Additional IRFs

We plot the IRFs of the benchmark model in the absence of investment adjustment costs (i.e., when $\psi_K = 0$). The calibration of the remaining parameters is the same present at the beginning of the current section.

From Figures 9 and 10 one can see that both shocks exert a negative influence on investment in new firms, *independently of the calibration for the elasticity of substitution*, since now transferring resources to investment in costless.

As an additional exercise, we show that the same results hold also when a richer specification of entry is modeled. We define a congestion effect as in Lewis and Stevens (2015), where it is assumed that not all firm startups are successful: only a fraction F_N becomes operational one period later. The success of a startup is defined as

$$F_{N,t}(N_{e,t}N_{e,t-1}) = 1 - S_N \left(\frac{N_{e,t}}{N_{e,t-1}} \right) \tag{25}$$

where S_N is the failure rate of entrants, which is an increasing function of the change in entry and is specified in our case as a quadratic adjustment cost function with a shock that reduces it:

$$S_N \left(\frac{N_{e,t}}{N_{e,t-1}} \right) = \frac{\psi_N}{2} \left(\frac{N_{e,t}}{N_{e,t-1}} - 1 \right)^2 \tag{26}$$

The representative household can now choose how much to invest directly into startups, besides investing in equity, which now includes only productive firms.

The timing is as follows (to simplify, we use end-of-the-period notation): on revenue side, the households has an amount $N_{e,t-1}$ of previously financed startups, of which a fraction $(1 - \delta_N)F_{N,t-1}$ is successfully established and survives the death shock. This amount of established productive startups is now worth v_t - its stream of future profits - and earns a profit equal to $P_t d_t$. A similar reasoning goes for the incumbent firms N, that are owned by the household through share holdings e_t; these pay a dividend equal to their profits and are worth the firms' value, as for the startups. Moreover, the death shock is the same for the incumbents, so that on the revenue side we can write, in nominal terms

$$P_t(v_t + d_t)e_{t-1}(1 - \delta_N)N_{t-1} + P_t(v_t + d_t)(1 - \delta_N)F_{N,t-1}N_{e,t-1}$$

$$(1 - \delta_N)P_t(v_t + d_t)(E_{t-1} + F_{N,t-1}N_{e,t-1})$$

where $E_t = N_t e_t$ Then on the expenditure side, the household at time t decides how much to invest in new firms $N_{e,t}$ and in shares of incumbents to be carried into next period. The cost of a share is given by the value of the representative firm today, v_t, while the cost of a startup is given by its creation cost in terms of effective labor, $w_t \frac{f_e}{Z_{l,t}}$, so that

the expenditure side will have, among the usual terms,

$$P_t v_t E_t + P_t w_t \frac{f_e}{Z_{l,t}} N_{e,t}$$

Figures 11 to 14 plot the IRFs in presence and absence of investment adjustment costs, where it's clear that the results area qualitatively unchanged. Note, moreover, that given the increasing cost of entry - in terms of failure probability - now the MEI has a sluggish, negative dynamic of investment in startups when $\psi_K > 0$.

2.4 Conclusions

We extended the benchmark model of firm dynamics with the introduction of input-specific productivity shocks and allowing labor and capital to differ in their degree of substitutability; this was made possible through the use of the Constant Elasticity of Substitution (CES) production function. The exercise allowed us - through the process of normalization of the production function - to shed light on how the elasticity of substitution influences the firm dynamics.

Contrary to the bulk of the literature on this topic (Bilbiie et al. (2012), Lewis and Poilly (2012) among others), we find that capital-augmenting and MEI shocks can lead to an outflow of investment in new firms towards physical capital and consumption when the production function is non-Cobb-Douglas, reducing the total number of firm in the system in spite of rising output and profits. This counter-cyclicality of product creation represents another comovent puzzle that is worth inspecting. The result is analyzed in detail through Monte Carlo Filtering (MCF) and Global Sensitivity Analysis (GSA) techniques to demonstrate that such outcome depends on the presence of investment adjustment costs. As long as investing in capital is costly, under gross complementarity among inputs - a microfounded definition of inputs - the capital-augmenting shock lowers firm entry. The consequence is that immediately after the shock the total number of firms decreases. The model dynamics decisively change when investment adjustment costs are null: both capital-augmenting and MEI shocks entail negative comovements between output and firm entry.

As this second chapter is theory-oriented and we are the first, to the best of our knowledge, to look at the implication of assuming a CES production function and input-specific shocks for firm dynamics, there are several directions for future research. The most in-

teresting ones in the short term are empirical: on the one hand, model validation by means of second moments analysis against the benchmark Cobb-Douglas framework will provide another measure of the properties of our model, while on the other hand we point at an estimation exercise similar to the one presented in the previous chapter on a more detailed model in the spirit of Lewis and Stevens (2015); given the recorded importance of product creation on the economy, we think that considering a shock to the efficiency of firm creation, which might enter in the competition between capital-augmenting and MEI disturbances, is worth investigating.

Figure 3: Growth rates: GDP, net entry and profits for the U.S. economy.Sample period: 1947-1998. Source: Bilbiie et al. (2007)
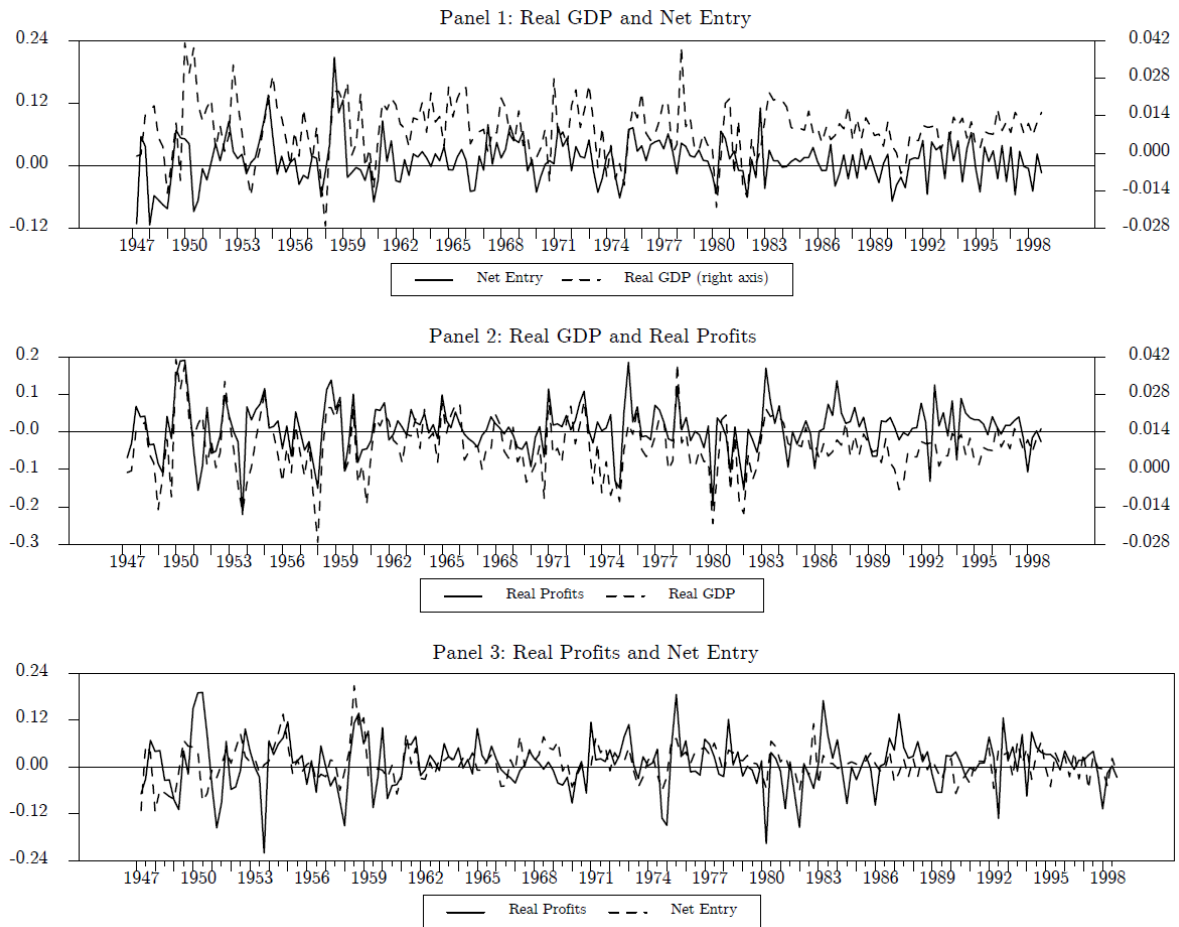
Figure 4: Impulse responses to a labor-augmenting shock
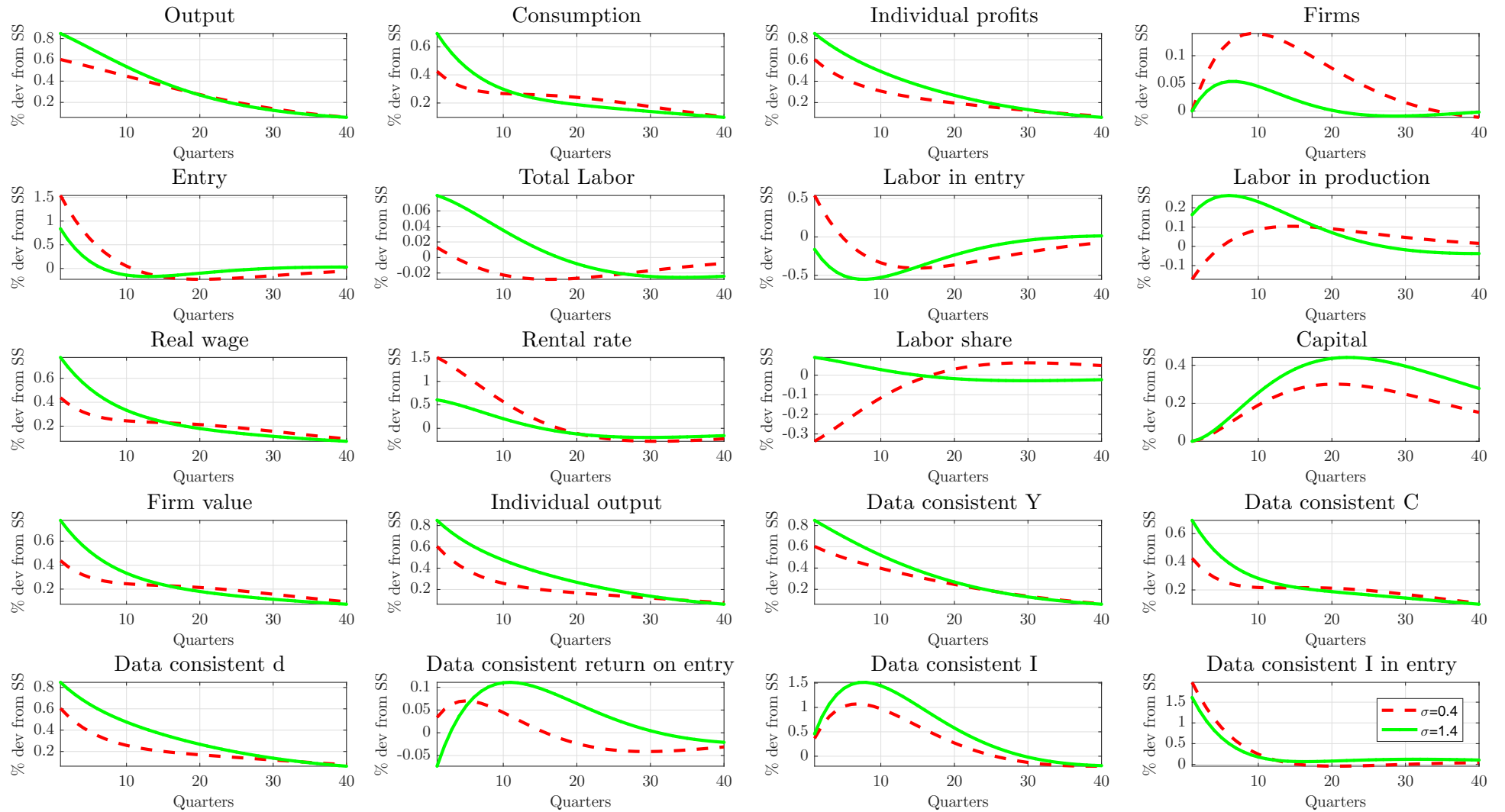
Figure 5: Impulse responses to a capital-augmenting shock
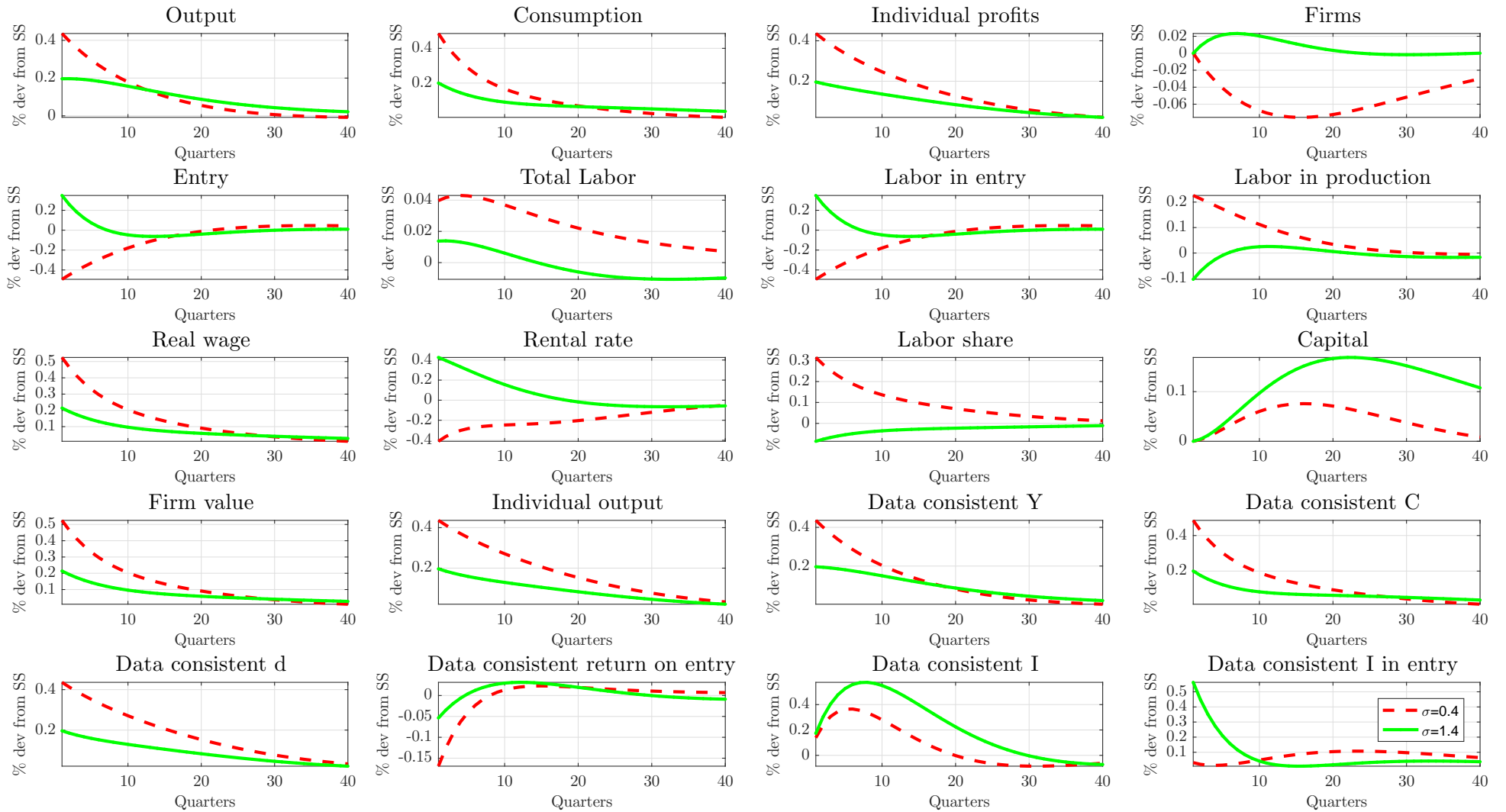
Figure 6: Global Sensitivity analysis on the response of N_e after the shock Z_k

The blue dots indicate the combination of σ and ψ_K yielding positive response of N_e
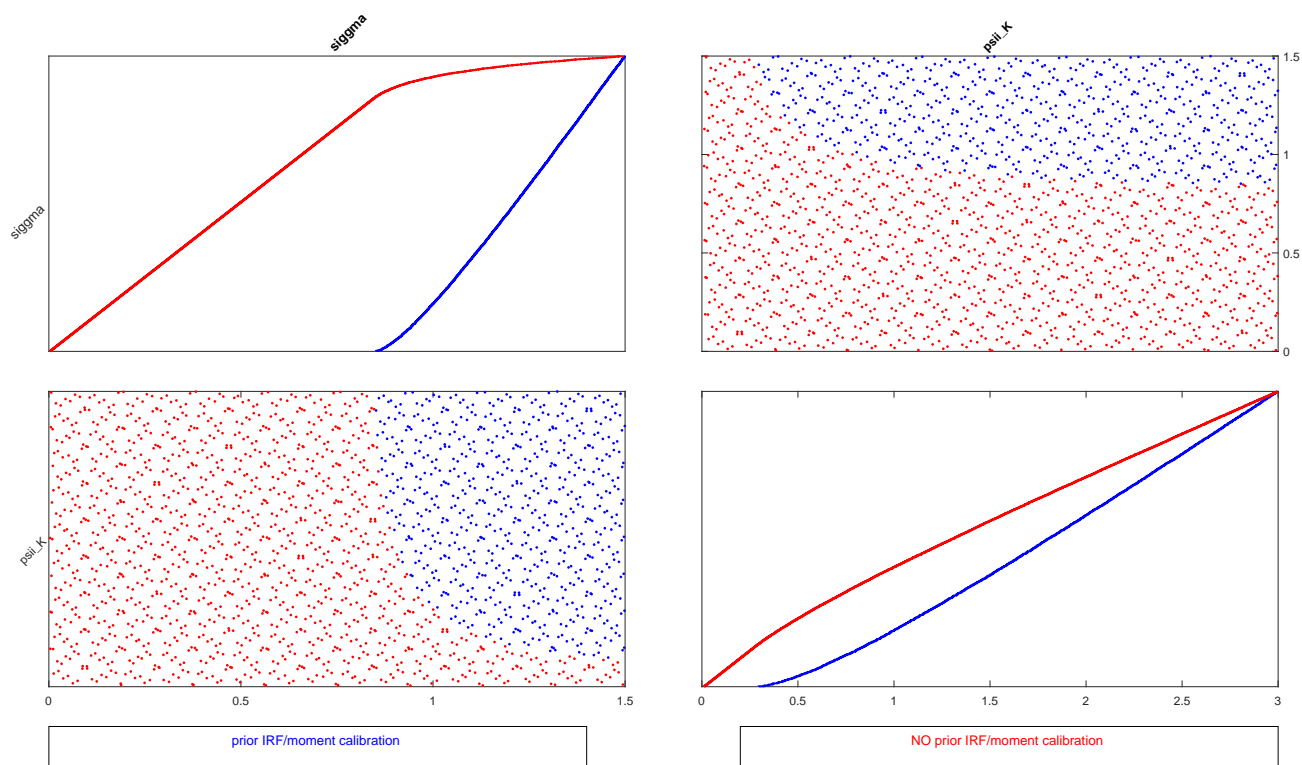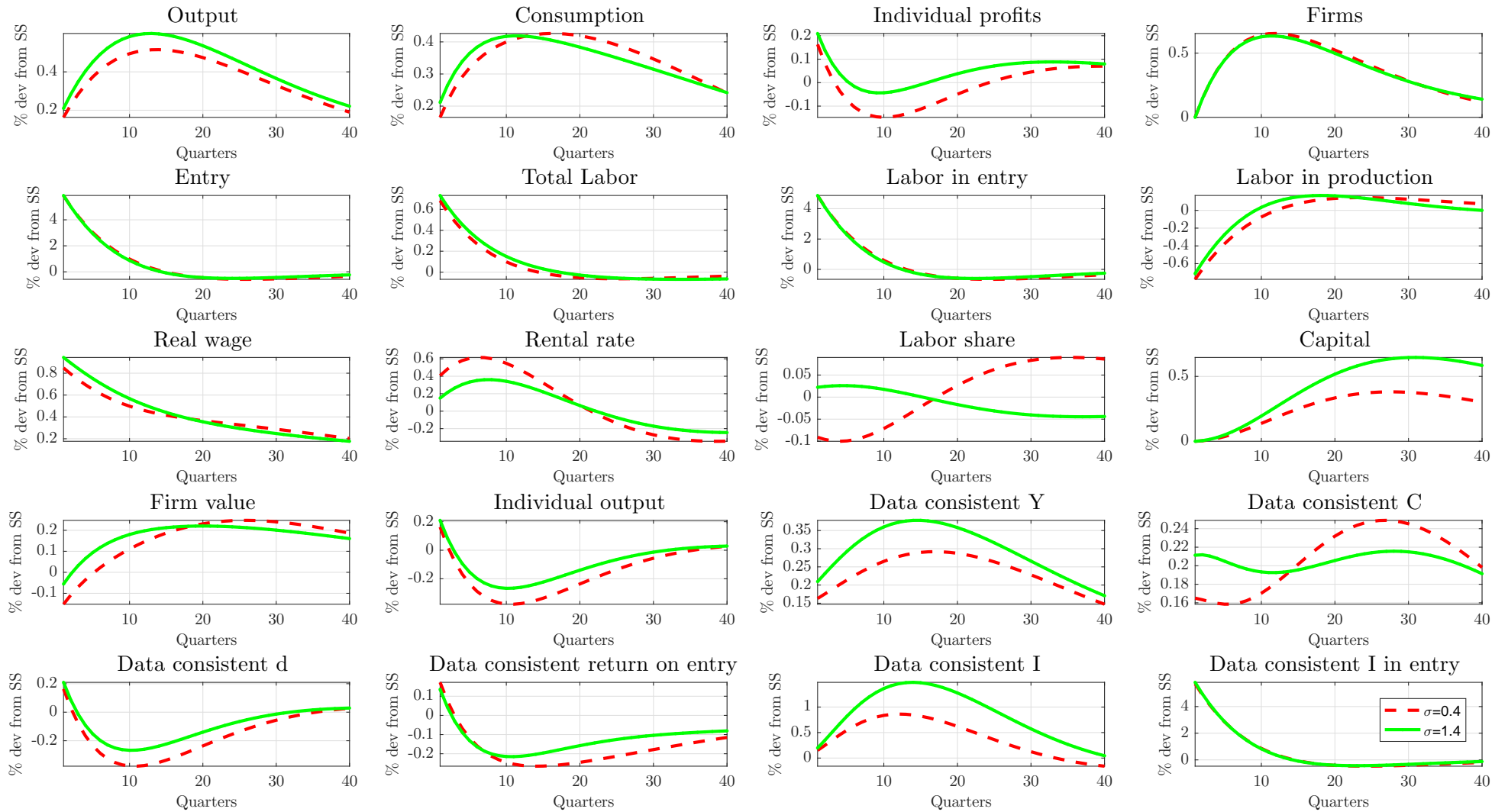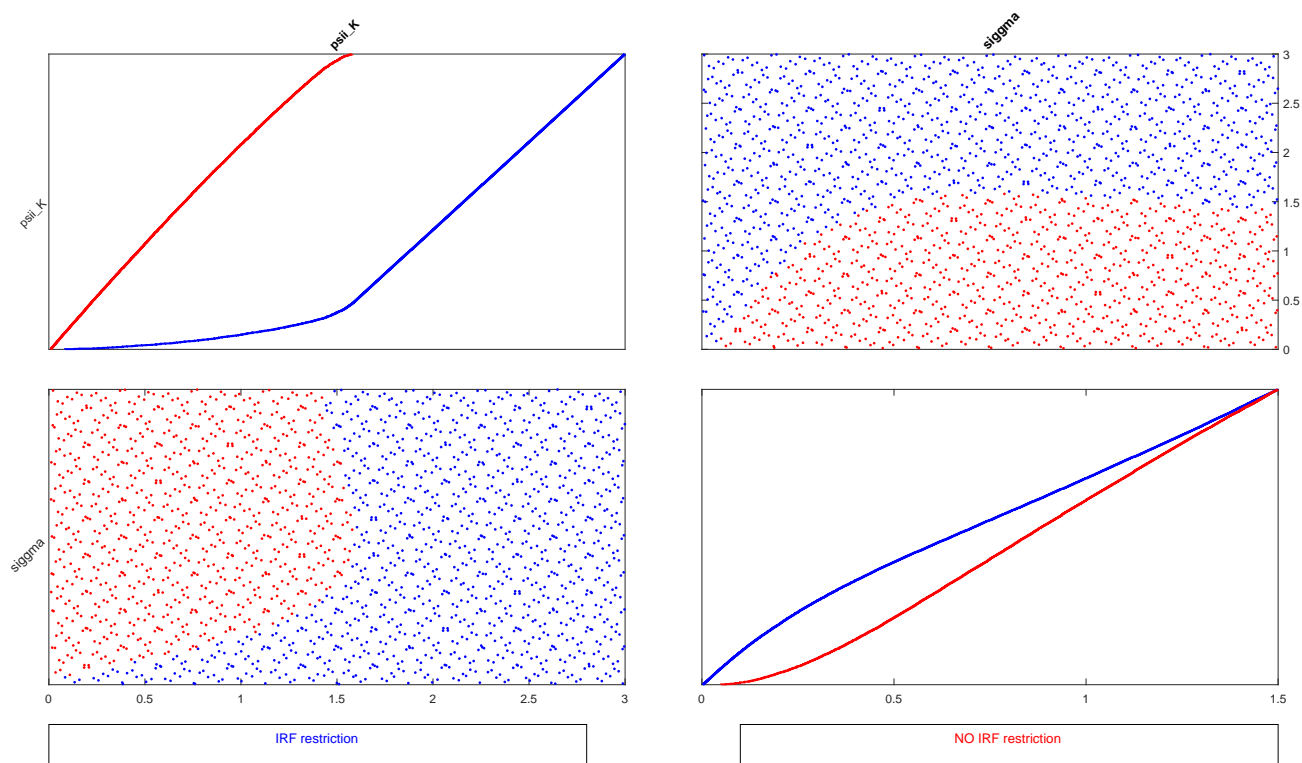
Figure 7: Impulse responses to a MEI shock

Figure 8: Global Sensitivity analysis on the response of N_e after the shock Z_i

The blue dots indicate the combination of σ and ψ_K yielding positive response of N_e

Figure 9: Impulse responses to a capital-augmenting shock. $\psi_K = 0$
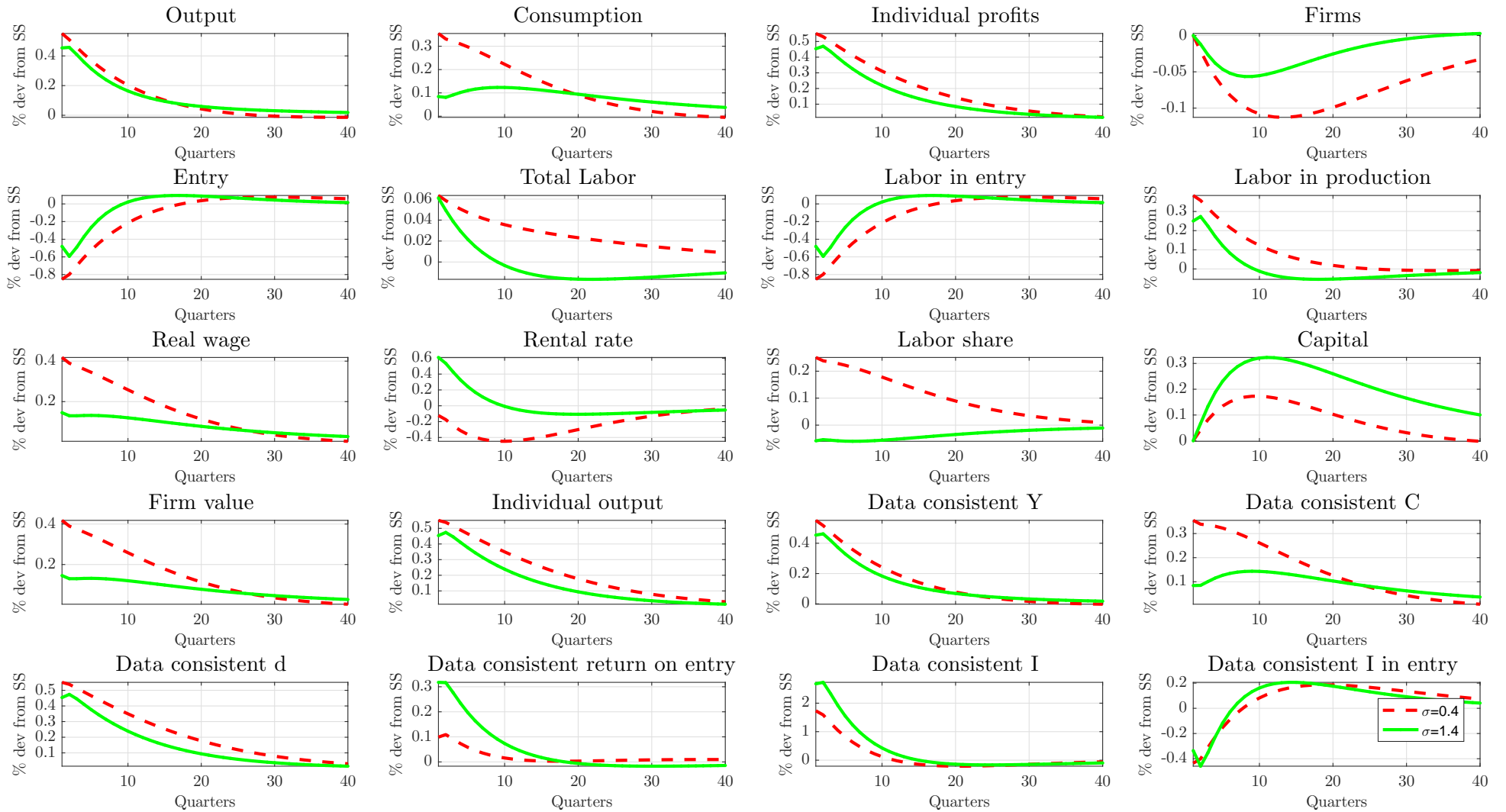
Figure 10: Impulse responses to a MEI shock. $\psi_K = 0$
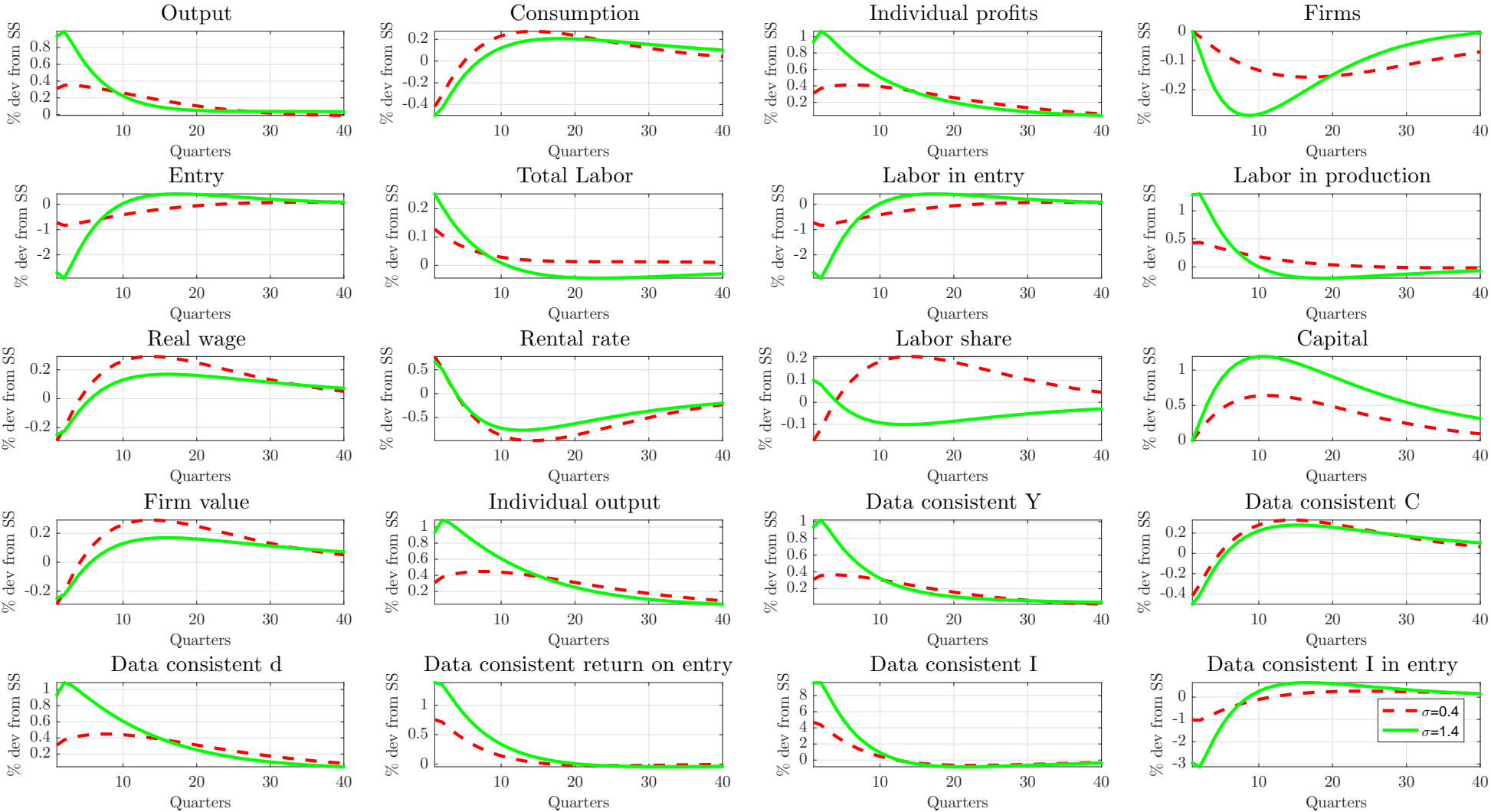
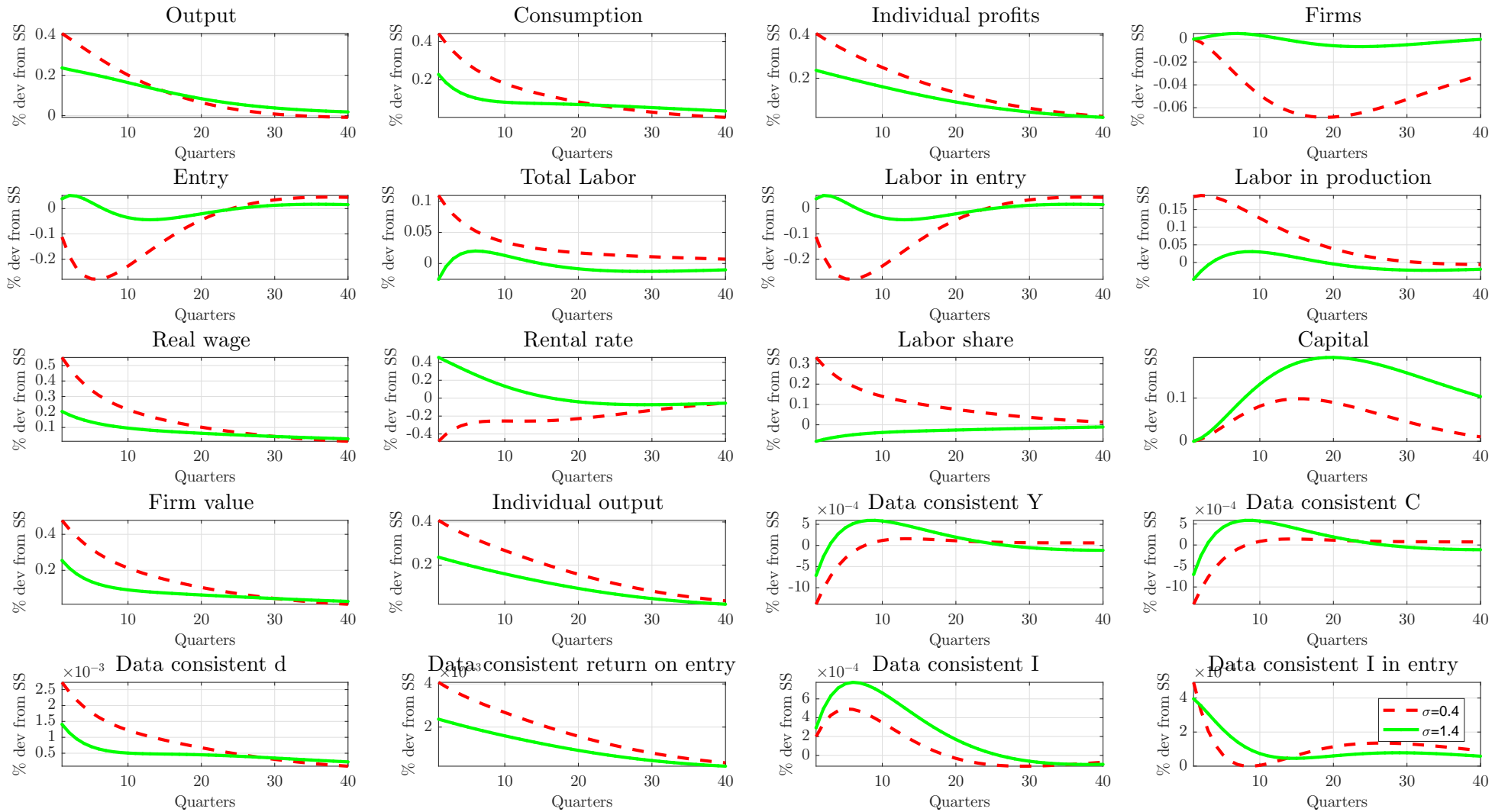Figure 11: Impulse responses to a capital-augmenting shock with congestion effects

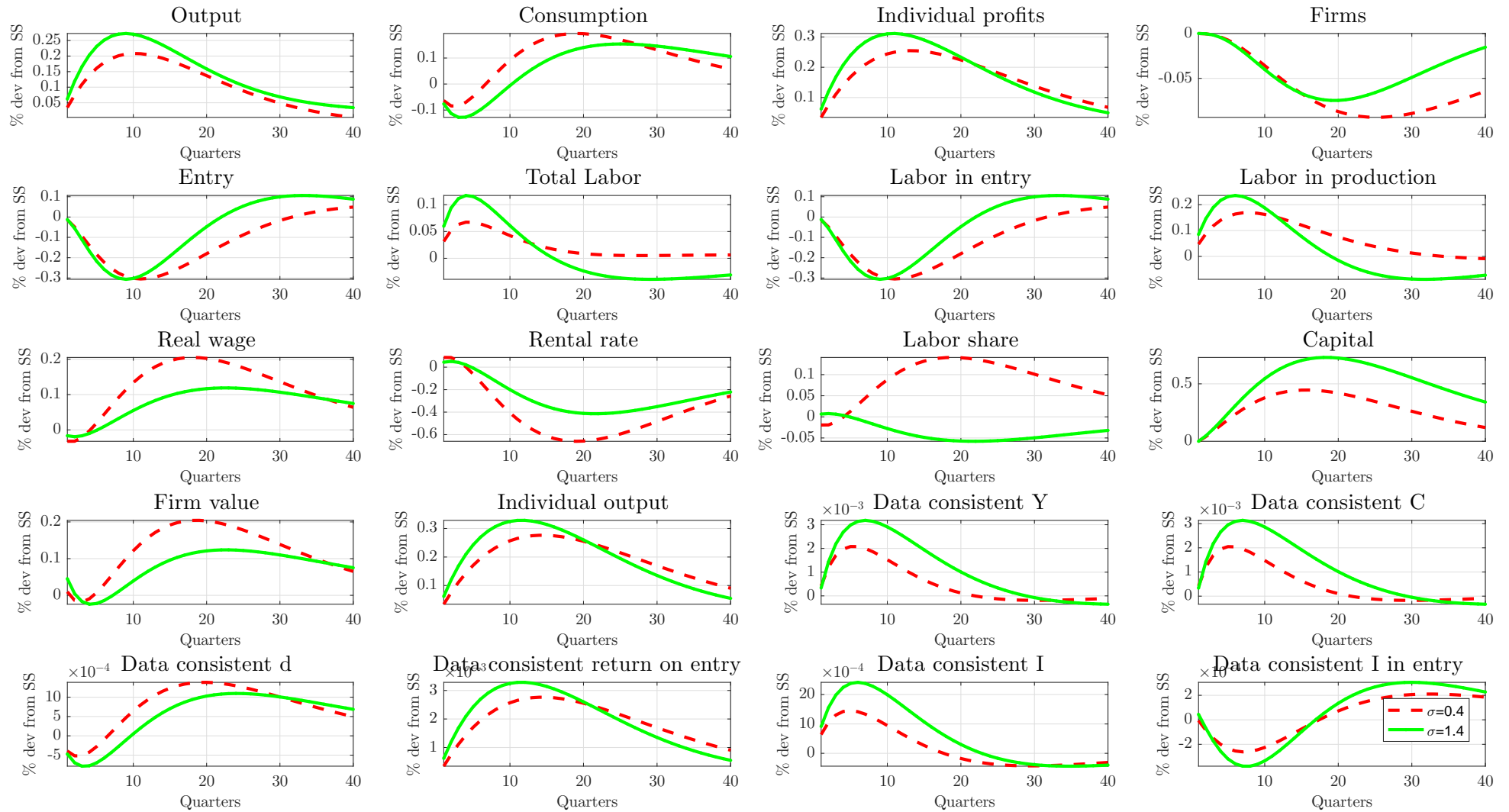Figure 12: Impulse responses to a MEI shock with congestion effects

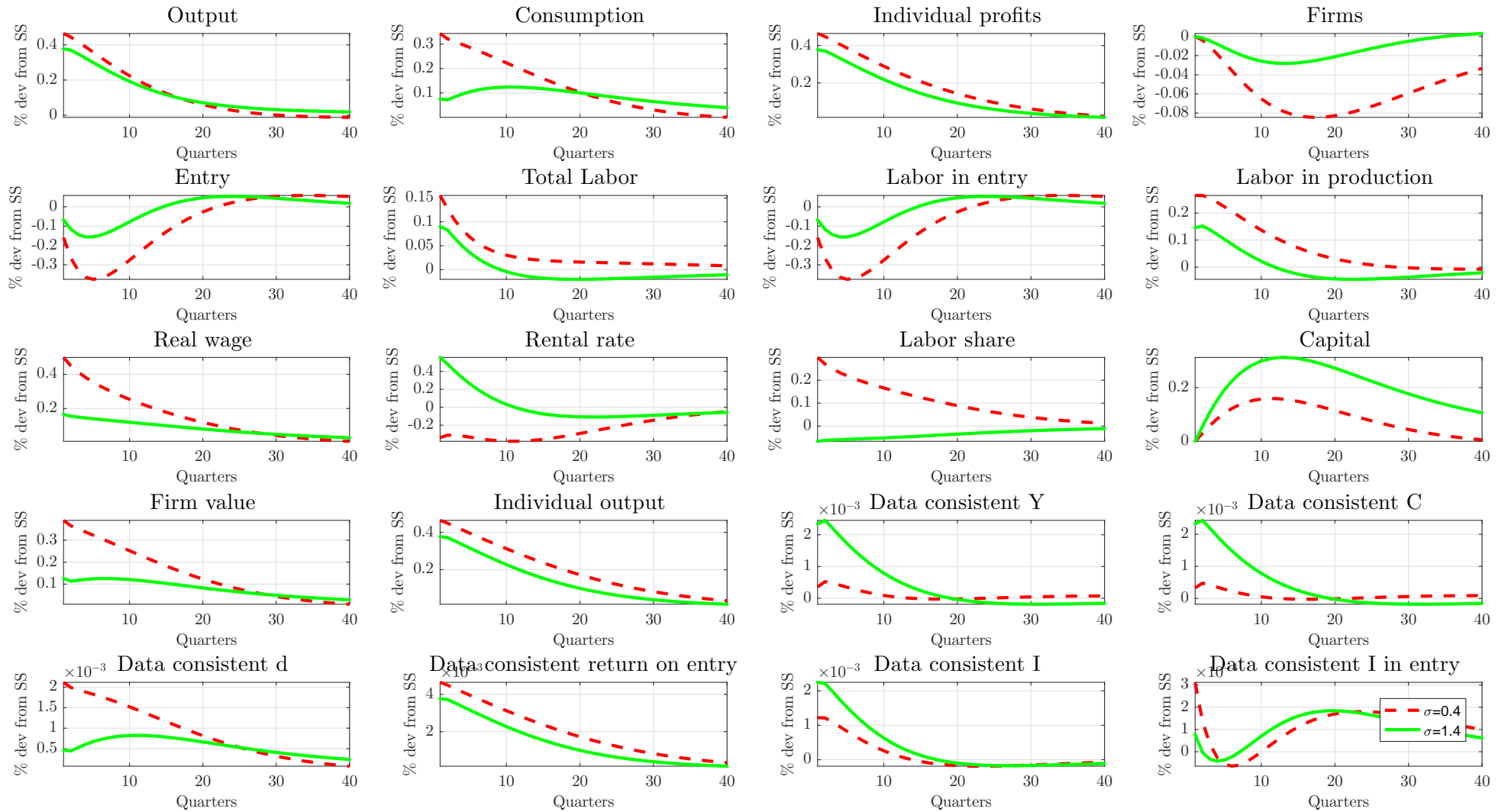Figure 13: Impulse responses to a capital-augmenting shock with congestion effects. $\psi_K = 0$

Figure 14: Impulse responses to a MEI shock with congestion effects. $\psi_K = 0$
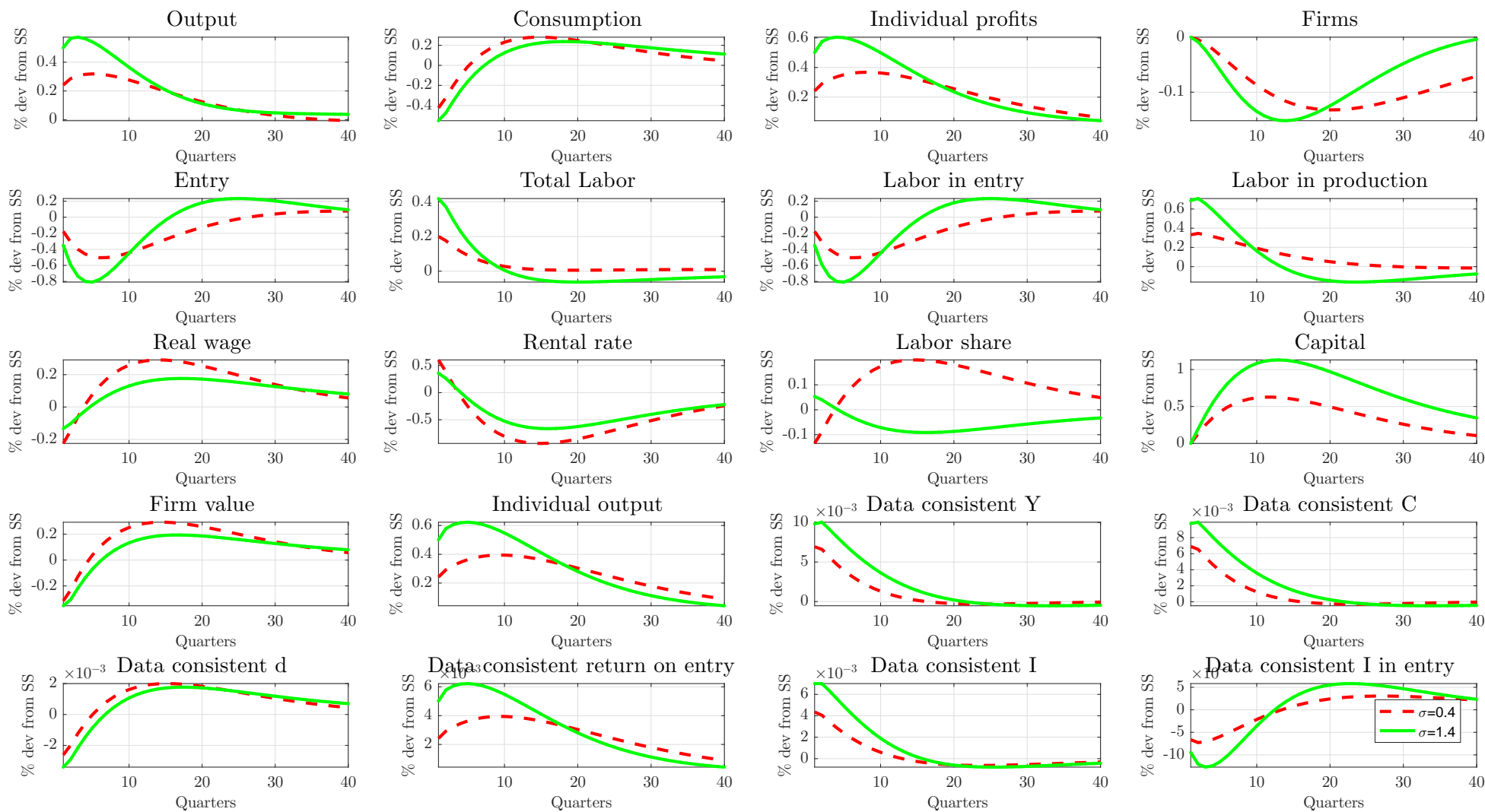
Variable	Definition
Production function	$Y_t = \frac{\varrho_t}{\varrho_0} Y_0 \left[\alpha_0 \left(\frac{K_t}{K_0} Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_0) \left(\frac{L_{c,t}}{L_{c,0}} Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$
Profits	$d_t = \left(1 - \frac{1}{\mu} \right) \frac{Y_t}{N_t}$
Real wage	$w_t = \frac{(1-\alpha_0)}{\mu} \left(\frac{\varrho_t}{\varrho_0} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_{c,t}} \right)^{\frac{1}{\sigma}} \left(\frac{Y_0}{L_{c,0}} Z_{l,t} \right)^{\frac{\sigma-1}{\sigma}}$
Rental rate	$r_{k,t} = \frac{\alpha_0}{\mu} \left(\frac{\varrho_t}{\varrho_0} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}} \left(\frac{Y_0}{K_0} Z_{k,t} \right)^{\frac{\sigma-1}{\sigma}}$
Markup	$\mu = \frac{\theta}{\theta-1}$
Entry technology	$N_{e,t} = Z_{l,t} \frac{L_{e,t}}{f_e}$
Love for variety	$\varrho_t = N_t^{\frac{1}{\theta-1}}$
Aggregate income identity	$C_t + I_t + N_{e,t} v_t = w_t L_t + r_t^K K_t + N_t d_t$
Capital law of motion	$K_{t+1} = (1 - \delta_K) K_t + I_t \left[1 - \frac{\psi_K}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] Z_{i,t}$
Firms law of motion	$N_{t+1} = (1 - \delta)(N_t + N_{e,t})$
Euler on capital	$\zeta_t = \beta \lambda_{t+1} r_{t+1}^K + \beta \zeta_{t+1}(1 - \delta_K)$
Euler on shares	$v_t = \beta(1 - \delta) \frac{\lambda_{t+1}}{\lambda_t} (d_{t+1} + v_{t+1})$
Euler on consumption	$\frac{Z_{b,t}}{(C_t - hC_{t-1})^{\sigma_C}} - \frac{\beta h Z_{pref,t+1}}{(C_{t+1} - hC_t)^{\sigma_C}} = \lambda_t$
Euler on investment	$\lambda_t = \zeta_t Z_{i,t} \left[1 - \frac{\psi^K}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \psi^K \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] + $ $+ \beta Z_{i,t+1} \psi^K \zeta_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$
Euler on bonds	$\lambda_t = \beta \lambda_{t+1} (1 + r_t)$
Labor supply	$w_t = \frac{\chi}{\lambda_t} L_t^{\frac{1}{\phi}}$
Total labor	$L_t = L_{c,t} + L_{e,t}$
Free entry	$v_t = w_t \frac{f_e}{Z_{l,t}}$

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