

Review and Comparison of Random Spray Retinex and of its variants STRESS and QBRIX

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ABSTRACT

In this paper, we review and compare three spatial color algorithms of the Milano Retinex family: Random Spray Retinex (RSR) and its subsequent variants STRESS and QBRIX. These algorithms process the colors of any input image in line with the principles of the Retinex theory, introduced about 50 years ago by Land and McCann to explain how humans see colors. According to this theory, RSR, STRESS and QBRIX re-scale independently the color intensities of each pixel by a quantity, named local reference white, which depends on the spatial arrangement of the colors in the pixel surround. The output is a new color enhanced image that generally has a higher brightness and more visible details than the input one. RSR, STRESS and QBRIX adopt different models of spatial arrangement and implement different equations for the computation of the *local reference white*, so that they produce different enhanced images. We propose a comparative analysis of their performance based on numerical measures of the image brightness, details and dynamic range. In order to enable result repeatability and further comparisons, we use a set of images publicly available on the net.

KEYWORDS

Retinex, Spatial Color Algorithms, Color Image Enhancement.

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1. INTRODUCTION

The human color sensation deriving from the observation of a certain point of a scene may differ from the physical luminance of that point. In fact, several experiments revealed that the color sensation at a point depends not only on the photometric properties of that point but also on the physical luminance of the colors surrounding that point and on their spatial arrangement (Albers, 2009), (McCann and Rizzi, 2012), (De Valois and De Valois, 2008).

Retinex (Land and McCann, 1971) is the earliest computational model that attempts to estimate the human color sensation by taking into account this empirical evidence. Retinex assumes that the color signal is processed separately by the retina photoreceptors and that there exists a spatial interaction among the colors of the viewed scene. In agreement with these hypotheses, when applied to a digital color picture, Retinex works separately on the three image chromatic channels and processes the color of each image pixel based on the surrounding colors. In this procedure, the digital color values of the input image undergo two so-called color calibration phases, which transform them to a scale of human color appearances and vice-versa. The resulting image is an estimate of the human color sensation. When calibrations are skipped, Retinex outputs an enhanced color image, where the chromatic dominant of the light and possible smooth shadows are lowered, while scene details and edges are enhanced.

Many implementations of Retinex are available in the literature (McCann and Rizzi, 2012). They differ from each other in the way they spatially explore the neighborhood of each pixel and in the way the colors adjacent to that pixel are processed.

Among the many variants of Retinex, here we focus on three algorithms belonging to the so-called Milano Retinex family (Rizzi and Bonanomi, 2017). Milano Retinexes are spatial color processing algorithms implementing the Retinex principles and mainly tailored to image enhancement. In this application field, Milano Retinex algorithms do not perform any calibration of the color signal. In the next, we will refer to the image output by Milano Retinexes as *color sensation* to distinguish it from the *human color sensation* computed by the original Retinex algorithm.

The first Milano-Retinex implementation, analyzed in details in (Provenzi et al., 2005), scans the neighborhood of each image pixel x by a set of random paths ending in x . The chromatic intensities of the color sensation at x are obtained as the average, among the random paths, of the relative ratios of the pixel's chromatic intensities along each path, where division by zero is of

course prevented. The one-dimensional, path-based scanning approach of the original Retinex has been adopted by many other subsequent Retinex implementations, e.g. (Marini and Rizzi, 2000), (Montagna and Finlayson, 2011), (Simone et al., 2014), (Lecca, Rizzi and Gianini, 2015), (Simone et al., 2017), (Gianini, Rizzi and Damiani, 2016). Here we describe and compare three algorithms, that implement a different scheme for image sampling and a different equation for the color sensation. These algorithms are: the Random Spray Retinex (RSR) algorithm presented in (Provenzi et al., 2007), and its two subsequent variants STRESS (Kolås, Farup and Rizzi, 2011) and QBRIX (Gianini, Manenti and Rizzi, 2014). Our attention to RSR and to its variants is justified by the success of this algorithm that has been widely used for many applications, e.g. (Rizzi and Parraman, 2010), (Fierro et al., 2013), (Banić and Lončarić, 2013a), (Banić and Lončarić, 2013b) (Gianini, Lecca and Rizzi, 2016).

In RSR, STRESS and QBRIX the color sensation is computed by processing the R, G, B channels of the input image independently. For each channel, the intensity of any pixel x is rescaled by the so-called *local reference white*, which is an intensity value depending on the spatial distributions of the intensities in a neighborhood of x .

RSR is the first Retinex implementation that replaces the random path scanning approach with a bi-dimensional random sampling. RSR originated from the need to solve some problems raised up by the use of the random paths. In fact, the path-based sampling mechanism of the original Retinex presents three main disadvantages: first, the color filtering strongly depends on the number and on the geometry of the used paths; second, the randomness of the paths introduces chromatic noise in the estimate of the color sensation; third, in order to reduce this noise, many paths have to be scanned, resulting in long computational times. RSR overcomes these problems by introducing a novel spatial sampling that explores the region around each image pixel x with a *random spray*. A random spray is a set of pixels randomly selected from a circular neighbor of x with a radially decreasing density, accounting for the fact that the pixels closer to x are more relevant to human color sensation than the others (McCann and Rizzi, 2012), (Creutzfeldt, Lange-Malecki and Wortmann, 1987). For any spray, the color sensation at x is computed as the ratio between the intensity of x and the maximum intensity in the spray. In order to reduce the chromatic noise due to the random sampling, many sprays are generated, and the final color sensation is the average value over the sprays' color sensations. Again, the algorithm precludes division by zero. STRESS (Kolås, Farup and Rizzi, 2011) (Spatio-

Temporal Retinex-inspired Envelope with Stochastic Sampling) is a variant of RSR, particularly suitable for local contrast stretching, automatic color correction, spatial color gamut mapping, and efficient color to grayscale conversion, e.g. (Islam and Farup, 2010), (Islam and Farup, 2011). As RSR, STRESS explores the neighborhood of each image pixel by the use of random sprays, but it estimates the color sensation in a different way. Precisely, for each chromatic channel, and for each image pixel x , STRESS computes the lightest and the darkest pixel in each spray centered at x and uses these values to define two functions, said the minimum and the maximum envelope, that contain the chromatic signal. The color sensation at x is obtained by stretching the chromatic intensities at x between the corresponding minimum and maximum values in the envelopes.

QBRIX (Gianini, Manenti and Rizzi, 2014) (Quantile-Based approach to Retinex) is a probabilistic formulation of RSR. It removes the sampling procedure and models the spatial arrangement of the image colors by a suitable distribution function. There are two implementations of QBRIX. The first one is a global filter (here indicated as G-QBRIX), based on the fact that chromatic intensities rarely occurring in the image are not relevant to the color sensation, thus they can be ignored. According to this principle, the probability density function (pdf) of each chromatic channel is computed, and the intensity value corresponding to a quantile, fixed by the user, is set up as reference white. The color sensation is obtained by rescaling the chromatic values of the image by the input quantile that controls the percentage of discarded colors. The second implementation is a local filter (here termed L-QBRIX), that takes into account the spatial distance between the image colors: for each chromatic channel and for each pixel x , the algorithm computes a pdf at x where the contribution of each image pixel is weighted by its distance from x . The color sensation at x is then computed as in the G-QBRIX. Disregarding the random sampling, QBRIX estimates a color sensation image with a negligible chromatic noise (Gianini, 2016) (Gianini, 2017).

The present paper presents a review and a comparative analysis of RSR, STRESS and QBRIX, carried out on a public dataset of color pictures (Rizzi, Gatta, and Marini, 2003b) and on an image usually employed to test color filtering algorithms. We evaluate the performance by means of different measures concerning the capability to enhance the contrast and luminance of the input image, the spatial local properties and the chromatic noise possibly generated by the filtering. The evaluation scheme and the obtained results can be used for the analysis

and the comparison of other filtering algorithms. This paper is mainly based on the work (Lecca, Gianini and Rizzi, 2016) originally presented at the 12th Colour Conference, held in Torino, Italy, in 2016.

2. NOTATION

Let I be a color image with size $A \times B$, where A, B are positive integer numbers. Let C_0, C_1, C_2 be the chromatic channels of I . Each of them is regarded as a function $C_i : \text{Supp}(I) \rightarrow [0, 1]$, where $\text{Supp}(I)$ is the image support, i.e. the set of the spatial coordinates (a, b) of the image pixels ($a \in \{1, \dots, A\}, b \in \{1, \dots, B\}$). Here we assume that the chromatic intensities of I , that in a standard RGB image range over $\{0, \dots, 255\}$, have been normalized in $[0, 1]$. This assumption is introduced for numerical reasons. Here we refer to the color sensation as to a color image L defined on $\text{Supp}(I)$ and having chromatic components L_0, L_1, L_2 .

3. RANDOM SPRAY RETINEX

Random Spray Retinex (RSR) (Provenzi et al., 2007) is an efficient implementation of Retinex, used in many applications. As pointed out in Section 1, RSR was born from the need to decrease the computational cost of Retinex and to partially remove the chromatic noise possibly introduced by the random paths in the estimate of the color sensation. These objectives have been reached by adopting a novel sampling scheme and by simplifying the equation for computing the color sensation. The algorithm works as follows.

For each chromatic channel C_i and for each pixel $x \in \text{Supp}(I)$, with $C_i(x) \neq 0$, RSR generates $N (> 0)$ random sprays centered at x . The k -th random spray $S_k(x)$ ($k = 1, \dots, N$) is a set of image pixels $\{y_k^1, \dots, y_k^M\}$, belonging to a circular neighborhood centered at x . The parameters N and M are non-null positive numbers and $M (> 0)$ is smaller or equal to the number of image pixels. The pixel y_k^j is defined as

$$y_k^j = (x_0 + \rho \cos(\theta), x_1 + \rho \sin(\theta)) \quad (3.1)$$

where (x_0, x_1) are the spatial coordinates of x , ρ and θ are uniformly randomly chosen over the sets $[0, R]$ and $[0, 2\pi)$ respectively, and R is the radius of the spray. Experiments reported in (Provenzi et al., 2007) showed that the optimal value for R is the length of the image diagonal. Fig. 1 shows a random spray centered at the middle point of an image.

The i -th chromatic intensity of the color channel at x is computed by the following Equation:

$$L_i(x) = \frac{1}{N} \sum_{i=1}^M \frac{C_i(x)}{C_i(x_k)} \quad (3.2)$$

where x_k is a pixel of $S_k(x)$ such that

$$C_i(x_k) = \max\{C_i(y) : y \in S_k(x)\}. \quad (3.3)$$

Since we assume that $S_k(x)$ contains x , and $C_i(x) \neq 0$, $C_i(x_k)$ is greater than zero, and thus Equation (3.2) makes sense. When $C_i(x) = 0$, $L_i(x)$ is set to zero.

The parameters N and M control respectively chromatic noise and locality of color filtering. The optimal values of N and M depend on the input image and they can be empirically determined as a trade-off between good image quality and computational time (Provenzi et al., 2007).

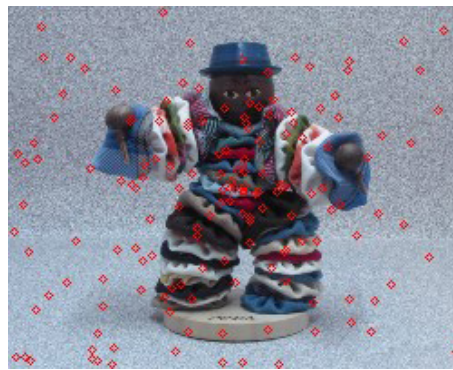


Figure 1 - A random spray centered in the middle of a color image from YACCD. The pixels of a spray are highlighted in red

4. STRESS: SPATIO-TEMPORAL RETINEX-INSPIRED ENVELOPE WITH STOCHASTIC SAMPLING

STRESS (Spatio-Temporal Retinex-inspired Envelope with Stochastic Sampling) (Kolås, Farup and Rizzi, 2011) inherits from RSR the image exploration mechanism, while it implements a different equation for estimating the color sensation.

For each chromatic channel C_i and for each pixel $x \in \text{Supp}(I)$, with $C_i(x) \neq 0$, STRESS computes N random sprays centered at x . For each spray $S_k(x)$ ($k = 1, \dots, N$), it estimates the local reference lightest and darkest chromatic intensities $E_{min}^k(x)$ and $E_{max}^k(x)$, defined as

$$\begin{aligned} E_{min}^k(x) &= \min\{C_i(y) : y \in S_k(x)\}; \\ E_{max}^k(x) &= \max\{C_i(y) : y \in S_k(x)\}. \end{aligned}$$

E_{min}^k and E_{max}^k can be regarded as two smooth functions, i.e. the *minimum* and *maximum envelopes*, defined from the image support to the chromatic intensities, and containing the entire chromatic channel. Fig. 2 shows an example of such envelopes.

The values $E_{min}^k(x)$ and $E_{max}^k(x)$ are used to compute a local chromatic sensation $v_k(x)$ from $S_k(x)$ by the following equation:

$$v_k(x) = \begin{cases} \frac{1}{2} & \text{if } R_k(x) = 0 \\ \frac{C_i(x) - E_{min}^k(x)}{R_k(x)} & \text{otherwise} \end{cases} \quad (3.4)$$

where $R_k(x) = E_{max}^k(x) - E_{min}^k(x)$.

The final chromatic sensation $L_i(x)$ is then obtained as

$$L_i(x) = \begin{cases} \frac{1}{2} & \text{if } E_M(x) = E_m(x) \\ \frac{C_i(x) - E_m(x)}{E_M(x) - E_m(x)} & \text{otherwise} \end{cases} \quad (3.5)$$

where

$$\begin{aligned} R(x) &= \frac{1}{N} \sum_{k=1}^N R_k(x); \quad v(x) = \frac{1}{N} \sum_{k=1}^N v_k(x) \\ E_m(x) &= C_i(x) - R(x)v(x); \quad E_M(x) = E_m(x) + R(x) \end{aligned}$$

Using the mean values $R(x)$ and $v(x)$ instead of the mean values of $E_{min}^k(x)$ and $E_{max}^k(x)$ over the number of sprays emphasizes the weight of the chromatic intensity of the pixels close to x , and thus produces a less noisy image, with few halving artifacts (Kolås, Farup and Rizzi, 2011).

4. QBRX: QUANTILE-BASED APPROACH TO RETINEX

QBRX (Quantile-Based approach to RetIneX) (Gianini, Manenti and Rizzi, 2014) is a probabilistic version of Retinex, based on RSR. QBRX relies on the observation that the color sensation at any image pixel x is poorly influenced by (1) colors rarely occurring in the image and (2) colors of pixel located far from x . The observation (1) leads to an implementation of a global color filter, named G-QBRX, while merging both (1) and (2) leads to a local color filter, named L-QBRX. Both these versions process the chromatic channels separately and replace the random sampling of RSR with a statistic approach that suppresses the chromatic noise that in RSR is due to the random sampling.

4.1 THE GLOBAL FILTER: G-QBRX

For each chromatic channel, G-QBRX computes the probability density function (pdf) of the chromatic intensities. Due to the discrete nature of the data, the pdf is approximated by a histogram, normalized so that it sums up to 1.0. The pdf of a certain chromatic value C is thus computed as

$$p(C) = \frac{\#\{y \in \text{Supp}(I), y: C_i(y) = C\}}{\#\{y \in \text{Supp}(I)\}} \quad (4.1)$$



CubeAA01

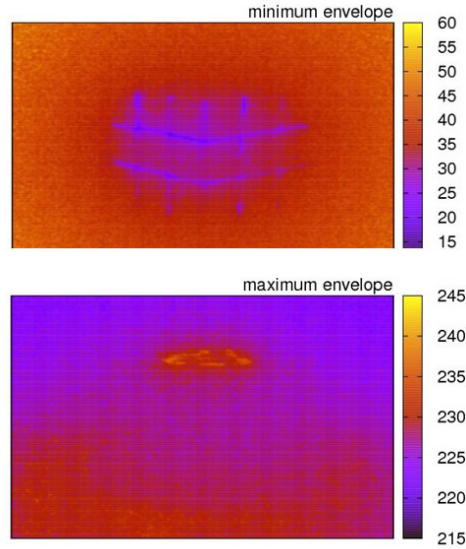


Figure 2 - The red component of an image (CubeAA01) from YACCD (Rizzi, Gatta, and Marini, 2003b) and its minimum and maximum envelopes. These results have been obtained by setting $N=35$ and $M=125$.

G-QBRIX is grounded in the observation that chromatic intensities rarely occurring are irrelevant to color sensation, regardless of their values. The observation holds for the highest luminance pixels which are to be used as reference white values: as a consequence G-QBRIX considers only a certain percentage of image pixels, corresponding to a quantile Q of the pdf p . We remind that the quantile Q of the pdf p is a real number ranging over $[0, 1]$ that partitions p in two parts at a chromatic intensity q such that $\sum_{z=0}^q p(z) = Q$.

The value of Q is a user input and controls the percentage of pixels to be disregarded: more precisely, the percentage of retained pixels is the $(100 \cdot Q)\%$. The value of $L_i(x)$ (where as usual x is an image pixel) is given by the following equation:

$$L_i(x) = \begin{cases} 0 & \text{if } C_i(x) = 0 \\ 1 & \text{if } C_i(x) > q, C_i(x) \neq 0 \\ \frac{C_i(x)}{q} & \text{otherwise} \end{cases} \quad (4.2)$$

The second condition defines the chromatic sensation when q is smaller than the value of $C_i(x)$. For instance, this is the case of an image with $C_i(x) = 1$ and a low value of $p(C_i(x))$.

4.2 THE LOCAL FILTER: L-QBRIX

L-QBRIX re-implements the procedure of G-QBRIX by taking into account also the influence of the spatial arrangement of the colors in the image. For each chromatic channel C_i and for each pixel x in the image support, the algorithm computes a pdf of the chromatic intensities at x , such that

$$p_x(C) = \frac{1}{W} \sum_{y \in \{\text{Supp}(I) \setminus \{x\}, C_i(y)=C\}} \left[\frac{d(x,y)}{D} \right]^{-\alpha} \quad (4.3)$$

In this equation, $d(x, y)$ denotes the Euclidean distance between x and y , D is the length of the image diagonal, the parameter α is a real number tuning the relevance of the distance in the color sensation, and $W = \sum_A p_x(A)$ is a normalization factor where A ranges over the possible chromaticity intensities.

As in G-QBRIX, also in L-QBRIX the chromatic sensation $L_i(x)$ is computed by selecting a quantile of the pdf $p_x(C)$ and by applying Equation (4.2).

5. EXPERIMENTS

The color sensation output by the Retinex algorithms is an enhanced color image, where the chromatic dominant of the light and possible slight shadows are partially removed and the scene details are emphasized. In our comparative analysis of RSR, STRESS and QBRIX, we define a set of evaluation measures taking into account three visual properties of the color sensation, already used in previous works, e.g. (Lecca and Rizzi, 2015), (Lecca, Rizzi and Gianini, 2016): brightness, contrast, chromatic dynamic range. The analysis of these quantities allow to evaluate the image enhancement produced by the algorithms of the Retinex family. We remark that here we do not consider any pre- or post-filtering calibration of the input and output images, which is usually required by other tasks, e.g. for modeling the human vision system (Rizzi and McCann, 2007).

The proposed comparison is performed on the public dataset YACCD (Rizzi, Gatta and Marini, 2003b) that consists in 168 pictures displaying seven objects portrayed against two textured backgrounds and under seven illuminants with and without shadows (see Fig. 3 for an example). The seven illuminants are: 01 - Philips Neon Natural Daylight 6500 K (TLD965); 02 - Philips Neon Fluotone 4100 K

(TLD840); 03 - Philips Neon Fluotone Daylight 5000 K (TLD950); 04 - OSRAM Bellalux Insecta (H529); 05 - OSRAM Concentra Red R80 (H640); 06 - OSRAM Concentra Green R80 (H649); 07 - OSRAM Concentra Blu R80 (H649).

The tests have been also performed on a simple image, named Test16.255, displaying a bright square (with uniform RGB color (255, 255, 255)) over a dark background (with uniform RGB color (16, 16, 16)). Due to its simple structure (see Fig. 4(a)), Test16.255 is commonly employed to visualize the filtering effects of spatial color algorithms.

5.1 EVALUATION MEASURES

Brightness - We evaluate the brightness variation between the input image I and its color filtered version L by comparing their brightness mean values. Precisely, the mean image brightness is defined as

$$\mu = \frac{1}{\#\{y \in \text{Supp}(I)\}} \sum_{y \in \text{Supp}(I)} (B_0(y) + B_1(y) + B_2(y)) \quad (5.1)$$

where $\text{Supp}(I)$ indicates the set of the image pixels, and for any $i = 0, 1, 2$, B_i indicates the chromatic values of the input image (i.e. $B_i = C_i$) or of a filtered version (i.e. $B_i = L_i$).

Contrast - We evaluate the improvement of the details visibility by comparing the mean value of a multi-resolution contrast of the brightness of I and L , as proposed in (Rizzi et al., 2004), which accounts for local chromatic intensity variations at different scales. The contrast measure is defined operatively by constructing a pyramid of H images I_0, \dots, I_{H-1} (with H an integer positive number) where I_0 is the brightness of I and for each $h = 1, \dots, H-1$, I_h is the image I_{h-1} rescaled by 0.5. For each image

I_h we evaluate the h th local contrast at a pixel $y \in \text{Supp}(I_h)$

$$c_h(y) = \frac{1}{8} \sum_{z \in N_8(y)} |I_h(z) - I(y)|$$

where $N_8(y)$ denotes the 8-connected neighborhood of y . The global contrast \bar{c}_h of I_h is then obtained as the mean values of c_k over the number of pixels of I_h , i.e.

$$\bar{c}_h = \frac{1}{\#\{y \in \text{Supp}(I_h)\}} c_k(y).$$

The global contrast of I is then obtained by averaging the values \bar{c}_h over h :

$$C = \frac{1}{H} \sum_{h=0}^{H-1} \bar{c}_h.$$

Histogram Flatness - The brightness and contrast enhancement of the Retinex algorithms modifies the shape of the pdfs of the chromaticity image channels. Precisely, the divisions performed by RSR, STRESS and QBRX stretch the pdf of the brightness of the input image. We quantify this effect by computing the flatness of the pdf. Flatness is defined as the L^1 distance of the pdf p of the image brightness from an uniform pdf (Rizzi, Gatta and Marini, 2003a).

5.2 RESULTS

Figure 4 shows the filtering results obtained on the image Test16.255. These results have been obtained by using $N = 15$ and $M = 200$ for RSR and STRESS. As highlighted by the histogram equalizations in (f) and (g), the results obtained by using RSR and STRESS are noisy because of the random sampling.

Tab. 1 reports the values of the evaluation

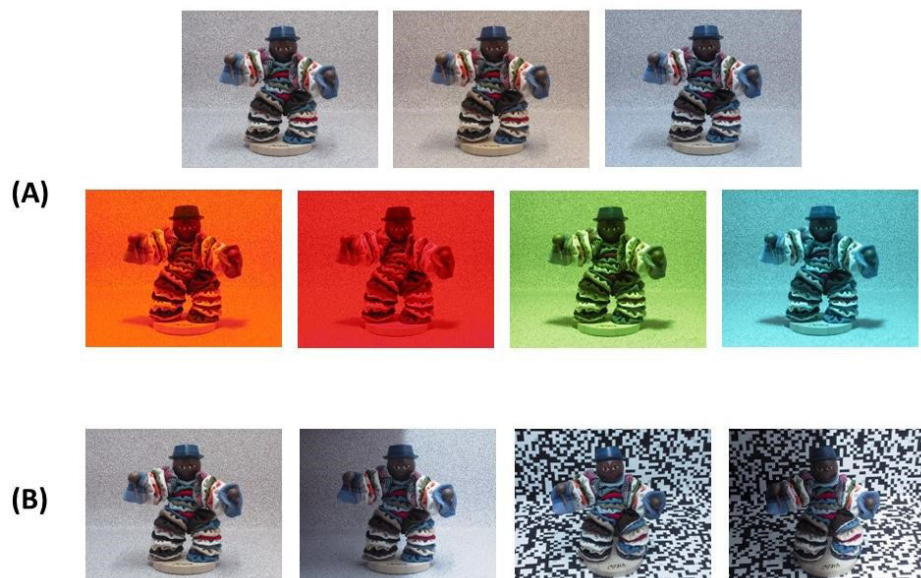


Figure 3 - An example from YACCD: (A) the same image has been captured under seven different illuminants; (B) for each illuminant, the object is displayed against two different backgrounds, with and without shadows.

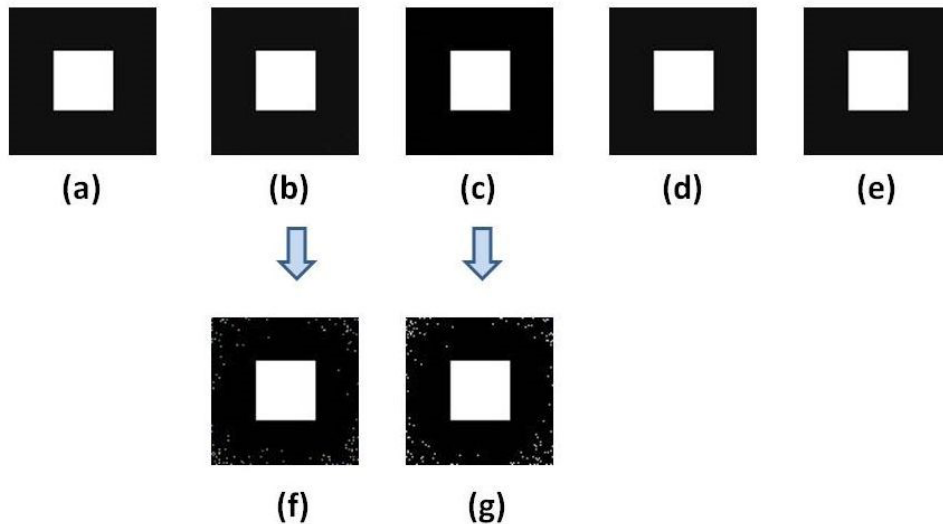


Figure 4 - Image Test16.255, and its filtered versions with (b) RSR; (c) STRESS; (d) G-QBRIX; (e) L-QBRIX). Visually the images look equal. Equalizing the histogram does not change the images (a), (d), (e) while the images (b) and (c) change: figures (f) and (g) show the chromatic noise added by the filtering process to (b) and (c) respectively.

ALGORITHM	MEAN LUMINANCE (μ)	MULTI-RESOLUTION CONTRAST (C)	FLATNESS [$\times 10^{-3}$] (F)
NONE	113.31	23.32	3.48
RSR	138.09	28.75	2.82
STRESS	126.40	32.04	2.44
G-QBRIX	138.01	28.70	2.79
L-QBRIX	143.57	30.00	2.84

Table 1 - Results on YACCD.

measures listed in Section 6.1 for the database YACCD. In these experiments, in order to reduce the computational time, we rescaled the YACCD images by 0.25, so that the size of the images considered here is 250 x 200 pixels. The parameters N and M input to RSR vary from image to image. They have been set up as suggested in [12]: each image has been processed with different increasing values of N and M , then we chose as final filtering the picture such that the CIE Lab difference between two subsequent filtering gets smaller than a fixed threshold. More details about the parameter set up for this dataset are provided in (Lecca and Rizzi, 2015). The pairs (N, M) used for RSR have been used also for STRESS.

In QBRIX, we report the results obtained by setting $Q = 0.99$.

All the considered algorithms output an enhanced image, with increased luminance and contrast, while the histogram flatness diminishes, meaning that the filtering widens the dynamic range of the chromatic intensities. On average, the results obtained by RSR and G-QBRIX are very similar. STRESS outputs images with a lower brightness and with a higher contrast than the other algorithms. L-QBRIX produces the brightest pictures.

Some examples of the images obtained by RSR, STRESS and the two versions of QBRIX are shown in Figure 5, Figure 6, Figure 7 and Figure 8. Precisely, the figures show the

results on an image depicting a rollerskate and captured respectively under the illuminants 01, 02, 04 and 06. In Figure 5 and Figure 6 (Figure 7, Figure 8, resp.) the background is almost uniform (textured, resp.); in Figure 6 and Figure 8 shadows are present.

Figure 9 shows the results on an image acquired under a light with a considerable red component, and reports the color filtering results obtained by running QBRIX with two different values of the quantile Q . The results obtained with the highest quantile ($Q = 0.99$) are closer to those output by RSR and STRESS. We remark that L-QBRIX may produce a halo around the object borders: this is due to the spatial locality of the algorithm. In general, for a very low value of Q , many pixels satisfy the second condition of Equation (4.2), thus the algorithm provides a too bright image, where many details are lost. When $Q=1$, G-QBRIX behaves like scale-by-max, while L-QBRIX realizes a local effect, where the pixels' intensity is weighted by the spatial distance of the max.

6. CONCLUSIONS

In this paper, we have reviewed and compared three Retinex implementations: RSR, and its variants STRESS and QBRIX. RSR and STRESS share the same spatial sampling strategy, performed by random points, while they differ from each other for the mathematical equation

Figure 5 - (a) Image SkateAA01 from YACCD and its filtered versions with (b) RSR; (c) STRESS; (d) G-QBRX; (e) L-QBRX.



(a) Input Image



(b) RSR



(c) STRESS



(d) G-QBRX



(e) L-QBRX

Figure 6 - (a) Image SkateAB02 from YACCD and its filtered versions with (b) RSR; (c) STRESS; (d) G-QBRX; (e) L-QBRX.



(a) Input Image



(b) RSR



(c) STRESS



(d) G-QBRX



(e) L-QBRX

Figure 7 - (a) Image SkateBA04 from YACCD and its filtered versions with (b) RSR; (c) STRESS; (d) G-QBRX; (e) L-QBRX.



(a) Input Image



(b) RSR



(c) STRESS



(d) G-QBRX



(e) L-QBRX

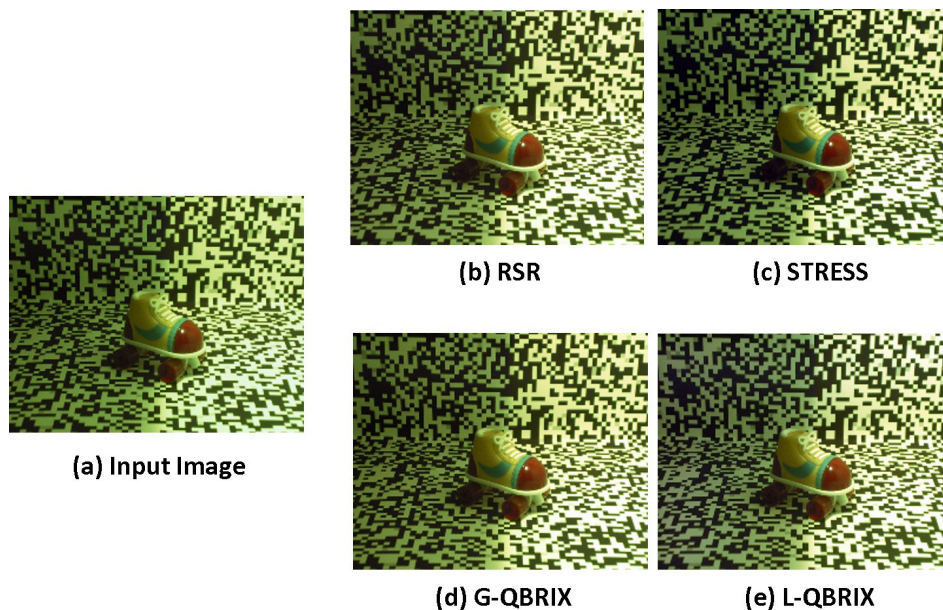


Figure 8 - (a) Image SkateBB06 from YACCD and its filtered versions with (b) RSR; (c) STRESS; (d) G-QBRIX; (e) L-QBRIX.

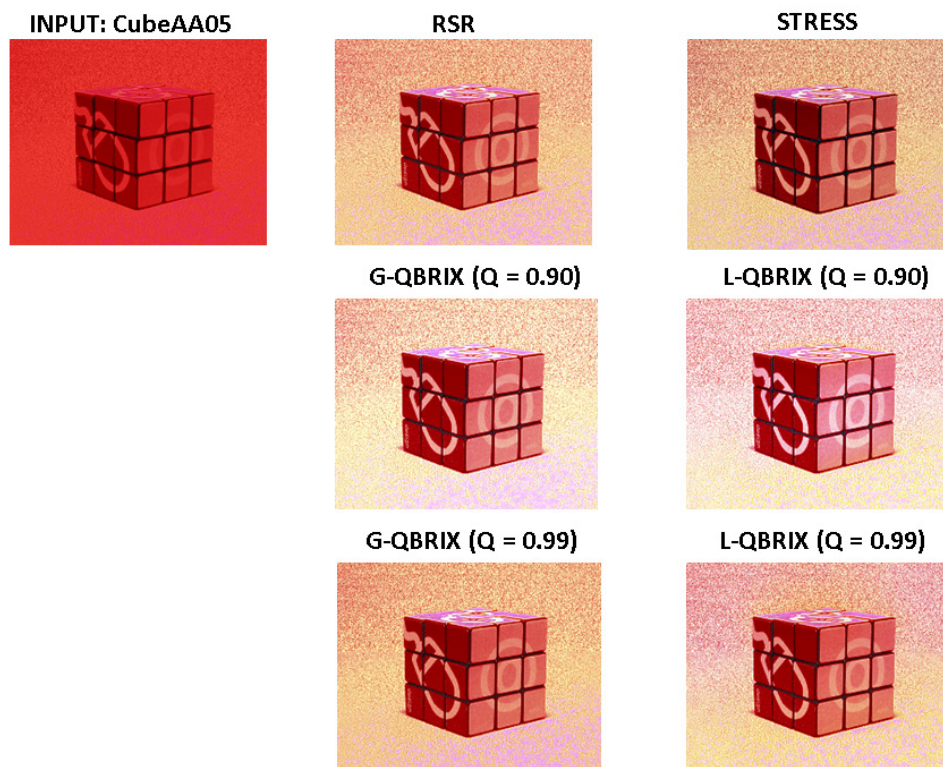


Figure 9 - Results of RSR, STRESS and QBRIX for two different values of the quantile.

used to compute the color sensation. QBRIX reformulates RSR in a probabilistic framework that avoids the random sampling, and performs color correction by considering the statistic distribution of the image chromatic channels. We have compared these methods by measuring their image enhancement capability on the public color database, YACCD that contains the pictures of six objects acquired under seven different lights, with and without shadows, and with two different background. The experiments show that all these methods enhance the input image by producing an image with higher brightness and contrast, and with a better use of the available dynamic range.

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CONFLICT OF INTEREST

The authors declare no conflict of interest. In particular, no financial/personal interest have affected their objectivity.

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