Application and validation of dynamic Poisson models to measure credit contagion

Applicazione e validazione di modelli di Poisson dinamici per misurare il contagio nel credito

Arianna Agosto and Emanuela Raffinetti

Abstract The growing importance of financial technology platforms, based on interconnectedness, makes necessary the development of credit risk measurement models that properly take contagion into account. To this aim, we propose to use a credit risk model that allows to investigate contagion through Poisson autoregressive stochastic processes. We apply this model to the quarterly count of defaulted loans in the Italian banking system, finding evidence of contagion effects in several economic sectors. To calculate the accuracy of the model we use a new measure, whose main advantage is being not dependent on the type of predicted variable.
Abstract *La diffusione di piattaforme finanziarie digitali, basate sull'interconnessione, rende necessario lo sviluppo di modelli per il rischio di credito che tengano in opportuna considerazione il contagio. A tale scopo proponiamo di utilizzare un modello per il rischio di credito che permette di studiare il contagio attraverso processi di Poisson autoregressivi. Applicando il modello alle serie trimestrali del numero di prestiti a default nel sistema bancario italiano, troviamo evidenza di effetti di contagio in diversi settori economici. Per calcolare l'accuratezza del modello utilizziamo una nuova misura, il cui principale vantaggio risiede nella non dipendenza dalla tipologia di variabile risposta considerata.*

Key words: Credit Risk, Poisson Autoregressive models, Systemic risk, Accuracy measures

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1 Introduction

Interconnectedness between the economic agents was recognized as a trigger of the great financial crisis in 2008-2009 and is increasing with the growth of innovative financial technologies (fintechs), especially peer to peer lending platforms. Measuring the credit risk arising from interconnectedness is necessary to safeguard investors and maintain financial stability.

To this aim, we apply a novel credit risk model that incorporates contagion through Poisson autoregressive stochastic processes.

The main systemic risk measures defined in the literature (see e.g. Adrian and Brunnermeier, 2011 and Acharya et al., 2012) have been applied to financial market data and are based on Gaussian processes.

We indeed study contagion through discrete data models for default counts. Examples of default counts modelling can be found in Lando and Nielsen (2010), Koopman et al. (2012) and, recently, Azizpour et al. (2018). Our contribution is proposing a credit risk assessment approach based on a dynamic model which includes autoregressive, contagion and exogenous effects in a time-varying Poisson intensity specification.

Validation is a critical issue in credit risk modelling, because of the interest in selecting indicators able to predict the default peaks. In our empirical application we validate the models applied to default counts using a newly developed predictive accuracy measure. Contrary to the main summary predictive accuracy indices, such as the root mean squared error and the area under the ROC curve, the new measure, which is called Rank Graduation measure, does not depend on the type of response variable being predicted. It is based on the distance between the observed response variable values and the same values re-ordered in terms of the predicted values given by the model and was found quite effective in two real machine learning applications characterised, respectively, by a binary and a continuous response variable. Our aim is to show how the Rank Graduation measure appears as an appropriate model predictive accuracy index also when dealing with discrete response variables, here represented by count variables.

The paper is organised as follows. Section 2 describes the proposed modelling approach. Section 3 provides the basic elements characterising the Rank Graduation measure. Section 4 presents the main empirical findings derived from the application and validation of the presented model for default counts. Section 5 concludes.

2 Proposal

We propose to use Poisson autoregressive models to investigate inter-sectoral correlation. Specifically, Agosto and Giudici (2019) have defined the following model for the number of defaults in economic sector j at time t:

$$\begin{aligned} y_{jt}|\mathscr{F}_{t-1} &\sim Poisson(\lambda_{jt}) \\ \log(\lambda_{jt}) &= \omega_j + \textstyle\sum_{i=1}^{p} \alpha_{ji}\log(1+y_{jt-i}) + \sum_{i=1}^{q} \beta_{ji}\log(\lambda_{jt-i}) \\ &\quad + \textstyle\sum_{i=1}^{r} \gamma_{ji}x_{t-i} + \sum_{i=1}^{s} \zeta_{ji}\log(1+y_{kt-i}) \end{aligned} \tag{1}$$

with $\omega_j, \alpha_{ji}, \beta_{ji}, \gamma_{ji}, \zeta_{ji} \in \mathbb{R}$ and $x_{t-i} := (x_{1t-i}, x_{2_t-i}, ..., x_{dt-i})' \in \mathbb{R}^d$ being a vector of lagged exogenous covariates. In other words it is assumed that the default count time series y_{jt} in a given sector called j, conditional on its past, follows a Poisson distribution with a time-varying autoregressive intensity λ_{jt} whose formulation also includes the past default counts y_{kt-i} of another sector called k and, possibly, a set of exogenous covariates x_t. The impact of other sectors' default counts is what we call contagion. Taking the $\log(\cdot)+1$ of the counts makes possible to deal with zero values. The inclusion of past values of intensity λ_{jt} allows for parsimonious modelling of long memory effects in a way analogous to the extension of standard ARCH processes to GARCH ones for Gaussian variables. It can be shown that including an autoregressive component in a Poisson process generates overdispersion (i.e. unconditional variance larger than the mean), an empirical feature of default count series, which typically cluster in time. The exogenous component x_t contains macroeconomic and financial variables affecting all companies' default probability, i.e. *systematic risk* factors.

Model (1) is called Contagion PARX as it is inspired by Poisson Autoregression with Exogenous Covariates (PARX) developed by Agosto et al. (2016), who studied the properties of the process and applied it to Moody's rated US default counts. Differently from the PARX and following Fokianos and Thjøstheim (2011), Contagion PARX uses a log-linear specification, rather than a linear one, for the intensity, allowing to consider negative dependence on the covariates, a likely possibility in application to default risk. Furthermore, the model includes an explicit contagion component, i.e. the default count in other sectors, while Agosto et al. (2016) only considered an autoregressive part and an exogenous (macroeconomic) one.

3 A brief overview of the Rank Graduation index

With the aim of assessing the predictive accuracy associated with the models described in the application section, an overview on the *RG* index is here presented. When the response variable Y is both binary and continuous there is not a unique measure. Giudici and Raffinetti (2019) have worked out one possible solution. The proposal is based on the C concordance curve, obtained by ordering the Y original values according to the ranks of the corresponding $\hat{Y}$ estimated values. Let Y be a (binary, continuous or also, as in the case discussed in this paper, discrete) response variable and let $X_1, \ldots, X_p$ be a set of p explanatory variables. Suppose to apply a model such that $\hat{y} = f^h(\boldsymbol{X})$, with $h = 1, \ldots, l$. The model predictive accuracy is assessed by measuring the distance between the set of points lying on the C concordance curve $(i/n, (1/(n\bar{y}))\sum_{j=1}^{i} y_{\hat{r}_j})$, where $\bar{y} = \sum_{i=1}^{n} y_i$ and $y_{\hat{r}_j}$ represents the *j-th* response variable value ordered by the rank of the corresponding predicted value, and the set of points lying on the bisector curve $(i/n, i/n)$ (see Figure 1).

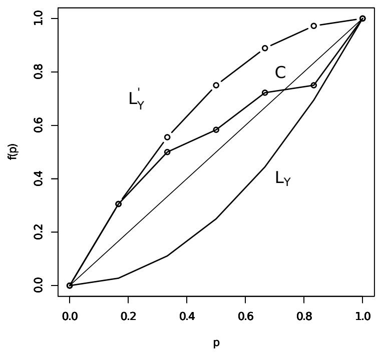

Fig. 1 The L_Y and $L_Y^{'}$ Lorenz curves and the C concordance curve, normalised.

If the C concordance curve overlaps with the bisector curve, the model has no predictive capability. On the contrary, if the C concordance curve overlaps with the response variable Lorenz curve L_Y (defined by the normalised Y values ordered in non-decreasing sense) or the dual Lorenz curve $L_Y^{'}$ (defined by the normalised Y values ordered in non-increasing sense), situations of concordance and discordance between Y and $\hat{Y}$ arise, meaning that the model perfectly explains the target variable.

The RG (Rank Graduation) index is defined as:

$$RG = \sum_{i=1}^{n} \frac{\left\{(1/(n\bar{y}))\sum_{j=1}^{i} y_{\hat{r}_j} - i/n\right\}^2}{i/n} = \sum_{i=1}^{n} \frac{\{C(y_{\hat{r}_i}) - i/n\}^2}{i/n}, \tag{2}$$

where $C(y_{\hat{r}_j}) = \frac{\sum_{j=1}^{i} y_{\hat{r}_j}}{\sum_{i=1}^{n} y_{r_i}}$ represents the cumulative values of the (normalised) response variable.

Note that the RG index takes values between 0 and RG_{max}, which is obtained when the predicted ranks order the response variable values in full concordance (or full discordance) with the observed ranks. In addition, we remark that when some of the $\hat{Y}$ values are equal to each other, the original Y values associated with the equal $\hat{Y}$ values are substituted by their mean, as suggested by Ferrari and Raffinetti (2015).

4 Application

In this section we provide the application of the model presented in Section 2 to Italian corporate default counts data.

As a proxy of default counts, we use the number of transitions to *bad loans* of Italian banks' credit exposures collected in Bank of Italy's Credit Register. Bad loans are exposures to insolvent debtors that the bank does not hope to recover anymore and must report as losses in its balance sheet. The data are quarterly and divided by economic sector. We use data covering the period March 1996 - June 2018 (90 observations).

Figure 2 shows the default count time series of two economic sectors of major importance: Households and Commercial corporate sectors. Both series exhibit clustering and a possible structural break in 2009, with an increase in both level and variability.

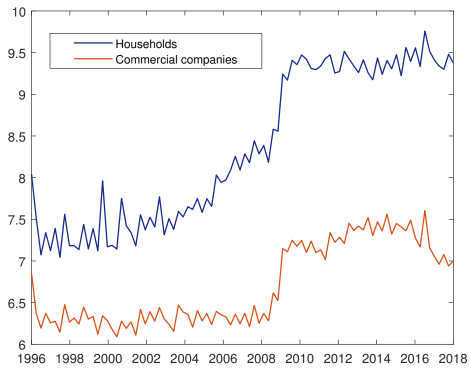

Fig. 2 Default count time series in the Italian banking system (logarithmic scale)

To investigate credit contagion effects, we apply model (1) to various economic sector default counts. As the number of available observations is limited, we carry out a sector pairwise analysis. Specifically, in each regression the dependent variable is the number of defaults in a given sector (y_{jt} in specification 1), while the covariate is the number of defaults in another sector (y_{kt}), lagged by one or two time periods. That is for now we use model (1) with $\gamma_{ji} = 0$. The results are shown in Table 1. Each row represents a model, in which the default counts of one sector are pairwise regressed on all the others, lagged by one or two periods, and on the autoregressive component, according to the presented specification.

From Table 1 note that Commercial and Financial sectors are the most influenced by the default dynamics of the others. As expected from an economic point of view, Households, which represent the consumer sector, influence all the other sectors. An interesting interaction is that involving Real Estate and Households sectors: the first, including both constructions and real estate companies, is affected by Households default risk, as expected, but in turn affects Households, which are typically highly engaged in the housing market.

	HH	MNF	RE	FIN	CMM
HH			0.093		
MNF	0.086				
RE	0.203	0.252			0.650
FIN	0.298	0.275	0.925		0.926
CMM	0.267	0.264	0.473		

Table 1 Significant effects between sectors, from the fitted Contagion PARX models (HH=Households, RE=Constructions and Real Estate, FIN=Financial, CMM=Commercial). The rows report the infected sectors (as response variables) and the columns the infecting sectors (as explanatory lagged variables. The reported effects are the sum of the first and second lag coefficients).

To provide an example of validation, we consider the model regressing Commercial sector default counts on their past values and on Household default counts, which is of major economic interest.

We then compare three different specifications. The first - that we call Full Model - includes both the contagion component and a macroeconomic covariate. The covariate we consider is the Italian GDP quarterly growth rate, which has turned out to be the preferable exogenous regressor through a preliminary model selection. In other words, referring to formulation (1), GDP growth rate is the x process while the Households default counts are the y_k one. The addition of an exogenous covariate in the intensity specification can be considered as a robustness step of our contagion analysis, verifying to what extent the macroeconomic and financial stress affecting all economic agents explains the default and contagion dynamics. The second model is the Contagion Model without other covariates than Households default counts (γ parameters equal to 0 in specification 1, like the models in Table 1). The third (PAR Model) is only autoregressive, without covariates (both γ and ζ parameters equal to 0 in formulation 1).

We compare the in-sample performances of the three models above by using the *RG* measure. The *RG* index computed on Full Model equals 3.826 against a value of 3.627 for PAR Model. Contagion Model provides a value of 3.790 for the *RG* index. It thus follows that PAR Model explains the 83% of the variable ordering, compared with the 86% of Contagion Model and the 87% of Full Model. This means that adding the contagion component leads to a remarkable increase in the predictive accuracy with respect to the only autoregressive specification. GDP turns out to be a significant regressor for default intensity: the coefficient associated to the second lag of GDP growth rate is -0.081 with a t-statistic of -3.340. Though, its inclusion improves model performance only slightly.

5 Conclusion

We have proposed a credit risk modelling approach which allows to investigate contagion through Poisson autoregressive stochastic processes.