Comment on "Roughness of Interfacial Crack Fronts: Stress-Weighted Percolation in the Damage Zone"

A recent Letter [1], by Schmittbuhl, Hansen, and Batrouni (SHB) addresses the question of how interfacial cracks roughen in the presence of disorder. SHB explain this process by a stress induced gradient percolation model that takes into account the damage accumulated, and translates that into a self-affine crack front profile. In this Comment, we point out that the results presented in Ref. [1] do not prove self-affinity but rather support self-similarity of the crack fronts. This result, however, would be in disagreement with experiments [2].

In the model of SHB the strain gradient induces a damage profile and a crack front results. As the load is raised the width of the front W increases approximately as a power law, and eventually saturates. As in gradient percolation [3], the saturated width W^* scales with the gradient of the damage profile $1/l_y$ as $W^* \sim l_y^{\alpha}$ with $\alpha = \nu/(1+\nu)$ where ν is the correlation exponent of the underlying percolation problem. Since in Ref. [1] $l_y \sim L_x$, where L_x is the lattice size parallel to the front, SHB combine the initial dynamic scaling with that of the saturated width into a "Family-Vicsek"-like scaling form $W(L_x, t) = L_x^{\alpha} f(t/L_x^{z})$, and conclude that the fronts are self-affine interfaces. Such an attempt is misleading, since presenting data in such a form does not imply that the fronts are self-affine. In gradient percolation α *cannot* be interpreted as a roughness exponent [3]: the front is self-similar (i.e., the scaling is isotropic) up to a length scale $\xi \sim W$ [4] and it is trivially flat on scales beyond ξ. Self-affinity implies instead that on *any* length scale $l < \xi$ the system rescales anisotropically. Although strain induced correlations could change the values of the critical exponents from the standard percolation ones, the basic picture remains the same.

Figure 1 shows the data of the corresponding Fig. 1 from [1], displaying the broken springs. We also include the hull of the (damage) gradient percolation cluster and the corresponding solid-on-solid (SOS) interface. Comparing these two shows that the SOS presentation is just an artificial projection from the fractal perimeter of the damage zone which is not self-affine. In particular, we see that the size of overhangs is of the same order of the width. We have also studied an effective medium model in the spirit of Ref. [5] in which the strain profile is computed similarly to Ref. [1], but the damage is replaced by its average along the transverse direction [6]. This model is able to reproduce the features of the Family-Vicsek data collapse of SHB, but the fronts are obviously described by standard gradient percolation. From our simulations we find that the gradient l_y depends on the elastic constants of the problem. In Ref. [1] the Green function G_{ij} is normalized so that $\sum_{ij} G_{ij}/(L_x L_y)$ is constant. Since L_y is kept constant this amounts to rescaling the elastic constant by L_x, producing an effective dependence of l_y on L_x.

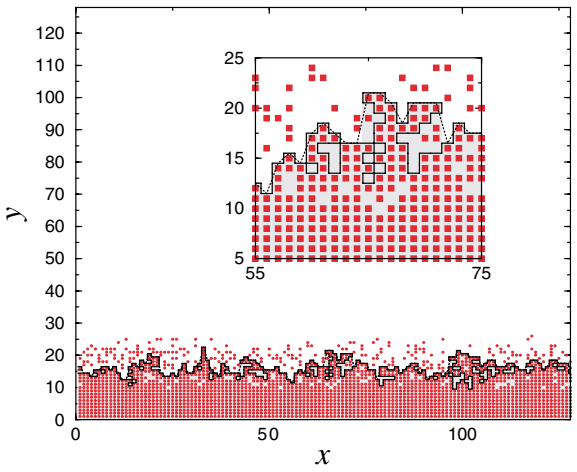

FIG. 1 (color online). The damage reported in Fig. 1 of Ref. [1] is plotted together with the front perimeter (solid line) and the SOS approximation (dotted line). As shown in the inset, the perimeter displays substantial overhangs, whose size is comparable with the width, and it is thus not self-affine.

In conclusion, a correct interpretation in the framework of gradient percolation of the data presented in Ref. [1] implies that fronts are self-similar rather than self-affine. Thus the model of Ref. [1] does not explain the roughness of planar cracks observed experimentally [2].

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