

**Loreto *et al* Reply:** The major criticism [1] to our work [2] is that the real space renormalization group (RSRG) transformation is inconsistent and therefore the method does not describe correctly the critical behavior of the model. We would like to point out that we did not present exact results and we clearly stated in our Letter the approximation involved and the range of validity of the method. The preceding Comment [1], however, does not suggest an alternative, more consistent, renormalization scheme for the forest-fire (FF) model.

The inconsistency of our method would lie in the fact that the coarse graining procedure should constitute a transformation law for the densities. The densities obtained in this way would be incompatible with the calculation of the densities from dynamic mean-field equations. A similar inconsistency could be claimed for the RSRG for equilibrium critical phenomena [3], based on the Kadanoff block transformation. In equilibrium systems the RSRG transformation acts on the partition function, not on the densities. Except for the case of deterministic fractal lattices, the transformation is approximate and the densities one obtains from the coarse grained partition function are in general different from those one would obtain applying directly the transformation to the densities. Despite these approximations, however, RSRG has provided reliable results for a large variety of models and is considered an important tool in understanding critical phenomena.

The major difficulty in applying real space RG to nonequilibrium critical systems, such as the FF model is that there is not a general prescription, such as the Gibbs measure in equilibrium phenomena, to assign a weight to a configuration of the system. We have proposed to apply the RG transformation to the dynamical evolution operator  $T$  which defines the probabilities for the system to evolve from one configuration to another. In the case of the FF,  $T$  can be directly written in terms of  $f$  and  $p$ . We show in [4] that the renormalization equations for  $f$  and  $p$  we presented in [2] can be extracted from the renormalization of the evolution operator. The coarse grained evolution operator (or equivalently  $f$  and  $p$ ) is obtained by averaging all the paths that lead from one coarse grained configuration to another. Different fine scale configurations correspond, however, to the same coarse grained configuration and it is therefore necessary to assign them a relative weight. We assign this weight by the simplest approximation to the unknown stationary probability distribution, that is, the product measure of mean-field densities.

By constructing explicitly the transformation for  $T$  one can see that the two limits mentioned in [1] are correctly taken into account. In fact the RG equations reported in [2] are valid *only* in the first limit since proliferations in

the parameter space change the equations in the second limit and higher order term in  $f$  and  $p$  are not relevant.

The fact that relevant microscopic features of modified versions of the original model [5] would become irrelevant under coarse graining may sound plausible but definitely remains to be vindicated by establishing the RG transformation for those models.

Finally we would like to comment on the definition of self-organized criticality (SOC) we use in [2]. We have defined SOC, as it is often done in the literature, as a critical stationary state without tuning parameters. In the RG language this statement corresponds to the absence of relevant parameters. Our RG analysis shows that this definition does not correspond to the behavior of the FF model and we agree with [1] that this statement also applies to other models claimed to display SOC.

From the RG point of view there is no difference between nonequilibrium phase transitions and SOC, scale invariance being reached by tuning a control parameter. However, there is a clear physical difference since the control parameter in the SOC system is the ratio of two different time scales, with the critical value being zero. The meaning of SOC is therefore related to the widespread existence of phenomena ruled by very different time scales rather than to the absence of fine tuning parameters as it is often reported in the literature.

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