Quantile Regression to Tackle the Heterogeneity on the Relationship between Economic Growth, Energy Consumption, and CO_2 Emissions.

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Abstract

Heterogeneity in the data is a common issue arising in research. When data are heterogeneous equal variation in the data to set up a model for the studied phenomena cannot be assumed. Ordinary least square regression does not consider the unequal variation which may provide inefficient estimation of the relationship between variables. On the contrary, quantile regression could efficiently tackle this problem by detecting the relationship between variables at different levels, and could be useful especially in applications where extreme values are important to consider, such as in environmental studies, where upper quantiles of pollution levels are critical from a public health perspective. The main purpose of this study is to model the relationship between CO_2, economic growth, energy consumption by considering the heterogeneity problem for developed and developing countries and applying the quantile regression at different quantile values (0.05, 0.25, 0.50, 0.75, and 0.95) on panel data. The panel data consists of 29 countries from two different economic development groups,- 17 developed versus 12 developing countries - over the period 1960-2008. QR results are then compared with those of the OLS model, resulting similar for developed and developing countries. In both cases countries having lower GDP release less CO_2 emissions.

Keywords: Quantile Regression, Heterogeneous Data, Environmental Phenomena.

1. Introduction

Recently, quantile regression (QR) is broadly applied on panel data covering a wide research area. Koenker (2004) suggested a general approach of QR into panel data model, defining the conditional quantile functions approach in which quantiles of the conditional distribution of the dependent variable are expressed as functions of observed covariates (Koenker and Hallock, (2001). QR is used to estimate the conditional median or any other quantile of the dependent variable. Sometimes it is called least absolute value (LAV), minimum absolute deviation (MAD) models, or L1-norm model. QR seeks to search for the regression model that minimizes the sum of the absolute residuals rather than the sum of the squared residuals as in the ordinary least squares (OLS) model. Gilchrist (2001) defined the quantile as the value that corresponds to a specified proportion of an ordered sample. For example, the 0.5 quantile from ordered data is the median *M*, which corresponds to a quantile with a probability of 0.5 of occurrence. QR measures the effects of unobserved heterogeneity in the included variables in the estimated model, but the panel data model properly controls the fixed effects of some unobserved independent variables. Moreover, if the distribution of the dependent variable changes together with the independent variables, the result is misleading when using the OLS regression, whereas QR shows how such changes in the independent variables affect the distribution shape of the dependent variable.

Most of the empirical research in the environmental area suffers from two common shortcomings. One of these shortcomings is the use of the OLS mean regression models to find the conditional mean of the dependent variable in response to the independent variables. Another shortcoming is that experimental data have often an heterogeneous distribution. Thus, OLS may not provide efficient estimations. QR is an important and well-established tool for planning and resource management which could provide a meaningful explanation of the environmental relationships. To the best of our knowledge, this study is the first environmental experiment study applying the QR for multiple quantiles (i.e., 0.05, 0.25, 0.50, 0.75, and 0.95) against the OLS model to detect the relationship between CO_2, energy consumption (EC) and gross domestic product (GDP) into two different economic groups of countries, developed and developing countries. QR enables us to evaluate the levels of CO_2 emissions at different points of the dependent variable distribution.

The remaining sections of this paper are organized as follows; Section 2 illustrates the related literature to QR. Section 3 explains the QR approach. Section 4 provides results and discussion. Section 5 concludes the paper.

2. Literature Review on Quantile Regression

Generally, many scientists focused on estimating the rates of changes in the mean of the response variable distribution. However, few studies have applied QR along with the upper and lower boundaries of the conditional distribution of response variables. OR also could measure the effects of independent variables on location, shape and scale of the distribution into the dependent variable. The person who pioneered the application of QR in science is Kaiser *et al*. (1994). Since after that, there were more researches applied QR in the other fields. Among them, Dunham et al. (2002) analysed the abundance of Lahontan cutthroat trout to the ratio of stream width (as the predictor variable) to depth. They constructed the value of the additional information provided by QR for different quantiles (0.95, 0.75, 0.50, 0.25, and 0.05). The QR reported a negative nonlinear relationship with the upper 30% of cutthroat densities across 13 streams and 7 years, while the OLS indicated no significant relationship in mean densities with stream width to depth. Cade and Guo (2000) examined the reduction in densities of mature plants with increasing germination densities of seedlings of annual plants in the Chihuahuan desert of the southwestern US. They estimated the QR for 0.99 and 0.90 quantile to measure the changing in the survival of Chihuahuan desert by modelling changes in mature plant density *(y*) as a function of germination density of seedlings (*x*). The conventional OLS regression was inaccurate for estimating the relationship. On the other hand, QR indicated that the effects of seeds density are best revealed at the higher plant densities associated with upper quantiles, where there is a strong decline in density of mature plants at higher germination densities. Vaz et al. (2008) applied the QR for five quantile intervals from the 75^{th} to the 95^{th} into 16 of the most abundant marine fish and cephalopods in the eastern English Channel. The purpose of applying the QR is to estimate the upper quantile model which could define the limiting factors and design the potential habitat given the environmental data available for model construction. The results of the experiment study indicated that QR provides effective and significant differences with a *p*-value less than 0.05 between the estimated coefficients for the different quantile values to detect that relationship. Taheripour et al. (2002) applied three different methods, i.e. OLS, QR and Tobit regression to detect the relationship between leasing and debt in farm capital structure in Illinois by including other factors in the model such as the age of farm's operator, soil quality and net worth of the farm. The results supported that all estimated parameters are highly significant. Also the QR could give a clearer idea than the OLS regression on the different effects in farm characteristics on the distribution of leased to assets ratio.

Hennings and Katchova (2005) applied QR approaches for different values (10^{th}, 20^{th} to 90^{th} percentiles) to examine the relationship between the business strategies employed by Illinois farms with equity growth. The Breusch-Pagan test for heterogeneity was applied. The results showed that the data is heterogeneous, meaning that the conditional variance of the equity growth distribution is not constant across different levels of equity growth ratios; hence the QR should be applied. The main results supported that the estimated coefficients of the 10^{th} and 90^{th} percentiles are significantly different from the OLS coefficients. In other words, the effect of

different business strategies on equity growth rates differs between values of quantiles. However, OLS regression showed significant positive effects of these variables on equity growth. Cade et al. (2005) applied the QR to estimate the effects of physical habitat resources on bivalve in spatially structured landscape on a sand flat in New Zealand. The results for the 75^{th} percentile were less biased than the estimated mean parameters by OLS. However, the variation of estimated parameters for modelling the spatial trend surface reduced the quantiles associated with heterogeneous effects of the habitat variable. Gorg, et al. (2000) employed QR to analyse the determinants of firm start-up size. They showed that QR can provide more precise information on the determinants of start-up size than OLS regression model. Jayachandram et al. (2002) investigated the dietary impact of nutrition from several factors: income, education and age. The results showed that quantile regression is effective in estimating conditional function and provides more information for that relationship than OLS regression. The results of QR for different percentiles (10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th}) suggested that income, education, and age have larger effects at intake levels where the risk of excess is greater compared to the intake levels where risk of excess is lower. In particular, people with higher income and education level may have benefitted more from nutrition information than people with a lower level of income and education level.

3. Quantile Regression in the Panel Data Approach.

In this study, two statistical methods are applied; the OLS and the QR models. QR allows the researcher to account for unobserved heterogeneity and heterogeneous independent variables effects, while the availability of panel data potentially allows the researcher to include fixed effects to control for some unobserved covariates. QR was introduced by Koenker and Bassett (1978) as a generalization of the sample quantiles for the estimation of conditional quantile functions, expressed as linear functions of the independent variables. QR is the extension of OLS regression allowing for the specification of conditional functions at any quantile. QR approach is more accurate to detect the effect of independent variables on the dependent variable than the OLS approach, particularly if data contain heterogeneity. OLS is based on the average relationship between a set of independent variables and the dependent variable by the conditional mean function $E(y/x)$, which provides only a partial view of the relationship. In contrast, QR could describe that relationship at different points in the conditional median or quantiles distribution of dependent variable $Q_q(y/x)$, where q is the quantiles or percentiles and the median is the 50^{th} percentile of the empirical distribution, and the dependent variable should be continuous with no zeros values or no many repeated observations (Chernozhukov and Hansen (2007)). For that, the QR is especially meaningful in environmental applications where extremes values or outliers are important to study, where upper quantiles of pollution levels are critical from a public health perspective. Median regression is more robust in presence of outliers than the OLS regression, and is a semiparametric method as it avoids the assumptions regarding the error process and the parametric distribution.

In conditional quantile models, the parameters of interest are assumed to vary based on a non-separable disturbance term (Koenker and Basset (1978)). However, when the additional variables are added, the interpretation of these parameters will change. The computation of QR uses the linear programming methods in contrast to that in OLS and maximum likelihood approach. In both OLS and QR, being based on the sum of square error $\sum_i e_i^2$ and the absolute-error $\sum_i |e_i|$ respectively, are symmetric, making the sign of the prediction error not relevant. However if the quantile q differs from the median (50^{th}), there is an asymmetric penalty with increasing asymmetry as q approaches 0 or 1.

3.1. Advantages of Quantile Regression

One of the advantages of QR versus OLS regression is to provide different estimators for each quantile, which may allow the analysis of the various effects of the independent variables on the dependent variable. Consequently, it allows for a clearer path to compare its estimated coefficients and standard errors with those of OLS. The other advantage of QR is that that it is less sensitive to the tail behaviour of the underlying random variables, thus it will be less sensitive to outliers and has a high breakdown point compared to the OLS regression (Montenegro, 2001).

Furthermore, QR is insensitive to any monotonic transformations, the later referring to a transformation by a strictly increasing function, such as log(.), so the $h(y)$ quantile of y-monotone transform is $h(Q_q(y))$ and by using the inverse transformation it could transform the results back to x-values. This characteristic cannot be used for the OLS mean regression, as $E[h(y)] \neq h[E(y)]$ (Buchinsky, 1998).

In addition, if the data has homogeneous distribution, then the estimated slopes by QR at each point of the dependent variable will be identical with each other and with the estimated slopes given by OLS. In other words, the QR will produce again the same values of the OLS estimated slopes at any point across the distribution of the dependent variable, being the only differences in the intercepts.

Further, if the data has heterogeneous conditional distributions (the error terms is not constant across a distribution, or the level of independent variables) and the distribution of errors is non-normal, then the QR provides efficient results, while OLS will be inefficient if the errors are highly non-normal, as one of the main assumption of OLS is that the errors must be normally distributed to estimate the coefficients, whereas QR does not assume that. As a result, the estimated slopes by conditional quantile functions will differ from each other and from the OLS slopes. Thus, estimating the conditional quantiles at different points of the dependent variable

will provide different marginal responses of the dependent variable itself, according to the change of the independent variables in these points (Cade & Noon, 2003).

3.2. The QR Model

QR becomes one of the most suitable methods to apply if the estimated coefficients are significantly different from zero and also from OLS coefficients, so showing different effects across the distribution of the dependent variable. The quantile q splits the data of dependent variable y into two proportions: a proportion q of the ordered data lies below the quantile and a proportion $(1 - q)$ above, being $F(y_q) = q$ and $y_q = F^{-1}(q)$ the inverse cumulative distribution function (CDF). The median $q = 0.5$ splits the data into two parts with equal proportions.,the quartiles split the data into four parts, the quintiles split the data into five parts, the deciles into ten parts, and so on.

QR minimizes the sum that gives asymmetric penalties for over-prediction $(1-q)|e_i|$ and $q|e_i|$ for under-prediction. If $\hat{y}$ is the predictor variable, and $e = y$ - ŷ is the prediction of error, then $L(e) = L(y$ - ŷ$)$ indicates the loss associated with the prediction errors. If $L(e) = e^2$ then the OLS results the optimal predictor.
QR has the following form;

$$y_{it} = \alpha + X_{it}^{'}\beta_q + e_i,$$

where β_q is a vector of parameters associated with the q^{th} quantile, $0 < q < 1$.
Assuming $E(e_i^2) < \infty$, so that the distribution of e_i is not too spread out, the median regression minimizes the sum of absolute deviation (LAD) $\sum_i e_i$, and QR minimize the equation

$$\sum_i q|e_i| + \sum_i (1-q)|e_i|.$$

The q^{th} quantile regression estimator β_q minimizes the objective function

$$Q(\beta_q) = \sum_{i:y_i \geq x_i^{'}\beta}^{N} q|y_i - x_i^{'}\beta_q| + \sum_{i:y_i < x_i^{'}\beta}^{N} (1-q)|y_i - x_i^{'}\beta_q|$$

4. Results and Discussion

The main purpose of this study is to apply the OLS estimator and QR for different quantiles; 5th, 25th, 50th, 75th, and 95th to detect the significant effects of GDP and EC on the CO_2 emissions at different levels for developed and developing countries. CO_2 emissions are indicated in metric kilogram per capita, but GDP and EC are indicated by USD$ per capita and kiloton of oil equivalent per capita respectively. The panel data includes 29 countries over the period 1960 to 2008. The countries are categorized into developing and developed countries according to the World Bank classification. The list of countries is shown in the appendix.

Figure 1 and Figure 2 illustrate the distribution of CO_2 in developed and developing countries respectively, both clearly appearing increasing. Note that the increasing rate in developed countries was slow from 10th to 90th quantile but after the latter point, the level of CO_2 increased dramatically. On the contrary,the increase of CO_2 in developing countries started from the 10th quantile and was quite gradual in all the distribution.

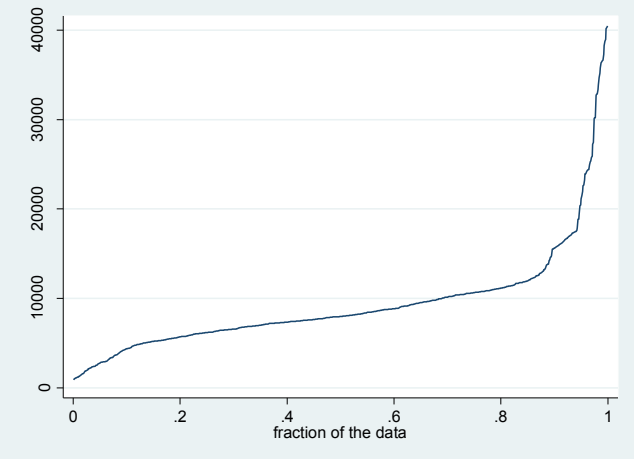

Figure (1) CO_2 distribution for developed countries

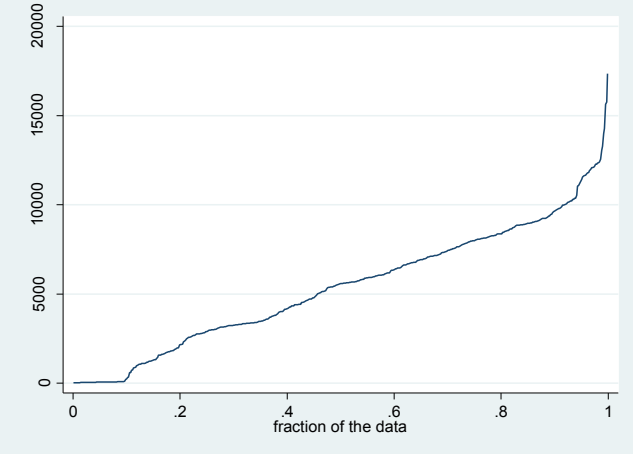

Figure (2) CO_2 distribution for developing countries

Before performing the QR analysis, the Modified Wald test and Breusch-Pagan test were performed to test the heterogeneity in the data. The results in Table (1) show that the data do not have constant variance, which supports the using of QR, as it could provide more information and accurate results than the OLS method in detecting the relationship between the variables, as may there is a strong relationship with some parts of the CO_2 emissions but there is no any significant relationship with other parts.

Table (1) heterogeneity Test

Diagnostic Tests	Statistics Test Value
Modified Wald test	74.07***
Breusch-Pagan test (χ^2)	18.31***

*** indicate the significant level of statistical test at the 1%, which support the rejection of null hypothesis and indicates the data is homogenous

Model Coefficient Interpretation on QR and OLS regression

Results of estimation using QR and OLS regression in the contaminated dataset are summarized in Table (2). The majority of the estimated coefficients under QR and OLS methods have significant effects on the CO_2 emissions. Besides, the lowest values of RMSE and MAD for developed countries estimated models are in favour of the 50^{th} quantile model with 4939.8 and 2812.4 respectively, whilst the lowest values of RMSE and MAD in the developing countries models are in favour of the 25^{th} quantile model with 2522.5 and 1423.3 respectively.

We can summarize the main results of our analysis as following. First, in the developed countries group, the GDP coefficient based on the OLS estimation is -0.11, which indicates that the GDP has a negative relationship with CO_2 emissions, as 1 USD$ increase of GDP in developed countries leads to a decline of 0.11 metric kilogram per capita in CO_2 emissions. The QR results show that the 25^{th} and 50^{th} quartiles of GDP have a significant stronger negative effect on CO_2 emissions than the other higher quantiles (75^{th} and 95^{th}), also larger than the effects of the OLS estimated model. In other words, 25% and 50% of the data from developed countries panel could show a stronger relationship between CO_2 and GDP than by using 75% or 95% of the data in the analysis. Further, the estimated coefficient of EC by OLS in developed countries model is 2.01, which indicates that EC has a positive relationship with CO_2 emissions, i.e. one unit increase in EC will lead to an increase of CO_2 emissions by 2.01 metric kilogram per capita. Moreover, QR results indicate that the effect of EC has similar effects on CO_2 emissions across the quantiles except in the highest quantile 95^{th} which indicates that the EC has about two times stronger positive effects on CO_2 emissions than that at lower quantiles and OLS estimation coefficients. Developed countries with low CO_2 emissions (at the lowest quantile considered, the 5^{th} percentile) have 2.03 unit increase of CO_2 emissions corresponding to one EC unit increase, whereas developed countries with a higher release of CO_2 emissions (at the higher quantile), have a significant 4.07 unit increase in CO_2 emissions for each unit increase in EC. In other words, the effect of EC is increasing for countries with higher CO_2 emissions (higher quantiles).

Second, in the developing countries group, the estimated coefficient of GDP by OLS estimation is -0.25, which illustrates that GDP has a negative relationship with CO_2, as 1 USD$ increase in GDP in developing countries leads to a decline of 0.25 metric kilogram per capita in CO_2 emissions. On the other hand, results of the QR models show that the 25^{th} and 50^{th} percentiles of GDP have a significantly stronger negative effect on CO_2 emissions than the other higher quantiles (75^{th} and 95^{th}), also larger than that effects of the OLS estimated coefficient. Therefore, these results are in line with the results obtained in the developed countries group. Moreover, the estimated coefficient for EC in the OLS model for the developing countries is 2.64, which indicates that EC has a positive relationship with CO_2 emissions, i.e. one unit increase in EC will lead to an increase of CO_2 emissions by 2.64 metric kilogram per capita. However, the QR results reveal that the effect of EC on the 50^{th} quartile of GDP have about two times the

significant stronger effect on CO_2 emissions than the other quantiles and OLS estimated coefficients. Developing countries with low CO_2 emissions (at the lowest quantile) release 1.76 unit of CO_2 emissions for one EC unit increase, whereas countries with a middle level of CO_2 emissions (i.e. at the 50^{th} percentile) release 4.13 unit of CO_2 emissions for one EC unit increase and countries at the highest quantile release 2.3 unit of CO_2 emissions for one EC unit increase. In other words, the effect of EC is higher on releasing CO_2 emissions for countries with a middle level of CO_2 emissions (50^{th} quantiles).

In addition of that, the best comparison between the QR estimated models for developed and developing countries could be made at level of the 50^{th} quantile models, as the most significant difference between the (GDP and EC) regression coefficients of the QR with respect to OLS can be also found at this level. The estimated coefficient of GDP in developing countries is -0.65, which is about five times larger than that negative effects in developed countries (-0.17). Tis indicates that the increase in one GDP unit in will affect negatively the CO_2 emissions in developing countries almost five times more than in developed countries. The developing countries tend to have five times lower CO_2 emissions by increasing one unit GDP in comparison to the developed countries at the 50^{th} percentile level. However, the estimated coefficient of EC in developing countries is 4.13, which is approximately two times larger than the one in developed countries (2.42). This means that one unit increase in EC will affect positively CO_2 emissions (i.e. countries release release more CO_2) about two times more in developing countries than in developed countries. In conclusion, the explanatory variables GDP and EC show different effects on CO_2 at different quantile levels for developed and developing countries.

Table (2) CO_2 Models by OLS and QR at different quantiles

CO_2	**OLS Model**	**Quantile models**				
		5^{th}	**25^{th}**	**50^{th}**	**75^{th}**	**95^{th}**
			Developed countries			
Intercept	3920.2*	-24.6*	336.0**	1905.9*	3940.9*	6369.7*
GDP	-0.11**	-0.14*	-0.18*+	-0.17**+	-0.13*	-0.12*
EC	2.01*	2.03**	2.55*	2.42*+	2.45*	4.07**+
RMSE	5744.1	6628.7	5674.7	**4939.8**	5165.7	9053.2
MAD	3129.4	4384.2	3236.7	**2812.4**	3565.2	7909.4
			Developing countries			
Intercept	1575.0*	-573.1*	-467.2*	-451.2*	4561.6*	5282.6*
GDP	-0.25**	-0.18*	-0.37*+	-0.65**+	-0.10*+	0.09**+
EC	2.64*	1.76*+	2.69*	4.13*+	2.01*	2.3*
RMSE	2741.0	2839.2	**2522.5**	2776.1	2711.1	2823.9
MAD	1845.2	1624.7	**1423.3**	1513.2	1623.2	1695.8

*, ** and ***: Significantly different quantile regression coefficient from zero at the 10%, 5% and 1% level. +: The QR coefficient is significantly different from OLS coefficient at 5% significance level (when the OLS coefficient is outside of the confidence interval of QR coefficient).

Figure 3 & 4 show the effects of GDP and EC for developed and developing countries with respect to the quantiles. The estimated coefficients with respect to various quantile levels clearly differ from the OLS coefficients and their confidence intervals. OLS coefficients are plotted as a horizontal dashed line with two horizontal dotted lines for the confidence intervals. The OLS coefficients do not vary along the distribution of the data (quantiles). The QR estimated coefficients are plotted as curve lines varying along the quantiles together with their confidence intervals (indicated by the shadowed area around them). In case the quantile coefficients are outside the OLS confidence interval borders, then they can be considered significantly different from those from the OLS models (significant differences are indicated with a + sign in Table 2).

The estimated QR coefficients in developed countries showed in Figure 3 for both GDP and EC as predictors of CO_2 emission are almost within the interval of the OLS estimation coefficient until about the location of the 90th quantile. This means that there is no significant difference between the estimated coefficients in the case of the QR model and the estimated coefficient in the case of the OLS model until a certain level which is about the location of the 90th quantile and above; in this part of the distribution the QR coefficients become significantly different from those of the OLS model.

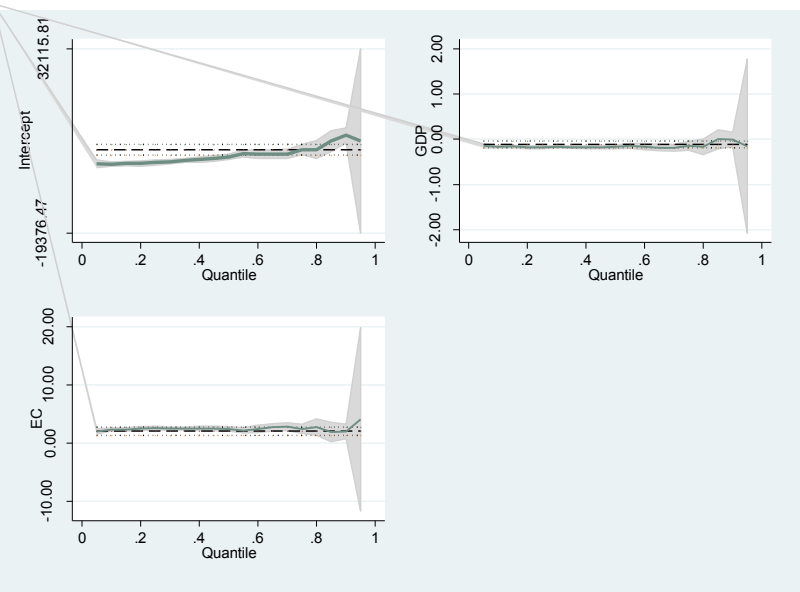

Figure 3 Quantile regression coefficients for developed countries

On the other hand, in Figure 4.4 which shows the estimated QR coefficients for the developing countries model, the effect of GDP on CO_2 decreases within the OLS estimation interval until around the location of the 25th quantile; after this threshold it exceeds the OLS estimation interval and becomes significantly different from the OLS estimation until the area around the

75th quantile where it starts to increase for countries with higher release of CO_2 emissions (higher quantiles), while the increasing effect of EC on CO_2 becomes significantly different for the OLS coefficients estimation after the 25th quantile until around the location of 75th quantile, where it lies within the OLS interval.

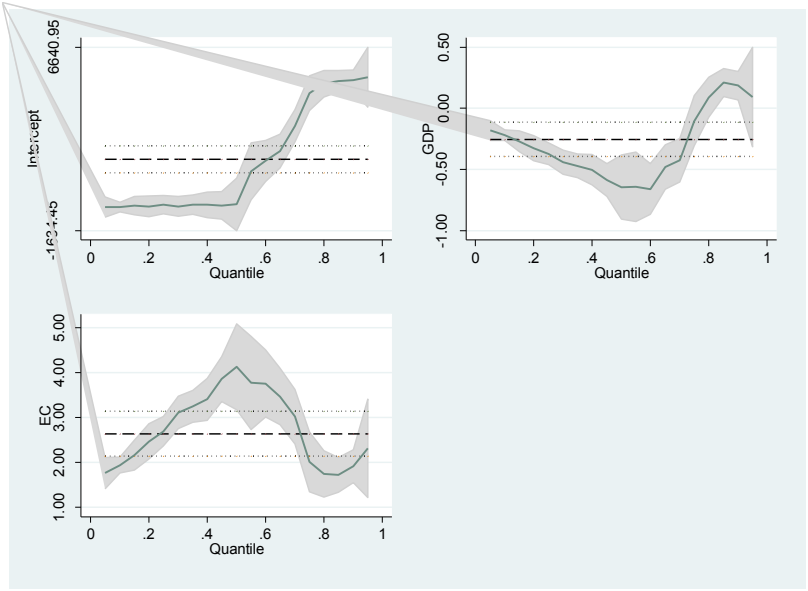

Figure 4 Quantile regression coefficients for developing countries

5. Conclusion

The topic of the relationship between CO_2 emission, EC and economic growth has got much efforts by many researchers, but conflicting results are often obtained due to using different approaches/efficiency. In environmental experimental studies it is often the case that the collected data suffer from the heterogeneous problem, which could cause inaccurate results by using the OLS regression, which may provide a weak or no relationship between the variables, while there could exist a stronger and useful relationship in some parts of the dependent variable distribution. Therefore, in this paper the quantile regression was applied to estimate the coefficients of this relationship by tackling at the same time the heterogeneous problem in the data. The main focus objective was therefore to detect the effects of the economic growth and energy consumption towards the CO_2 emissions at different release amount for developed and developing countries and at different quantile levels (0.05, 0.25, 0.50, 0.75, and 0.95) and compare these effects with those found in the OLS model. The panel data used consisted of 29 countries from two different economic development groups, 17 developed versus 12 developing countries over the period 1960-2008.

Results of QR for developed developing countries showed similar patterns. Both groups showed that the (25th and 50th) quartiles of GDP have a significantly stronger negative effect on CO_2 emissions than the other higher quantiles (75th and 95th), and these effects are also larger than those detected in the OLS regression. However, the effect of EC in developed countries had

similar positive effects on CO_2 emission all across the quantiles except in the highest quantile (95^{th}). Results also revealed a tendency of developing countries to have five times lower CO_2 emission by increasing one unit of GDP compared to developed countries. Increasing one unit in EC had a positive effect on CO_2 emission (countries release more CO_2) within the developing country group. This group had coefficients twice larger than those within the developed country group, based on the 50^{th} quantile model. In conclusion, results differ significantly across the two groups of countries, with EC contributing to a higher environmental degradation in CO_2 emissions. Thus, EC monitoring is a key factor for an environmentally balanced and sustainable development especially within the developing countries.

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