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Direct determination of $\sin^2\vartheta^{\ell}_{\rm eff}$ at hadron colliders

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We discuss the renormalization of the electroweak Standard Model at one loop using the leptonic effective weak mixing angle as one of the input parameters. We evaluate the impact of this choice in the prediction of the forward-backward asymmetry for the neutral current Drell-Yan process. The proposed input scheme is suitable for a direct determination of the effective leptonic weak mixing angle from the experimental data.

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I. INTRODUCTION

The weak mixing angle [1–4] is a fundamental parameter of the theory of the electroweak (EW) interaction, as it determines the combination of the gauge fields associated to the third component of the weak isospin and to the hypercharge, yielding the photon and the Z boson fields. The leptonic effective weak mixing angle $\sin^2\vartheta^{\ell}_{\rm eff}$, defined at the Z resonance, has been proposed [5–10] as a quantity sensitive to new physics, offering the opportunity of a stringent test of the Standard Model (SM). The measurement at LEP/SLD [11,12] was later challenged by the CDF and D0 determinations [13] at the Fermilab Tevatron and more recently by the results from the LHC collaborations ATLAS [14], CMS [15], and LHCb [16]. Two conceptually different strategies can (and should) be pursued for the direct determination of $\sin^2\vartheta^{\ell}_{\rm eff}$: with, whenever possible, a model independent as well as a pure SM approach. The latter will be useful as an internal self-consistency check of the SM, through the comparison of the direct determination with the most precise available calculations of $\sin^2\vartheta^{\ell}_{\rm eff}$. In this paper we discuss the renormalization of the EW SM at one-loop level, using $\sin^2\vartheta^{\ell}_{\rm eff}$, as defined at LEP/SLD, as one of the input parameters in the EW gauge sector. Any simulation code implementing such a scheme will be able to provide theoretical templates for a direct sensible comparison with the experimental data, with the leptonic effective weak mixing angle used as a fit parameter and consistently treated in the evaluation of next-to-leading-order (NLO) and higher-order corrections. The use of $\sin^2\vartheta^{\ell}_{\rm eff}$ as input parameter of the electroweak sector has also been proposed in Refs. [17–21] in the framework of the high-precision measurements at the Z boson resonance and higher energies at future e^+e^- colliders.

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II. INPUT SCHEMES AND RENORMALIZATION

The choice of an input scheme in the EW gauge sector of the SM is relevant for two distinct reasons:

(i) In a theoretical perspective, the prediction of an observable should be affected by the smallest possible parametric uncertainty. This goal can be achieved by using the best known measured constants, like, for instance, the fine structure constant α, the Fermi constant G_μ, and the Z boson mass M_Z. Furthermore, the convergence of the perturbative expansion used to predict an observable is an additional criterium to judge whether the chosen inputs describe the process already in lowest order with good accuracy and reabsorb in their definition large radiative corrections. This is the case, for instance, of the scheme which uses G_μ, M_Z, and the W boson mass M_W, to describe processes at the electroweak and higher scales.

(ii) The determination of a fundamental constant at high-energy colliders can be achieved through the comparison of kinematical distributions computed in a theoretical model, the so-called templates, with the experimental data. The fundamental constant must be a free parameter of the model and is varied in the fitting procedure. Only the input parameters of the model can be unambiguously determined, because they are the only ones which can be freely varied without spoiling the accuracy of the calculation, while any other quantity is a prediction expressed in terms of them. Typical examples have been M_Z at LEP1 and M_W at LEP2, Tevatron, and LHC.

Following the second perspective, we discuss in this paper the formulation of a renormalization scheme which includes the leptonic effective weak mixing angle $\sin^2\vartheta_{\rm eff}^{\ell}$ [5] as one of the input parameters. Such a scheme will allow us to exploit the Tevatron and LHC (and in particular the future HL-LHC) potential to provide very high precision measurements of the neutral channel (NC) Drell-Yan (DY) process and, in turn, of $\sin^2\vartheta_{\rm eff}^{\ell}$.

A. Input scheme definitions

A set of three commonly adopted SM Lagrangian input parameters in the gauge sector is e, M_W, M_Z; they have to be expressed in terms of three measured quantities, whose choice defines a renormalization scheme. The relation between e, M_W, M_Z and the reference measured quantities has to be evaluated at the same perturbative order of the scattering amplitude calculation at hand and allows us to fix the renormalization conditions. The usual sets of reference measured quantities are α, M_W, M_Z, which defines the on-shell scheme; $\alpha(M_Z), M_W, M_Z$, which is a variant of the on-shell scheme and reabsorbs the large logarithmic contributions due to the running of the electromagnetic coupling from the scale 0 to M_Z [22]; G_μ, M_W, M_Z, which defines the G_μ scheme and is particularly suited to describe DY processes at hadron colliders because it allows one to include a large part of the radiative corrections in the LO predictions, guaranteeing a good convergence of the perturbative series. For a detailed description of these schemes see Ref. [26]. The presence of M_W among the input parameters is a nice feature in view of a direct M_W determination at hadron colliders via a template fit method, as described above. On the other hand, these schemes are not suited for high-precision predictions, because of the "large" parametric uncertainties stemming from the present experimental precision on the knowledge of M_W. In fact, for NC DY precise predictions, a LEP style scheme with α, G_μ, M_Z would be preferred. However, in view of a direct SM determination of the quantity $\sin^2\vartheta_{\rm eff}^{\ell}$, also this scheme has its own shortcomings, because $\sin^2\vartheta_{\rm eff}^{\ell}$ is a calculated quantity and cannot be treated as a fit parameter. With the aim of a direct $\sin^2\vartheta_{\rm eff}^{\ell}$ SM determination, we discuss an alternative scheme, which includes the weak mixing angle as a SM Lagrangian input parameter, $\sin^2\vartheta$, together with e and M_Z. The experimental reference data are the Z boson mass value measured at LEP, the fine structure constant α, and $\sin^2\vartheta_{\rm eff}^{\ell}$ as defined at LEP at the Z resonance. An additional possibility discussed in the following is to replace α with G_μ. We will refer to these two choices as the $(\alpha, M_Z, \sin^2\vartheta_{\rm eff}^{\ell})$ and the $(G_\mu, \sin^2\vartheta_{\rm eff}^{\ell}, M_Z)$ input schemes. At tree level $\sin^2\vartheta = \sin^2\vartheta_{\rm eff}^{\ell}$. The quantity $\sin^2\vartheta_{\rm eff}^{\ell}$ is defined in terms of the vector and axial-vector couplings of the Z boson to leptons $g_{V,A}^{\ell}$, measured at the Z boson peak, or alternatively the chiral electroweak couplings $g_{L,R}^{\ell}$ and reads (at tree level) [27]

$$\sin^2\vartheta = \sin^2\vartheta_{\rm eff}^{\ell} = \frac{I_3^l}{2Q_l}\left(1 - \frac{g_V^{\ell}}{g_A^{\ell}}\right) = \frac{I_3^l}{Q_l}\left(\frac{-g_R^{\ell}}{g_L^{\ell} - g_R^{\ell}}\right), \qquad (1)$$

where

$$g_L^{\ell} = \frac{I_3^l - \sin^2\vartheta_{\rm eff}^{\ell} Q_l}{\sin\vartheta_{\rm eff}^{\ell}\cos\vartheta_{\rm eff}^{\ell}}, \qquad g_R^{\ell} = -\frac{\sin\vartheta_{\rm eff}^{\ell}}{\cos\vartheta_{\rm eff}^{\ell}} Q_l. \qquad (2)$$

$I_3^l = -\frac{1}{2}$ is the third component of the weak isospin and Q_l is the electric charge of the lepton in units of the positron charge.

B. Renormalization

We implement the one-loop renormalization of the three input parameters by splitting the bare ones into renormalized parameters and counterterms

$$M_{Z,0}^2 = M_Z^2 + \delta M_Z^2, \qquad (3)$$

$$\sin^2\vartheta_0 = \sin^2\vartheta_{\rm eff}^{\ell} + \delta\sin^2\vartheta_{\rm eff}^{\ell}, \qquad (4)$$

$$e_0 = e(1 + \delta Z_e), \qquad (5)$$

where the bare parameters are denoted with subscript 0. The counterterms δM_Z^2 and δZ_e are defined as in the usual on-shell scheme. Complete expressions are given in Eqs. (3.19) and (3.32) of Ref. [28]. The counterterm $\delta\sin^2\vartheta_{\rm eff}^{\ell}$ is defined by imposing that the tree-level relation equation (1) holds to all orders. Considering the $Z\ell^+\ell^-$ vertex and neglecting the masses of the lepton ℓ, the couplings $g_{L,R}^{\ell}$ are replaced by the form factors $\mathcal{G}_{L,R}^{\ell}(q^2)$ [9] once radiative corrections are accounted for. The effective weak mixing angle has been defined at LEP/SLD by taking the form factors at $q^2 = M_Z^2$:

$$\sin^2\vartheta_{\rm eff}^{\ell} \equiv \frac{I_3^l}{Q_l}\mathrm{Re}\left(\frac{-\mathcal{G}_R^{\ell}(M_Z^2)}{\mathcal{G}_L^{\ell}(M_Z^2)-\mathcal{G}_R^{\ell}(M_Z^2)}\right). \tag{6}$$

The form factors $\mathcal{G}_i^{\ell}$ can be computed in the SM in any input scheme that does not contain $\sin^2\vartheta_{\rm eff}^{\ell}$ as input parameter, yielding in turn, via Eq. (6), a prediction for $\sin^2\vartheta_{\rm eff}^{\ell}$, as discussed at length in Refs. [29,30].

In this paper instead we consider the weak mixing angle as an input parameter. In order to fix its renormalization condition, we write Eq. (6) at one loop,

$$\sin^2\vartheta_{\rm eff}^{\ell} = \frac{I_3^l}{Q_l}\mathrm{Re}\left(\frac{-g_R^{\ell}-\delta g_R^{\ell}}{g_L^{\ell}-g_R^{\ell}+\delta g_L^{\ell}-\delta g_R^{\ell}}\right), \tag{7}$$

where $\delta g_{L,R}^{\ell}$ represent the effect of radiative corrections, expressed in terms of renormalized quantities and related counterterms, including $\delta\sin^2\vartheta_{\rm eff}^{\ell}$. We do not consider NLO QED corrections because they factorize on form factors and therefore do not affect the $\sin^2\vartheta_{\rm eff}^{\ell}$ definition. The effective weak mixing angle is defined to all orders by the request that the measured value coincides with the tree-level expression. The counterterm $\delta\sin^2\vartheta_{\rm eff}^{\ell}$ is fixed by imposing that the one-loop corrections to Eq. (1) vanish, namely,

$$\frac{1}{2}\frac{g_L^{\ell}g_R^{\ell}}{(g_L^{\ell}-g_R^{\ell})^2}\mathrm{Re}\left(\frac{\delta g_L^{\ell}}{g_L^{\ell}}-\frac{\delta g_R^{\ell}}{g_R^{\ell}}\right)=0. \tag{8}$$

We remark that at one loop the condition in Eq. (8) holds also if $\sin^2\vartheta_{\rm eff}^{\ell}$ is defined from the ratio of the real parts of $\mathcal{G}_V$ and $\mathcal{G}_A$. Moreover, Eq. (8) remains unchanged if the complex-mass scheme [31–33] is used for the treatment of unstable particles. In fact, M_Z becomes complex but $\sin^2\vartheta_{\rm eff}^{\ell}$ and, consequently, g_L^{ℓ} and g_R^{ℓ} remain real. Therefore Eq. (8) is valid also in the complex-mass scheme without modifications.

From the $\mathcal{O}(\alpha)$ corrections to the vertex $Z\ell^+\ell^-$ we obtain

$$\frac{\delta\sin^2\vartheta_{\rm eff}^{\ell}}{\sin^2\vartheta_{\rm eff}^{\ell}} = \mathrm{Re}\left\{-\frac{1}{2}\frac{\cos\vartheta_{\rm eff}^{\ell}}{\sin\vartheta_{\rm eff}^{\ell}}\delta Z_{AZ}+\left(1-\frac{Q_\ell}{I_3^{\ell}}\sin^2\vartheta_{\rm eff}^{\ell}\right)\times[\delta Z_L^{\ell}+\delta V^L-\delta Z_R^{\ell}-\delta V^R]\right\}, \tag{9}$$

where δZ_{AZ} contains the fermionic and bosonic contributions to the γZ self-energy corrections, while the second line of Eq. (9) stems from the vertex corrections and counterterm contributions. We remark that the γZ self-energy does not contain enhanced terms proportional to m_t^2. The bosonic contributions in Eq. (9) form a gauge invariant set because they are a linear combination of the corrections to the left- and right-handed components of the Z decay amplitude into a lepton pair. The expressions of δZ_{AZ} and $\delta Z_{L/R}^{\ell}$ are given in Eqs. (3.19) and (3.20) of Ref. [28], respectively. If the complex-mass scheme is used, the expression of δZ_{AZ} can be found in Eqs. (4.7) and (4.30) of Ref. [32]. In $\delta Z_{L/R}^{\ell}$ we suppressed the lepton family indices. The vertex corrections $\delta V^{L/R}$ are given by

$$\begin{aligned}\delta V^L &= (g_L^{\ell})^2\frac{\alpha}{4\pi}\mathcal{V}_a(0,M_Z^2,0,M_Z,0,0)\\ &\quad+\frac{1}{2s_W^2}\frac{g_L^{\nu}}{g_L^{\ell}}\frac{\alpha}{4\pi}\mathcal{V}_a(0,M_Z^2,0,M_W,0,0)\\ &\quad-\frac{c_W}{s_W}\frac{1}{2s_W^2}\frac{1}{g_L^{\ell}}\frac{\alpha}{4\pi}\mathcal{V}_b(0,M_Z^2,0,0,M_W,M_W)\\ \delta V^R &= (g_R^{\ell})^2\frac{\alpha}{4\pi}\mathcal{V}_a(0,M_Z^2,0,M_Z,0,0)\end{aligned} \tag{10}$$

and the vertex functions $\mathcal{V}_a$ and $\mathcal{V}_b$ are given in Eqs. (C.1) and (C.2) of Ref. [28], respectively.

The renormalization condition that the measured effective leptonic weak mixing angle matches the tree-level expression to all orders in perturbation theory applies, following the LEP definition, to the real part of the ratio of the vector and axial-vector form factors. The latter develop, order by order, an imaginary part which is computed in terms of the input parameters and contributes to the scattering amplitude.

C. The G_μ scheme

The muon decay amplitude allows to establish a relation between α, G_μ, M_Z, and $\sin^2\vartheta_{\rm eff}^{\ell}$ which reads

$$\sin^2\vartheta_{\rm eff}^{\ell}\cos\vartheta_{\rm eff}^2 M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1+\Delta\tilde{r}), \tag{11}$$

with the following expression for $\Delta\tilde{r}$,

$$\Delta\tilde{r} = \Delta\alpha(s)-\Delta\rho+\Delta\tilde{r}_{\rm rem}, \tag{12}$$

$$\begin{aligned}\Delta\tilde{r}_{\rm rem} &= \frac{\mathrm{Re}\Sigma^{AA}(s)}{s}-\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\Sigma_T^{ZZ}(0)}{M_Z^2}\right)\\ &\quad+\frac{s_W^2-c_W^2}{c_W^2}\frac{\delta s_W^2}{s_W^2}+2\frac{c_W}{s_W}\frac{\Sigma_T^{AZ}(0)}{M_Z^2}\\ &\quad+\frac{\alpha}{4\pi s_W^2}\left(6+\frac{7-4s_W^2}{2s_W^2}\log(c_W^2)\right),\end{aligned} \tag{13}$$

where $s_W = \sin\vartheta_{\rm eff}^{\ell}$ and $c_W = \cos\vartheta_{\rm eff}^{\ell}$, respectively. We note the appearance of the combination $\Delta\alpha(s)-\Delta\rho$, which differs from the corresponding one for Δr in the $(\alpha, M_W M_Z)$ on-shell scheme $\Delta\alpha(s)-\frac{c_W^2}{s_W^2}\Delta\rho$. The $\Delta\tilde{r}_{\rm rem}$ correction does not contain any terms enhanced by a $m_t{}^2$ factor, nor large logarithmically enhanced contributions.

For the sake of clarity, we report the expressions of $\Delta\alpha(s)$ and $\Delta\rho$:

$$\Delta\alpha(s) = \left(\frac{\partial \Sigma_T^{AA}(s)}{\partial s}\right)_{|_{s=0}} - \frac{\mathrm{Re}(\Sigma_T^{AA}(s))}{s}, \quad (14)$$

$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{W}(0)}{c_W^2 M_Z^2}, \quad (15)$$

where $\Sigma_T^{AA}(s)$, $\Sigma_T^{AZ}(s)$, $\Sigma_T^{ZZ}(s)$, and $\Sigma_T^{W}(s)$ can be found in Eqs. (B.1)–(B.4) of Ref. [28].

Using Eq. (11) to derive an effective electromagnetic coupling, it is possible to convert results computed in the (α, M_Z, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$) scheme to the corresponding ones in the (G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$, M_Z) scheme. The $\Delta\rho$ term present at $\mathcal{O}(\alpha)$ in Eq. (11) accounts for one-loop quantum corrections growing like $m_t{}^2$. For convenience we can introduce the quantity $\Delta\rho^{(1)} = \frac{\alpha}{4\pi}\frac{3}{4 s_W^2 c_W^2}\frac{m_t^2}{M_Z^2}$, which represents the m_t^2 dependent part of $\Delta\rho$ defined in Eq. (15). This can be resummed to all orders, together with the irreducible two-loop contributions $\Delta\rho^{(2)}$, computed in the heavy top limit in Ref. [34]. In the following predictions for the (G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$, M_Z) scheme, we include the effect of the universal $m_t{}^2$ corrections at two loops with the replacement $G_\mu \to G_\mu(1+\Delta\rho^{(1)}+\Delta\rho^{(2)})$ after subtracting the $\Delta\rho^{(1)}$ contributions already included in the one-loop calculation.

III. THE DRELL-YAN PROCESS

We study at NLO EW the NC DY process, in the setup described in [35] but without acceptance cuts on the lepton transverse momentum and pseudorapidity, with $M_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV, $m_t = 173.5$ GeV, $M_H = 125$ GeV, and $\sin^2\vartheta_{\mathrm{eff}}^{\ell} = 0.23147$. For the numerical simulations with the (G_μ, M_W, M_Z) scheme, we adopt $M_W = 80.385$ GeV.

The distributions are simulated with the POWHEG code (Z_BMNNPV processes svn revision 3652, under the POWHEG BOX v2 framework) [36], focusing on the lepton-pair invariant mass forward-backward asymmetry $A_{FB}(M^2_{\ell^+\ell^-})$, defined as $(F-B)/(F+B)$, where $F = \int_0^1 dc d\sigma/dc$ and $B = \int_{-1}^0 dc d\sigma/dc$, for a given value of $M^2_{\ell^+\ell^-}$ with c the cosinus of the scattering angle in the Collins-Soper frame. Given the gauge invariant separation of photonic and weak corrections, we focus on the latter to discuss the main features of the (G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$, M_Z) schemes, in view of a direct determination of $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$.

The absolute change of A_{FB} computed with two $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$ values differing by $\Delta\sin^2\vartheta_{\mathrm{eff}}^{\ell} = 5\times 10^{-4}$, for a fixed choice of all the other inputs, is shown in Fig. 1. The observed A_{FB} shift sets the precision goal of a measurement that aims at the determination of $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$ at the level of $\Delta\sin^2\vartheta_{\mathrm{eff}}^{\ell}$. Taking as a reference $\Delta\sin^2\vartheta_{\mathrm{eff}}^{\ell} = 1\times 10^{-4}$ as a final precision goal at the LHC, the results of Fig. 1 must be rescaled, in first approximation, by a factor 5.

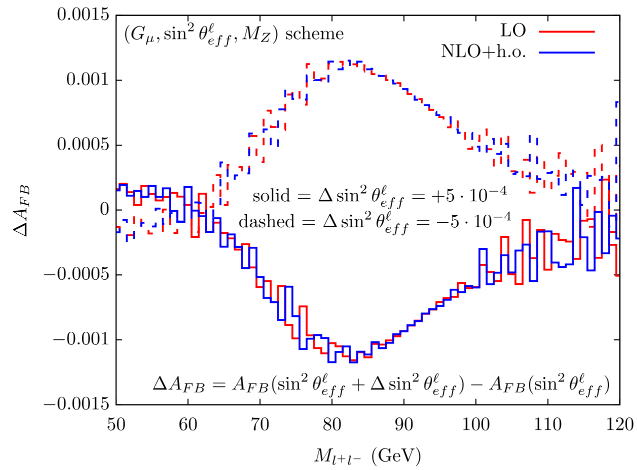

FIG. 1. The absolute variation of the predictions for the forward-backward asymmetry by changing $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$ by $\Delta = \pm 5\times 10^{-4}$ with respect to the value 0.23147d0, using the (G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$, M_Z) scheme, at NLO-h.o. and LO accuracies.

The absolute change ΔA_{FB} of $A_{FB}(M_Z^2)$ computed with NLO weak virtual corrections with respect to the LO result, and the variation obtained with improved couplings with respect to the NLO case are shown in Fig. 2 for the (G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$, M_Z) scheme (red lines) and for the (G_μ, M_W, M_Z) scheme (blue lines). The comparison of the blue and red lines shows a reduction by almost 1 order of magnitude in the (G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$, M_Z) scheme for the value of ΔA_{FB} due to the inclusion of the NLO corrections; we observe a negligible residual correction due to higher-order terms (h.o.), at variance with the (G_μ, M_W, M_Z) case, where we have a shift at the few parts 10^{-4} level in the Z peak region. The universal h.o. corrections in the (G_μ, M_W, M_Z) scheme are estimated according to Ref. [26].

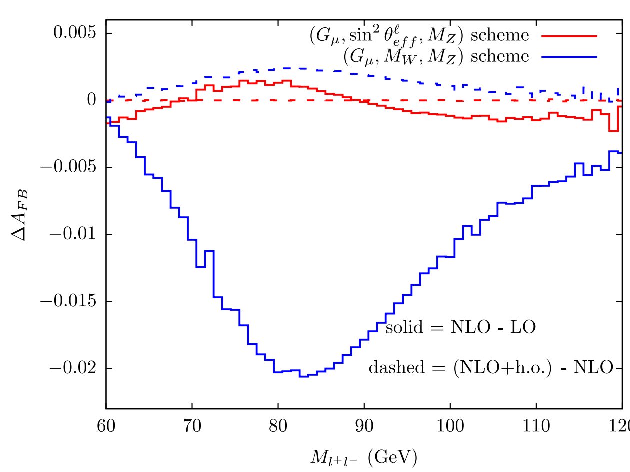

FIG. 2. The absolute deviation of NLO (NLO-h.o.) with respect to LO (NLO) predictions on the lepton forward-backward asymmetry, in the renormalization scheme with G_μ, $\sin^2\vartheta_{\mathrm{eff}}^{\ell}$ as input.

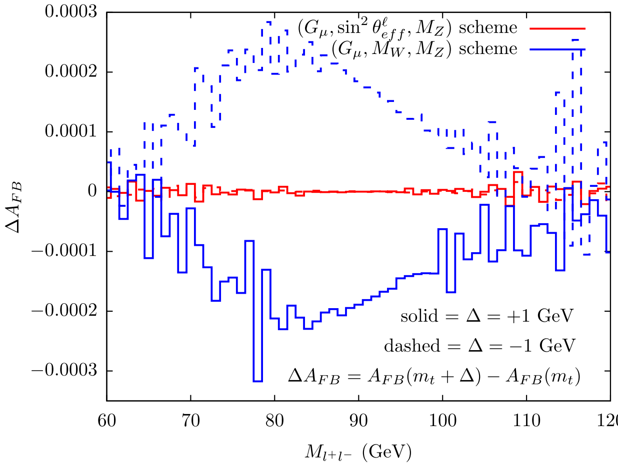

FIG. 3. The absolute deviation between predictions on the lepton pair A_{FB} as a function of $M_{\mu^+\mu^-}$, in the renormalization scheme with G_μ, $\sin^2\vartheta^{\ell}_{\mathrm{eff}}$ as input, with a variation of m_t of ± 1 GeV around the value $m_t = 173.5$ GeV. The precision of the calculation is NLO.

The size of NLO and higher-order radiative corrections, smaller than in the (G_μ, M_W, M_Z) case, can be ascribed to the choice as input parameters of the quantities that parametrize the Z resonance in terms of normalization (G_μ), position (M_Z) and shape $(\sin^2\vartheta^{\ell}_{\mathrm{eff}})$, the latter two being defined at the Z resonance and thus reabsorbing a good fraction of the quantum corrections.

One of the main sources of parametric uncertainties is given, in any scheme with G_μ as input, by the value of m_t. In Fig. 3 we show the absolute variation of ΔA_{FB} with respect to a change of ± 1 GeV of m_t around its central value, taken at $m_t = 173.5$ GeV, using the NLO accuracy with higher-order effects included, evaluated in the $(G_\mu$, $\sin^2\vartheta^{\ell}_{\mathrm{eff}}$, $M_Z)$ (red lines) and (G_μ, M_W, M_Z) (blue lines) schemes. In the $(G_\mu$, $\sin^2\vartheta^{\ell}_{\mathrm{eff}}$, $M_Z)$ scheme, the effect is well below the 2×10^{-5} scale for A_{FB} in the [60, 120] GeV mass range, almost vanishing in the Z peak region, while in the (G_μ, M_W, M_Z) case a variation of m_t by ± 1 GeV induces a shift ΔA_{FB} of order 2×10^{-4}. The very small dependence of A_{FB} on the m_t value is due to the cancellation of the overall normalization factor of the squared matrix element, between numerator and denominator of A_{FB}, where the factor with the m_t^2 dependence is present. Radiative corrections, logarithmic in m_t, are by construction small at the Z peak, so that also the residual m_t dependence is milder than in other invariant mass regions. In the (G_μ, M_W, M_Z) case instead the m_t^2 dependence enters via the corrections to M_W and affects the precise value of the on-shell weak mixing angle and, in turn, the shape of the A_{FB} distribution.

In conclusion, we have presented an EW scheme that has $\sin^2\vartheta^{\ell}_{\mathrm{eff}}$, with exactly the same definition adopted at LEP/SLD, among the input parameters of the gauge sector and discussed its one-loop renormalization. In such a scheme the predictions of the NC DY process exhibit a faster convergence of the perturbative expansion and smaller m_t parametric uncertainties, with respect to the (G_μ, M_W, M_Z) scheme. The presence of $\sin^2\vartheta^{\ell}_{\mathrm{eff}}$ among the inputs allows its direct determination at hadron colliders and a closure test with a comparison against its best theoretical prediction in the SM based on the (α, G_μ, M_Z) input scheme. Such a scheme will allow the preparation of templates and the quantitative evaluation of the impact of radiative corrections and other theoretical uncertainties, in analogy with the study presented in Ref. [37] in the M_W case. We implemented the scheme in the `Z_BMNNPV` svn revision 3652 processes under the POWHEG BOX v2 framework, but it can be easily implemented in any other code.

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